Lectures on NRQCD Factorization for Quarkonium Production and Decay

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I. Nonrelativistic QCD
II. Annihilation decays
III. Inclusive hard production

NRQCD Factorization for Quarkonium Production and Decay

I. Nonrelativistic QCD

II. Annihilation decays

- Introduction
- Color-singlet Model
- Electromagnetic annihilation
- Hadronic annihilation
- NRQCD Factorization
- Applications

III. Inclusive hard production

Introduction

Quarkonium Decays

Transitions

- radiative: $H' \rightarrow H\,\gamma$
- hadronic: $H' \rightarrow H\,\pi\pi$, $H\,\eta$, etc
- heavy meson pairs: $H \rightarrow M'M$

  where M', M are mesons

  containing a heavy quark

Annihilation

- electromagnetic: $H \rightarrow e^+e^-$, $\gamma\,\gamma$
- hadronic: $H \rightarrow$ light hadrons

Introduction

Transition Decays

involve only soft scales $\lesssim Mv$

- radiative: $H' \rightarrow H\,\gamma$

  matrix element $\propto \langle H \,|\, \bar{\Psi}\gamma^\mu\Psi(0) \,|\, H' \rangle$

  can be calculated using lattice QCD (or lattice NRQCD)

- hadronic: $H' \rightarrow H\,\pi\pi$, $H\,\eta$, etc

  difficult to calculate using lattice QCD

- heavy meson pairs: $H \rightarrow M'M$

  difficult to calculate using lattice QCD

Introduction

Annihilation Decays

- electromagnetic: $H \rightarrow e^+e^-,\ \gamma\gamma$
  involves hard scale M from $Q\bar{Q}$ annihilation
  and soft scales $\lesssim Mv$ from wavefunction of H

- hadronic: $H \rightarrow$ light hadrons
  involves hard scale M from $Q\bar{Q}$ annihilation
  and soft scales $\lesssim Mv$ from wavefunction of H
  and from light hadrons

difficult to calculate directly using lattice QCD
but can use NRQCD Factorization
to separate scales and simplify calculation

Color-Singlet Model

for annihilation decays into light hadrons

- onium = color-singlet $Q\bar{Q}$
  in appropriate angular momentum state

$$\eta_c = c\bar{c}(\underline{1}, {}^1S_0)$$
$$J/\psi = c\bar{c}(\underline{1}, {}^3S_1)$$
$$\chi_{cJ} = c\bar{c}(\underline{1}, {}^3P_J)$$

- annihilation rate Γ into light hadrons
  = annihilation rate of $Q\bar{Q}$ into partons

$$\eta_c \rightarrow gg,\ ggg,\ q\bar{q}g,\ \ldots$$
$$J/\psi \rightarrow ggg,\ \ldots$$
$$\chi_{c0,2} \rightarrow gg,\ ggg,\ q\bar{q}g,\ \ldots$$
$$\chi_{c1} \rightarrow ggg,\ q\bar{q}g,\ \ldots$$

Color-Singlet Model

- Q and $\bar{Q}$ must be very close to annihilate

$\Gamma \;\propto\;$ probability density near origin

$\Gamma \;\propto\; |R(0)|^2$ for S-waves

$\propto\; |R'(0)|^2$ for P-waves

- $Q\bar{Q}$ annihilate into partons

with hard momenta of order M

Q, $\bar{Q}$ have soft momenta of order Mv

can be neglected compared to M

$\Gamma \;\propto\;$ annihilation rate of $Q\bar{Q}$ at rest

can be calculate as power series in $\alpha_s(M)$

Color-Singlet Model

Conjectured factorization formula
for decay rates into light hadrons

S-waves: $$\Gamma = \begin{pmatrix} \text{power series} \\ \text{in } \alpha_s(M) \end{pmatrix} \times \frac{|R(0)|^2}{M^2}$$

P-waves: $$\Gamma = \begin{pmatrix} \text{power series} \\ \text{in } \alpha_s(M) \end{pmatrix} \times \frac{|R'(0)|^2}{M^4}$$

For S-waves,
factorization formula verified to NLO in $\alpha_s(M)$

$$\Gamma[\eta_c] = \left[\frac{2}{3}\alpha_s^2(2m_c) + \frac{572 - 31\pi^2}{24\pi}\alpha_s^3\right]\frac{|R(0)|^2}{m_c^2}$$

For P-waves,
infrared divergences at order $\alpha_s(M)^3$!

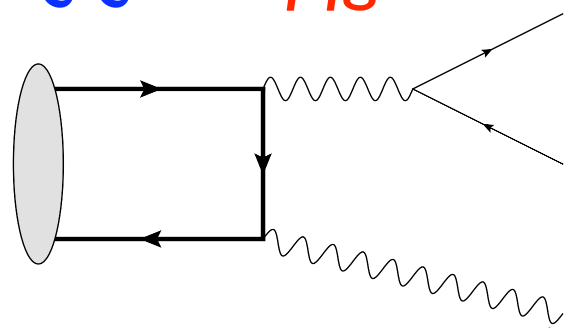
Color-Singlet Model

P-wave Problem of the Color-Singlet Model

infrared divergences

in the decay rates of P-wave states into light hadrons

at order α_s^3 from $Q\bar{Q} \to q\bar{q}g$



$$\Gamma[\chi_{c0}] = \left[6\alpha_s^2 + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

$$\Gamma[\chi_{c1}] = \left[\;0\; + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

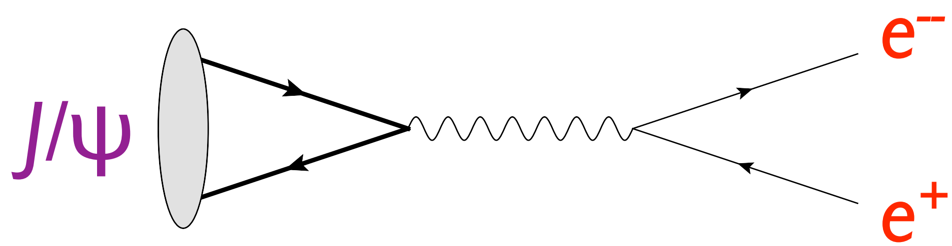
$$\Gamma[\chi_{c2}] = \left[\frac{8}{5}\alpha_s^2 + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

μ = infrared cutoff on real gluon

Electromagnetic annihilation

$J/\psi \rightarrow e^+e^-$

amplitude:



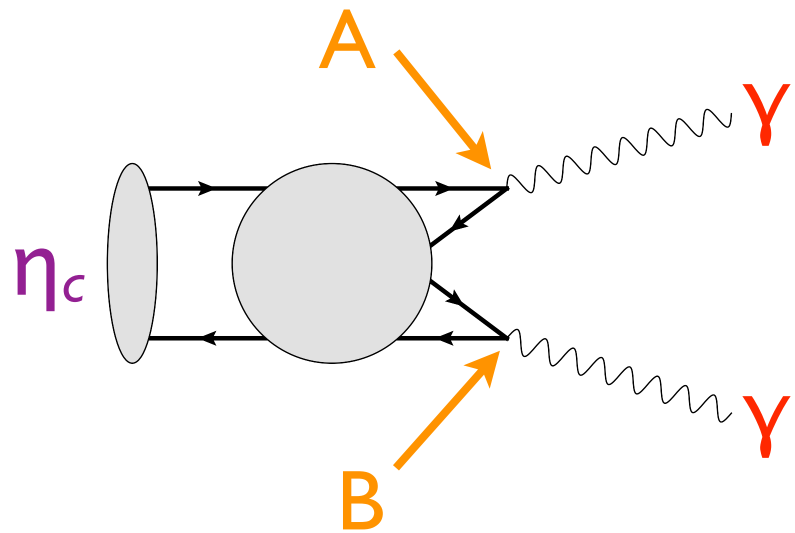
QCD factor: matrix element of local operator
calculate using lattice QCD
or (approximately) using lattice NRQCD

$$\langle 0|\bar{\Psi}\vec{\gamma}\Psi(0)|J/\psi\rangle = \langle 0|\chi^\dagger\vec{\sigma}\psi(0)|J/\psi\rangle \times \left[1 + O(\alpha_s) + O(v^2)\right]$$

Electromagnetic annihilation

$\eta_c \rightarrow \gamma\gamma$

amplitude:



QCD factor: matrix element of bilocal operator

$$\langle 0 | \bar{\Psi} \gamma^{\mu} \Psi(x_A) \bar{\Psi} \gamma^{\nu} \Psi(x_B) | \eta_c \rangle$$

dominated by short distances: $x_A - x_B \sim 1/M$

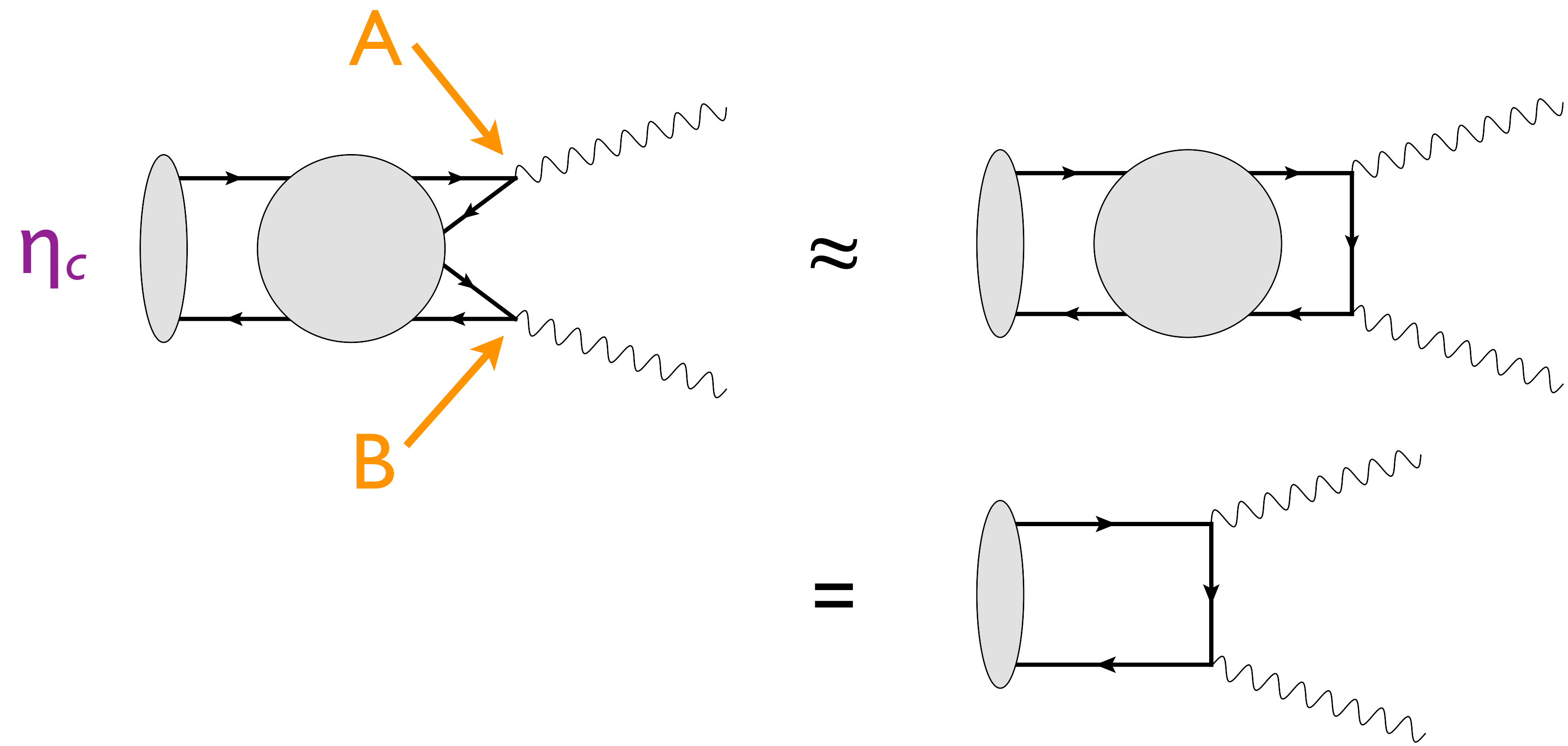
$\Rightarrow$ difficult to calculate using lattice QCD

Electromagnetic annihilation

$\eta_c \to \gamma\gamma$ (cont.)

charm quark propagating from A to B is off-shell by $\sim m_c$

$\Rightarrow$ coupling of gluons to virtual c suppressed by $\alpha_s(m_c)$



(use bound-state equation for wavefunction)

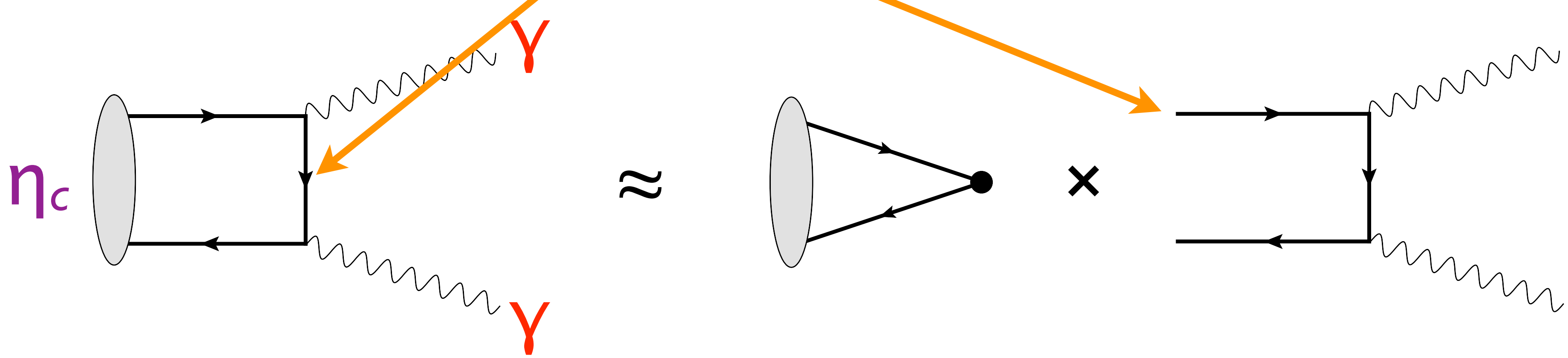
Electromagnetic annihilation

$\eta_c \rightarrow \gamma\gamma$ (cont.)

Factor amplitude into soft part and hard part

soft factor: insensitive to separation of currents
can shrink virtual c line to a point

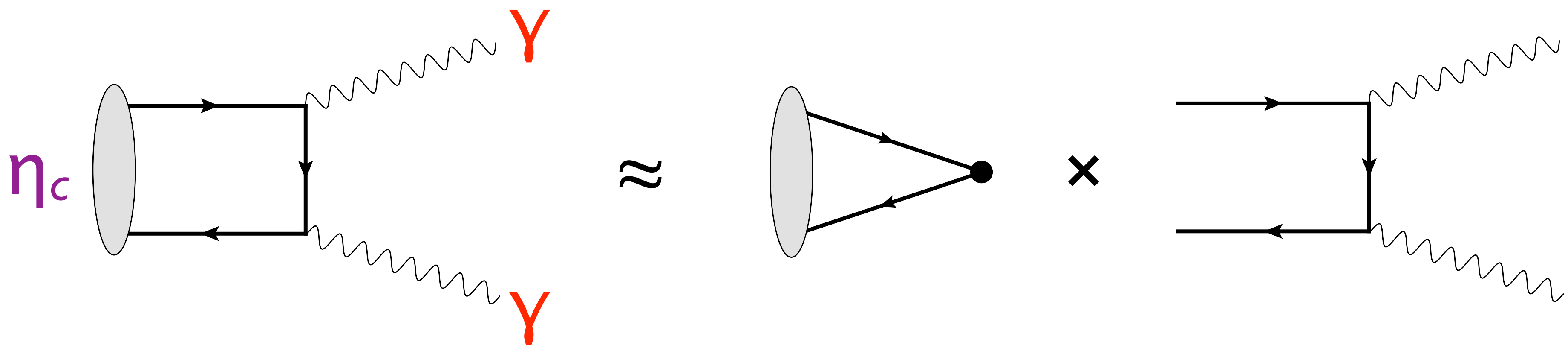
hard factor: insensitive to small momenta of c, $\bar{c}$
can set momenta = 0



Electromagnetic annihilation

$\eta_c \rightarrow \gamma\gamma$ (cont.)

Factorization of amplitude:



$$\mathcal{M}[\eta_c \to \gamma\gamma] = \langle 0|\chi^\dagger \psi(0)|\eta_c\rangle \; \mathcal{M}[c\bar{c} \to \gamma\gamma]$$

Factorization of decay rate:

$$\Gamma[\eta_c \to \gamma\gamma] = \frac{32\pi\alpha^2}{81 m_c^2} \, \left|\langle 0|\chi^\dagger \psi(0)|\eta_c\rangle\right|^2$$

Electromagnetic annihilation

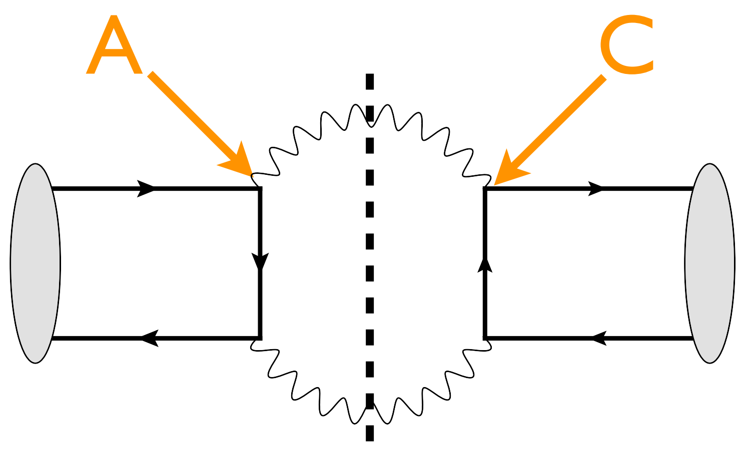
$\eta_c \to \gamma\gamma$ (cont.)

Decay rate:

$$\Gamma[\eta_c \to \gamma\gamma] = \frac{32\pi\alpha^2}{81 m_c^2} \langle \eta_c | \psi^\dagger \chi(0) | 0 \rangle \; \langle 0 | \chi^\dagger \psi(0) | \eta_c \rangle$$

same space-time point

4M Γ = 2 Im [



+ 1 diagram]

source of real photon of energy *M* can be localized to within wavelength *1/M* $\Longrightarrow$ $x_A - x_C \sim 1/M$

Electromagnetic annihilation

$\eta_c \to \gamma\gamma$ (cont.)

Factorization of decay rate:

$$\Gamma[\eta_c \to \gamma\gamma] = \frac{32\pi\alpha^2}{81 m_c^2} \left|\langle 0|\chi^\dagger \psi(0)|\eta_c\rangle\right|^2$$

Express in terms of wavefunction:

$$\left|\langle 0|\chi^\dagger \psi(0)|\eta_c\rangle\right|^2 = \frac{3}{2\pi} |R(0)|^2$$

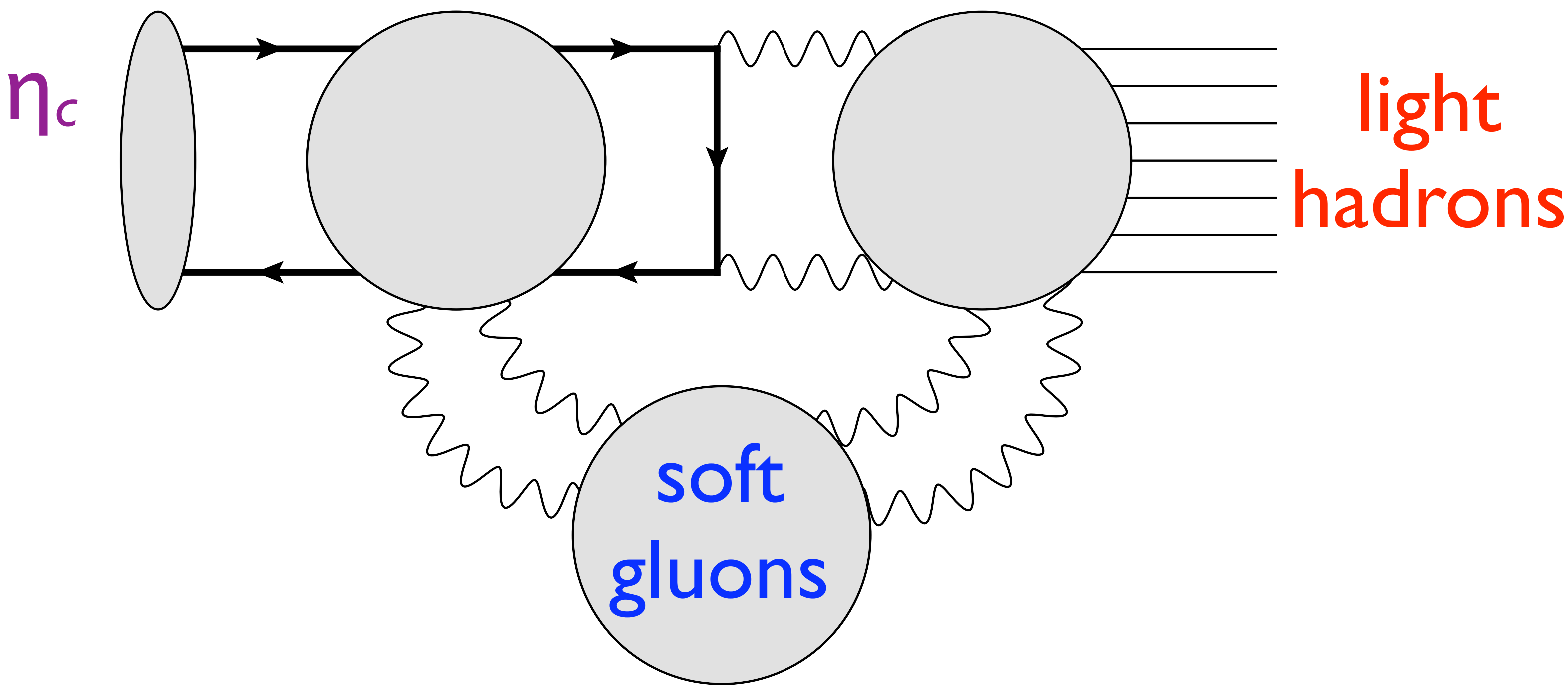
$$\Gamma[\eta_c \to \gamma\gamma] = \frac{16\alpha^2}{27 m_c^2} |R(0)|^2$$

agrees with Color-Singlet Model

Hadronic Annihilation

$\eta_c \rightarrow$ light hadrons

at leading order in $\alpha_s(m_c)$, proceeds through $c\bar{c} \rightarrow gg$



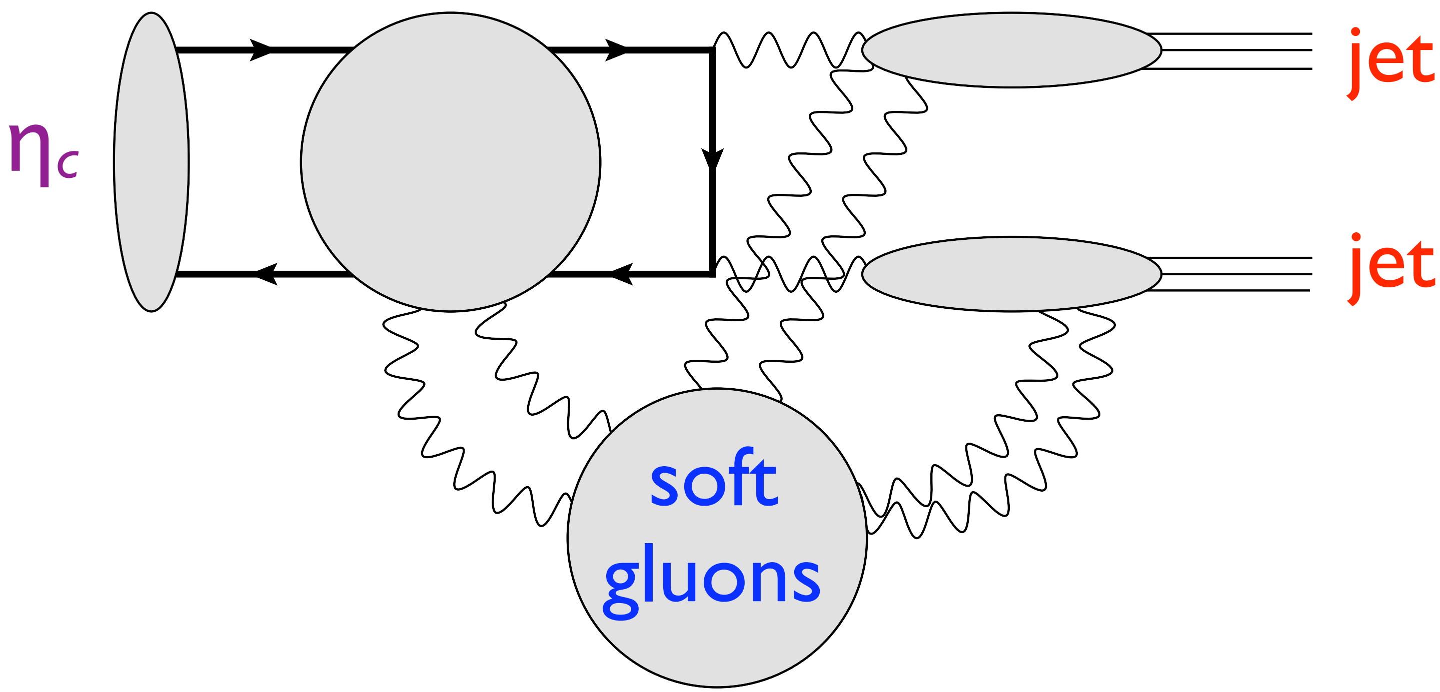
soft gluons couple to $c\bar{c}$ pair and to light hadrons

Hadronic annihilation

$\eta_c \longrightarrow$ light hadrons

at leading power of $1/M$,

energetic light hadrons form 2 jets



amplitude does not factor into soft × hard

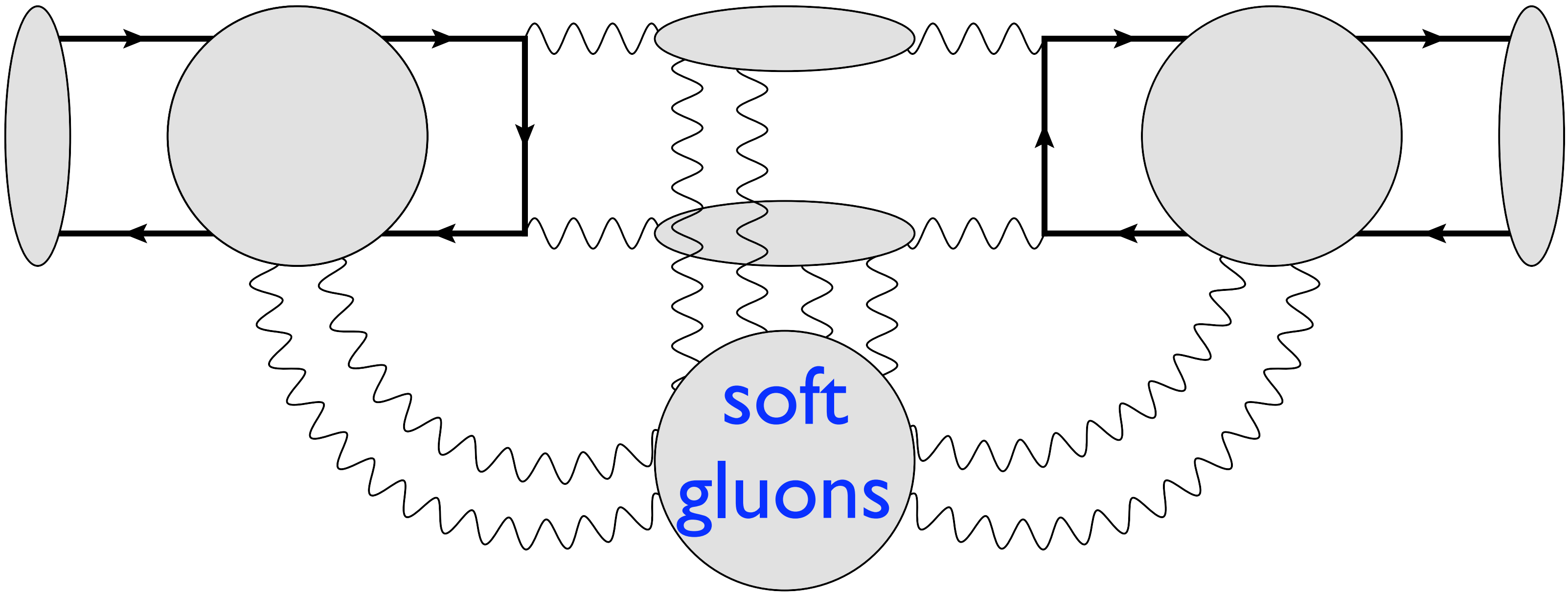
However decay rate factors!

Hadronic annihilation

$\eta_c \rightarrow$ light hadrons

decay rate (1 of 2 diagrams)

$4M\,\Gamma_1 \;=\; 2\,\mathrm{Im}$



Kinoshita-Lee-Nauenberg Theorem:

infrared divergences from coupling of soft gluons to jets cancel after summing over final-state cuts

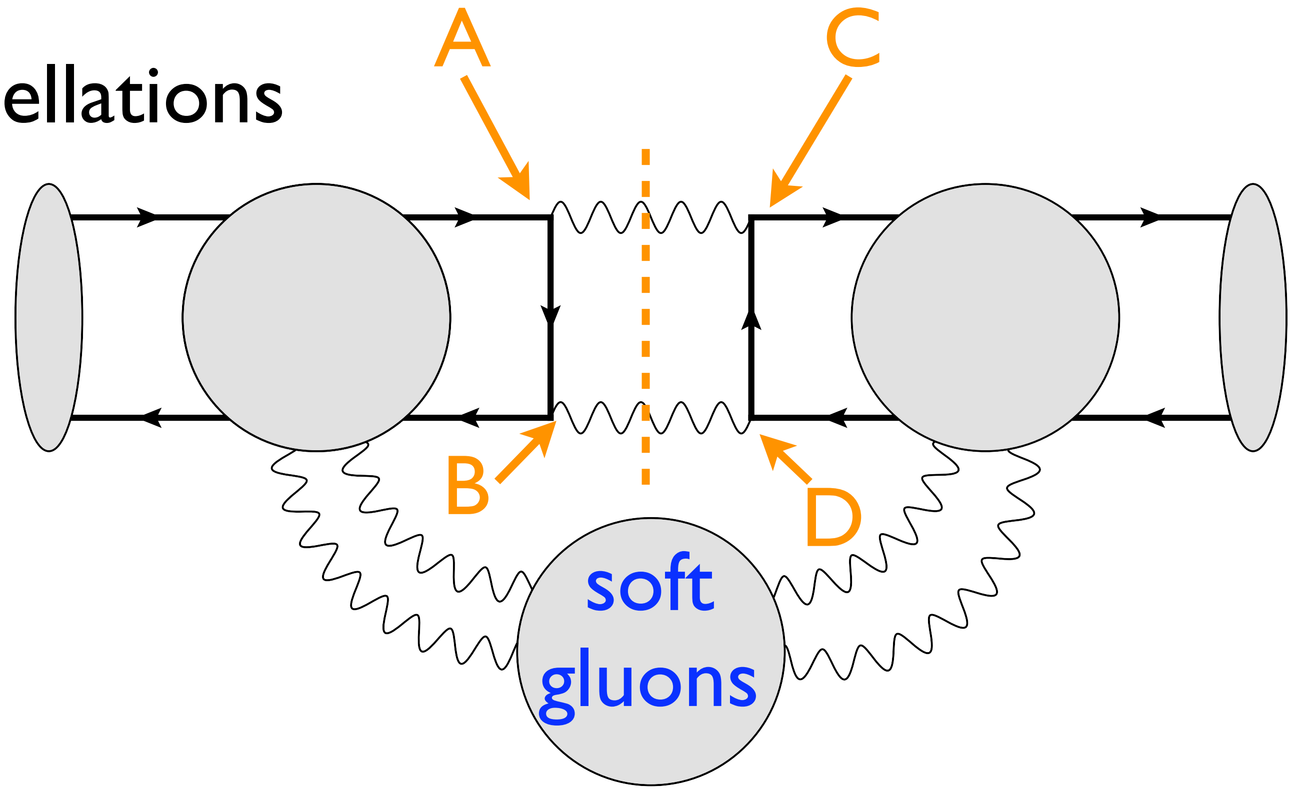
$\Rightarrow$ soft gluons decouple from jets

Hadronic annihilation

$\eta_c \rightarrow$ light hadrons

after KLN cancellations

$4M\,\Gamma_l = 2\,\mathrm{Im}$



virtual *c* is off-shell by $\sim 1/M \;\Rightarrow\; x_A - x_B \sim 1/M$

source of real gluon of energy M can be localized to within wavelength $1/M \;\Rightarrow\; x_A - x_C \sim 1/M$

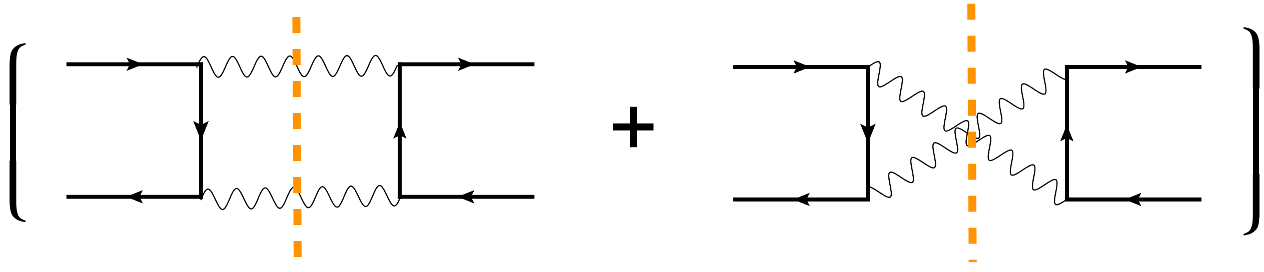
$Q\bar{Q}$ annihilation occurs within region of size $1/M$

Hadronic annihilation

$\eta_c \to$ light hadrons

Factorization of decay rate

$$4M\,\Gamma = 2\,\mathrm{Im}$$



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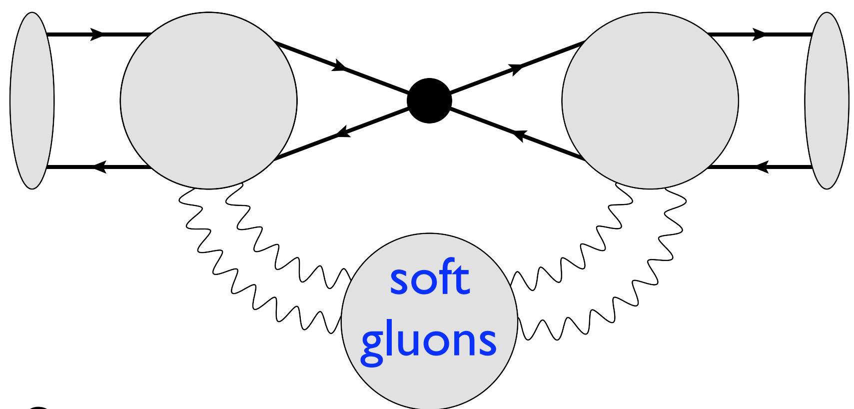


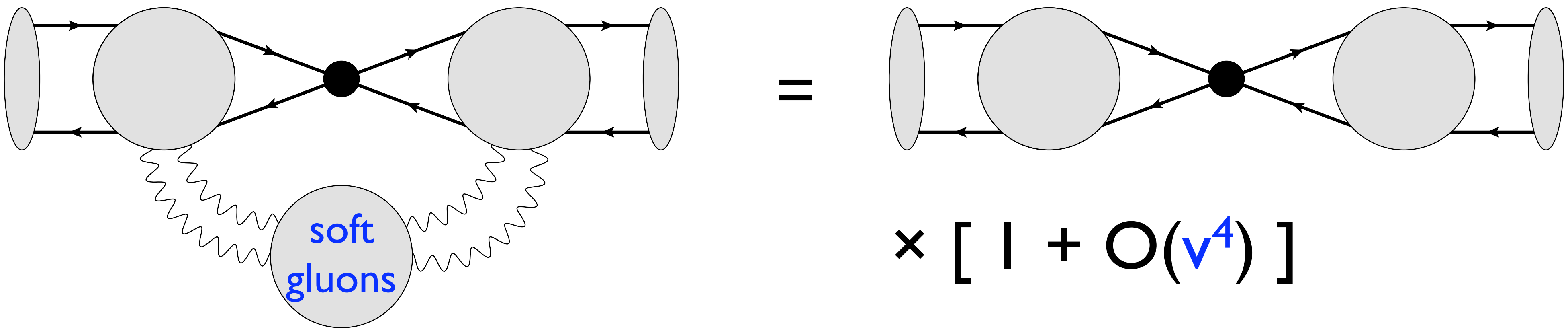
$$\Gamma[\eta_c] = \frac{4\pi\alpha_s^2(m_c)}{9m_c^2}\,\langle\eta_c|\psi^\dagger\chi\chi^\dagger\psi(0)|\eta_c\rangle$$

local operator

Hadronic annihilation

$\eta_c \rightarrow$ light hadrons

Transitions from soft gluon emission suppressed by powers of *v*



$$\Gamma[\eta_c] = \frac{4\pi\alpha_s^2(m_c)}{9m_c^2}\langle\eta_c|\psi^\dagger\chi\chi^\dagger\psi(0)|\eta_c\rangle$$

$$= \frac{2\alpha_s^2(m_c)}{3m_c^2}|R(0)|^2$$

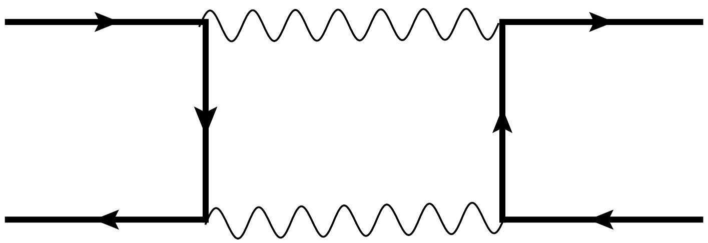
NRQCD Factorization

effects of $Q\bar{Q} \to gg \to Q\bar{Q}$

from annihilation into hard gluons

are taken into account in NRQCD

through local 4-fermion operators



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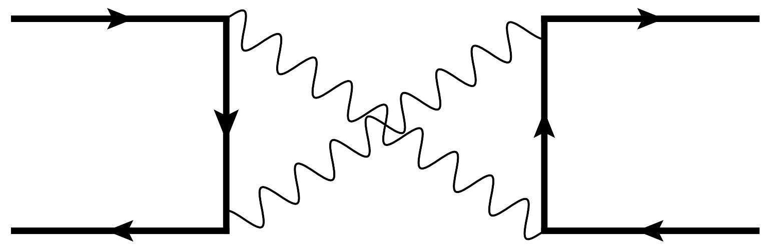
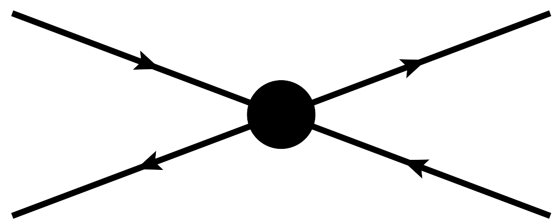


= $F(\alpha_s)$ ×



$$\mathrm{Re}F(\alpha_s) = ??, \quad \mathrm{Im}F(\alpha_s) = \frac{2\pi\alpha_s^2}{9m_c^2}$$

$$\Delta\mathcal{L}_{\mathrm{eff}} = F(\alpha_s)\ \psi^\dagger\chi\chi^\dagger\psi$$

NRQCD Factorization

Effective Lagrangian

that takes into account $Q\bar{Q} \to gg \to Q\bar{Q}$

$$\Delta\mathcal{L}_{\mathrm{eff}} = F(\alpha_s)\ \psi^\dagger\chi\chi^\dagger\psi$$

Effective Hamiltonian

$$\Delta\mathcal{H}_{\mathrm{eff}} = -F(\alpha_s)\ \psi^\dagger\chi\chi^\dagger\psi$$

gives energy shift $\Delta E - i\,\Delta\Gamma/2$

$$\Delta\Gamma = 2\,\mathrm{Im}F(\alpha_s)\ \langle\eta_c|\psi^\dagger\chi\chi^\dagger\psi|\eta_c\rangle$$

$$\Gamma[\eta_c] = \frac{4\pi\alpha_s^2(m_c)}{9m_c^2}\ \langle\eta_c|\psi^\dagger\chi\chi^\dagger\psi(0)|\eta_c\rangle$$

NRQCD Factorization

generalization to all orders in α_s and to all orders in v

NRQCD Factorization Formula

for decay rate into light hadrons

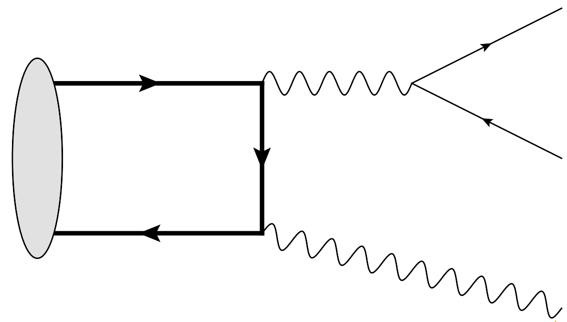
$$\Gamma[H] = \sum_n 2\,\mathrm{Im} F_n(\alpha_s)\ \langle H|\mathcal{O}_n|H\rangle$$

- sum is over all 4-fermion operators of the form $\psi^\dagger(\ldots)\chi\ \chi^\dagger(\ldots)\psi$
- short-distance coefficients $2\ \mathrm{Im}\ F_n(\alpha_s)$ are power series in $\alpha_s(M)$
- NRQCD matrix elements $\langle H \mid O_n \mid H\rangle$ have definite scaling with v

NRQCD Factorization

P-wave Problem of the Color-Singlet Model

Decay rates into light hadrons for P-wave states have infrared divergences at order α_s^3 from $Q\bar{Q} \to q\bar{q}g$



$$\Gamma[\chi_{c0}] = \left[6\alpha_s^2 + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

$$\Gamma[\chi_{c1}] = \left[\;0\; + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

$$\Gamma[\chi_{c2}] = \left[\frac{8}{5}\alpha_s^2 + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \ldots\right)\alpha_s^3\right]\frac{|R'(0)|^2}{m_c^4}$$

μ = infrared cutoff on real gluon

Solution: NRQCD Factorization!

NRQCD Factorization

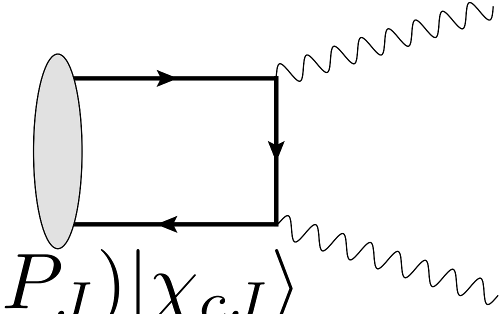
NRQCD Factorization Formula

for decays of P-wave states into light hadrons
at leading order in *v*

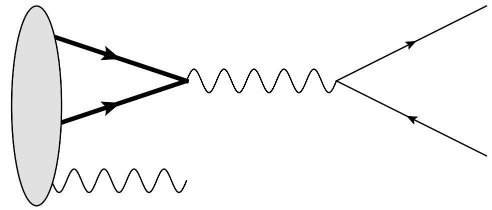
$$\Gamma[\chi_{cJ}] = 2\,\mathrm{Im}F(\underline{1},{}^3P_J)\,\langle\chi_{cJ}|\mathcal{O}(\underline{1},{}^3P_J)|\chi_{cJ}\rangle + 2\,\mathrm{Im}F(\underline{8},{}^3S_1)\,\langle\chi_{cJ}|\mathcal{O}(\underline{8},{}^3S_1)|\chi_{cJ}\rangle$$

- 1st term is Color-Singlet Model
- 2nd term takes into account $c\bar{c}g$ Fock state of χ_{cJ} (color-octet mechanism)
- both NRQCD matrix elements scale like v^5
- infrared divergence in 1st term at order $\alpha_s(M)^3$ can be absorbed into 2nd term

NRQCD Factorization



$$\Gamma[\chi_{cJ}] = 2\,\mathrm{Im}F(\underline{1},{}^3P_J)\ \langle\chi_{cJ}|\mathcal{O}(\underline{1},{}^3P_J)|\chi_{cJ}\rangle$$
$$+2\,\mathrm{Im}F(\underline{8},{}^3S_1)\ \langle\chi_{cJ}|\mathcal{O}(\underline{8},{}^3S_1)|\chi_{cJ}\rangle$$



both NRQCD matrix elements scale like v^5

- color-singlet matrix element $\langle\chi_{cJ}|\mathcal{O}(\underline{1},{}^3P_J)|\chi_{cJ}\rangle$ receives contribution from dominant $c\bar{c}$ Fock state suppressed by v^2 from P-wave operator

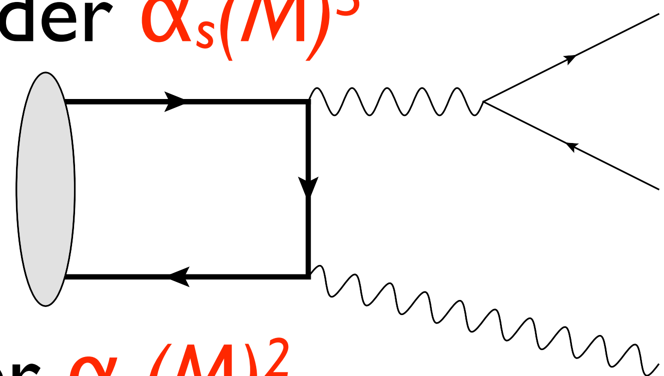
- color-octet matrix element $\langle\chi_{cJ}|\mathcal{O}(\underline{8},{}^3S_1)|\chi_{cJ}\rangle$
  where $\mathcal{O}(\underline{8},{}^3S_1) = \psi^\dagger\vec{\sigma}T^a\chi\cdot\chi^\dagger\vec{\sigma}T^a\psi^\dagger$
  receives contribution from $c\bar{c}g$ Fock state suppressed by v^2 from probability of $c\bar{c}g$ Fock state

NRQCD Factorization

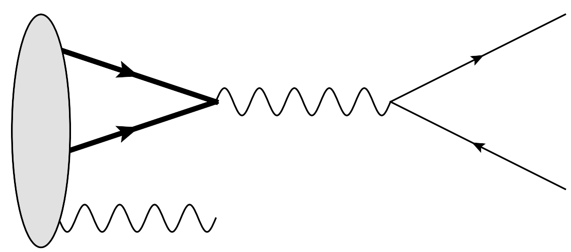
$$\Gamma[\chi_{cJ}] \quad = \quad 2\,\mathrm{Im}F(\underline{1},{}^3P_J)\ \langle\chi_{cJ}|\mathcal{O}(\underline{1},{}^3P_J)|\chi_{cJ}\rangle$$
$$+2\,\mathrm{Im}F(\underline{8},{}^3S_1)\ \langle\chi_{cJ}|\mathcal{O}(\underline{8},{}^3S_1)|\chi_{cJ}\rangle$$

infrared divergence in color-singlet term at order $\alpha_s(M)^3$ can be absorbed into color-octet term

- color-singlet term at order $\alpha_s(M)^3$



- color-octet term at order $\alpha_s(M)^2$



NRQCD Factorization

NRQCD Factorization Formula

at leading order in *v* and leading order in $\alpha_s(m_c)$

$$
\begin{aligned}
\Gamma[\chi_{c0}] &= \frac{4\pi\alpha_s^2(m_c)}{3m_c^4}\langle\mathcal{O}(\underline{1},{}^3P_0)\rangle_{\chi_{c0}} + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle\mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c0}} \\
\Gamma[\chi_{c1}] &= 0 + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle\mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c1}} \\
\Gamma[\chi_{c2}] &= \frac{16\pi\alpha_s^2(m_c)}{45m_c^4}\langle\mathcal{O}(\underline{1},{}^3P_2)\rangle_{\chi_{c2}} + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle\mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c2}}
\end{aligned}
$$

5 NRQCD matrix elements are related by heavy-quark spin symmetry

NRQCD Factorization

Heavy-quark spin symmetry

$\langle \mathcal{O}(\underline{1},{}^3P_J)\rangle_{\chi_{cJ}}$ is same for *J = 0,1,2*

$\langle \mathcal{O}(\underline{8},{}^3S_1)\rangle_{\chi_{cJ}}$ is same for *J = 0,1,2*

$\Rightarrow$ only two independent NRQCD matrix elements

$$\Gamma[\chi_{c0}] = \frac{4\pi\alpha_s^2(m_c)}{3m_c^4}\langle \mathcal{O}(\underline{1},{}^3P_0)\rangle_{\chi_{c0}} + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle \mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c0}}$$

$$\Gamma[\chi_{c1}] = 0 + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle \mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c0}}$$

$$\Gamma[\chi_{c2}] = \frac{16\pi\alpha_s^2(m_c)}{45m_c^4}\langle \mathcal{O}(\underline{1},{}^3P_0)\rangle_{\chi_{c0}} + \frac{\pi n_f \alpha_s^2(m_c)}{6m_c^2}\langle \mathcal{O}(\underline{1},{}^3S_1)\rangle_{\chi_{c0}}$$

short-distance coefficients

have been calculated to next-to-leading order in $\alpha_s(m_c)$

Applications

S-wave annihilation decays

S-wave multiplet

spin-triplet: J/ψ

spin-singlet: η_c

5 annihilation decays:

$J/\psi \rightarrow e^+e^-$

$\eta_c \rightarrow \gamma\gamma$

$J/\psi \rightarrow$ light hadrons (via gluons)

$J/\psi \rightarrow$ light hadrons (via virtual photon)

$\eta_c \rightarrow$ light hadrons

(also $J/\psi \rightarrow \gamma\gamma\gamma$)

Applications

S-wave annihilation decays (cont.)

at leading order in v^2,
one independent NRQCD matrix element:

$$\left|\langle 0|\chi^\dagger\psi(0)|\eta_c\rangle\right|^2 = \frac{3}{2\pi}\,|R_{1S}(0)|^2$$

at next-to-leading order in v^2,
three independent NRQCD matrix elements:

$$\left|\langle 0|\chi^\dagger\psi(0)|\eta_c\rangle\right|^2 = \frac{3}{2\pi}\,|R_{\eta_c}(0)|^2$$

$$\left|\langle 0|\chi^\dagger\vec{\sigma}\psi(0)|J/\psi\rangle\right|^2 = \frac{3}{2\pi}\,|R_{J/\psi}(0)|^2$$

$$\langle v^2\rangle = m_c^2\,\frac{\langle 0|\chi^\dagger\vec{D}^2\psi(0)|\eta_c\rangle}{\langle 0|\chi^\dagger\psi(0)|\eta_c\rangle}$$

Applications

S-wave annihilation decays (cont.)

perturbative QCD corrections

to short-distance coefficients can be large!

$$\Gamma[J/\psi \to e^+e^-] = \frac{8\pi\alpha^2}{27m_c^2}\left(1 - \frac{16\alpha_s(m_c)}{3\pi}\right)|\langle 0|\chi^\dagger\vec{\sigma}\psi(0)|J/\psi\rangle|^2$$

40% correction if $\alpha_s(m_c)$ = 0.25

relativistic corrections

of order v^2 can be large

$$\Gamma[J/\psi \to e^+e^-] = \frac{8\pi\alpha^2}{27m_c^2}|\langle 0|\chi^\dagger\vec{\sigma}\psi(0)|J/\psi\rangle|^2\left(1 - \frac{4}{3}\langle v^2\rangle\right)$$

40% correction if v^2 = 0.3

Applications

P-wave annihilation decays

P-wave multiplet

spin-triplet: χ_{c0}, χ_{c1}, χ_{c2}
spin-singlet: h_c

6 annihilation decays:

$\chi_{c0} \rightarrow \gamma\gamma$
$\chi_{c2} \rightarrow \gamma\gamma$
$\chi_{c0} \rightarrow$ light hadrons
$\chi_{c1} \rightarrow$ light hadrons
$\chi_{c2} \rightarrow$ light hadrons
$h_c \rightarrow$ light hadrons (see talk by Ying Fan)

Applications

P-wave annihilation decays (cont.)

at leading order in v^2,
two independent NRQCD matrix elements

$$\left|\langle 0|\chi^\dagger \vec{D}\psi(0) - \vec{D}\chi^\dagger\psi(0)|h_c\rangle\right|^2 = \frac{18}{\pi}\,|R'_{1P}(0)|^2$$

$$\langle h_c|\psi^\dagger T^a \chi\chi^\dagger T^a \psi^\dagger|h_c\rangle$$

short-distance coefficients have been calculated to NLO in $\alpha_s(m_c)$

expected accuracy in decay rates: $v^2 \approx 30\%$

at next-to-leading order in v^2,
6 independent NRQCD matrix elements

Comparisons of NRQCD Factorization with Experiment

Inclusive P-Wave Quarkonium Decays

- IR finite predictions first possible with NRQCD factorization.

Ratio	PDG04	PDG00	LO	NLO
$\frac{\Gamma(\chi_{c0} \to \gamma\gamma)}{\Gamma(\chi_{c2} \to \gamma\gamma)}$		12.6$\pm$9.7	$\approx$ 3.75	$\approx$ 5.43
$\frac{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$		271$\pm$196	$\approx$ 347	$\approx$ 383
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$		3545$\pm$2576	$\approx$ 1300	$\approx$ 2781
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c2} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$		12.1$\pm$3.2	$\approx$ 2.75	$\approx$ 6.63
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$		13.1$\pm$3.3	$\approx$ 3.75	$\approx$ 7.63

$m_c = 1.5$ GeV. $\alpha_s(2m_c) = 0.245$.

Compiled by A. Vairo.

Mainly from E835 (χ_{c0} total width and $\chi_{c0} \to \gamma\gamma$).

Also from Belle ($\chi_{c0} \to \gamma\gamma$), CLEO, and BES.

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$\frac{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	404±98	271±196	$\approx$ 347	$\approx$ 383
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	3609±752	3545±2576	$\approx$ 1300	$\approx$ 2781
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c2} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$	7.9±1.4	12.1±3.2	$\approx$ 2.75	$\approx$ 6.63
$\frac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$	8.9±1.5	13.1±3.3	$\approx$ 3.75	$\approx$ 7.63

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Phenomenological analysis -- Maltoni

- Global fit (Maltoni):
  - χ^2/d.o.f. $= 15.0/10$.

$$\langle \chi_{cJ} | \mathcal{O}_1(^3P_J) | \chi_{cJ} \rangle = (7.2 \pm 0.9) \times 10^{-2}\ \text{GeV}^5.$$

$$\langle \chi_{cJ} | \mathcal{O}_8(^3S_1) | \chi_{cJ} \rangle = (4.3 \pm 0.9) \times 10^{-3}\ \text{GeV}^3.$$

Lattice NRQCD calculations

- Lattice (GTB, D.K. Sinclair, S. Kim):

$$\langle \chi_{cJ} | \mathcal{O}_1(^3P_J) | \chi_{cJ} \rangle = (8.0 \pm 1.7) \times 10^{-2}\ \text{GeV}^5.$$

$$\langle \chi_{cJ} | \mathcal{O}_8(^3S_1) | \chi_{cJ} \rangle = (4.6 \pm 2.5) \times 10^{-3}\ \text{GeV}^3.$$

Bodwin, Sinclair, and Kim 1996

Summary

NRQCD Factorization

- rigorous formalism for calculating
  - electromagnetic annihilation rates
  - decay rates into light hadrons

- separates hard scale $\sim M$ from soft scales $\lesssim Mv$

- reduces problem to
  - perturbative QCD calculation of short-distance coefficients
  - lattice NRQCD calculation of matrix elements (or treat them as phenomenological parameters)

Summary

- for S-wave and P-wave charmonium states, agreement with experiment
  to within expected theoretical errors

- can theoretical errors be decreased?

  - NLO perturbative QCD corrections are not small
    calculate NNLO corrections?

  - NLO v^2 corrections are not small
    include NRQCD matrix elements to next order in v^2?
    must calculate them using lattice NRQCD
    (too many to be treated as phenomenological parameters)