Lectures on NRQCD Factorization for Quarkonium Production and Decay

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I. Nonrelativistic QCD
II. Annihilation decays
III. Inclusive hard production

NRQCD Factorization for Quarkonium Production and Decay

I. NonRelativistic QCD

- Introduction
- Quarkonium
- Separation of scales
- NonRelativistic QCD

II. Annihilation decays

III. Inclusive hard production

Introduction

Heavy quark: $Q = c$ or b (but not t)

mass: $M = m_c$ or m_b

$m_c \approx 1.5$ GeV

$m_b \approx 4.5$ GeV

Quarkonium

hadron containing heavy quark Q and heavy antiquark $\bar{Q}$

charmonium: $c\,\bar{c}$

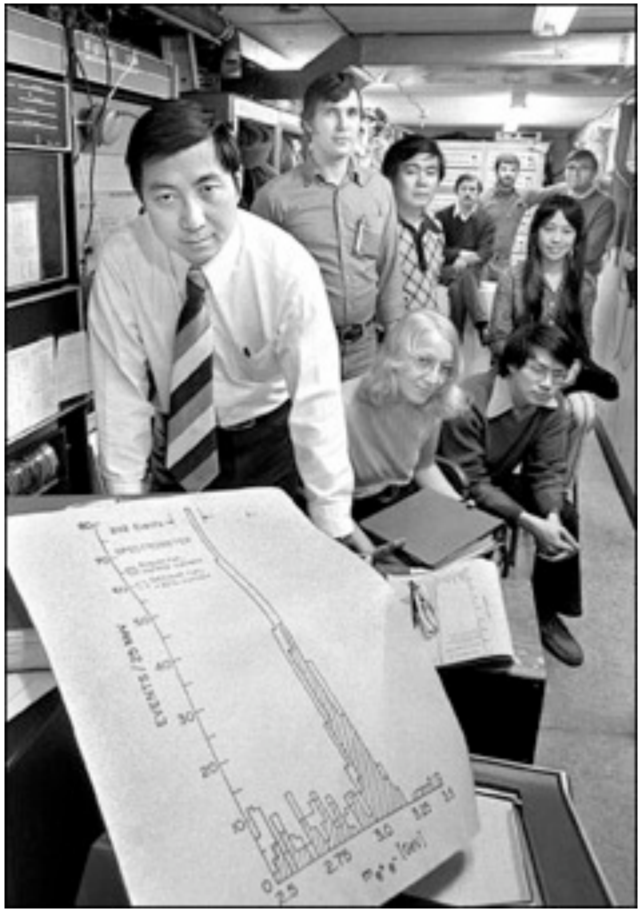
bottomonium: $b\,\bar{b}$

B_c meson: $b\,\bar{c}$

Discovery of Charmonium

November 1974

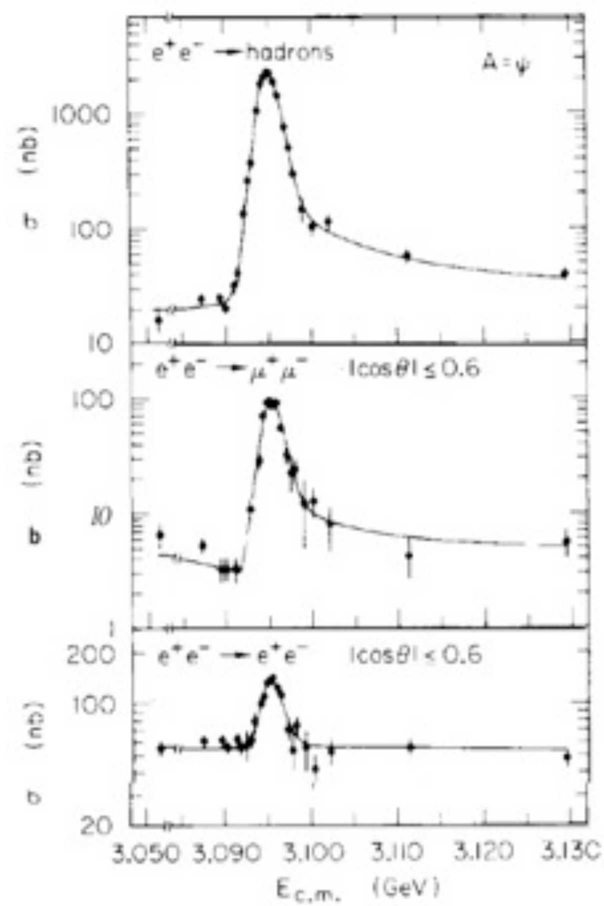
$\bar{p}$ on Be target at Brookhaven



$\bar{p}\,N \rightarrow J/\psi + X,$

$J/\psi \rightarrow e^+e^-$

e^+e^- annihilation at SLAC



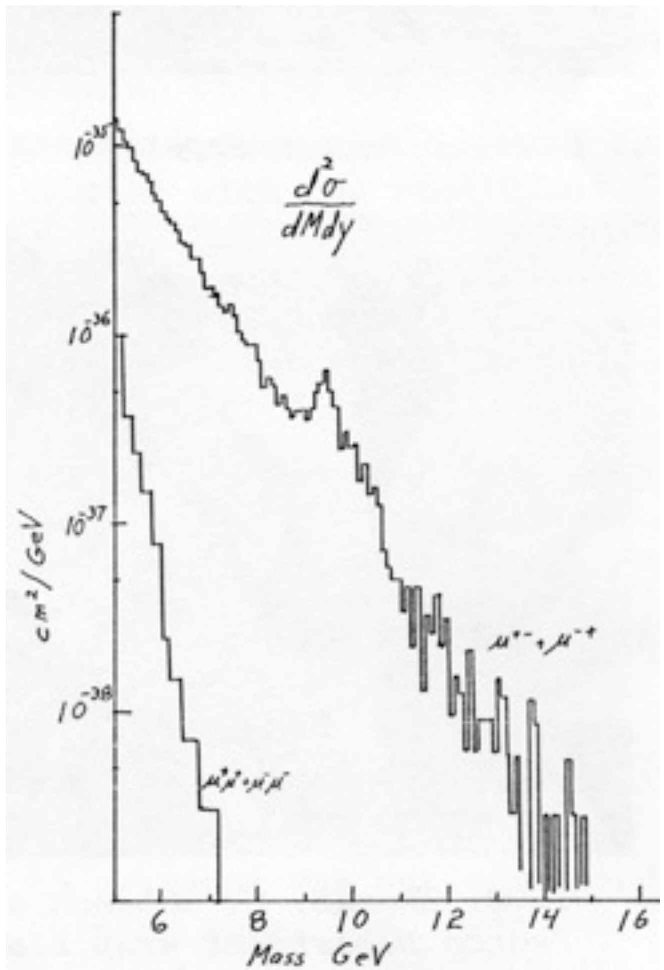
$e^+e^- \rightarrow J/\psi,$

$J/\psi \rightarrow$ hadrons

Discovery of Bottomonium

August 1977

p on Be target at Fermilab



$p\,N \rightarrow \Upsilon + X,\ \Upsilon \rightarrow \mu^+\mu^-$

Introduction

Nonrelativistic QCD

effective field theory for $Q\bar{Q}$ sector of QCD

Factorization: separation of scales

NRQCD Factorization

separate hard momenta $\gtrsim M$

from soft momenta $<< M$

applications to quarkonium:

annihilation decays

inclusive hard production

Quarkonium

Quarkonium is nonrelativistic bound state

v: typical relative velocity of heavy quark in CM frame

charmonium: $v^2 \approx 0.3$
bottomonium: $v^2 \approx 0.1$

can exploit $v^2 << 1$

Quarkonium

Quarkonium is multi-scale system

important momentum scales

- M : heavy quark mass
- Mv : typical momentum of heavy quark
- Mv^2 : typical kinetic energy
  or potential energy of heavy quark
- Λ_{QCD} : momentum scale below which
  gluons are strongly interacting

idealized hierarchy of momentum scales ($M \to \infty$)

$$\Lambda_{QCD} \ll Mv^2 \ll Mv \ll M$$

<-------- "soft" --------> "hard"

Quarkonium is multi-scale system

mass splittings between radial excitations $\sim Mv^2$

$\psi(2S)$ -- J/ψ : 589 MeV

$\Upsilon(2S)$ -- $\Upsilon(1S)$: 563 MeV

$\Rightarrow$ $Mv^2 \approx$ 500 MeV for both charmonium and bottomonium

realistic hierarchy of momentum scales

$$\Lambda_{QCD} \lesssim Mv^2 << M$$

QCD interactions

hard momenta $\sim M$

$\alpha_s(m_c) \approx 0.25$

$\alpha_s(m_b) \approx 0.18$

can use perturbative QCD

soft momenta $\lesssim Mv$

$\alpha_s(Mv) \approx v$

must sum interactions to all orders

$\alpha_s(\Lambda_{QCD}) >> 1$

must use Monte Carlo methods

Separation of scales

Quantum field theory with hierarchy of scales

$$K_{soft} << K_{hard}$$

Strategies for separating scales

1. Partial Integration

2. Effective Field Theory

Separation of scales

Hierarchy of scales: $K_{soft} << K_{hard}$

<u>Partial Integration strategy</u>

- introduce separation scale: $K_{soft} << \Lambda < K_{hard}$
- integrate out hard momenta $p > \Lambda$
  to get effective lagrangian for soft momenta $p < \Lambda$
  with ultraviolet cutoff Λ

conceptually simple, but difficult in practice

Separation of scales

Hierarchy of scales: K_{soft} << K_{hard}

Effective Field Theory strategy

- identify soft degrees of freedom and introduce corresponding fields
- construct most general effective lagrangian L_{eff} for soft fields with no hard scale K_{hard}
  ultraviolet cutoff Λ >> K_{soft}
- identify relative importance of terms in L_{eff} and truncate to desired accuracy
- determine parameters of L_{eff} by matching to original theory at soft momenta

QCD with heavy quark

degrees of freedom: gluons
light quarks, antiquarks
heavy quark, antiquark

Lagrangian:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm glue} + \mathcal{L}_{\rm light} + \bar{\Psi}(i\gamma^\mu D_\mu - M)\Psi$$

Ψ: 4-component Dirac spinor
for heavy quark, antiquark

parameters: α_s, $m_q \approx 0$, M

QCD symmetries

SU(3) gauge symmetry

Poincare symmetry: space-time translation invariance
Lorentz invariance

discrete symmetries: charge conjugation *C*
parity *P*

flavor conservation: heavy quark flavor
light quark flavors

Separation of scales

$Q\bar{Q}$ sector of QCD near theshold $2M$

idealized hierarchy of momentum scales

$$\Lambda_{QCD} \ll Mv^2 \ll Mv \ll M$$

preferred Lorentz frame: CM frame of $Q\bar{Q}$

Strategies for separating hard scale M
from soft scales Λ_{QCD}, Mv^2, Mv

1. Partial Integration

2. Effective Field Theory

Separation of scales

$Q\bar{Q}$ sector of QCD near threshold $2M$

Partial Integration strategy

- introduce separation scale Λ:

$$\Lambda_{QCD},\ Mv^2,\ Mv \ll \Lambda < M$$

- integrate out hard momenta, energies:

gluons, light quarks: $p > \Lambda,\ E > \Lambda$

heavy quarks: $p > \Lambda,\ |E - M| > \Lambda^2/M$

- eliminate energy scale M by

$$\Psi(\vec{r},t) \longrightarrow e^{-iMt}\ \Psi(\vec{r},t)$$

Separation of scales

<u>Partial Integration</u> strategy (cont.)

- effective Lagrangian*

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_{\text{glue}} + \mathcal{L}_{\text{light}} \\ &+ \bar{\Psi}[i\gamma^0 D_0 - ic_0 \vec{\gamma} \cdot \vec{D}]\Psi \\ &+ \frac{1}{M}\bar{\Psi}\left[c_1 D_0^2 - c_2 \vec{D}^2 + c_3 g \vec{B} \cdot \vec{\gamma}\gamma_5 + c_4 g \vec{E} \cdot [\gamma_0, \vec{\gamma}]\right]\Psi \\ &+ \frac{1}{M^2}(\text{dimension-6 terms}) + \ldots\end{aligned}$$

comparable in importance: $Mv^2 \sim (Mv)^2/M$

- relative importance of terms?

* if cutoff respects SU(3) gauge symmetry

Separation of scales

Partial Integration strategy (cont.)

- 4-component Dirac field for heavy quark can be replaced by 2-component Pauli fields:

$$\Psi(\vec{r},t) = \begin{pmatrix} \psi(\vec{r},t) \\ \chi(\vec{r},t) \end{pmatrix}$$

- Foldy-Wouthuysen transformation can be used to eliminate cross terms:

~~$\psi^\dagger(...)\chi$~~ ~~$\chi^\dagger(...)\psi$~~

$\Longrightarrow$ separate conservation of heavy quark number
heavy antiquark number

NonRelativistic QCD

$Q\bar{Q}$ sector of QCD near threshold $2M$

idealized hierarchy of momentum scales

$$\Lambda_{QCD} \ll Mv^2 \ll Mv \ll M$$

separate hard scale M

from soft scales Λ_{QCD}, Mv^2, Mv

Effective Field Theory strategy

NonRelativistic QCD! Caswell and Lepage 1986

organizing principle:

scaling with typical velocity v of heavy quark

NonRelativistic QCD

Effective Field Theory strategy (cont.)

- identify soft degrees of freedom

  gluons

  light quarks, antiquarks

  nonrelativistic heavy quark
    annihilated by Pauli spinor field ψ

  nonrelativistic heavy antiquark
    created by Pauli spinor field χ

NonRelativistic QCD

Effective Field Theory strategy (cont.)

- construct most general effective lagrangian L_{eff}
  for soft fields with no hard scale M

  ultraviolet cutoff Λ >> Mv

Symmetries

SU(3) gauge symmetry
space-time translations
rotations invariance
discrete symmetries: C, P
light quark flavors
heavy quark number (= one)
heavy antiquark number (= one)

NonRelativistic QCD

Effective Field Theory strategy (cont.)

- identify relative importance of terms in L_{eff}
  and truncate to desired accuracy

relative importance of operator O
is determined by expectation value $\langle O \rangle$
in quarkonium state

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_2 + \ldots$$

L_n consists of operators with scaling behavior

$$\langle O \rangle \sim v^{5+n}$$

Effective Field Theory strategy (cont.)

- truncate to desired accuracy in powers of v

Leading Order in v^2

$$\mathcal{L}_0 = \mathcal{L}_{\rm glue} + \mathcal{L}_{\rm light} + \psi^\dagger(iD_0 + \vec{D}^2/2M)\psi + \chi^\dagger(iD_0 - \vec{D}^2/2M)\chi$$

only parameters are M and α_s

respects heavy-quark spin symmetry
$\Longrightarrow$ spin-flip amplitudes suppressed by v^2

NonRelativistic QCD

Effective Field Theory strategy (cont.)

- truncate to desired accuracy in powers of v

Next-to-Leading Order in v^2

$$\begin{aligned}\mathcal{L}_2 \;=\;& \frac{c_1}{8M^3}\left[\psi^\dagger(\vec{D}^2)^2\psi - \chi^\dagger(\vec{D}^2)^2\chi\right]\\ &+\frac{c_2}{8M^2}\left[\psi^\dagger(\vec{D}\cdot g\vec{E} - g\vec{E}\cdot\vec{D})\psi + \chi^\dagger(\vec{D}\cdot g\vec{E} - g\vec{E}\cdot\vec{D})\chi\right]\\ &+\frac{c_3}{8M^2}\left[\psi^\dagger(i\vec{D}\times g\vec{E} - g\vec{E}\times i\vec{D})\cdot\vec{\sigma}\psi + \chi^\dagger(i\vec{D}\times g\vec{E} - g\vec{E}\times i\vec{D})\cdot\vec{\sigma}\chi\right]\\ &+\frac{c_4}{2M}\left[\psi^\dagger(g\vec{B}\cdot\vec{\sigma})\psi - \chi^\dagger(g\vec{B}\cdot\vec{\sigma})\chi\right]\end{aligned}$$

4 adjustable parameters: c_1, c_2, c_3, c_4

last 2 terms contain Pauli matrix σ

break heavy-quark spin symmetry

NonRelativistic QCD

Effective Field Theory strategy (cont.)

- determine parameters of L_{eff}
  by matching to QCD at soft momenta

  calculate scattering amplitudes
  - using perturbative QCD
    (depends on momenta, M, and α_s)
  - using perturbative NRQCD
    (depends on momenta, M, α_s, c_1, c_2, c_3, c_4, ...)

  determine parameters c_i as power series in $\alpha_s(M)$
  by demanding that scattering amplitudes
  in QCD and NRQCD agree

Applications

Lattice formulation of NRQCD

Lepage et al. hep-lat/9205007

Lattice NRQCD calculations pioneered by

NRQCD Collaboration (1992 - 2000)
HPQCD Collaboration (2001 - ?)
(Davies, Lepage, Shigemitsu, ...)

spectrum:

ground states for S-wave and P-wave
excited states below open-flavor threshold

precise determination of QCD parameters: α_s, m_b

Applications

Lattice NRQCD

semi-quantitative calculations for charmonium

only quantitative calculations for bottomonium

v^2-improved NRQCD Lagrangian

short-distance coefficients: LO in $\alpha_s(M)$ (tree level)

lattice spacing a restricted to $aM < 1$

spin-averaged spectrum

example: $\left(3M_{J/\psi} + M_{\eta_c}\right)/4$

accuracy: NLO in v^2 (10% for charmonium, 1% for bottomonium)

spin splittings

example: $M_{J/\psi} - M_{\eta_c}$

accuracy: LO in v^2 (30% for charmonium, 10% for bottomonium)

Applications

Fock-State Expansion for Quarkonium

Quark model:
Quarkonium is pure color-singlet $Q\bar{Q}$
with definite angular momentum $^{2S+1}L_J$

NRQCD:
angular momentum of $Q\bar{Q}$
can be changed by emission of soft gluon

emission amplitude is suppressed by v

Quarkonium (in Coulomb gauge) has Fock state expansion
$|Q\bar{Q}\rangle$ $|Q\bar{Q}+g\rangle$ $|Q\bar{Q}+gg\rangle$ ...

coefficients have definite scaling with v

Applications

Fock-State Expansion for Charmonium (Coulomb gauge)

onium	order 1	order v	order v^2
η_c	$c\bar{c}(\underline{1},{}^1S_0)$	$c\bar{c}(\underline{8},{}^1P_1)+g$	$c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^1S_0)+gg$ $c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^1D_2)+gg$ $c\bar{c}(\underline{8},{}^3S_1)+g$
J/ψ	$c\bar{c}(\underline{1},{}^3S_1)$	$c\bar{c}(\underline{8},{}^3P_J)+g$	$c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^3S_1)+gg$ $c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^3D_J)+gg$ $c\bar{c}(\underline{8},{}^1S_0)+g$
χ_{cJ}	$c\bar{c}(\underline{1},{}^3P_J)$	$c\bar{c}(\underline{8},{}^3S_1)+g$ $c\bar{c}(\underline{8},{}^3D_{J'})+g$	$c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^3P_1)+gg$ $c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^3D_{J'})+gg$ $c\bar{c}(\underline{1}\,\mathrm{or}\,\underline{8},{}^3F_{J'})+gg$ $c\bar{c}(\underline{8},{}^1P_1)+g$

Applications

Other applications of NRQCD:

annihilation decays

inclusive hard production

require NRQCD Factorization

Final comments

Alternatives to NRQCD

a ``more effective” EFT?

Potential NRQCD

Brambilla, Pineda, Soto, and Vairo hep-ph/0410047

integrate out soft momentum scale Mv

to get EFT for ultrasoft scales Λ_{QCD}, Mv^2

Q and $\bar{Q}$ interact through nonlocal potentials

weakly-coupled pNRQCD: $\Lambda_{QCD} << Mv^2$

applications limited to $t\,\bar{t}$

$\eta_b(1S)$, $\Upsilon(1S)$?

strongly-coupled pNRQCD: $\Lambda_{QCD} \sim Mv^2$

Final comments

Alternatives to NRQCD

a ``less effective'' EFT?

<u>Fermilab heavy quarks</u>

El-Khadra, Kronfeld, and Mackenzie hep-ph/9604004

EFT with Dirac spinor field Ψ

instead of Pauli spinor fields ψ and χ

Q and $\bar{Q}$ have relativistic dispersion relation

lattice gauge theory calculations for charmonium

Fermilab Lattice Collaboration

other applications?