Charmonium: Spectroscopy and Decays

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- Lecture 1: Spectroscopy
- Lecture 2: Transitions
- Lecture 3: Above Threshold and the X,Y,Z States

General References

[1] N. Brambilla *et al.* [Quarkonium Working Group], arXiv:hep-ph/0412158.

[2] N. Brambilla *et al.* , 'Heavy quarkonium: progress, puzzles, and opportunities" (in preperation)

[3] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008) [arXiv:0711.4556 [hep-ph]].

[4] E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008) [arXiv:hep-ph/0701208].

[5] S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. **58**, 51 (2008) [arXiv:0801.3867 [hep-ph]].

Charmonium I: Spectroscopy

- QCD with heavy quarks
  - QED -> QCD
  - Velocity expansion
- Static energy
  - pNRQCD + QCD string
  - Potential models + Lattice
- Spin Splittings
- Open issues in Spectroscopy

QCD with Heavy Quarks

- For heavy quarks $m_Q \gg \Lambda_{QCD}$, nonrelativistic ($Q\bar{Q}$) bound states form with masses M near $2m_Q$:
  - systematic expansion in powers of v/c
  - heavy quark velocity: $p_Q/m_Q \approx v/c \ll 1$
  - binding energy: $2m_Q - M \approx m_Q v^2/c^2$
- QED - positronium (e^+e^-), (true) muonium ($\mu^-\mu^+$), muonium ($e^-\mu^+$), hydrogen atom (e^-P);
- QCD - charmonium (c-cbar), bottomoniun (b-bbar), "toponium" (t-tbar), (b-cbar), ...
- Let's look at QED first:
  - Nonrelativistic fermions: four component Ψ -> two component ψ and $\sigma_2\chi^*$

$$\mathcal{H} = \int d^3x \Psi^\dagger[\alpha \cdot (\mathbf{p} - \mathbf{A}) + A^0 + \beta m]\Psi$$

$$i\frac{\partial}{\partial t}\Psi = [\mathcal{H}, \Psi] \to i\frac{\partial}{\partial t}\Psi = H\Psi$$

$$\text{where} \quad H = \alpha \cdot (\mathbf{p} - \mathbf{A}) + A^0 + \beta m$$

Odd Even

$$\alpha^i = \gamma^0\gamma^i$$

$$\beta = \gamma^0$$

- Foldy-Wouthuysen transformation: $\exp{(iS)}\Psi = \Psi' = \begin{bmatrix} \psi \\ \sigma_2 \chi^* \end{bmatrix}$

$$\begin{aligned} i\frac{\partial}{\partial t}\exp(-iS)\Psi' &= H\exp(-iS)\Psi' \rightarrow i\frac{\partial}{\partial t}\Psi' = \{[\exp(-iS)]^\dagger[H - i\frac{\partial}{\partial t}]\exp(-iS)\}\Psi' = H'\Psi' \\ H' &= H + i[S,H] + \frac{(i)^2}{2}[S,[S,H]] + \frac{(i)^3}{6}[S,[S,[S,H]]] + ... \\ & \qquad - \frac{\partial S}{\partial t} - \frac{i}{2}[S, \frac{\partial S}{\partial t}] + ... \end{aligned}$$

- Goal is to remove odd terms in H'

- Solve iteratively in powers of 1/m

$$\exp{(-iS)} = \exp{(-iS_1)}\exp{(-iS_2)}\exp{(-iS_3)}...$$

$$\text{To kth order}: \ S_k \sim O(m^{-k}) \text{ and } \mathcal{H}'(\text{leading odd term}) \sim O(m^{-k})$$

- In lowest order:

$$H = H(even) + H(odd) \text{ where } [H(even), \beta] = 0$$

S_1 is given by solving:

$$H_1 = \beta m + A^0 + \alpha \cdot (\mathbf{p} - \mathbf{A}) + i[S_1, \beta]m + O(1/m)$$

giving

$$S_1 = -i\beta[\alpha \cdot (\mathbf{p} - \mathbf{A})]/(2m)$$

then use S_1 to determine the full H_1

- To third order:

Gauge $\quad A^0 j_0$

$$\mathcal{H}_{\rm QED} = \mathcal{H}_{\rm gauge} + \mathcal{H}_\psi = \int d^3x \left\{ \frac{1}{2e^2}(\mathbf{E}^2 + \mathbf{B}^2) + A^0\psi^\dagger\psi \right.$$

Kinetic + corrections $\quad$ magnetic

$$+\psi^\dagger[m + \frac{(\mathbf{p}-\mathbf{A})^2}{2m} - \frac{(\mathbf{p^2})^2}{8m^3}]\psi - \frac{1}{2m}\sigma\cdot\mathbf{B}$$

$$\left. - \frac{i}{8m^2}\sigma\cdot(\nabla\ \mathbf{x}\ \mathbf{E}) - \frac{1}{4m^2}\sigma\cdot(\mathbf{E}\ \mathbf{x}\ \mathbf{p}) - \frac{1}{8m^2}\nabla\cdot\mathbf{E} + ...]\psi \right\}$$

spin orbit $\quad$ Darwin

Bjorken+Drell,
"Relativistic Quantum Mechanics", Chap. 4 (1964)

- So for a nonrelativistic ($Q\,Q_c$) system: (Q = e-, μ-); (Q_c = e+, μ+, P):

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm gauge} + \mathcal{H}_Q + \mathcal{H}_{Q_c}$$

- In Coulomb gauge: $\quad \nabla\cdot\mathbf{A} = 0; \qquad A^0$ dependent

$$\mathbf{E} = \mathbf{E_T} + \mathbf{E_L}; \qquad \nabla\cdot\mathbf{E_T} = \mathbf{0}$$

$$(\mathbf{E_T}, \mathbf{A}); \quad (\psi^\dagger, \psi) \quad \text{ConjugateVariables}$$

- Eliminate A^0

$$\int d^3x \left\{ \frac{1}{2e^2}(\vec{\nabla}A^0)^2 + A^0[Q^\dagger Q - Q_c^\dagger Q_c] \right\} \rightarrow \frac{e^2}{8\pi}\int d^3x d^3y \left\{ j^0_{\rm EM}(x)\frac{1}{|\mathbf{x}-\mathbf{y}|}j^0_{\rm EM}(y) \right\}$$

$A^0 j_{EM}{}^0 \quad$ since $\quad < x|\frac{1}{\nabla^2}|y> = \frac{1}{4\pi}\frac{1}{|\mathbf{x}-\mathbf{y}|}$

- Effective Hamiltonian

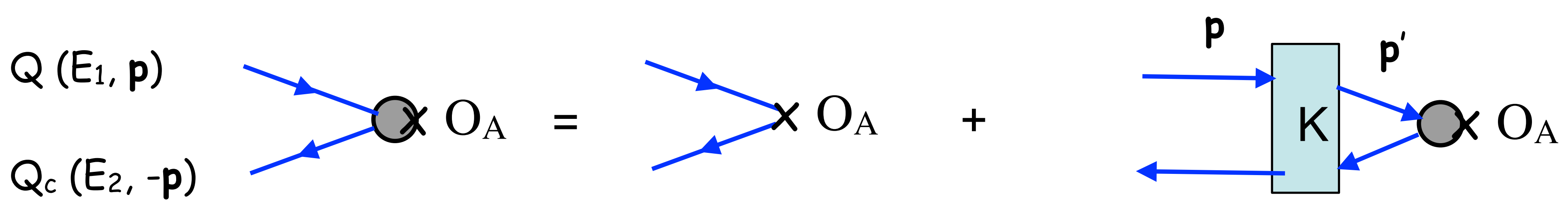
$$
\begin{aligned}
\mathcal{H}_{\text{eff}} \;=\; & \int d^3x \left\{ \frac{1}{2e^2} (\mathbf{E_T}^2 + \mathbf{B}^2) \right\} \\
& + \int d^3x \Bigg\{ Q^\dagger [m + \frac{(\mathbf{p}-\mathbf{A})^2}{2m} + ...] Q \\
& \qquad + \; Q^\dagger [-\frac{1}{2m}\sigma \cdot \mathbf{B} - \frac{i}{8m^2}\sigma \cdot (\nabla \;\mathbf{x}\; \mathbf{E_T}) - \frac{1}{4m^2}\sigma \cdot (\mathbf{E_T} \;\mathbf{x}\; \mathbf{p}) + ...] Q \Bigg\} \\
& + \mathcal{H}_{Q_c} \\
& - \frac{e^2}{4\pi} \int d^3x \int d^3y \left\{ Q^\dagger Q(x) \frac{1}{|\mathbf{x}-\mathbf{y}|} Q_c^\dagger Q_c(y) + ... \right\}
\end{aligned}
$$

Gauge

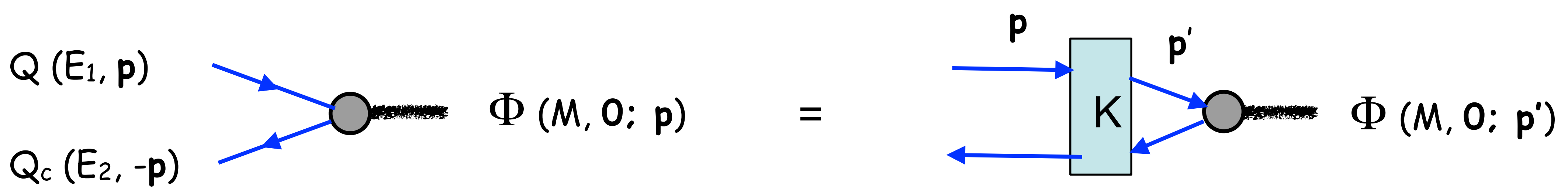
H_Q

Coulomb interaction

- Threshold bound states
  - let O_A be a local operator coupling to QQc with bare vertex Γ the full vertex function satisfies the equation:



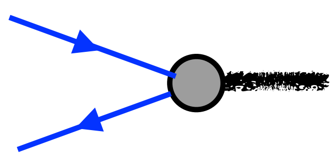
  - where K is the two body irreducible kernel
  - if a bound state exists in this channel then for $E_1 + E_2$ near the mass M of the bound state the pole should dominate.
  - So the bound state equation becomes



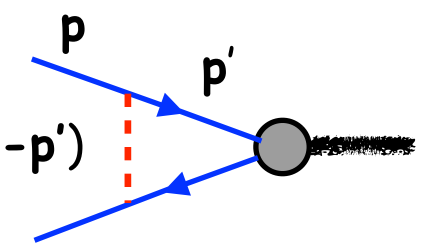
- Using the nonrelativistic H_{eff}, to leading order in (1/m)
  - the kernel K is just the Coulomb exchang

Q (E_1, **p**)

Qc (E_2, -**p**)



Φ (M, 0; **p**) = α_{EM} (**p**-**p**')



Φ (M, 0; **p**')

$$\Phi(M,\mathbf{0},\mathbf{p}) = -\frac{\alpha_{\rm EM}}{2\pi^2}\int d^3p'\frac{1}{(\mathbf{p}-\mathbf{p}')^2}\Big[\frac{1}{\sqrt{m_1^2+\mathbf{p}'^2}+\sqrt{m_2^2+\mathbf{p}'^2}-M}\Big]\Phi(M,\mathbf{0},\mathbf{p}')$$

- But α_{EM} << 1 -> no solutions unless:

$$\begin{aligned}\sqrt{m_1^2+\mathbf{p}'^2}+\sqrt{m_2^2+\mathbf{p}'^2}-M &= (m_1+m_2-M)+\frac{\mathbf{p^2}}{\mu_R}+\ldots << m_1+m_2\\ E(\text{binding}) &\sim \frac{v_{\rm rel}^2}{2\mu_R}\\ \mathbf{p},\mathbf{p}' &\sim v_{\rm rel}\mu_R\end{aligned}$$

- Nonrelativistic states with natural expansion in v/c ≈ α_{EM}

- Schrödinger Equation:

$$\Psi(M,\mathbf{0},\mathbf{p}) \equiv \sqrt{m_1^2+\mathbf{p}^2}+\sqrt{m_2^2+\mathbf{p}^2}-M\Big]\Phi(M,\mathbf{0},\mathbf{p})$$

$$\Big[-\frac{\vec{\nabla}^2}{2\mu_R}-\frac{\alpha_{\rm EM}}{r}\Big]\Psi(M,\mathbf{x}) = M\Psi(M,\mathbf{x})$$

with

$$\begin{aligned}\mu_R &= \frac{m_1m_2}{m_1+m_2}\\ r &= |\mathbf{x}|\end{aligned}$$

- QCD with heavy quarks we can follow the same steps to obtain:

  - Notation

$$(A^0, \mathbf{A}) = gA^\mu_a t^a \quad \mathbf{D} = \nabla + i\mathbf{A} \qquad -i[\mathbf{D}^0, \mathbf{D}^i] = E^j$$

$$\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab} \qquad -i[\mathbf{D}^j, \mathbf{D}^k] = \epsilon^{jkl} B^l$$

with

$$\nabla \cdot \mathbf{A} = 0$$
$$\nabla \cdot \mathbf{E} = 0$$

$$J^0_a(\mathbf{x}) = \sum_{Q=c,b,t} [J^0_a(\mathbf{x})]_Q + [J^0_a(\mathbf{x})]_{\text{gauge}}$$

Color current

$$[J^0_a(\mathbf{x})]_{\text{gauge}} = g f_{abc} \sum_i E^i_b(\mathbf{x}) A^i_c(\mathbf{x})$$
$$[J^0_a(\mathbf{x})]_Q = Q^\dagger(\mathbf{x}) t_a Q(\mathbf{x}) - Q^\dagger_c(\mathbf{x}) t^*_a Q_c(\mathbf{x})$$

$$G_{ab}(\mathbf{x}, \mathbf{y}) = 4\pi < \mathbf{x}, a| \frac{1}{\nabla \cdot \mathbf{D}} \nabla^2 \frac{1}{\nabla \cdot \mathbf{D}} |\mathbf{y}, b > \to \frac{\delta_{ab}}{|\mathbf{x} - \mathbf{y}|} \text{ as } g \to 0$$

Gribov ambiguity

- Result NRQCD

Gauge

$$\mathcal{H}^{\rm eff}_{\rm QCD} = \frac{1}{g^2}\int d^3x\, \left[{\rm Tr}(\mathbf{E}^2) + {\rm Tr}(\mathbf{B}^2)\right] + \sum_{Q=c,b,t} [\mathcal{H}_Q + \mathcal{H}_{Q_c}]$$
$$+ \frac{g^2}{4\pi}\int d^3x \int d^3y\, \{J^0_a(x)\mathcal{G}(\mathbf{x},\mathbf{y})_{ab}J^0_b(y)\}$$

Potential

$$\mathcal{H}_Q = \int d^3x\, Q^\dagger\left((m_Q + \delta m_Q) - \frac{\mathbf{D}^2}{2m_Q}\right)Q$$

Kinetic

Darwin

$$- \int d^3x\, Q^\dagger\left[c_4\frac{1}{8m_Q^3}(\mathbf{D}^2)^2 + c_d\frac{1}{8m_Q^2}(\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D})\right]Q$$

Spin Independent

Relativistic corrections

$$- \int d^3x\, Q^\dagger\left[c_f\frac{1}{2m_Q}\sigma\cdot\mathbf{B} + c_s\frac{i}{8m_Q^2}\sigma\cdot(\mathbf{D}\ \mathbf{x}\ \mathbf{E} + \mathbf{E}\ \mathbf{x}\ \mathbf{D})\right]Q + \ldots$$

Spin Dependent

Magnetic

Spin- Orbit

- Comments

- The coefficients (δm_Q, c_4, c_d, c_f, c_8) are (0, 1, 1, 1, 1) plus perturbative corrections associated with matching at scale m_Q

- Gribov copies, ordering issues and $1/m_Q^2$ contact terms ignored

- Adding light quarks

  - relativistic form appropriate for (u,d,s) quarks

$$J_a^0(\mathbf{x}) \to [J_a^0(\mathbf{x})]_{\text{gauge}} + \sum_{Q=c,b,t} [J_a^0(\mathbf{x})]_Q + \sum_{f=u,d,s} [J_a^0(\mathbf{x})]_f \quad \text{with} \quad [J_a^0(\mathbf{x})]_f = \psi_f^\dagger(\mathbf{x}) t_a \psi_f(\mathbf{x})$$

$$\mathcal{H}_{\text{QCD}} = \mathcal{H}_{\text{QCD}}^{\text{eff}} + \sum_{f=u,d,s} \mathcal{H}_f \quad \text{where} \quad \mathcal{H}_f = \int d^3x \; \psi_f^\dagger \left[-i\alpha\cdot\mathbf{D} + \beta m_f\right]\psi_f$$

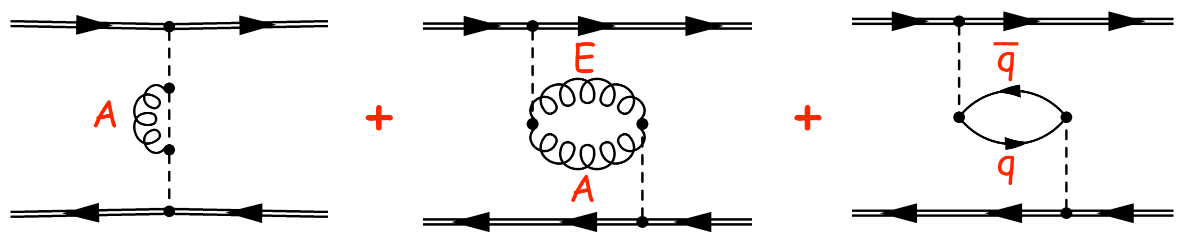
  - Light quarks have both important effects on charmonium physics
    - » modify properties of narrow states
    - » above threshold allow strong decays to charmed meson pairs
    - » more details later

- QCD with sufficiently heavy quarks: $\alpha_{\text{QCD}}((p-p')^2)$ will be small

  - charmonium ~ $\langle v^2/c^2 \rangle \approx 0.24$ bottomonium ~ $\langle v^2/c^2 \rangle \approx 0.08$

Static Energy

- Can compute the potential at short distance using NRQCD
  - In coulomb gauge (using H^{eff})



$$-\frac{12}{3}N_c \qquad \frac{1}{3}N_c \qquad \frac{2}{3}n_f$$

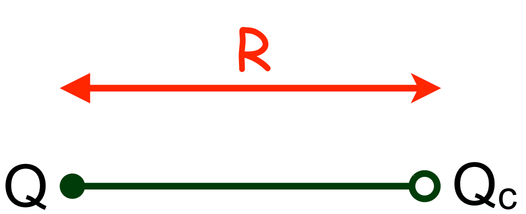
$$-\frac{4}{3}\frac{\alpha_s(\mathbf{k^2})}{\mathbf{k^2}}$$

Coupling runs

$$\alpha_s(\mu^2) = \frac{4\pi}{(11-\frac{2}{3}n_f)\ln(\mu^2/\Lambda^2_{\rm QCD})}$$

  - Three loop calculation: A. V. Smirnov, V. A. Smirnov and M. Steinhauser, arXiv:1006.5513

- Large distance - expect string-like behavior
  - $\mathcal{E}(\mathrm{R}) = \sigma\mathrm{R}$



- Potential models - V(R) = - κ /R + σ R + C (Cornell Model)

Practical realities

- The scales that enter NR calculations are :
  - for heavy quarks: $m_Q^2 - p^2 \sim m_Q v^2 \sim$ (binding energy), $\mathbf{p} \sim m_Q \mathbf{v}$
  - for gauge fields: $k \sim m_Q v$ ("soft" or "potential" gluons) ; $k \sim m_Q v^2$ ("ultrasoft")
- In QED $v \sim \alpha \ll 1$ so bound states are always nonrelativistic: $mv^2 \ll mv \ll 1$.

  Corrections to the NR limit Coulomb interaction can be calculated in perturbation theory.
- In QCD there is a strong interaction scale Λ_{QCD} .
  - $\Lambda_{QCD} \ll m_Q v^2 \ll m^Q v \ll 1$ only for the $(t\bar{t})$ system. but the top quark decays before toponium states form.
  - For the $(c\bar{c})$, $(b\bar{b})$ and $(b\bar{c})$ systems: $\Lambda_{QCD} \sim m_Q v^2 < m_Q v < m_Q$ at best.
  - Integrate out perturbative scales: m_Q (NRQCD) and $m_Q\, v$ (pNRQCD). Still left with nonperturbative theory that must be modeled on computed on the lattice.

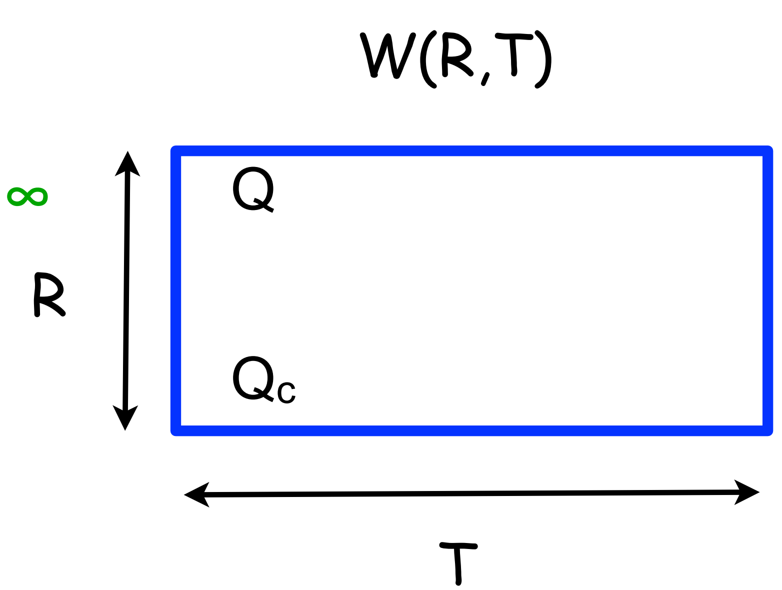
Static Energy

- Wilson loops W(R,T) and static energy E(R)
  - Q and Q_c separated by distance R for time T:
  - Static states: $H_0|n, \mathbf{x}_1, \mathbf{x}_2\rangle = E_n\,|n, \mathbf{x}_1, \mathbf{x}_2\rangle$ where Q at $\mathbf{x}_1$, Q_c at $\mathbf{x}_2$; H_0 = all gauge interactions and no 1/mQ corrections included.
    - Translation invariance: depends on $R = \mathbf{x}_1 - \mathbf{x}_2$
    - Ground state for gluonic degrees of freedom: E_0
    - States with excited gluonic degrees of freedom (hybrids): E_n n=1,2,...
  - Wilson loop: W(R,T) = << 1 >> with $\mathrm{U}(\mathrm{x},\hat{\mathrm{n}}^\mu) = P\exp\left\{i\int_x^{x+a\hat{n}^\mu} A^\mu\cdot\hat{n}_\mu dx\right\}$ and

$$<< O_G >>= \int \Pi\mathrm{dU}\; O_G\; Tr\left[\Pi_{C(R,T)}\mathrm{U}(\mathrm{x},\hat{\mathrm{n}}^\mu)\right]\;\exp\left(-\mathrm{S}_{\mathrm{gauge}}\right)$$

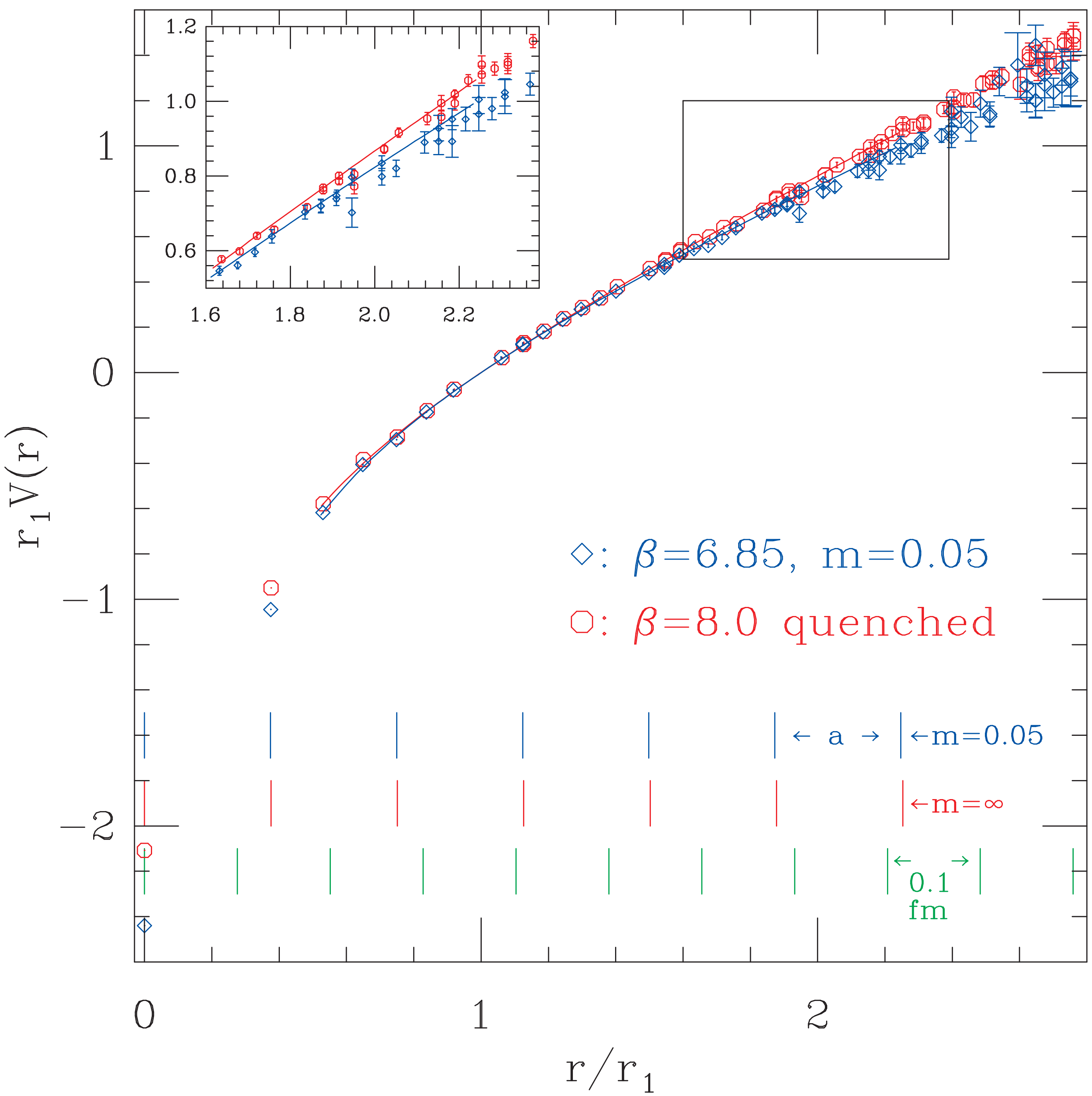
  - Euclidean space: $W(R,T) = \sum_n C_n\, e^{-E_n(R)T} \to C_0\, e^{-E_0(R)T}$ as $T \to \infty$

$$\mathcal{E}(R) = -\lim_{T\to\infty}\frac{1}{T}\log\left(W(R,T)\right)$$



Static Energy

- LQCD calculations with and without 3 light quark loops



C. W. Bernard *et al.*, Phys. Rev. D **62**, 034503 (2000) [arXiv:hep-lat/0002028]

Narrow States Below Threshold

- Basic to this NR picture is the adiabatic assumption
  - Heavy quarks react slowly relative to the interactions of the gauge degrees of freedom
  - This allows the velocity expansion in NRQCD and the $1/m_Q$ expansion in HQET

– So for the $(Q\, Q_c)$ system at rest:

$$\Psi(\mathbf{R}) = \frac{u_{nl}(r)}{r} Y_{lm}(\theta, \phi) \qquad\qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

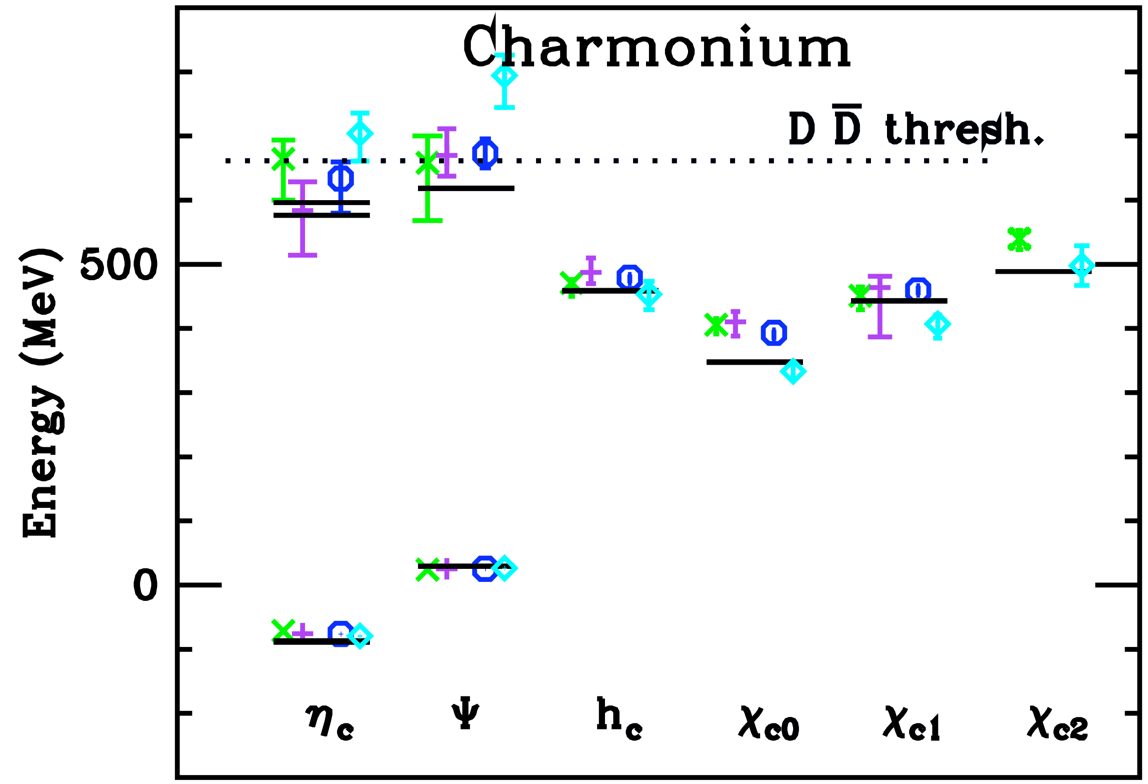
$$-\frac{1}{2\mu}\frac{d^2 u_{nl}(r)}{dr^2} + \left\{\frac{\mathbf{L}^2}{2\mu r^2} + V_{Q\bar{Q}}(r)\right\} u_{nl}(r) = E_{nl} u_{nl}(r) \qquad\qquad M = E_o + E_{nl}$$

– Phenomenological potential models:

- V(r) [Cornell, Richardson, Buchmueller-Tye, Godfrey-Isgur, ...]
- Many successful predictions

Narrow States Below Threshold

- Below threshold for heavy flavor meson pair production
  - Narrow states allow precise experimental probes of the subtle nature of QCD.
  - Lattice QCD supports and will supplant potential models
  - *A variety of lattice approaches*
    - Low-lying states directly calculated in Lattice QCD
    - Use Lattice calculations to determine non-perturbative potentials and continuum NRQCD to calculate masses and properties of states. ✓



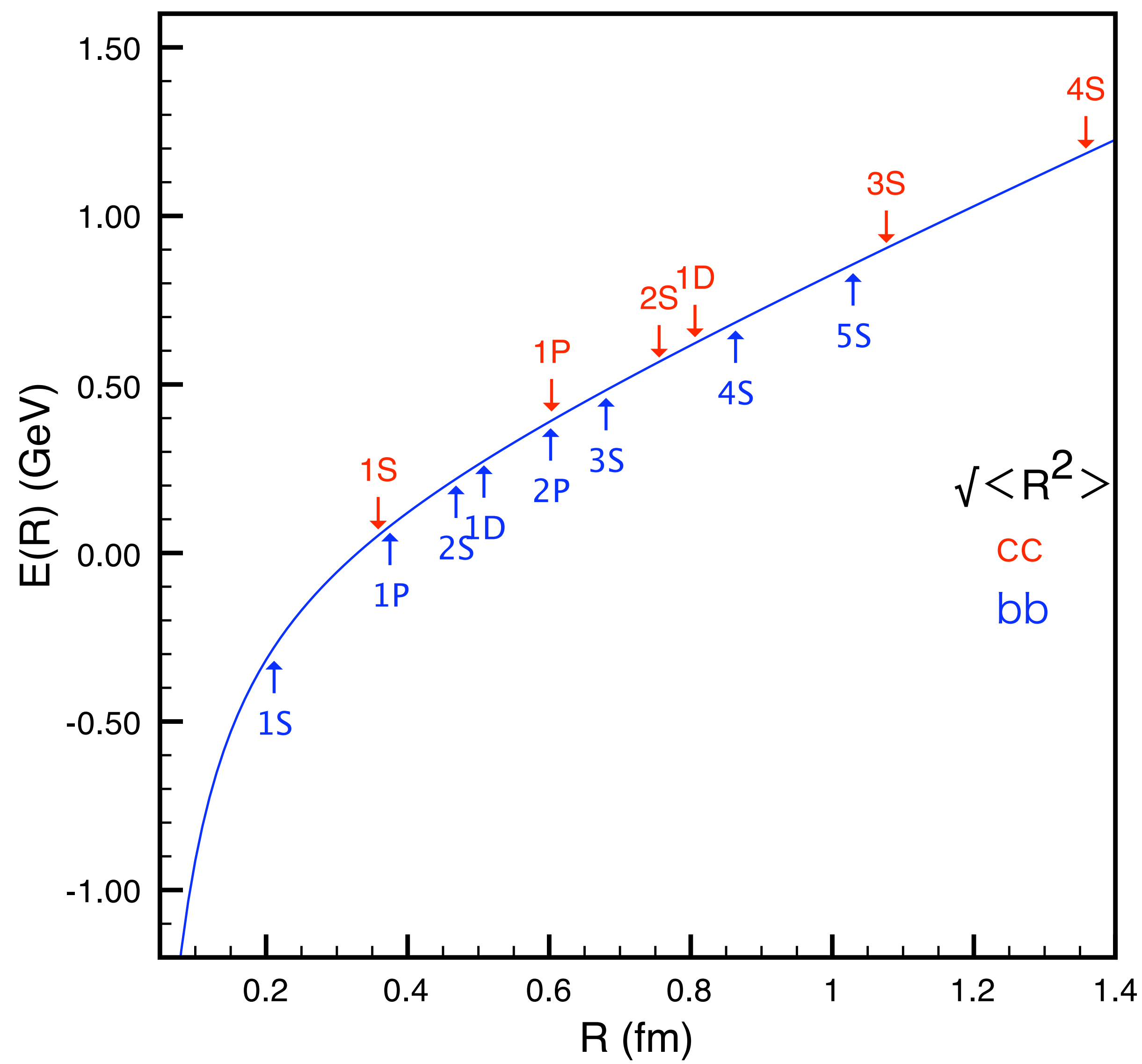
S. Gottlieb et al., PoS LAT2006

Figure 5: Summary of charmonium spectrum.

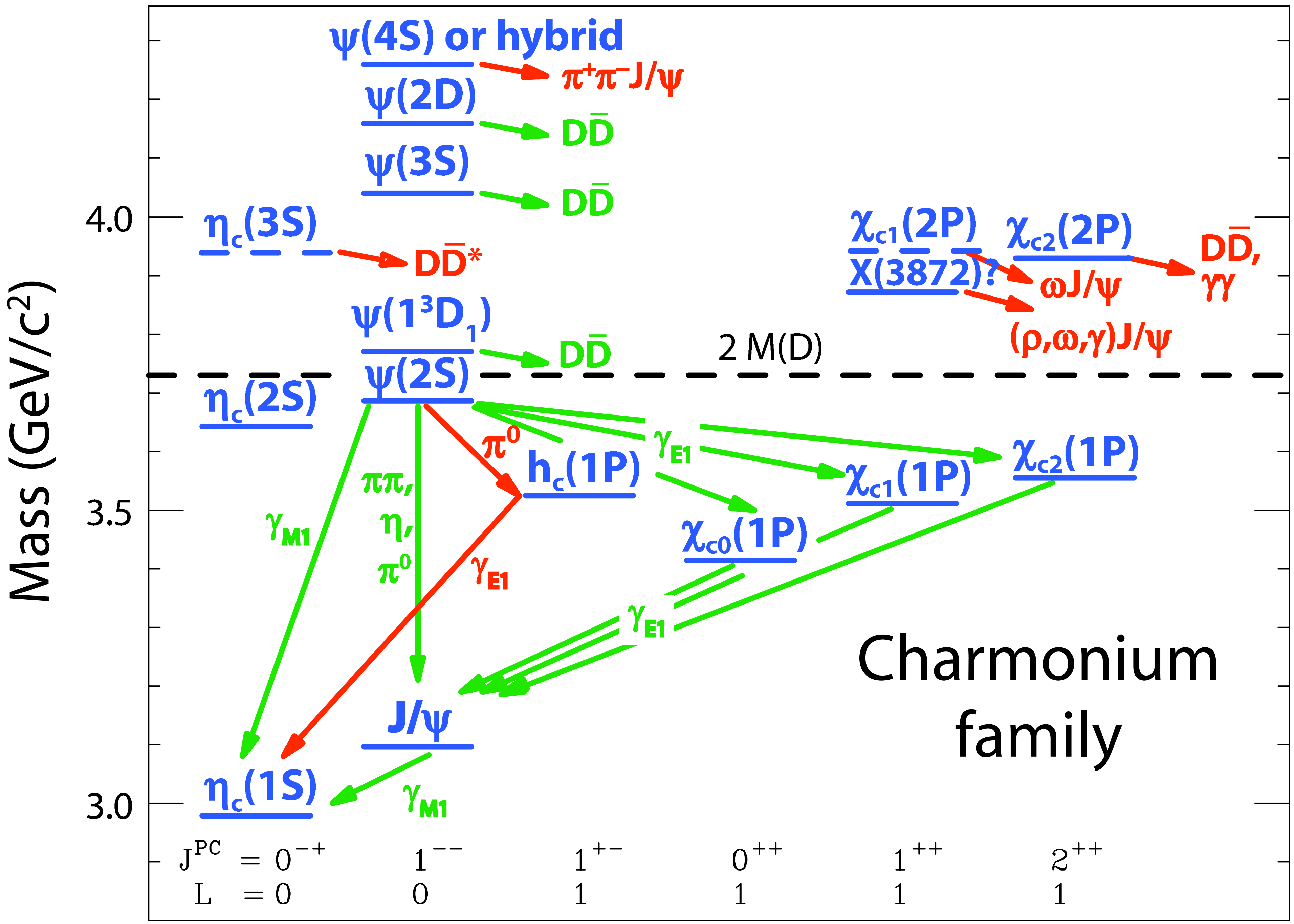
Narrow States Below Threshold

- Consistency between $(b\bar{b})$ and $(c\bar{c})$ systems validates NRQCD approach.
  - masses (pNRQCD, LQCD)
  - spin splittings (pNRQCD, LQCD)
  - EM transitions (ME, LQCD)
  - hadronic transitions (ME)
  - direct decays (pQCD)

Cornell Potential Model

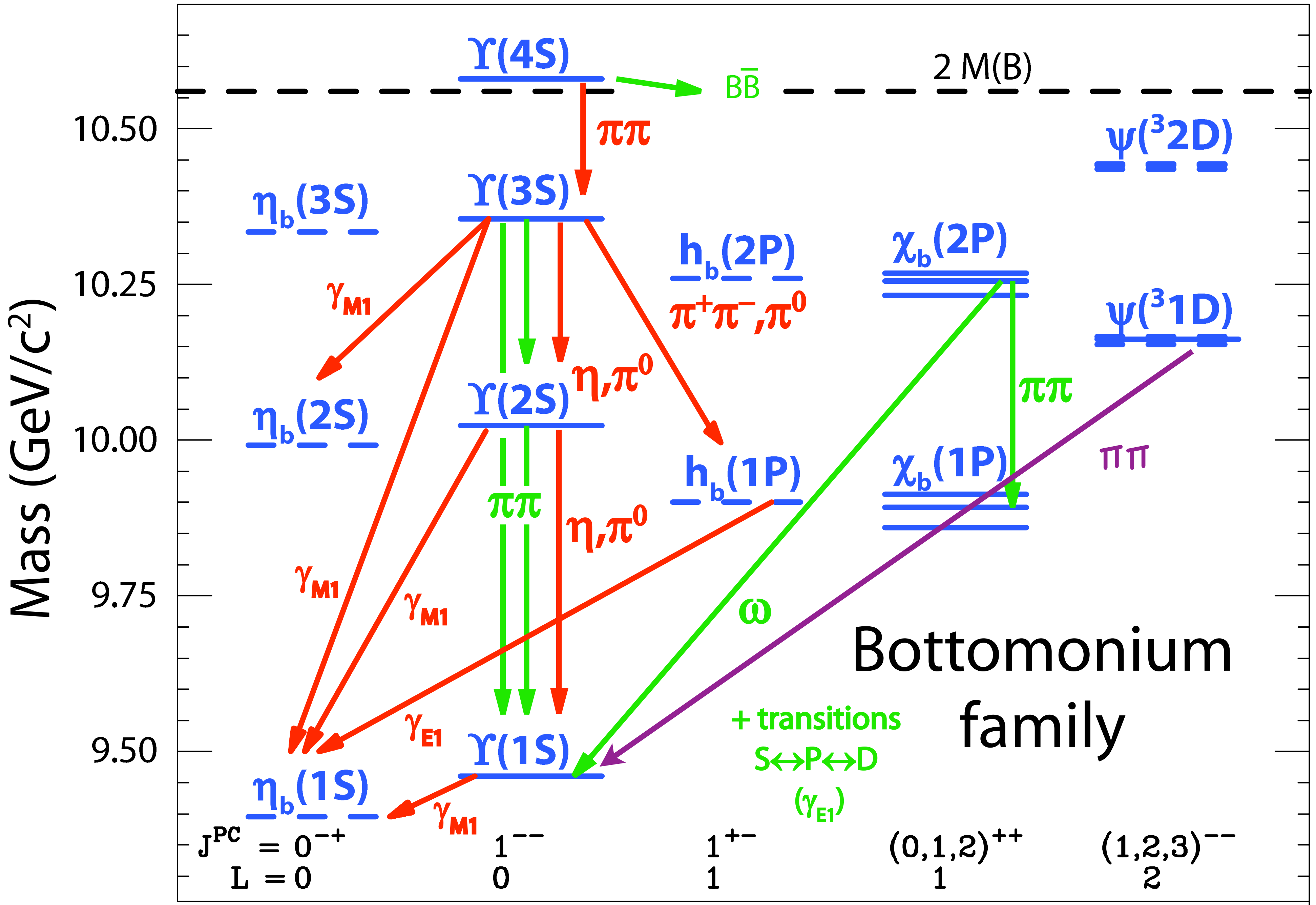


Narrow States Below Threshold



Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E. [Rev. Mod. Phys. 80, 1161 (2008)]

Narrow States Below Threshold



Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E. [Rev. Mod. Phys. 80, 1161 (2008)]

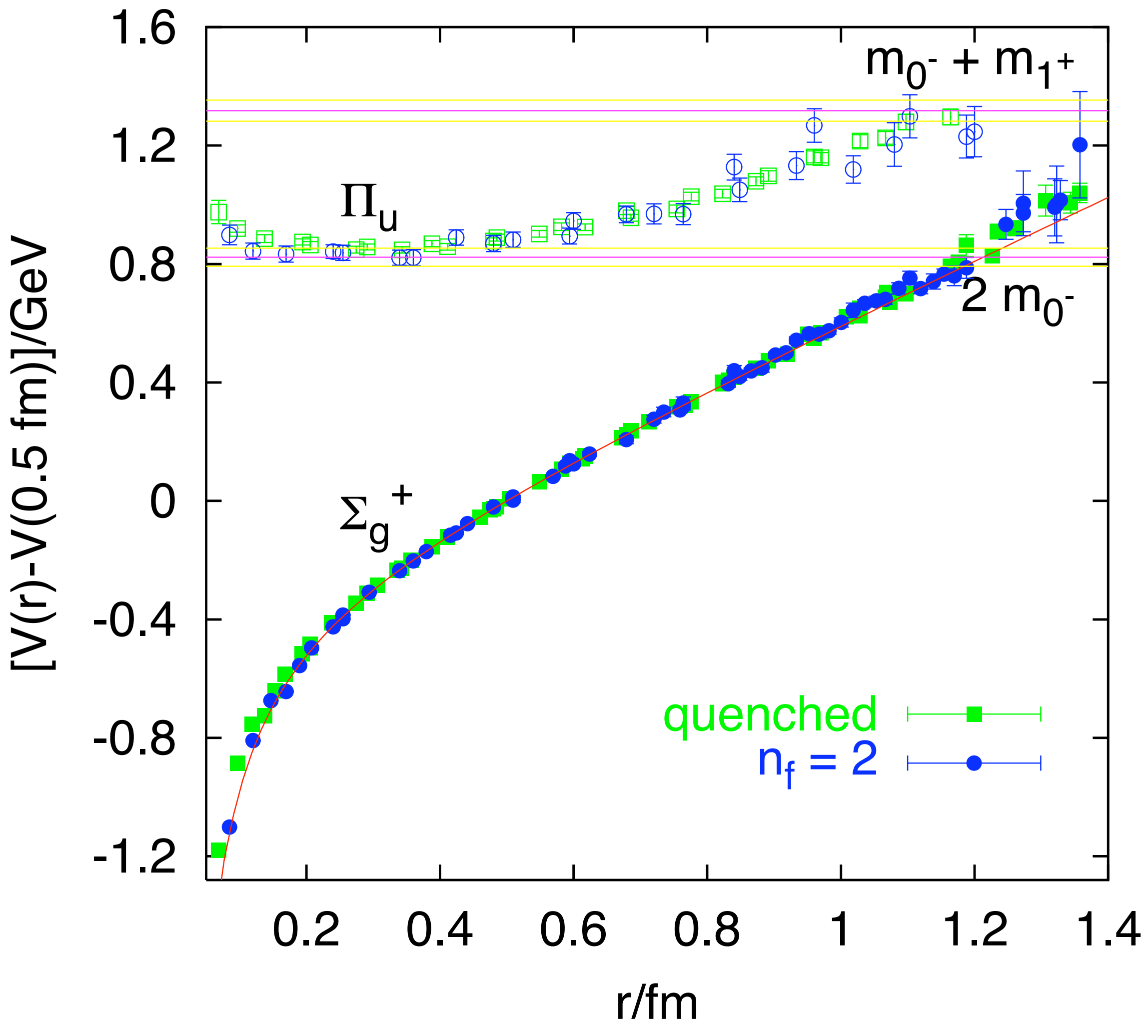
Why it works so well

- Lattice calculation V(r), then SE

$$-\frac{1}{2\mu}\frac{d^2u(r)}{dr^2}+\left\{\frac{\langle \boldsymbol{L}^2_{Q\bar{Q}}\rangle}{2\mu r^2}+V_{Q\bar{Q}}(r)\right\}u(r)=E\,u(r)$$

- What about the gluon and light quark degrees of freedom of QCD?
- Two thresholds:
  - Usual $(Q\bar{q})+(q\bar{Q})$ decay threshold
  - Excite the string - hybrids
- Hybrid states will appear in the spectrum associated with the potential Π_u, ...
- In the static limit this occurs at separation: r ≈ 1.2 fm. Between 3S-4S in $(c\bar{c})$; just above the 5S in $(b\bar{b})$.

LQCD calculation of static energy



Relativistic Corrections

- In order $1/m_Q^2$ masses have both spin dependent and spin independent corrections. First the spin dependent terms:

$$H_1(\mathbf{x_1}) = -c_f \frac{1}{2m_1}\mathbf{s_1}\cdot\mathbf{B} - \frac{\mathbf{D_1}^2}{2m_1} - c_s \frac{i}{8m_1^2}\sigma\cdot(\mathbf{D_1}\ \mathbf{x}\ \mathbf{E} + \mathbf{E}\ \mathbf{x}\ \mathbf{D_1}) + \ldots$$
$$H_2(\mathbf{x_2}) = -c_f \frac{1}{2m_2}\mathbf{s_2}\cdot\mathbf{B} - \frac{\mathbf{D_2}^2}{2m_2} - c_s \frac{i}{8m_2^2}\sigma\cdot(\mathbf{D_2}\ \mathbf{x}\ \mathbf{E} + \mathbf{E}\ \mathbf{x}\ \mathbf{D_2}) + \ldots$$

Magnetic Spin-Orbit

Q m1 t_a
Q_c m2 t_a^*

– D_1 term

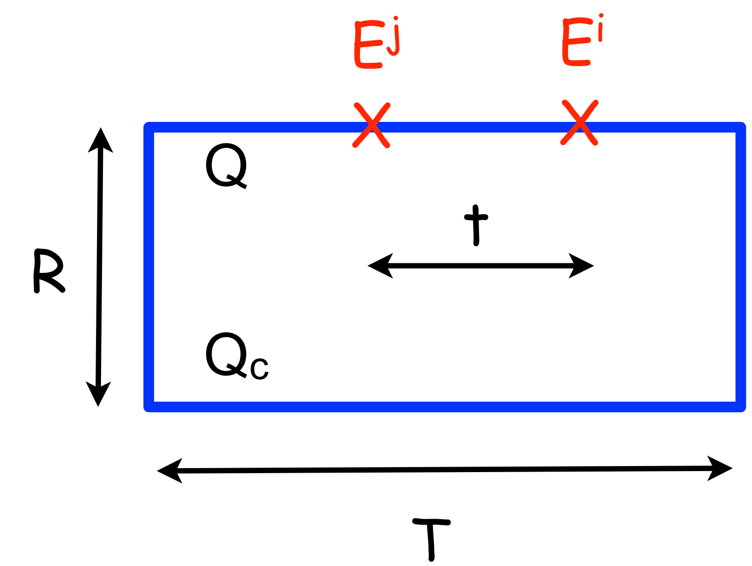
Denote the static state $|n, \mathbf{x_1}, \mathbf{x_2}>$ by $|n>$

$$\mathbf{D_1}|0> = \nabla_1|0> + i\sum_{n\neq 0}|n><n|\mathbf{A}|0>$$
$$= \nabla_1|0> + \sum_{n\neq 0}\frac{1}{(E_n - E_0)}|n><n|\mathbf{E}|0>$$

$$<0|\mathbf{D_1}^2|0> = <0|\nabla_1^2|0> + \sum_{n\neq 0}\frac{1}{(E_n - E_0)}<0|\mathbf{E_1^i}|n><n|\mathbf{E_1^i}|0>$$

- Define in terms of Wilson loop

$$V^{(1,0)}(r) = -\frac{1}{2}\lim_{T\to\infty}\int_0^T dt\, t\, \langle\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\rangle_c.$$



A. Pineda and A. Vairo, Phys. Rev. D **63**, 054007 (2001) [Erratum-ibid. D **64**, 039902 (2001)] [arXiv:hep-ph/0009145].

Relativistic Corrections

- $$V_{SD}^{(1,1)} = V_{L_1S_2}^{(1,1)}(r)\mathbf{L}_1\cdot\mathbf{S}_2 - V_{L_2S_1}^{(1,1)}(r)\mathbf{L}_2\cdot\mathbf{S}_1 + V_{S^2}^{(1,1)}(r)\mathbf{S}_1\cdot\mathbf{S}_2 + V_{\mathbf{S}_{12}}^{(1,1)}(r)\mathbf{S}_{12}(\hat{\mathbf{r}}),$$
  $$V_{SD}^{(2,0)} = V_{L_1,S_1}^{(2,0)}(r)\mathbf{L_1}\cdot\mathbf{S_1}$$

- Calculations in perturbation theory:

S. F. Radford and W. W. Repko, Phys. Rev. D **75**, 074031 (2007) [arXiv:hep-ph/0701117].

$$\begin{aligned}
V_{HF} &= \frac{32\pi\alpha_S\vec{S}_1\cdot\vec{S}_2}{9m^2}\left\{\left[1-\frac{\alpha_S}{12\pi}(26+9\ln 2)\right]\delta(\vec{r})\right. \\
&\left. -\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]+\frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln mr+\gamma_E}{r}\right]\right\} \\
V_{LS} &= \frac{2\alpha_S\vec{L}\cdot\vec{S}}{m^2r^3}\left\{1-\frac{\alpha_S}{6\pi}\left[\frac{11}{3}-(33-2n_f)\left(\ln\mu r+\gamma_E-1\right)+12\left(\ln mr+\gamma_E-1\right)\right]\right\} \\
V_T &= \frac{4\alpha_S(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)}{3m^2r^3}\left\{1+\frac{\alpha_S}{6\pi}\left[8+(33-2n_f)\left(\ln\mu r+\gamma_E-\frac{4}{3}\right)\right.\right. \\
&\left.\left. -18\left(\ln mr+\gamma_E-\frac{4}{3}\right)\right]\right\}
\end{aligned}$$

- General results:
  - The spin-spin terms

$$< 0|\mathbf{B_1^i B_2^j}|0 > \;=\; \sum_{n\neq 0}\frac{1}{(E_n-E_0)^2} < 0|\mathbf{B_1^i}|\mathbf{n} >< \mathbf{n}|\mathbf{B_2^j}|\mathbf{0} >$$

Relativistic Corrections

- Define in terms of Wilson loops

Hyperfine interactions

Spin-Spin

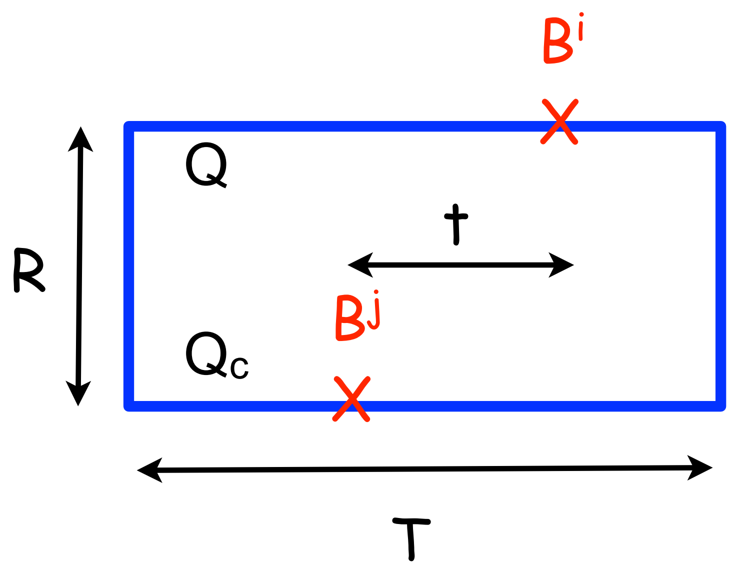
$$< 0|\mathbf{B_1^i B_2^j}|0 > \;=\; \sum_{n\neq 0} \frac{1}{(E_n - E_0)^2} < 0|\mathbf{B_1^i}|\mathbf{n} >< \mathbf{n}|\mathbf{B_2^j}|\mathbf{0} >$$

$$V_{S^2}^{(1,1)}(r) = \frac{2c_F^{(1)} c_F^{(2)}}{3} i \lim_{T\to\infty} \int_0^T dt \, \langle\langle g\mathbf{B}_1(t) \cdot g\mathbf{B}_2(0) \rangle\rangle$$

Tensor

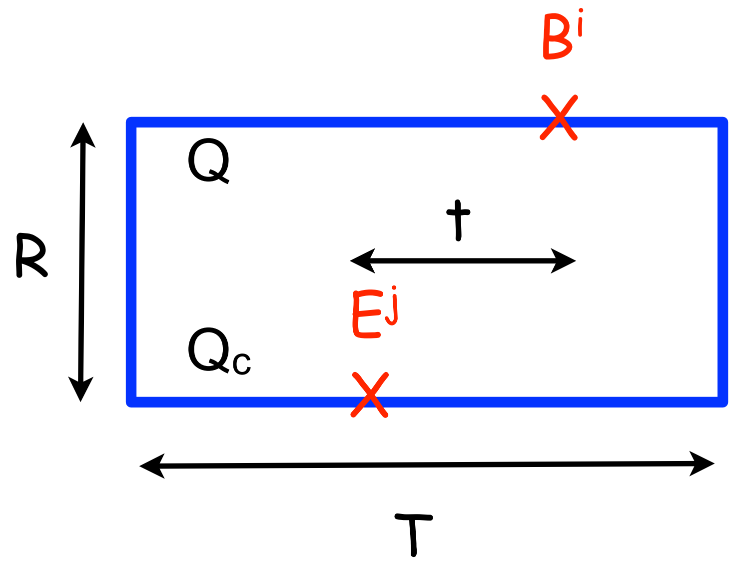
$$< 0|\mathbf{B_1^i B_2^j}|0 > \;=\; \sum_{n\neq 0} \frac{1}{(E_n - E_0)^2} < 0|\mathbf{B_1^i}|\mathbf{n} >< \mathbf{n}|\mathbf{B_2^j}|\mathbf{0} >$$

$$V_{\mathbf{S}_{12}}^{(1,1)}(r) = \frac{c_F^{(1)} c_F^{(2)}}{4} i\hat{\mathbf{r}}^i\hat{\mathbf{r}}^j \lim_{T\to\infty} \int_0^T dt \left[\langle\langle g\mathbf{B}_1^i(t) g\mathbf{B}_2^j(0) \rangle\rangle - \frac{\delta^{ij}}{3} \langle\langle g\mathbf{B}_1(t) \cdot g\mathbf{B}_2(0) \rangle\rangle \right]$$

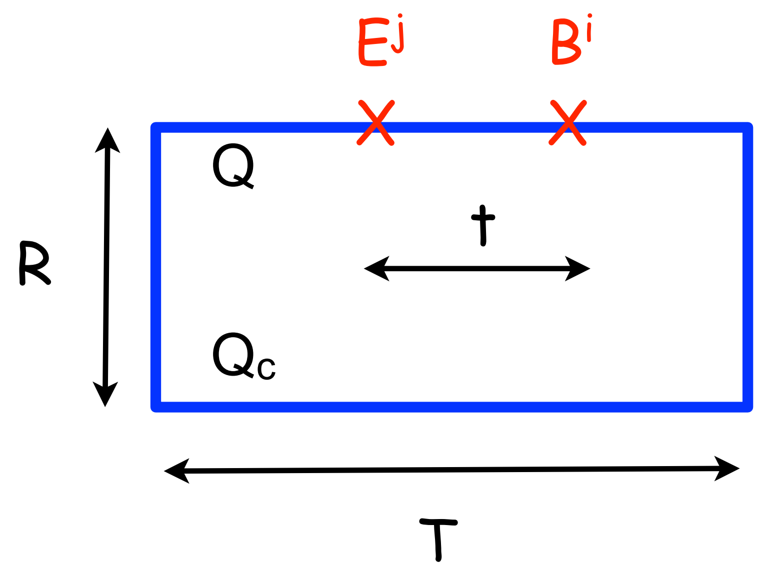


- The spin-orbit terms more complicated but same procedure

$$V^{(1,1)}_{L_2S_1}(r) = -\frac{c_F^{(1)}}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\langle g\mathbf{B}_1(t) \times g\mathbf{E}_2(0)\rangle\rangle \,,$$



$$V^{(2,0)}_{LS}(r) = -\frac{c_F^{(1)}}{r^2} i\mathbf{r} \cdot \lim_{T\to\infty} \int_0^T dt\, t\, \langle\langle g\mathbf{B}_1(t) \times g\mathbf{E}_1(0)\rangle\rangle + \frac{c_S^{(1)}}{2r^2}\mathbf{r} \cdot (\boldsymbol{\nabla}_r V^{(0)}).$$



- Spin independent terms - Analysis exists - many terms

A. Pineda and A. Vairo, Phys. Rev. D **63**, 054007 (2001) [Erratum-ibid. D **64**, 039902 (2001)] [arXiv:hep-ph/0009145].

Lattice QCD Calculation of Spin Dependent Terms

M. Koma, Y. Koma and H. Wittig, PoS **CONFINEMENT8**, 105 (2008).

V' = dV/dr

$$V_{SD}(r) = \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2} - \frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r} + 2\frac{V_1'(r)}{r}\right) + \left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1m_2} - \frac{\vec{s}_1\cdot\vec{l}_2}{2m_1m_2}\right)\frac{V_2'(r)}{r}$$

Long range component

$$+ \frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r) + \frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r),$$

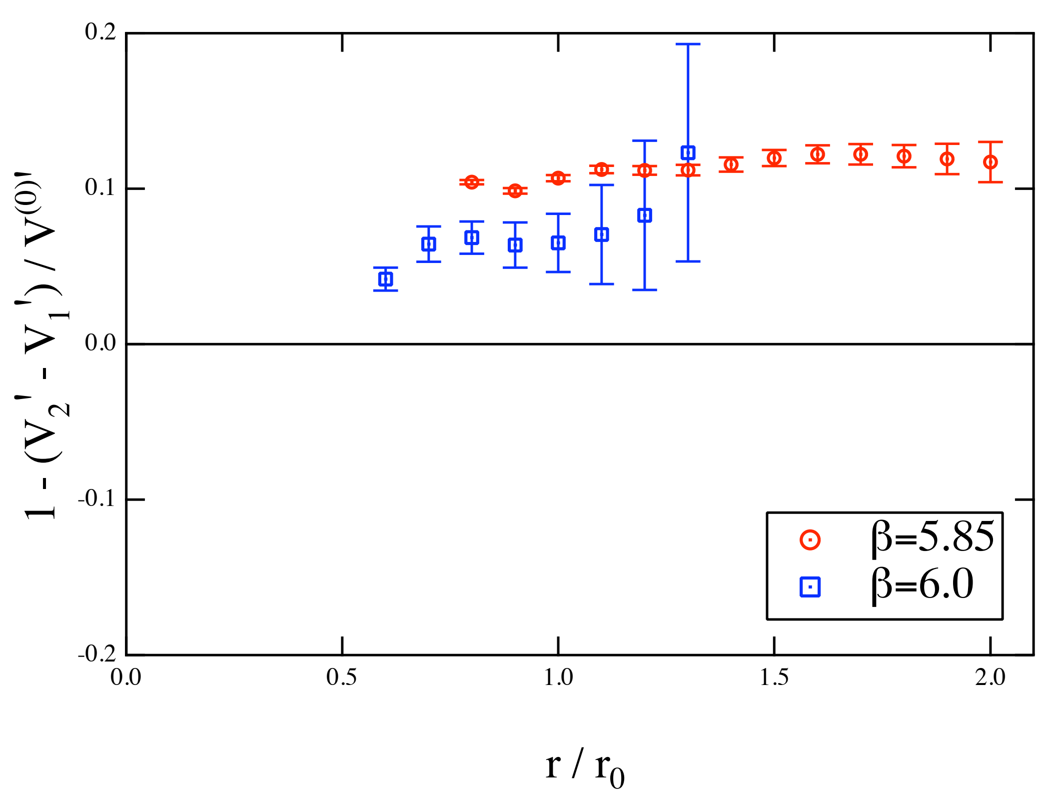
Short range

Gromes relation:

$$\frac{dV_2(r)}{dr} - \frac{dV_1(r)}{dr} = \frac{d\mathcal{E}(r)}{dr}$$

Follows from Lorentz invariance under infinitesimal boosts

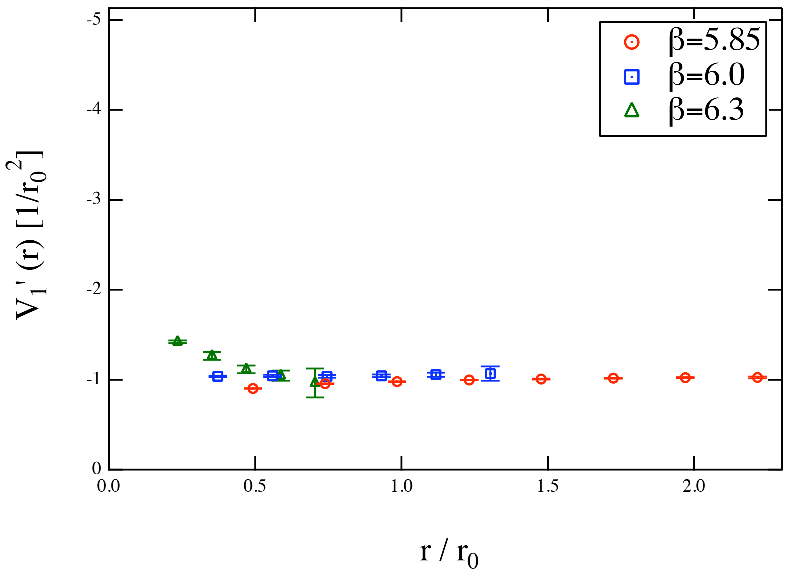
D. Gromes, Z. Phys. C 22, 265 (1984)



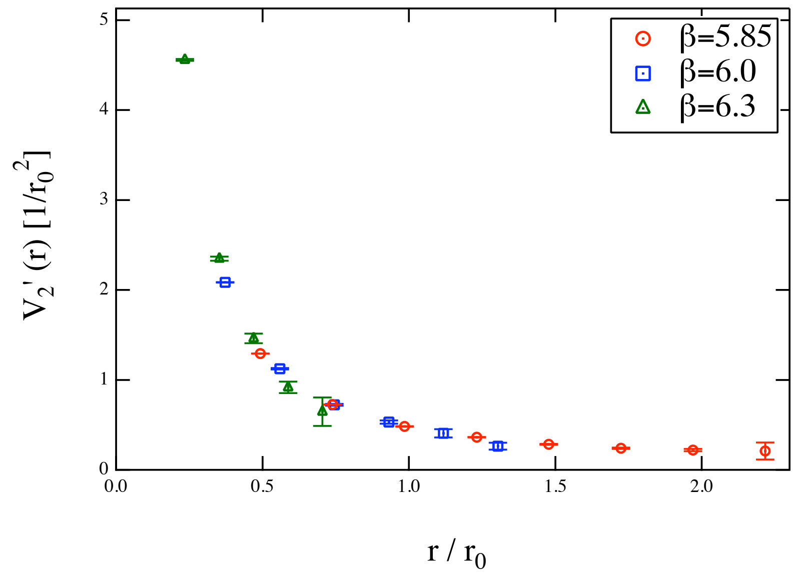
Lattice QCD Calculation of Spin Dependent Terms

M. Koma, Y. Koma and H. Wittig, PoS **CONFINEMENT8**, 105 (2008).

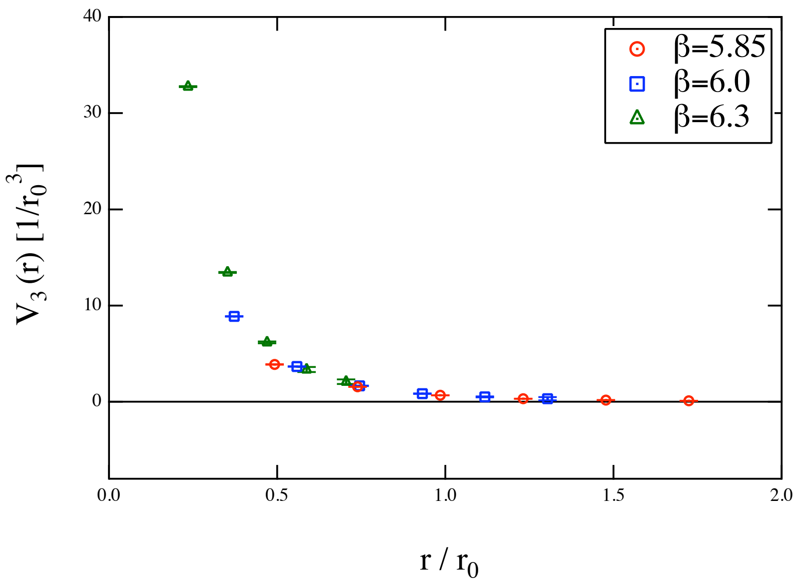
dV₁/dr



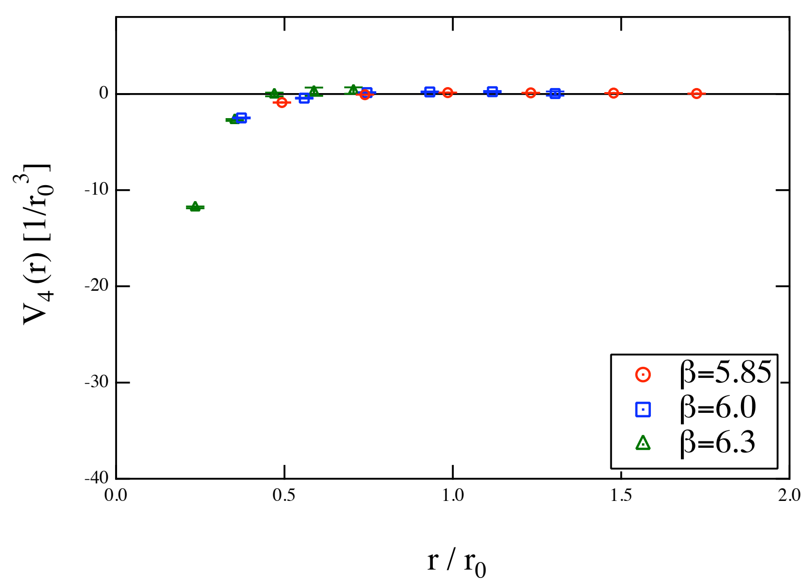
dV₂/dr



V₃



V₄



- Calculated from r = 0.25 to 1.2 fm
- The spin-spin (V_3) and tensor (V_4) potentials are indeed short range
- The spin-orbit potential V'_2 shows a long range piece

Open issues in Spectroscopy

- h_c ($^1P_1$) spin singlet charmonium state
  - Observation: CLEOc, BESIII

$$M(h_c) = 3525.28 \pm 0.19 \pm 0.12 \text{ MeV} \quad \text{CLEOc}$$

$$M(h_c) = 3525.40 \pm 0.13 \pm 0.18 \text{ MeV} \quad \text{BESIII}$$

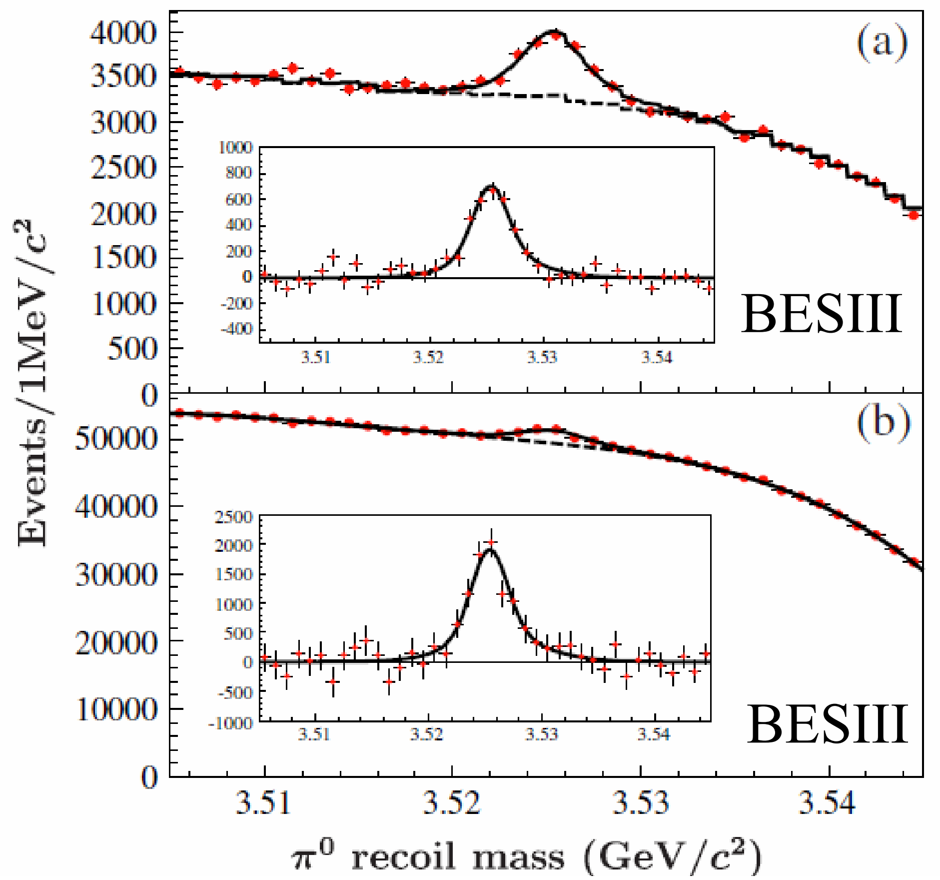
$$\Gamma(h_c) = 0.73 \pm 0.45 \pm 0.28 \text{ MeV}$$

$$\mathcal{B}(\psi(2S) \to \pi^0 h_c) = [8.4 \pm 1.3 \pm 1.0] \times 10^{-4} \quad \text{BESIII}$$

$$\mathcal{B}(h_c) \to \gamma\eta_c) = [54.3 \pm 6.7 \pm 5.2]\%$$

$$e^+e^- \to \psi(2S) \to \pi^0 h_c, \quad h_c \to \gamma\eta_c, \quad \pi^0 \to \gamma\gamma.$$

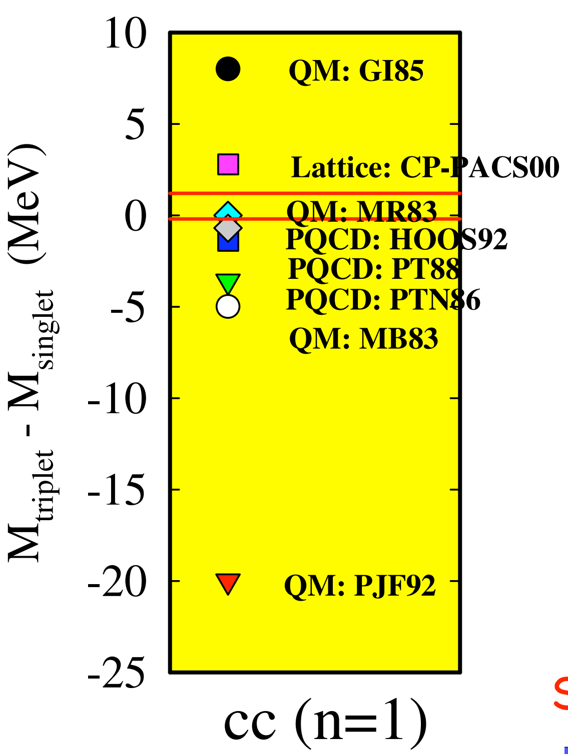
BES III: PRL 104, 132002 (2010)



  - Partial widths and decay modes agree with expectations:

    J. L. Rosner et al., PRL 95, 102003 (2005)

  - Spin -dependent forces:



$$\Delta M_{hf}(< M(^3P_J) > - M(^1P_1)) = -0.4 \pm 0.25 \text{ MeV}$$

Confirms the short range nature of spin-spin and tensor potentials. Phenomenological models which closely follow pert QCD are best.

S. Godfrey [hep-ph/0501083]

- $(1^3D_J)_b$ spin-triplet bottomonium state
  - Observation: $(1^3D_J)_b \to \pi\pi\,\Upsilon$ (BABAR)

$$M(^3D_3) = 10172.9 \pm 1.7 \pm 0.5 \text{ MeV}$$
$$M(^3D_2) = 10161.1 \pm 0.6 \pm 1.6 \text{ MeV} \quad *$$
$$M(^3D_1) = 10151.6 \pm 1.4 \pm 0.5 \text{ MeV}$$

    - Useful to check spin-splittings in charmonium system

P. del Amo Sanchez et al. (BABAR Collaboration) arXiv:1004.0175 [hep-ex]

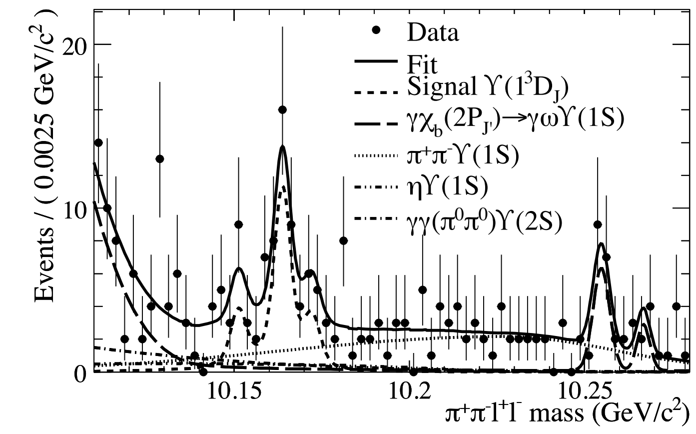
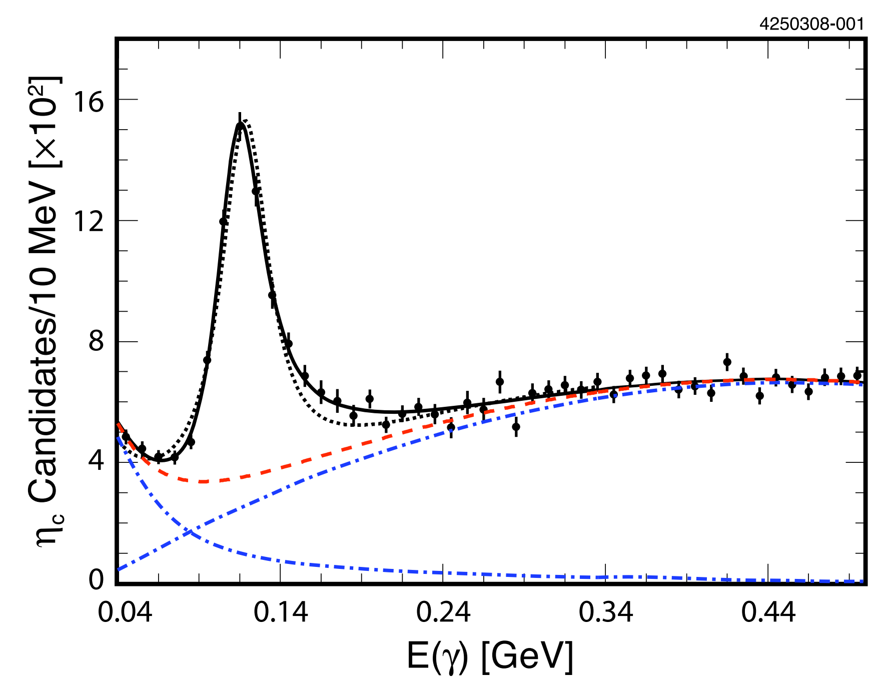


FIG. 1: The $\pi^+\pi^-\ell^+\ell^-$ mass spectrum and fit results. The two peaks near 10.25 GeV/c^2 arise from $\chi_{bJ'}(2P) \to \omega\Upsilon(1S)$ background events with $\omega \to \pi^+\pi^-$.

- η_c (1^1S_0) spin-singlet charmonium state
  - Mass splitting (CLEO)

$$M(\eta_c) = 2976.7 \pm 0.6 \text{ MeV}/c^2 \quad \text{Breit} - \text{Wigner}$$
$$= 2982.2 \pm 0.6 \text{ MeV}/c^2 \quad \text{Modified Breit} - \text{Wigner}$$

long tail



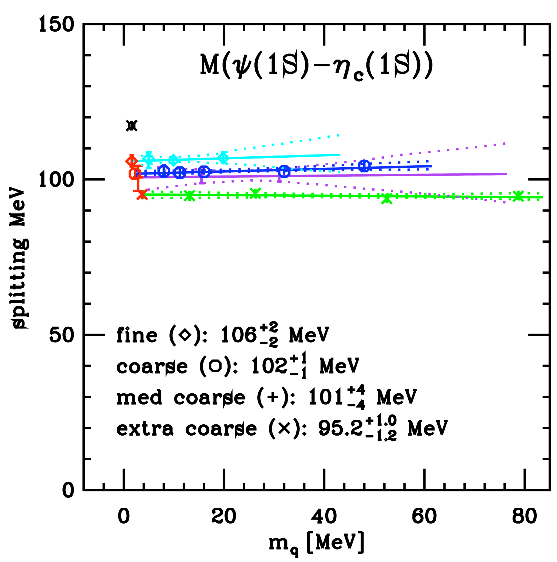
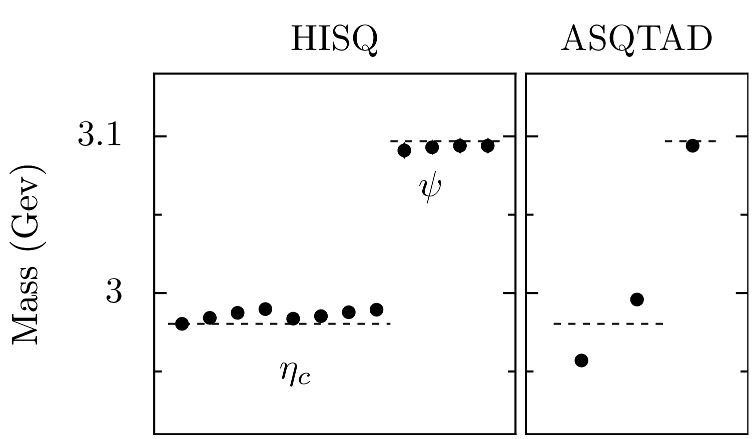


Figure 4: Hyperfine splitting of the 1S states.



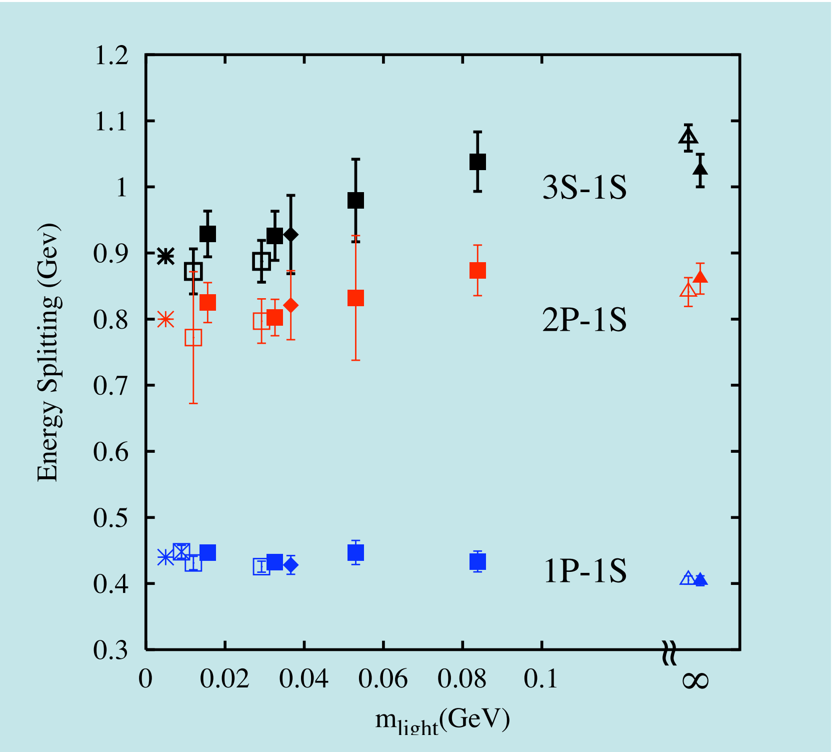
E. Follana et al.,[HPQCD] PR D 75, 054502 (2006)

S. Gottlieb et al., PoS LAT2006

- η'_c (2^1S_0) spin-singlet charmonium state
  - Spin splitting
  - Effects of light quark loops significant

$$\Delta M \;=\; M(\psi(2S)) - M(\eta'_c) = 49 \pm 4 \text{ MeV}/c^2 \quad \text{PDG 2008}$$

Too small - scaling from 1S; most models.
Are we seeing threshold effects?



Effects on spectrum seen in LQCD

C. T. H. Davies et al. [HPQCD, Fermilab Lattice, MILC, and UKQCD Collaborations], PRL 92, 022001 (2004)

- Strong coupling to virtual decay channels induces spin-dependent forces in charmonium near threshold, because M(D*) > M(D)
- Spin dependent shifts small far below threshold

Less that 1 MeV shift ⇒

Reduces ΔM(2S) by 21 MeV ⇒

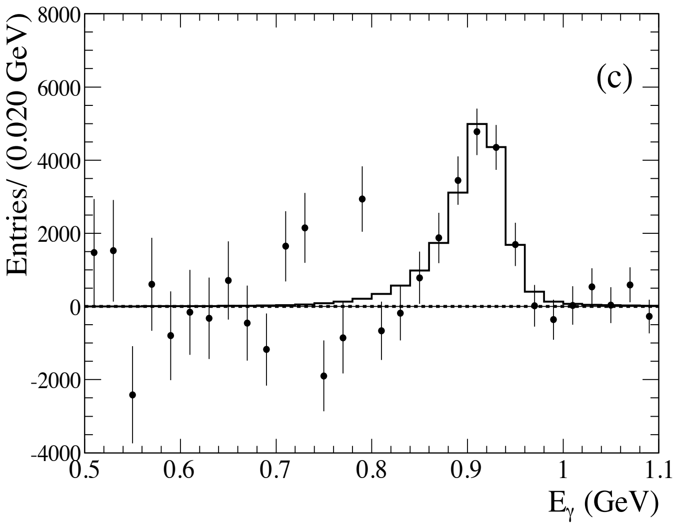
State	Mass	Centroid	Splitting (Potential)	Splitting (Induced)
1^1S_0	$2\,979.9^a$	$3\,067.6^b$	-90.5^e	$+2.8$
1^3S_1	$3\,096.9^a$		$+30.2^e$	-0.9
1^3P_0	$3\,415.3^a$	$3\,525.3^c$	-114.9^e	$+5.9$
1^3P_1	$3\,510.5^a$		-11.6^e	-2.0
1^1P_1	$3\,524.4^f$		$+0.6^e$	$+0.5$
1^3P_2	$3\,556.2^a$		$+31.9^e$	-0.3
2^1S_0	$3\,638^a$	$3\,674^b$	-50.1^e	$+15.7$
2^3S_1	$3\,686.0^a$		$+16.7^e$	-5.2
1^3D_1	$3\,769.9^a$	$(3\,815)^d$	-40	-39.9
1^3D_2	$3\,830.6$		0	-2.7
1^1D_2	$3\,838.0$		0	$+4.2$
1^3D_3	$3\,868.3$		$+20$	$+19.0$
2^3P_0	$3\,881.4$	$(3\,922)^d$	-90	$+27.9$
2^3P_1	$3\,920.5$		-8	$+6.7$
2^1P_1	$3\,919.0$		0	-5.4
2^3P_2	$3\,931^g$		$+25$	-9.6
3^1S_0	$3\,943^h$	$(4\,015)^i$	-66^e	-3.1
3^3S_1	$4\,040^a$		$+22^e$	$+1.0$

ELQ PRD 73:014014 (2006)

- η_b (1^1S_0) spin-singlet bottomonium state
  - Observed by BaBar in Υ(3S) radiative decays

$$E_\gamma = 921.2\ ^{+2.1}_{-2.8} \pm 2.4$$
$$M(\eta_b) = 9388.9\ ^{+3.1}_{-2.3} \pm 2.7 \text{ MeV}$$

BaBar [PRL 101, 071801



  - Hyperfine splitting:

$$M(\Upsilon(1S)) - M(\eta_b) = 71.4\ ^{+2.3}_{-3.1} \pm 2.7 \text{ MeV}$$

$$\text{Naive}: \frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)} \frac{4\Gamma_{e^+e^-}(\Upsilon)}{\Gamma_{e^+e^-}(J/\Psi)} [M(J/\Psi) - M(\eta_c)] \approx 68 \text{ (MeV)}$$

$$\text{QCD NNL}: 39 \pm 11\ ^{+\ 9}_{-\ 8} \text{ (MeV)} \quad [\text{PRL 92 242001 (2004)}]$$

$$\text{LQCD}: 61 \pm 14 \text{ (MeV)} \quad [\text{PR D72}: 094507 \text{ (2005)}]$$

  - Hindered M1 Transitions:
    - Relativistic corrections poorly understood. Phenomenological models for Υ(3S) -> $\gamma\eta_b$ and Υ(2S) -> $\gamma\eta_b$ varied greatly.

- Narrow states still missing
  - Charmonium - 3 - 1D2, 3D2, and 3D3
  - Bottomonium - 21 - 13FJ, 23DJ, 13GJ, 33PJ, 11P1, 21S0, 11D2, 21P1, 31S0, 11F3, 21D2, 11G4, 31P1
- The wealth of precision data has solidified our confidence in the NRQCD approach for narrow states below threshold.
- In lecture 2 we will look into the more detailed properties of the states probed by radiative and hadronic transitions