Charmonium II: Transitions

- QCD and the Multipole Expansion
- Electromagnetic Transitions - E1, M1
  - Overlaps and relativistic corrections
  - New developments
- Hadronic Transitions
  - Two pion transitions
  - π, η and ω transitions
- Issues and opportunities

QCD and the Multipole Expansion

In QCD the effective interaction for heavy quarks is

HQET $\frac{\Lambda}{m_Q}$

NRQCD V

$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m_Q} \right) \psi + \frac{c_F}{2m_Q} \psi^\dagger \boldsymbol{\sigma} \cdot g\mathbf{B}\psi + o(\frac{1}{m_Q^2}) + [\psi \to i\sigma^2\chi^*, A_\mu \to -A_\mu^T]$$

ψ =Q ; $i\sigma_2 \chi^*$ = Q_c

- Again begin by considering EM multipole interactions:

Including EM interactions: Replace D -> D - ie_Q (eA)

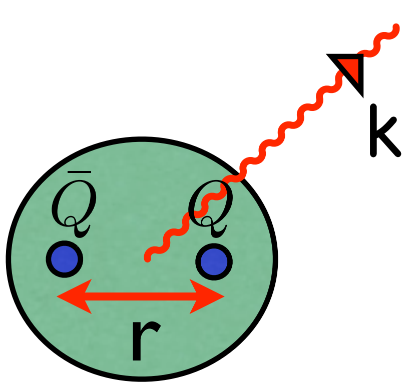
$$\mathcal{H}_I = ie_Q \psi^\dagger \left(\frac{\mathbf{D}\cdot \mathbf{e}\mathbf{A} + \mathbf{e}\mathbf{A}\cdot\mathbf{D}}{2m_Q} \right) \psi + \frac{c_F e_Q}{2m_Q} \psi^\dagger \boldsymbol{\sigma} \cdot e\mathbf{B}\psi + \ldots$$

Electric Magnetic

Theory of quarkonium transitions relies on the multipole expansion

$$\mathbf{A}(R_{\rm cm}, r, t) = \mathbf{A}(R_{\rm cm}, t) + \mathbf{x}\cdot\nabla\mathbf{A}(R_{\rm cm}, t) + \ldots$$

expansion kr/2



- Electric

$$\frac{1}{m_Q}\{\mathbf{p}, \mathbf{A}(R_{\rm cm}, t) + ...\} = \mathbf{r}\cdot\mathbf{E}(R_{\rm cm}, t) + ...$$

CM expand in kr/2

Siegert's theorem

using $i[\mathrm{H}, \mathbf{r}] = \frac{2\mathbf{p}}{m_Q}$ and $i[\mathrm{H}, \mathbf{A}] = \frac{\partial}{\partial t}\mathbf{A} = \mathbf{E}$

E1

$$\mathbf{p} \sim \frac{1}{\mathbf{r}} \sim m_Q\mathbf{v} \qquad k \sim \frac{m_Q v^2}{2}$$

$$e_Q\psi^\dagger\mathbf{r}\cdot\mathbf{e}\mathbf{E}\psi + \cdots$$

power counting $\frac{\mathrm{rk}}{2} \sim \mathrm{v}$

- Magnetic

anomalous magnetic moment

M1

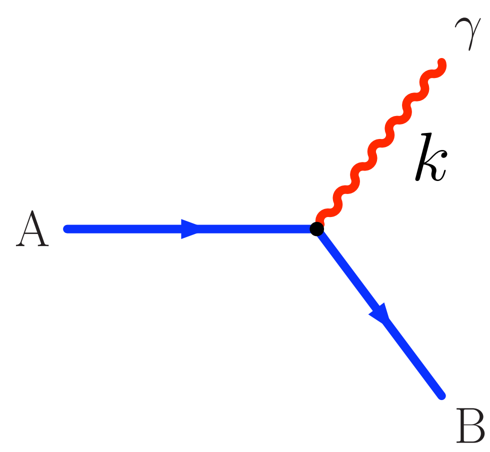
$$c_F = 1 + \kappa_Q$$

$$\frac{c_F e_Q}{2m_Q}\psi^\dagger\boldsymbol{\sigma}\cdot e\mathbf{B}\psi$$

spin flip

power counting $\frac{\mathrm{k}}{m_Q} \sim \mathrm{v}^2$

- Higher order terms



Electric $$\frac{1}{m_Q}\{\mathbf{p}, \mathbf{A}(R_{\rm cm}, r, t)\} = \mathbf{r}\cdot\mathbf{E}(R_{\rm cm}, t) + \cdots$$

Magnetic $$\frac{c_F e_Q}{2m_Q}\psi^\dagger \boldsymbol{\sigma}\cdot e\mathbf{B}\psi$$

Include $$\mathrm{e}^{\frac{\mathbf{ir}\cdot\mathbf{k}}{2}} = \sum_n \frac{1}{n!}\frac{(\mathbf{ir}\cdot\mathbf{k})}{2}^n$$

E1, E2, E3, ...
M1, M2, M3, ...

Selection Rules

Resulting radial wavefunction overlap integrals

Electric S<->P $$r \to \frac{3}{k}\left[\frac{kr}{2}j_0(\frac{kr}{2}) - j_1(\frac{kr}{2})\right]$$

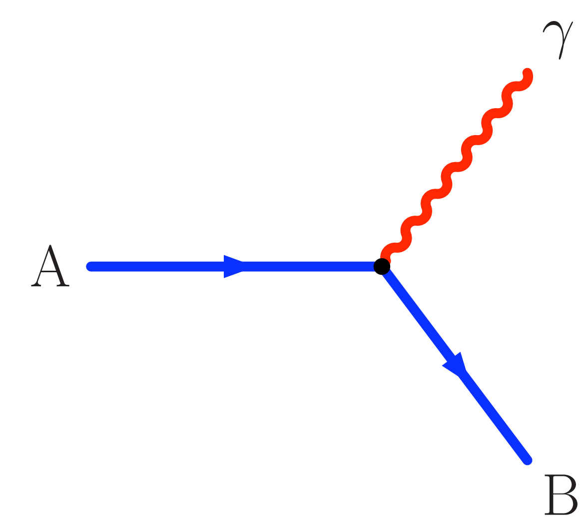
Magnetic S<->S $$1 \to j_0(\frac{kr}{2}) = r\left[1 - \frac{(kr)^3}{24} + \cdots\right]$$

expansion coefficients are small: $$\frac{1}{(2n+1)!!}$$

Photon Transitions

- E1 transition rates

$$\Gamma(i \xrightarrow{\mathrm{E1}} f+\gamma) = \frac{4\alpha e_Q^2}{3}(2J_f+1)\mathrm{S}_{if}^{\mathrm{E}} k^3 |\mathcal{E}_{if}|^2$$



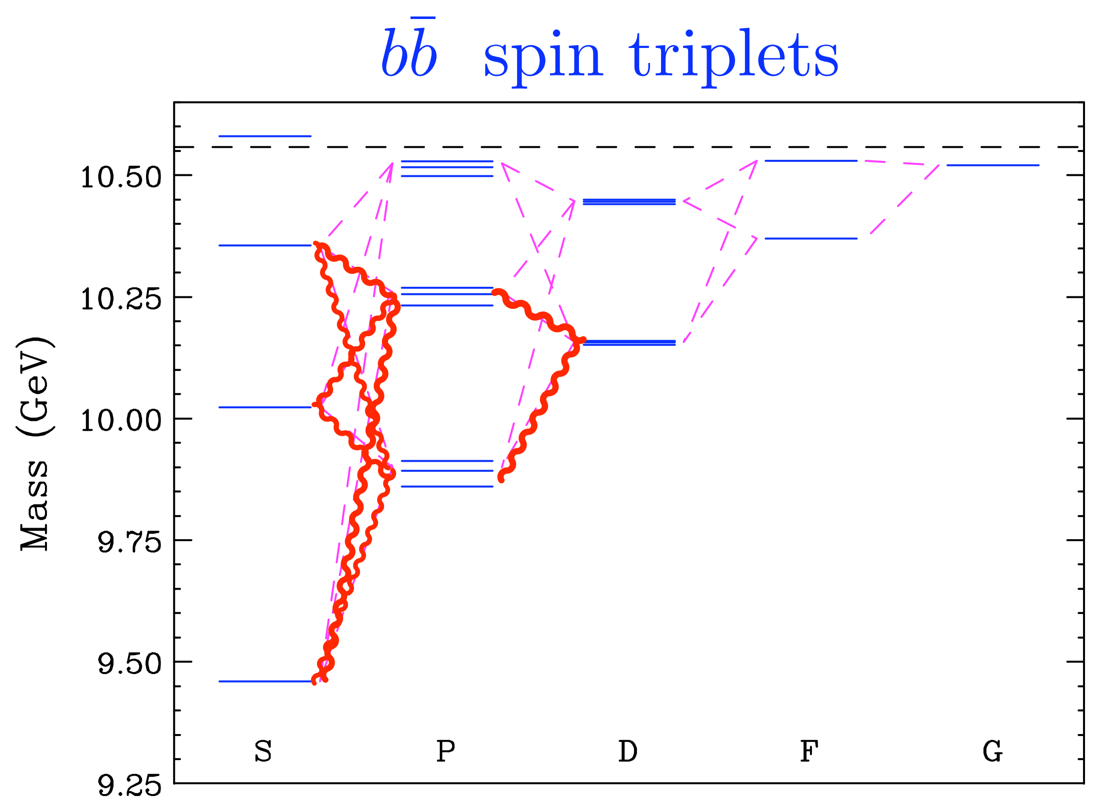
  - CG factor

$$\mathrm{S}_{if}^{\mathrm{E}} = \max(L_i, L_f) \begin{Bmatrix} J_i & 1 & J_f \\ L_f & S & L_i \end{Bmatrix}^2$$

  - Wavefunction overlap

$$\mathcal{E}_{if} = \int r^2 dr\, R_{n_i \mathrm{L}_i}(r) r R_{n_f \mathrm{L}_f}(r)$$

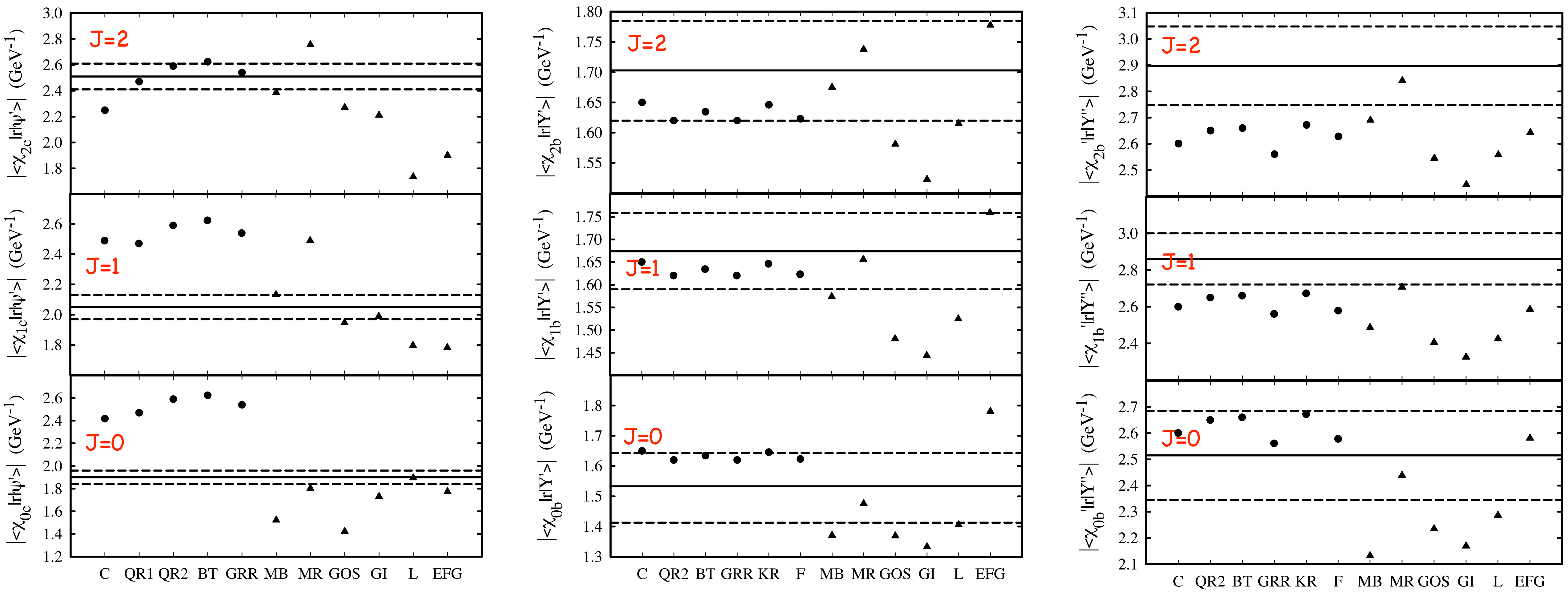
Sensitive to detailed dynamics for transitions involving radially excited states



- S states -> P states
  - Potential models with various phenomenologically motivated relativistic correction term.
  - Need better theoretical guidance.

$\mathcal{E}_{if}$

$2^3S_1 \to 1^3P_J$ ($c\bar{c}$) $2^3S_1 \to 1^3P_J$ ($b\bar{b}$) $3^3S_1 \to 2^3P_J$ ($b\bar{b}$)



Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E. [Rev. Mod. Phys. 80, 1161 (2008)]

- Ratios of overlaps test the importance of relativistic corrections

$$R^{nm}_{J/J'} = \left[\frac{\mathcal{E}(n\ ^3S_1 \to m\ ^3P_J)}{\mathcal{E}(n\ ^3S_1 \to m\ ^3P'_J)}\right]$$

Ratio	$(J=2)/(J'=1)$	$(J=1)/(J'=0)$
$R_{21}(c\bar{c})$	1.18	1.06
$R_{21}(b\bar{b})$	1.01	1.10
$R_{32}(b\bar{b})$	0.98	1.14

$< v^2 >_{c\bar{c}} \sim 0.3$

$< v^2 >_{b\bar{b}} \sim 0.1$

- Relativistic corrections 10%-20% effects in charmonium.
- The $3^3S_1 \to 1^3P_J$ (bb) E1 transitions highly suppressed

$$\left|\frac{\mathcal{E}(3^3S_1 \to 1^3P_0)}{\mathcal{E}(3^3S_1 \to 2^3P_0)}\right| = 0.044 \pm 0.008$$

- Cancellation in wavefunction overlap
  - Nodes in 3S radial wavefunction leads to large cancellations in overlap integrals
  - Hence effects of relativistic corrections to wavefunctions are magnified.
  - Sensitivity varies within multiplets. Ratios not immune.

- Example of accidental (dynamical) suppression of leading behavior

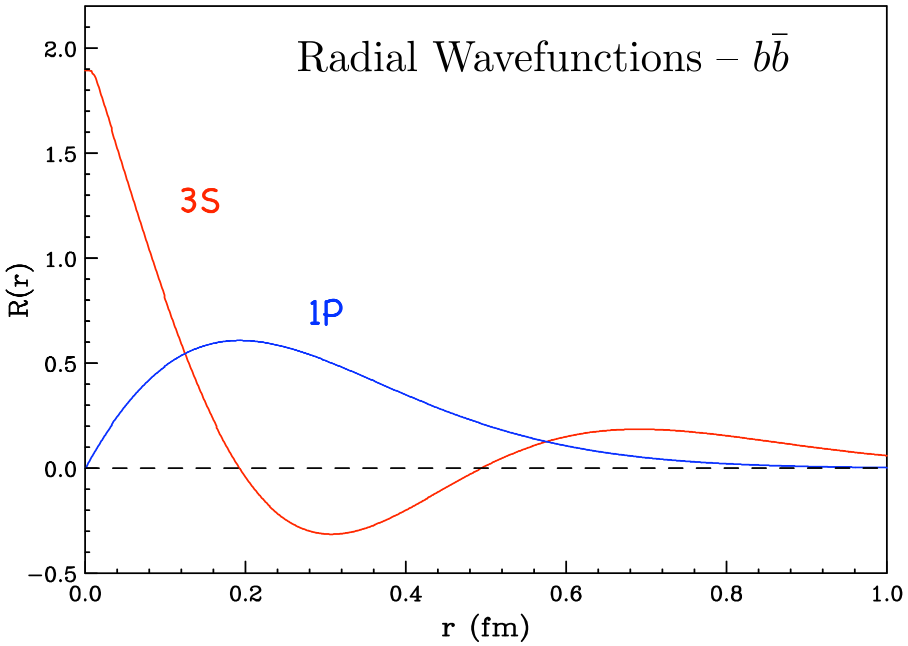


TABLE II: $\mathcal{E}_{if}$ by node regions for initial $b\bar{b}$ state.

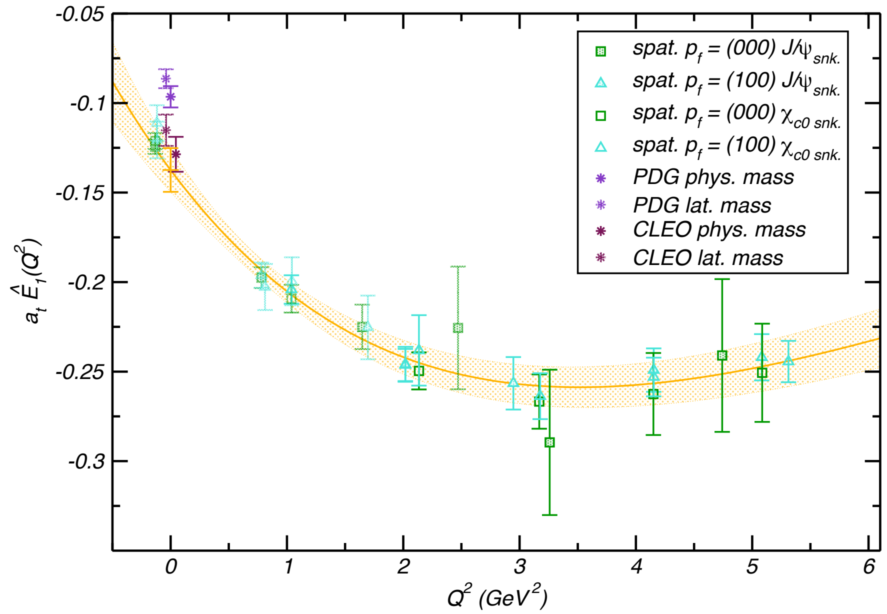
Transition	< 1st	1st → 2nd	2nd → 3rd	total
$2S \to 1P$	0.07	−1.68		−1.61
$3S \to 2P$	0.04	−0.12	−2.43	−2.51
$3S \to 1P$	0.04	−0.63	0.65	0.06

Cornell model wavefunctions

- 1P states -> 1S state
  - Potential models with various phenomenological relativistic correction terms. No clear improvement
  - Alternative approach needed
- Direct Lattice QCD calculation

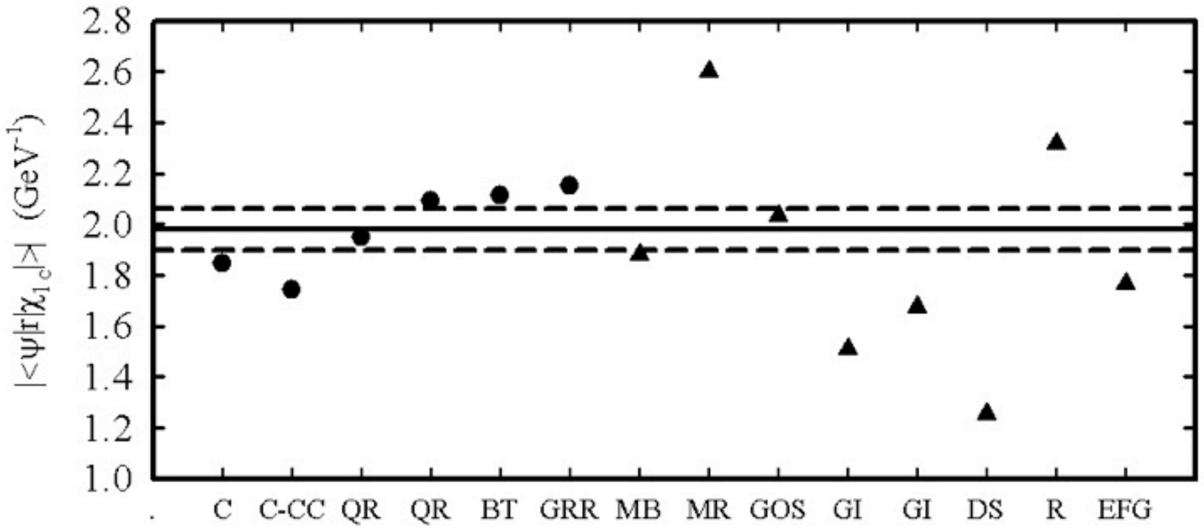
$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V, r)\rangle$$

$$\chi_c^0 \rightarrow \gamma + J/\psi$$

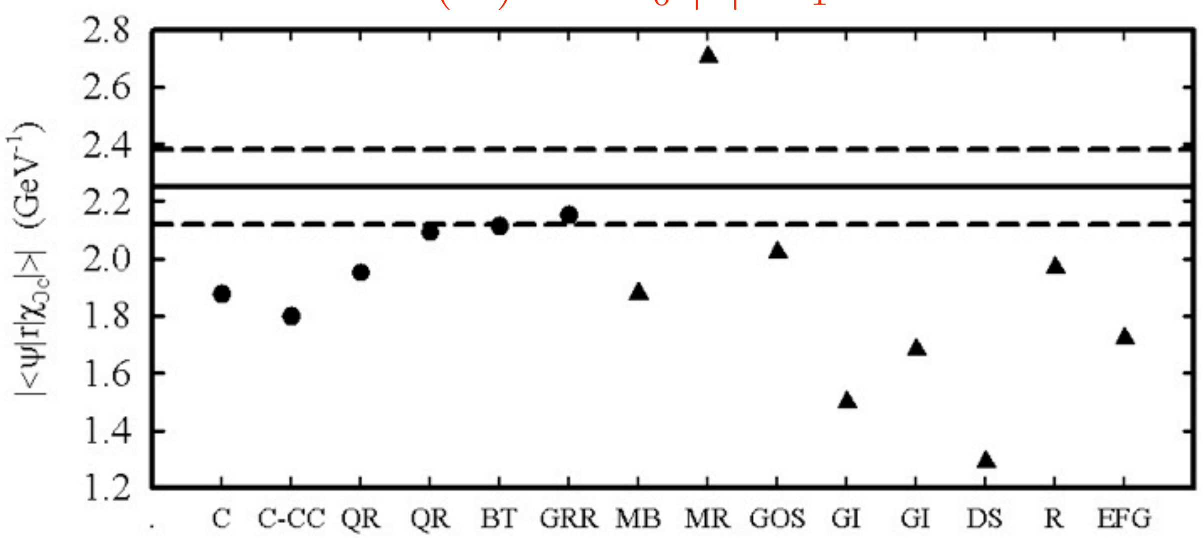


Dudek, Edwards, Richards and Mathur [PR D73:074507(2006)}

$(c\bar{c}) < {}^3P_1\ |r|\ {}^3S_1 >$



$(c\bar{c}) < {}^3P_0\ |r|\ {}^3S_1 >$



Timed ordered

- NR models
- Models with relativistic corrections

  - Promising start

- $h_c(1^1P_1) \to \eta_c(1^1S_0)$
  - Observed at CLEO, BESIII
  - The dependence on dynamics cancels in the ratio of singlet to triplet transitions

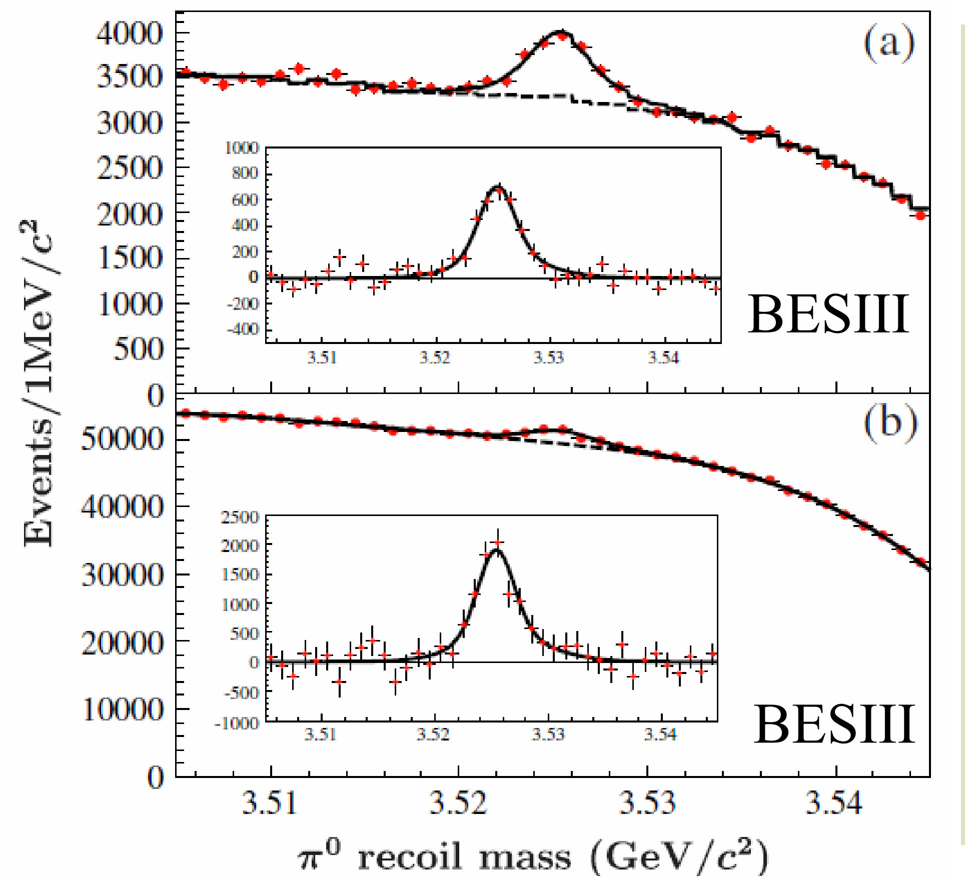
$$\frac{\Gamma(h_c \to \gamma(k) + \eta_c)}{\Gamma(\chi_{c1} \to \gamma(k') + J/\psi)} = [\frac{k}{k'}]^3 = 2.16$$

  - Using the known χ_{c1} properties

$$\Gamma(h_c \to \gamma + \eta_c) = 638 \pm 37 \text{ keV}$$

$$e^+e^- \to \psi(2S) \to \pi^0 h_c, \quad h_c \to \gamma\eta_c, \quad \pi^0 \to \gamma\gamma.$$

BES III: PRL 104, 132002 (2010)



- ψ(3770) -> 1^3P_J transitions:
  - Can study relativistic effects including coupling to decay channels.
    - Effects of nearby open charmed thresholds contribute significant mixing between the 1^3D_1 and 2^3S_1 states:

$$\psi(3770) = \cos(\phi)|1^3D_1 > + \sin(\phi)|2^3S_1 >$$

$$\frac{\Gamma(\psi(3770) \to \gamma\chi_{c1})}{\Gamma(\psi(3700) \to \gamma\chi_{c0})} = 1.32 \left(\frac{-\frac{\sqrt{3}}{2} + x\tan(\phi)}{\sqrt{3} + x\tan(\phi)} \right)^2$$

$$\frac{\Gamma(\psi(3770) \to \gamma\chi_{c2})}{\Gamma(\psi(3700) \to \gamma\chi_{c0})} = 1.30 \left(\frac{\frac{\sqrt{3}}{10} + x\tan(\phi)}{\sqrt{3} + x\tan(\phi)} \right)^2$$

$$\mathbf{x} = |\frac{< 2S|r|1P >}{< 1D|r|1P >}| \sim 0.82$$

  - $\phi \sim -10^\circ \Rightarrow$ Small J=2 E1 rate
  - BESIII can measure the J=2 transition?

CLEO [PR D74 (2006) 031106]	$\Gamma(\psi(3770) \to \gamma\chi_{cJ})$ in keV		
	$J = 2$	$J = 1$	$J = 0$
Our results	< 21	70 ± 17	172 ± 30
Rosner (non-relativistic) [7]	24 ± 4	73 ± 9	523 ± 12
Ding-Qin-Chao [6]			
non-relativistic	3.6	95	312
relativistic	3.0	72	199
Eichten-Lane-Quigg [8]			
non-relativistic	3.2	183	254
with coupled-channels corrections	3.9	59	225
Barnes-Godfrey-Swanson [9]			
non-relativistic	4.9	125	403
relativistic	3.3	77	213

- Higher Multipoles: $\psi'(2S) \to \chi_{cJ}(1P)$ and $\chi_{cJ}(1P) \to J/\psi$
  - Measure helicity amplitudes A

$$\begin{pmatrix} A_0^{J=1} \\ A_1^{J=1} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} a_1^{J=1} \\ a_2^{J=1} \end{pmatrix},$$

$$\begin{pmatrix} A_0^{J=2} \\ A_1^{J=2} \\ A_2^{J=2} \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{10}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{2}{5}} \\ \sqrt{\frac{3}{10}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{8}{15}} \\ \sqrt{\frac{3}{5}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{15}} \end{pmatrix} \begin{pmatrix} a_1^{J=2} \\ a_2^{J=2} \\ a_3^{J=2} \end{pmatrix}.$$

Extract multipole coefficients:
a_1 = E1/|a| ; a_2 = M2/|a|; a_3 = E3/|a|
$|a|^2 = E1^2 + M2^2 + E3^2$

E1 for J=0,1,2
M2 for J=1,2
E3 for J=2

  - Theory [Karl, Meshkov &Rosner PRL 45,215 (1980)] [Sebastian, Grotch & Ridener PR D45, 3136 (1992)]
    - M2/E1 and E3/E1 ~ $O(v^2)$
    - Anomalous quark magnetic moment: κ_c

$$X = \frac{\mathcal{E}_{1P,2S}}{\mathcal{E}_{1P,2D}} \tan(\phi)$$

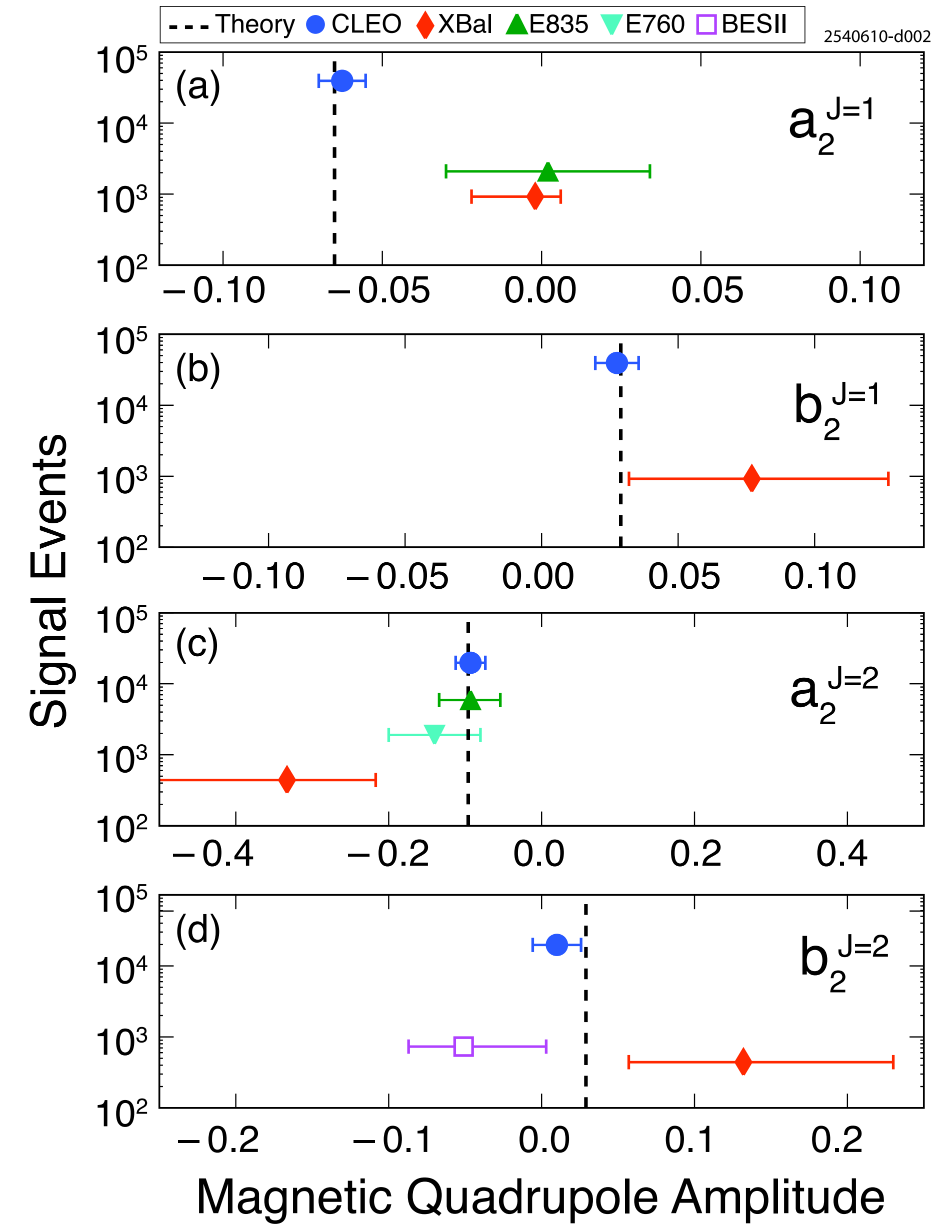
$$Y = \frac{\int r^3 dr R_{1P}(r)[r\frac{d}{dr}R_{2S}dr - 2R_{2S}(r)]}{\mathcal{E}_{1P,2S}}$$

    - No S-D mixing => X=0:
      - $a_3 = 0$
      - ratios of multipole coefficients are independent of κ_c

	$\chi_{cJ} \to J/\psi + \gamma$		
J	theory	E835	PDG
2	$a_2 \approx -\frac{\sqrt{5}}{3}\frac{k}{4m_c}(1+\kappa_c)$	$-0.093^{+0.039}_{-0.041} \pm 0.006$	-0.140 ± 0.006
2	$a_3 \approx 0$	$0.020^{+0.055}_{-0.044} \pm 0.009$	$0.011^{+0.041}_{-0.033}$
1	$a_2 \approx -\frac{k}{4m_c}(1+\kappa_c)$	$0.002 \pm 0.032 \pm 0.004$	$-0.002^{+0.008}_{-0.017}$

J	$\psi' \to \chi_{cJ} + \gamma$ theory
2	$a_2 \approx -\frac{\sqrt{3}}{2\sqrt{10}}\frac{k}{m_c}[(1+\kappa_c)(1+\frac{\sqrt{2}}{5}X) - i\frac{1}{5}X]/[1-\frac{1}{5\sqrt{2}}X]$
2	$a_3 \approx -\frac{12\sqrt{2}}{175}\frac{k}{m_c}X[1+\frac{3}{8}Y]/[1-\frac{1}{5\sqrt{2}}X]$
1	$a_2 \approx -\frac{k}{4m_c}[(1+\kappa_c)(1+\frac{2\sqrt{2}}{5}X) + i\frac{3}{10}X]/[1+\frac{1}{\sqrt{2}}X]$

M. Artuso *et al.* [CLEO Collaboration], Phys. Rev. D **80**, 112003 (2009) [arXiv:0910.0046 [hep-ex]].



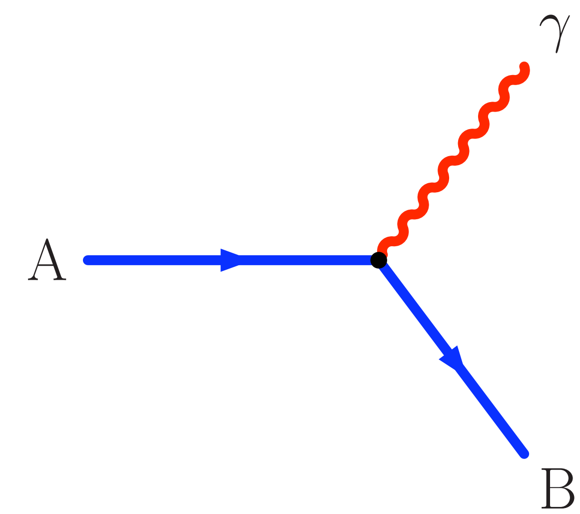
- CLEO has recent results
  - Assumes a_3, b_3 = 0
  - theory: $\kappa_c = 0$, m_c =1.5 GeV
- In excellent agreement with theory.

$b_2^J = a_2\,(\psi'(2S) \to \chi_{cJ}(1P))$

$a_2^J = a_2\,(\chi_{cJ}(1P) \to J/\psi')$

- M1 transition rates

$$\Gamma(i \xrightarrow{\rm M1} \gamma + f) = \frac{16}{3}\alpha\, e_Q^2 \frac{E_\gamma^3}{M_i^2} (2J' + 1)\, \mathrm{S}_{if}^{\rm M} |\mathcal{M}_{if}|^2$$



- CG factor

$$S_{if}^{\rm M} = 6(2S_i+1)(2S_f+1)\left\{ \begin{array}{ccc} J_i & 1 & J_f \\ S_f & 1 & S_i \end{array} \right\}^2 \left\{ \begin{array}{ccc} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & S_f & S_i \end{array} \right\}^2$$

- Wavefunction overlap

$$\mathcal{M}_{if} = (1+\kappa_Q) \int_0^\infty dr\; u_{nl}(r)\, u'_{n'l}(r)\; j_0\left(\frac{E_\gamma\, r}{2}\right)$$

  - To date the only M1 transitions observed are between states with L = 0 (S states)
  - As $E_\gamma \to 0$ $\mathcal{M}_{if} \to \delta_{nn'}$ by wavefunction orthogonality. Transitions with n ≠ n' are greatly suppresses (hindered). Corrections to this behavior are $O(v^2)$

- $J/\psi \to \eta_c$ M1 transition was a disaster
  - Theoretically clean - no dependence on potential model wavefunctions:
    - Naive expectations

$$\mathcal{M}_{if} = \int r^2 dr\, R_{n_i L_i}(r) j_0(\frac{rk}{2}) R_{n_f L_f}(r) = 1 + O(v^2) \quad (i = f = 1\ L_i = L_f = 0)$$

    - pNRQCD — Model independent - completely accessible by perturbation theory to o(v^2)

$$\Gamma(J/\psi \to \eta_c \gamma) = \frac{16}{3}\alpha e_c^2 \frac{k_\gamma^3}{M_{J/\psi}^2}\left[1 + C_F \frac{\alpha_s(M_{j/\psi}/2)}{\pi} + \frac{2}{3}(C_F \alpha_s(p_{J/\psi}))^2\right]$$

Brambilla, Jia & Vairo [PR D73:054005 (2006)]

$$\Gamma(J/\psi \to \eta_c \gamma) = (1.5 \pm 1.0)\ \text{keV}.$$

No large anomalous magnetic moment

No scalar long range interaction

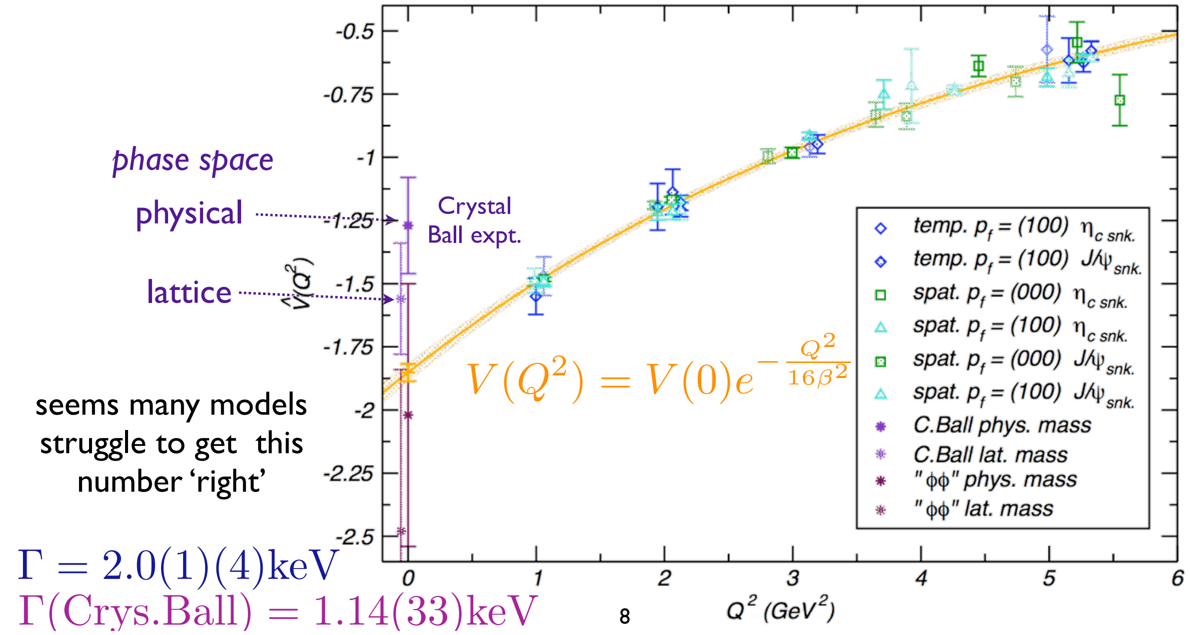
    - Lattice QCD

$$\Gamma(J/\psi \to \eta_c \gamma) = (2.0 \pm 0.1 \pm 0.4)\ \text{keV}$$

  - Crystal Ball measurement (1986)

$$1.19 \pm 0.33\ \text{keV}$$

half the expected theoretical result

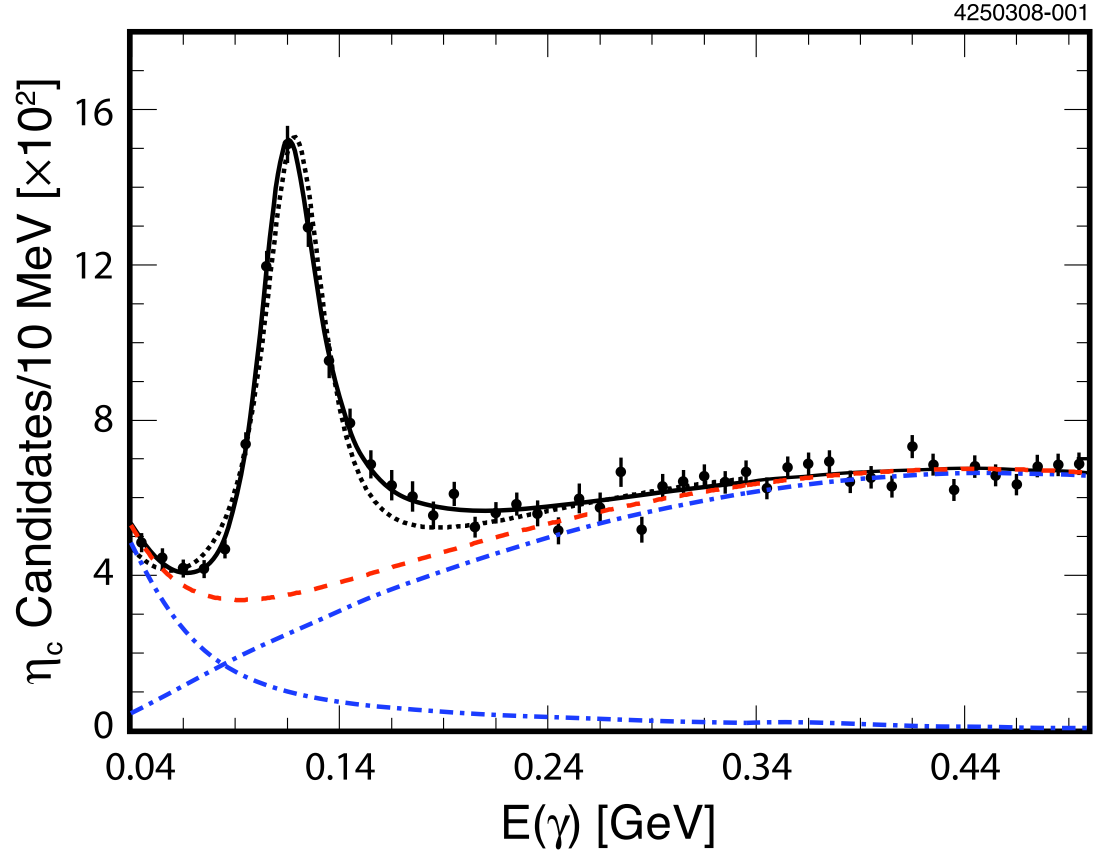


Dudek, Edwards, Richards and Mathur [PR D73:074507(2006)}

- CLEO measurement solves the issue

R. E. Mitchell *et al.* [CLEO Collaboration], Phys. Rev. Lett. **102**, 011801 (2009) [arXiv:0805.0252 [hep-ex]].

$$\mathcal{B}(\psi(2S) \to \gamma\eta_c) = (4.32 \pm 0.16 \pm 0.60) \times 10^{-3}$$
$$\mathcal{B}(J/\psi \to \gamma\eta_c) = (1.98 \pm 0.09 \pm 0.30) \times 10^{-3}$$



- photon line shape has long tail.
- Recently explained (Vairo QWG 2010). Same effect well understood in positronium transitions.
- Will resolve the lineshape uncertainity

- ψ' -> η'_c
  - experimental limit from CLEO

$$\mathcal{B}(\psi(2S) \to \gamma\eta_c(2S)) < 7.6 \times 10^{-4}$$

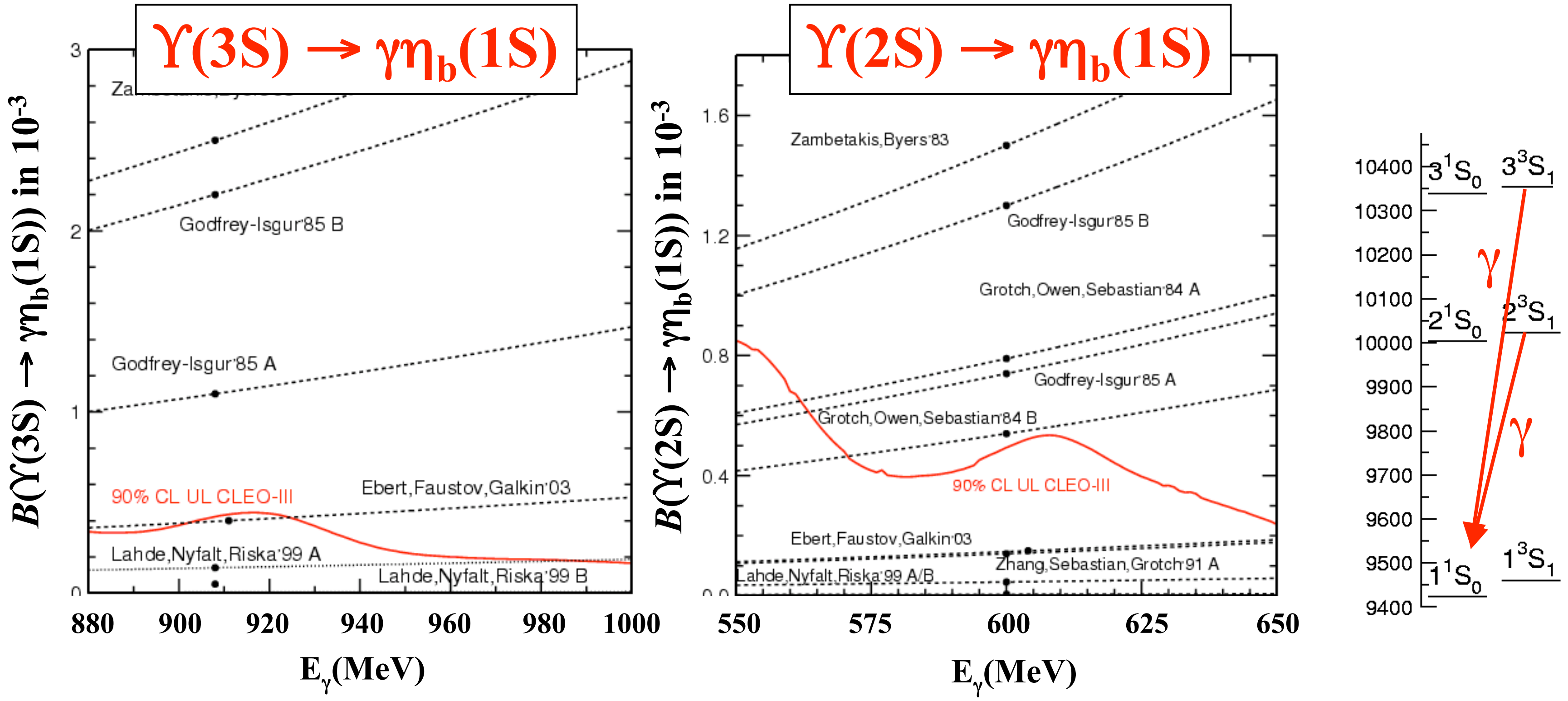
  - expectation from scaling from J/ψ

$$(3.9 \pm 1.1) \times 10^{-4}.$$

BESIII ?

- $\Upsilon(3S) \to \eta_b$ and $\Upsilon(2S) \to \eta_b$ hindered M1 transition
  - Leading order zero -> order v^2 corrections determine rate
  - Relativistic corrections poorly understood

Phenomenological models made widely varying predictions



pNRQCD

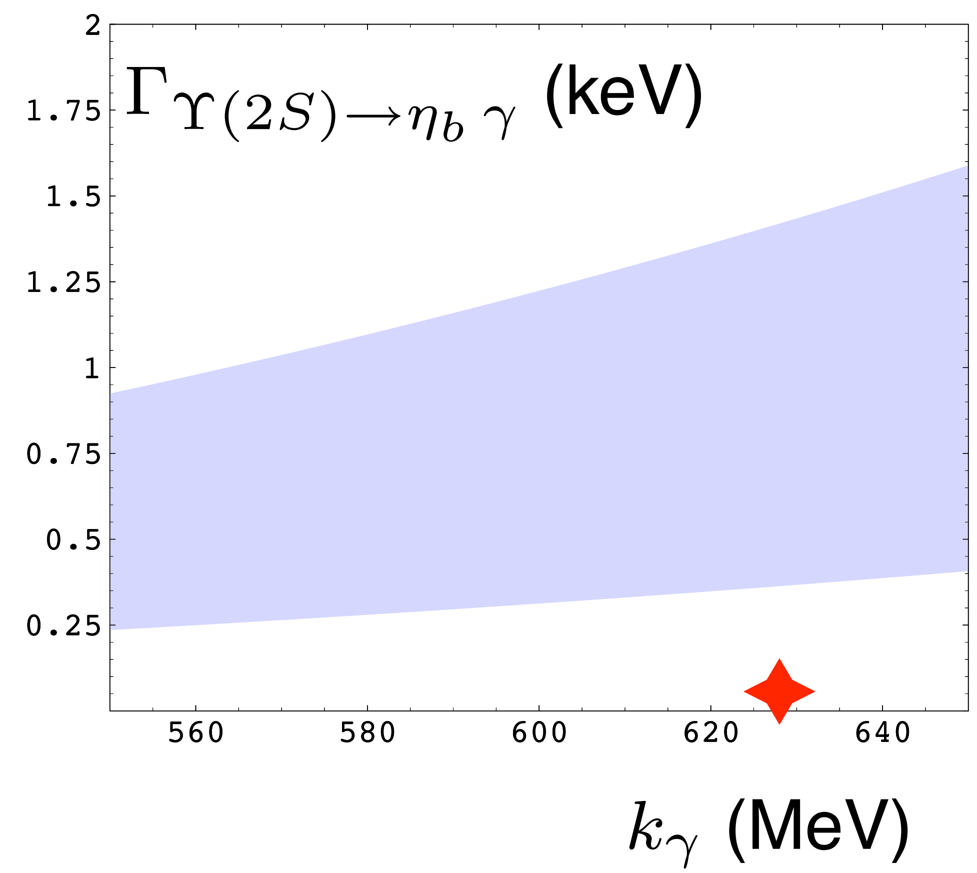
- New operators contribute :

$$-\frac{1}{16m^2}\, c_S^{\rm em}\, \left[S^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}\times, ee_Q\mathbf{E}^{\rm em}\right]\right] S$$

$$-\frac{1}{16m^2}\, c_S^{\rm em}\, \left[S^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}^i ee_Q\mathbf{E}^{\rm em})\right]\right] S$$

- wavefunction corrections:
  - induced by spin-spin potential
  - recoil correction induced by spin-orbit potential
- Can only calculate in weak coupling region.

N. Brambilla, Y. Jia and A. Vairo, Phys. Rev. D 73, 054005 (2006) [arXiv:hep- ph/0512369].



- Recently observed by BABAR:

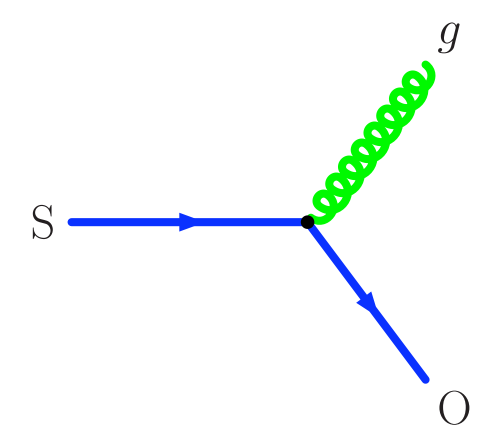
$$\mathcal{B}(\Upsilon(3S)\to\gamma\eta_b(1S)) = (4.8\pm0.5\pm0.6)\times10^{-4}$$
$$\mathcal{B}(\Upsilon(2S)\to\gamma\eta_b(1S)) = (4.2^{+1.1}_{-1.0}\pm0.9)\times10^{-4}\,.$$

- Far below theoretical expectations

Multipole Expansion for Hadronic Transitions

- For lowest order gluon emission:

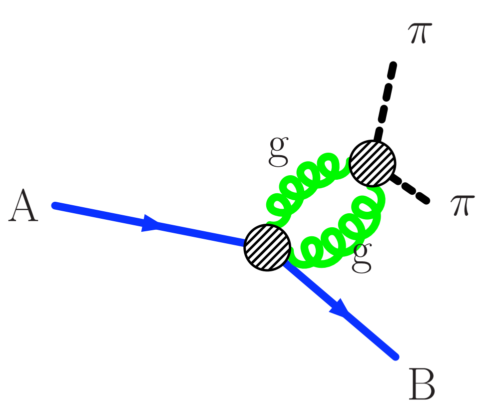
$$\mathcal{H}_I = i\psi^{\dagger\prime}\frac{\mathbf{r}}{\mathbf{2}}\cdot \mathbf{g}\mathbf{E}'_{\mathbf{a}}\mathbf{t}^{\mathbf{a}}\psi' + \frac{\mathbf{c_F}}{\mathbf{m_Q}}\psi^{\dagger\prime}\mathbf{s_Q}\cdot \mathbf{g}\mathbf{t}^{\mathbf{a}}\mathbf{B}'_{\mathbf{a}}\psi' + [\mathbf{Q}- > \bar{\mathbf{Q}}] + \cdots$$



for dressed fields $\psi' = U^{-1}\psi$, $\mathbf{t}^{\mathbf{a}}\mathbf{A}'^{\mu}_{\mathbf{a}} = U^{-1}\mathbf{t}^{\mathbf{a}}\mathbf{A}^{\mu}_{\mathbf{a}}U - \frac{i}{g}U^{-1}\partial^{\mu}U$

But single emission takes color singlet state (S) to unphysical octet state (O).

- Double transitions dominate: E1 E1, E1 M1, M1 M1
- Factorization:



$$\delta_{\rm ab}$$

E1 E1 $\frac{g_{\rm E}^2}{8} < B|\mathbf{r_i}gt^a\mathcal{G}\mathbf{r_j}gt^b|A > \quad < \pi\pi|\mathbf{E}^{\mathbf{i}}_{\mathbf{a}}\mathbf{E}^{\mathbf{i}}_{\mathbf{b}}|\mathbf{0} >$

electric polarizability chiral methods

Brown & Cahn (1975), ...

model $\mathcal{G} = (\mathrm{E_A} - \mathcal{H}^0_{\rm NR})^{-1} = \sum_{\rm KL} \frac{|\mathrm{KL}><\mathrm{KL}|}{\mathrm{E_A} - \mathrm{E_{KL}}}$ $(Q\bar{Q}\ \text{octet})$

Y.P. Kuang & T.M. Yan [PR D24, 2874 (1981)]

quark confining string

E1 E1, E1 M1, M1 M1

power counting	v	v^2	v^3	→

leading EM and hadronic transitions remain proportional as c->b

Hadronic Transitions

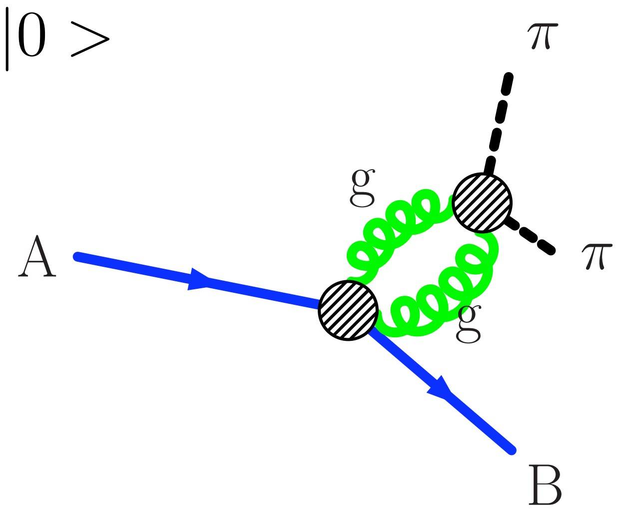
- Two pion transitions

$$\mathcal{M}_{if}^{gg} = \frac{1}{16} < B|\mathbf{r_i}\xi^a \mathcal{G} \mathbf{r_j}\xi^a|A > \quad \frac{g_E^2}{6} < \pi_\alpha \pi_\beta | Tr(\mathbf{E^i E^j})|0 >$$

α_{AB}^{EE}

Model: Kuang & Yan

Hadronize



S-wave D-wave

$$\frac{\delta_{\alpha\beta}}{\sqrt{(2\omega_1)(2\omega_2)}} \left[C_1 \delta_{kl} q_1^\mu q_{2\mu} + C_2 \left(q_{1k} q_{2l} + q_{1l} q_{2k} - \frac{2}{3}\delta_{kl} \ (q_1 \cdot q_2) \right) \right]$$

S state -> S state

$$d\Gamma \sim K \sqrt{1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}} (M_{\pi\pi}^2 - 2m_\pi^2)^2 \ dM_{\pi\pi}^2 \qquad K \equiv \frac{\sqrt{(M_A + M_B)^2 - M_{\pi\pi}^2}\sqrt{(M_A - M_B)^2 - M_{\pi\pi}^2}}{2M_A}$$

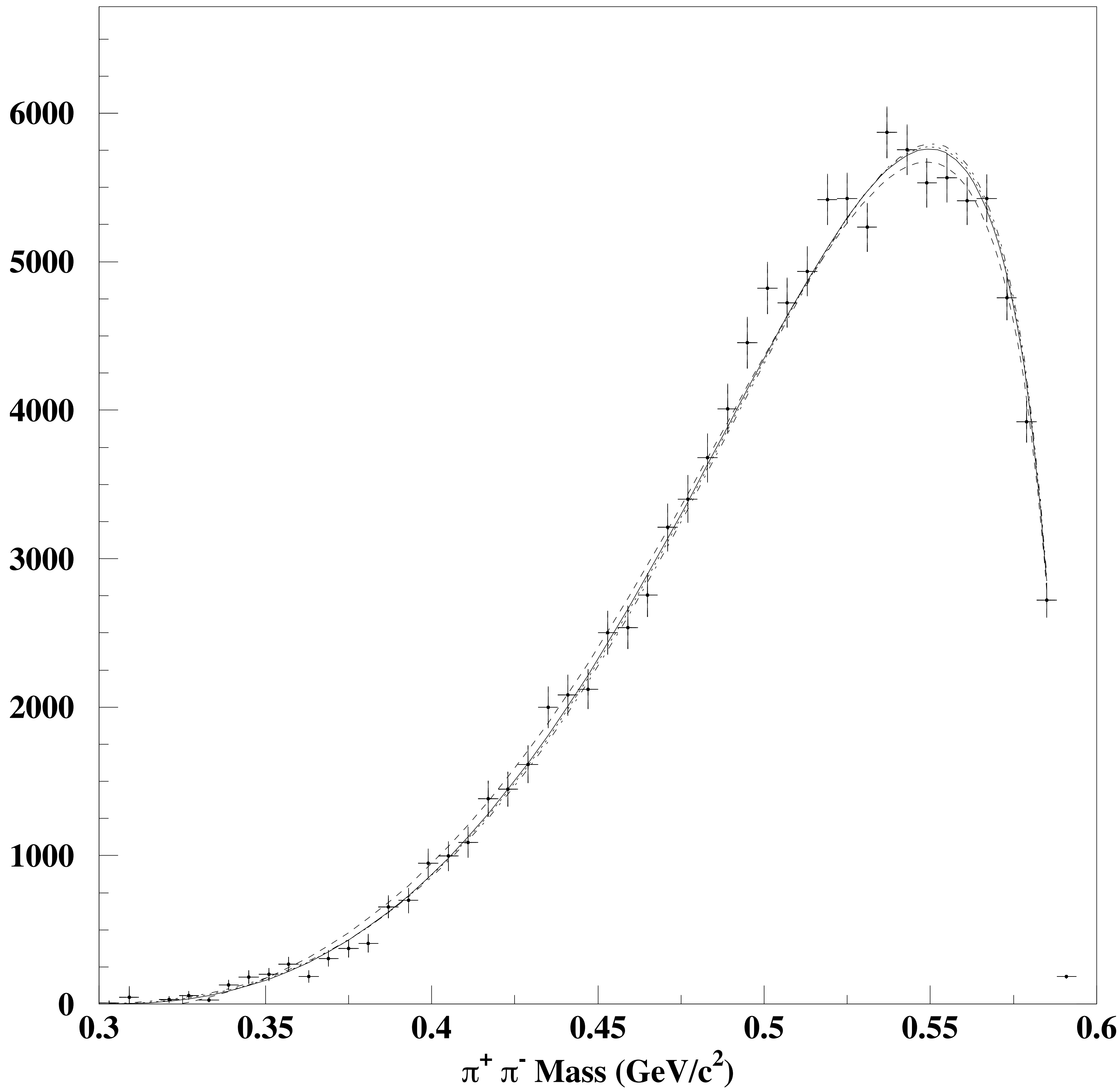
$$\Gamma = \mathrm{G} \ |\alpha_{\mathrm{AB}}^{\mathrm{EE}} \ \mathrm{C_1}|^2$$

Phase Space Overlap - Vibrating String Model

BES

$$\psi' \to J/\psi + \pi^+\pi^-$$

detailed study



D state -> S state

Determines

$$C_2/C_1 = 1.52^{+0.35}_{-0.45}$$

CLEO

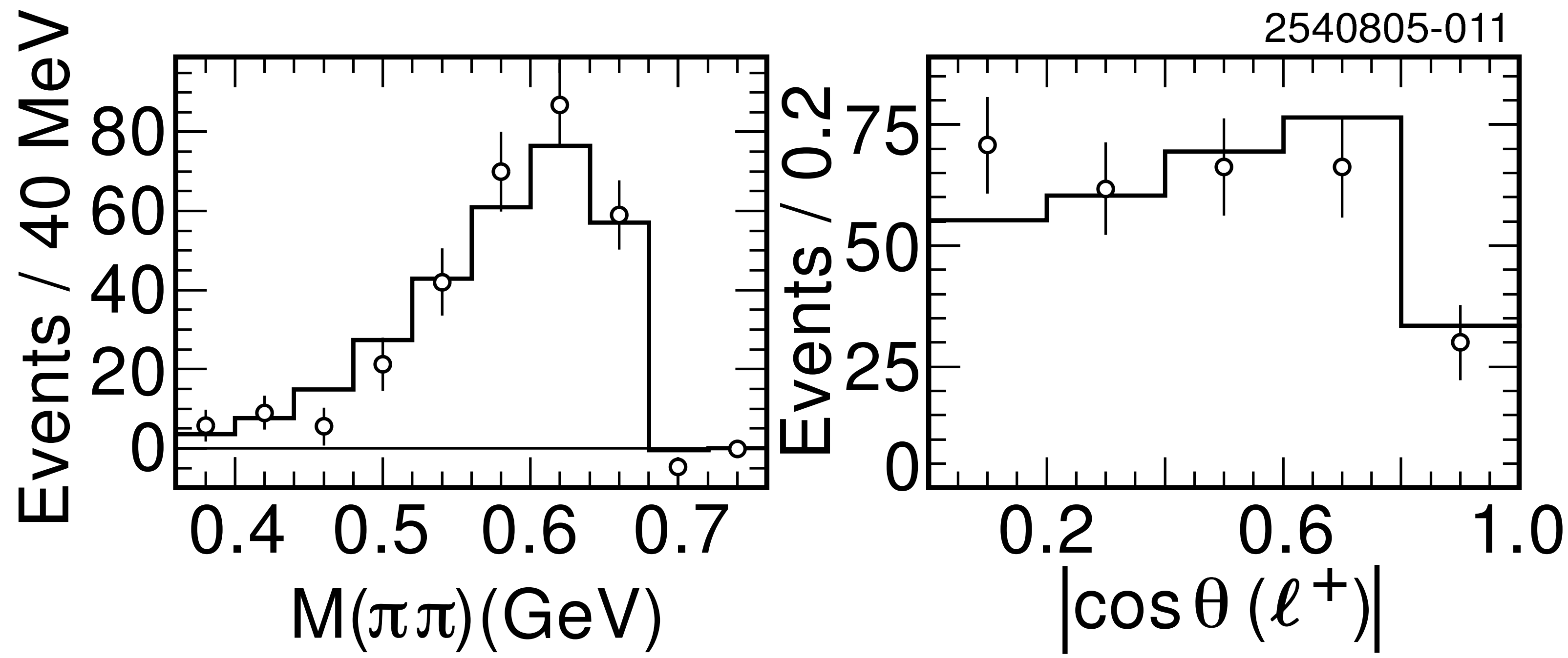


FIG. 4: Distributions in $\pi^+\pi^-\ell^+\ell^-$ events of the $\pi^+\pi^-$ mass (left) and polar angle (right) of the positively charged lepton from data (open circles) and MC (solid line line).

P state -> P state

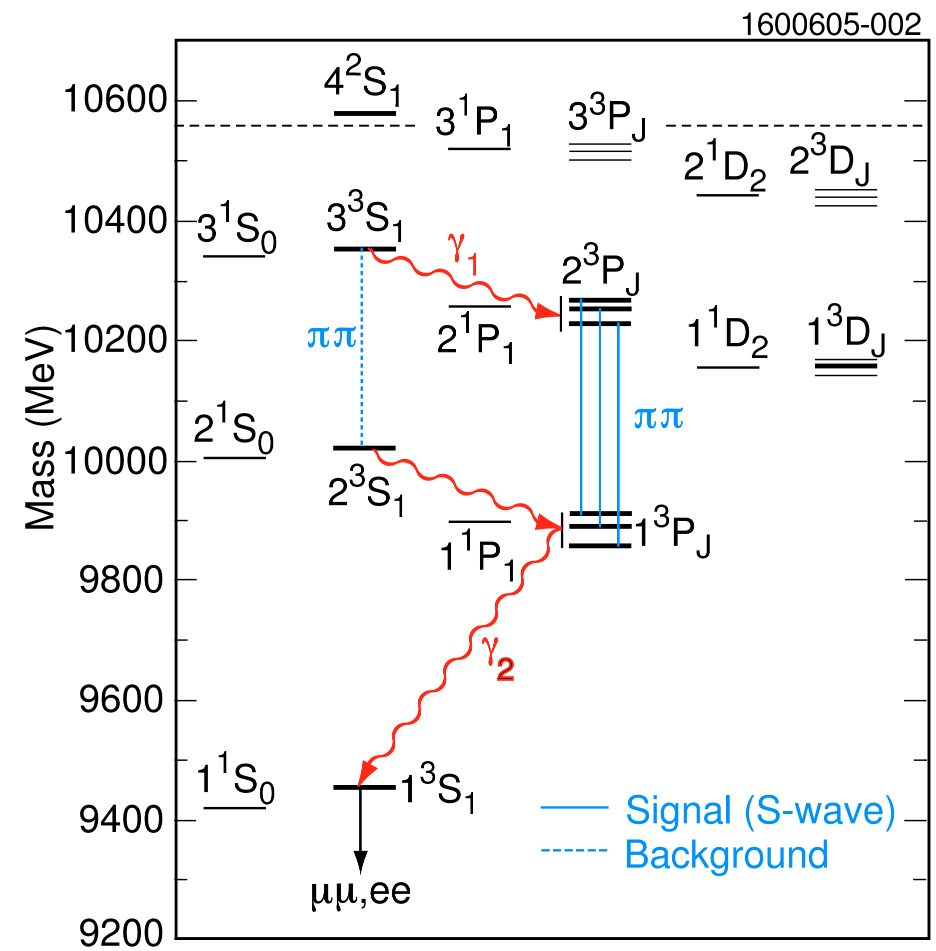
Assume only S wave term => J = J'

$$\Gamma_{\pi\pi} = (0.83 \pm 0.22 \pm 0.08 \pm 0.19)\ \mathrm{keV}$$

CLEO

$2P_J$ -> $1P_{J'}$ + 2π - First observation[CLEO]

Results agree with Kuang and Yan (1988)



Known hadronic transitions

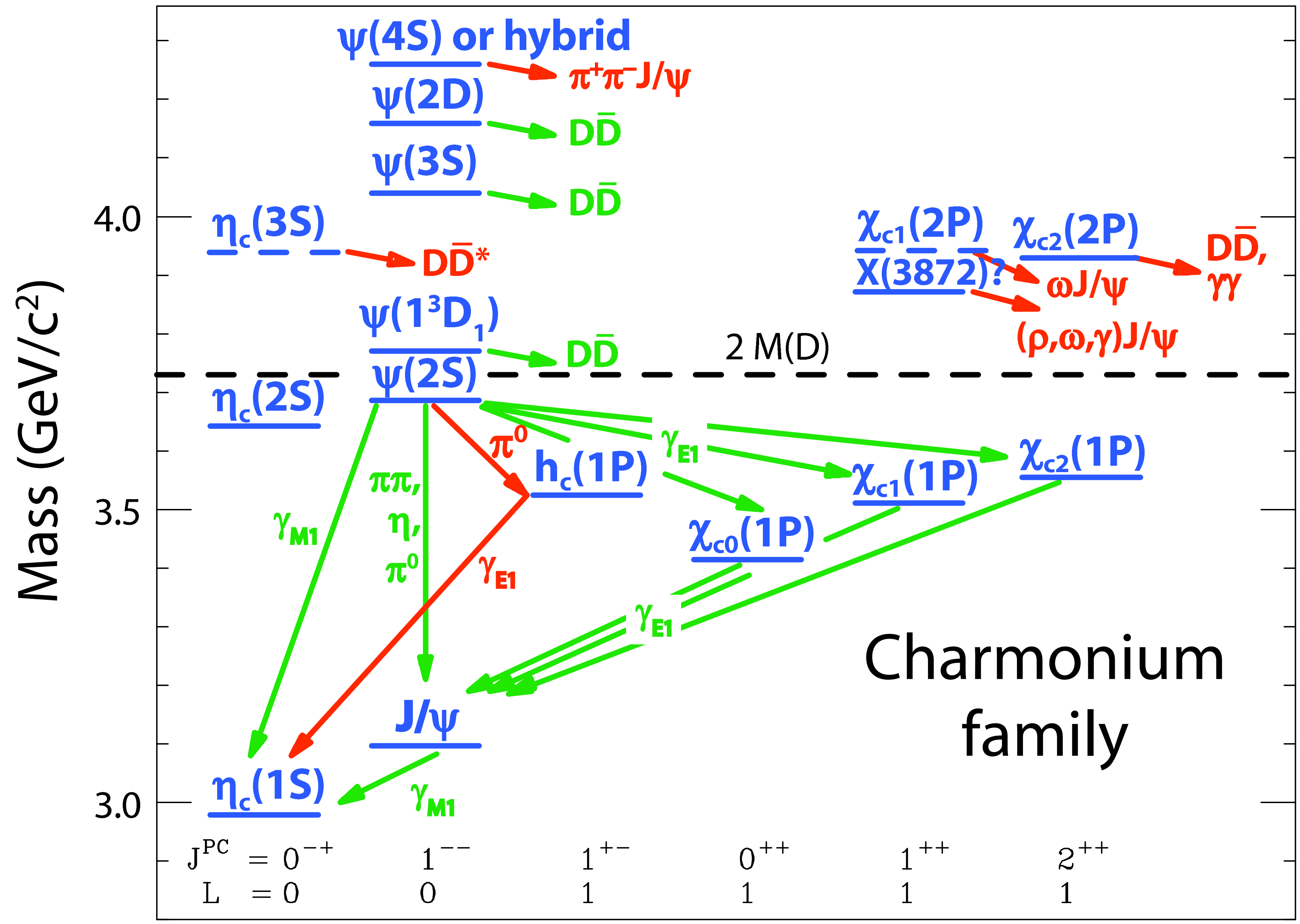
Transition	Γ_{partial} (keV) (Experiment)	Γ_{partial} (keV) (KY Model)
$\psi(2S)$		
$\to J/\psi + \pi^+\pi^-$	102.3 ± 3.4	input ($\|C_1\|$)
$\to J/\psi + \eta$	10.0 ± 0.4	input (C_3/C_1)
$\to J/\psi + \pi^0$	0.411 ± 0.030 [435]	0.64 [456]
$\to h_c(1P) + \pi^0$	0.26 ± 0.05 [43]	0.12-0.40 [376]
$\psi(3770)$		
$\to J/\psi + \pi^+\pi^-$	52.7 ± 7.9	input ($C_2/C_\|$)
$\to J/\psi + \eta$	24 ± 11	
$\psi(3S)$		
$\to J/\psi + \pi^+\pi^-$	< 320 (90% CL)	
$\Upsilon(2S)$		
$\to \Upsilon(1S) + \pi^+\pi^-$	5.79 ± 0.49	8.7 [461]
$\to \Upsilon(1S) + \eta$	$(6.7 \pm 2.4) \times 10^{-3}$	0.025 [455]
$\Upsilon(1^3D_2)$		
$\to \Upsilon(1S) + \pi^+\pi^-$	0.188 ± 0.046 [59]	0.07 [462]
$\chi_{b1}(2P)$		
$\to \chi_{b1}(1P) + \pi^+\pi^-$	0.83 ± 0.33 [457]	0.54 [463]
$\to \Upsilon(1S) + \omega$	1.56 ± 0.46	
$\chi_{b2}(2P)$		
$\to \chi_{b2}(1P) + \pi^+\pi^-$	0.83 ± 0.31 [457]	0.54 [463]
$\to \Upsilon(1S) + \omega$	1.52 ± 0.49	
$\Upsilon(3S)$		
$\to \Upsilon(1S) + \pi^+\pi^-$	0.894 ± 0.084	1.85 [461]
$\to \Upsilon(1S) + \eta$	$< 3.7 \times 10^{-3}$	0.012 [455]
$\to \Upsilon(2S) + \pi^+\pi^-$	0.498 ± 0.065	0.86 [461]
$\Upsilon(4S)$		
$\to \Upsilon(1S) + \pi^+\pi^-$	1.64 ± 0.25	4.1 [461]
$\to \Upsilon(1S) + \eta$	4.02 ± 0.54	
$\to \Upsilon(2S) + \pi^+\pi^-$	1.76 ± 0.34	1.4 [461]

Known hadronic transition

$\Upsilon(5S)$	
$\rightarrow \Upsilon(1S) + \pi^+\pi^-$	228 ± 33
$\rightarrow \Upsilon(1S) + K^+K^-$	26.2 ± 8.1
$\rightarrow \Upsilon(2S) + \pi^+\pi^-$	335 ± 64
$\rightarrow \Upsilon(3S) + \pi^+\pi^-$	206 ± 80

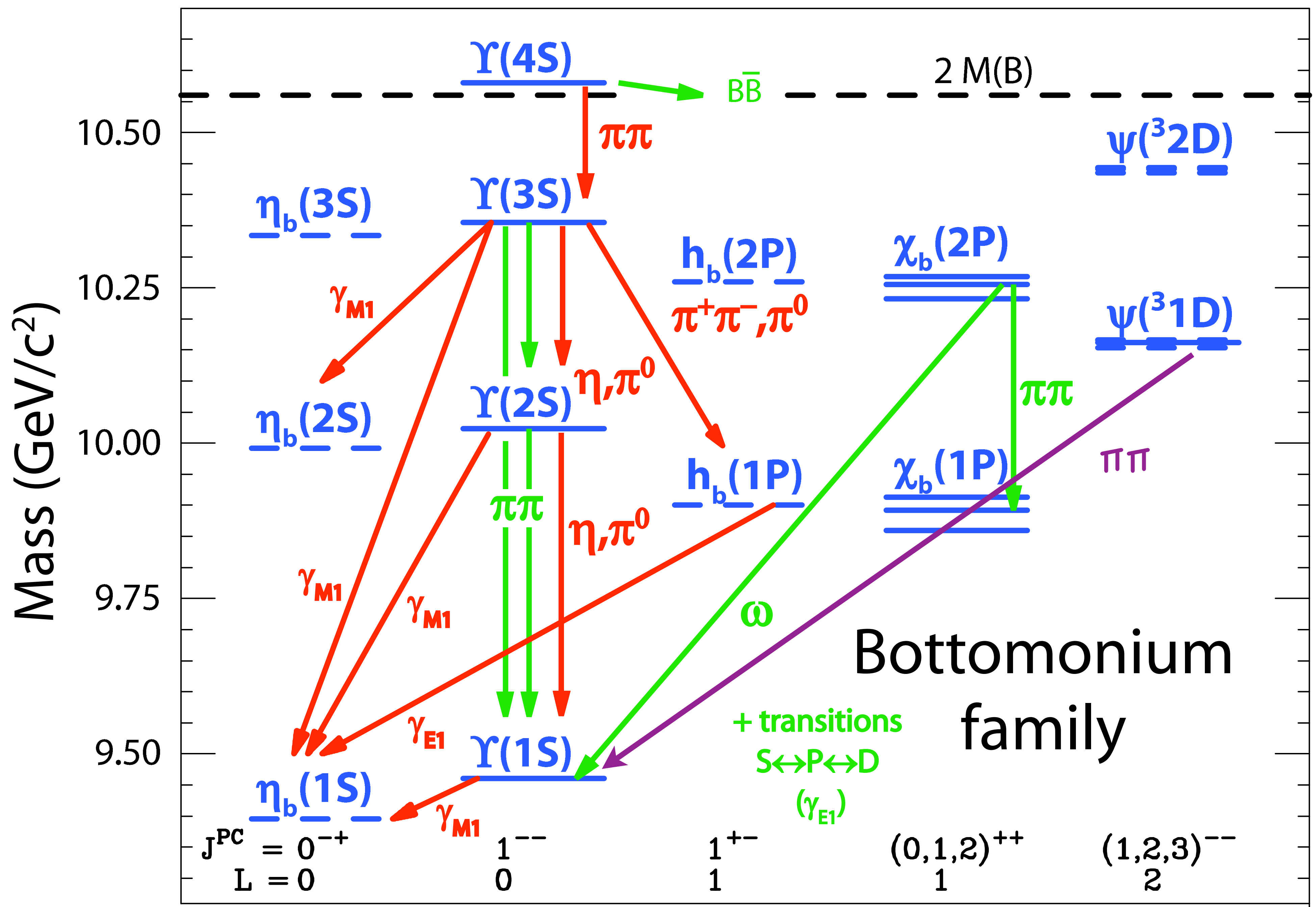
- Will return to this at the end.

Narrow States Below Threshold



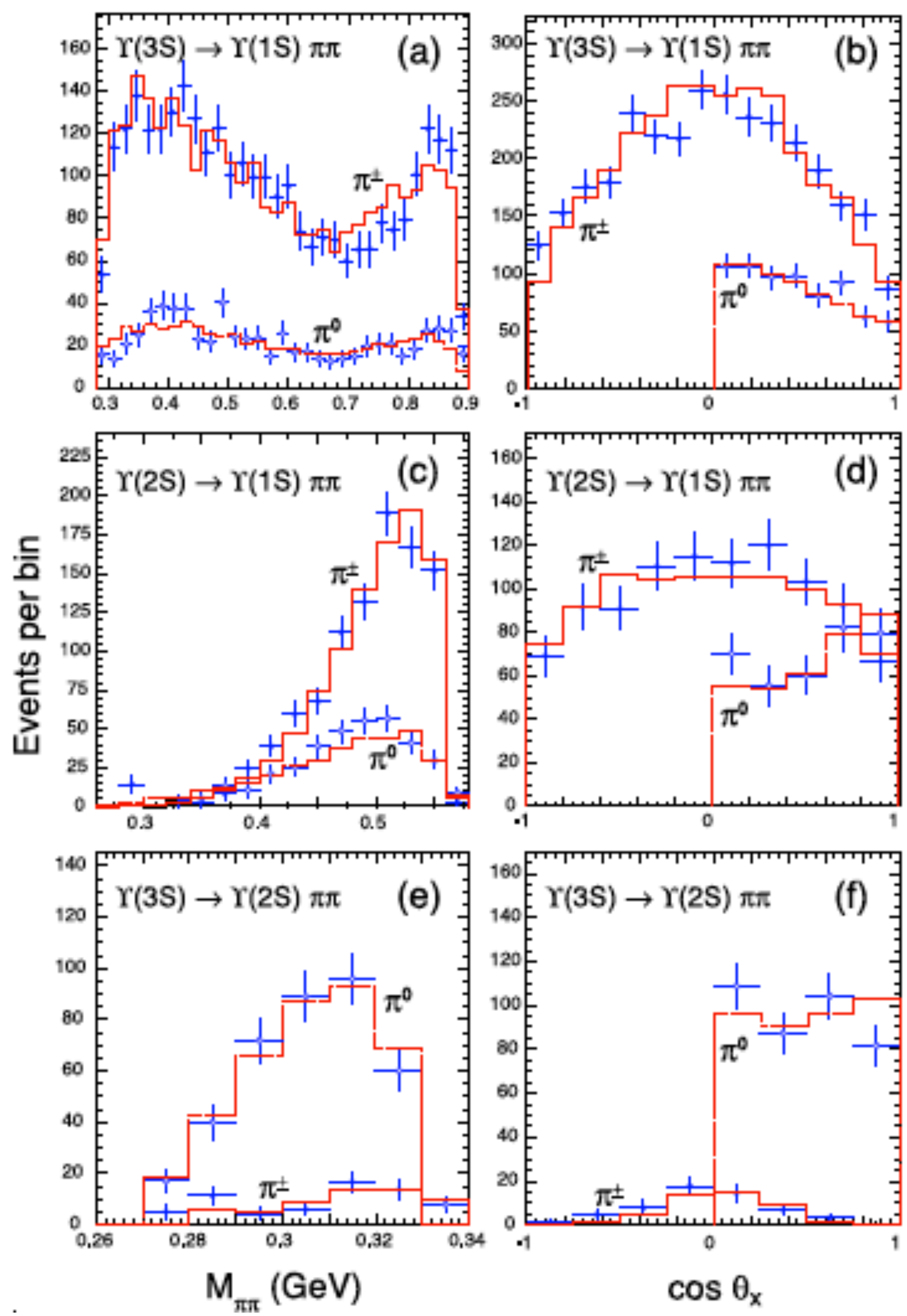
Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E. [Rev. Mod. Phys. 80, 1161 (2008)]

Narrow States Below Threshold



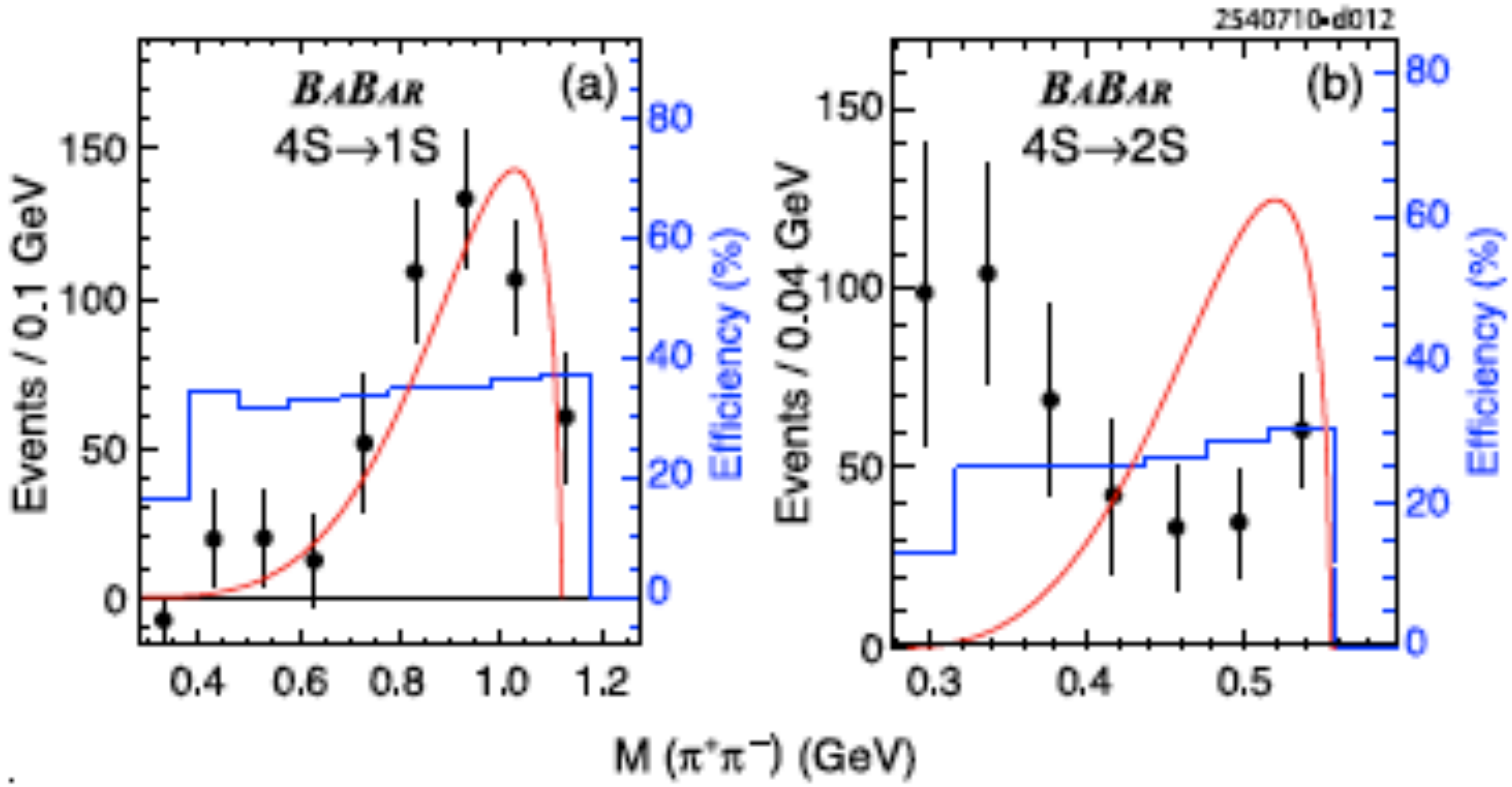
Stephen Godfrey, Hanna Mahlke, Jonathan L. Rosner and E.E. [Rev. Mod. Phys. 80, 1161 (2008)]

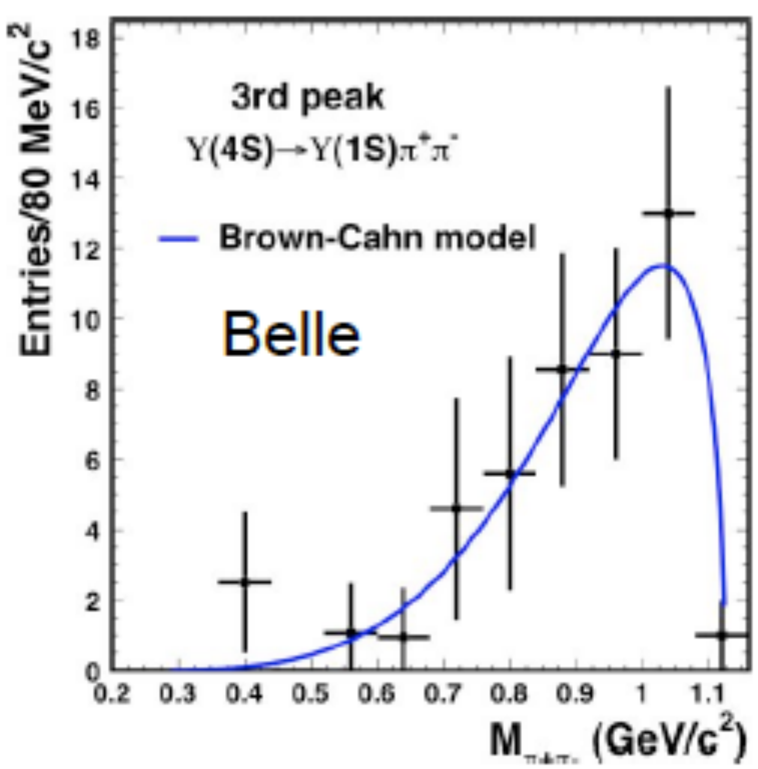
$M_{\pi\pi}$ distributions



CLEO

3S -> 1S ?
3S -> 2S ✓
2S -> 1S ✓


2540710-d012
BABAR
4S→1S
(a)
BABAR
4S→2S
(b)
Events / 0.1 GeV
Events / 0.04 GeV
Efficiency (%)
M (π⁺π⁻) (GeV)


3rd peak
Υ(4S)→Υ(1S)π⁺π⁻
Brown-Cahn model
Belle
Entries/80 MeV/c²
(GeV/c²)

4S -> 1S ✓

4S -> 2S ?

Puzzles

$$\Upsilon(3S) \rightarrow \Upsilon + \pi\pi$$
$$\Upsilon(4S) \rightarrow \Upsilon(2S) + \pi\pi$$

don't show leading (S-wave) two pion invariant mass distribution

Many proposals for explaining the Υ(3S)->Υ transition but most don't survive new results for Υ(4S):

- Final State Interactions

  ➤ Problem: Compare Υ(4S)->Υ(2S), Υ(2S)->Υ(1S) and ψ(2S) -> J/ψ

  essentially the same phase space but different distributions.

- coupling to decay channels

  Problem: Compare Υ(3S)->Υ(1S) to ψ(2S)->J/ψ ,Υ(4S)->Υ(1S)

  Coupled channel effects should be larger in second set.

- exotic intermediate state

  Problem: Compare Υ(4S)->Υ(2S), Υ(3S)->Υ(1S)
  similiar distributions but shifted masses

- dynamical accident - suppress the leading E1E1 term

Worth a closer look.

◇ dynamical accident - suppress the leading E1E1 term

Like the E1 case ?

Δn =2 overlaps suppressed.

Toy model

Below lowest threshold

$$\sum_{nl} \frac{|\Psi_{nl}><\Psi_{nl}|}{E_i - E_{nl}} \sim \frac{1}{E_i - E^{\rm TH}_{\rm string}} + \cdots$$

Predicted for Υ(3S)->Υ(1S)

3. The rate for $\Upsilon'' \to \Upsilon\pi\pi$ is surprisingly small. If we compare the phase-space integrals (2.4) for the two transitions $\Upsilon'' \to \Upsilon\pi\pi$ and $\Upsilon' \to \Upsilon\pi\pi$, their ratio is large,

$$\frac{G(\Upsilon'' \to \Upsilon\pi\pi)}{G(\Upsilon' \to \Upsilon\pi\pi)} \approx 33\ . \qquad (2.24)$$

The matrix element for $\Upsilon'' \to \Upsilon\pi\pi$ is tremendously suppressed:

$$\left|\frac{f^1_{if}(\Upsilon'' \to \Upsilon\pi\pi)}{f^1_{if}(\Upsilon' \to \Upsilon\pi\pi)}\right|^2 \approx (2-4)\times 10^{-3}\ . \qquad (2.25)$$

The large suppression is due to two effects. First, there is a great deal of cancellation among different terms in the series for $f^1_{if}(\Upsilon'' \to \Upsilon\pi\pi)$. Second, many high vibrational levels contribute, so the mean distance from these levels to Υ'' is large. Because of the delicate cancellations, we cannot expect our results to be very reliable.

Kuang &Yan (1981)

Transition	G(GeV^5)	$< f\|r^2\|i >$ (GeV^{-2})	$G(< f\|r^2\|i >)^2 \times 10^{+2}$
$\psi(2S) \to J/\psi$	2.3×10^{-2}	-3.0	20
$\Upsilon(2S) \to \Upsilon$	2.1×10^{-2}	-1.19	3.0
$\Upsilon(3S) \to \Upsilon$	3.3×10^{-1}	-2.4×10^{-1}	1.9
$\Upsilon(3S) \to \Upsilon(2S)$	9.1×10^{-4}	-3.7	1.2
$\Upsilon(4S) \to \Upsilon$	1.2	-9.8×10^{-2}	1.2
$\Upsilon(4S) \to \Upsilon(2S)$	2.0×10^{-2}	-4.6×10^{-1}	0.21

If leading <E1 E1> suppressed, can <M1 M1> be significant

Detailed study: S-wave

Voloshin hep-ph/606258

$$S(\psi_2 \to \pi^+\pi^-\psi_1) = \qquad (25)$$
$$-\frac{4\pi^2}{b}\,\alpha_0^{(12)}\left[(1-\chi_M)\,(q^2+m^2) - (1+\chi_M)\,\kappa\left(1+\frac{2m^2}{q^2}\right)\left(\frac{(q\cdot P)^2}{P^2} - \frac{1}{2}\,q^2\right)\right](\psi_1\cdot\psi_2)\ ,$$

and three D-waves

$$D_1(\psi_2 \to \pi^+\pi^-\psi_1) = -\frac{4\pi^2}{b}\,\alpha_0^{(12)}\,(1+\chi_M)\,\frac{3\kappa}{2}\,\frac{\ell_{\mu\nu}P^\mu P^\nu}{P^2}\,(\psi_1\cdot\psi_2)\ ,$$ spin independent

$$D_2(\psi_2 \to \pi^+\pi^-\psi_1) = \frac{4\pi^2}{b}\,\alpha_0^{(12)}\left(\chi_2+\frac{3}{2}\,\chi_M\right)\frac{\kappa}{2}\left(1+\frac{2m^2}{q^2}\right)q_\mu q_\nu\psi^{\mu\nu}$$

spin dependent

$$D_3(\psi_2 \to \pi^+\pi^-\psi_1) = \frac{4\pi^2}{b}\,\alpha_0^{(12)}\left(\chi_2+\frac{3}{2}\,\chi_M\right)\frac{3\kappa}{4}\,\ell_{\mu\nu}\psi^{\mu\nu}$$

$$\psi^{\mu\nu} = \psi_1^\mu\psi_2^\nu + \psi_1^\nu\psi_2^\mu - (2/3)\,(\psi_1\cdot\psi_2)\,(P^\mu P^\nu/P^2 - g^{\mu\nu}) \qquad P_\mu = M_A\delta_\mu^0$$

$$\ell_{\mu\nu} = r_\mu r_\nu + \frac{1}{3}\left(1-\frac{4m^2}{q^2}\right)(q^2\,g_{\mu\nu} - q_\mu q_\nu) \qquad r_\mu = (k_{1\mu} - k_{2\mu})$$

$$\chi_M = \frac{\alpha_M}{\alpha_0}\ , \qquad \chi_2 = \frac{\alpha_2}{\alpha_0}$$

magnetic   S-D mixing

Expect noticeable presence of D2 and D3 in Υ (3S) ->Υ +$\pi\pi$ decay

BUT - In addition to the suppression of the M1-M1 term by $\langle v^2 \rangle$ relative to the dominate E1-E1 term:

Radial overlap amplitude:

$$\sum_{n,l} \frac{< f|\Psi_{nl}) >< \Psi_{nl}|i >}{E_i - E_{X(nl)}}$$

with the hybrid states

$$\Psi_{nl} = \Pi_u^+(nP)$$

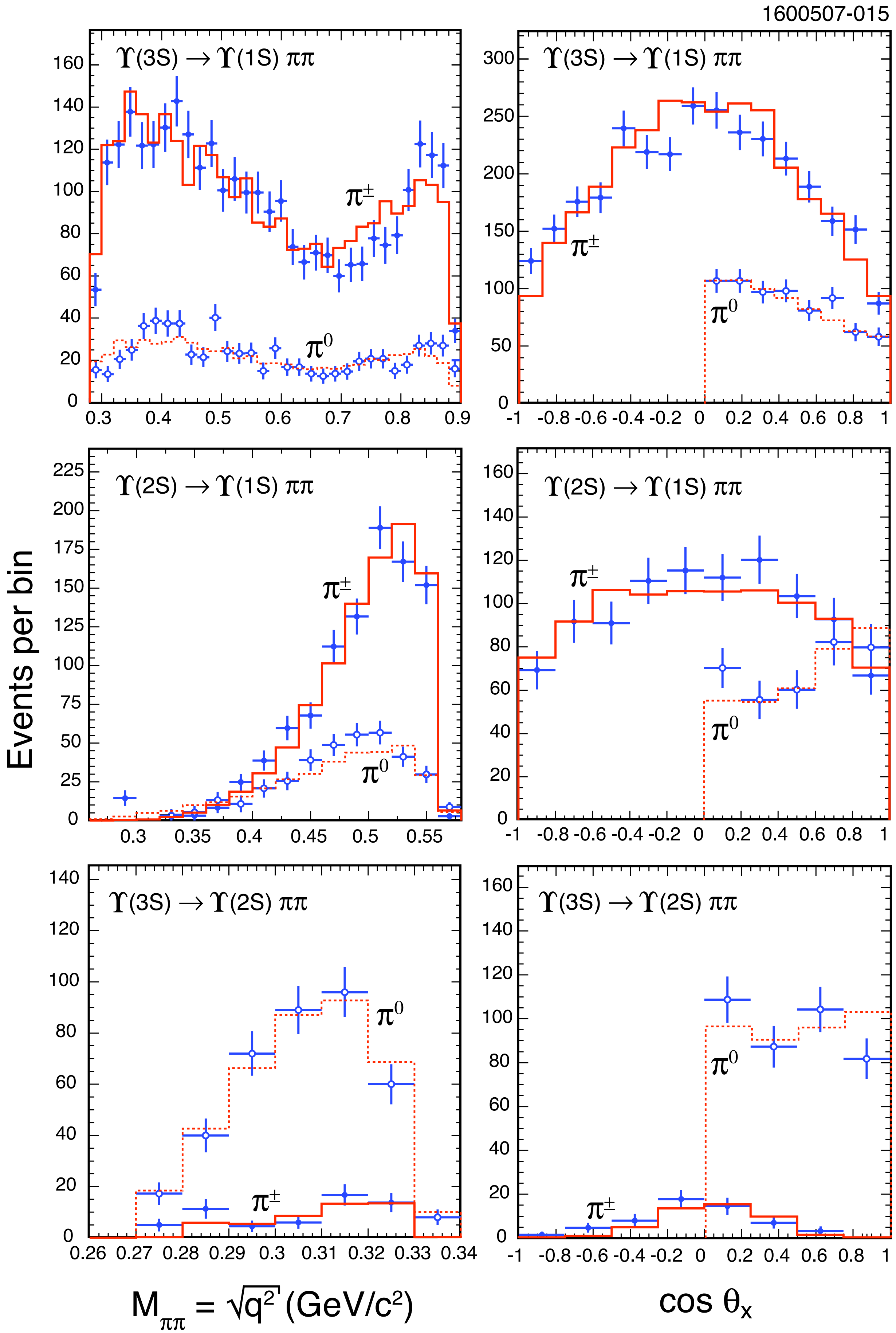
Again below lowest intermediate state threshold

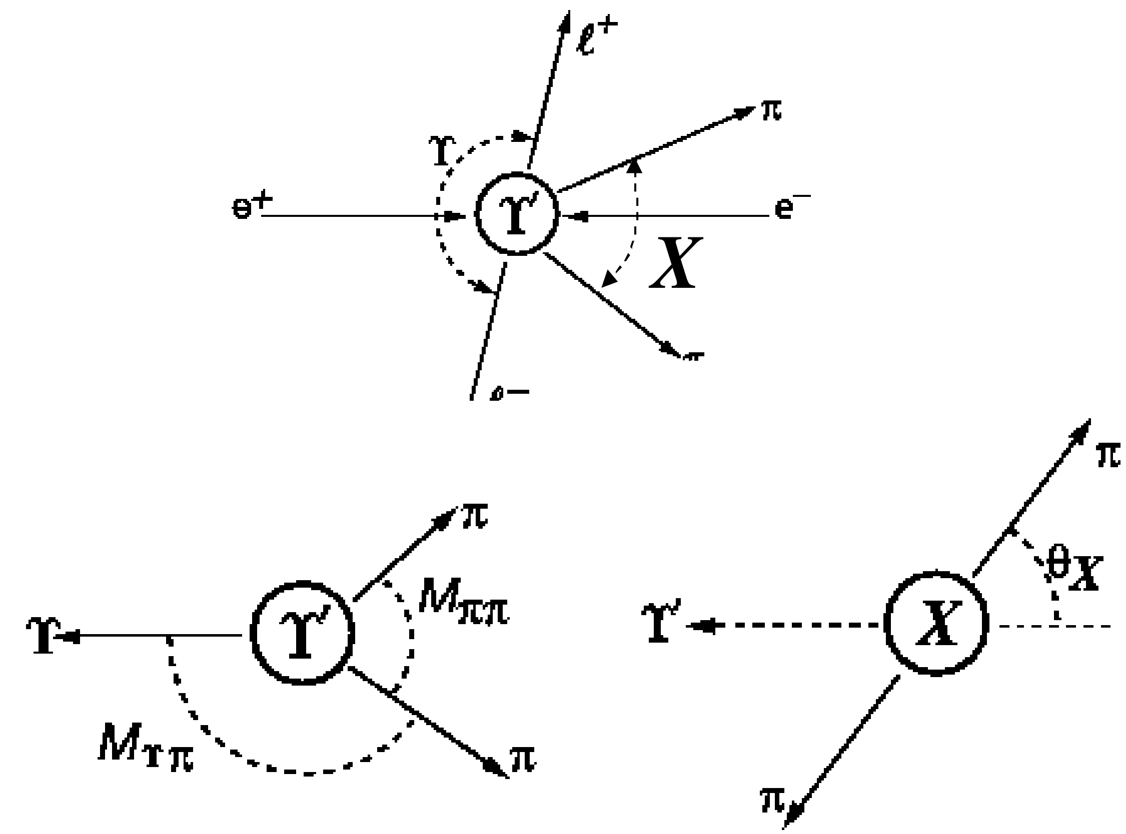
$$\sum_{nl} \frac{|\Psi_{nl} >< \Psi_{nl}|}{E_i - E_{nl}} \sim \frac{1}{E_i - E^{\mathrm{TH}}_{\mathrm{string}}} + \cdots$$

In this limit the overlap vanishes since $\langle f|i\rangle=0$ $(i\neq f)$

The M1-M1 term is highly suppressed !

CLEO





$$q^2 = M_{\pi\pi}{}^2$$

$$r^2 = M_{\Upsilon\pi}{}^2$$

$$\cos\theta_X = \frac{M_{\Upsilon(2S)}{}^2 + M_{\Upsilon(1S)}{}^2 + 2m_\pi{}^2 - q^2 - 2r^2}{\sqrt{\Lambda_3(M_{\Upsilon(2S)}{}^2, M_{\Upsilon(1S)}{}^2, q^2)\,\frac{q^2 - 4m_\pi{}^2}{q^2}}}$$

QQ(n^3S_1) -> QQ(m^3S_1) + $\pi^+\pi^-$

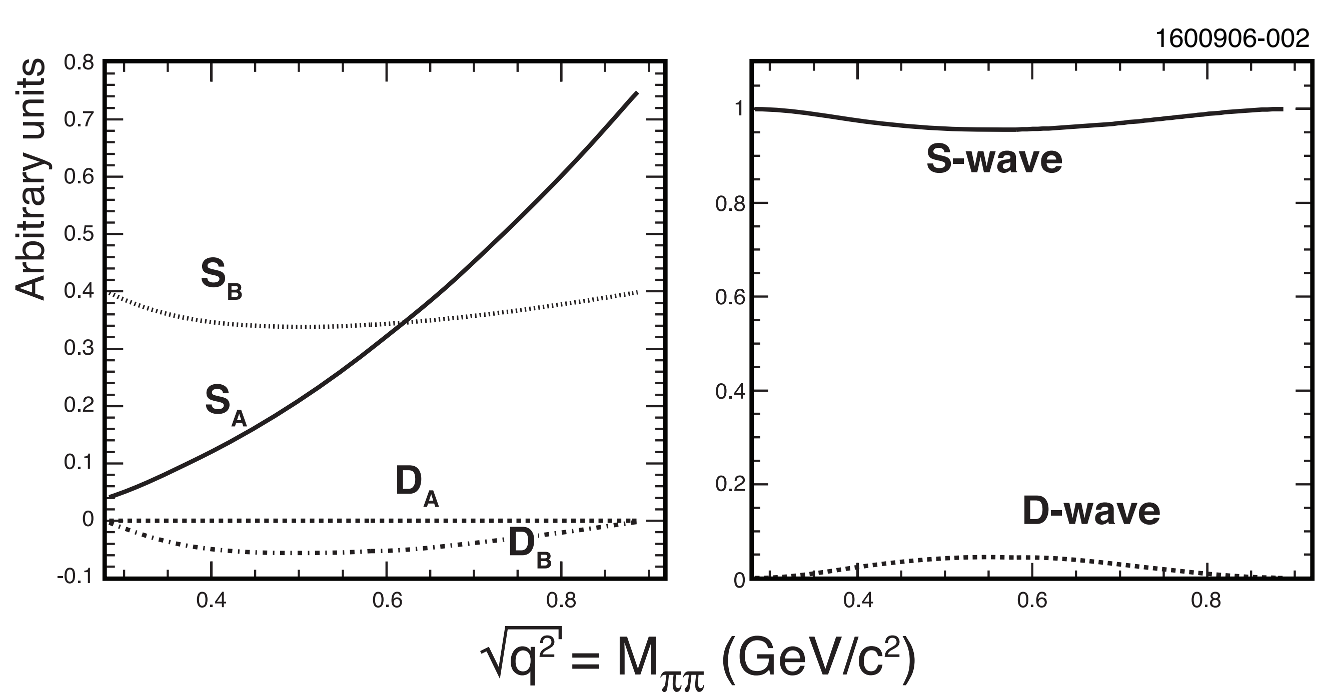
$$M = \mathrm{A}(\varepsilon'\cdot\varepsilon)(q^2 - 2m_\pi^{\ 2}) + \mathrm{B}(\varepsilon'\cdot\varepsilon)E_1E_2 + \mathrm{C}\left[(\varepsilon'\cdot q_1)(\varepsilon\cdot q_2) + (\varepsilon'\cdot q_2)(\varepsilon\cdot q_1)\right]$$

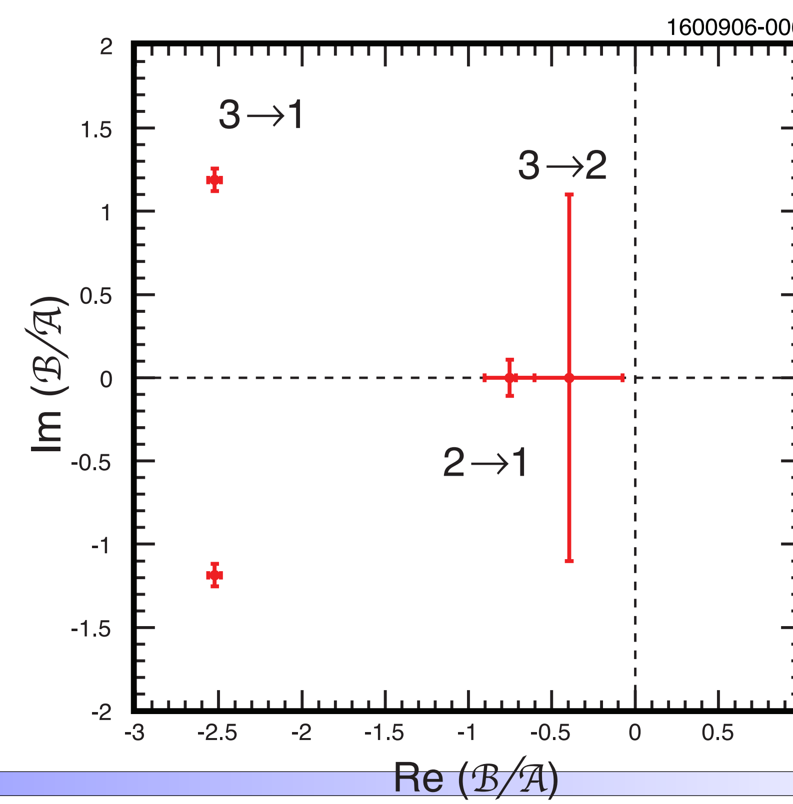
- Hindered M1-M1 term => C≈0. Consistent with CLEO results.
- Small D-wave contributions
- Useful to look at polarization info.
  Dubynskiy & Voloshin [hep-ph/0707.1272]

CLEO

Fit, No $\mathcal{C}$			stat.	effcy. ($\pi^\pm$)	effcy.(π^0)	bg. sub.
$\Upsilon(3S)\to\Upsilon(1S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-2.523	±0.031	±0.019	±0.011	±0.001
	$\Im(\mathcal{B}/\mathcal{A})$	±1.189	±0.051	±0.026	±0.018	±0.015
$\Upsilon(2S)\to\Upsilon(1S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-0.753	±0.064	±0.059	±0.035	±0.112
	$\Im(\mathcal{B}/\mathcal{A})$	0.000	±0.108	±0.036	±0.012	±0.001
$\Upsilon(3S)\to\Upsilon(2S)\pi\pi$	$\Re(\mathcal{B}/\mathcal{A})$	-0.395	±0.295		±0.025	±0.120
	$\Im(\mathcal{B}/\mathcal{A})$	±0.001	±1.053		±0.180	±0.001
Fit, float $\mathcal{C}$			stat.	effcy. ($\pi^\pm$)	effcy.(π^0)	bg. sub.
$\Upsilon(3S)\to\Upsilon(1S)\pi\pi$	$\lvert\mathcal{B}/\mathcal{A}\rvert$	2.89	±0.11	±0.19	±0.11	±0.027
	$\lvert\mathcal{C}/\mathcal{A}\rvert$	0.45	±0.18	±0.28	±0.20	±0.093

3S->1S





Single light hadron transitions

- Need higher order: E1-M1 O(v) CP = -- couples to ω; E1-M1 and E1-E2 O(v) amd M1-M1, E1-M2 in O(v2) CP = +- couples to π, η; M1-M1, E1-E3, E2-E2 CP=++; ...
- final states π, η ,η' proceed from two gluon component of η'
- SU(3) and chiral symmetry breaking - chiral effective theory

$$\tilde{\pi}^0 = \pi^0 + \epsilon\eta + \epsilon'\eta'$$

$$\tilde{\eta} = \eta - \epsilon\pi^0 + \theta\eta'$$

$$\tilde{\eta}' = \eta' - \theta\eta - \epsilon'\pi^0,$$

$$\epsilon = \frac{(m_d - m_u)\sqrt{3}}{4(m_s - \frac{m_u + m_d}{2})}, \qquad \epsilon' = \frac{\tilde{\lambda}(m_d - m_u)}{\sqrt{2}(m^2_{\eta'} - m^2_{\pi^0})}, \qquad \theta = \sqrt{\frac{2}{3}} \; \frac{\tilde{\lambda}\left(m_s - \frac{m_u + m_d}{2}\right)}{m^2_{\eta'} - m^2_{\eta}}.$$

- Many puzzles in relative strengths for η in the charmonium and bottomium systems. Opportunites for new theoretical insights.

Transitions for states above threshold

- Rare processes
- ψ(3770) E1 transitions and $\pi\pi$ transitions already
- ψ(4040): Γ ~ 80 MeV
  - Rate for ψ (4040) -> J/ψ + $\pi\pi$ expected to be about 25 keV [scaling from the Υ(3S) -> Υ(1S) + $\pi\pi$] (<340 KeV at present). Compare $M_{\pi\pi}$ distributions with ψ(4040) -> J/ψ + $\pi\pi$ with Υ(3S) -> Υ(1S) + $\pi\pi$]. Others? η and π^0
  - E1 rates (naive):

$$
\begin{aligned}
\psi(4040) \quad &\to \quad \chi'_{c2}(2^3P_2)(Z(3927)?) + \gamma = 46\text{keV} \\
&\to \quad \chi'_{c1}(2^3P_1)(X(3872)?) + \gamma = 85\text{keV} \\
&\to \quad \chi'_{c0}(2^3P_0)(3820) + \gamma = 58\text{keV} \\
&\to \quad \chi'_{c2}(1^3P_2) + \gamma = 5.9\text{keV} \\
&\to \quad \chi'_{c1}(1^3P_1) + \gamma = 6.6\text{keV} \\
&\to \quad \chi'_{c0}(1^3P_0) + \gamma = 6.5\text{keV}
\end{aligned}
$$

- ψ(4160): Γ ~ 100 MeV
  - Hadronic transitions: $\pi\pi$, π, n
  - E1 rates:

$$
\begin{aligned}
\psi(4160) &\to \chi'_{c2}(2^3P_2)(Z(3927)?) + \gamma = 7.7\text{keV} \\
&\to \chi'_{c1}(2^3P_1)(X(3872)?) + \gamma = 193\text{keV} \\
&\to \chi'_{c0}(2^3P_0)(3820) + \gamma = 372\text{keV} \\
&\to \chi'_{c2}(1^3P_2) + \gamma = 2.8\text{keV} \\
&\to \chi'_{c1}(1^3P_1) + \gamma = 57.2\text{keV} \\
&\to \chi'_{c0}(1^3P_0) + \gamma = 134\text{keV}
\end{aligned}
$$

-> 1^3F_2 + γ if this transition is kinematically allow

Issues and Outlook

- Multipole expansion approach for EM and hadronic transitions works well with some initially puzzling exceptions:
  - E1 transitions rate for $\Upsilon(3S) \to \chi_b(1P_J) + \gamma$
  - M1 transition rates for $\Upsilon(3S) \to \eta_b(1P) + \gamma$ and $\Upsilon(2S) \to \eta_b(1P) + \gamma$
  - The two pion invariant mass distributions for the $\Upsilon(3S) \to \Upsilon(1S)+\pi\pi$ and $\Upsilon(4S) \to \Upsilon(2S)+\pi\pi$ transitions do not show the expected strong S-wave leading order E1-E1 behavior.
- In all these cases the leading order MPE coefficient is dynamically suppressed [as predicted]
  - E1 rates - Cancellations in overlap for states with nodes in radial wavefunctions. Here nearly complete. [Moxhay & Rosner PR D28, 1132 (1983); McClary and Byers PR D28, 1692 (1983)]
  - M1 rates - Hindered M1 transitions. Zero in leading order.
  - Two pion: - Again suppressed overlap. [Kuang and Yan PR D24, 2874 (1981)]

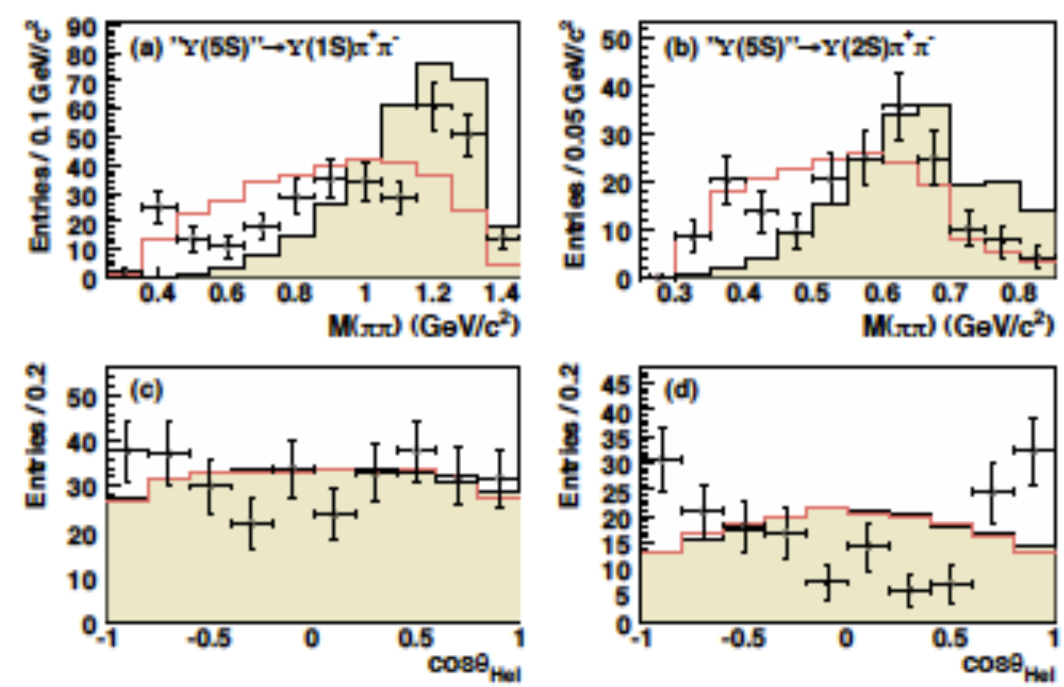
- Theoretical control of the non leading terms in the MPE is still poor. Modern tools (effective field theories and nonperturbative LQCD) combined with more detailed high statistics experimental data (BESIII, LHCb, super B factories, ...) with be needed to pin down these terms.
- Studying the EM and hadronic transitions for states well above threshold will add to our understanding of these issues (3S and 2D states in charmonium)

One more thing

- The $\Upsilon(5S) \to \Upsilon(1S)+\pi\pi$, $\Upsilon(5S) \to \Upsilon(2S)+\pi\pi$ transitions
- Very large rates and nonstandard $M_{\pi\pi}$

Belle

$\Upsilon(5S)$	
$\to \Upsilon(1S)+\pi^+\pi^-$	228 ± 33
$\to \Upsilon(1S)+K^+K^-$	26.2 ± 8.1
$\to \Upsilon(2S)+\pi^+\pi^-$	335 ± 64
$\to \Upsilon(3S)+\pi^+\pi^-$	206 ± 80



- Failure of usual multipole expansion
  - $\langle r^2 \rangle$ too large?
  - too relativistic?
  - nearby hybrid state ✓

$$\Gamma(\Upsilon(5S) \to \pi^+\pi^- + \Upsilon(nS)) \propto G(n)|f(n)|^2$$

$$\text{with } f(n) = \sum_l \frac{<\Upsilon(5S)|r|\Sigma_g^{+'}(lP)><\Sigma_g^{+'}(lP)|r|\Upsilon(nS)>}{M_{\Upsilon(5S)} - E_l(\Sigma) + i\Gamma_l(\Sigma)}|^2$$

phase space (GeV^{-7})

$$G(n) = 28.7,\ 0.729,\ 1.33 \times 10^{-2}$$

$$\text{for } n = 1, 2, 3$$

theory - hadronic transition rates

- If lowest hybrid mass near Υ(5S) a few states dominate sum. Results sensitive to mass value.
- Overall scale of transitions more than an order of magnitude larger than theory expects.

Will discuss Above Threshold in the last lecture