Some Issues on Charm Physics

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I. Introduction

A brief history of charm

Originally there are three quarks in the quark model , a theory based on the SU(3) symmetry of hadrons, in 1960s.

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad s$$

But there are at least two leptons and two neutrinos since 1962 when people knew ν_e and ν_μ are not the same.

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

The number of quarks and leptons are unsymmetric

A theory of weak interaction with four quarks was proposed by Bjorken, Glashow, Illiopoulos and Maiani in the mid-1960 and early 1970s

Phys. Lett. 11 (1964) 255;

Phys. Rev. D2 (1970) 1285

L REVIEW D VOLUME 2, NUMBER 7 1 OCTOBE

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139

(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading

The fourth quark charm was inroduced.

Then two families are formed due to the structure of the charged current of weak interaction of quarks.

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

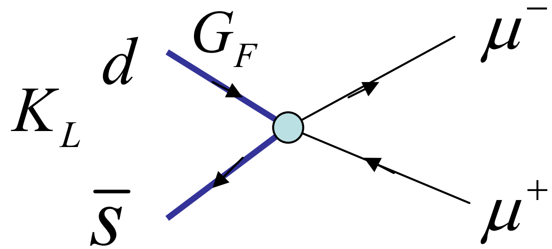
The properties of charm quark are the same as up, except for the mass

The weak interactions of quarks and leptons are highly symmetric in this theory.

Fine features in the theory with charm

The flavor-changing-neutral-current (FCNC) naturally cancel in tree level.

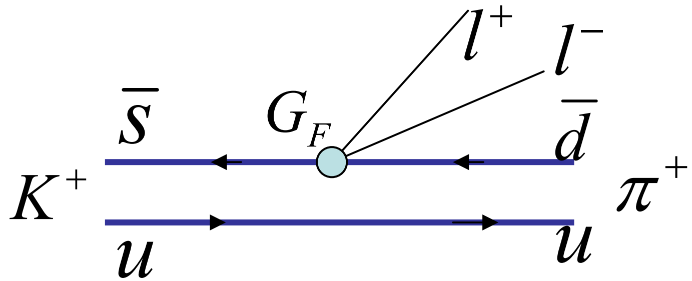
$$\cancel{\bar{s}\gamma_\mu(1-\gamma_5)d} \qquad \cancel{\bar{d}\gamma_\mu(1-\gamma_5)s}$$



With FCNC, it will predict that the decays

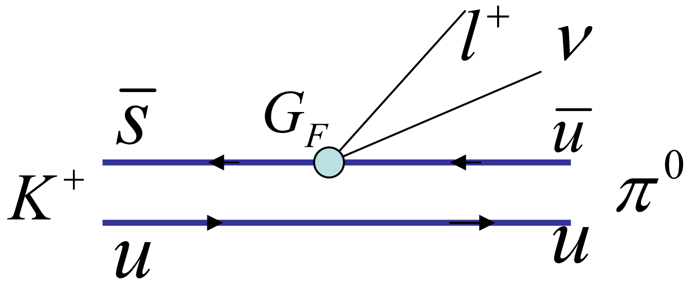
$$K_L \to \mu^+\mu^-$$

$$K^+ \to \pi^+ l^+ l^-$$



happen with the magnitude the same order as the charged current induced process

$$K^+ \to \pi^0 l^+ \nu$$



In experiment:

CC

$$\begin{cases} BR(K^+ \to \pi^0 e^+ \nu_e) = (4.98 \pm 0.07)\% \\ BR(K^+ \to \pi^0 \mu^+ \nu_\mu) = (3.32 \pm 0.06)\% \end{cases}$$

FCNC

$$BR(K^+ \to \pi^+ \mu^+ \mu^-) = (8.1 \pm 1.4) \times 10^{-8}$$

$$BR(K_L \to \mu^+ \mu^-) = (6.87 \pm 0.11) \times 10^{-9}$$

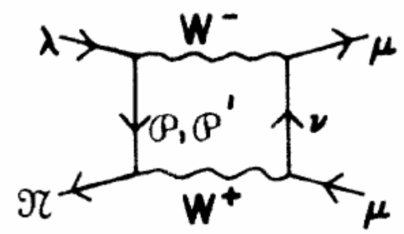
Introduing charm quark makes the theory of weak interaction more beautiful.

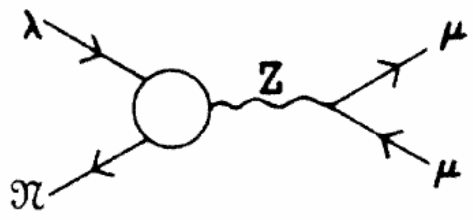
Charm quark was predicted in theory several years before it was discovered in experiment.

The eve before the discovery of charm

In March of 1974, M.K. Gaillard and B. Lee studied how charm quark affect kaon decays through loop effects:

$$K \to \mu\bar{\mu},\ K \to \pi\nu\bar{\nu},\ K \to \gamma\gamma,\ K \to \pi\gamma\gamma,\ \text{and}\ K \to \pi e\bar{e}$$





$$\begin{array}{cccc} \mathcal{P}, & \mathfrak{N}, & \lambda; & \mathcal{P}' \\ \downarrow & \downarrow & \downarrow & \downarrow \\ u & d & s & c \end{array}$$

From experimental information, they got:

$$m_u << m_c$$

$$m_c < 5\text{GeV}$$

Phys. Rev. D10 (1974) 897

In the October of 1974 a heavy, narrow width particle was discovered simultaneously by two groups, one is led by Samuel Chao Chung Ting, the other by Burton Richter.

Ting

J/ψ : M=3.1GeV, $\Gamma \leqslant$1.3MeV

Brookhaven, 30-GeV alternating-gradient synchrotron

$$p + \mathrm{Be} \rightarrow \underbrace{\mathrm{e}^{+} + e^{-}} + x$$

measuring $m_{e^+e^-}$

Richter

SLAC, electron-positron stroge ring SPEAR

$$e^{+} + e^{-} \rightarrow \text{hadrons}, \ e^{+}e^{-}, \ \mu^{+}\mu^{-}, \cdots$$

The ADONE e+e- collider at Frascati in Italy was designed at a maximum center-of-mass energy 3.0GeV.

That was badly unfortunate, the energy was just below the edge of discovering this particle.

Immediately after receiving the news of J/ ψ observation, they boosted the currents beyond designed limits, …

Phys. Rev. Lett. 33 (1974) 1404;

Phys. Rev. Lett. 33 (1974) 1406;

Phys. Rev. Lett. 33 (1974) 1408;

- The new particle J/ ψ is heavier than 3 protons

- Its life is surprisingly longer than any resonances found at that time

This can only be explained by assuming it is a bound state of a new kind quark-antiquark pair :

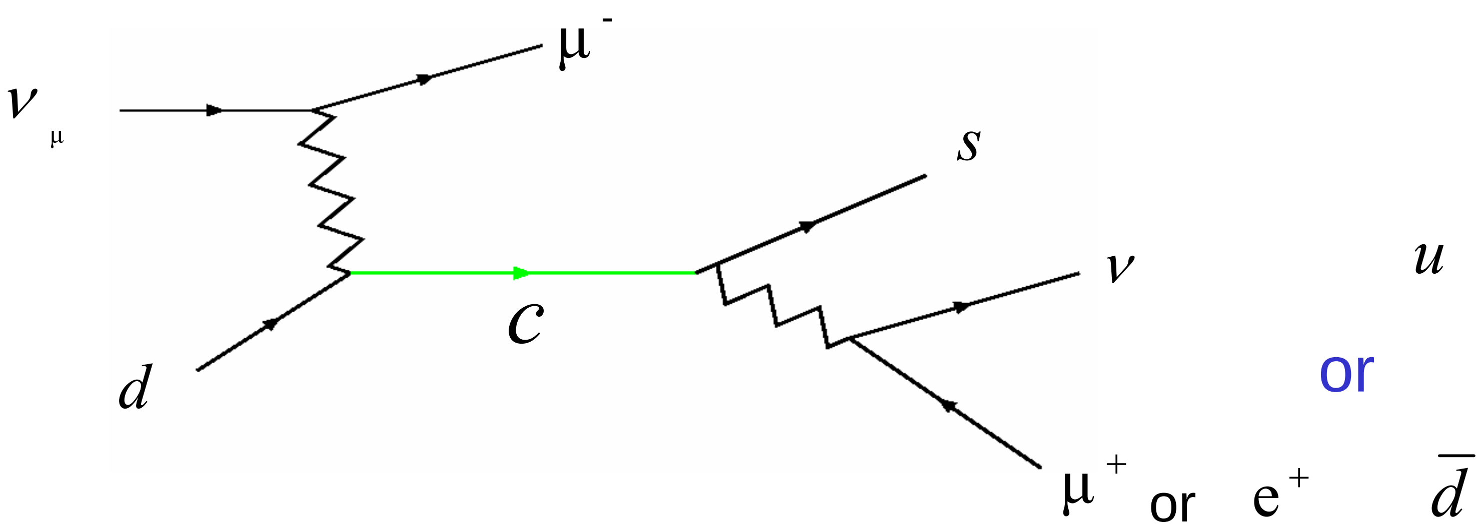
$$C\overline{C}$$

The open charm

Hadrons with open charm had to be found before charm could be acccepted as an established explanation of J/ Ψ .

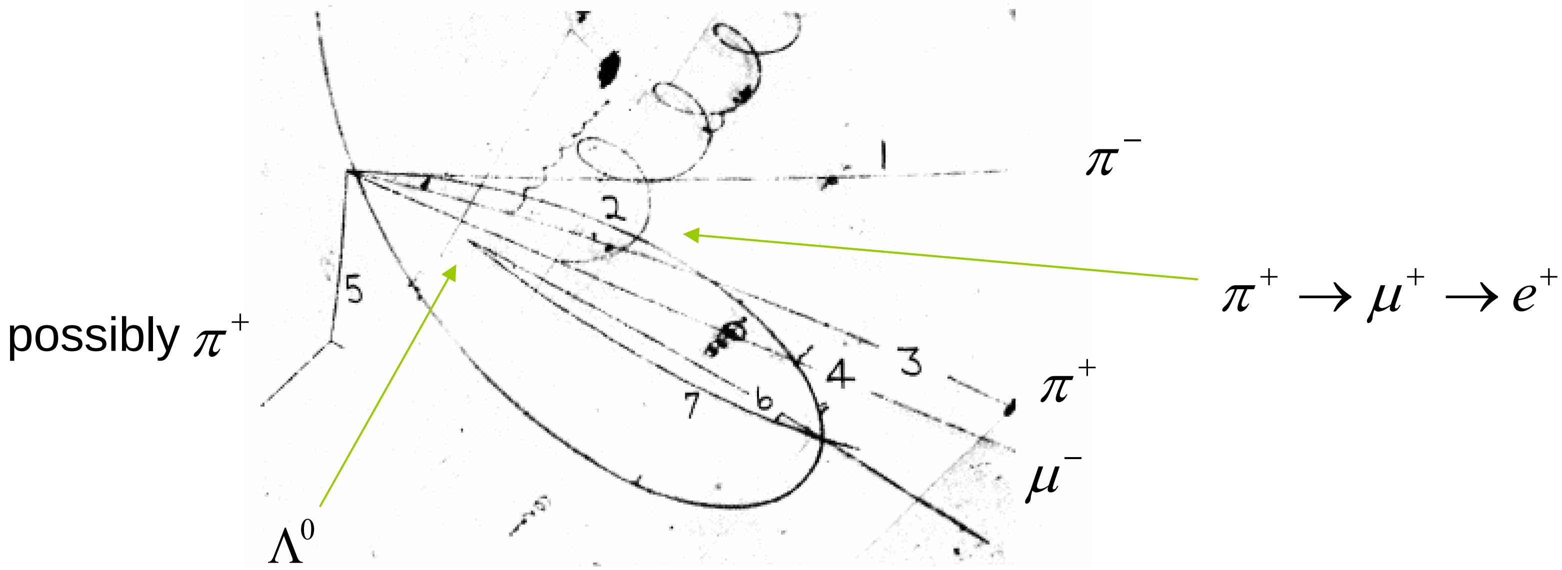
Then the question is how to find hadrons with open charm?

Then possible production and decay process for charm is through weak interactions induced by charged current:



Evidence for the production and decay of charm

Brookhaven, bubble chamber exposed to broad-band neutrino beam, 1975



$$\nu p \rightarrow \mu^- \Lambda^0 \pi^+ \pi^+ \pi^+ \pi^-$$

PRL34(1975)1125

The observation

The charmed-meson D was finally observed at SLAC-Spear, at e+e- annihilation at center-of-mass energy between 3.9GeV and 4.6GeV in 1975 and 1976.

Mass: $1.865 \pm 15\ \text{MeV}$

PRL37(1976)255

It is D^0, composed of $c\bar{u}$

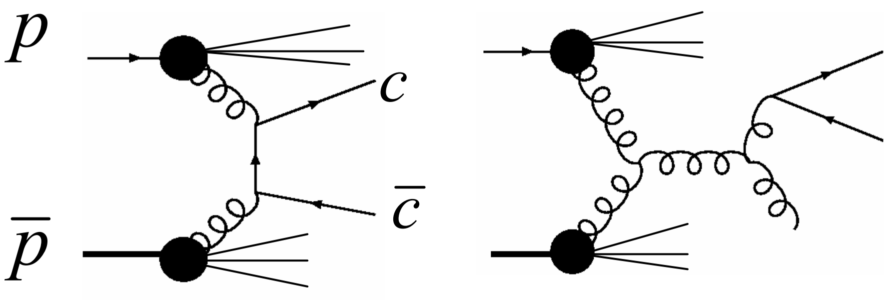
PRL37(1976)569

It decays to : $K^{\pm}\pi^{\mp}$

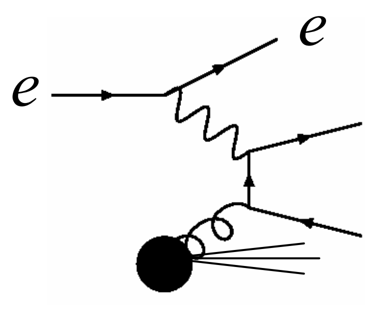
$K^{\pm}\pi^{\mp}\pi^{\pm}\pi^{\mp}$

A little summary for

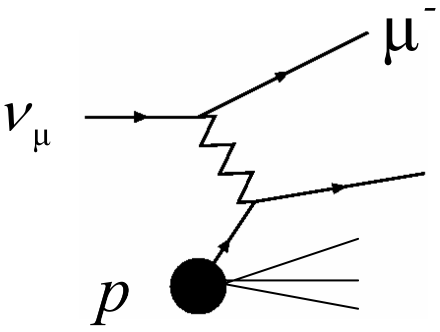
the production environment of D mesons



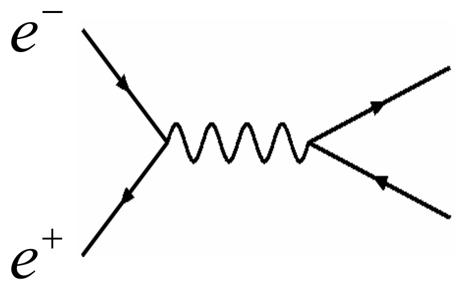
(a) Hadronproduction



(b) e+p production



(c) Neutrino interaction



(d) electron-positron collision

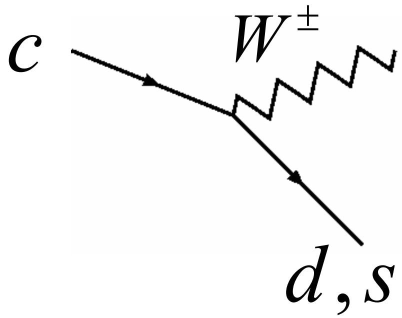
Experiments for charm

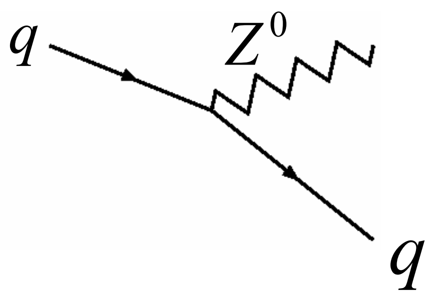
	beam	Sample
E791	π 500 GeV	2.5×10^5 D
CDF	$p\bar{p}$ 1 GeV	1.5×10^6 D
FOCUS	γ 200 GeV	1×10^6 D
CLEO	e^+e^- $(\Upsilon(4s))$	1.5×10^5 D
CLEO-C	e^+e^- $(\psi(3770))$	10 nb $c\bar{c}$
BABAR	e^+e^- $(\Upsilon(4s))$	1.5×10^5 D
LEP	e^+e^- $(Z^0))$	1×10^5 D
BELLE	e^+e^- $(\Upsilon(4s))$	1.5×10^6 D
BES	$e^+e^-(J/\psi)$	

II. What can we do with charm?

(1) To reveal the features of weak interaction of up-type quarks

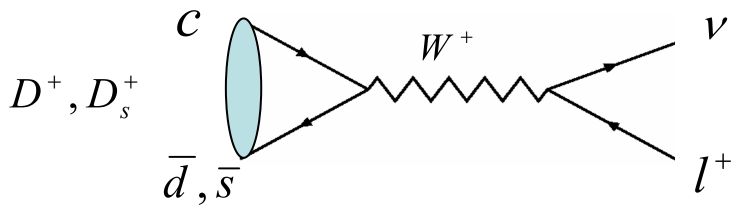
The weak interaction of charm quark in the SM:





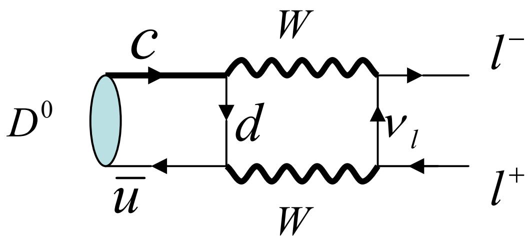
This can be tested with D decays:

a) Leptonic decay

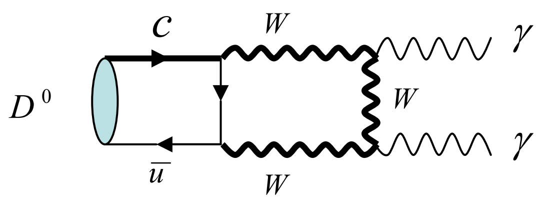


$$D^+ \rightarrow \mu^+ \nu_\mu \qquad D_s^+ \rightarrow \mu^+ \nu_\mu$$

b) Rare decay

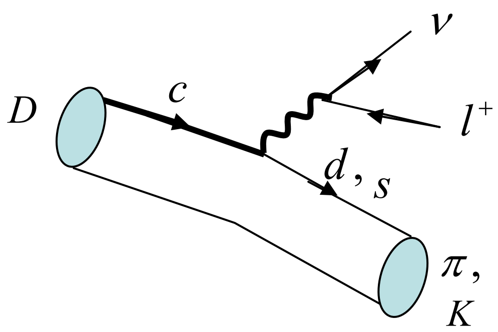


$$D^0 \rightarrow l^+ l^-$$



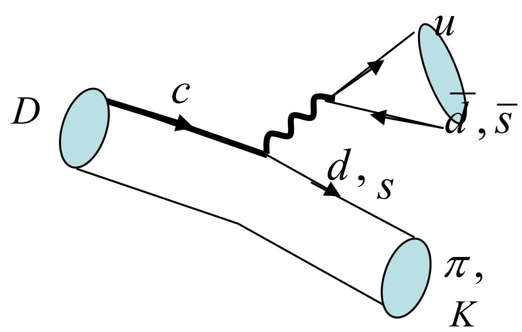
$$D^0 \rightarrow \gamma\gamma$$

c) Semi-leptonic decay



$$D \to \pi\ l^+\nu, \quad D \to K\ l^+\nu$$

d) Hadronic decay



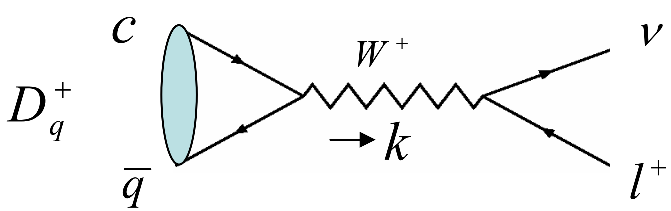
$$D \to h_1 h_2$$

(2) the methods to treat strong interaction

Some methods:

- Perturbative QCD
- Lattice QCD
- QCD sum rule
- Potential model

III. Leptonic decays



$$T = \left\langle \nu\ l^+ \middle| \bar{\nu} \frac{ig}{2\sqrt{2}} \gamma_\mu (1-\gamma_5) l \frac{i}{k^2 - m_W^2} \bar{q} \frac{ig}{2\sqrt{2}} V_{cq}^* \gamma^\mu (1-\gamma_5) c \middle| D_q^+ \right\rangle$$

$$\boxed{k^2 << m_W^2}$$

$$= \frac{ig^2}{8m_W^2} V_{cq}^* \left\langle \nu\ l^+ \middle| \bar{\nu} \gamma_\mu (1-\gamma_5) l \bar{q} \gamma^\mu (1-\gamma_5) c \middle| D_q^+ \right\rangle$$

$$\boxed{\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}}$$

$$= \frac{iG_F}{\sqrt{2}} V_{cq}^* \bar{u}_\nu^{\lambda_1} \gamma_\mu (1-\gamma_5) v_l^{\lambda_2} \left\langle 0 \middle| \bar{q} \gamma^\mu (1-\gamma_5) c \middle| D_q^+ \right\rangle$$

The vector current contribution to the hadronic matrix element is zero due to the parity property of the matrix element

$$\langle 0 | \bar{q}\gamma^{\mu} c | D_q^+ \rangle = 0$$

Only the axial current contribute!

The hadronic matrix element induced by the axial current is conventionally parametrized as

$$\langle 0 | \bar{q}\gamma^{\mu}\gamma_5 c | D_q^+ \rangle = i f_{D_q} p^{\mu}$$

decay constant

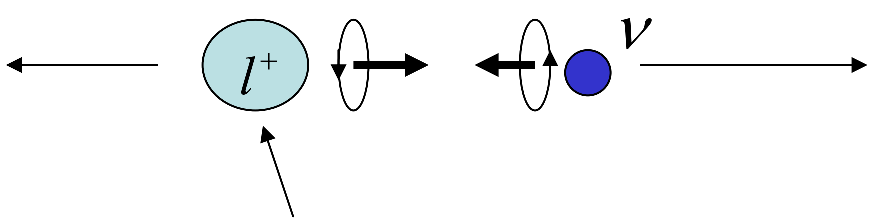
The amplitude

$$T = \frac{G_F}{\sqrt{2}} f_D V_{cq}^{*} \overline{u}_{\nu}^{\lambda_1} \gamma_{\mu} (1 - \gamma_5) v_l^{\lambda_2} p^{\mu}$$

The total decay width is then given by

$$\Gamma(D_q^+ \to l^+ \nu) = \frac{G_F^2}{8\pi} f_{D_q}^2 \, |V_{cq}|^2 \, m_l^2 \left(1 - \frac{m_l^2}{m_{D_q}^2}\right)^2 m_{D_q}$$

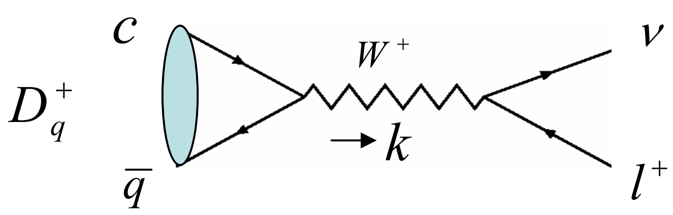
Helicity suppression



antiparticle

The physical explanation of the decay constant

For the D meson to decay leptonically, c and $\overline{q}$ have to come together to annihilate due to the practically zero range of the weak interactions.



The decay amplitude is therefore proporational to

$$\psi_{c\overline{q}}(0)$$

The wavefunction at zero separation

Then

$$f_{D_q} \propto \psi_{c\overline{q}}(0)$$

Recent experimental result

Decay mode	exp. BR.	
$D^+ \to \mu^+ \nu_\mu$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$	[1]
$D_s^+ \to \tau^+ \nu_\tau$	$(4.5 \pm 0.5 \pm 0.4 \pm 0.3)\,\%$	[2]
	$(5.30 \pm 0.47 \pm 0.22)\,\%$	[3]
	$(5.52 \pm 0.57 \pm 0.21)\,\%$	[4]

CLEO-c: [1] PRD78 (2008)052003

BABAR: [2] arxiv: 1003.3063

CLEO-c: [3] PRD79 (2009)052002
[4] PRD80 (2009)112004

Taking $|V_{cs}| = 0.97425,\qquad |V_{cd}| = 0.2256,$

the decay constants can be extracted from the measured branching ratios of leptonic decays:

$$f_{D^+} = (205.8 \pm 8.5 \pm 2.5)\ \text{MeV}$$

$$f_{D_s^+} = (233 \pm 13 \pm 10 \pm 7)\ \text{MeV}$$

$$(252.5 \pm 11.1 \pm 5.2)\ \text{MeV}$$

$$(259.0 \pm 6.2 \pm 3.0)\ \text{MeV}$$

$$f_{D_s^+} / f_{D^+} = 1.14$$

Comparision with theoretical predictions

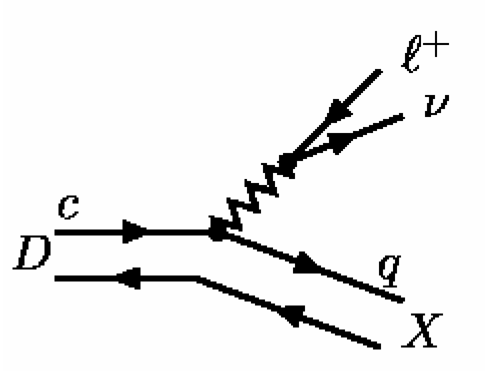
Theoretical predictions of f_{D^+} and $f_{D_S^+}/f_{D^+}$. QL indicates quenched lattice calculations.

Model	$f_{D_s^+}$ (MeV)	f_{D^+} (MeV)	$f_{D_s^+}/f_{D^+}$
Lattice (n_f=2+1) [1]	$249 \pm 3 \pm 16$	$201 \pm 3 \pm 17$	$1.24 \pm 0.01 \pm 0.07$
QL (Taiwan) [2]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [3]	$236 \pm 8^{+17}_{-14}$	$210 \pm 10^{+17}_{-16}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [4]	$231 \pm 12^{+6}_{-1}$	$211 \pm 14^{+2}_{-12}$	1.10 ± 0.02
QCD Sum Rules [5]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [6]	235 ± 24	203 ± 20	1.15 ± 0.04
Quark Model [7]	268	234	1.15
Quark Model [8]	248 ± 27	230 ± 25	1.08 ± 0.01
Potential Model [9]	241	238	1.01
Isospin Splittings [10]		262 ± 29	
CLEO-c	$233 \pm 13 \pm 10 \pm 7$	$205.8 \pm 8.5 \pm 2.5$	1.14

[1] PRL95(2005)122002. [2] PLB624(2005)31. [3] PRD64 (2001)094501.
[4] PRD60 (1999) 074501. [5] hep-ph/0507241. [6] hep-ph/0202200.
[7] PLB635 (2006)93. [8] PLB596(2004)84.
[9] NPA744 (2004) 156; J.Phys.34(2004)297. [10] PRD47(1993) 3059

III Semi-leptonic decays

D meson can decay into light hadrons by emitting a pair of leptons $l^+\nu$ through weak interaction.



Leptons do not involve strong interactions, they can be factored out from the hadronic matrix element

$$A = \frac{G_F}{\sqrt{2}} V_{cq}^* \bar{\nu}\gamma_\mu(1-\gamma_5) l \underbrace{\langle X | \bar{q}\gamma^\mu(1-\gamma_5)c | D\rangle}$$

All the strong interactions are included in the hadronic matrix element.

$$\langle X | \bar{q}\gamma^{\mu}(1-\gamma_5)c | D \rangle$$

- The hadronic matrix element can be decomposed into several form factors.

- The form factors are controled by non-perturbative dynamics, where perturbative QCD can not apply.

Several metholds are developed:

Lattice QCD

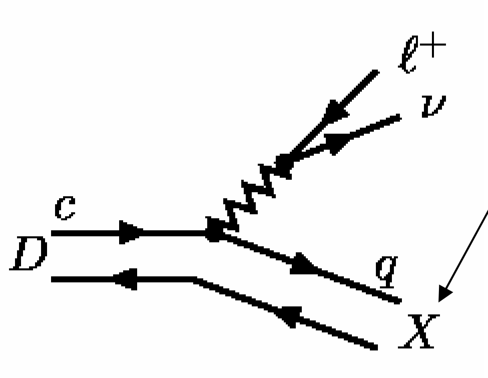
QCD Sum Rule

QCD Light-cone Sum Rule

Quark Model (potential)

Light-Front approach

Large-Energy Effective theory



The final mesin X can be pseudoscalar, vector, or scalar meson due to the quantum number of the final quark-antiquark syetem.

pseudoscalar

$$D \to P l^+ \nu$$

$$D \to V\, l^+ \nu$$

$$D \to S l^+ \nu$$

vector

scalar

(1) Transitions to pseudoscalar mesons $D \to P\ell^+\nu$

According to the Lorentz structure, the hadrnoic matrix element can be decomposed as

$$\langle \underbrace{P}_{p_2}|\bar{q}\gamma^\mu(1-\gamma_5)c|\underbrace{D}_{p_1}\rangle = (p_1+p_2)_\mu F_+(q^2) + (p_1-p_2)_\mu F_-(q^2)$$

Form Factors

$$q = p_1 - p_2$$

Why do F_+ and F_- only depend on q^2 ?

In general, $F\pm$ should depend on all the Lorentz scalars formed by p_1 and p_2 :

$$\underbrace{p_1^2}_{m_D^2}, \quad p_1 \cdot p_2, \quad \underbrace{p_2^2}_{m_P^2}$$

$$q^2 = p_1^2 - 2p_1 \cdot p_2 + p_2^2.$$

Equivalently the decomposition can be in another form

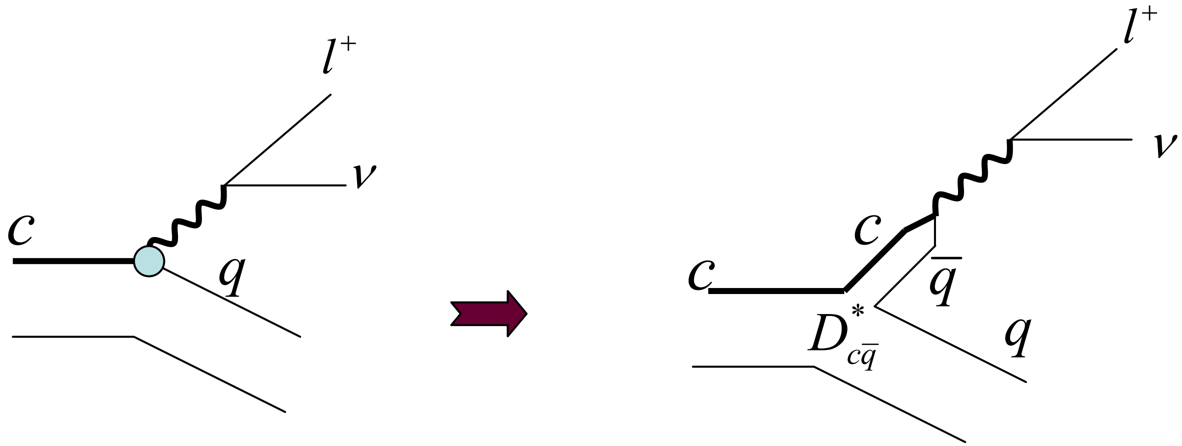
$$\langle P|\bar{q}\gamma^{\mu}(1-\gamma_5)c|D\rangle = \left(p_1 + p_2 - \frac{m_D^2 - m_P^2}{q^2} q\right)_{\mu} F_+(q^2) + \frac{m_D^2 - m_P^2}{q^2} q_{\mu} F_0(q^2)$$

transverse longitudinal

●It is difficult to calculate the form factor for the whole range of the q2 analytically in theory.

● One usually calculate the form factor for a set of discrete points for q^2, then fit these point with an assumed formula for the q^2 dependence due to some phenomenological considerations.

Pole Dominance



Z.Phys.C29(1985)637

Z.Phys.C34(1987)103

$$F_{\pm}(q^2) = \frac{F_{\pm}(0)}{1 - q^2/m^2_{\text{pole}}}$$

The quantum munber J^P of the pole resonance should be the same as that of the weak current, which induces the transition c→q

$$\langle P|\bar{q}\gamma^{\mu}(1-\gamma_5)c|D\rangle \rightarrow \langle P|\bar{q}\gamma^{\mu}c|D\rangle \longrightarrow J^P = 1^-$$

Vector current

The modified pole model

There may be other contributions out side the single pole, then the formula should be modified, one proposal in the literature is

$$F_\pm(q^2) = \frac{F_\pm(0)}{(1 - q^2/m^2_{D^*_{(s)}})(1 - \alpha q^2/m^2_{D^*_{(s)}})}$$

PLB478 (2000) 417

PRD62(2000) 114002

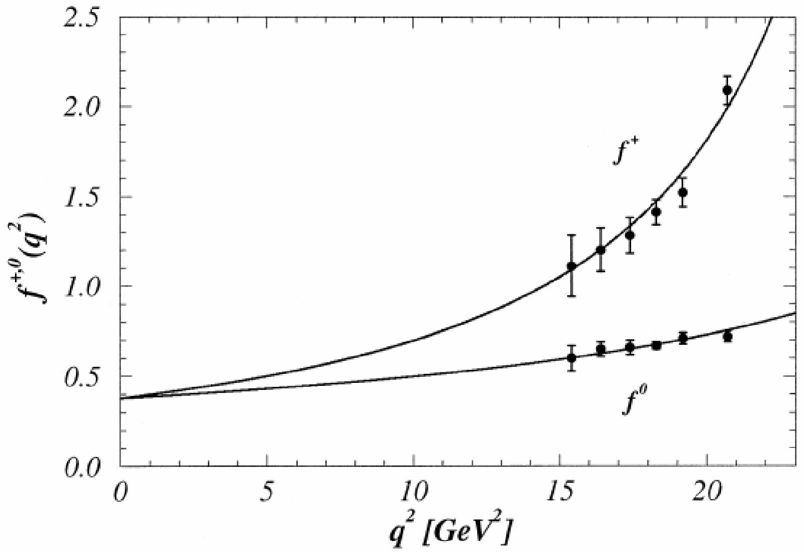


Fig. 1. Fitting the lattice data using the parametrization (19). Note that the fit of f^+ form factor is constrained by the precise data for f^0. For easiness, only the central curves (without errors in parameters) are displayed.

$$F^{B\pi}(q^2)$$

$$m_{B^*} = 5.32 \text{ GeV}$$

$$F^{+,0}(0) = 0.38$$

$$\alpha = 0.54(17)$$

There are also other models for the q^2 dependence of the form factors, for example

$$F_{\pm}(q^2) = F_{\pm}(0)e^{\alpha q^2}$$

The q^2 dependence of the form factors can be tested by experiment.

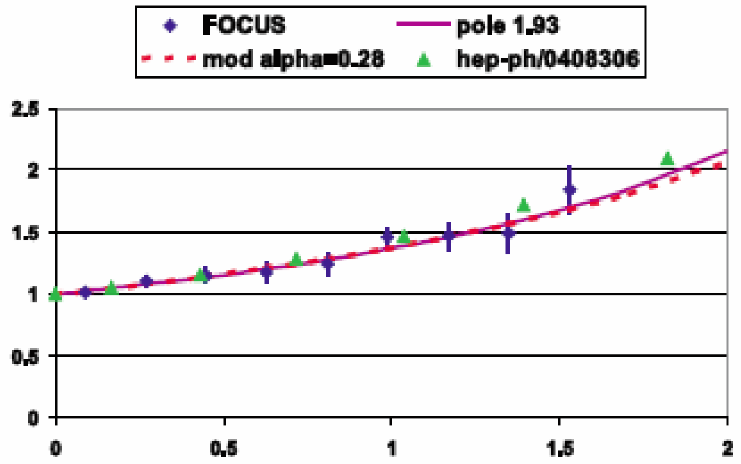
Calculation of the differential decay width

The differential decay width of $D \to P\ell^+\nu$ with lepton mass neglected is

$$\frac{d\Gamma}{dq^2}(D \to P\ell^+\nu) = \frac{G_F^2}{192\pi^3 m_D^3}|V_{cq}|^2[(m_D^2 + m_P^2 - q^2)^2 - 4m_D^2 m_P^2]^{3/2}|F_+(q^2)|^2$$

The range of q^2 is: $$0 < q^2 < (m_D - m_P)^2$$

The branching ratio: $$BR = \frac{\Gamma}{\Gamma_{total}} = \tau_D \cdot \Gamma$$



Hep-ex/0410015

Fig. 1. The overlay of the unquenched LQCD calculation of the form factor $f_+(q^2)$[1] over the preliminary FOCUS preliminary result. The solid line represents a pole form fit to the FOCUS data while the dotted line represents a modified pole form fit. All the data points and the fitted lines are normalized to have $f_+(0) = 1$.

The models should be tested by exp. data. Unfortunately, the statistics and detected range of q^2 of the present data are not large enough, the different models can not be differentiate yet.

The comparasion of theoretical prediction and exp. Data for the form factor

Table 1: Form factors of $F_+^{D\pi}(0)$ and $F_+^{DK}(0)$.

	$F_+^{D\pi}(0)$	$F_+^{DK}(0)$
LQCD1[1]	$0.57 \pm 0.06^{+0.01}_{-0.00}$ $0.57 \pm 0.06^{+0.02}_{-0.00}$	$0.66 \pm 0.04^{+0.01}_{-0.00}$
LQCD2[2]	$0.64 \pm 0.03 \pm 0.06$	$0.73 \pm 0.03 \pm 0.07$
QCD SR[5]	0.5 ± 0.1	$0.6^{+0.15}_{-0.10}$
LCSR[8]	0.65 ± 0.11	0.785 ± 0.11
LCSR[9]	0.67 ± 0.19	0.67 ± 0.20
Quark Model[13]	0.69	0.78
Light-Front[14]	0.67	
BES[18](Exp.)	$0.73 \pm 0.14 \pm 0.06$	$0.78 \pm 0.04 \pm 0.03$
CLEO[19](Exp.)	$\frac{\lvert F_+^{D\pi}(0)\rvert^2 \lvert V_{cd}\rvert^2}{\lvert F_+^{DK}(0)\rvert^2 \lvert V_{cs}\rvert^2} = 0.038^{+0.007+0.005}_{-0.007-0.003}$ Input $\lvert V_{cd}\rvert^2/\lvert V_{cs}\rvert^2 = 0.052 \pm 0.001$ [22] $\Rightarrow \frac{\lvert F_+^{D\pi}(0)\rvert^2}{\lvert F_+^{DK}(0)\rvert^2} = 0.86 \pm 0.07^{+0.06}_{-0.04} \pm 0.01$	

Single pole fit

modified pole fit

Table 2: Branching ratios of $D^0 \to \pi^- \ell^+ \nu$ and $D^0 \to K^- \ell^+ \nu$.

	$Br(D^0 \to \pi^- \ell^+ \nu)(\%)$	$Br(D^0 \to K^- \ell^+ \nu)(\%)$
LQCD1[1]	0.23 ± 0.06	2.83 ± 0.45
	0.24 ± 0.06	2.99 ± 0.45
LQCD2[2]	$0.32 \pm 0.02 \pm 0.06 \pm 0.03$	$3.77 \pm 0.29 \pm 0.74 \pm 0.08$
QCD SR[5]	0.16 ± 0.03	2.7 ± 0.6
LCSR[8]	0.27 ± 0.10	3.6 ± 1.4
LCSR[9]	0.30 ± 0.09	3.9 ± 1.2
BES[18](Exp.)	$0.33 \pm 0.13 \pm 0.03$	$3.82 \pm 0.40 \pm 0.27$
CLEO[20](Exp.)	$0.262 \pm 0.025 \pm 0.008$	$3.44 \pm 0.10 \pm 0.10$

(2) Transitions to vector mesons $D \to V\ell^+\nu$

The decomposition of D→V hadronic matrix element according to its Lorentz structure:

$$\langle V(\varepsilon, p_2)|\bar{q}\gamma_\mu(1-\gamma_5)c|D(p_1)\rangle = \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_D+m_V}$$
$$-i(\varepsilon^*_\mu - \frac{\varepsilon^*\cdot q}{q^2}q_\mu)(m_D+m_V)A_1(q^2) + i[(p_1+p_2)_\mu - \frac{m_D^2-m_V^2}{q^2}q_\mu]$$
$$\times\varepsilon^*\cdot q\frac{A_2(q^2)}{m_D+m_V} - i\frac{2m_V\varepsilon^*\cdot q}{q^2}q_\mu A_0(q^2)$$

$V(q^2)$ ---------------------- vector current $\bar{q}\gamma_\mu c$,

$A_i(q^2)$ ---------------------- axial-vector current $\bar{q}\gamma_\mu\gamma_5 c$

Calculation of the Form Factors in QCD Sum Rule

The idea of QCD sum rule:

1) Constructing a correlation function, which can be calculated within QCD in the quark level. At the same time it can be also described in the hadronic level.

2) By assuming quark-hadron duality, these two descriptions of the correlation function are equal .

3) Solve the above equation, get the form factors

Calculation of the form factors in $D_s^+ \to \phi \bar{l} \nu$

The correlation function

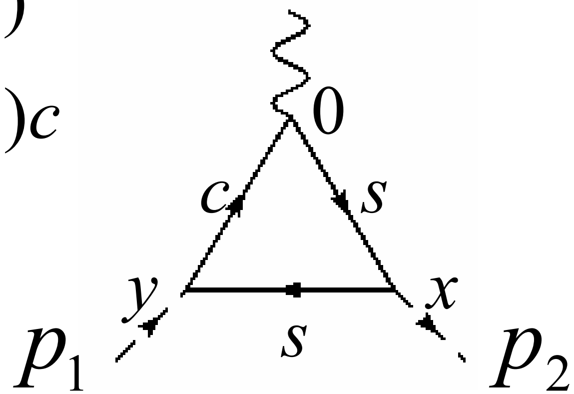
$$\Pi_{\mu\nu} = \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{e}^{\mathrm{i}p_2\cdot x - \mathrm{i}p_1\cdot y} \langle 0|T\{j_\nu^\phi(x) j_\mu(0) j_5^D(y)\}|0\rangle,$$

The currents have the same quantum numbers as the relevant particles:

1) the currents for D_s : $j_5^D(y) = \bar{c}(y) i\gamma_5 s(y)$

2) the currents for ϕ : $j_\nu^\phi(x) = \bar{s}(x)\gamma_\nu s(x)$

3) c→s weak current: $j_\mu(0) = \bar{s}\gamma_\mu(1-\gamma_5)c$



D.S. Du, J.W. Li and M.Z. Yang, EPJC37(2004) 173

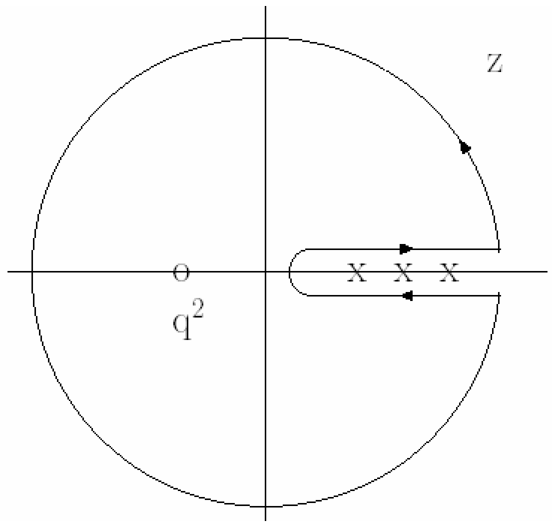
Dispersion relation

Due to the Cauchy formula, any analytic function $\Pi(q^2)$ can be expressed as contour integration

$$\prod\left(q^2\right)=\frac{1}{2\pi i}\oint_C dz\,\frac{\prod(z)}{z-q^2}$$

$$=\frac{1}{2\pi i}\oint_{|z|=R} dz\,\frac{\prod(z)}{z-q^2}$$

$$+\frac{1}{2\pi i}\int_0^R dz\,\frac{\prod(z+i\varepsilon)-\prod(z-i\varepsilon)}{z-q^2}$$



If as $\left|q^2\right| \sim R \to \infty,$ the function Π(q2) →0 fast enough,

the integration along the round contour would be zero

Then

$$\prod\left(q^2\right) = \frac{1}{2\pi i}\int_0^\infty dz\,\frac{\prod\left(z+i\varepsilon\right) - \prod\left(z-i\varepsilon\right)}{z-q^2}$$

$$\because \prod\left(z+i\varepsilon\right) - \prod\left(z-i\varepsilon\right) = 2i\,\mathrm{Im}\prod\left(z\right)$$

$$\prod\left(q^2\right) = \frac{1}{\pi}\int_{\min}^\infty ds\,\frac{\mathrm{Im}\prod\left(s\right)}{s-q^2-i\varepsilon}$$

———— Dispersion relation

Define the hadronic spectrum density:

$$\rho(s) = \frac{1}{\pi} \operatorname{Im} \prod (s)$$

Then the dispersion relation can be written as

$$\prod \left(q^2\right) = \int_{\min}^{\infty} ds \, \frac{\rho(s)}{s - q^2 - i\varepsilon}$$

Calculate the correlation function in the hadronic level

Insert two complete sets of hadronic states into the time-ordered product

$$\Pi_{\mu\nu} = \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{e}^{\mathrm{i}p_2\cdot x - \mathrm{i}p_1\cdot y} \langle 0|T\{j_\nu^\phi(x) j_\mu(0) j_5^D(y)\}|0\rangle,$$

$$\sum_X |X\rangle\langle X| \qquad \sum_Y |Y\rangle\langle Y|$$

And using the dispersion relation due to the two variables p_1^2 and p_2^2

$$\Pi_{\mu\nu} = \int \mathrm{d}s_1 \mathrm{d}s_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)},$$

where

$$\rho(s_1, s_2, q^2) = \sum_{XY} \langle 0|j_\nu^\phi|X\rangle \langle X|j_\mu|Y\rangle \langle Y|j_5^D|0\rangle \times \delta(s_1 - m_Y^2)\delta(s_2 - m_X^2)\theta(p_X^0)\theta(p_Y^0),$$

$$\Pi_{\mu\nu} = \sum_{XY} \frac{\langle 0|j_\nu^\phi|X\rangle\langle X|j_\mu|Y\rangle\langle Y|j_5^D|0\rangle}{(m_Y^2 - p_1^2)(m_X^2 - p_2^2)} + \text{continuum states.}$$

Separate the ground states:

$$\Pi_{\mu\nu} = \frac{\langle 0|j_\nu^\phi|\phi\rangle\langle \phi|j_\mu|D_s\rangle\langle D_s|j_5^D|0\rangle}{(m_{D_s}^2 - p_1^2)(m_\phi^2 - p_2^2)} + \text{higher resonances and continuum states.}$$

Calculate the correlation function by OPE in QCD

According to OPE, the time-ordered operator in the correlation function can be expanded as

$$\begin{aligned} &\mathrm{i}^2\int \mathrm{d}^4x\mathrm{d}^4y\mathrm{e}^{\mathrm{i}p_2\cdot x-\mathrm{i}p_1\cdot y}T\{j_\nu^\phi(x)j_\mu(0)j_5^D(y)\} \\ &= C_{0\mu\nu}I + C_{3\mu\nu}\bar{\Psi}\Psi + C_{4\mu\nu}G^a_{\alpha\beta}G^{a\alpha\beta} \\ &+ C_{5\mu\nu}\bar{\Psi}\sigma_{\alpha\beta}T^aG^{a\alpha\beta}\Psi + C_{6\mu\nu}\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi + \cdots \end{aligned}$$

1) I is the unity operator

2) $\overline{\Psi}\Psi$ is the local Fermion field operator

3) $G^a_{\alpha\beta}$ is the gluonic strength tensor

4) C's are the Wilson coefficients

The vacuum expectation of the above OPE will give the correlation function:

$$\begin{aligned}\Pi_{\mu\nu} &= \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{e}^{\mathrm{i}p_2\cdot x - \mathrm{i}p_1\cdot y} \langle 0|T\{j_\nu^\phi(x) j_\mu(0) j_5^D(y)\}|0\rangle \\ &= C_{0\mu\nu} I + C_{3\mu\nu}\langle 0|\bar{\Psi}\Psi|0\rangle + C_{4\mu\nu}\langle 0|G^a_{\alpha\beta}G^{a\alpha\beta}|0\rangle \\ &+ C_{5\mu\nu}\langle 0|\bar{\Psi}\sigma_{\alpha\beta}T^a G^{a\alpha\beta}\Psi|0\rangle \\ &+ C_{6\mu\nu}\langle 0|\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi|0\rangle + \cdots\end{aligned}$$

The Wilson coefficients can be calculated in QCD perturbatively

The expressions of the correlation function in hadronic level and in quark level should be the same thing, then

$$\frac{\langle 0|j_\nu^\phi|\phi\rangle\langle\phi|j_\mu|D_s\rangle\langle D_s|j_5^D|0\rangle}{(m_{D_s}^2-p_1^2)(m_\phi^2-p_2^2)}$$

$$+ \text{ higher resonances and continuum states.}$$

$$\begin{aligned}&= C_{0\mu\nu}I + C_{3\mu\nu}\langle 0|\bar{\Psi}\Psi|0\rangle + C_{4\mu\nu}\langle 0|G^a_{\alpha\beta}G^{a\alpha\beta}|0\rangle \\ &+ C_{5\mu\nu}\langle 0|\bar{\Psi}\sigma_{\alpha\beta}T^aG^{a\alpha\beta}\Psi|0\rangle \\ &+ C_{6\mu\nu}\langle 0|\bar{\Psi}\Gamma\Psi\bar{\Psi}\Gamma'\Psi|0\rangle + \cdots\end{aligned}$$

● With the assumption of quark-hadron duality, the contribution of higher resonances and continuum state are canceled in the above equation.

$$\int_{s_1^h}^{\infty} ds_1 \int_{s_2^h}^{\infty} ds_2 \frac{\rho^{had.}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} = \int_{s_1^0}^{\infty} ds_1 \int_{s_2^0}^{\infty} ds_2 \frac{\rho^{pert.}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

● With Borel transformation, the dependence on the quark-hadron duality is suppressed.

$$\hat{B}\Big|_{p^2, M^2} f(p^2) = \lim_{\substack{n \to \infty \\ p^2 \to -\infty \\ -p^2/n = M^2}} \frac{(-p^2)^n}{(n-1)!} \frac{\partial^n}{\partial (p^2)^n} f(p^2)$$

The form factors for $D_s^+ \to \phi \bar{l} \nu$ are calculated to be

$$
\begin{aligned}
V(0) &= 1.21 \pm 0.33, \\
A_0(0) &= 0.42 \pm 0.12, \\
A_1(0) &= 0.55 \pm 0.15, \\
A_2(0) &= 0.59 \pm 0.17,
\end{aligned}
$$

$$
\begin{aligned}
r_V &\equiv \frac{V(0)}{A_1(0)} = 2.20 \pm 0.85, \\
r_2 &\equiv \frac{A_2(0)}{A_1(0)} = 1.07 \pm 0.43.
\end{aligned}
$$

The branching ratio:

$$\mathrm{Br}(D_s^+ \to \phi \bar{\ell} \nu) = (1.8 \pm 0.5)\%,$$

$$\mathrm{Br}(D_s^+ \to \phi \ell \nu)^{\mathrm{exp}} = (2.0 \pm 0.5)\%$$

V D^0 -$\overline{D}^0$ Mixing

1) Mixing under weak interaction

2) The basic formulas

3) The mixing parameters in the SM

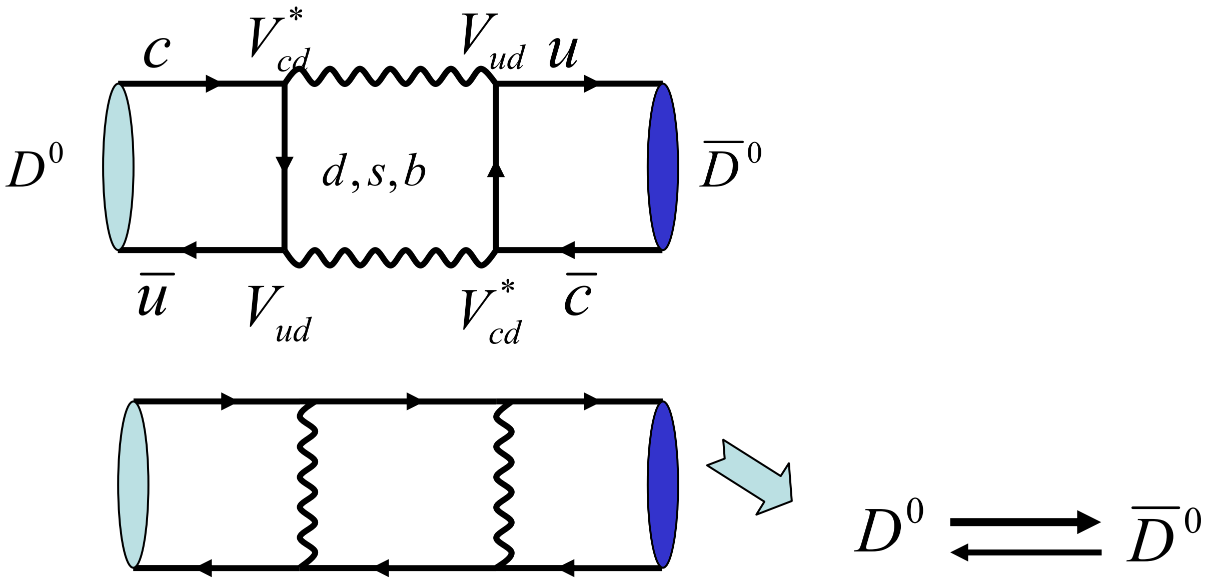
4) Decays of physical neutral D system

5) Time-dependent measurement of $D^0\overline{D}^0$ mixing

6) Summary

1) Mixing under weak interaction

D^0 and $\overline{D}^0$ can transform into each under weak interaction



D^0 and $\overline{D}^0$ can not be separated absolutely

- The D^0-$\overline{D}^0$ mixing occurs via loop diagrams involving intermediate down-type quarks, it provides unique information about weak interaction
- In the standard model, the mixing amplitude is quite small

It is severely suppressed by the GIM mechanisms

$$A \propto \sum_{i,j=d,s,b} f(m_i, m_j)(V_{ui}V_{ci}^*)(V_{uj}V_{cj}^*)$$

Loop-integration function

The b-quark contribution is highly suppressed by the CKM factor

$$\begin{array}{cc} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \end{array}$$

The CKM suppression factor

$$|V_{ub}V_{cb}^*|/|V_{us}V_{cs}^*| = O(3\times 10^{-4})$$

The b-quark contribution in the loop diagram can be neglected

- Thus, the mixing in D system involves only the first two generations. CP violation is absent in both the mixing and decay amplitudes, and therefore can be neglected.

- The mixing amplitude vanishes in the limit of SU(3) flavor symmetry , $m_s=m_d$, due to the GIM suppression.

- Mixing is only the effect of SU(3) breaking

$$T_{mixing} \sim \sin^2\theta_C \times [SU(3)\ \text{breaking}]$$

2) The basic formulas

In general the neutral D meson exists as a mixture state of D^0 and $\overline{D}^0$

$$|D\rangle = a|D^0\rangle + b|\overline{D}^0\rangle$$

Assume there is a neutral D state at t=0:

$$|\psi(0)\rangle = a(0)|D^0\rangle + b(0)|\overline{D}^0\rangle$$

Then at any time t, the state evolves into

$$|\psi(t)\rangle = \underbrace{a(t)|D^0\rangle + b(t)|\overline{D}^0\rangle}_{\text{Oscillation within neutral D states}} + \underbrace{c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots}_{\text{States D decays into}}$$

Oscillation within neutral D states

States D decays into

If we only consider the oscillation within the neutral D state, then we can consider the evolution of the following state

$$\left| D(t) \right\rangle = a(t) \left| D^0 \right\rangle + b(t) \left| \overline{D}^0 \right\rangle$$

which can be written in the form of matrix product

$$D(t) = \begin{pmatrix} D^0 & \overline{D}^0 \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

then we can use $\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$ to stand for the wave function of the neutral D meson state

The Shrödinger equation for the evolution of the wave function is

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

2×2

H needs not be Hermite because D meson can decay in the evolution

$$H \equiv M - \frac{i}{2}\Gamma$$

$M^{+} = M$ $\Gamma^{+} = \Gamma$

$$H = \frac{H+H^{+}}{2} + \frac{H-H^{+}}{2}$$

M $-\frac{i}{2}\Gamma$

The matrix H can be expressed explicitly in terms of the matrix elements

$$H = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

The matrix elements are determined by the Hamiltonians of strong, electromagnetic and weak interactions

$$H_{total} = H_{st} + H_{em} + H_{w}$$

The magnitude of weak interaction is greatly smaller than the strong and electromagnetic interaction

$$H_{w} << H_{st} + H_{em}$$

The weak interaction can be treated as a perturbation over the strong and EM interaction.

Then the matrix elements $M_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta}$ can be solved perturbatively

$$M_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta} = \frac{1}{2m_D}\langle\alpha|H_w^{\Delta c=2}|\beta\rangle + \frac{1}{2m_D}\sum_n \frac{\langle\alpha|H_w^{\Delta c=1}|n\rangle\langle n|H_w^{\Delta c=1}|\beta\rangle}{E_D - E_n + i\varepsilon}$$
$$+ O(H_w^3) + \cdots$$

The eigenstates of H_{st}+H_{em}

$$\alpha,\ \beta \sim D^0,\ \overline{D}^0$$

$$n \sim 2\pi,\ 3\pi,\ K\pi,\ K\pi\pi,\ K\pi\pi\pi,\ \pi e\nu,\ \cdots$$

Using the formula

$$\frac{1}{f(x)+i\varepsilon} = P\frac{1}{f(x)} - i\pi\delta(f(x))$$

$$\Downarrow$$

$$\frac{1}{E_D - E_n + i\varepsilon} = P\frac{1}{E_D - E_n} - i\pi\delta(E_D - E_n)$$

Then from

$$M_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta} = \frac{1}{2m_D}\left\langle \alpha \left| H_w^{\Delta c=2} \right| \beta \right\rangle + \frac{1}{2m_D}\sum_n \frac{\left\langle \alpha \left| H_w^{\Delta c=1} \right| n \right\rangle \left\langle n \left| H_w^{\Delta c=1} \right| \beta \right\rangle}{E_D - E_n + i\varepsilon} + \cdots$$

$$\left\{ \begin{aligned} M_{\alpha\beta} &= \frac{1}{2m_D}\left\langle \alpha \left| H_w^{\Delta c=2} \right| \beta \right\rangle + \frac{1}{2m_D}\sum_n P\frac{\left\langle \alpha \left| H_w^{\Delta c=1} \right| n \right\rangle \left\langle n \left| H_w^{\Delta c=1} \right| \beta \right\rangle}{E_D - E_n} + \cdots \\ \frac{1}{2}\Gamma_{\alpha\beta} &= \frac{1}{2m_D}\sum_n \pi \left\langle \alpha \left| H_w^{\Delta c=1} \right| n \right\rangle \left\langle n \left| H_w^{\Delta c=1} \right| \beta \right\rangle \delta(E_D - E_n) + \cdots \end{aligned} \right.$$

Theorems:

① If CPT is conserved, then $M_{11}=M_{22}$, and $\Gamma_{11}=\Gamma_{22}$.

② If T is conserved, then $$\frac{\Gamma_{12}^*}{\Gamma_{12}}=\frac{M_{12}^*}{M_{12}}$$

The proof can be performed with the formulas in the previous page

The eigen-equation

$$\begin{pmatrix} M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{21}-\frac{i}{2}\Gamma_{21} & M_{22}-\frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \mu\begin{pmatrix} a \\ b \end{pmatrix}$$

Solve the equation, one can get the eigenvalues and eigenfunctions

$$\mu_{1,2} = \frac{1}{2}[M_{11}+M_{22}-\frac{i}{2}(\Gamma_{11}+\Gamma_{22})$$
$$\mp\sqrt{(\delta m-\frac{i}{2}\delta\Gamma)^2+4(M_{12}-\frac{i}{2}\Gamma_{12})(M_{21}-\frac{i}{2}\Gamma_{21})}]$$

$$\boxed{\delta m \equiv M_{11}-M_{22}} \qquad \boxed{\delta\Gamma \equiv \Gamma_{11}-\Gamma_{22}}$$

- The real parts of $\mu_{1,2}$ will be the masses m_1 and m_2 of the two eigenstates
- The imaginary parts will be decay-widths of the two eigenstates: Γ_1, and Γ_2

That is

$$\mu_{1,2} \equiv m_{1,2} - \frac{i}{2}\Gamma_{1,2}$$

The two relevant eigenstates are

$$|D_1\rangle = p\sqrt{1+z}\,|D^0\rangle + q\sqrt{1-z}\,|\overline{D}^0\rangle \rightarrow \boxed{\mu_1 = m_1 - \frac{i}{2}\Gamma_1}$$

$$|D_2\rangle = p\sqrt{1-z}\,|D^0\rangle - q\sqrt{1+z}\,|\overline{D}^0\rangle \rightarrow \boxed{\mu_2 = m_2 - \frac{i}{2}\Gamma_2}$$

$$\frac{q}{p} = \sqrt{\frac{M_{21} - \frac{i}{2}\Gamma_{21}}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

$$z = \frac{M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})}{\sqrt{(\delta m - \frac{i}{2}\delta\Gamma)^2 + 4(M_{12} - \frac{i}{2}\Gamma_{12})(M_{21} - \frac{i}{2}\Gamma_{21})}}$$

If CPT is conserved, then z=0

Then

$$|D_1\rangle = p|D^0\rangle + q|\overline{D}^0\rangle$$

$$|D_2\rangle = p|D^0\rangle - q|\overline{D}^0\rangle$$

p and q satisfy the normalization condition

$$|p|^2 + |q|^2 = 1$$

3) The estimation of the mixing parameters in the SM

Two physical parameters which characterize the mixing are

$$x \equiv \frac{\Delta m}{\Gamma} = \frac{m_2 - m_1}{\Gamma}$$

$$y \equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

Where Γ is the averaged decay widths of the two eigenstates D_1 and D_2

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

Using the eigenvalues of D_1 and D_2, we can obtain with CPT conservation and neglecting CP violation in mixing

$$\Delta m = 2M_{12}$$

$$= \frac{1}{m_D}\left\langle D^0 \middle| H_w^{\Delta c=2} \middle| \overline{D}^0 \right\rangle + \frac{1}{m_D}\sum_n P\,\frac{\left\langle D^0 \middle| H_w^{\Delta c=1} \middle| n \right\rangle \left\langle n \middle| H_w^{\Delta c=1} \middle| \overline{D}^0 \right\rangle}{E_D - E_n} + \cdots$$

$$\Delta\Gamma = 2\Gamma_{12} = \frac{1}{m_D}\sum_n 2\pi \left\langle D^0 \middle| H_w^{\Delta c=1} \middle| n \right\rangle \left\langle n \middle| H_w^{\Delta c=1} \middle| \overline{D}^0 \right\rangle \delta(E_D - E_n) + \cdots$$

where $(2\pi)^3 \delta^3(\vec{p}_D - \vec{p}_n)$ is implicitly included

Define the correlator

$$\Pi \equiv \int dx^4 \left\langle D^0 \left| T\{H_w^{\Delta c=1}(x) H_w^{\Delta c=1}(0)\} \right| \overline{D}^0 \right\rangle$$

Insert a complete set of final states $\sum_n |n\rangle\langle n|$

One can obtain

$$\Pi = \sum_n \left\langle D^0 \left| H_w^{\Delta c=1} \right| n \right\rangle \left\langle n \left| H_w^{\Delta c=1} \right| \overline{D}^0 \right\rangle \{ \sum_n P \frac{2i}{E_D - E_n} + 2\pi\delta(E_D - E_n) \}$$
$$\cdot (2\pi)^3 \delta^3 (\vec{p}_D - \vec{p}_n)$$

Compare the above result with the expression of

$$\Delta m, \quad \Delta\Gamma$$

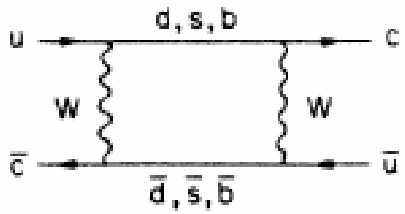
One can obtain

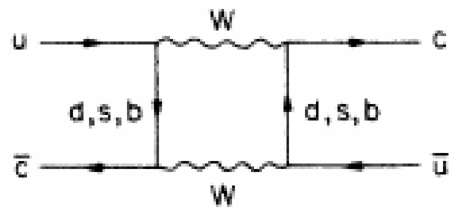
$$\Delta m = \frac{1}{m_D}\left\langle D^0 \left| H_w^{\Delta c=2} \right| \overline{D}^0 \right\rangle$$

$$+ \frac{1}{2m_D}\mathrm{Re}[\frac{1}{i}\int dx \left\langle D^0 \left| T\{H_w^{\Delta c=1}(x) H_w^{\Delta c=1}(0)\} \right| \overline{D}^0 \right\rangle] + \cdots$$

$$\Delta\Gamma = \frac{1}{m_D}\mathrm{Im}[i\int dx \left\langle D^0 \left| T\{H_w^{\Delta c=1}(x) H_w^{\Delta c=1}(0)\} \right| \overline{D}^0 \right\rangle\} + \cdots$$

The contribution of the box diagrams





$$H_w^{\Delta c=2} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{8\pi \sin^2\theta_W} \xi_s \xi_d \frac{(m_s{}^2 - m_d{}^2)^2}{M_W{}^2 m_c{}^2} (O + 2O')$$

$$\xi_i = V_{ic}^* V_{iu}$$

$$O = \bar{u}\gamma^\mu(1+\gamma_5)c\,\bar{u}\gamma_\mu(1+\gamma_5)c \ ,$$

$$O' = \bar{u}(1-\gamma_5)c\,\bar{u}(1-\gamma_5)c$$

[1] A. Datta and D. Kumbhakar, Z. Phys. C **27**, 515 (1985); H. Y. Cheng, Phys. Rev. D **26**, 143 (1982).

[2] J. F. Donoghue, E. Golowich, B. Holstein, and J. Trampetic, Phys. Rev. D **33**, 179 (1986).

[3] J. F. Donoghue, E. Golowich, and G. Valencia, Phys. Rev. D **33**, 1387 (1986).

The box contribution to Δm is given by

$m_d \sim 0$

$$\Delta m_D^{\text{box}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi \sin^2\theta_W} \xi_s \xi_d \frac{{m_s}^4}{{M_W}^2 {m_c}^2} \frac{8}{3}$$

$$\times m_D {F_D}^2 (B_D - 2B_D') ,$$

where the quantities B_D, B_D' are defined by

$$\langle D^0 | O | \bar{D}^0 \rangle = \frac{16}{3} \frac{{m_D}^2 {F_D}^2}{2m_D} B_D ,$$

$$\langle D^0 | O' | \bar{D}^0 \rangle = -\frac{10}{3} \left(\frac{m_D}{m_c} \right)^2 \frac{{m_D}^2 {F_D}^2}{2m_D} B_D' ,$$

Numerically, by simply taking $B_D = B'_D = 1$

$$\Delta m_D^{\text{box}} \approx 2.5 \times 10^{-17}\ \text{GeV}$$

which leads to

$$x^{\text{box}} \approx 1.6 \times 10^{-5}$$

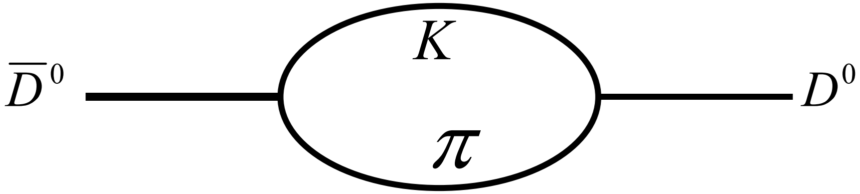
The bare quark loop contribution to $\Delta\Gamma$ is even further suppressed by additional powers of m_s / m_c

Numerically, one finds

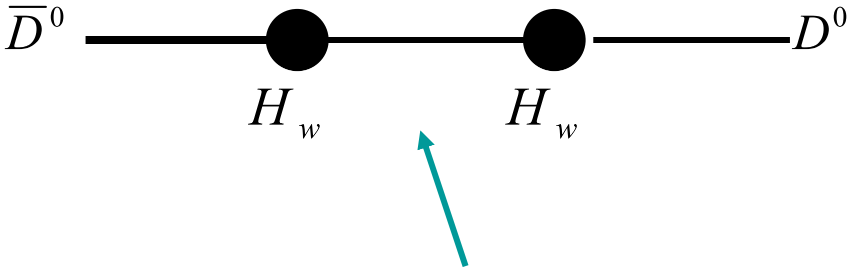
$$y^{\text{box}} \sim \text{few} \times 10^{-7}$$

The small result of box diagram can be enhanced by various long-distance effects, or by contributions of higher-dimension operator in the OPE

Long-distance effects



$$\pi\pi,\ KK,\ K\pi\pi,\ K\pi\pi\pi,\ K\pi\pi\pi\pi,\ etc.$$



$$K(1460),\ \eta(1760),\ \pi(1800),\ K(1830),\cdots$$

Long-distance contributions can severely enhance the mixing parameters, although it is difficult to calculate them accurately.

It is estimated that long-distance dynamics can enhance the mixing parameters to be

$$x,\ \ y \sim 10^{-4} - 10^{-3}$$

J.F. Donoghue et.al, Phys. Rev. D33, 179 (1986)

E. Golowich, A.A. Petrov, Phys. Lett. B427, 172 (1998)

Contribution of higher-dimension operator in operator product expansion (OPE)

$$\Delta m = \frac{1}{m_D}\left\langle D^0 \left| H_w^{\Delta c=2} \right| \overline{D}^0 \right\rangle$$

$$+\frac{1}{2m_D}\mathrm{Re}[\frac{1}{i}\int dx \left\langle D^0 \left| T\{H_w^{\Delta c=1}(x) H_w^{\Delta c=1}(0)\} \right| \overline{D}^0 \right\rangle] + \cdots$$

$$\Delta\Gamma = \frac{1}{m_D}\mathrm{Im}[i\int dx \left\langle D^0 \left| T\{H_w^{\Delta c=1}(x) H_w^{\Delta c=1}(0)\} \right| \overline{D}^0 \right\rangle\} + \cdots$$

The time-ordered product can be expanded in local operators of increasing dimension.

The higher dimension operators are suppressed by powers of

$$\Lambda / m_c$$

The leading contribution comes from the dimension-6 $|\Delta C| = 2$ four-quark operators corresponding to the short distance box diagram,

$$O_1 = \overline{u}_\alpha \gamma_\mu P_L c_\alpha \overline{u}_\beta \gamma_\mu P_L c_\beta\,, \quad O_1' = \overline{u}_\alpha P_L c_\alpha \overline{u}_\beta P_L c_\beta\,,$$

$$O_2 = \overline{u}_\alpha \gamma_\mu P_L c_\beta \overline{u}_\beta \gamma_\mu P_L c_\alpha\,, \quad O_2' = \overline{u}_\alpha P_L c_\beta \overline{u}_\beta P_L c_\alpha$$

where $P_L = \frac{1}{2}(1 - \gamma_5)$

Higher order terms in the OPE can be important, because chiral suppression can be lifted by the quark condensates, which lead to contribution proportional to m_s^2, rather than m_s^4.

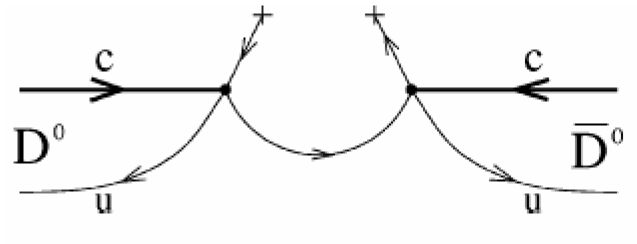


Diagram for 6-quark operator (D=9)

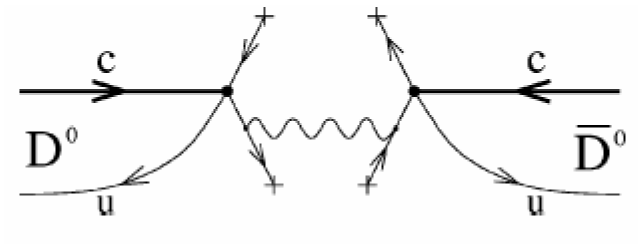


Diagram for 8-quark operator (D=12)

I.I. Bigi, N.G. Uraltsev, Nucl. Phys. B592 (2001)92

A. Falk et al., Phys. Rev. D65 (2002) 054034

Explicitly, the contribution of 6-quark operator with D=9 is given by

$$\Delta M_D^{(D=9)} = 2\sin^2\theta_C \cos^2\theta_C \frac{G_F^2 m_s^2}{m_c^3} \times \frac{1}{2M_D}\langle \overline{D}\big|\bar{u}^i\gamma_\mu(1-\gamma_5)c^j\bar{u}^k\gamma_\nu(1-\gamma_5)\gamma_0 c^i \times \left(\bar{s}^j\gamma^\mu\gamma_0\gamma^\nu(1-\gamma_5)s^k - \bar{d}^j\gamma^\mu\gamma_0\gamma^\nu(1-\gamma_5)d^k\right)\big|D\rangle.$$

The dominant contribution to x is from 6- and 8-quark operators

For y, the dominant contribution is from 8-quark operators

Numerically, the resulting estimates are

$$x,\ \ y \sim O(10^{-3})$$

This estimate means that the observation of mixing parameters severely larger than 10^{-3} would reveal the existence of new physics beyond the SM

But this statement can not be conclusive because the uncertainty in this estimation is still large.

- The main uncertainty comes from the size of the relevant hadronic matrix elements.

- Larger observed values of x and y might indicate serious underestimation of the hadronic matrix elements.

4) Decays of physical neutral D system

How to measure the mixing parameters in experiments?

To measure them, one has to study the evolution and various decays of the neutral D meson system.

The evolution of neutral D meson state

For any neutral D state at any time t, its wave function can be expanded as linear combination of the eigenstates

$$\left|D(t)\right\rangle = \underbrace{c_1}_{\text{constant}} e^{-i(m_1-\frac{i}{2}\Gamma_1)t}\left|D_1\right\rangle + \underbrace{c_2}_{\text{constant}} e^{-i(m_2-\frac{i}{2}\Gamma_2)t}\left|D_2\right\rangle$$

c1 and c2 can be determined by initial and normalization condition

$$\left|D(t)\right\rangle = c_1 e^{-i(m_1-\frac{i}{2}\Gamma_1)t}\left|D_1\right\rangle + c_2 e^{-i(m_2-\frac{i}{2}\Gamma_2)t}\left|D_2\right\rangle$$

$$= c_1 e^{-i(m_1-\frac{i}{2}\Gamma_1)t}(p\left|D^0\right\rangle + q\left|\overline{D}^0\right\rangle) + c_2 e^{-i(m_2-\frac{i}{2}\Gamma_2)t}(p\left|D^0\right\rangle - q\left|\overline{D}^0\right\rangle)$$

$$= p(c_1 e^{-i(m_1-\frac{i}{2}\Gamma_1)t} + c_2 e^{-i(m_2-\frac{i}{2}\Gamma_2)t})\left|D^0\right\rangle - q(c_2 e^{-i(m_2-\frac{i}{2}\Gamma_2)t} - c_1 e^{-i(m_1-\frac{i}{2}\Gamma_1)t})\left|\overline{D}^0\right\rangle$$

For a state initially $\left|D^0\right\rangle$,i.e., $\left|D(t)\right\rangle_{|t=0} = \left|D^0\right\rangle$

$$\Rightarrow \quad c_1 = c_2 = \frac{1}{2p}$$

Denote such a state as $\left|D^0_{phys}(t)\right\rangle$

Then

$$\left|D^0_{phys}(t)\right\rangle = \frac{1}{2}\left(e^{-i(m_2-\frac{i}{2}\Gamma_2)t} + e^{-i(m_1-\frac{i}{2}\Gamma_1)t}\right)\left|D^0\right\rangle$$

$$-\frac{q}{2p}\left(e^{-i(m_2-\frac{i}{2}\Gamma_2)t} - e^{-i(m_1-\frac{i}{2}\Gamma_1)t}\right)\left|\overline{D}^0\right\rangle$$

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-i(m_2-\frac{i}{2}\Gamma_2)t} \pm e^{-i(m_1-\frac{i}{2}\Gamma_1)t}\right)$$

$$\left|D^0_{phys}(t)\right\rangle = g_+(t)\left|D^0\right\rangle - \frac{q}{p}g_-(t)\left|\overline{D}^0\right\rangle$$

Similarly

$$\left|\overline{D}^0_{phys}(t)\right\rangle = g_+(t)\left|\overline{D}^0\right\rangle - \frac{p}{q}g_-(t)\left|D^0\right\rangle$$

The time-dependent decay rate of a physical neutral D meson state

The time-dependent amplitude of initial D^0 physical state to any final state f

$$T_f(t) = \langle f | H | D^0_{phys}(t) \rangle$$

$$= g_+(t) \underbrace{\langle f | H | D^0 \rangle}_{\boxed{A_f}} - \frac{q}{p} g_-(t) \underbrace{\langle f | H | \overline{D}^0 \rangle}_{\boxed{\overline{A}_f}}$$

$$= g_+(t) A_f - \frac{q}{p} g_-(t) \overline{A}_f$$

$$\left|T_f(t)\right|^2 = \frac{1}{2}e^{-\Gamma t}\{(\left|A_f\right|^2 + \left|\frac{q}{p}\bar{A}_f\right|^2)\cosh(y\Gamma t) + (\left|A_f\right|^2 - \left|\frac{q}{p}\bar{A}_f\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[A_f(\frac{q}{p}\bar{A}_f)^*]\sinh(y\Gamma t) - 2\,\mathrm{Im}[\frac{q}{p}A_f^*\bar{A}_f]\sin(x\Gamma t)]\}$$

$$\frac{d\Gamma[D^0_{phys}(t) \to f]}{dt} \propto \left|T_f(t)\right|^2$$

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}, x = \frac{\Delta m}{\Gamma} = \frac{m_2 - m_1}{\Gamma},$$

$$y = \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

$\therefore$

$$\frac{d\Gamma[D^0_{phys}(t) \to f]/dt}{e^{-\Gamma t}N_f} = \{(\left|A_f\right|^2 + \left|\frac{q}{p}\bar{A}_f\right|^2)\cosh(y\Gamma t) + (\left|A_f\right|^2 - \left|\frac{q}{p}\bar{A}_f\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[A_f(\frac{q}{p}\bar{A}_f)^*]\sinh(y\Gamma t) - 2\,\mathrm{Im}[\frac{q}{p}A_f^*\bar{A}_f]\sin(x\Gamma t)]\}$$

A normalization factor

We can get the decay rate of an initial $\overline{D}^0$ state by

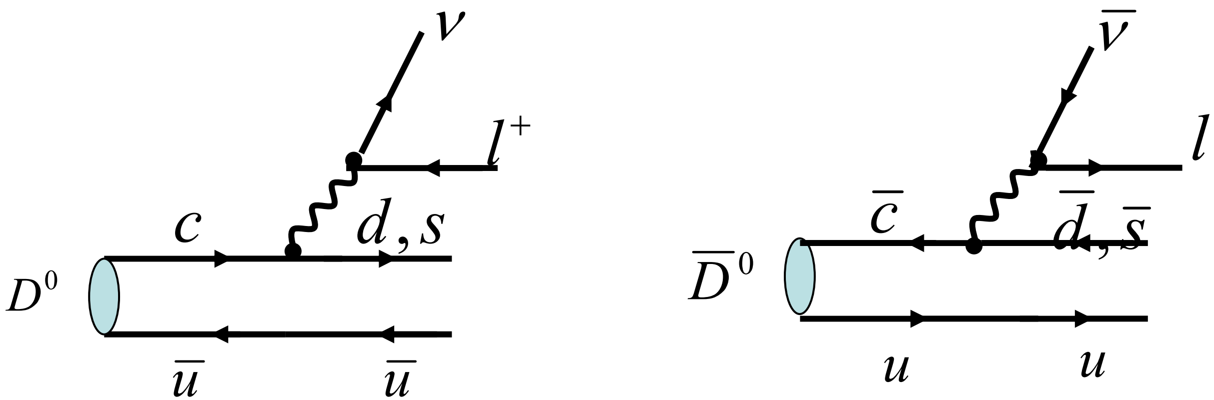
$$\begin{cases} \left|D^0\right\rangle \leftrightarrow \left|\overline{D}^0\right\rangle \\ q \leftrightarrow p \\ A_f \leftrightarrow \overline{A}_f \end{cases}$$

$$\frac{d\Gamma[\overline{D}^0_{phys}(t) \to f]/dt}{e^{-\Gamma t} N_f} = \{(\left|\frac{p}{q} A_f\right|^2 + \left|\overline{A}_f\right|^2)\cosh(y\Gamma t) - (\left|\frac{p}{q} A_f\right|^2 - \left|\overline{A}_f\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\frac{p}{q} \overline{A}_f^{\,*} A_f]\sinh(y\Gamma t) - 2\,\mathrm{Im}[\frac{p}{q} \overline{A}_f^{*} A_f]\sin(x\Gamma t)]\}$$

(1) Semi-leptonic decay

At leading order of weak interaction



$D^0 \to l^+ \nu X$ $\qquad$ $\bar{D}^0 \to l \bar{\nu} X$

$D^0 \not\to l \bar{\nu} X$ $\qquad$ $\bar{D}^0 \not\to l^+ \nu X$

$$\therefore \quad A_l = 0 \qquad \bar{A}_{l^+} = 0$$

With mixing present, the wrong-sign decay can occur

Let us consider the wrong-sign decays

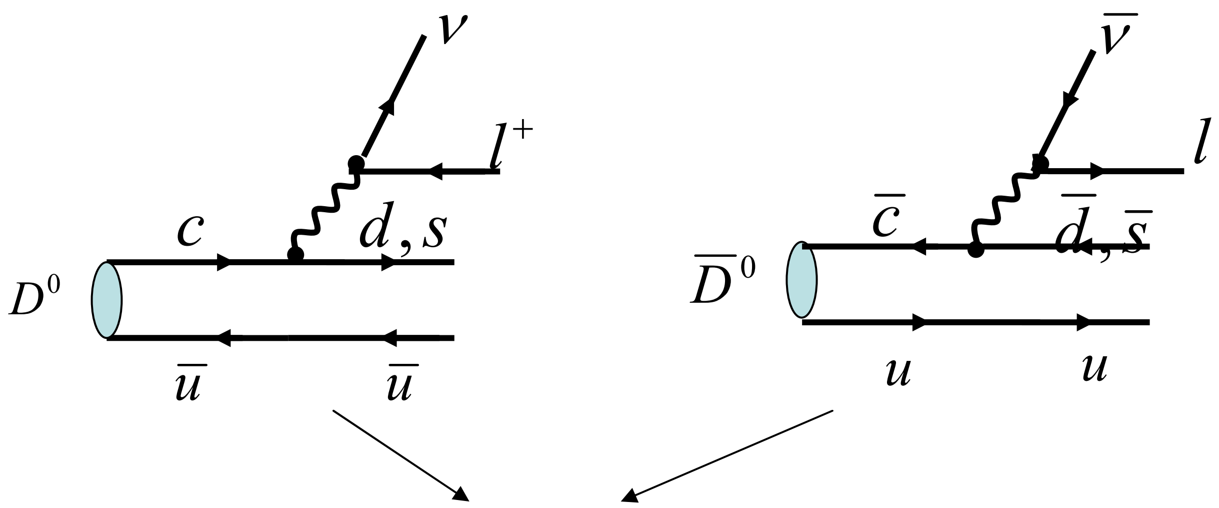
$$\begin{cases} D^0_{phys}(t) \to l\,\overline{\nu} X \\ \overline{D}^0_{phys}(t) \to l^+ \nu X \end{cases} \quad CP$$

$$\frac{d\Gamma[D^0_{phys}(t) \to l\,\overline{\nu} X]}{dt} = N_f e^{-\Gamma t} \left| \frac{q}{p} \overline{A}_l \right|^2 [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

$$\frac{d\Gamma[\overline{D}^0_{phys}(t) \to l^+ \nu X]}{dt} = N_f e^{-\Gamma t} \left| \frac{p}{q} A_{l^+} \right|^2 [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

$$\frac{d\Gamma[D^0_{phys}(t) \to l\,\overline{\nu} X]}{dt} \neq \frac{d\Gamma[\overline{D}^0_{phys}(t) \to l^+ \nu X]}{dt}$$

CP is violated!



$$\left|A_{l^+}\right| = \left|A_l\right|$$

CP violation in wrong-sign semileptonic decay

$$A_{CP}^{SL}(t) \equiv \frac{d\Gamma(\overline{D}^0_{phys}(t) \to l^+ \nu X)/dt - d\Gamma(D^0_{phys}(t) \to l \overline{\nu} X)/dt}{d\Gamma(\overline{D}^0_{phys}(t) \to l^+ \nu X)/dt + d\Gamma(D^0_{phys}(t) \to l \overline{\nu} X)/dt}$$

We can get

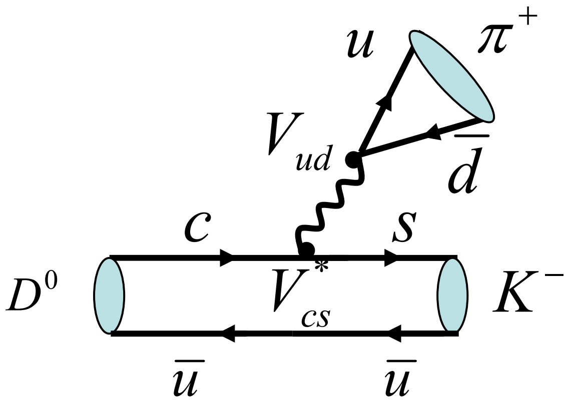
$$A_{CP}^{SL} = \frac{1 - \left|q/p\right|^4}{1 + \left|q/p\right|^4}$$

The condition for mixing induced CP violation

$$|q| \neq |p|$$

(2) Wrong-sign decay to hadronic final state f

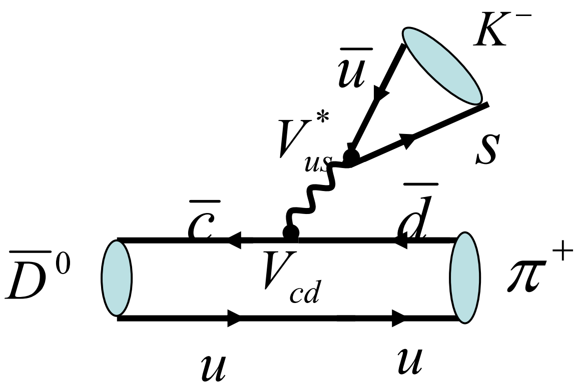
For $f = K^- \pi^+$



A_f

$$A(D^0 \to K^- \pi^+) \propto V_{ud} V_{cs}^* \propto 1$$

Cabibbo favored



$$A(\bar{D}^0 \to K^- \pi^+) \propto V_{cd} V_{us}^* \propto A^2 \lambda^4$$

$\bar{A}_f$

Doubly Cabibbo suppressed

Let us consider the doubly cabibbo suppressed decay

$$\overline{D}^0_{phys}(t) \to K^-\pi^+$$

Using the formua we have derived

$$\frac{d\Gamma[\overline{D}^0_{phys}(t) \to f]/dt}{e^{-\Gamma t} N_f} = \{(\left|\frac{p}{q} A_f\right|^2 + \left|\overline{A}_f\right|^2)\cosh(y\Gamma t) - (\left|\frac{p}{q} A_f\right|^2 - \left|\overline{A}_f\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\frac{p}{q}\overline{A}_f^{\,*} A_f]\sinh(y\Gamma t) - 2\,\mathrm{Im}[\frac{p}{q}\overline{A}_f^{\,*} A_f]\sin(x\Gamma t)]\}$$

We can obtain

$$\frac{d\Gamma[\overline{D}^0_{phys}(t) \to K^-\pi^+]/dt}{e^{-\Gamma t} N_f}$$

$$= \{(\left|\frac{p}{q} A_{K^-\pi^+}\right|^2 + \left|\overline{A}_{K^-\pi^+}\right|^2)\cosh(y\Gamma t) - (\left|\frac{p}{q} A_{K^-\pi^+}\right|^2 - \left|\overline{A}_{K^-\pi^+}\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\frac{p}{q}\overline{A}_{K^-\pi^+}{}^* A_{K^-\pi^+}]\sinh(y\Gamma t) - 2\,\mathrm{Im}[\frac{p}{q}\overline{A}^*_{K^-\pi^+} A_{K^-\pi^+}]\sin(x\Gamma t)]\}$$

$$= \left|\frac{p}{q} A_{K^-\pi^+}\right|^2 \{(1 + \left|\frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}}\right|^2)\cosh(y\Gamma t) - (1 - \left|\frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}}\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}}]\sinh(y\Gamma t) + 2\,\mathrm{Im}[\frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}}]\sin(x\Gamma t)]\}$$

define $\boxed{\lambda_{K^-\pi^+} \equiv \frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}}}$

$$\frac{d\Gamma[\overline{D}^0_{phys}(t) \to K^-\pi^+]/dt}{e^{-\Gamma t} N_f}$$

$$= \left|\frac{p}{q} A_{K^-\pi^+}\right|^2 \{(1+\left|\lambda_{K^-\pi^+}\right|^2)\cosh(y\Gamma t) - (1-\left|\lambda_{K^-\pi^+}\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\lambda_{K^-\pi^+}]\sinh(y\Gamma t) + 2\,\mathrm{Im}[\lambda_{K^-\pi^+}]\sin(x\Gamma t)]\}$$

Similarly the CP conjugated process

$$\frac{d\Gamma[D^0_{phys}(t) \to K^+\pi^-]/dt}{e^{-\Gamma t} N_f}$$

$$= \left|\frac{q}{p} \overline{A}_{K^+\pi^-}\right|^2 \{(\left|\overline{\lambda}_{K^+\pi^-}\right|^2 + 1)\cosh(y\Gamma t) - (1-\left|\overline{\lambda}_{K^+\pi^-}\right|^2)\cos(x\Gamma t)$$

$$+ 2\,\mathrm{Re}[\overline{\lambda}_{K^+\pi^-}]\sinh(y\Gamma t) + 2\,\mathrm{Im}[\overline{\lambda}_{K^+\pi^-}]\sin(x\Gamma t)]\}$$

$$\overline{\lambda}_{K^+\pi^-} \equiv \frac{p}{q}\frac{A_{K^+\pi^-}}{\overline{A}_{K^+\pi^-}}$$

To simplify the above result, we can make the following definitions

$$\frac{q}{p} \equiv (1 + A_m) e^{-i\beta}, \quad \frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}} \equiv -\sqrt{r} e^{-i\alpha}, \quad \frac{A_{K^+\pi^-}}{\overline{A}_{K^+\pi^-}} \equiv -\sqrt{r'} e^{-i\alpha'}$$

Mixing induced CPV

In order to demonstrate the CP violation in decay, we define

$$\sqrt{r} \equiv \sqrt{R_D} \frac{1}{1+A_D} \text{ and } \sqrt{r'} \equiv \sqrt{R_D}(1 + A_D).$$

$$\lambda_{K^-\pi^+} \equiv \frac{q}{p}\frac{\overline{A}_{K^-\pi^+}}{A_{K^-\pi^+}} = -\sqrt{R_D}\,\frac{1+A_M}{1+A_D}\,e^{-i(\delta+\phi)}$$

$$\overline{\lambda}_{K^+\pi^-} \equiv \frac{p}{q}\frac{A_{K^+\pi^-}}{\overline{A}_{K^+\pi^-}} = -\sqrt{R_D}\,\frac{1+A_D}{1+A_M}\,e^{-i(\delta-\phi)}$$

where $\delta = \dfrac{\alpha+\alpha'}{2}$, $\qquad \phi = \dfrac{\alpha-\alpha'}{2}+\beta$

1. A_D — CP violation in decay

$$D^0 \to f \Longleftrightarrow \overline{D}^0 \to \bar{f}$$

2. A_M — CP violation in mixing

$$\begin{array}{c} D^0 \to \overline{D}^0 \to f \\ \Updownarrow \\ \overline{D}^0 \to D^0 \to \bar{f} \end{array}$$

3. ϕ — CP violation in the interference between decays with and without mixing

$$\begin{tikzcd} D^0 \arrow[rr] \arrow[dr] & & f \\ & \overline{D}^0 \arrow[ur] & \end{tikzcd}$$

Then the time-dependent decay rates are

$$\bar{r}(t) \equiv \frac{d\Gamma[\overline{D}^0_{phys}(t) \to K^-\pi^+]\big/dt}{e^{-\Gamma t} N_f}$$

$$= \left|\frac{p}{q} A_{K^-\pi^+}\right|^2 \left\{\frac{(1+A_M)^2}{(1+A_D)^2} R_D - \frac{1+A_M}{1+A_D}\sqrt{R_D}(y\cos(\delta+\phi) - x\sin(\delta+\phi))(\Gamma t) + \frac{x^2+y^2}{4}(\Gamma t)^2\right\}$$

$$= \left|\frac{p}{q} A_{K^-\pi^+}\right|^2 \left\{R_D^+ - \sqrt{R_D^+}\, y'_+(\Gamma t) + \frac{x'^2_+ + y'^2_+}{4}(\Gamma t)^2\right\}$$

Note the sign

$$R_D^+ = \frac{(1+A_M)^2}{(1+A_D)^2} R_D, \quad R_D^- = \frac{(1+A_D)^2}{(1+A_M)^2} R_D$$

$$x'_\pm = x\cos(\delta \pm \phi) + y\sin(\delta \pm \phi)$$

$$y'_\pm = y\cos(\delta \pm \phi) - x\sin(\delta \pm \phi)$$

$$r(t) \equiv \frac{d\Gamma[D^0_{phys}(t) \to K^+\pi^-]\big/dt}{e^{-\Gamma t} N_f}$$

$$= \left|\frac{q}{p} \overline{A}_{K^+\pi^-}\right|^2 \left\{R_D^- - \sqrt{R_D^-}\, y'_-(\Gamma t) + \frac{x'^2_- + y'^2_-}{4}(\Gamma t)^2\right\}$$

If CP is conserved, then

$$A_M \to 0,\ A_D \to 0,\ \phi \to 0$$

Then

$$R_D^{\pm} = R_D$$

$$x'_{\pm} = x' = x\cos\delta + y\sin\delta$$

$$y'_{\pm} = y' = y\cos\delta - x\sin\delta$$

$$r(t) = \bar{r}(t) \propto \{R_D - \sqrt{R_D}\,y'\Gamma t + \frac{x'^2 + y'^2}{4}(\Gamma t)^2\}$$

The coefficients of the time-dependent terms can be fitted if the time dependent decay rate can be measured.

~~CP~~: $\{R_D^{\pm}, x'_{\pm}, y'_{\pm}\}$ for $\begin{cases} D^0_{phys}(t) \to K^+\pi^- \\ \overline{D}^0_{phys}(t) \to K^-\pi^+ \end{cases}$

CP: $\{R_D, x', y'\}$

In the early stage of 2007, BaBar and Belle found the evidence of D^0-$\bar{D}^0$ mixing in the decay modes

$$D^0 \to K^+\pi^- \quad \text{and} \quad D^0 \to K_S^0\pi^+\pi^-$$

respectively. Later CDF also found the evidence.

BaBar Collaboration, PRL98,211802 (2007)

Fit type	Parameter	Fit results (10^{-3})
No CP violation	R_D	$3.03 \pm 0.16 \pm 0.10$
	x'^2	$-0.22 \pm 0.30 \pm 0.21$
	y'	$9.7 \pm 4.4 \pm 3.1$
CPV allowed	R_D	$3.03 \pm 0.16 \pm 0.10$
	A_D	$-21 \pm 52 \pm 15$
	x'^2_+	$-0.24 \pm 0.43 \pm 0.30$
	x'^2_-	$-0.20 \pm 0.41 \pm 0.29$
	y'_+	$9.8 \pm 6.4 \pm 4.5$
	y'_-	$9.7 \pm 6.1 \pm 4.3$

CPV is consistent with zero

TABLE I. Fit results and 95% C.L. intervals for x and y, including systematic uncertainties. The errors are statistical, experimental systematic, and decay-model systematic, respectively. For the CPV-allowed case, there is another solution as described in the text.

Fit case	Parameter	Fit result	95% C.L. interval
No	$x(\%)$	$0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14}$	(0.0, 1.6)
CPV	$y(\%)$	$0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08}$	$(-0.34, 0.96)$
CPV	$x(\%)$	$0.81 \pm 0.30^{+0.10+0.09}_{-0.07-0.16}$	$\lvert x \rvert < 1.6$
	$y(\%)$	$0.37 \pm 0.25^{+0.07+0.07}_{-0.13-0.08}$	$\lvert y \rvert < 1.04$
	$\lvert q/p \rvert$	$0.86^{+0.30+0.06}_{-0.29-0.03} \pm 0.08$	$\cdots$
	$\arg(q/p)(^\circ)$	$-14^{+16+5+2}_{-18-3-4}$	$\cdots$

Table from Belle collaboration, PRL99, 131803 (2007)

- The data disfavor the no-mixing point x=0, y=0 significantly.

- CP violation is also searched for in mixing and decays, but no evidence for CPV is found.

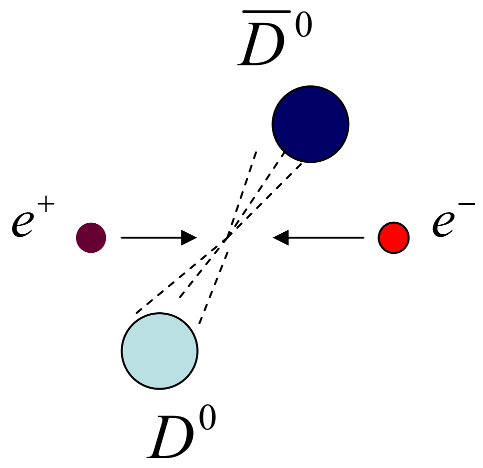
  The comparison of the data and theoretical prediction is premature, because the errors of the data and the uncertainty of the theoretical prediction are both too large.

- No new physics has to be invoked right now.

The decay rate of a correlated state

For a physical process producing $D^0\,\bar{D}^0$ such as

$$e^+e^- \rightarrow \psi'' \rightarrow D^0\overline{D}^0$$



The $D^0\,\bar{D}^0$ pair will be a quantum-correlated state

The quantum number of ψ'' is $J^{PC} = 1^{--}$

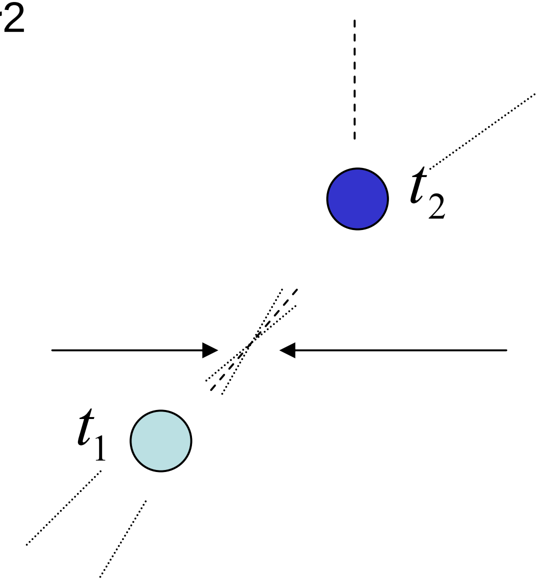
∴ The C number of $D^0\,\bar{D}^0$ pair in this process is $C = -$

For a correlated state with $C = -$

$$\psi_- = \frac{1}{\sqrt{2}}\left(\left|D^0\right\rangle\left|\overline{D}^0\right\rangle - \left|\overline{D}^0\right\rangle\left|D^0\right\rangle\right)$$

$$\hat{C}\left|D^0\right\rangle = \left|\overline{D}^0\right\rangle$$
$$\hat{C}\left|\overline{D}^0\right\rangle = \left|D^0\right\rangle$$

Let us consider the decay of such a correlated state, one D decays to f_1 at time t_1 , the other decays to f_2 at time t_2



The correlated wave function at time t_1 and t_2

$$\psi_-(t_1,t_2)=\frac{1}{\sqrt{2}}(\left|D^0_{phys}(t_1)\right\rangle\left|\overline{D}^0_{phys}(t_2)\right\rangle-\left|\overline{D}^0_{phys}(t_1)\right\rangle\left|D^0_{phys}(t_2)\right\rangle)$$

The amplitude of such state decaying to final state $f_1 f_2$ is

$$\begin{aligned}
&\left\langle f_1 f_2\left|H\right|\psi_-(t_1,t_2)\right\rangle \\
&=\frac{1}{\sqrt{2}}(\left\langle f_1\left|H\right|D^0_{phys}(t_1)\right\rangle\left\langle f_2\left|H\right|\overline{D}^0_{phys}(t_2)\right\rangle \\
&-\left\langle f_1\left|H\right|\overline{D}^0_{phys}(t_1)\right\rangle\left\langle f_2\left|H\right|D^0_{phys}(t_2)\right\rangle)
\end{aligned}$$

$$\left|\left\langle f_1 f_2 \middle| H \middle| \psi_-(t_1,t_2)\right\rangle\right|^2$$

$$= \frac{1}{4} e^{-\Gamma(t_1+t_2)}[(|a_+|^2 + |a_1|^2)\cosh(y\Gamma\Delta t) + (|a_+|^2 - |a_1|^2)\cos(x\Gamma\Delta t)$$

$$-2\,\mathrm{Re}(a_+ a_-^*)\sinh(y\Gamma\Delta t) - 2\,\mathrm{Im}(a_+ a_-^*)\sin(x\Gamma\Delta t)]$$

$$a_+ = \overline{A}_{f_1} A_{f_2} - A_{f_1}\overline{A}_{f_2}$$

$$a_- = \overline{A}_{f_1} A_{f_2} + A_{f_1}\overline{A}_{f_2}$$

$$\Delta t = t_2 - t_1$$

5) The time-dependent measurement of $D^0\bar{D}^0$ Mixing parameters

The most promising place to produce $D^0\bar{D}^0$ pair is

$$e^+e^- \to \psi(3770) \to D^0\bar{D}^0$$

However, the time-dependent information can not be used here

Many interesting ideas and phenomenological studies for measuring the mixing parameters have been presented in the literature

Z.Z. Xing (1996); Z.Z. Xing, S. Zhou (2007); Y. Grossman, A. Kagan, Y. Nir (2007); N. Sinha, R. Sinha, T. Browder, N. Deshpanda, S. Pakvasa (2007);

We proposed a new method to measure the mixing parameters in the correlated time-dependent processes through

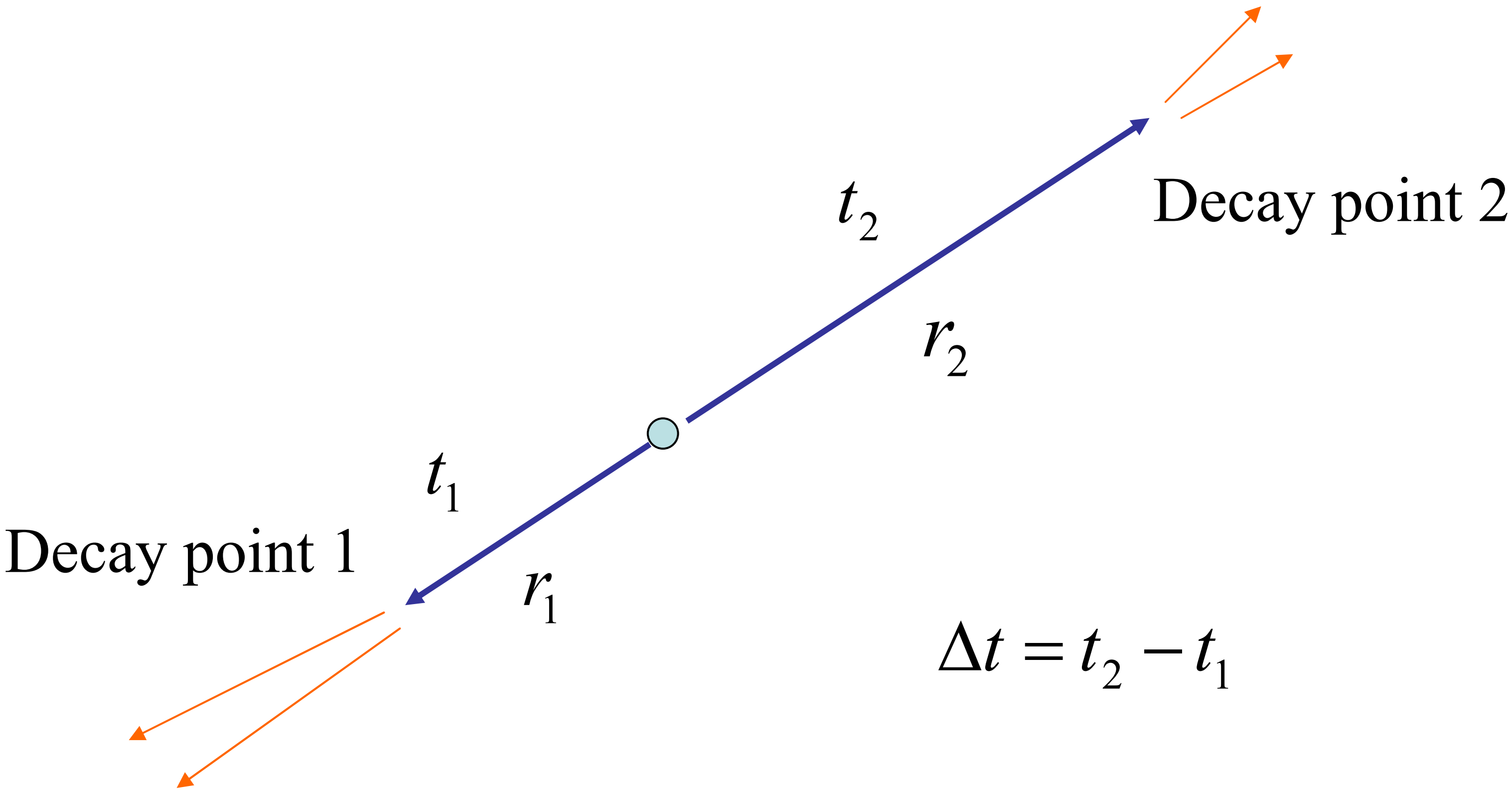
$$e^+e^- \to \Upsilon(1S) \to D^0\overline{D}^0$$

at super-B factor

H.B Li and M.Z. Yang PRD74 (2006) 094016;
PRD75 (2007) 094015

$$\Upsilon(1S) \to D^0 \overline{D}^0$$

The D mesons are strongly boosted



$$\Delta t = t_2 - t_1$$

Proper time interval Δt can be precisely determined

$$|D^0\bar{D}^0\rangle^{C=-1} = \frac{1}{\sqrt{2}}[|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle].$$

The time-dependent evolution

$$\begin{aligned}|D^0\bar{D}^0(t_1, t_2)\rangle = \frac{1}{\sqrt{2}}[&|D^0_{\rm phys}(\mathbf{k_1}, t_1)\rangle|\bar{D}^0_{\rm phys}(\mathbf{k_2}, t_2)\rangle \\ &- |\bar{D}^0_{\rm phys}(\mathbf{k_1}, t_1)\rangle|D^0_{\rm phys}(\mathbf{k_2}, t_2)\rangle],\end{aligned}$$

$$\frac{d\Gamma(\Upsilon(1S) \to D^0_{\text{phys}} \bar{D}^0_{\text{phys}} \to f_1 f_2)}{dt}$$

$$= \mathcal{N} e^{-\Gamma|t|} \times [(|a_+|^2 + |a_-|^2) \cosh(y\Gamma t)$$

$$+ (|a_+|^2 - |a_-|^2) \cos(x\Gamma t) - 2\,\mathcal{R}e(a_+^* a_-)$$

$$\times \sinh(y\Gamma t) + 2\,\mathcal{I}m(a_+^* a_-) \sin(x\Gamma t)],$$

here $\mathcal{N}$ is a common normalization factor, in]

$$a_+ \equiv \bar{A}_{f_1} A_{f_2} - A_{f_1} \bar{A}_{f_2} = A_{f_1} A_{f_2} \frac{p}{q} (\lambda_{f_1} - \lambda_{f_2}),$$

$$a_- \equiv \frac{p}{q} A_{f_1} A_{f_2} - \frac{q}{p} \bar{A}_{f_1} \bar{A}_{f_2} = A_{f_1} A_{f_2} \frac{p}{q} (1 - \lambda_{f_1} \lambda_{f_2}),$$

with $A_{f_i} \equiv \langle f_i | \mathcal{H} | D^0 \rangle$, $\bar{A}_{f_i} \equiv \langle f_i | \mathcal{H} | \bar{D}^0 \rangle$, and define

$$\lambda_{f_i} \equiv \frac{q}{p} \frac{\langle f_i | \mathcal{H} | \bar{D}^0 \rangle}{\langle f_i | \mathcal{H} | D^0 \rangle} = \frac{q}{p} \frac{\bar{A}_{f_i}}{A_{f_i}},$$

$$\bar{\lambda}_{\bar{f}_i} \equiv \frac{p}{q} \frac{\langle \bar{f}_i | \mathcal{H} | D^0 \rangle}{\langle \bar{f}_i | \mathcal{H} | \bar{D}^0 \rangle} = \frac{p}{q} \frac{A_{\bar{f}_i}}{\bar{A}_{\bar{f}_i}}.$$

$$R(f_1, f_2; t) \equiv \frac{d\Gamma(\Upsilon(1S) \to D^0_{\text{phys}}\bar{D}^0_{\text{phys}} \to f_1 f_2)}{dt}.$$

Now we consider the following cases for the D meson decays to various final states, such as semileptonic, hadronic, and CP eigenstates.

(1) $(l^- X^+, K^+\pi^-; t)$:

$$R(l^- X^+, K^+\pi^-; t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2\left|\frac{q}{p}\right|^2 e^{-\Gamma|t|} \times ((1+|\bar{\lambda}_{K^+\pi^-}|^2)\cosh(y\Gamma t) - (1-|\bar{\lambda}_{K^+\pi^-}|^2)\cos(x\Gamma t)$$
$$+ 2\,\mathcal{R}e(\bar{\lambda}_{K^+\pi^-})\sinh(y\Gamma t) + 2\,\mathcal{I}m(\bar{\lambda}_{K^+\pi^-})\sin(x\Gamma t)).$$

(2) $(l^+ X^-, K^-\pi^+; t)$:

$$R(l^+ X^-, K^-\pi^+; t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2\left|\frac{p}{q}\right|^2 e^{-\Gamma|t|} \times ((1+|\lambda_{K^-\pi^+}|^2)\cosh(y\Gamma t) - (1-|\lambda_{K^-\pi^+}|^2)\cos(x\Gamma t)$$
$$+ 2\,\mathcal{R}e(\lambda_{K^-\pi^+})\sinh(y\Gamma t) + 2\,\mathcal{I}m(\lambda_{K^-\pi^+})\sin(x\Gamma t)).$$

(3) $(l^{+}X^{-}, K^{+}\pi^{-}; t)$

(4) $(l^{-}X^{+}, K^{-}\pi^{+}; t)$

(5) $(l_1^{\mp}X^{\mp}, l_2^{\mp}X^{\pm}; t)$

(6) $(l^{\pm}X^{\mp}, S_{\eta}; t)$

(7) $(K^{-}\pi^{+}, S_{\eta}; t)$

(8) $(K^{-}\pi^{+}, K^{+}\pi^{-}; t)$

(9) $(K^{-}\pi^{+}, K^{-}\pi^{+}; t)$

(10) $(K^{+}\pi^{-}, K^{+}\pi^{-}; t)$

(11) $(l_1^{+}X^{-}, l_2^{+}X^{-}; t)$

(12) $(l_1^{+}X^{-}, l_2^{-}X^{+}; t)$

Taking into account that $\lambda_{K^-\pi^+}, \bar{\lambda}_{K^+\pi^-} \ll 1$ and $x, y \ll 1$, keeping terms up to order x^2, y^2, and R_D in the expressions, neglecting CP violation in mixing, decay, and the interference between decay with and without mixing ($A_M = 0, A_D = 0$, and $\phi = 0$), expanding the time dependent for $xt, yt \lesssim \Gamma^{-1}$, we can write the results as

(1) $(l^- X^+, K^+\pi^-; t)$:

$$R(l^- X^+, K^+\pi^-; t) = \mathcal{N}|A_l|^2|\bar{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|} \times (2R_D - 2\sqrt{R_D}y'\Gamma t + R_M\Gamma^2 t^2), \quad (31)$$

where $R_M \equiv \frac{x^2+y^2}{2}$ is the mixing rate, and $y' \equiv y\cos\delta - x\sin\delta$.

$$R(l^- X^+, K^+\pi^-; t) = N|A_l|^2|\overline{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|}(\underline{2R_D} - \underline{2\sqrt{R_D}y'}\Gamma t + \underline{R_M}\Gamma^2 t^2)$$

Fitting data, three numbers can be measured

(2) $(l^+X^-, K^-\pi^+; t)$:

$$R(l^+X^-, K^-\pi^+; t) = \mathcal{N}|A_l|^2|A_{K^-\pi^+}|^2 e^{-\Gamma|t|} \times (2R_D - 2\sqrt{R_D}y'\Gamma t + R_M\Gamma^2 t^2), \tag{32}$$

The same thing as case (1)

$$(3) \quad (l^+ X^-, K^+ \pi^-; t)$$

$$R(l^+ X^-, K^+ \pi^-; t)$$

$$= \mathcal{N} |A_l|^2 |\bar{A}_{K^+\pi^-}|^2 e^{-\Gamma|t|} \left(2 - 2\sqrt{R_D} (y \cos(\delta) + x \sin(\delta)) \Gamma t + \frac{y^2 - x^2}{2} \Gamma^2 t^2 \right).$$

$$= C e^{-\Gamma|t|} \times (1 - \underline{\sqrt{R_D} (y \cos\delta + x \sin\delta)} \Gamma t + \underline{\frac{y^2 - x^2}{4}} \Gamma^2 t^2)$$

Fitting data, these two numbers can be measured

(4) $(l^{\pm}X^{\mp}, s_{\eta}; t)$

$$R(l^{\pm}, S_{\eta}; t) = \mathcal{N}|A_l|^2|A_{S_\eta}|^2 e^{-\Gamma|t|} \times (2 - 2\eta(y\cos\beta \mp x\sin\beta)\Gamma t + y^2\Gamma^2 t^2),$$

the phase $\beta = arg[(V_{us}V^*_{cs}/(V_{cs}V^*_{us})] \sim 0.$

$$= Ce^{-\Gamma|t|} \times (1 - \eta y\Gamma t + y^2\Gamma^2 t^2)$$

y can be measured

(5) $(K^-\pi^+, S_\eta; t)$

$$R(K^-\pi^+, S_\eta; t)$$

$$= \underbrace{N\left|A_{K^-\pi^+}\right|^2 \left|A_{S_\eta}\right|^2 (\eta - \sqrt{R_D}\cos\delta)^2}_{C} \times e^{-\Gamma|t|}(1 - \eta y\Gamma t + \frac{1}{2}y^2(\Gamma t)^2)$$

$$= C \times e^{-\Gamma|t|}(1 - \underline{\eta y}\Gamma t + \frac{1}{2}y^2(\Gamma t)^2)$$

y can be measured

- In summary, the numbers that can be fitted are:

$$R_D, \qquad \sqrt{R_D}(y\cos\delta + x\sin\delta), \qquad \frac{x^2+y^2}{4},$$

$$\frac{x^2-y^2}{4}, \qquad y$$

- There are four unknown variables in these fitted numbers：

$$x,\ \ y,\ \ \delta,\ \ R_D$$

- Therefore, the mixing parameters can be measured in the these correlated decay processes.

CP violation

● In the SM CP violation is produced by the complex phase of the CKM matrix, current experimental constraint for CKM matrix element implies that CP violation in D system is very small

● However, New physics may induce new source of CP violation

● One possibility is the new CPV phase from

$$\phi = \arg(q\bar{A} / pA)$$

$$A_{CP}^{+-}(t) \equiv \frac{R(l^+X^-, K^+\pi^-;t) - R(l^-X^+, K^-\pi^+;t)}{R(l^+X^-, K^+\pi^-;t) + R(l^-X^+, K^-\pi^+;t)}$$

One can obtain

$$A_{CP}^{+-}(t) = -\sqrt{R_D}\,(y\sin\delta - x\cos\delta)\sin\phi \times \Gamma t$$

Very small in the SM,
unless there is New Physics

In experiment

KEK-B can move to $\Upsilon(1S)$ peak

about $7.1\times10^{9}\,\Upsilon(1S)$ events 1 year

Super-B about $10^{12}\,\Upsilon(1S)$ 1 year

if the luminosity is about $10^{36}\,cm^{-2}s^{-1}$

Theoretical estimate

$$Br(\Upsilon(1S) \to D^+ D^-) \approx 10^{-4} \sim 10^{-5}$$

At super-B: $10^7 \sim 10^8 \; D^0 \overline{D}^0$ pairs

It is possible to measure D$\overline{\text{D}}$ mixing parameters and CPV in charm by using the time-dependent information in coherent neutral D$\overline{\text{D}}$ decays

Summary

- The mixing parameters can be predicted to be

$$x, \quad y \sim 10^{-3}$$

without introducing new physics beyond the standard model.

- The present uncertainty in both theoretical prediction and the exp. data are still large.

- Various decays of the neutral D-meson system are studied, a new method to measure the mixing parameter by using the correlation and time-dependent information is proposed.