Estimate of the h_c decay width in NRQCD

Ying Fan
Peking University

In collaboration with Kuang-Ta Chao

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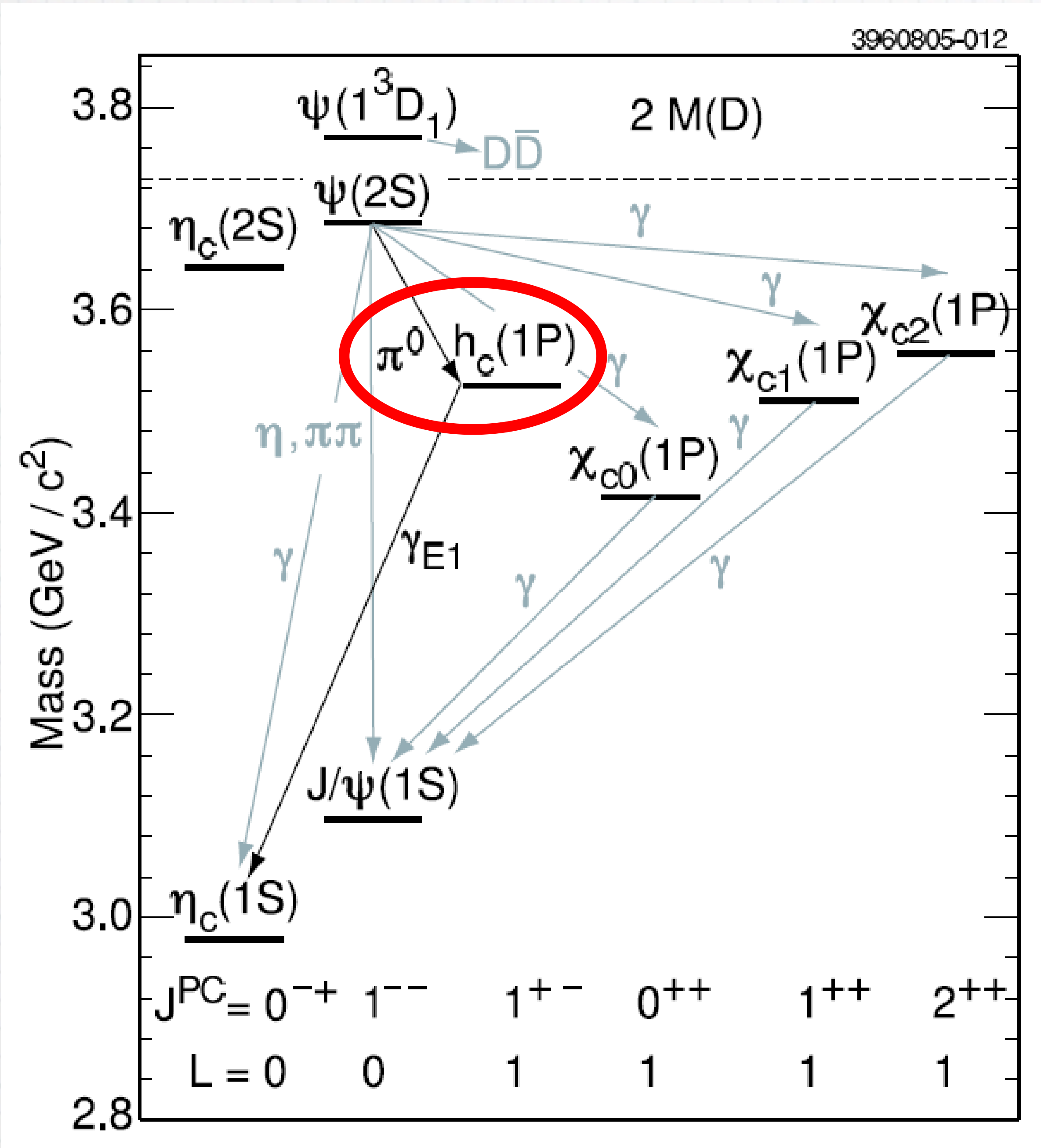
Outline

* Introduction
* Determination of h_c light hadronic and total decay width in NRQCD
* P-wave spin-triplet χ_{cJ} (J=0,1,2) light hadronic decay widths revisited
* Summary

Introduction



See talk by BESIII Collaboration at QWG7, Fermilab, 2010 for this figure

- P-wave spin-singlet charmonium state
- Mass 3525.42±0.29 MeV (PDG 10)
- Produced in exclusive decay $\psi(2S) \to \pi^0 h_c$
- E1 transition & light hadronic (LH) decay are main decay modes of h_c

Introduction

	BESIII	NRQCD	Comparison
$\Gamma(h_c)$	0.73±0.45±0.28 MeV <1.44 MeV (90% confidence level) BESIII Collaboration,PRL104, 132002 Talk by BESIII Collaboration at QWG7, Fermilab, 2010	1.1 MeV (0.53MeV LH) Bodwin et.al.,PRD46,R1914 Kuang,PRD65, 094024	consistent @ LO
		NLO LH exists Huang et.al.,PRD54, 3065 Petrelli et.al., NPB 514, 245 Maltoni, arXiv: hep-ph/0007003	NLO more persuasive

LH Decay width in NRQCD

$$\Gamma(h_c \to LH) = \sum_n \frac{2Im f_n(\mu)}{m^{d_n - 4}} \langle h_c | \mathcal{O}_n(\mu) | h_c \rangle$$

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$$|^1P_1\rangle = \mathcal{O}(1)|Q\bar{Q}(^1P_1^{[1]})\rangle + \mathcal{O}(v)|Q\bar{Q}(^1S_0^{[8]})g\rangle + \cdots$$

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typical velocity of quark,v<<1, v²≈0.3

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Color Singlet Model

LH Decay width in NRQCD

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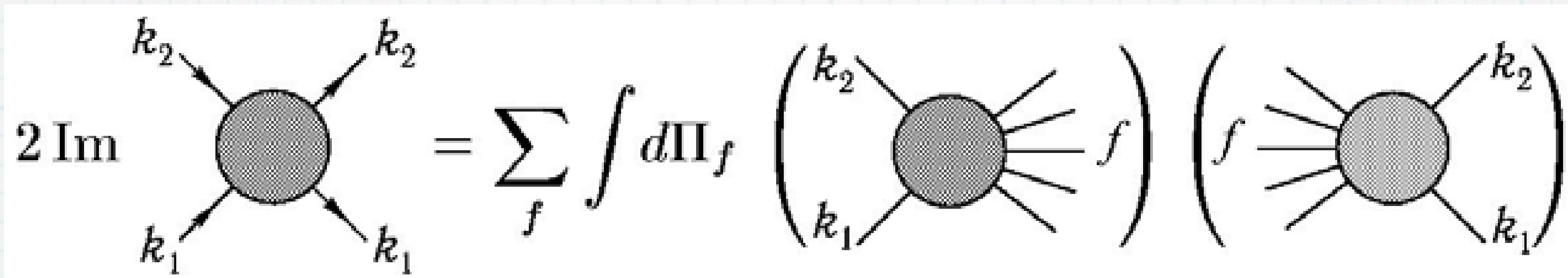
Color Singlet Model

Color-Octet Mechanism

How to determine $\mathrm{Im} f_n$

How to determine Imf_n



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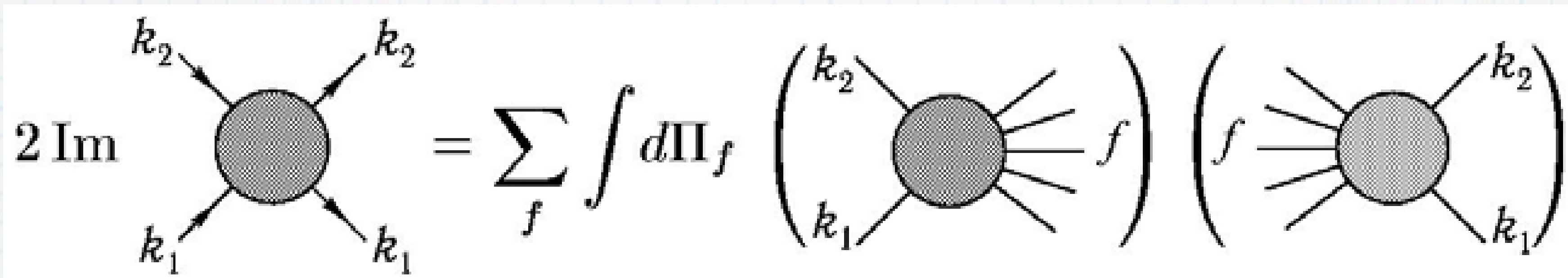

adopt optical theorem

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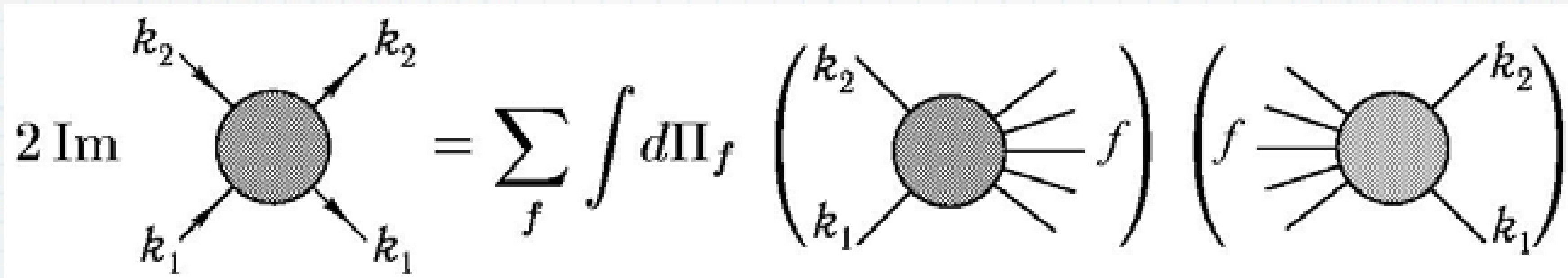

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f: all possible intermediate particles, gluons, light quarks.

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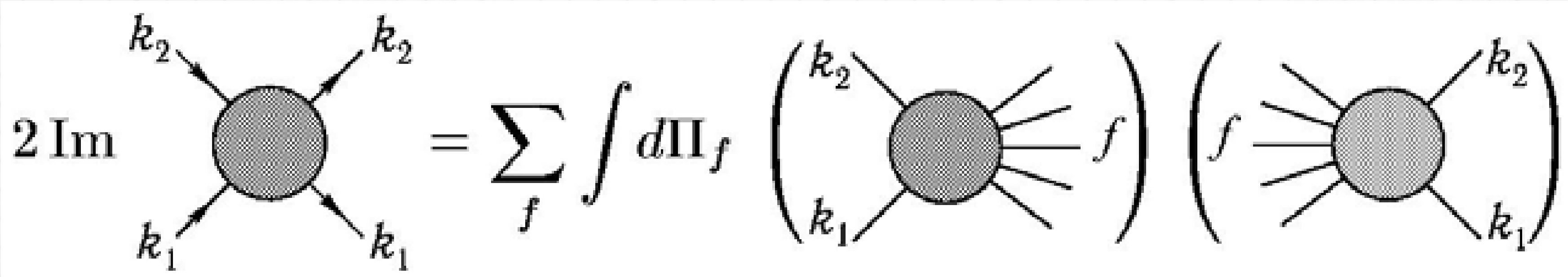

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$$\mathcal{A}(Q\bar{Q} \to Q\bar{Q})|_{pertQCD} = \sum_n \frac{f_n(\mu)}{m^{d_n-4}} \langle Q\bar{Q}|\mathcal{O}_n(\mu)|Q\bar{Q}\rangle|_{pertNRQCD}$$

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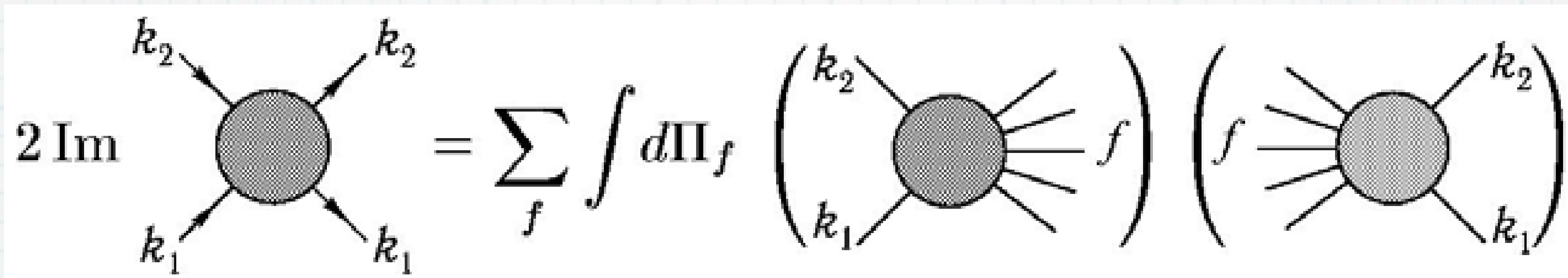

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matching condition: connecting full theory and effective theory

The h_c light hadronic decay

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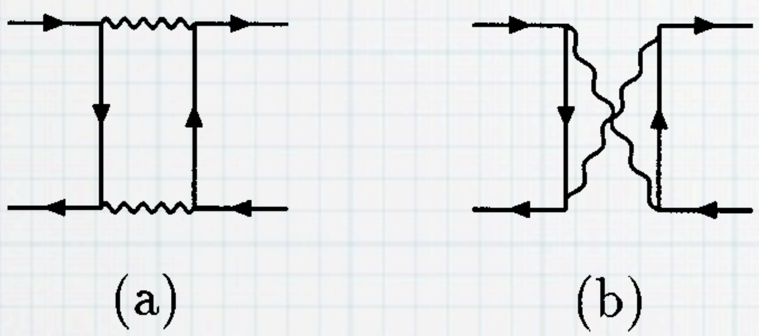
Representative example: divergence cancelled by introducing color-octet mechanism

$\mathrm{Im} f_1({}^1P_1)$: full QCD

Huang et.al.,PRD54, 3065 (1996)



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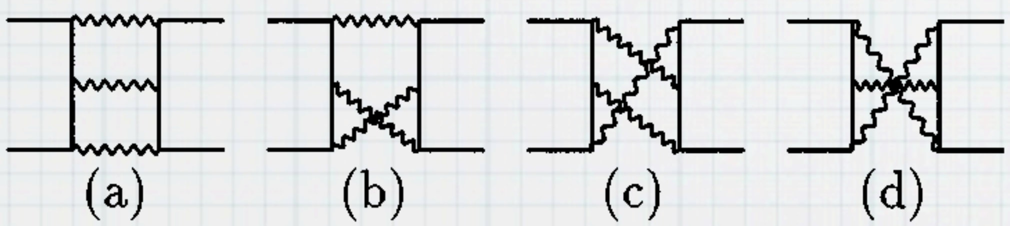

$$\frac{[\mathrm{Im} f_8({}^1S_0)]_0}{m^2}\,\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{IR}}-\gamma_E+\ln\frac{4\pi\mu_{IR}^2}{4m^2}\right)+\frac{7\pi^2-112}{48}\right]$$

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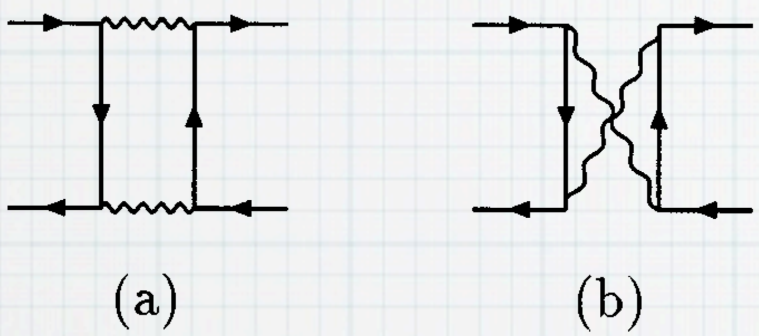

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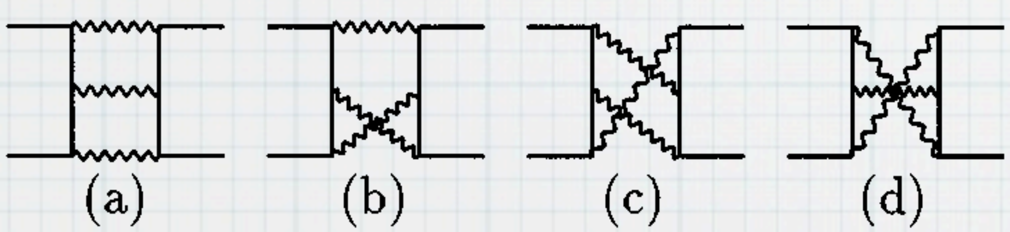

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residual divergence

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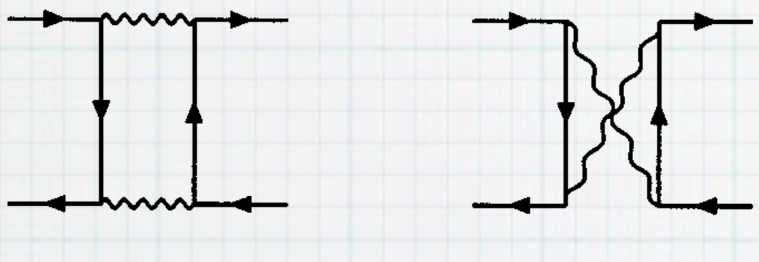

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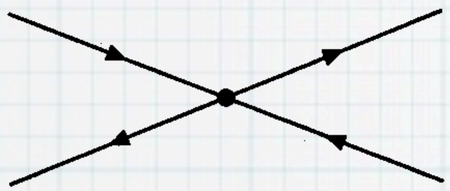
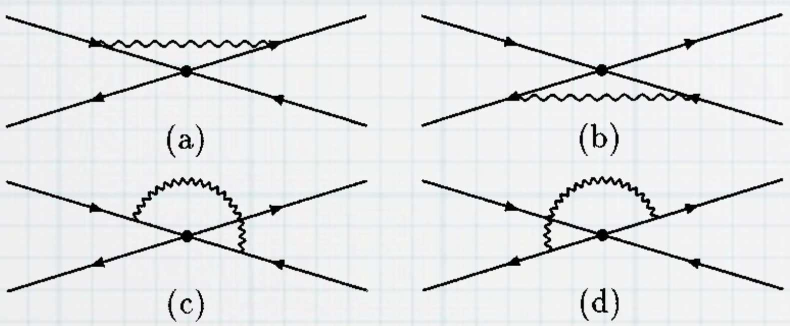
expression 1

$\mathrm{Im} f_1(^1P_1)$: NRQCD



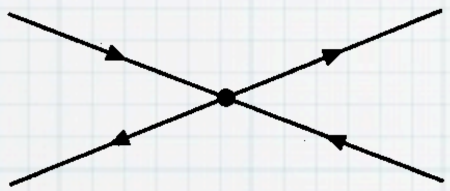
$\mathcal{O}_1(^1P_1)$



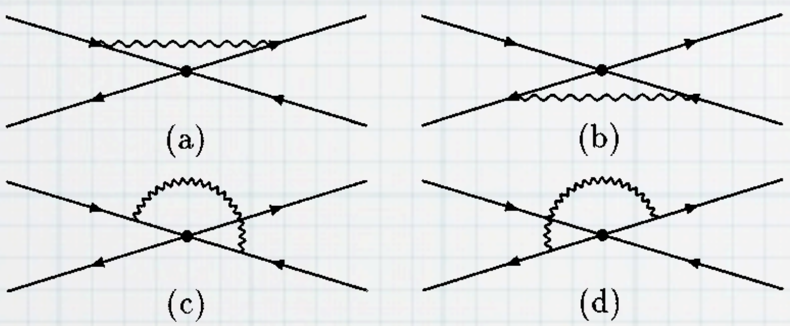
$$\frac{\mathrm{Im} f_8(^1S_0)}{m^2}\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{UV}}^2}{4m^2}\right)\right]$$

$$\frac{\mathrm{Im} f_1(^1P_1)}{m^2}+\frac{[\,\mathrm{Im} f_8(^1S_0)]_0}{m^2}\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\ln\frac{\mu_{\mathrm{UV}}}{2m}\right]$$

Imf$_1$($^1P_1$): NRQCD



$\mathcal{O}_1(^1P_1)$ tree level matrix



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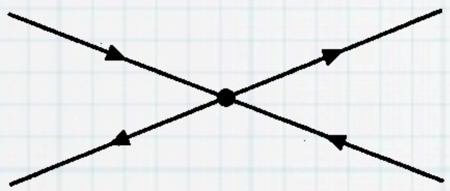

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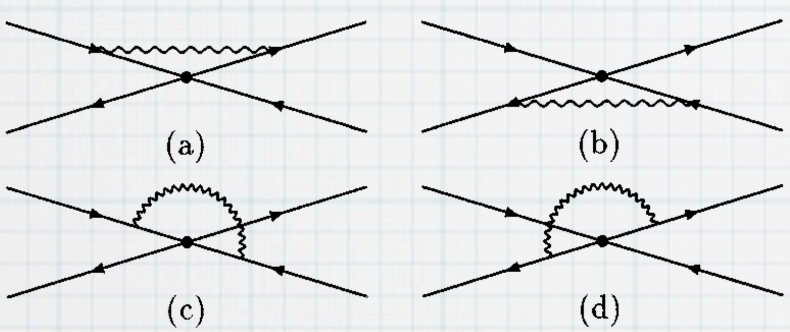

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real corrections: gluon line connecting with incoming and outgoing quark or antiquark lines

$$\frac{\mathrm{Im}f_1(^1P_1)}{m^2}+\frac{[\,\mathrm{Im}f_8(^1S_0)]_0}{m^2}\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\ln\frac{\mu_{\mathrm{UV}}}{2m}\right]$$

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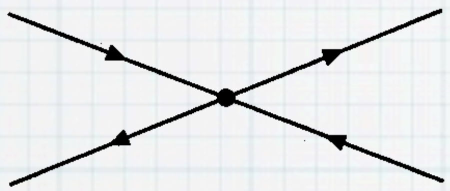

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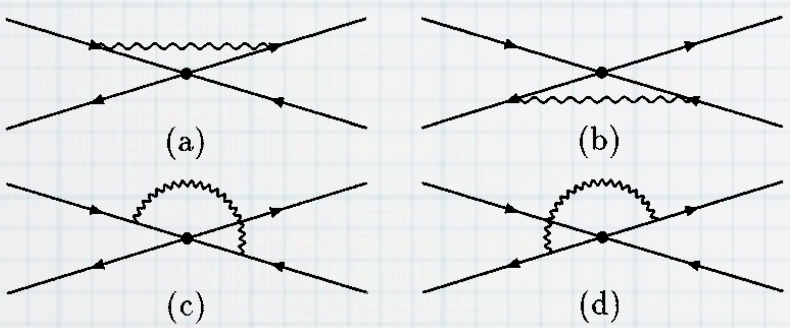

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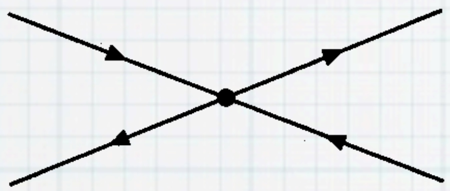

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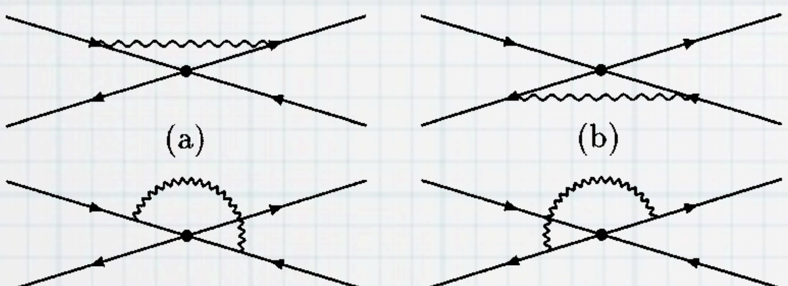

the same divergence

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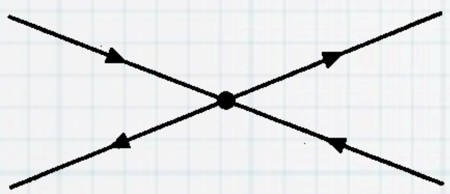

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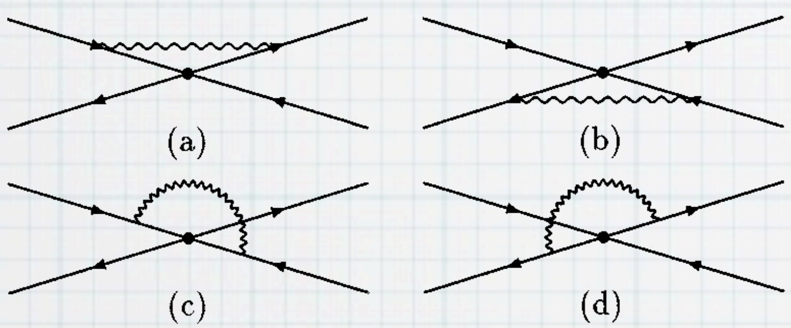
the same divergence



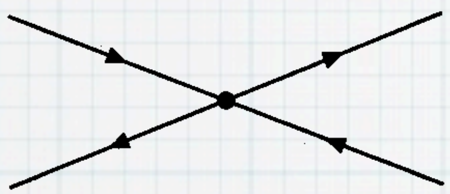
$$\frac{\mathrm{Im} f_8(^1S_0)}{m^2}\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{UV}}^2}{4m^2}\right)\right]$$

real corrections: gluon line connecting with incoming and outgoing quark or antiquark lines

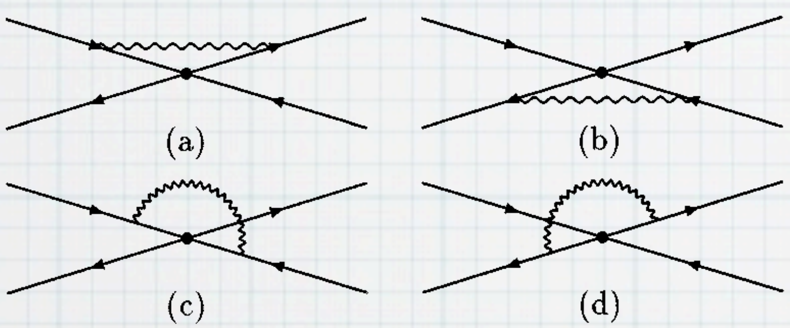
NRQCD result:

$$\frac{\mathrm{Im} f_1(^1P_1)}{m^2}+\frac{[\,\mathrm{Im} f_8(^1S_0)]_0}{m^2}\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\ln\frac{\mu_{\mathrm{UV}}}{2m}\right]$$

$\mathrm{Im}f_1({}^1P_1)$: NRQCD



$\mathcal{O}_1({}^1P_1)$ tree level matrix



the same divergence

$$\frac{\mathrm{Im}f_8({}^1S_0)}{m^2}\,\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{UV}}^2}{4m^2}\right)\right]$$

real corrections: gluon line connecting with incoming and outgoing quark or antiquark lines

NRQCD result:

$$\frac{\mathrm{Im}f_1({}^1P_1)}{m^2}+\frac{[\,\mathrm{Im}f_8({}^1S_0)]_0}{m^2}\,\frac{4C_F\alpha_s}{3N_c\pi}\left[-\frac{1}{2}\left(\frac{1}{\epsilon_{\mathrm{IR}}}-\gamma_E+\ln\frac{4\pi\mu_{\mathrm{IR}}^2}{4m^2}\right)+\ln\frac{\mu_{\mathrm{UV}}}{2m}\right]$$

expression 2

finite $\mathrm{Im} f_1(^1P_1)$:

$$\mathrm{Im} f_1(^1P_1) = \frac{(N_c^2-4)C_F\alpha_s^3}{3N_c^2}\left(\frac{7\pi^2-112}{48} - \ln\frac{\mu}{2m}\right)$$

finite $\mathrm{Im} f_1(^1P_1)$:

matching expression 1 and expression 2

$$\mathrm{Im} f_1(^1P_1) = \frac{(N_c^2-4)C_F\alpha_s^3}{3N_c^2}\left(\frac{7\pi^2-112}{48} - \ln\frac{\mu}{2m}\right)$$

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residual divergence cancelled by introducing color-octet mechanism,finite NLO short-distance coefficient of color-singlet $^1P_1$ component

Im$f_8(^1S_0)$: full QCD

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right.\right.$$

$$\left.\left.+4b_0\ln\frac{\mu}{2m}+A\right]\right\}$$

$$A=C_F\left(\frac{\pi^2}{4}-5\right)+C_A\left(\frac{122}{9}-\frac{17\pi^2}{24}\right)-\frac{8}{9}n_f$$

$\mathrm{Im} f_8({}^1S_0)$: full QCD

Real & virtual corrections

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right.\right.$$

$$\left.\left.+4b_0\ln\frac{\mu}{2m}+A\right]\right\}$$

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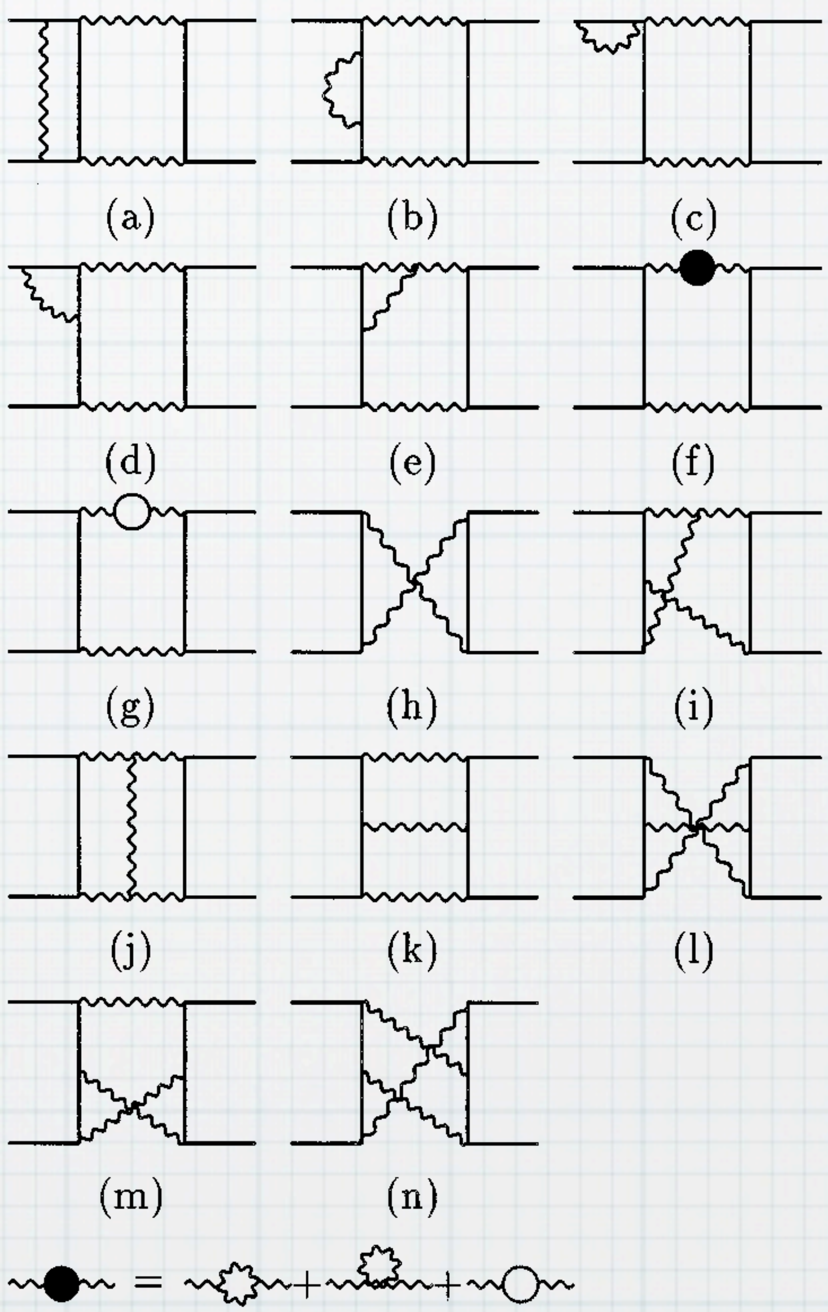
Im $f_8(^1S_0)$: full QCD

Real & virtual corrections



$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right.\right.$$

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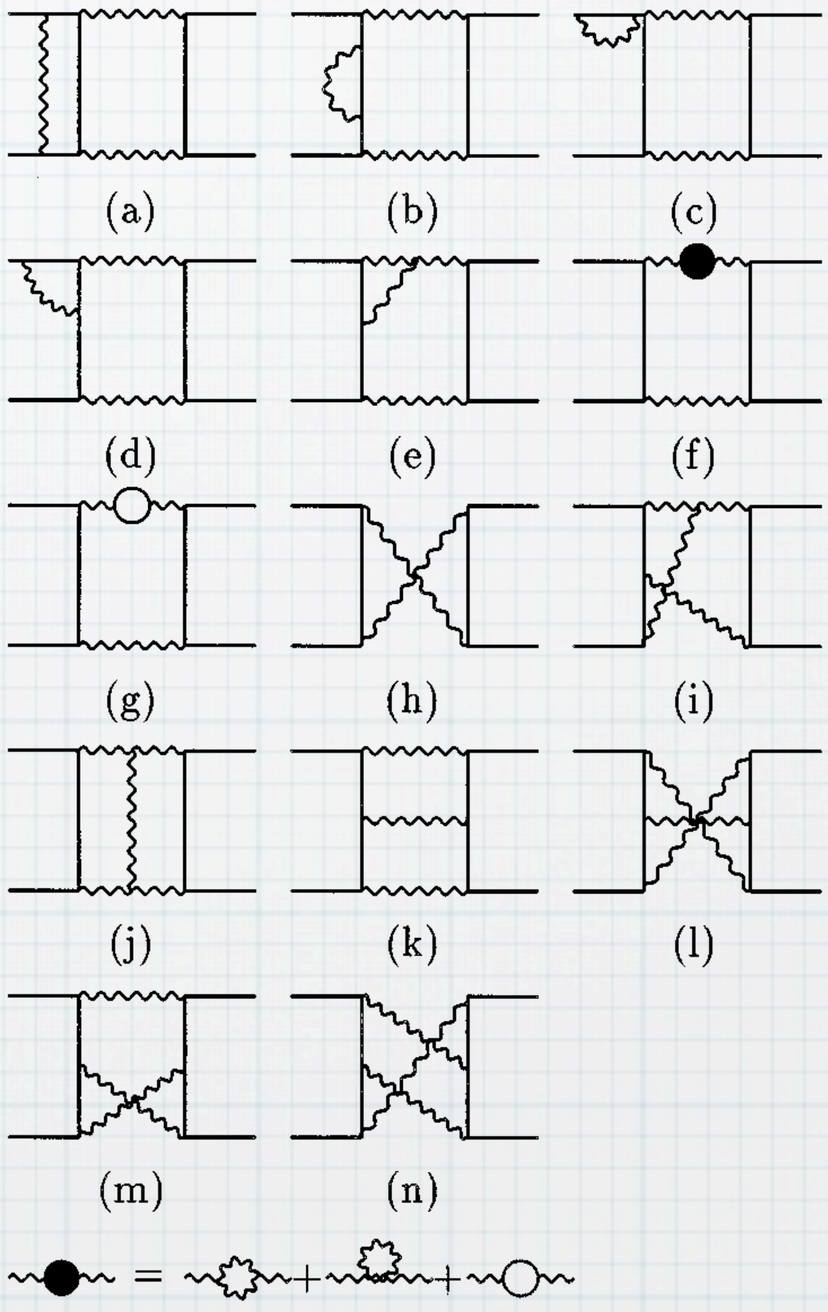
$\mathrm{Im}f_8(^1S_0)$: full QCD

Real & virtual corrections



After renormalized

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right.\right.$$

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Im$f_8(^1S_0)$: full QCD

Real & virtual corrections

After renormalized

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}+4b_0\ln\frac{\mu}{2m}+A\right]\right\}$$

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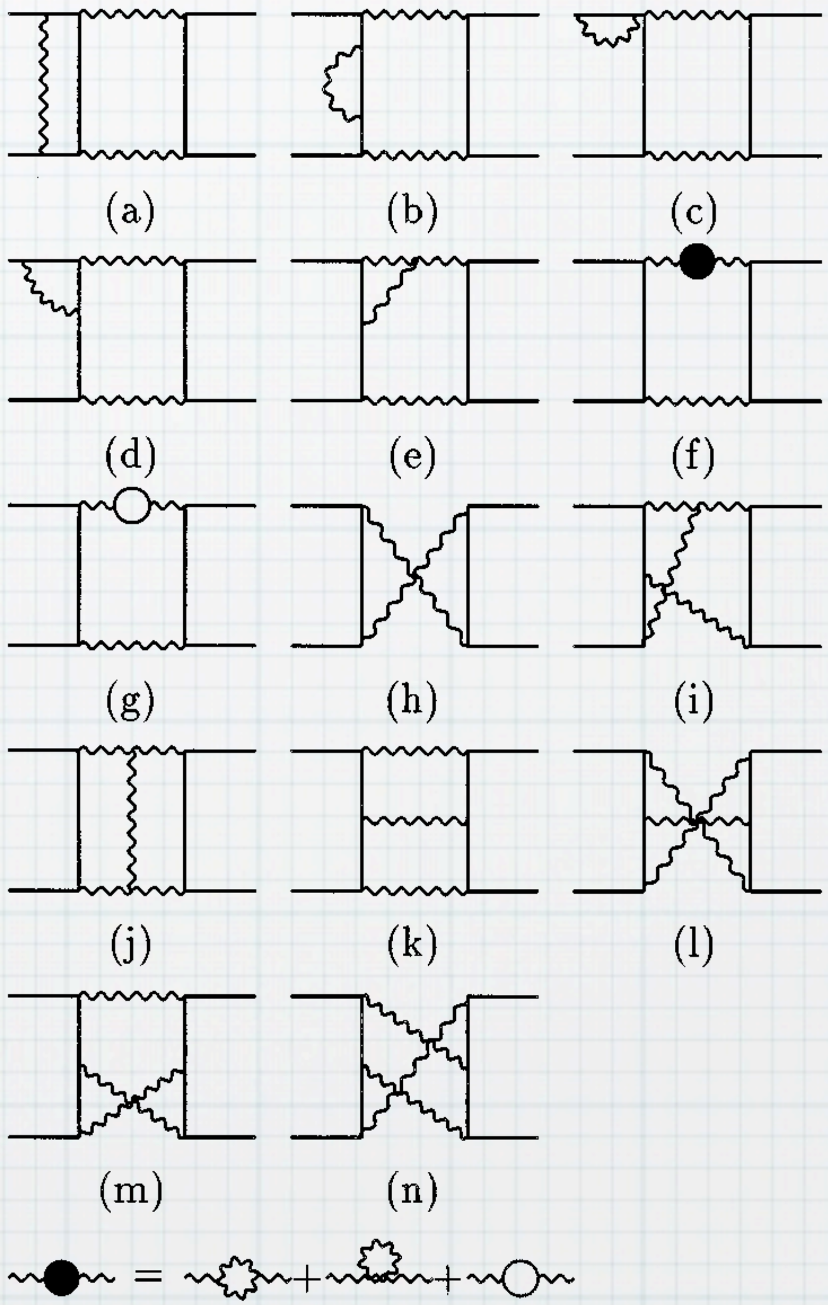
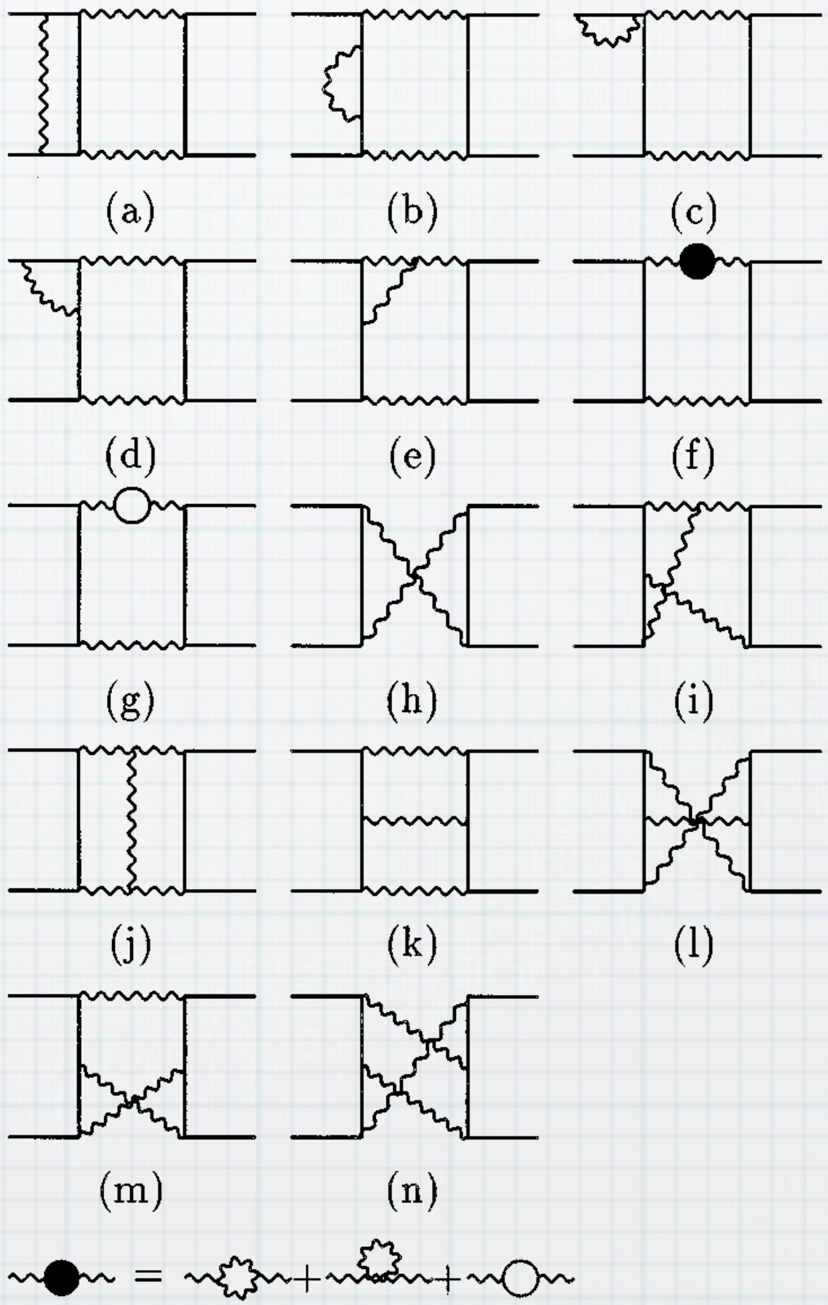


$\mathrm{Im} f_8({}^1S_0)$: full QCD

Real & virtual corrections



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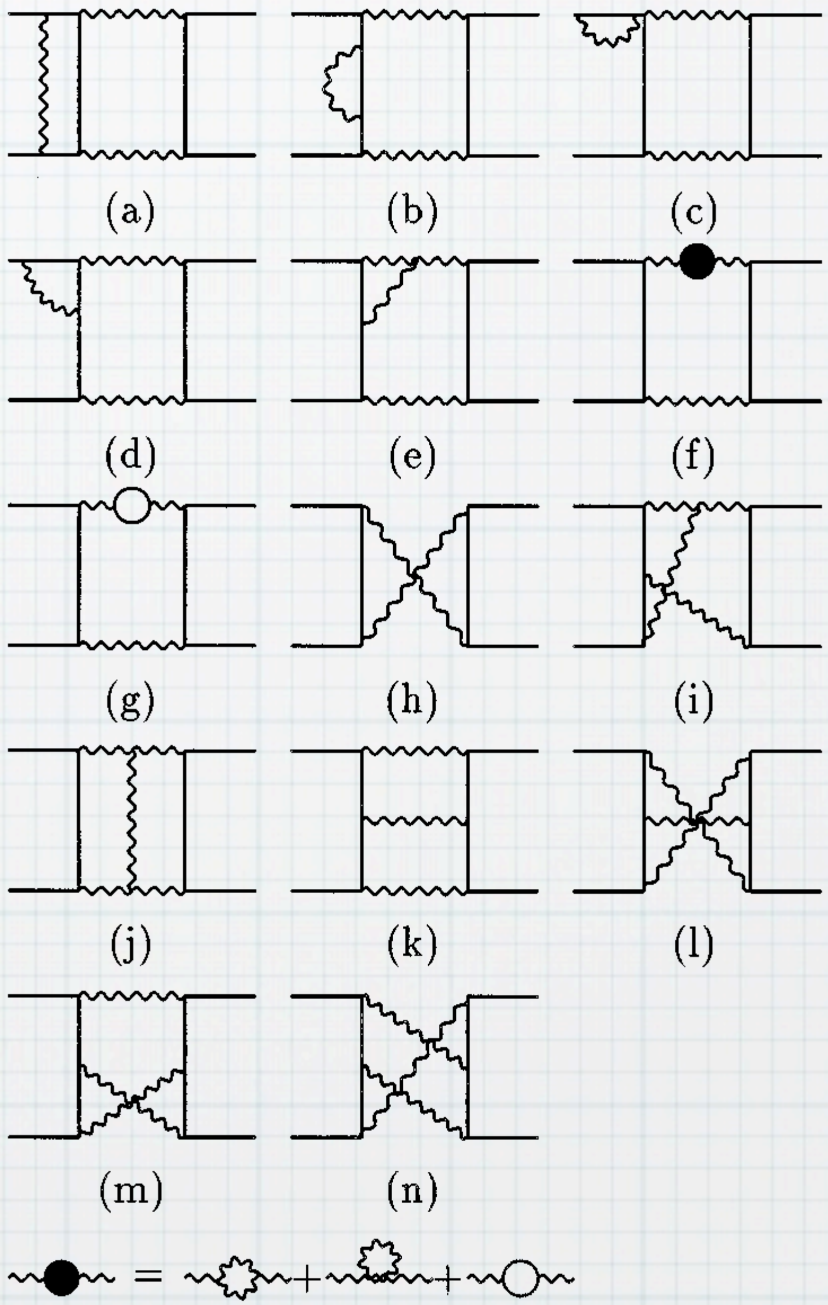
Im$f_8(^1S_0)$: full QCD

Real & virtual corrections



After renormalized

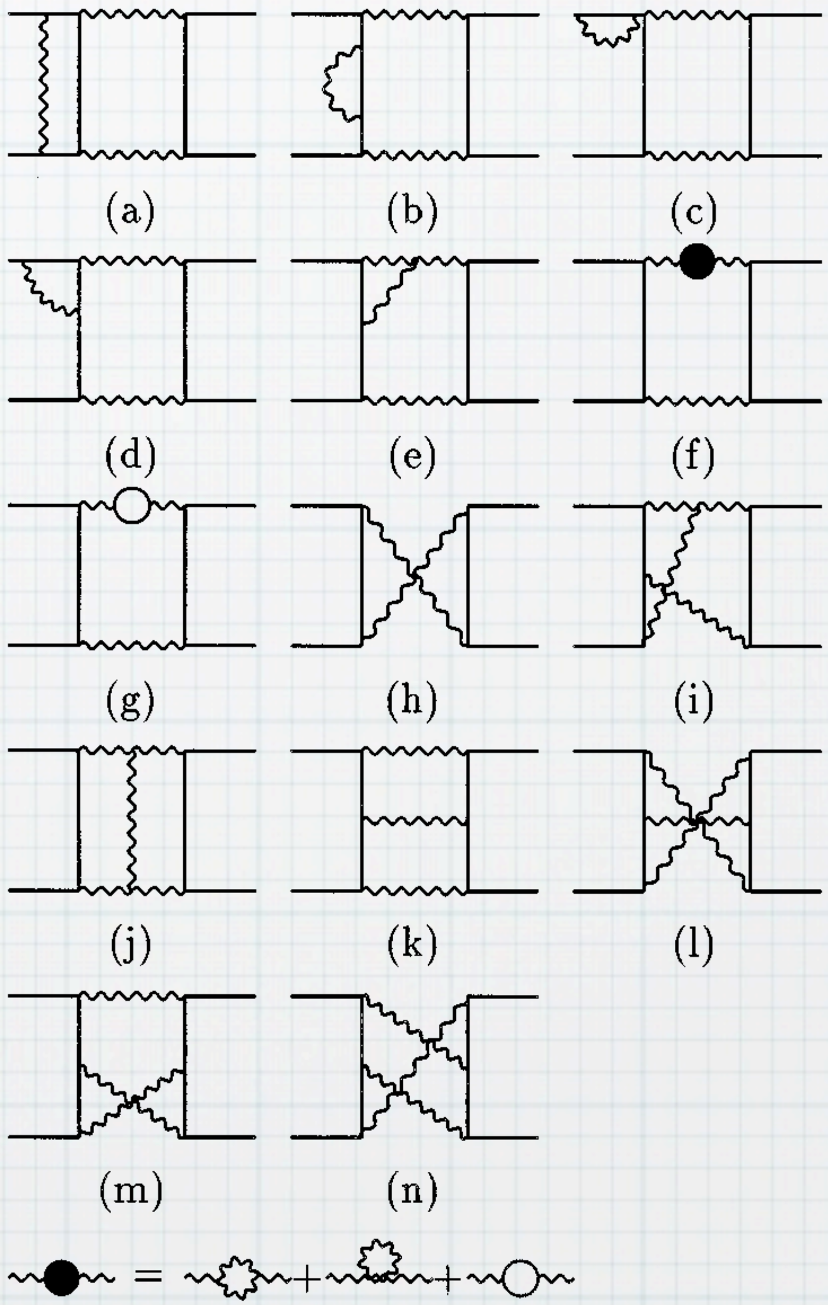
Coulomb singularity

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}+4b_0\ln\frac{\mu}{2m}+A\right]\right\}$$

$$A=C_F\left(\frac{\pi^2}{4}-5\right)+C_A\left(\frac{122}{9}-\frac{17\pi^2}{24}\right)-\frac{8}{9}n_f$$

$\mathrm{Im} f_8(^1S_0)$: full QCD

Real & virtual corrections



Coulomb singularity

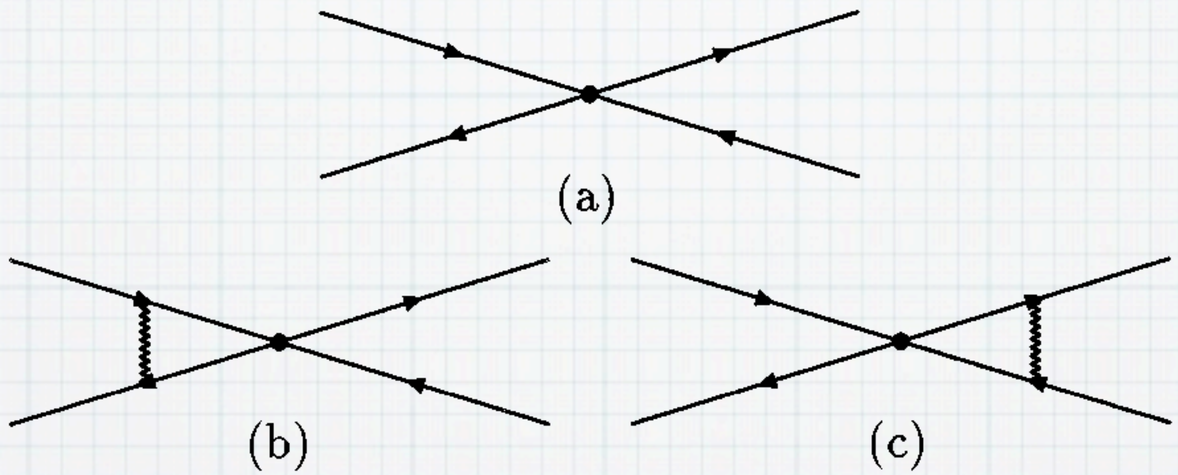
After renormalized

$$\frac{\pi(N_c^2-4)}{4N_c m^2}\alpha_s^2\left\{1+\frac{\alpha_s}{\pi}\left[\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}+4b_0\ln\frac{\mu}{2m}+A\right]\right\}$$

$$A=C_F\left(\frac{\pi^2}{4}-5\right)+C_A\left(\frac{122}{9}-\frac{17\pi^2}{24}\right)-\frac{8}{9}n_f$$

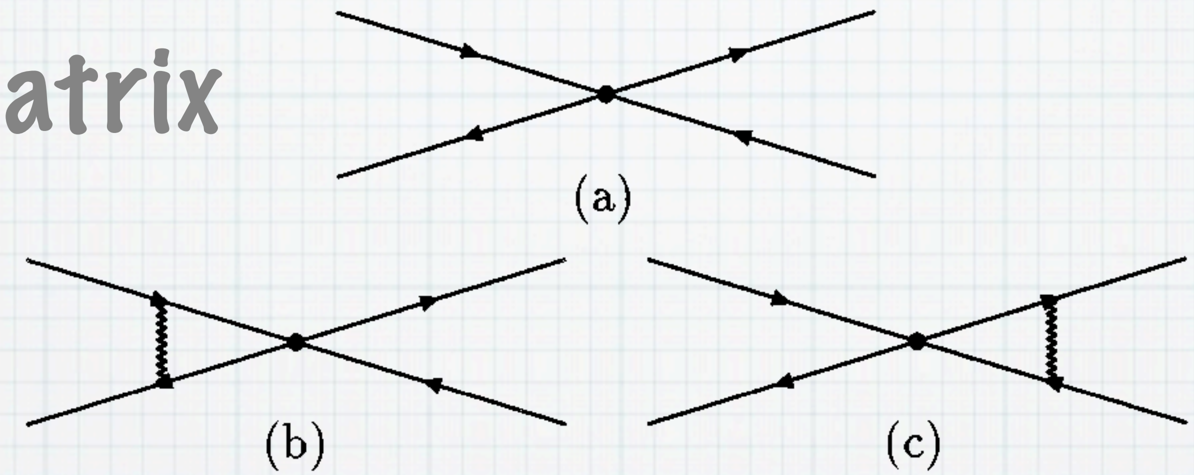
expression 3

$\mathrm{Im} f_8(^1S_0)$: NRQCD



Im $f_8(^1S_0)$: NRQCD

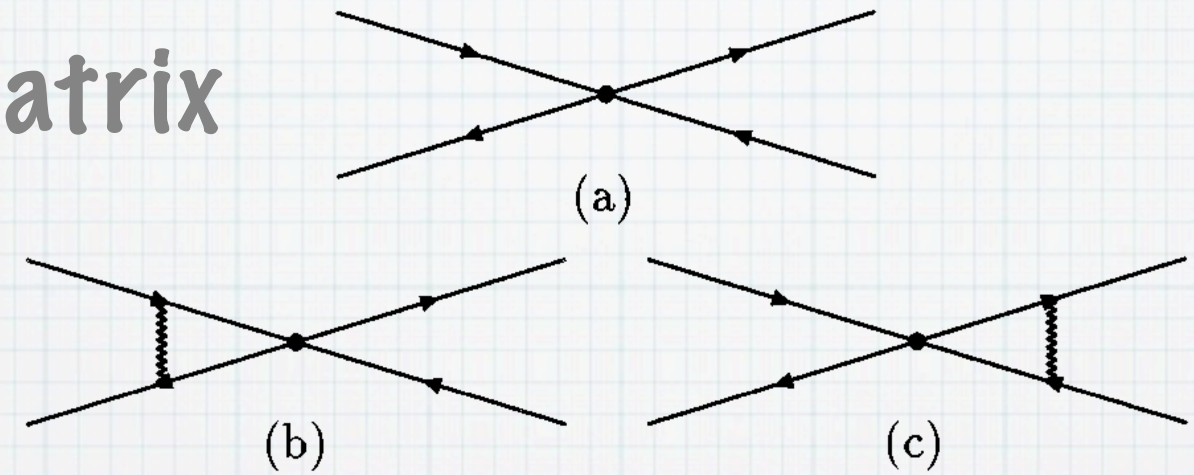
LO matrix



Im$f_8(^1S_0)$: NRQCD

LO matrix

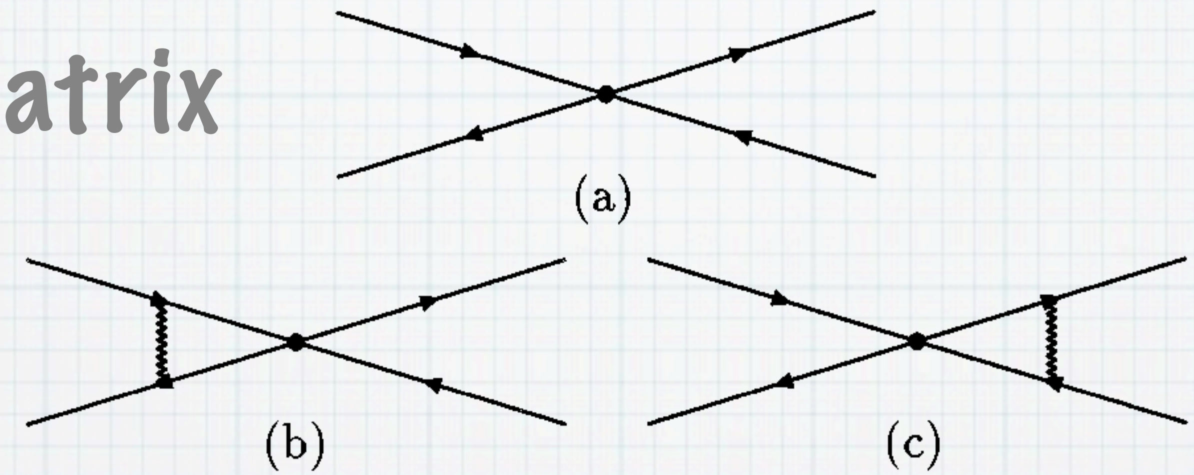
NLO matrix



$\mathrm{Im} f_8({}^1S_0)$: NRQCD

LO matrix

NLO matrix

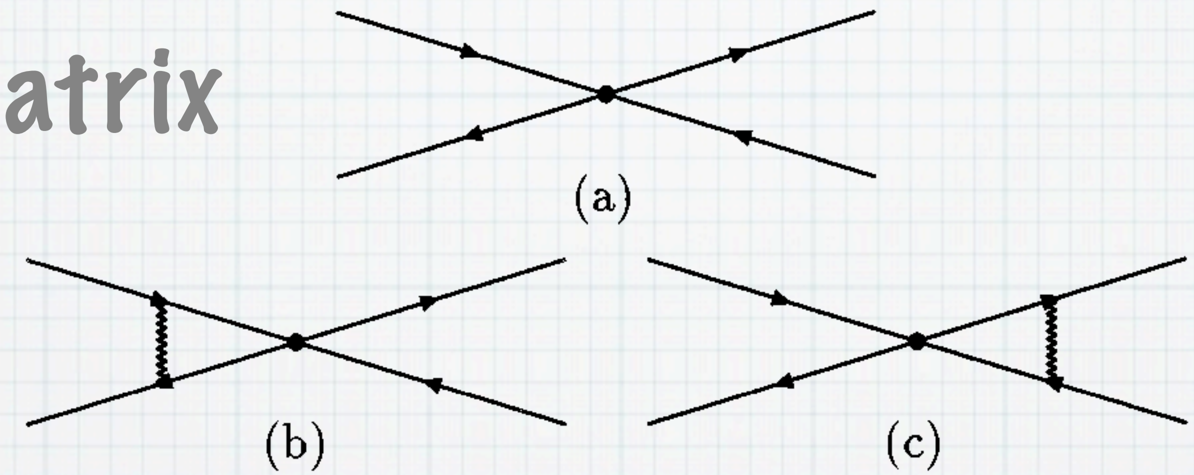


gluon lines connecting between incoming or outgoing heavy quark pairs

$\mathrm{Im} f_8(^1S_0)$: NRQCD

LO matrix

NLO matrix

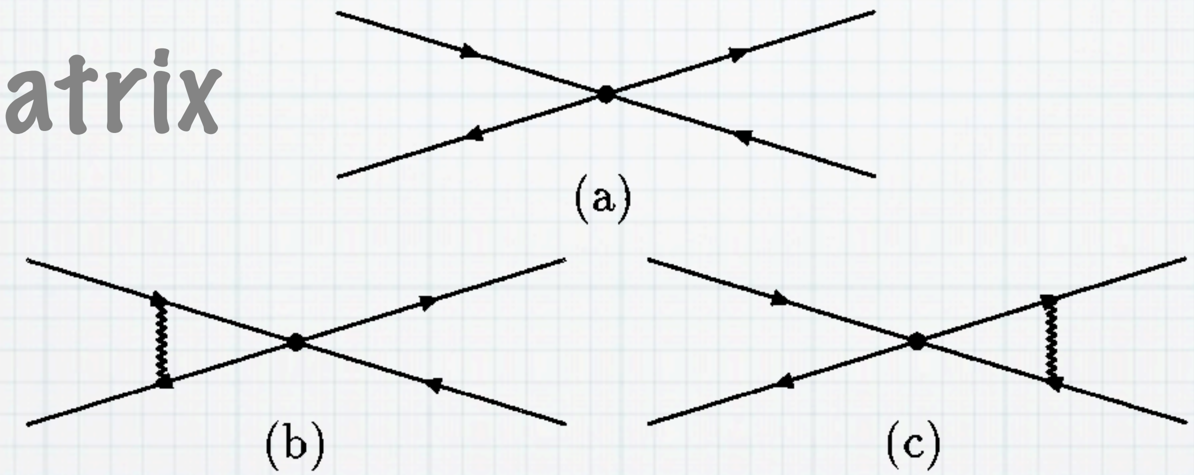


gluon lines connecting between incoming or outgoing heavy quark pairs

$$\frac{\mathrm{Im} f_8(^1S_0)}{m^2}\left[1+\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right]$$

$\mathrm{Im} f_8(^1S_0)$: NRQCD

LO matrix



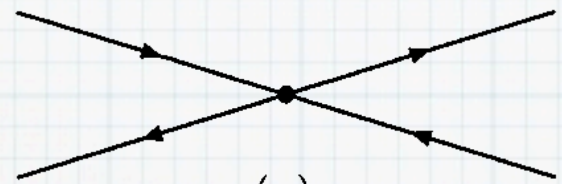
NLO matrix

gluon lines connecting between incoming or outgoing heavy quark pairs

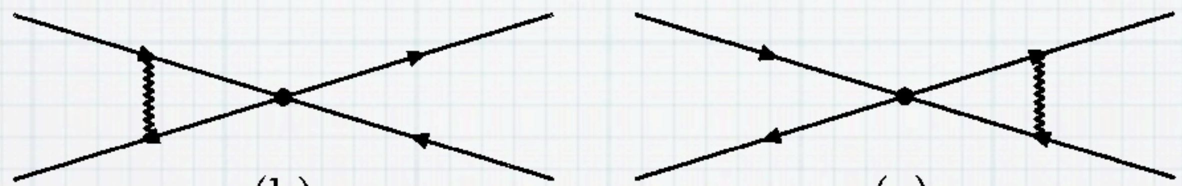
$$\frac{\mathrm{Im} f_8(^1S_0)}{m^2}\left[1+\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{\pi^2}{2v}\right]$$

$\mathrm{Im} f_8(^1S_0)$: NRQCD

LO matrix



NLO matrix



gluon lines connecting between incoming or outgoing heavy quark pairs

expression 4

$$\frac{\mathrm{Im} f_8(^1S_0)}{m^2}\left[1 + \frac{\alpha_s}{\pi}\left(C_F - \frac{C_A}{2}\right)\boxed{\frac{\pi^2}{2v}}\right]$$

finite $\mathrm{Im} f_8(^1S_0)$:

finite $\mathrm{Im} f_8(^1S_0)$:

matching expression 3 and expression 4

finite $\mathrm{Im} f_8(^1S_0)$:

matching expression 3 and expression 4

$$\mathrm{Im} f_8(^1S_0) = \frac{(N_c^2-4)\pi\alpha_s^2}{4N_c}\left[1+\frac{\alpha_s}{\pi}\left(4b_0 ln\frac{\mu}{2m}+A\right)\right]$$

finite $\mathrm{Im} f_8(^1S_0)$:

matching expression 3 and expression 4

$$\mathrm{Im} f_8(^1S_0) = \frac{(N_c^2-4)\pi\alpha_s^2}{4N_c}\left[1+\frac{\alpha_s}{\pi}\left(4b_0 ln\frac{\mu}{2m}+A\right)\right]$$

Coulomb singularity cancelled,finite NLO short-distance coefficient of color-octet $^1S_0$ component

finite LH decay width of h_c

$$\Gamma(h_c \to LH) = 2\mathrm{Im} f_1({}^1P_1^{[1]})H_1 + 2\mathrm{Im} f_8({}^1S_0^{[8]})H_8$$

$$= \frac{2(N_c^2-4)C_F\alpha_s^3}{3N_c^2}\left(\frac{7\pi^2-112}{48} - \ln\frac{\mu}{2m}\right)H_1$$

$$+ \frac{(N_c^2-4)\pi\alpha_s^2(\mu)}{2N_c}\left[1+\frac{\alpha_s}{\pi}\left(4b_0\ln\frac{\mu}{2m}+A\right)\right]H_8(\mu)$$

long-distance matrix elements

experimental extraction

operator evolution equation

long-distance matrix elements

Method I

experimental extraction

operator evolution equation

long-distance matrix elements

Method I

experimental extraction

Method II

operator evolution equation

long-distance matrix elements

Method I

experimental extraction

Method II

operator evolution equation

$$\Gamma(\chi_J \rightarrow LH) = 2\,\mathrm{Im} f_1(^3P_J) H_1 + 2\,\mathrm{Im} f_8(^3S_1) H_8 + O(v^2\Gamma)$$

long-distance matrix elements

Method I

experimental extraction

Method II

operator evolution equation

$$\Gamma(\chi_J \to LH) = 2\,\mathrm{Im} f_1(^3P_J) H_1 + 2\,\mathrm{Im} f_8(^3S_1) H_8 + O(v^2\Gamma)$$

$$\Gamma(h \to LH) = 2\,\mathrm{Im} f_1(^1P_1) H_1 + 2\,\mathrm{Im} f_8(^1S_0) H_8 + O(v^2\Gamma)$$

long-distance matrix elements

Method I

experimental extraction

Method II

operator evolution equation

$$\Gamma(\chi_J \to LH) = 2\,\mathrm{Im} f_1({}^3P_J) H_1 + 2\,\mathrm{Im} f_8({}^3S_1) H_8 + O(v^2\Gamma)$$

$$\Gamma(h \to LH) = 2\,\mathrm{Im} f_1({}^1P_1) H_1 + 2\,\mathrm{Im} f_8({}^1S_0) H_8 + O(v^2\Gamma)$$

long-distance matrix elements

Method I

experimental extraction

$$\Gamma(\chi_J \to LH) = 2\,\mathrm{Im} f_1({}^3P_J) H_1 + 2\,\mathrm{Im} f_8({}^3S_1) H_8 + O(v^2\Gamma)$$

$$\Gamma(h \to LH) = 2\,\mathrm{Im} f_1({}^1P_1) H_1 + 2\,\mathrm{Im} f_8({}^1S_0) H_8 + O(v^2\Gamma)$$

Method II

operator evolution equation

long-distance matrix elements

Method I

experimental extraction

$$\Gamma(\chi_J \to LH) = 2\mathrm{Im} f_1(^3P_J) H_1 + 2\mathrm{Im} f_8(^3S_1) H_8 + O(v^2\Gamma)$$

$$\Gamma(h \to LH) = 2\mathrm{Im} f_1(^1P_1) H_1 + 2\mathrm{Im} f_8(^1S_0) H_8 + O(v^2\Gamma)$$

Two input parameters:

$\chi_{c1}, \chi_{c2} \to LH$

two unknown ones:

H_1, H_8

process dependent

Method II

operator evolution equation

long-distance matrix elements

Method I

experimental extraction

$$\Gamma(\chi_J \to LH) = 2\,\mathrm{Im} f_1({}^3P_J) H_1 + 2\,\mathrm{Im} f_8({}^3S_1) H_8 + O(v^2\Gamma)$$

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Two input parameters:

$\chi_{c1}, \chi_{c2} \to LH$

two unknown ones:

H_1, H_8

process dependent

Method II

operator evolution equation

$$\mu \frac{\partial \mathcal{O}_8({}^1S_0)}{\partial \mu} = \alpha_s(\mu) \frac{4 C_F}{3\pi N_c m^2} \mathcal{O}_1({}^1P_1)$$

long-distance matrix elements

Method I

experimental extraction

$$\Gamma(\chi_J \to LH) = 2\,\mathrm{Im} f_1({}^3P_J) H_1 + 2\,\mathrm{Im} f_8({}^3S_1) H_8 + O(v^2\Gamma)$$

$$\Gamma(h \to LH) = 2\,\mathrm{Im} f_1({}^1P_1) H_1 + 2\,\mathrm{Im} f_8({}^1S_0) H_8 + O(v^2\Gamma)$$

Two input parameters:
$\chi_{c1}, \chi_{c2} \to LH$

two unknown ones:
H_1, H_8
process dependent

Method II

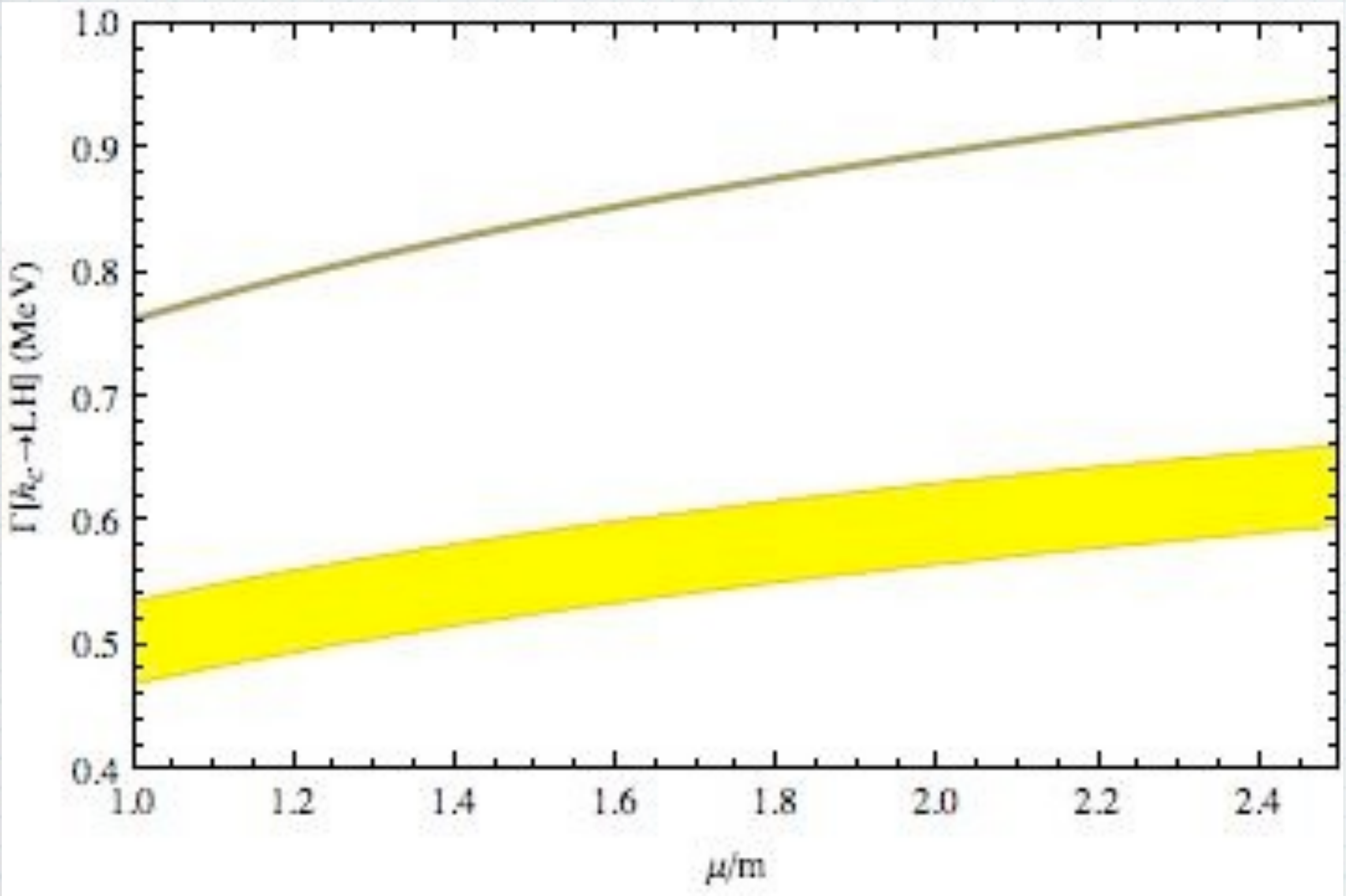
operator evolution equation

$$\mu \frac{\partial \mathcal{O}_8({}^1S_0)}{\partial \mu} = \alpha_s(\mu) \frac{4C_F}{3\pi N_c m^2} \mathcal{O}_1({}^1P_1)$$

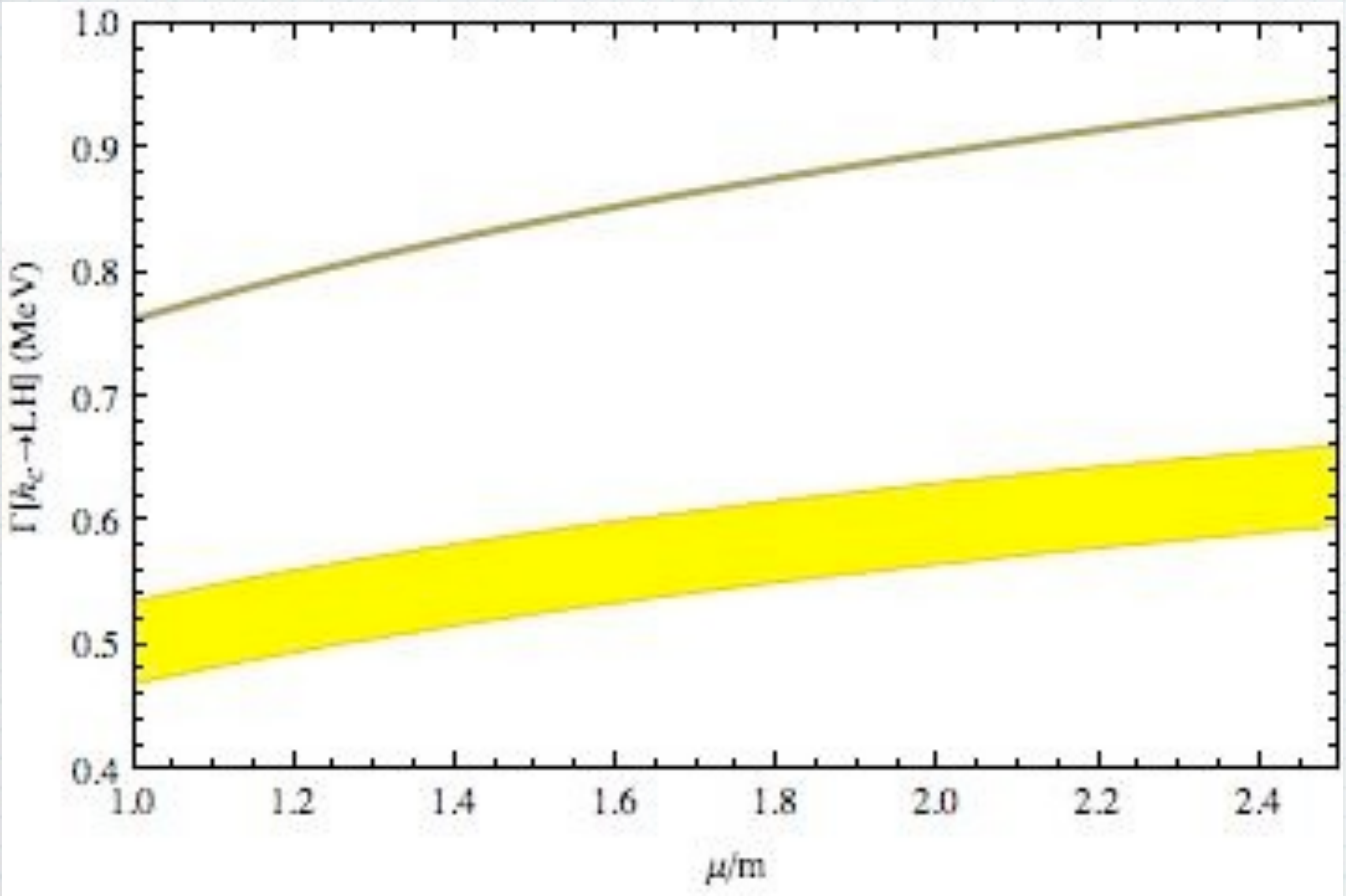
based on operator mixing

evolution by
renormalization scale μ

NLO LH decay width in NRQCD

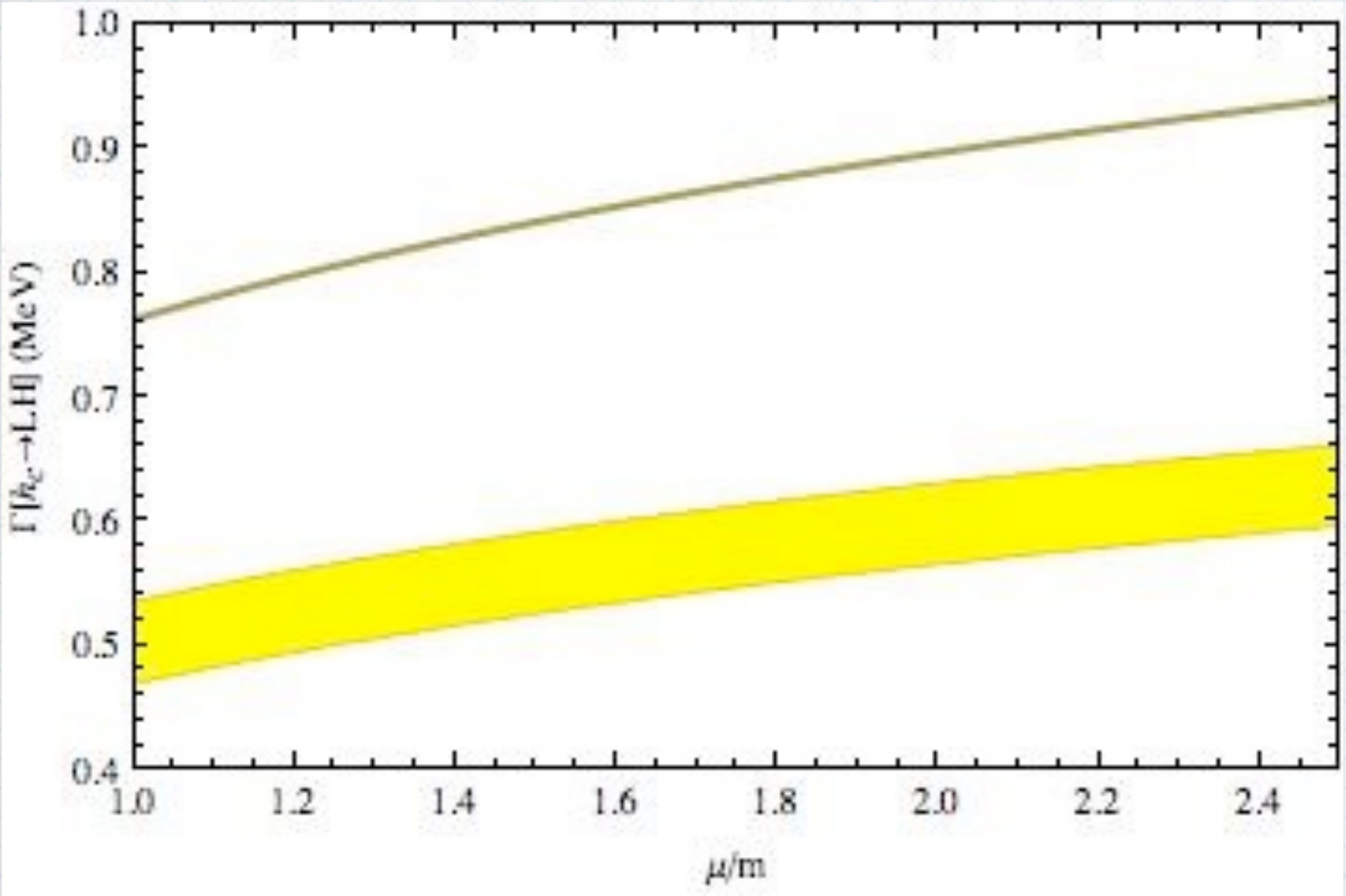


NLO LH decay width in NRQCD



shaded region: method I
single curve: method II

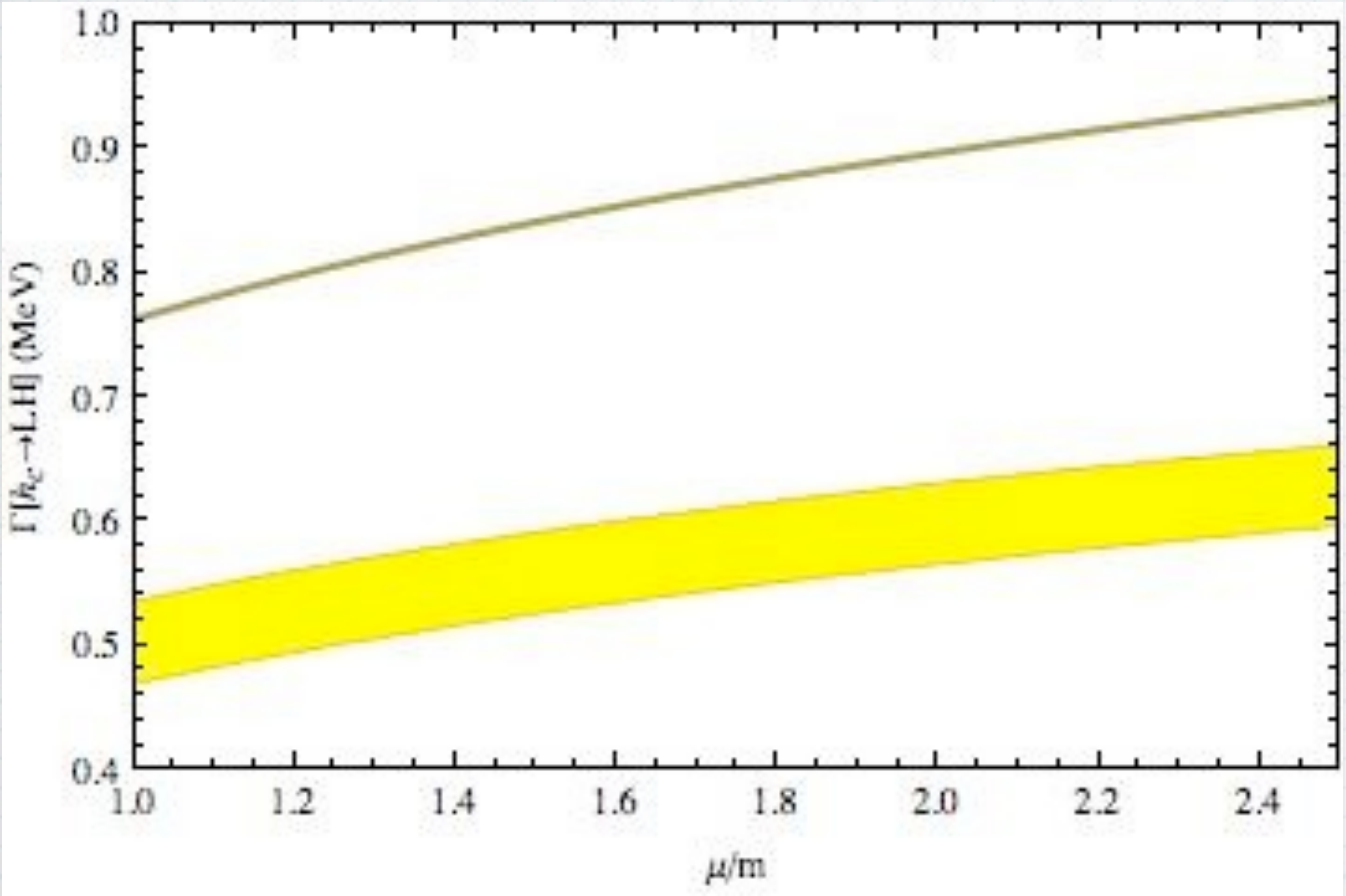
NLO LH decay width in NRQCD



At renormalization scale
$\mu=2m$

shaded region: method I
single curve: method II

NLO LH decay width in NRQCD



At renormalization scale
μ=2m

- 0.597±0.032 MeV by Method I
- 0.895 MeV by Method II

shaded region: method I
single curve: method II

E1 transition width

E1 transition width

spin-symmetry

E1 transition width

spin-symmetry

$$\Gamma({}^1P_1 \to \gamma\, {}^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma({}^3P_J \to \gamma\, {}^3S_1)$$

E1 transition width

spin-symmetry

$$\Gamma({}^1P_1 \to \gamma\, {}^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma({}^3P_J \to \gamma\, {}^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

average among χ_{cJ}

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

→

average among χ_{cJ}

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

average among χ_{cJ}

0.600 MeV

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10 → average among χ_{cJ} → 0.600 MeV

(Chao et.al., PLB301, 282)

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\,^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\,^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

average among χ_{cJ}

→ 0.600 MeV

Leading order: 0.646 MeV

(Chao et.al., PLB301, 282)

E1 transition width

spin-symmetry

$$\Gamma(^1P_1 \to \gamma\, ^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma(^3P_J \to \gamma\, ^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10

average among χ_{cJ}

0.600 MeV

Leading order: 0.646 MeV

relativistic correction

(Chao et.al., PLB301, 282)

E1 transition width

spin-symmetry

$$\Gamma({}^1P_1 \to \gamma\, {}^1S_0) = \left(\frac{E_\gamma^h}{E_\gamma^\chi}\right)^3 \Gamma({}^3P_J \to \gamma\, {}^3S_1)$$

(Maltoni, arXiv: hep-ph/0007003)

plug into PDG10 → average among χ_{cJ} → 0.600 MeV

relativistic correction

(Chao et.al., PLB301, 282)

Leading order: 0.646 MeV

next-to-leading order: 0.383 MeV

Total width of h_c in NRQCD

Total width of h_c in NRQCD

$$\Gamma_{\rm TOT} = \Gamma(h_c \to {\rm L}H) + \Gamma(h_c \to \gamma\eta_c)$$

Total width of h_c in NRQCD

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BESIII Collaboration,PRL104, 132002

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BESIII Collaboration,PRL104, 132002

larger than BESIII central value 0.73 MeV

Total width of h_c in NRQCD

0.895 MeV by Method II , error~30% compared to
0.597 MeV by Method I

Evolution equation is a good method to evaluate P-wave long-distance matrix element, and can be extended to D-wave LH decay, non-$D\bar{D}$ decay which are lack of data.

Error Estimate

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* For method I, experimental data errors;
* For method II, errors from first-order derivative of wave function at the origin and lower limits in evolution equation

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TABLE I. Radial wave functions at the origin and related quantities for $c\bar{c}$ mesons.

Level	$\lvert R_{n\ell}^{(\ell)}(0)\rvert^2$			
	QCD (BT) [5]	Power law [6]	Logarithmic [7]	Cornell [8]
$1S$	0.810 GeV^3	0.999 GeV^3	0.815 GeV^3	1.454 GeV^3
$2P$	0.075 GeV^5	0.125 GeV^5	0.078 GeV^5	0.131 GeV^5
$2S$	0.529 GeV^3	0.559 GeV^3	0.418 GeV^3	0.927 GeV^3
$3D$	0.015 GeV^7	0.026 GeV^7	0.012 GeV^7	0.031 GeV^7
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Eichten et.al., PRD52, 1726

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error~40%

Error Estimate

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$$H_8 = \frac{4C_F}{3N_c\beta_0} \ln[\frac{\alpha_s(\Lambda_0)}{\alpha_s(\Lambda)}]H_1$$

Error Estimate

$\Lambda_0 = mv = 0.822$ GeV, $\Lambda = 2m = 3$GeV

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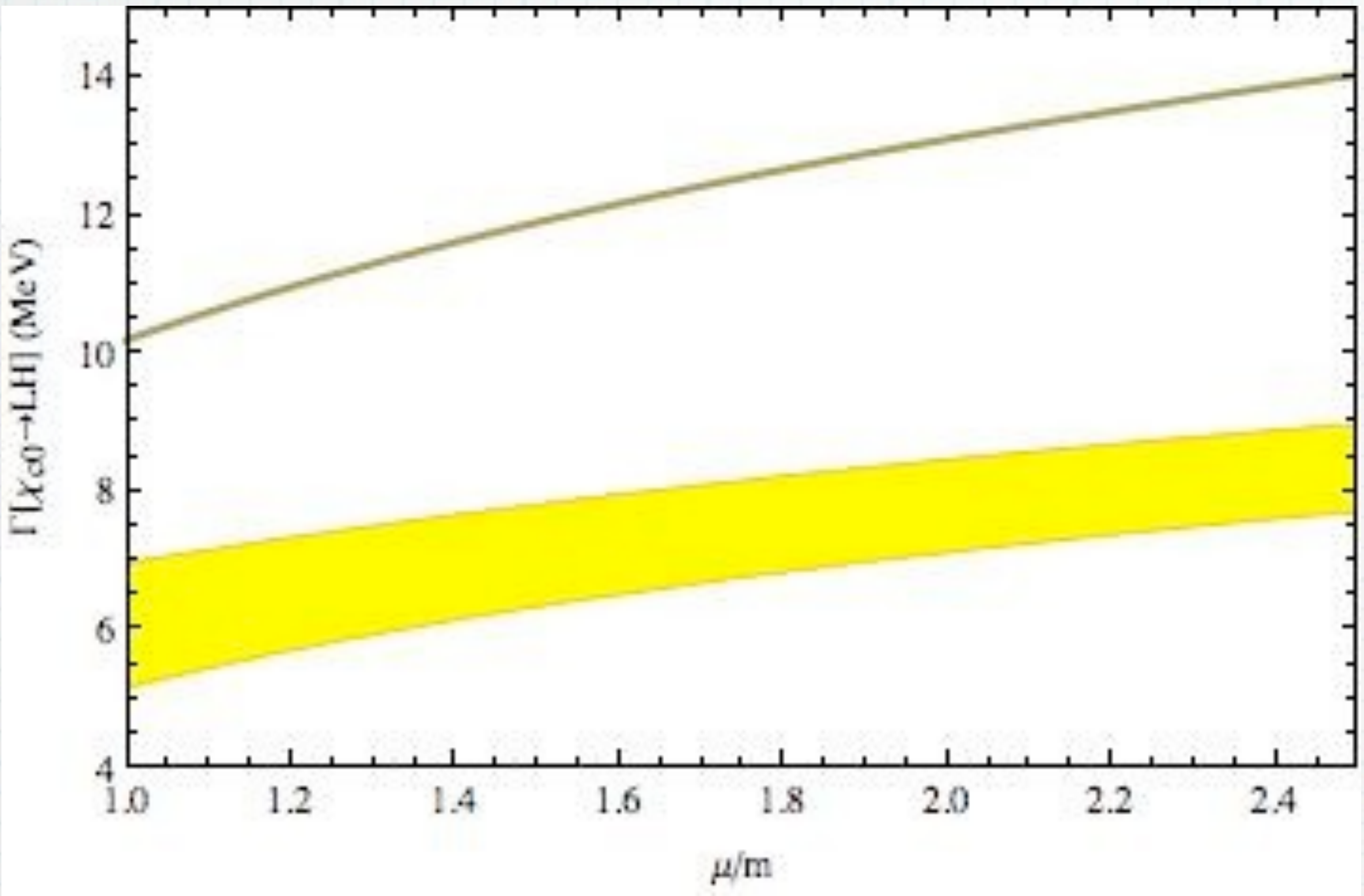
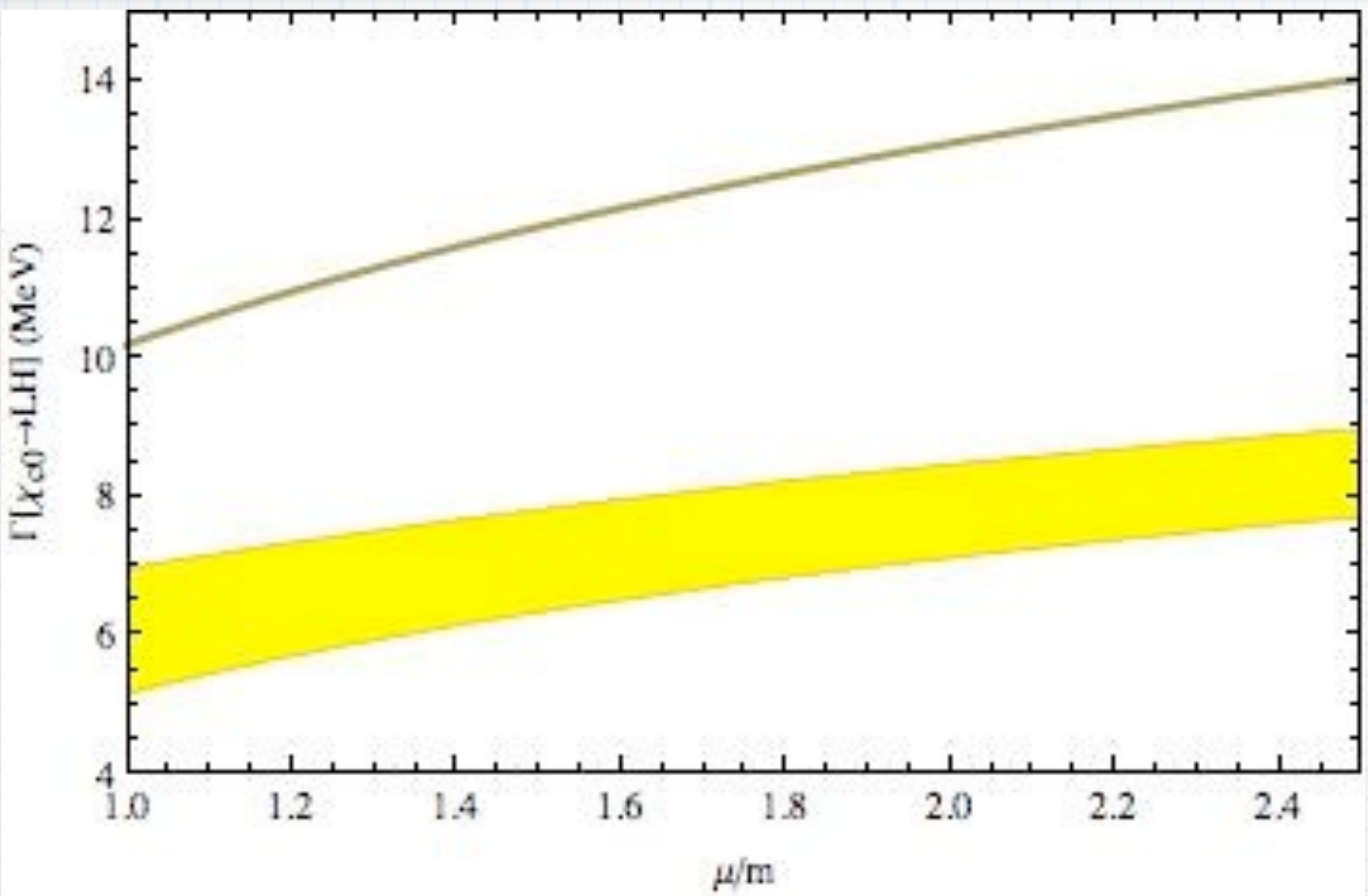
χ_{c0} LH decay width in NRQCD

χ_{c0} LH decay width in NRQCD



χ_{c0} LH decay width in NRQCD



- 7.76±0.67 MeV by Method I
- 13.1 MeV by Method II

χ_{c1}, χ_{c2} LH decay width in NRQCD

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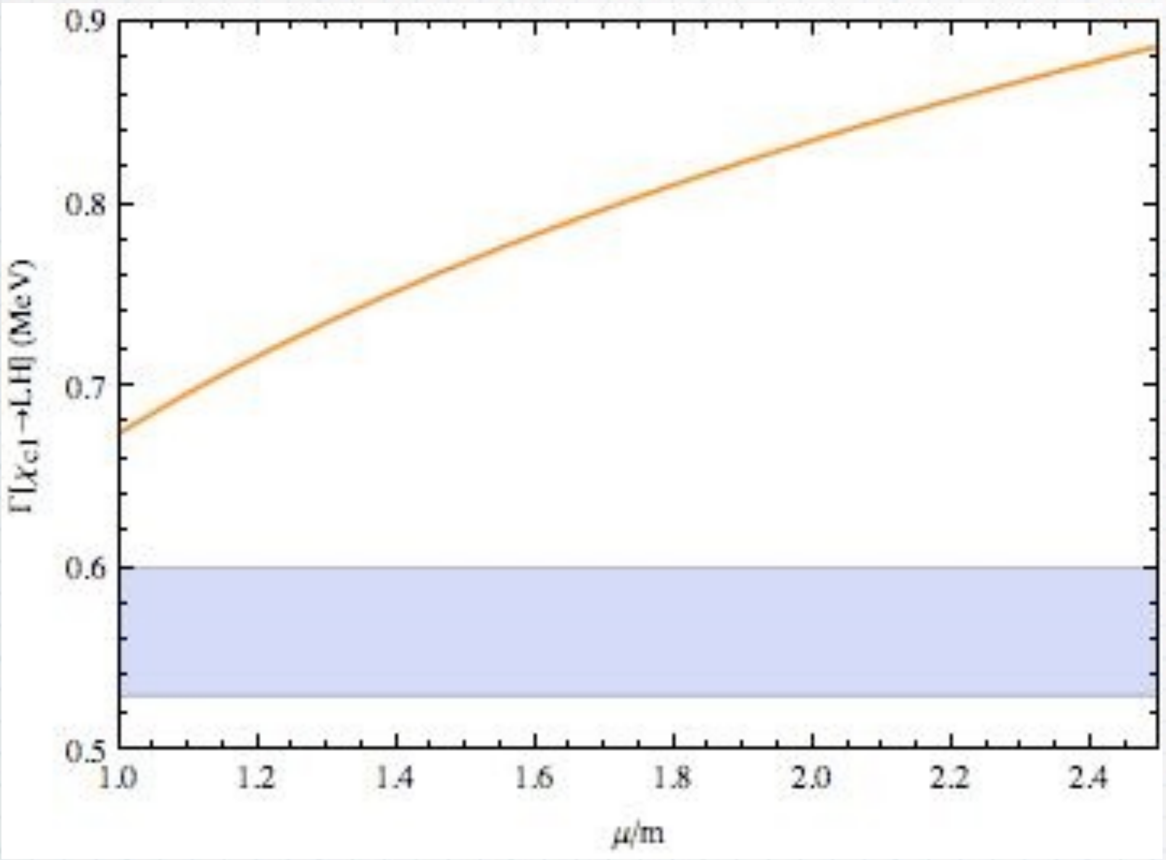
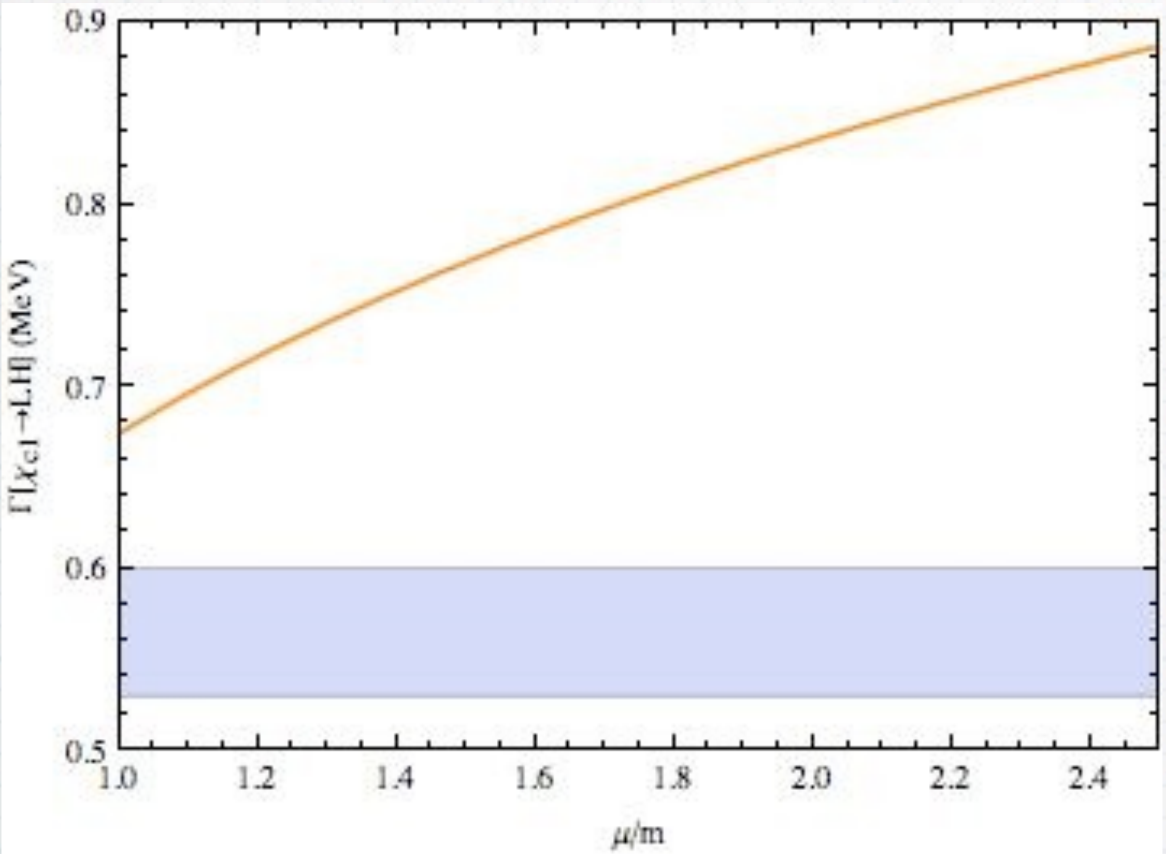
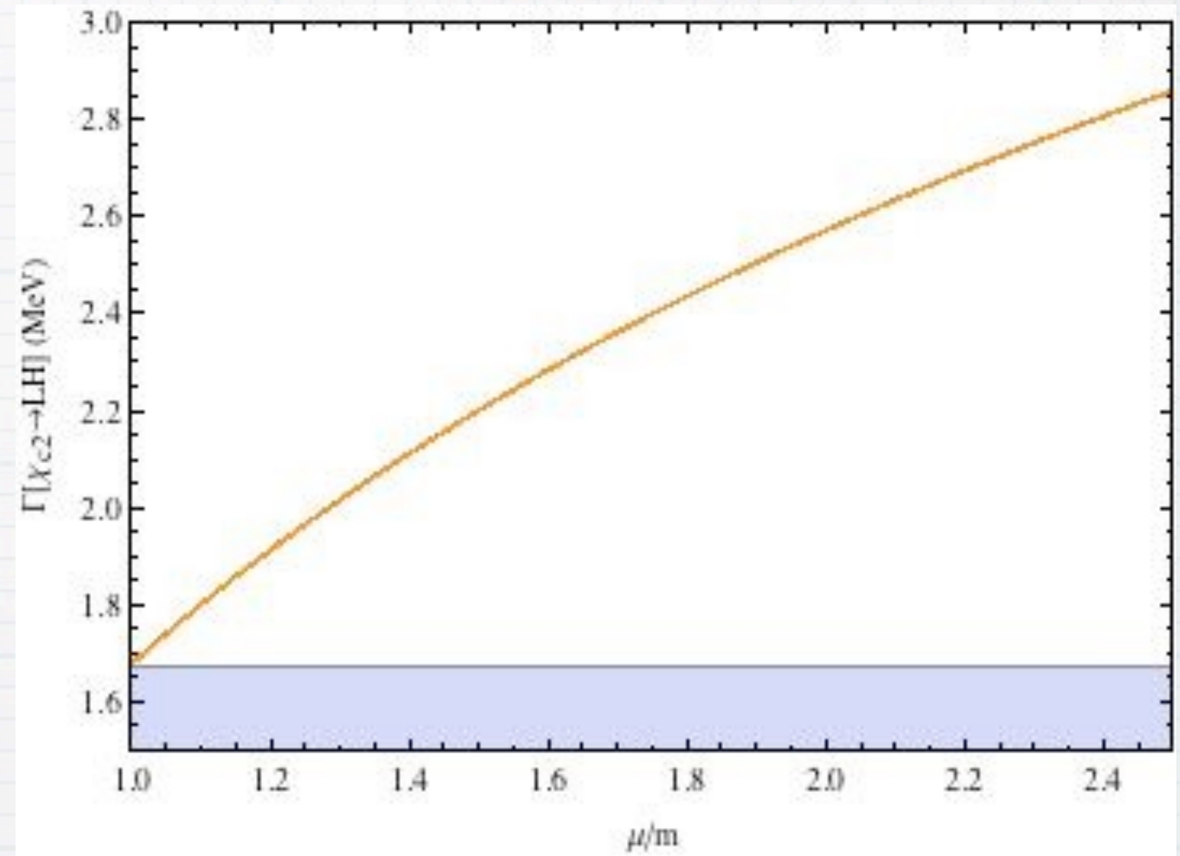
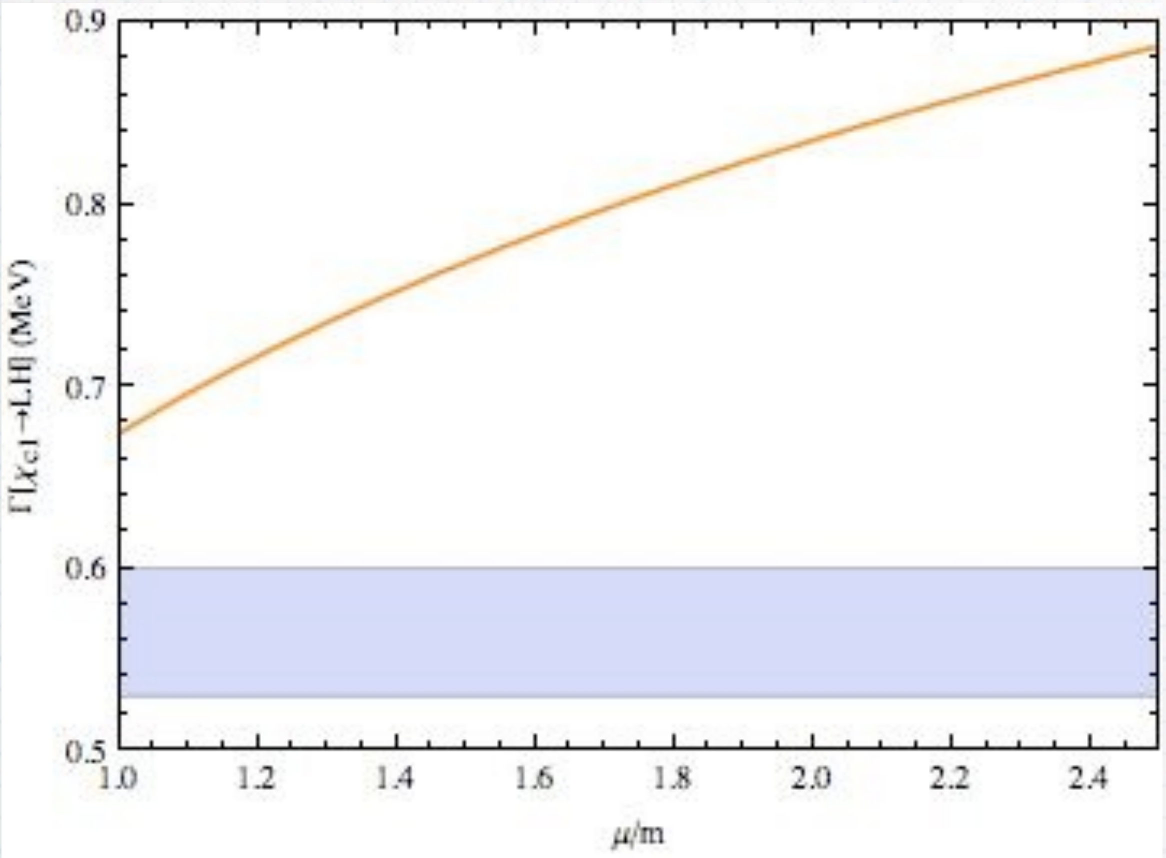


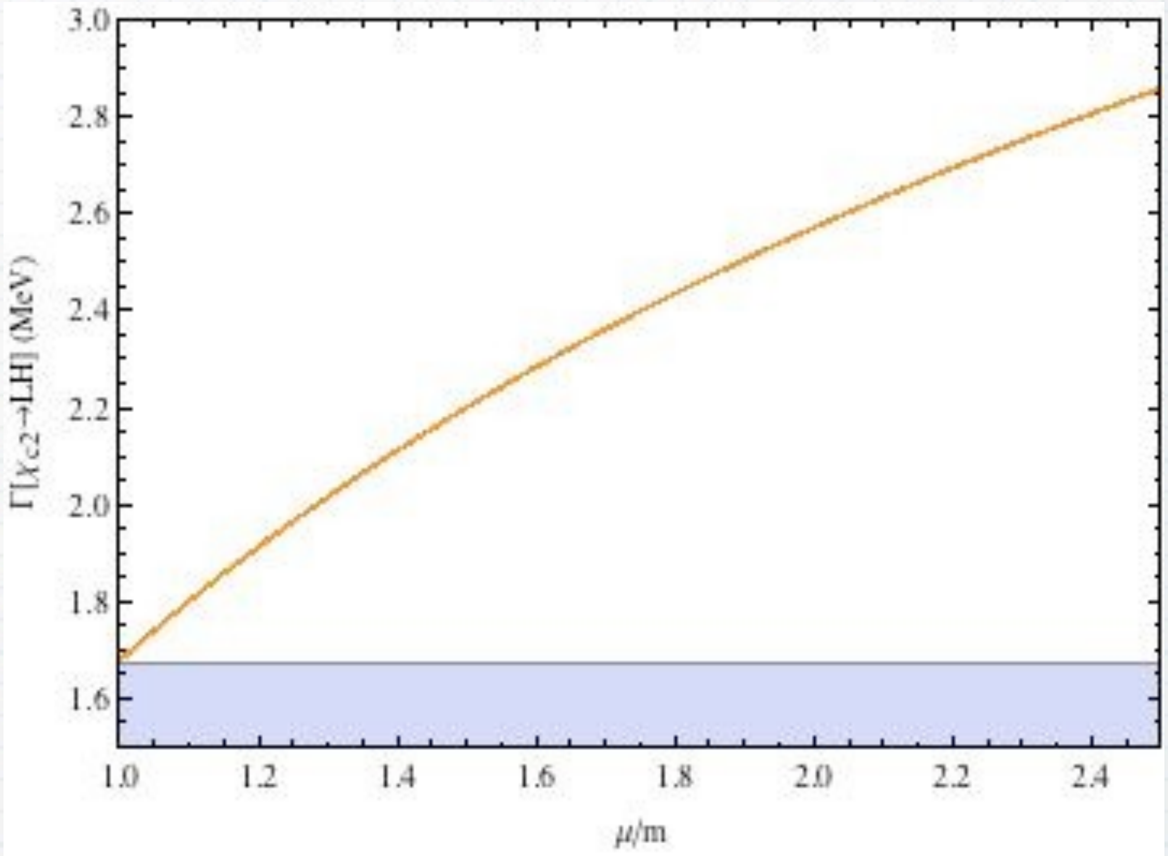
χc1,χc2 LH decay width in NRQCD





χ_{c1}, χ_{c2} LH decay width in NRQCD





filled belts :PDG 10 data;
curves: NRQCD predictions using Method II

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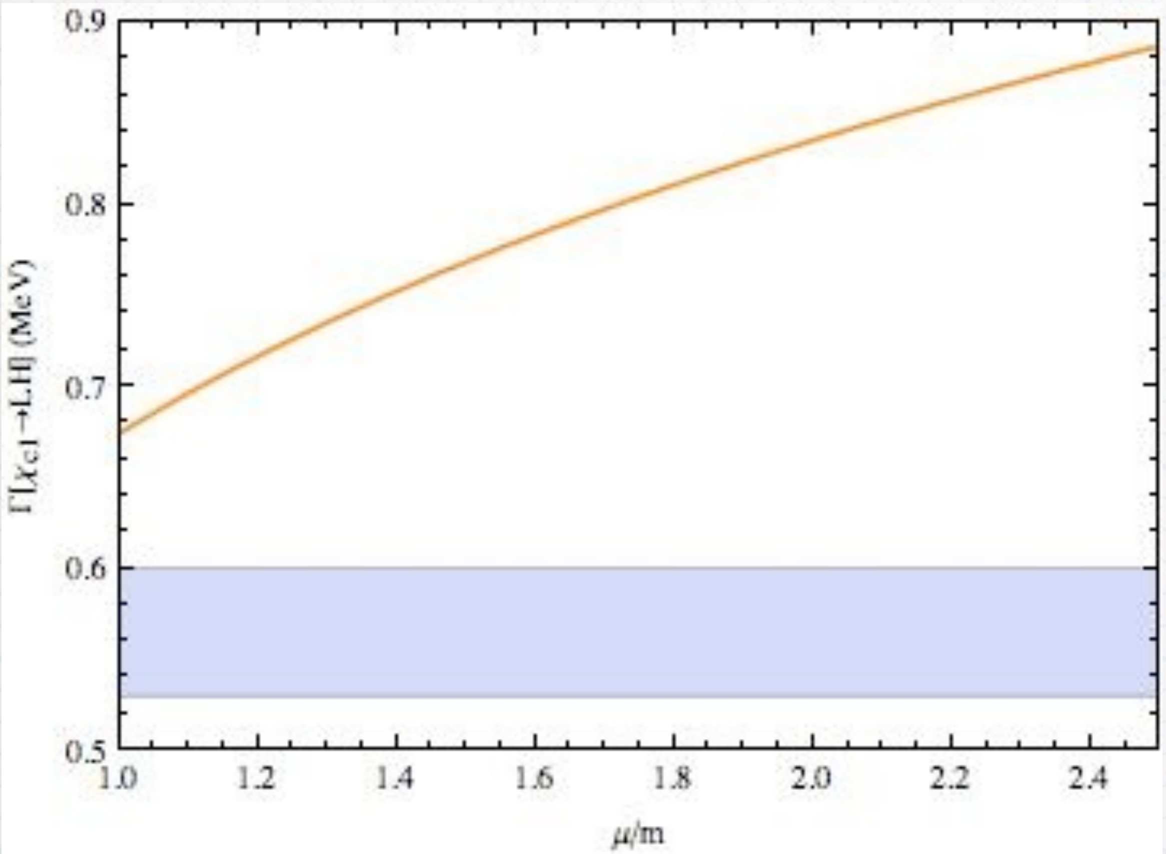

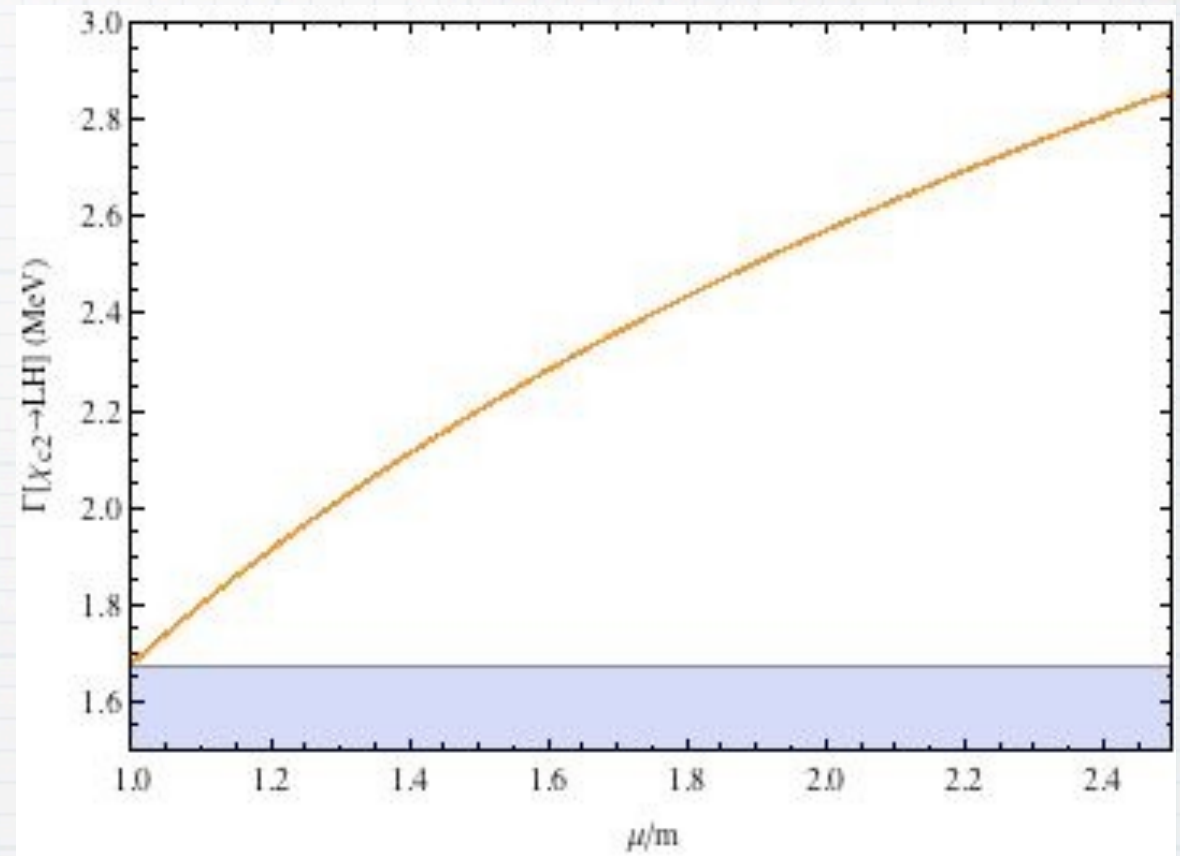

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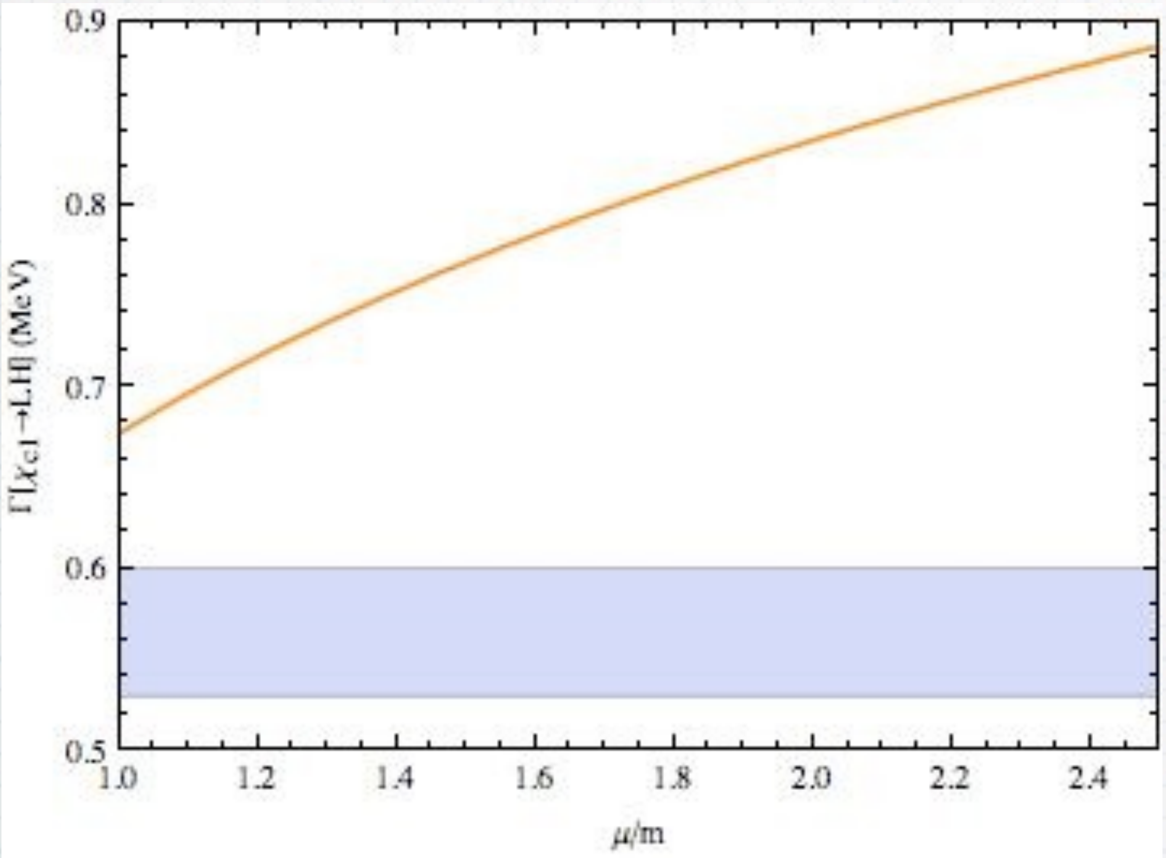
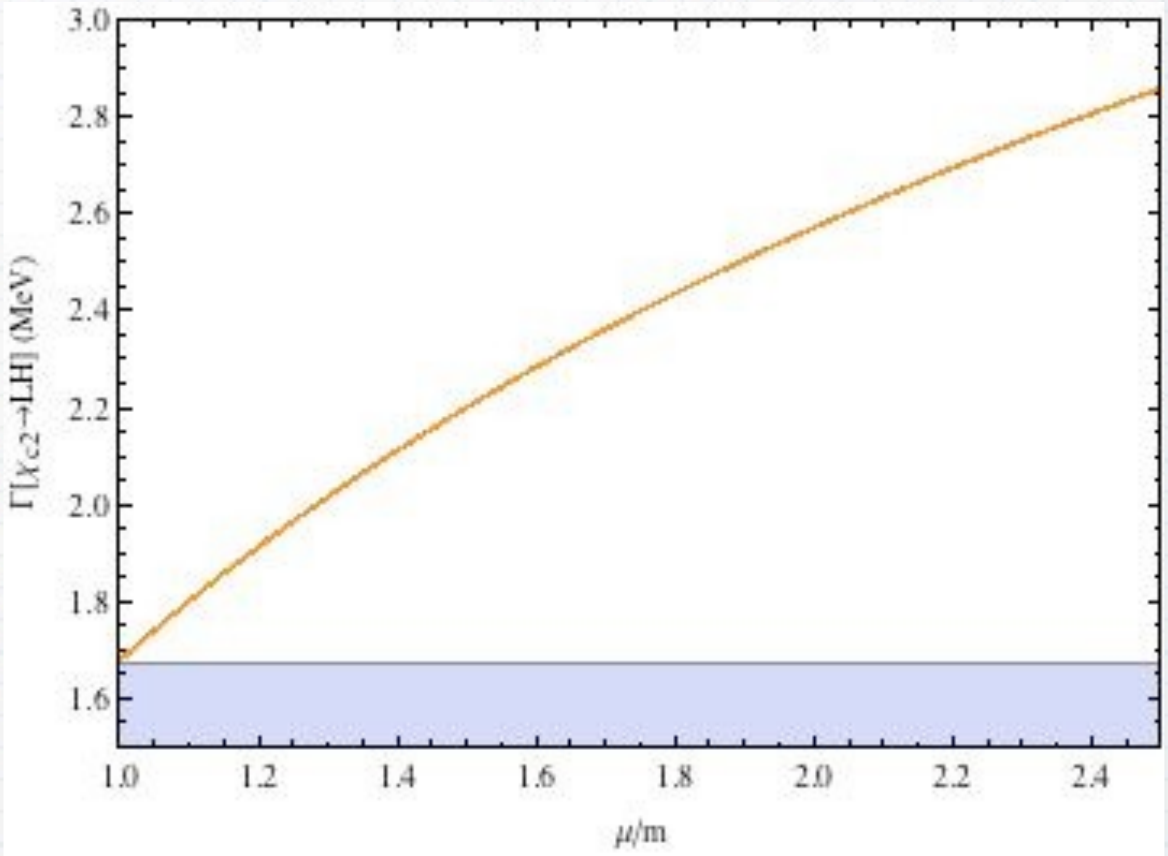
At perturbative energy scale $\mu = 2m$

χ_{c1},χ_{c2} LH decay width in NRQCD





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At perturbative energy scale μ = 2m

$$\Gamma(\chi_{c1} \to LH) = 0.834 \quad \text{MeV}$$

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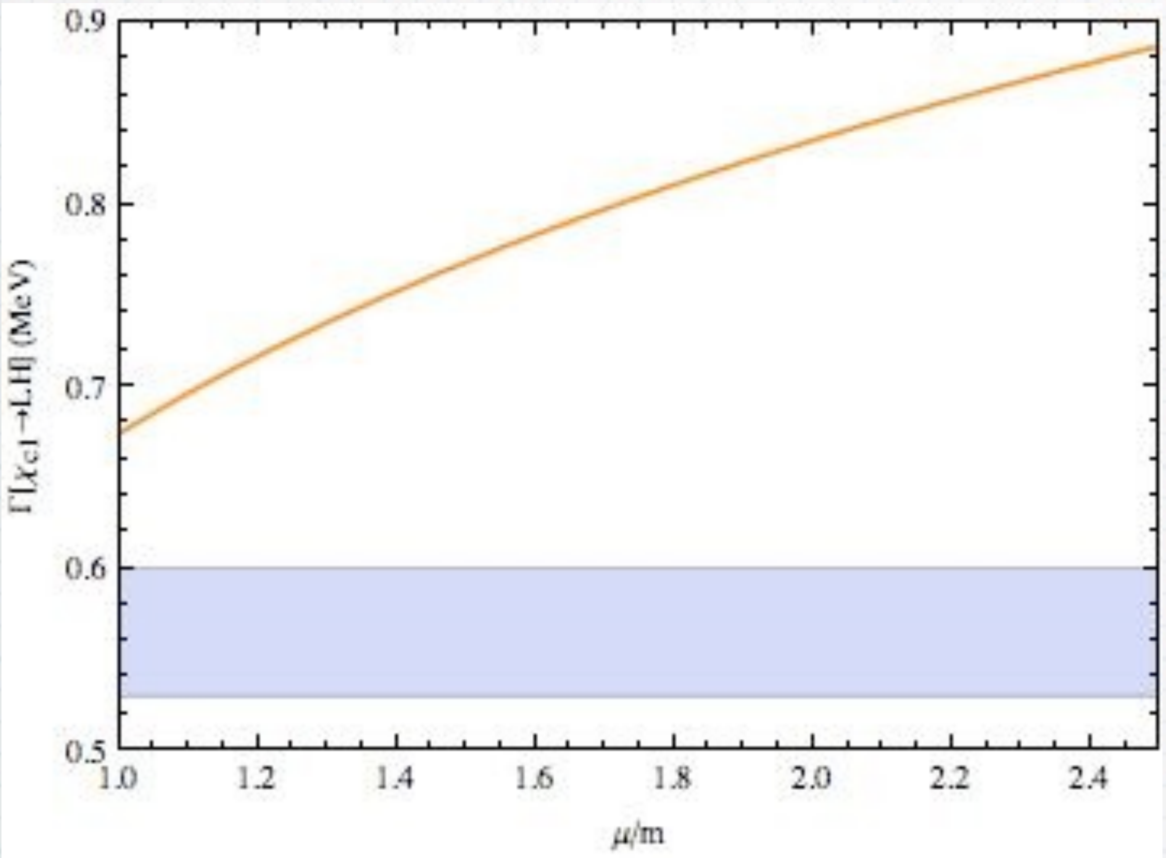
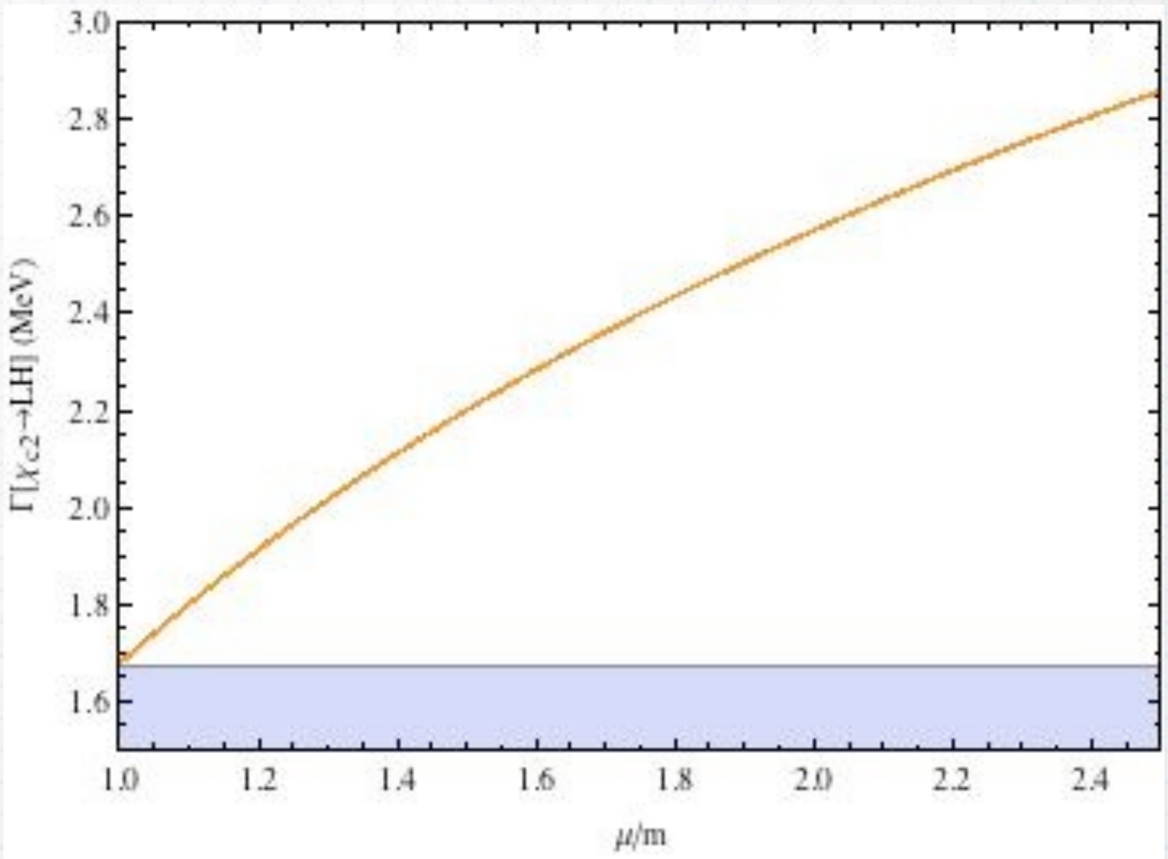
χ_{c1}, χ_{c2} LH decay width in NRQCD





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could be compared with experimental values by BESIII

Summary

* We estimate h_c light hadronic decay width up to NLO in α_s in NRQCD, and together with E1 transition width, the total width of h_c is larger than the central value of BESIII.
* Operator evolution equation is a good method to evaluate P-wave long-distance matrix element, and can be extended to D-wave case, which is lack of data.
* NLO NRQCD predictions for χ_{cJ} (J=0,1,2) are also given, which could be compared with BESIII results.

Thank you !