BabaYaga: an event generator for luminometry at flavour factories

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(Balossini, Barzè, Bignamini, Montagna, Nicrosini, Piccinini)

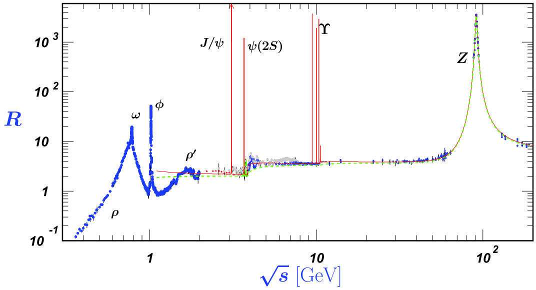
Outline

- ⋆ Motivations for precise luminometry
- ⋆ QED scattering processes & radiative corrections
- ⋆ The event generator **BabaYaga**
  - theoretical framework
  - from **BabaYaga 2.0** to **BabaYaga@NLO**
- ⋆ Phenomenological results
- ⋆ Independent calculations and tuned comparisons
- ⋆ Theoretical accuracy
- ⋆ Example run
- ⋆ Conclusions and outlook

- I do apologize for not citing complete references in the slides, but please refer to the recently published report
  S. Actis *et al.* (Working Group on RC and MC Generators for Low Energies), **Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data**, Eur. Phys. J. C **66** (2010) 585

Why precision luminosity generators?

- Precision measurements require a precise knowledge of the machine luminosity.
- e.g. the measurement of the $R(s)$ ratio is a key ingredient for the predictions of $a_\mu = (g_\mu - 2)/2$ and $\Delta\alpha_{\rm had}(M_Z)$ and in turn for SM precision tests



$$a_\mu = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} {\rm d}s\, K(s)\frac{R(s)}{s} \qquad \Delta\alpha^{(5)}_{\rm had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}{\rm Re}\int_{m_\pi^2}^{\infty}\frac{R(s){\rm d}s}{s(s-M_Z^2-i\epsilon)}$$

Reference processes for luminosity

- Instead of getting it from machine parameters, it's more effective to exploit the relation (e.g. LEP!)

$$L = \int \mathcal{L} dt = \frac{N}{\sigma_{th}} \qquad \frac{\delta L}{L} = \frac{\delta N}{N} \oplus \frac{\delta \sigma_{th}}{\sigma_{th}}$$

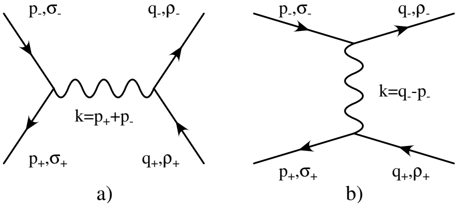
- Normalization processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes to achieve a typical precision at the level of $0.1\% \div \mathcal{O}(1\%)$

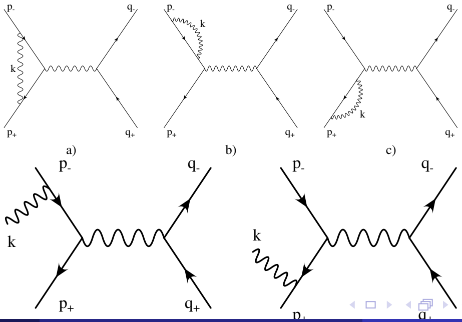
- High theoretical accuracy and comparison with data require precision Monte Carlo (MC) tools, which must include (QED) radiative corrections (RC) at the highest standard as possible
- **BabaYaga** has been developed to this purpose

Example of QED RC

⋆ Born (leading order, LO) Bhabha diagrams



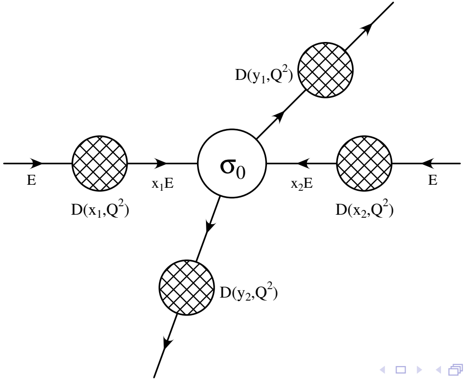
⋆ some of QED corrected diagrams (next-to-LO, NLO), virtual & real RC



The Structure Function (SF) approach

- an effective way to account for [some of the] QED RC is the SF approach

$$\sigma_{corrected} = \int dx_- dx_+ dy_- dy_+ \int d\Omega D(x_-, Q^2) D(x_+, Q^2) \times D(y_-, Q^2) D(y_+, Q^2) \frac{d\sigma_0}{d\Omega}(x_- x_+ s, \theta) \Theta(cuts)$$



The Structure Function approach

- $D(x, Q^2)$ is the QED SF, to account for QED virtual and real RC up to all orders in α in leading-log (LL) and collinear approximation. It has a probabilistic interpretation
- the (non-singlet) SF is the solution of the DGLAP eq. in QED

$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_+(y) D(\frac{x}{y}, Q^2)$$

$$P_+(x) = \frac{1+x^2}{1-x} - \delta(1-x) \int_0^1 dt P(t)$$

- $D(x, Q^2)$ resums (exponentiates) all the (numerically) large collinear, $L = \log \frac{s}{m_e^2}$, and infrared logarithms, which are "universal" and factorize over the kernel x-section
- solutions are analitically known (approximated, or exact with Mellin transform), but "exclusive" information are lost

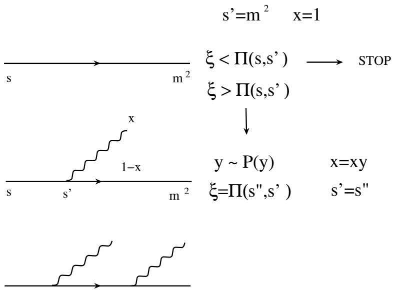
The QED Parton Shower

- alternatively, DGLAP eq. can be numerically (and exactly) solved by means of the Parton Shower (PS) MC algorithm
- $e \to e' + \gamma$ branching kinematics recoverable (exclusive photons generation)

⋆ Sudakov Form Factor $\Pi(s,s') = \exp\left[-\frac{\alpha}{2\pi}\ln\frac{s}{s'}\int_0^{x^+} P(x)dx\right]$

⋆ iterative solution of DGLAP equation

$$D(x,s) = \Pi(s,m^2)\delta(1-x)$$
$$+\frac{\alpha}{2\pi}\int_{m^2}^{s}\Pi(s,s')\frac{ds'}{s'}\Pi(s',m^2)\times$$
$$\int_0^{x_+} dy P(y)\delta(x-y)+$$
$$+2\,branchings\ +\ 3\ +\ \cdots$$



The QED Parton Shower

⋆ Advantages:

- PS is an exact numerical solution of DGLAP eq. $\rightarrow$ QED RC are accounted for up to at all orders (at least in LL approx.) $\rightarrow$ multiple photon effects
- at each branching, kinematical variables are generated (energies, virtualities) to reconstruct the emitted photons' momenta $\rightarrow$ fully exclusive event generation
- it can be truncated at $\mathcal{O}(\alpha)$, to consistently compare with exact $\mathcal{O}(\alpha)$ calculations

⋆ Disadvantages:

- initial-final (I-F) state radiation interference effects not naturally included (but they can be included!)
- the theoretical error on the x-section starts already at $\mathcal{O}(\alpha)$, at the level of non-log contributions $\rightarrow$ this requires the *matching* with exact $\mathcal{O}(\alpha)$ (NLO) RC
- a matching algorithm has been implemented in **BabaYaga@NLO**

Improving photon angular distribution in QED PS

- **BabaYaga 2.0** was based on a pure LL PS, without I-F state radiation interference effects
- these were added in **BabaYaga 3.5**

⋆ LL recipe ($p_{1,2,3,4} \to$ lepton momenta, $k \to$ photon momentum)

$$\cos\vartheta_\gamma \sim \frac{1}{p_1 \cdot k} + \frac{1}{p_2 \cdot k} + \frac{1}{p_3 \cdot k} + \frac{1}{p_4 \cdot k}$$

⋆ Coherent Radiation recipe for the emission of n soft photons

$$d\sigma_n \approx d\sigma_0 \frac{1}{n!} \prod_{l=1}^{n} \frac{e^2 d^3\mathbf{k}_l}{(2\pi)^3 2k_l^0} \sum_{i,j} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k_l)(p_j \cdot k_l)}$$

E. g., $n = 1$:

$$\frac{p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} + \frac{p_3 \cdot p_4}{(p_3 \cdot k)(p_4 \cdot k)} +$$

$$+ \frac{p_1 \cdot p_3}{(p_1 \cdot k)(p_3 \cdot k)} - \frac{p_1 \cdot p_4}{(p_1 \cdot k)(p_4 \cdot k)} - \frac{p_2 \cdot p_3}{(p_2 \cdot k)(p_3 \cdot k)} + \frac{p_2 \cdot p_4}{(p_2 \cdot k)(p_4 \cdot k)}$$

LL + I.-F. INTERFERENCES !!

Assessing PS theoretical accuracy

- the PS can be truncated at $\mathcal{O}(\alpha)$ (1-photon LL RC), allowing a consistent comparison with exact NLO calculations (which are implemented e.g. in **LABSPV**) to assess its theoretical accuracy
- the exact NLO x-section can be written

$$\sigma^{(\alpha)}_{exact} = \sigma^{(\alpha)}_{S+V}(E_\gamma < k_0) + \sigma^{(\alpha)}_{H}(E_\gamma > k_0, \text{cuts})$$

where

1. $\sigma^{(\alpha),i}_{S+V} = \sigma_0 \left\{1 + 2\,(\beta_e + \beta_{int}) \ln k_0/E + C^i_F\right\}, \quad i = s, t, s\text{-}t$

   M. Caffo, E. Remiddi et al., CERN Report **89-08**

   M. Greco, *Riv. Nuovo Cim.* **11** (1988) 1

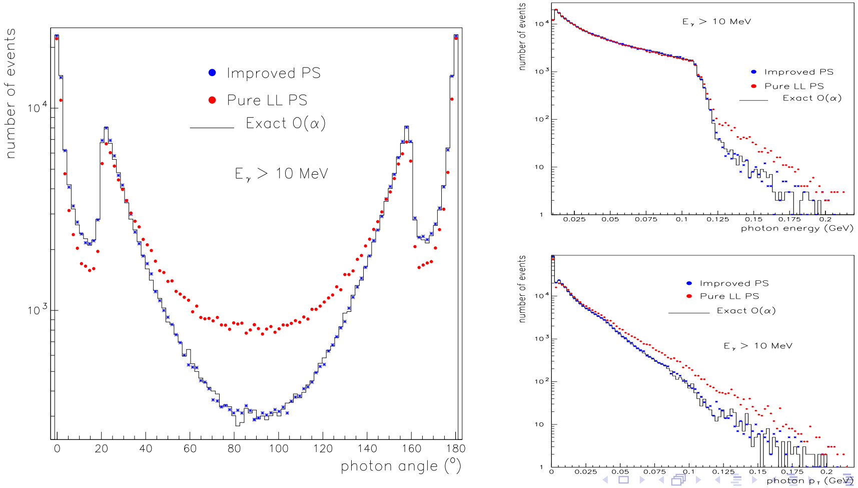
   F.A. Berends and R. Kleiss, *Nucl. Phys.* **B** 186 (1981) 22
2. $\sigma^{(\alpha)}_{H}$ "exact" hard bremsstrahlung matrix elements

   F.A. Berends et al., *Nucl. Phys.* **B** 202 (1981) 63

- k_0 is a fictitious (and arbitrary) soft-hard photon separator and $\beta_e \propto \frac{\alpha}{\pi} \log \frac{s}{m_e^2}$

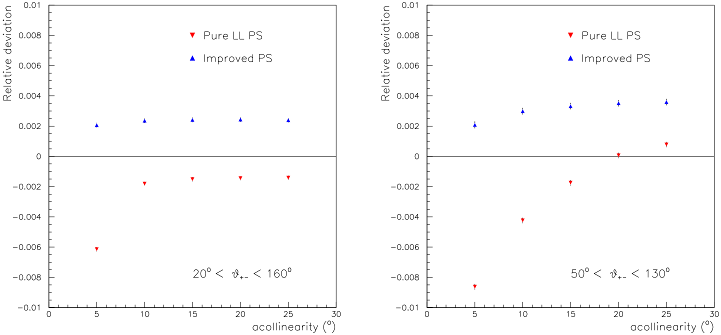
PS vs. exact NLO

- experimental setup as for KLOE luminometry is considered, $\sqrt{s} = 1.019$ GeV, $E_{min}^{\pm} = 0.4$ GeV, $20° \leq \vartheta_{\pm} \leq 160°$, $\xi \leq 10°$



PS accuracy

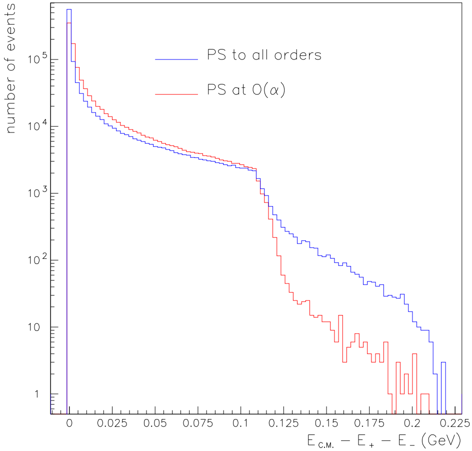
- distributions can be improved a lot by including I-F interferences
- the th. error is still at the level of NLO non-log corrections
- e.g. in typical setup for KLOE (as a function of the acollinearity cut, for $20^\circ - 160^\circ$ and $50^\circ - 130^\circ$ acceptances) the missing $\mathcal{O}(\alpha)$ terms amount to $\sim 0.5\%$

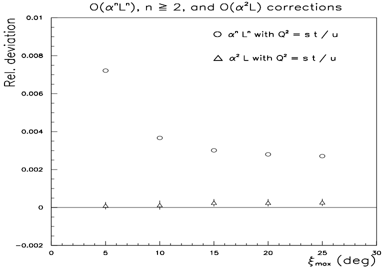


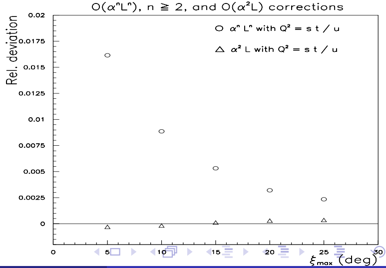
BabaYaga 2.0 and 3.5 accuracy (for Bhabha) = 0.5%

Multiple-photon effects (higher-order corrections)

- Nevertheless, the PS allows to take into account multiple-photon emissions, which are not negligible at the 1% level







Matching PS with NLO

- In order to achieve the desired theoretical accuracy (a few 0.1%), both non-log $\mathcal{O}(\alpha)$ and h.o. RC must be included
- This requires the matching of the PS with NLO RC (widely discussed also in QCD)
- A number of highly non-trivial technical issues arises and must be solved (negative weights, double counting, exact n-body phase space integration, ...)
- The matching algorithm must preserve the advantages of both PS and exact calculations
  - ⋆ complete exact $\mathcal{O}(\alpha)$ corrections
  - ⋆ multiple-photon emission (h.o. corrections), at least in LL approx
  - ⋆ exclusive event generation
  - ⋆ such an algorithm has been devised and implemented in **BabaYaga@NLO**

PS and exact NLO RC in BabaYaga@NLO

PS and exact $\mathcal{O}(\alpha)$ (NLO) matrix elements must be combined and matched. How?

- $d\sigma_{LL}^{\infty} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{SV}(\varepsilon) + d\sigma_H(\varepsilon)$
- $d\sigma_{exact}^{\alpha} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$
- $d\sigma_{exact}^{\alpha} \overset{\text{at } \mathcal{O}(\alpha)}{=} F_{SV}(1 + C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1$

$$d\sigma_{matched}^{\infty} = F_{SV} \, \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$$

PS and exact NLO RC in BabaYaga@NLO

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$$d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

PS and exact NLO RC in **BabaYaga@NLO**

PS and exact $\mathcal{O}(\alpha)$ (NLO) matrix elements must be combined and matched. How?

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$$d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

PS and exact NLO RC in BabaYaga@NLO

PS and exact $\mathcal{O}(\alpha)$ (NLO) matrix elements must be combined and matched. How?

- $d\sigma_{LL}^{\infty} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{SV}(\varepsilon) + d\sigma_H(\varepsilon)$
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$$d\sigma_{matched}^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

PS and exact NLO RC in BabaYaga@NLO

PS and exact $\mathcal{O}(\alpha)$ (NLO) matrix elements must be combined and matched. How?

- $d\sigma^{\infty}_{LL} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{LL} = [1 + C_{\alpha,LL}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{SV}(\varepsilon) + d\sigma_H(\varepsilon)$
- $d\sigma^{\alpha}_{exact} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1$
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$$d\sigma^{\infty}_{matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

Contents of the *matched* formula

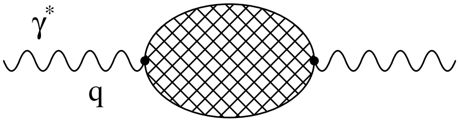
- F_{SV} and $F_{H,i}$ are infrared safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{exact}$
- resummation of higher orders LL contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV \mid H,i} \times LL$

G. Montagna et al., **PLB** 385 (1996)

- the th. error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m^2} \sim 5\times 10^{-4}$$

Vacuum Polarization (and Z exchange)



- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\Delta\alpha = \Delta\alpha_{e,\mu,\tau,\text{top}} + \Delta\alpha^{(5)}_{\text{had}}$
- $\Delta\alpha^{(5)}_{\text{had}}$ is a non-perturbative contribution. Evaluated with recently updated **HADR5N** by F. Jegerlehner or **HMNT** by Hagiwara, Teubner et al. They return also an error associated with exp. data.
- VP included both in lowest order and (at best) in one-loop diagrams $\Rightarrow$ part of the 2 loop factorizable corrections are included
- Z exchange included at lowest order in e^+e^- and $\mu^+\mu^-$ FS. Its effect is $\mathcal{O}(0.1\%)$ @ 10 GeV for Bhabha

- The matching procedure is now implemented in **BabaYaga@NLO**
- It is applied to Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$ final states
- The (**Fortran 77** [!]) code can be downloaded from **http://www.pv.infn.it/hepcomplex/babayaga.html**
- Relevant papers for all the details and phenomenological studies:

1. G. Balossini et al., **Matching perturbative and Parton Shower corrections to Bhabha process at flavour factories**, Nucl. Phys. **B** 758, 227 (2006)
2. G. Balossini et al., **Photon pair production at flavour factories with per mille accuracy**, Phys. Lett. **B** 663:209 (2008)
3. C.M. Carloni Calame, **An improved Parton Shower algorithm in QED**, Phys. Lett. **B** 520, 16 (2001)
4. C.M. Carloni Calame et al., **Large-angle Bhabha scattering and luminosity at flavour factories**, Nucl. Phys. **B** 584, 459 (2000)

Results with BabaYaga@NLO

- as examples to show the features of the EG, the following setups and definitions are used (for Bhabha)

  a. $\sqrt{s} = 1.02 \text{ GeV}, \; E_{min} = 0.408 \text{ GeV}, \; 20^\circ < \theta_\pm < 160^\circ, \; \xi_{max} = 10^\circ$
  b. $\sqrt{s} = 1.02 \text{ GeV}, \; E_{min} = 0.408 \text{ GeV}, \; 55^\circ < \theta_\pm < 125^\circ, \; \xi_{max} = 10^\circ$
  c. $\sqrt{s} = 10 \text{ GeV}, \; E_{min} = 4 \text{ GeV}, \; 20^\circ < \theta_\pm < 160^\circ, \; \xi_{max} = 10^\circ$
  d. $\sqrt{s} = 10 \text{ GeV}, \; E_{min} = 4 \text{ GeV}, \; 55^\circ < \theta_\pm < 125^\circ, \; \xi_{max} = 10^\circ$

$$\delta_{VP} \equiv \frac{\sigma_{0,VP} - \sigma_0}{\sigma_0} \qquad \delta_\alpha \equiv \frac{\sigma_\alpha^{NLO} - \sigma_0}{\sigma_0}$$

$$\delta_{HO} \equiv \frac{\sigma_{matched}^{PS} - \sigma_\alpha^{NLO}}{\sigma_0} \qquad \delta_{HO}^{PS} \equiv \frac{\sigma^{PS} - \sigma_\alpha^{PS}}{\sigma_0}$$

$$\delta_\alpha^{non\text{-}log} \equiv \frac{\sigma_\alpha^{NLO} - \sigma_\alpha^{PS}}{\sigma_0} \qquad \delta_\infty^{non\text{-}log} \equiv \frac{\sigma_{matched}^{PS} - \sigma^{PS}}{\sigma_0}$$

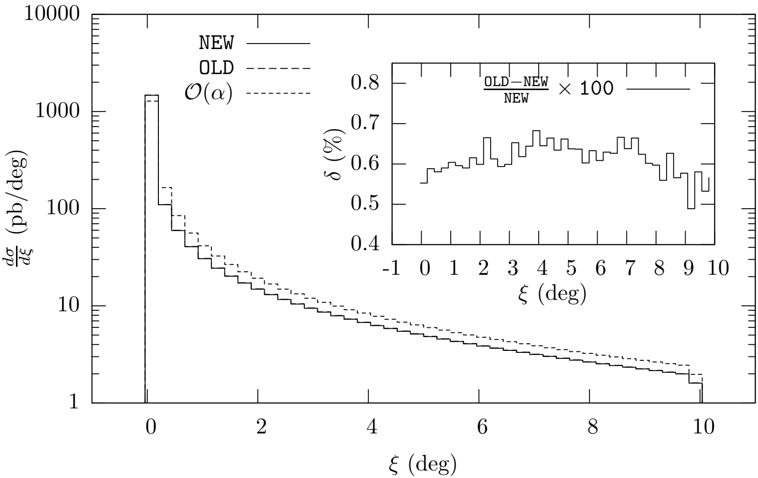
set up	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\infty}^{non\text{-}log}$	-0.30	-0.49	-0.29	-0.46

Table: Relative corrections (in per cent) to the Bhabha cross section for the four setups

- ⋆ in short, the fact that $\delta_{\alpha}^{non\text{-}log} \simeq \delta_{\infty}^{non\text{-}log}$ and $\delta_{HO} \simeq \delta_{HO}^{PS}$ means that the matching algorithm preserves both the advantages of exact NLO calculation and PS approach:
  - → it includes the missing NLO RC to the PS
  - → it adds the missing higher-order RC to the NLO

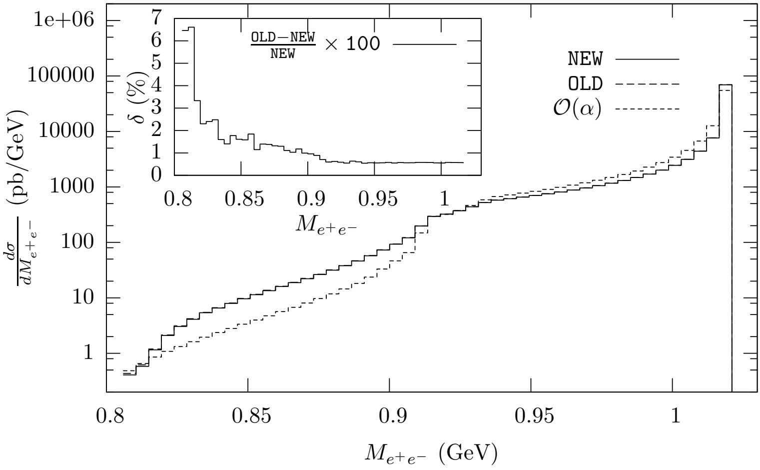
Results with **BabaYaga@NLO**

- acollinearity distribution, setup (a)



Results with **BabaYaga@NLO**

- e^+e^- invariant-mass distribution, setup (a)



Results with **BabaYaga@NLO** for $\gamma\gamma$ final state

- $\gamma\gamma$ final state has a lower x-section, but it does not depend on hadronic VP, which is a source of th. error
- Similar setups and definitions were used to study $\gamma\gamma$ FS

$$\begin{cases} \sqrt{s} = 1. \ - \ 3. \ - \ 10.\mathrm{GeV} \\ E_\gamma^{\mathrm{min}} = 0.3 \ \times \sqrt{s} \\ \vartheta_\gamma^{\mathrm{min}} = 45^\circ, \quad \vartheta_\gamma^{\mathrm{max}} = 135^\circ \\ \xi_{\mathrm{max}} = 10^\circ \end{cases}$$

$$\delta_\alpha = 100 \times \frac{\sigma_\alpha^{\mathrm{NLO}} - \sigma}{\sigma} \qquad \delta_\infty = 100 \times \frac{\sigma_{\mathrm{exp}} - \sigma}{\sigma}$$

$$\delta_{\mathrm{exp}} = 100 \times \frac{\sigma_{\mathrm{exp}} - \sigma_\alpha^{\mathrm{NLO}}}{\sigma_\alpha^{\mathrm{NLO}}} \qquad \delta_\alpha^{\mathrm{NLL}} = 100 \times \frac{\sigma_\alpha^{\mathrm{NLO}} - \sigma_\alpha^{\mathrm{PS}}}{\sigma_\alpha^{\mathrm{PS}}}$$

$$\delta_\infty^{\mathrm{NLL}} = 100 \times \frac{\sigma_{\mathrm{exp}} - \sigma_{\mathrm{exp}}^{\mathrm{PS}}}{\sigma_{\mathrm{exp}}^{\mathrm{PS}}}$$

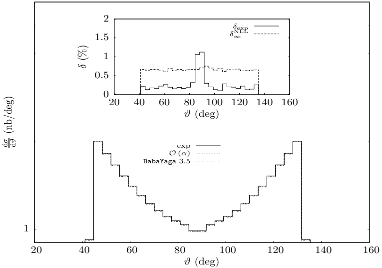
Results with **BabaYaga@NLO** for $\gamma\gamma$

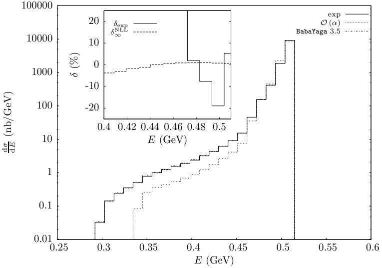
$\sqrt{s}$ (GeV)	1	3	10
σ	137.53	15.281	1.3753
$\sigma_{\alpha}^{\mathrm{PS}}$	128.55	14.111	1.2529
$\sigma_{\alpha}^{\mathrm{NLO}}$	129.45	14.211	1.2620
$\sigma_{\mathrm{exp}}^{\mathrm{PS}}$	128.92	14.169	1.2597
σ_{exp}	129.77	14.263	1.2685
δ_{α}	-5.87	-7.00	-8.24
δ_{∞}	-5.65	-6.66	-7.77
δ_{exp}	0.24	0.37	0.51
$\delta_{\alpha}^{\mathrm{NLL}}$	0.70	0.71	0.73
$\delta_{\infty}^{\mathrm{NLL}}$	0.66	0.66	0.69

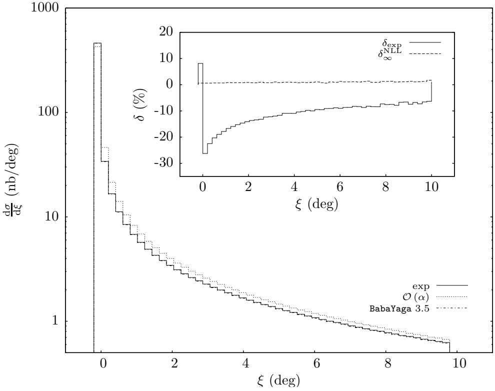
Table: Photon pair production cross sections (in nb) to different accuracy levels and relative corrections (in per cent)

Results with BabaYaga@NLO for $\gamma\gamma$

- most-energetic photon angle and energy, acollinearity distribution







Estimating the theoretical accuracy

- It is of utmost importance to compare with independent calculations/implementations, in order to
  - ⋆ asses the technical precision, spot bugs (with the same th. ingredients)
  - ⋆ estimate the theoretical “error” when including partial/incomplete higher-order corrections
- Generators exist on the marked and are used by exp. coll., some of them including QED h.o. and NLO corrections according to different approaches (collinear SF + NLO, YFS exponentiation,. . .)

Generator	Processes	Theory	Accuracy	Web address
BHAGENF/BKQED	$e^+e^-/\gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$	1%	www.lnf.infn.it/˜graziano/bhagenf/bhabha.html
BabaYaga v3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	Parton Shower	$\sim\ 0.5\%$	www.pv.infn.it/˜hepcomplex/babayaga.html
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + PS	$\sim 0.1\%$	www.pv.infn.it/˜hepcomplex/babayaga.html
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	0.5%(LEP1)	placzek.home.cern.ch/placzek/bhwide
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + SF	$< 0.2\%$	cmd.inp.nsk.su/˜sibid

Tuned comparisons

Without vacuum polarization, to compare consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga people, Wang Ping and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_{\mp} \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_{\mp} \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

★ Agreement well below 0.1%! ★

At BaBar (cross sections in nb)

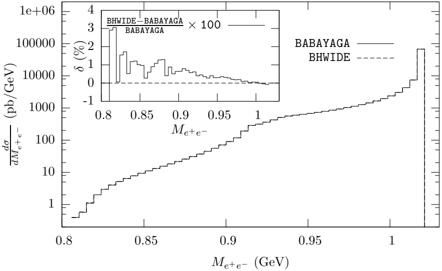
By A. Hafner and A. Denig

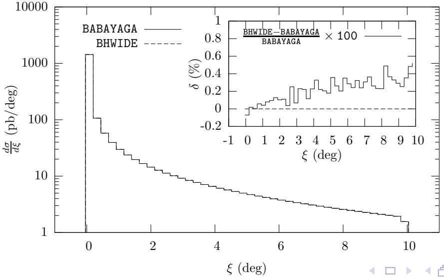
angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

★ Agreement at the $\sim$ 0.1% level! ★

Tuned comparisons

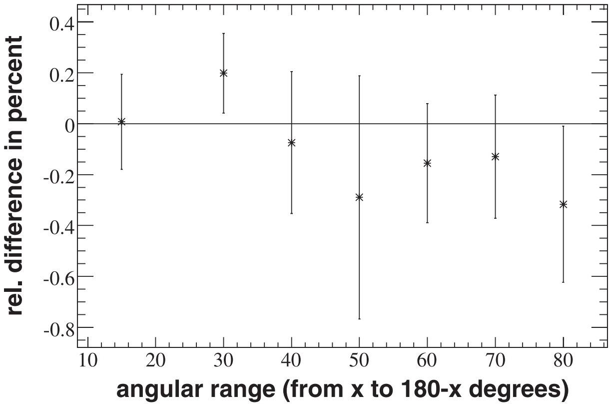
- distributions: **BabaYaga@NLO** vs. **Bhwide** (at KLOE)





Tuned comparisons

- **BabaYaga@NLO** vs. **Bhwide** (at BABAR)



Tuned comparisons

- **MCGPJ** vs. **BabaYaga@NLO** and **Bhwide** (at CMD2)

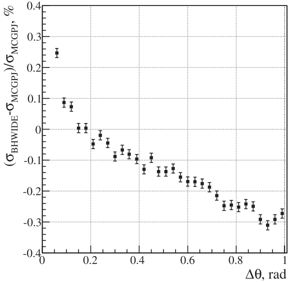


Fig. 23 Relative differences between BHWIDE and MCGPJ Bhabha cross sections as a function of the acollinearity cut, for the CMD-2 experiment at VEPP-2M

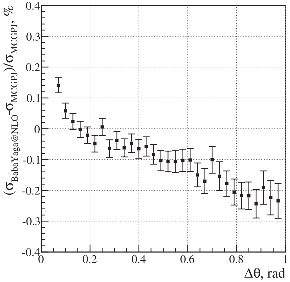


Fig. 24 Relative differences between BabaYaga@NLO and MCGPJ Bhabha cross sections as a function of the acollinearity cut, for the CMD-2 experiment at VEPP-2M

- The three generators agree within 0.1% for the typical experimental acollinearity cut $\Delta\theta \sim 0.2 \div 0.3$ rad

★ Main conclusion from tuned comparisons: technical precision of the generators well under control, the small remaining differences being due to slightly different details in the calculation of the same theoretical ingredients [additive vs factorized formulations, different scales for higher–order leading log corrections]

Theoretical accuracy, comparisons with NNLO calculations

- After including the exact NLO RC, the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) (although large NNLO corrections are already included by means of multiple photon emission)

⋆ The NNLO QED corrections to Bhabha scattering have been calculated in the last years $\rightarrow$ it's very important to measure the impact of the missing (non-leading) terms of order α^2 within typical setups for luminometry, to asses the MC accuracy

- The estimate of the theoretical accuracy will be sound and robust
- e.g., **BabaYaga** formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently compared with all the classes of NNLO corrections

$\mathcal{O}(\alpha^2)$ expansion

- the $\mathcal{O}(\alpha^2)$ content of **BabaYaga** cross section can be cast in the form

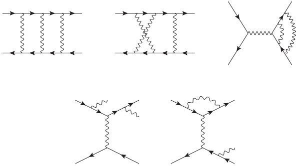
$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

where

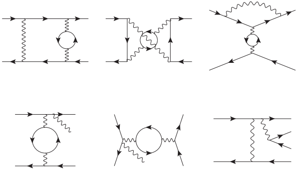
- $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$ $\longrightarrow$ compared with the corresponding available NNLO QED calculation
- $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung $\longrightarrow$ presently estimated relying upon existing (partial) results
- $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung $\longrightarrow$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

NNLO calculations

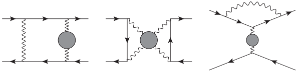
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280 / S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

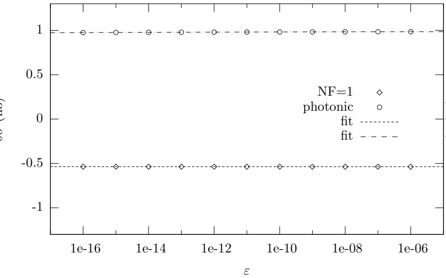


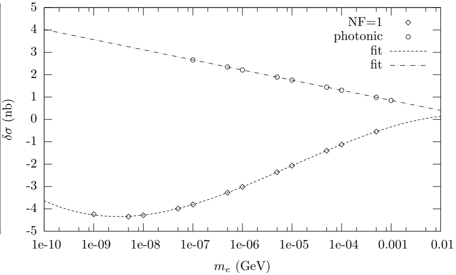
- Heavy fermion and hadronic corrections R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601 / S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602 / J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



Differences from Penin & Bonciani et al.

- diff. between Penin and Bonciani et al. and the corresponding **BabaYaga** content, as $f(\varepsilon)$ and $g(\log(m_e))$. E.g. LABS at 1 GeV





- ⋆ differences are infrared safe
- ⋆ $\delta\sigma(phot.)/\sigma_0 \propto \alpha^2 L$ $\quad$ $\delta\sigma(N_F = 1)/\sigma_0 \propto \alpha^2 L^2$
- ⋆ Numerically, in LABS and VLABS,

$$\delta\sigma(phot.) + \delta\sigma(N_F = 1) < 0.015\% \times \sigma_0$$

Pair corrections

- A Desy–Zeuthen & Katowice collaboration [H. Czyz, J. Gluza, M. Gunia, T. Riemann and M. Worek] did a new, exact calculation of pair corrections, based on exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-, l = e, \mu, \tau), e^+e^-(\pi^+\pi^-)$
- Results: in comparison with the approximation of BabaYaga@NLO and using realistic KLOE and BaBar luminosity cuts (cross sections in nb)

Electron pair corrections

	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-1.794	-1.570	0.04
BaBar	6.744	-0.008	-0.008	0.00

Muon pair corrections

	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-0.241	-0.250	0.002
BaBar	6.744	-0.004	-0.003	0.015

Pion pair corrections

	$\sigma_{\rm Born}$	$\sigma^{\rm exact}_{\rm pairs}$	$\sigma^{\rm BabaYaga@NLO}_{\rm pairs}$	$(\sigma^{\rm ex.} - \sigma^{\rm BabaYaga})/\sigma_{\rm Born}(\%)$
KLOE	529.469	-0.186	in progress	–
BaBar	6.744	-0.003	in progress	–

$\star$ The uncertainty due to lepton and hadron pair corrections is at the level of a few units in 10^{-4} [further comparisons in progress] $\star$

$\Delta\alpha^{(5)}_{\text{had}}$ and other $\mathcal{O}(\alpha^2)$ uncertainties

- the exact 1-loop virtual corrections to the 1-photon real emission for Bhabha only recenlty has been made available (comparisons in progress)

  Actis *et al.*, Phys. Lett. **B** 682 (419) 2010

  ⋆ relying on the LEP experience and being the error at the $\alpha^2 L$ level, the missing corrections are $\leq 0.05\%$

- the double real bremsstrahlung contribution is in principle approximated in **BabaYaga@NLO**

  ⋆ observed really negligible differences with the exact matrix elements, calculated with the **ALPHA** (Caravaglios and Moretti ('95)) algorithm/routine

- $\Delta\alpha^{(5)}_{\text{had}}$ is affected by the experimental error, which is returned by the routines in use (**HADR5N** and **HMNT**)

⋆ The total error budget for Bhabha can be summarized as follow (from G. Montagna's talk at the "International Workshop on e^+e^- collisions from Φ to Ψ", Beijing, October 2009)

Status of the MC theoretical accuracy

Main conclusion of the Luminosity Section of the WG Report "Radiative Corrections & MC Tools"

Putting the various sources of uncertainties (for large–angle Bhabha) all together...

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta_{\rm VP}^{\rm err}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta_{\rm VP}^{\rm err}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta_{\rm SV,\alpha^2}^{\rm err}\rvert$	0.02	0.02	0.02
$\lvert\delta_{\rm HH,\alpha^2}^{\rm err}\rvert$	0.00	0.00	0.00
$\lvert\delta_{\rm SV,H,\alpha^2}^{\rm err}\rvert$ [conservative?]	0.05	0.05	0.05
$\lvert\delta_{\rm pairs}^{\rm err}\rvert$ [in progress]	$\sim$0.05	$\sim$0.1[1]	$\sim$0.02[2]
$\lvert\delta_{\rm total}^{\rm err}\rvert$ linearly	0.12÷0.14	0.18	0.11÷0.12
$\lvert\delta_{\rm total}^{\rm err}\rvert$ in quadrature	0.07÷0.08	0.11	0.06÷0.07

- Comparisons with the Novosibirsk $\Delta\alpha_{\rm had}^{(5)}(q^2)$ parameterization routine and with the calculation by Actis *et al.* for $e^+e^-\gamma$ at one loop would put the evaluation of the $\lvert\delta_{\rm VP}^{\rm err}\rvert$ and $\lvert\delta_{\rm SV,H,\alpha^2}^{\rm err}\rvert$ uncertainties on firmer grounds

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved about ten years ago for small–angle Bhabha luminosity monitoring at LEP/SLC

[1] Very preliminary, work in progress using realistic BES-III and CLEO-c luminosity cuts

[2] Preliminary and assuming BaBar cuts. Work in progress for BELLE event selection

Resummation beyond α^2

- with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

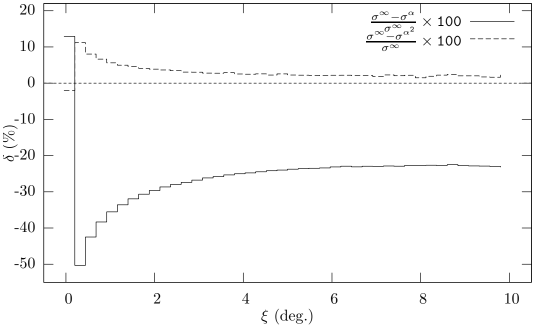


Figure: Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution

- resummation beyond α^2 still important!

The “dark” side of **BabaYaga**

- Recently, the process $e^+e^- \to \gamma, U \to e^+e^-\gamma$ (or $\to \mu^+\mu^-\gamma$) has been implemented, including LL collinear RC, for the search of a light, weakly-interacting, photon-like vector boson at flavour factories

⋆ the U boson is a candidate for dark matter

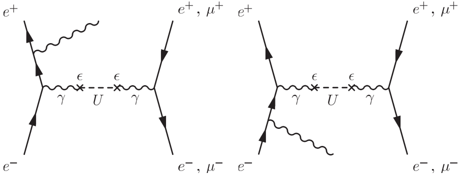


FIG. 1. Examples of Feynman diagrams with dark photon exchange contributing to the process $e^+e^- \to \gamma, U \to l^+l^-\gamma$, $l = e, \mu$.

- The details can be found in `arXiv:1007.4984 [hep-ph]` by L. Barzè et al., submitted to PRD
- for the moment being, forget about it!

Using BabaYaga@NLO

- Download the package `babayaga-NLO.tar.gz`
- Unpack it `tar -xzvf babayaga-NLO.tar.gz`
- `cd babayaga-NLO/`
- Read the file `README` !
- `./configure`
- `make`
- `./babayaga`
- On the shell prompt, it should appear an “interactive” menu like this…

```
*************************************************************
**********                                         **********
********                                             ********
******              Welcome to BabaYaga                ******
****               ^^^^^^^^^^^^^^^^^^^^^^^               ****
**            It is an event generator for QED processes    **
****           at low energies, matching a QED PS with    ****
******              exact order alpha corrections       ******
********                                             ********
**********                                         **********
*************************************************************
 [[ it simulates: e+e- -->>  g  -->> e+e- or mu+mu- or gg ]]
 [[             : e+e- -->> g,U -->> e+e-g or mu+mu-g     ]]

Principal Menu:
 [ type "run" to start generation,
   "legenda" for help or "quit" to quit ]
 [ fs      ] final state =  ee
 [ ecms    ] CoM energy    =    1.020  GeV
 [ thmin   ] min. angle    =   20.000  deg
 [ thmax   ] max. angle    =  160.000  deg
 [ zmax    ] acollinearity =   10.000  deg
 [ emin    ] min. energy   =    0.408  GeV
 [ nev     ]    10000000. events will be generated
 [ path    ] files saved in test-run/
 [ ntuple  ] ntuple creation no
 [ menu2   ] the second menu is off
 [ menud   ] the dark matter menu is off

Insert "variable value":
```

Using **BabaYaga@NLO**

- In the main menu a number of parameter can be modified, most notably
  - ⋆ final state: `ee`, `mm` or `gg`
  - ⋆ center of mass energy
  - ⋆ acceptance cuts (modify `cuts.f` for more elaborate event selection criteria)
  - ⋆ directory where to save the outputs
  - ⋆ enabling a 2nd menu, to modify inner parameters (order of RC, running of α, ...)
  - ⋆ type `legenda` for a quick explanation of the parameters and possible options, type `run` to start **BabaYaga** run
- The results are saved in a separate directory (`test-run/` by default), which contains
  - ⋆ the unweighted events if their storage was requested
  - ⋆ a number of simple text files with some relevant distributions (files to be fed to **gnuplot** for example)
  - ⋆ the file `statistics.txt` where the information (statistics, input parameters, results) of the run is saved $\longrightarrow$

The statistics.txt file

```
~ Generating    10000000. weighted events ~

 ::::::>>>>>>> weighted events <<<<<<::::::
 0 photons:    3324.86157302 +-    1.09820485    ( 53.6282 %)
 1 photons:    2079.28199279 +-    1.20356346    ( 33.5377 %)
 2 photons:     641.97792104 +-    0.87462503    ( 10.3547 %)
 3 photons:     131.27112769 +-    0.48134411    (  2.1173 %)
 4 photons:      19.88891842 +-    0.22678749    (  0.3208 %)
 5 photons:       2.30523354 +-    0.08724946    (  0.0372 %)
 6 photons:       0.22980637 +-    0.02961113    (  0.0037 %)
 7 photons:       0.02691640 +-    0.01406802    (  0.0004 %)
 8 photons:       0.00000183 +-    0.00000183    (  0.0000 %)
 total:      6199.84349111 +-    1.20348763 nb

 n.     10000000 of           10000000.
 cut points  11.78 %
 ::::::>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<<<<<::::::


 ::::::>>>>>>> unweighted events <<<<<<::::::
 hit or miss efficiency   3.03 %
 unweighted events generated      151323.

 total (nb):      6205.88699138 +-     15.71006625 +
        !!!!         0.00493666 +-     0.00391866 (bias over fmax) +
        !!!!        -0.00000000 +-     0.00000000 (bias negative)
 total + biases:    6205.89192804 +-     15.71398491

 N. points with w > fmax (bias):             2
 bias/hit and bias/(hit+missed): 0.0013217 % and  0.0000400 %
 N. points with w < 0:           0
 biases for w < 0:       0.00000 % and     0.00000 %

 Upper limits fmax and sdifmax  205054.320605  223691.434065
 when there were   4 photons
 ::::::>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<<<<<::::::
```

Conclusions & Outlook

- Remarkable progress to reduce the theoretical error in luminosity measurement at flavour factories down to $\sim 0.1\%$
- ⋆ Both exact NLO and multiple photon corrections are needed to reach such an accuracy and they are implemented in the most precise MC tools
- ⋆ At least 3 EG for Bhabha scattering (**BabaYaga@NLO**, **Bhwide**, **MCGPJ**) agree within 0.2% for integrated x-section and $\sim 1\%$ (or better) for distributions
- Precision generators are also available for $\gamma\gamma$ and $\mu^+\mu^-$ final states
- NNLO QED calculations allow to assess the MC theoretical accuracy at the 0.1% level
- Possible and in progress improvements concern
  - → Tuned comparisons: extend the study done in Bhabha to $\gamma\gamma$ and $\mu^+\mu^-[\gamma]$ processes
  - → Theoretical accuracy: deeper analysis of pair corrections, 1-loop RC to $e^+e^- \to e^+e^-\gamma$ and hadronic VP