Charmonium and Charm physics on lattice (I)

Ying Chen
Institute of High Energy Physics, CAS

Outline

I. An overview of lattice QCD
II. Present Status of LQCD
II. Charm quark on lattice
III. Charmonium and D meson spectrum
IV. Charmonium radiative transitions
V. Summary and perspectives

I. Overview of lattice QCD

- LQCD is QCD formulated on a discrete Euclidean space-time grid.
- LQCD is in a formulism of Feynmann path integral quantization.
- LQCD is a theory from the first principle, it retains the fundamental character of QCD.
- The functional integrals are calculated through the numerical Monte Carlo simulation instead of the perturbative expansion.
- Therefore, LQCD is a non-perturbative method for solving QCD.
- The numerical simulation of LQCD becomes the third branch of high energy study parallel to the theoretical and experimental approach.

Wick Rotation from Minkowski Space to Euclidean Space

A D-dimensonal Minkowski field theory is connected to a D-dimensional Euclidean field theory through analytical continuation----Wick rotation:

$$x_0 \equiv t \to -ix_4 \equiv -i\tau\ ,$$

$$p_0 \equiv E \to ip_4\ .$$

$$x_E^2 = \sum_{i=1}^{4} x_i^2 = \boldsymbol{x}^{\,2} - t^2 = -x_M^2\ ,$$

$$p_E^2 = \sum_{i=1}^{4} p_i^2 = \boldsymbol{p}^{\,2} - E^2 = -p_M^2\ .$$

$$e^{iS_M} \equiv e^{i\int dx_M^4 L(x_M)} = e^{\int dx_E^4 L(x_E)} \equiv e^{-S_E}$$

Path Integral Quantization in Euclidean Space

- The generating functional of QCD in the Euclidean Space

$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$

$$S = \int d^4x \; (\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \overline{\psi} M \psi) \; .$$

- Integrating out the fermion fields, we have,

$$Z = \int \mathcal{D}A_\mu \; \det M \; e^{\int d^4x \; (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu})} .$$

$$S = S_{gauge} + S_{quarks} = \int d^4x \; \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det} M_i)$$

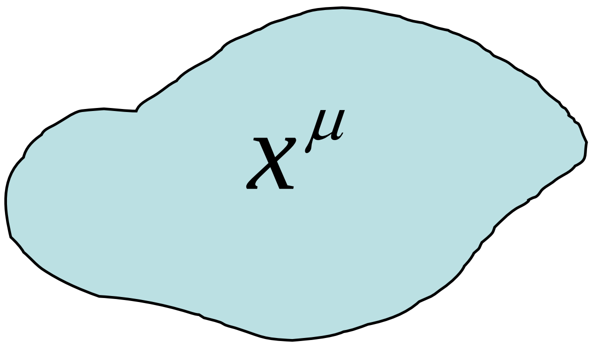
- The physical observables are obtained by calculating the expectation value of a field operator

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S} \; .$$

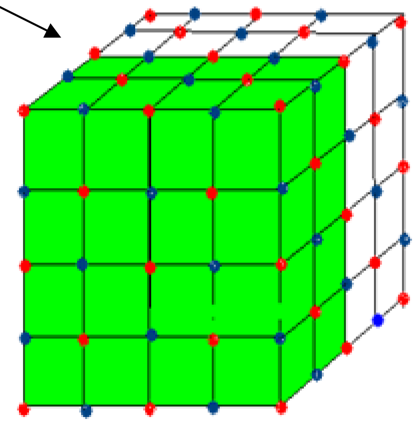
QCD on a Euclidean Space-time Grid

- Space-time discretization

Lattice spacing a



→



$$A_{\mu}(x) \rightarrow U_{\mu}(x) = e^{iagA_{\mu}(x)}$$

$$\psi(x) \rightarrow \psi(x)$$

The symmetry group of the continuum theory – Poincaré invariance – is reduced to a discrete group. On a hypercubic lattice rotations by only 90^o are allowed so the continuous rotation group is replaced by the discrete hypercubic group [34]. Translations have to be by at least one lattice unit, so the allowed momenta are discrete

$$k = \frac{2\pi n}{La} \qquad n = 0, 1, \ldots L$$

or equivalently

$$k = \pm \frac{2\pi n}{La} \qquad n = 0, 1, \ldots L/2 \; .$$

On the lattice momentum is conserved *modulo* 2π.

It is easily seen that, the largest momentum on the lattice is $\frac{\pi}{a}$
The finite lattice spacing provides a natural UV cutoff.

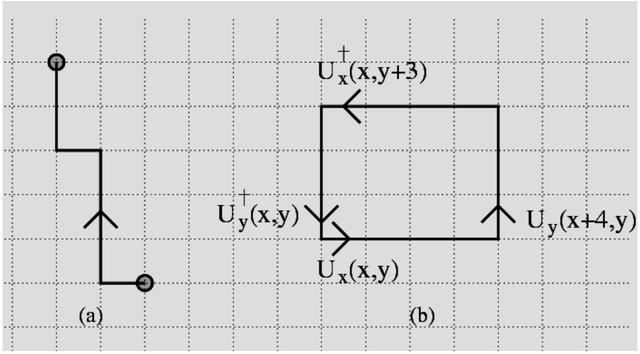
The Local gauge symmetry

- Local gauge transformation:

$$\psi(x) \rightarrow V(x)\psi(x)$$

$$\overline{\psi}(x) \rightarrow \overline{\psi}(x)V^\dagger(x)$$

$$U_\mu(x) \rightarrow V(x)U_\mu(x)V^\dagger(x+\hat{\mu})$$



- There exist only two types of gauge invariant quantities:

$$\mathrm{Tr}\ \overline{\psi}(x)\ U_\mu(x)\ U_\nu(x+\hat{\mu})\ldots U_\rho(y-\hat{\rho})\ \psi(y)$$

$$W_{\mu\nu}^{1\times1} = \mathrm{Re}\ \mathrm{Tr}\ (U_\mu(x)\ U_\nu(x+\hat{\mu})\ U_\mu^\dagger(x+\hat{\nu})\ U_\nu^\dagger(x))$$

The simplest gauge action

- The action should be gauge invariant, so it is constructed by Wilson loops.

$$W_{\mu\nu}^{1\times1} = U_\mu(x)U_\nu(x+\hat\mu)U_\mu^\dagger(x+\hat\nu)U_\nu^\dagger(x)$$
$$= 1 + ia^2 gF_{\mu\nu} - \frac{a^4g^2}{2}F_{\mu\nu}F^{\mu\nu} + O(a^6) + \dots$$

$$e^A e^B = e^{A+B+[A,B]/2+\cdots}$$

$$\mathrm{Re\,Tr}(1-W_{\mu\nu}^{1\times1}) = \frac{a^4g^2}{2}F_{\mu\nu}F^{\mu\nu} + \textit{terms higher order in } a$$

$$\frac{1}{g^2}\sum_x\sum_{\mu<\nu}\mathrm{Re\,Tr}(1-W_{\mu\nu}^{1\times1}) = \frac{a^4}{4}\sum_x\sum_{\mu,\nu}F_{\mu\nu}F^{\mu\nu} \to \frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$$

$$S_g = \frac{6}{g^2}\sum_x\sum_{\mu<\nu}\mathrm{Re\,Tr}\,\frac{1}{3}(1-W_{\mu\nu}^{1\times1}).$$

Naïve fermion action

- Replace the derivatives to differentiate

$$\partial_x \phi(x) \to \Delta_x \phi(x) = \frac{1}{2a}(\phi(x+a) - \phi(x-a))$$

$$\overline{\psi} \not{D} \psi = \frac{1}{2a} \overline{\psi}(x) \sum_\mu \gamma_\mu [U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\,\psi(x-\hat{\mu})]$$

$$\mathcal{S}^N = m_q \sum_x \overline{\psi}(x)\psi(x) \qquad \gamma_\mu = \gamma_\mu^\dagger$$

$$+ \frac{1}{2a} \sum_x \overline{\psi}(x)\gamma_\mu [U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})]$$

$$\equiv \sum_x \overline{\psi}(x) M_{xy}^N[U]\psi(y)$$

$$M_{i,j}^N[U] = m_q \delta_{ij} + \frac{1}{2a} \sum_\mu [\gamma_\mu U_{i,\mu}\delta_{i,j-\mu} - \gamma_\mu U_{i-\mu,\mu}^\dagger \delta_{i,j+\mu}]$$

- Satisfying the chiral symmetry

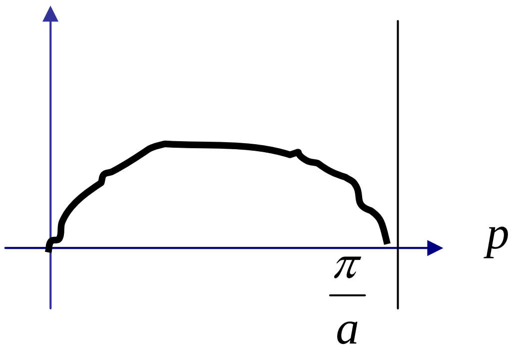
$$\gamma_5 M + M\gamma_5 = 0$$

The properties and problems of naïve fermion action

"Fermion doubling"

$$S^{-1}(p) = m_q + \frac{i}{a}\sum_{\mu}\gamma_\mu \sin p_\mu a$$

$$S^{-1}(p, m=0) = \frac{i}{a}\sum_{\mu}\gamma_\mu \sin p_\mu a$$



- **Sixteen poles can be divided into two groups with different chiral charge.**

- **These doublers cancel the axial anomaly which exists in the continuum.**

No-Go theorem on finite lattices

On a four-torus

- **Locality, hermicity, correct low-momentum limit**
- **Chiral symmetry and free of fermion douling cannot be satisfied simultaneously**

Wilson fermion action

- Adding an additiong dimension 5 term to the conventional action,

$$\partial^2 \phi(x) \rightarrow \frac{1}{a^2}\left(\phi(x+a) + \phi(x-a) - 2\phi(x)\right)$$

$$\begin{aligned}
A^W &= m_q \sum_x \overline{\psi}(x)\psi(x) \\
&+ \frac{1}{2a} \sum_{x,\mu} \overline{\psi}(x)\gamma_\mu [U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})] \\
&- \frac{r}{2a} \sum_{x,\mu} \overline{\psi}(x)[U_\mu(x)\psi(x+\hat{\mu}) - 2\psi(x) + U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})] \\
&\equiv \sum_{x,y} \overline{\psi}_x^L M_{xy}^W \psi_y^L
\end{aligned}$$

$$m_q a = \frac{1}{2\kappa} - 4r$$

$$M_{x,y}^W[U]a = \delta_{xy} - \kappa \sum_\mu [(r-\gamma_\mu)U_{x,\mu}\delta_{x,y-\mu} + (r+\gamma_\mu)U_{x-\mu,\mu}^\dagger \delta_{x,y+\mu}]$$

Properites of Wilson Fermion

- **Free of fermion doubling**

$$S_F(p) = M_W^{-1}(p) = \frac{a}{1 - 2\kappa \sum_\mu (r \cos p_\mu a - i\gamma_\mu \sin p_\mu a)}$$

The 15 extra states get heavy masses propotional to 2r/a and decouple from the theory when a is small enough.

- **Discretization errors**

$$e^{Ea} = \frac{(ma + r) \pm \sqrt{(1 + 2mar + m^2 a^2)}}{1 + r}$$

Naïve fermion (r=0): $E = m + O(m^2 a^2)$

Wilson fermion (r=1): $Ea = \log(1 + ma)$

Chiral Fermions

- Ginsburg-Wilson relation---chiral symmetry on lattice

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Chiral tranformation in the continuum

$$\psi \rightarrow e^{i\theta\gamma_5}\psi;$$

$$\overline{\psi} \rightarrow \overline{\psi}e^{i\theta\gamma_5}$$

- Chiral transfromation on the lattice

$$\psi \rightarrow e^{i\theta\gamma_5(1-aD/2)}\psi;$$

$$\overline{\psi} \rightarrow \overline{\psi}e^{i\theta\gamma_5(1-aD/2)}$$

The lattice action is invariant if GW relation holds for D

- Two types of fermion actions that satisfy GW relation:

overlap fermion
domain-wall fermion

- Free of fermion doubling + chiral
- But computation is much more expensive.

Monte Carlo Simulation of Lattice QCD

The equivalences between a Euclidean field theory and Classical Statistical Mechanics.

Euclidean Field Theory	Classical Statistical Mechanics
Action	Hamiltonian
unit of action h	units of energy $\beta = 1/kT$
Feynman weight for amplitudes $e^{-S/h} = e^{-\int \mathcal{L} dt/h}$	Boltzmann factor $e^{-\beta H}$
Vacuum to vacuum amplitude $\int \mathcal{D}\phi e^{-S/h}$	Partition function $\sum_{conf.} e^{-\beta H}$
Vacuum energy	Free Energy
Vacuum expectation value $\langle 0 \vert \mathcal{O} \vert 0 \rangle$	Canonical ensemble average $\langle \mathcal{O} \rangle$
Time ordered products	Ordinary products
Green's functions $\langle 0 \vert T[\mathcal{O}_1 \ldots \mathcal{O}_n] \vert 0 \rangle$	Correlation functions $\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle$
Mass M	correlation length $\xi = 1/M$
Mass-gap	exponential decrease of correlation functions
Mass-less excitations	spin waves
Regularization: cutoff Λ	lattice spacing a
Renormalization: $\Lambda \to \infty$	continuum limit $a \to 0$
Changes in the vacuum	phase transitions

Due to the similarity, we can borrow the methods of statistical mechanics to study lattice QCD, such as Monte Carlo simulation.

MC Simulation——Importance Sampling

- Taking $e^{-S[U,\psi,\overline{\psi}]}$ as a probability distribution, an ensemble of configurations are generated from MC simulation. This is the procedure that eat the computation resources mostly.
- After the generation of configurations, the functional integral

$$\langle O(A_\mu,\overline{\psi},\psi)\rangle = \frac{1}{Z}\int [DA_\mu D\overline{\psi}D\psi]\exp(-S(A_\mu,\overline{\psi},\psi))O(A_\mu,\overline{\psi},\psi)$$

becomes the much simpler arithmetic average:

$$\langle O \rangle_{MC} = \frac{1}{N}\sum_{i=1}^{N} O_i$$

- Generally speaking, the quantites that are most commonly calculated are Green's function, say, the vacuum expectation values of field operators defined at different space-time points.

Quenched and Unquenched

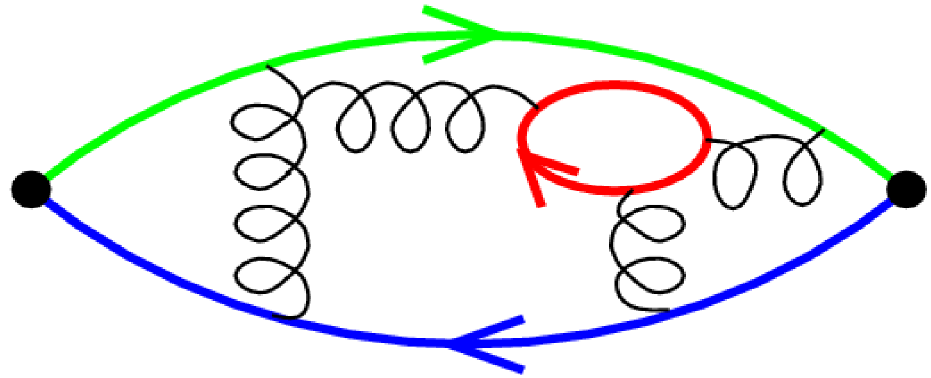
$$Z = \int DUD\,\overline{\psi}D\,\psi e^{-S_g + \overline{\psi}M\psi} = \int DU \det M e^{-S_g}$$

$$= \int DUe^{-S_g + Tr \ln M}$$

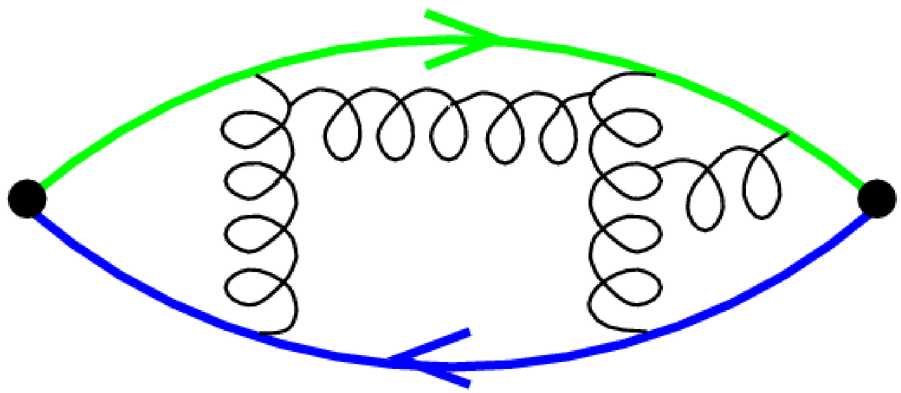
On the lattice, M is a very large matrix, such that the calculation of its trace is very expensive in the MC simulation. A way out this difficulty is to take the approximation

$$\det M[U] = const\,.$$

Theoretically, this means that we set the sea quark mass to be infinitely large such that they decouple from the gauge field. In other words, we will ignore the vacuum polarization diagram, say, the effects of sea quarks.



(B) Full QCD



(A) Quenched QCD: quark loops neglected

Extracting Physical Quantites from MC Simulation

Example: LQCD calculation of pion mass and pion decay constant

• **Two point function:**

$$C(t) = \langle 0|T[\sum_x \mathcal{O}_f(\boldsymbol{x},t)\mathcal{O}_i(\mathbf{o},0)]|0\rangle \quad \mathcal{O}_f = \mathcal{O}_i = A_4 = \overline{\psi}\gamma_4\gamma_5\psi$$

Theoretically, we have the following relations

$$C(t) = \langle 0|\sum_x \mathcal{O}_f(\boldsymbol{x},t)\mathcal{O}_i(0)|0\rangle = \sum_n \frac{\langle 0|\mathcal{O}_f|n\rangle\langle n|\mathcal{O}_i|0\rangle}{2E_n}\, e^{-E_n t}\,.$$

$$C(t) = \langle 0|\sum_x \mathcal{O}_f(x,t)\mathcal{O}_i(0)|0\rangle \underset{t\to\infty}{=} \frac{\langle 0|\mathcal{O}_f|\pi\rangle\langle \pi|\mathcal{O}_i|0\rangle}{2M_\pi}\, e^{-M_\pi t}\,.$$

$$\langle 0|A_4(\breve{\boldsymbol{p}} = 0)|\pi\rangle = M_\pi f_\pi\,.$$

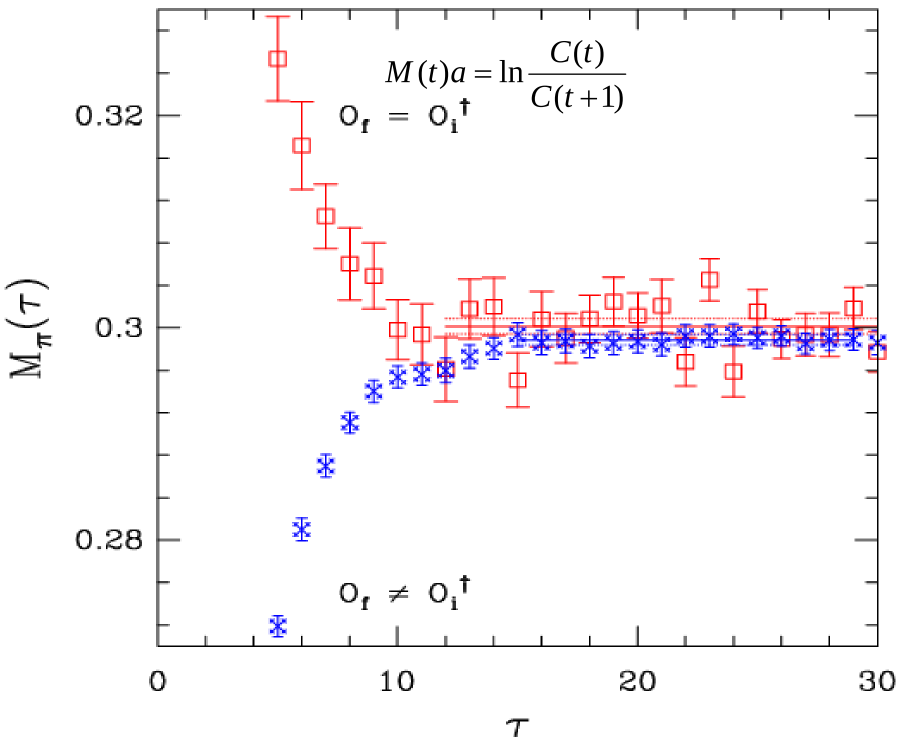
The local interpolating field operators for mesons and baryons in Wilson-like theories. Projection to zero momentum states is obtained by summing over the points x on a time slice. The C-parity is only relevant for flavor degenerate meson states. The () in baryon operators denote spin trace. $C = \gamma_2 \gamma_4$ and the symmetry properties of flavor indices for nucleons are discussed in the text. The decuplet baryon operator is completely symmetric in flavor index

State	$I^G(J^{PC})$	Operator
Scalar(σ)	$1^-(0^{++})$	$\overline{u}(x)d(x)$
	$1^-(0^{++})$	$\overline{u}(x)\gamma_4 d(x)$
Pseudoscalar	$1^-(0^{-+})$	$\overline{u}(x)\gamma_5 d(x)$
	$1^-(0^{-+})$	$\overline{u}(x)\gamma_4\gamma_5 d(x)$
Vector	$1^+(1^{--})$	$\overline{u}(x)\gamma_i d(x)$
	$1^+(1^{--})$	$\overline{u}(x)\gamma_i\gamma_4 d(x)$
Axial (a_1)	$1^-(1^{++})$	$\overline{u}(x)\gamma_i\gamma_5 d(x)$
Tensor(b_1)	$1^+(1^{+-})$	$\overline{u}(x)\gamma_i\gamma_j d(x)$
Nucleon octet	$\frac{1}{2}(\frac{1}{2}^-)$	$(u_a^T C d_b)\gamma_5 s_c \epsilon^{abc}$
	$\frac{1}{2}(\frac{1}{2}^-)$	$(u_a^T C \gamma_5 d_b) s_c \epsilon^{abc}$
Delta decuplet	$\frac{3}{2}(\frac{3}{2}^+)$	$(u_a^T C \gamma_i d_b) s_c \epsilon^{abc}$

Mumerically,

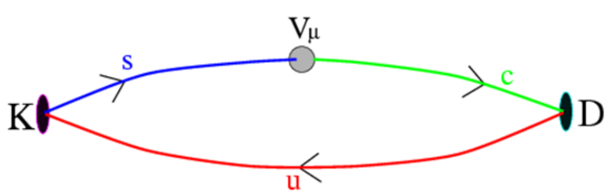
1. Generate the gauge configurations by MC.
2. Calculate the quark propagators for each gauge configuration.
3. Calculate the two-piont function.
4. Fit the data using the theoretical formula.

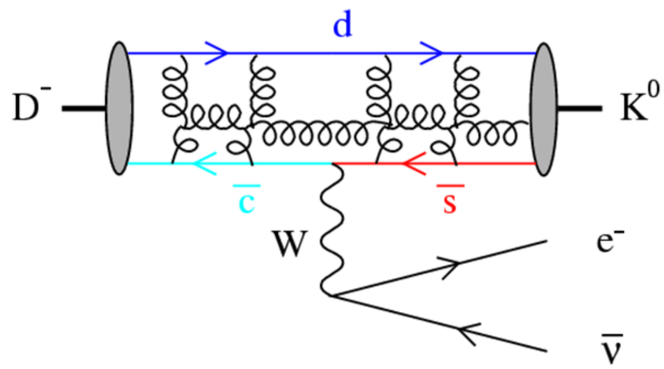
$$m(t)a = \ln\frac{C(t)}{C(t+1)}$$



The mass plateau

• Three-point function:





$$
\begin{aligned}
C_{3,\mu}(t_x,\mathbf{p},t_y;\mathbf{q}) &= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+qy)} \langle 0|T\left[K^+ V_\mu D^0\right]|0\rangle \\
&= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+qy)} \langle 0|\bar{s}(0)\gamma_5 u(0)\bar{c}(y)\gamma_\mu s(y)\bar{u}(x)\gamma_5 c(x)|0\rangle \\
&\sim \frac{\exp\left(E_K(\mathbf{p}+\mathbf{q})t_y - E_D(\mathbf{p})(t_x-t_y)\right)}{4E_K(\mathbf{p}+\mathbf{q})E_D(\mathbf{q})} \\
&\times \langle 0|\bar{s}\gamma_5 u|K(\mathbf{p}+\mathbf{q})\rangle \langle K(\mathbf{p}+\mathbf{q})|\bar{c}\gamma_\mu s|D(\mathbf{p})\rangle \langle D(\mathbf{p}|\bar{u}\gamma_5 c|0\rangle
\end{aligned}
$$

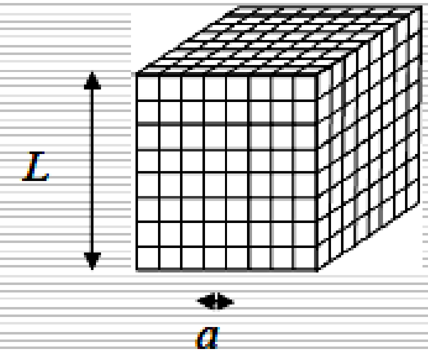
⬇

Form factors

Control of systematical uncertainties

parameters:

- Quark mass — m_q or m_π / m_ρ
- Lattice size — $L\,(fm)$
- Lattice spacing — $a\,(fm)$



Sources of systematical uncertainties:

- **Pion masses are higher than the physical value**
  chiral extrapolation-ChPT
- **Finite lattice spacing:** continuum extrapolation
- **Finite volume effects.** $m_\pi L \geq 3.5$

Going back to the Contimuum

- Dimensionful quantities measured on the lattice are all in unit of lattice spacing a, since the only dimensionful parameter in LQCD actions is the lattice spacing, apart from quark masses.

- The continuum limit can be reach by extrapolating the measured values at several finite lattice spacings.

$$Q_{phy.} = \lim_{a \to 0} Q_{lat}(a)$$

- LQCD has the correct continuum limit, continuum QCD. This is guaranteed by the renormlization group theory.

- The only tunable parameter in the action of LQCD is the bare couping constant, g, which directly relates to the lattice spacing according to the RG equation,

$$a = \frac{1}{\Lambda_L} f(g) \qquad f(g) = \left(\beta_0 g^2\right)^{-\frac{\beta_1}{2\beta_0^2}} e^{-\frac{1}{2\beta_0 g^2}}$$

II. Present Status of LQCD

Lattice QCD: A path integral formalism of QCD on Euclidean spacetime

$$Z = \int [DU] e^{-S_g(U) + \mathrm{Tr} \ln M[U]}$$

$\mathrm{Tr} \ln M[U] \sim \mathrm{const.} \quad \Rightarrow$ **Queched Approximation**

$m_q^{val} \neq m_q^{sea} \quad \Rightarrow$ **Partially Quenched**

$M_{val}[U] \neq M[U] \quad \Rightarrow$ **Mixed Action**

Otherwise, a unitary theory of full QCD on the lattice

} Dynamical Calculation

Observables: VEV of operators, such as Green's functons.

$$\langle O \rangle = \int [DU] O(U) e^{-S_g(U) + Tr \ln M[U]} \Rightarrow \frac{1}{N} \sum_i O_i$$

Monte Carlo simulation, importance sampling

Ukawa, KITPC 2009

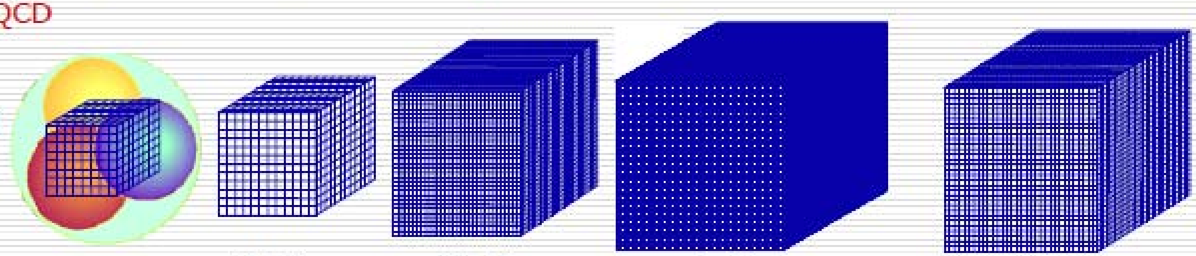
Lattice QCD over the years…

1975 — 1980 — 1985 — 1990 — 1995 — 2000 — 2005 — 2010

1973 QCD
1974 lattice QCD

Physics

1^{st} spec calculation
1981
Hamber-Parisi
Weingarten



Lattice size L	0.8fm	1.6fm	2.4fm	3.0fm	3.0fm
	8^3x16	16^3x32	24^3x48	64^3x118	32^3x64

Algorithms

Nf =#sea quarks

Nf=0 quenched

Nf=2 u,d

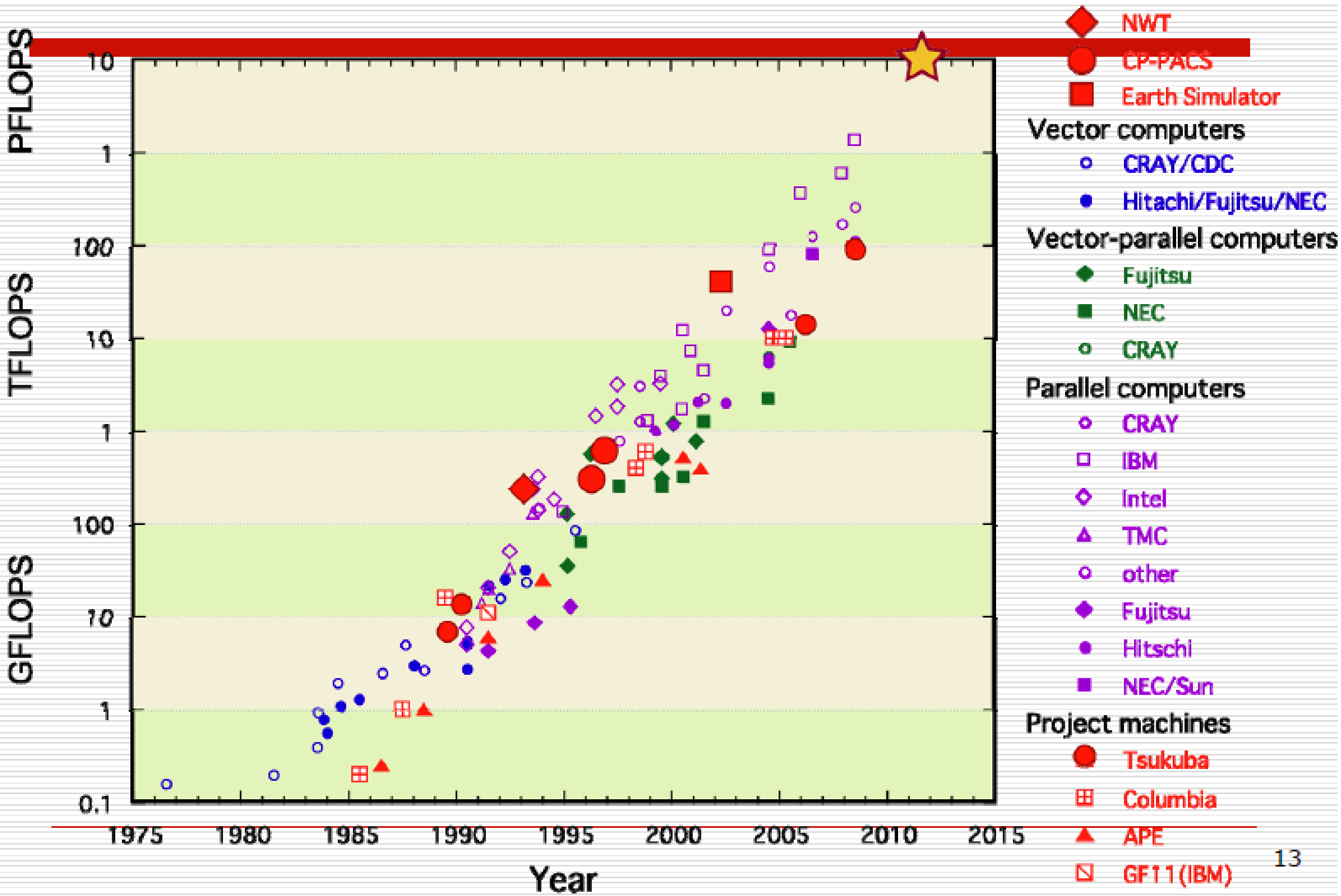
Nf=2+1 u,d,s

Machines

1st generation 1Gflops
2nd generation 10Gflops
3rd generation 1Tfops
4th generation 10Tfops
100Tflops

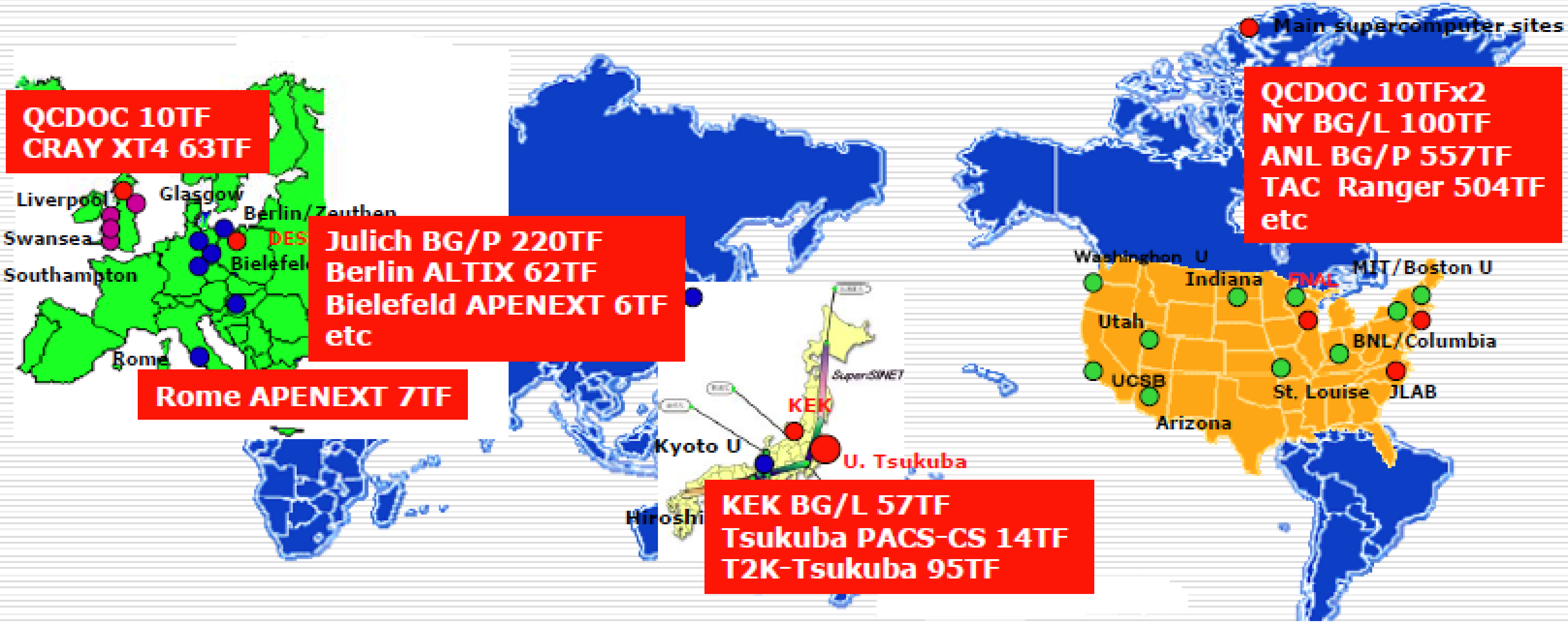
Ukawa, KITPC 2009

Development of supercomputers

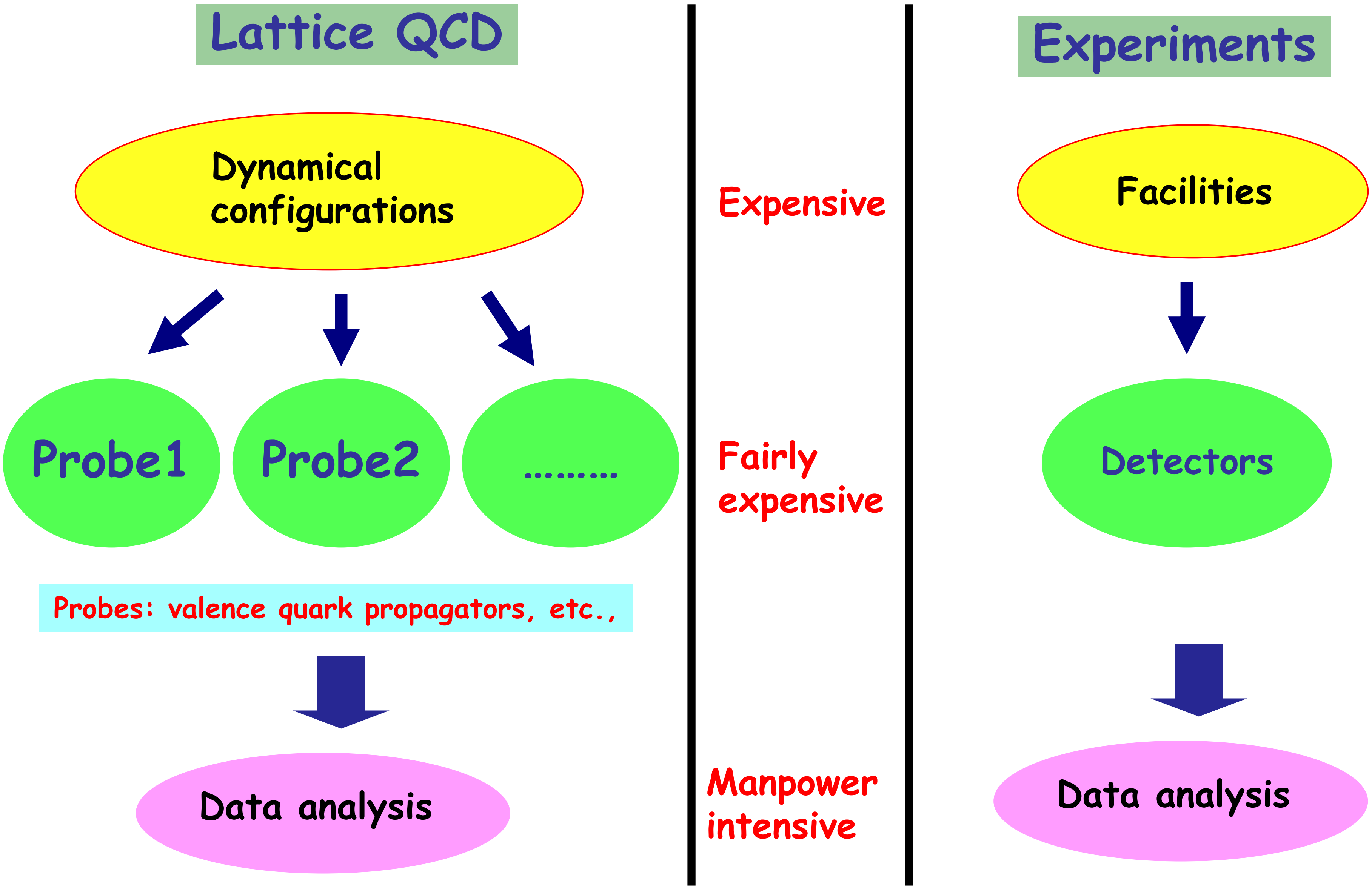


Ukawa, KITPC 2009

Current computing resources for lattice QCD in the World



- About 10 major cites scattered in USA, EU(UK, Germany, Italy etc), Japan
- In total 500~600Tflops in peak speed (US300Tf、EU150Tf、Japan100Tf)
- Data sharing through *ILDG (International Lattice Data Grid)*


Lattice QCD
Experiments
Dynamical configurations
Expensive
Facilities
Probe1
Probe2
....
Fairly expensive
Detectors
Probes: valence quark propagators, etc.,
Data analysis
Manpower intensive
Data analysis

Recent large-scale Nf=2+1 calculations

Collaborations	action	a (fm)	L (fm)	m_π (MeV)
MILC	Kogut-Susskind	0.06	4	220
PACS-CS	wilson-clover	0.09	3	155
BMW	wilson-clover	0.09	4	190
RBC-UKQCD	domain-wall	0.085	4	220
JLQCD	overlap	0.11	1.8	290

- **RBC-UKQCD Collaboration:**
  2+1 Domain Wall Fermions (DMF)

N_f	a[fm]	aL[fm]	$L^3 \times T$	m_π[MeV]
2+1	**0.121**	~ 1.9	$16^3 \times 32$	**400,526,627**
2+1	**0.114**	~ 2.7	$24^3 \times 64$	**328,417**
2+1	**0.084**	~ 3.0	$32^3 \times 64$	**295,350,397**

Advantage: chiral symmetry on the lattice

- **MILC Collaboration:**
  2+1 Staggered Fermions (Asqtad)

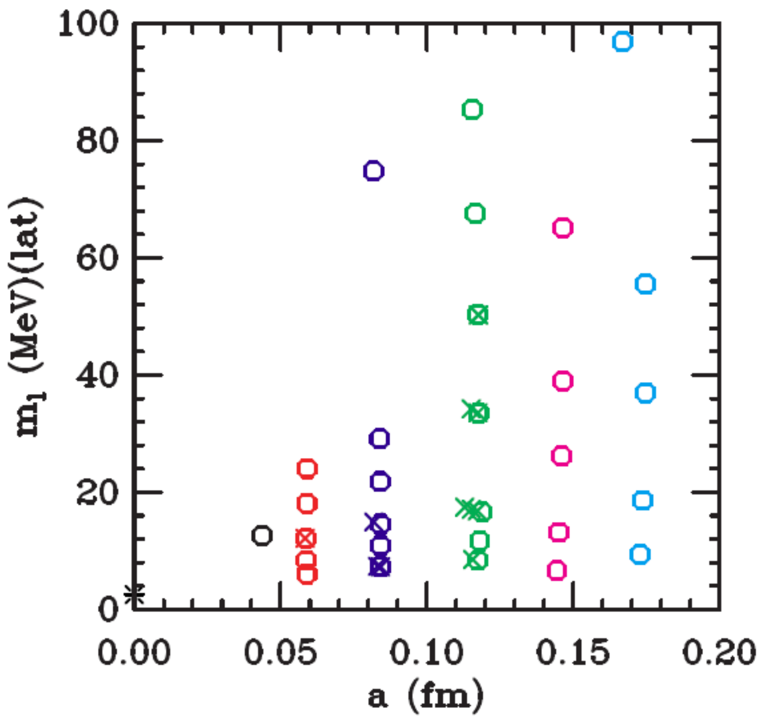
Chiral symmetry $\Rightarrow$ partially reserved

Each has 4 tastes

The Fourth Root Solution

Advantage: computationally cheap
Disadvantage:
the fourth-root-trick is controversial.

Ensemble	am_l	am_s	β	size	$m_\pi L$
$\approx$ 0.09 fm (F)	0.0062	0.031	7.09	$28^3 \times 96$	4.14
$\approx$ 0.09 fm (F)	0.00465	0.031	7.085	$32^3 \times 96$	4.10
$\approx$ 0.09 fm (F)	0.0031	0.031	7.08	$40^3 \times 96$	4.22
$\approx$ 0.09 fm (F)	0.00155	0.031	7.075	$64^3 \times 96$	4.80
$\approx$ 0.06 fm (SF)	0.0036	0.018	7.47	$48^3 \times 144$	4.50
$\approx$ 0.06 fm (SF)	0.0025	0.018	7.465	$56^3 \times 144$	4.38
$\approx$ 0.06 fm (SF)	0.0018	0.018	7.46	$64^3 \times 144$	4.27
$\approx$ 0.045 fm (UF)	0.0028	0.014	7.81	$64^3 \times 192$	4.56



• 2+1 Wilson-type Fermions (Clover,etc.)

Disadvantage: chiral symmetry can be restored only in the continuum limit.

Collaboration	BMW	PACS-CS	QCDSF		CLS
N_f	2+1	2+1	2	2+1(SLiNC)	2
Gauge action	Tree Symanzik	Iwasaki	Wilson	Tree Symanzik	Wilson
c_{sw}	Tree level	NP	NP		NP
Smearing	Stout			Stout	
Algorithm	(R)HMC	DDHMC +PHMC	(R)HMC		DDHMC +deflation
a(fm)	0.065-0.125	0.09	0.072-0.09	$\sim$0.08	0.04-0.08
m_π(Mev)	$>\sim$ 190	156-702	140 - 1010		360 - 520

- **ETMC Collaboration:**
  nf=2 (2+1+1) Twisted Mass Fermions (Asqtad)

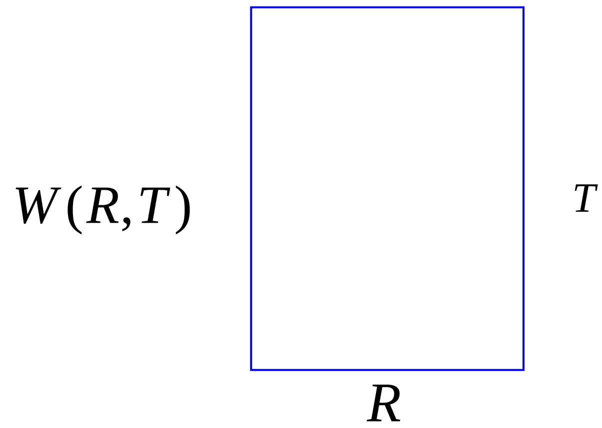
Ens.	β	a [fm]	V/a^4	$a\mu_{sea}$	m_π [MeV]	$m_\pi L$	N_{cfg}
A_2	3.8	0.10	$24^3 \times 48$	0.0080	410	5.0	240
A_3				0.0110	480	5.8	240
B_1	3.9	0.085	$24^3 \times 48$	0.0040	315	3.3	480
B_2				0.0064	390	4.0	240
B_3				0.0085	450	4.7	240
B_4				0.0100	490	5.0	240
B_7	3.9	0.085	$32^3 \times 64$	0.0030	270	3.7	240
B_6				0.0040	310	4.3	240
C_1	4.05	0.065	$32^3 \times 64$	0.0030	310	3.3	144
C_2				0.0060	430	4.6	128
C_3				0.0080	500	5.3	128

III. Selected Latest Results of LQCD

- Static potential----confinement
- Standard model parameters
  alpha_s
  current quark masses
  CKM matrix elements relevant
- Spectroscopy

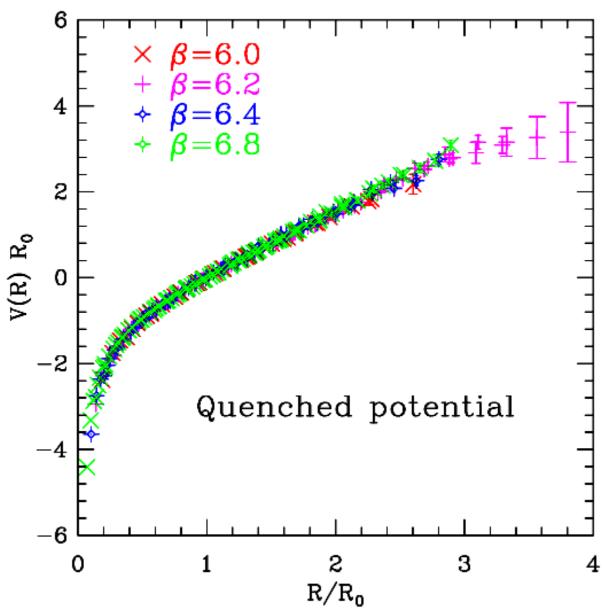
1. Static potential and confinement

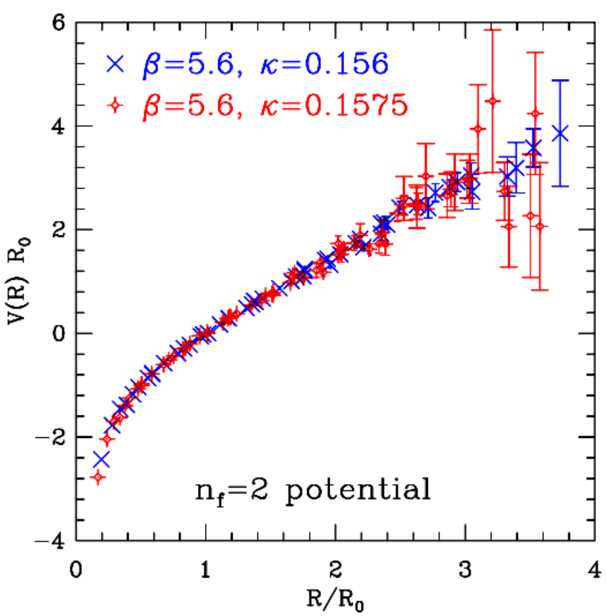
- **Derived from vacuum expectation values of Wilson loops**



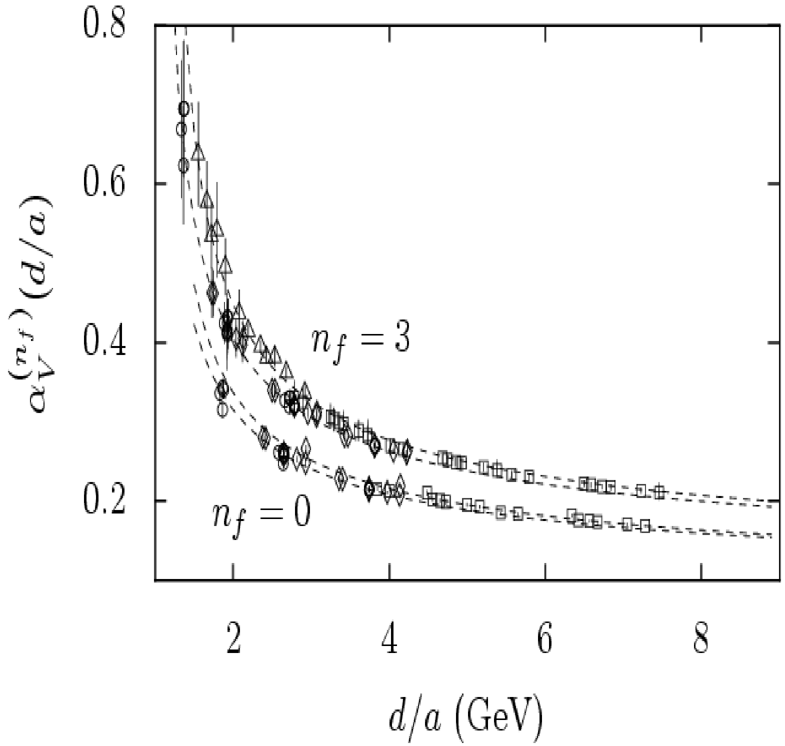
$$V(R) = \lim_{T\to\infty} -\frac{1}{T}\log\langle\mathcal{W}(R,T)\rangle\,.$$

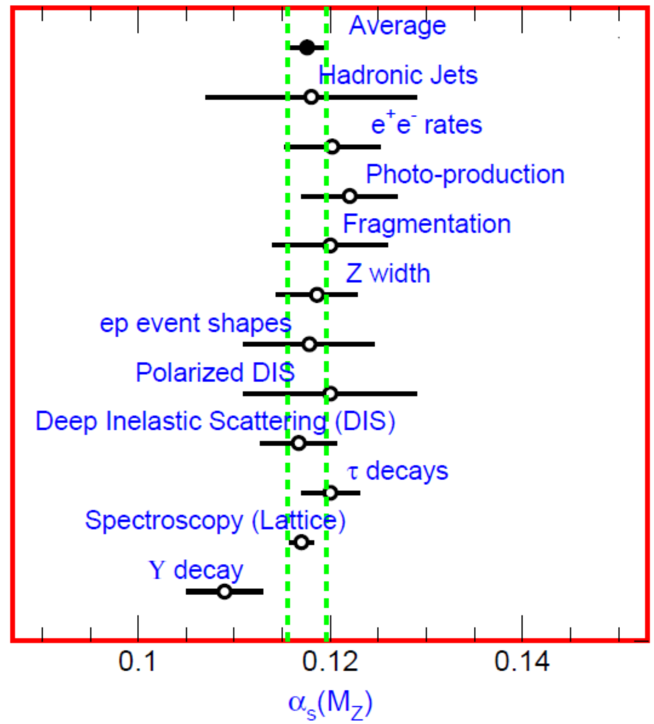
Cornell confinement potential $V(R) = V_0 + \sigma R - \frac{\alpha}{R}\,,$





2. Strong coupling constant





$\alpha_s(M_Z)$	Observable	Sea formulation	Reference
0.1183 ± 0.0008	Wilson loops, Creutz ratios, etc.	2+1 asqtad staggered	HPQCD [
0.1174 ± 0.0012	charmonium correlator	2+1 asqtad staggered	HPQCD [
0.1197 ± 0.0013	Schrödinger functional	2+1 improved Wilson	PACS-CS
0.1185 ± 0.0009	Adler function	2+1 overlap	JLQCD [4
0.1186 ± 0.0011	scattering, τ decay, etc.	2+1(+1+1) Dirac (!)	Bethke [45

3. Quark masses

(a) Pseudoscalar meson masses⇒Light quark masses. The lowest-order ChPT gives

$$m_\pi^2 \propto m_u + m_d.$$

(b) Up and down quark are always set to be degenerate in mass,

$$m_l = \frac{1}{2}(m_u + m_d)$$

(c) The bare quark mass should be renormalized. Z_m is always abtained by the The Ward identity of a flavour non-singlet vector current $J_\mu^a = \bar{\psi}\gamma_\mu\tau^a\psi$,

$$\langle \partial_\mu J_\mu^a \rangle = \langle \bar{\psi}[M, \tau^a]\psi \rangle + \text{ contact terms},$$

which gives the relation $Z_m Z_{\bar{\psi}\psi} = 1$, where M is the quark mass matrix and $Z_{\bar{\psi}\psi}$ is the renormalization factor for the scalar operator.

For heavy quark masses, HPQCD proposes a new solution

C. McNeile et al, arXiv:1004.4285

$$j_5 = \overline{\psi}_h \gamma_5 \psi_h$$

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

$$G_n \equiv \sum_t (t/a)^n G(t)$$

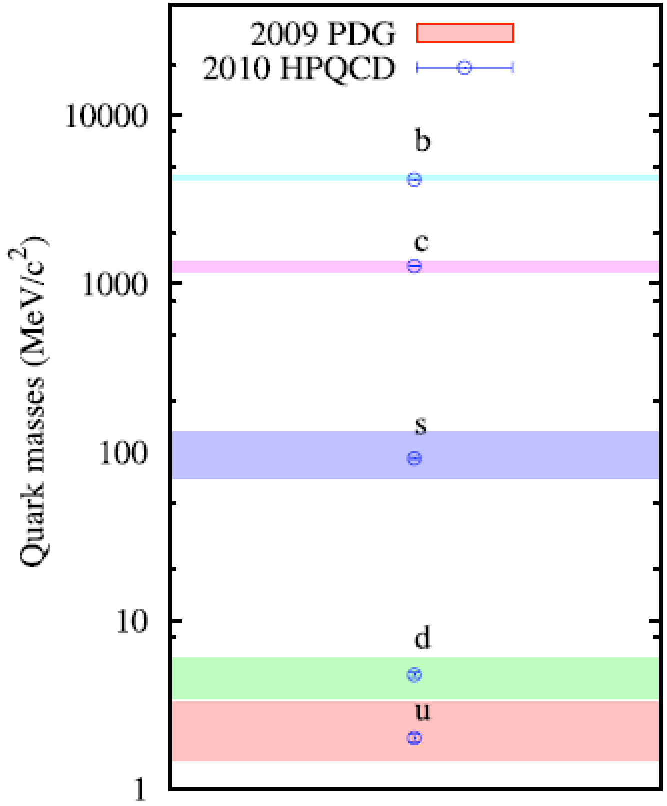
$$G_n = \frac{g_n(\alpha_{\overline{\rm MS}}(\mu), \mu/m_h)}{(am_h(\mu))^{n-4}} + \mathcal{O}((am_h)^m)$$

PT calculable

Lattice calculation

Latest results of quark masses

	m_u [MeV]	m_d [MeV]	m_u/m_d
MILC	1.96(0)(6)(10)(12)	4.53(1)(8)(23)(12)	0.432(1)(9)(0)(39)
Aubin et al.	1.7(0)(2)(2)(1)	4.4(0)(2)(4)(1)	0.39(1)(3)(0)(4)



C. T. H. Davies et al. (HPQCD Collab.), arXiv:0910.3102[hep-ph]

Quantity	HPQCD/MILC	PDG
m_u(2 GeV)	2.01 ± 0.10	2.55 ± 1.05
m_d(2 GeV)	4.77 ± 0.15	5.04 ± 1.54
m_s(2 GeV)	92.2 ± 1.3	105 ± 35
$m_c(m_c)$ GeV	1.273 ± 0.006	1.27 ± 0.11
$m_b(m_b)$ GeV	4.164 ± 0.023	4.20 ± 0.17
$m_t(m_t)$ GeV	-	160 ± 3
m_c/m_s	11.85 ± 0.16	-
m_u/m_d	0.42 ± 0.04	0.35 - 0.6
m_s/m_l	27.2 ± 0.03	25 - 30
m_b/m_c	4.51 ± 0.04	-

4. CKM matrix elements relevant

- CKM matrix and PDG number (Amsler2008)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix}$$

Lattice gold-plated processes which can be used to determine the CKM matrix elements

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

Master equation:

$$Expt.\# = (\text{SM parameters})\,(\text{matrix elements})\,(\text{kinematic factors})$$

Decay constants,

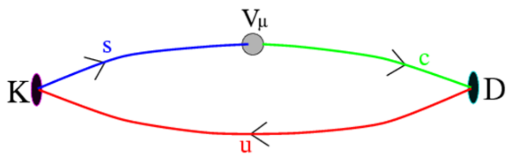
Form factors

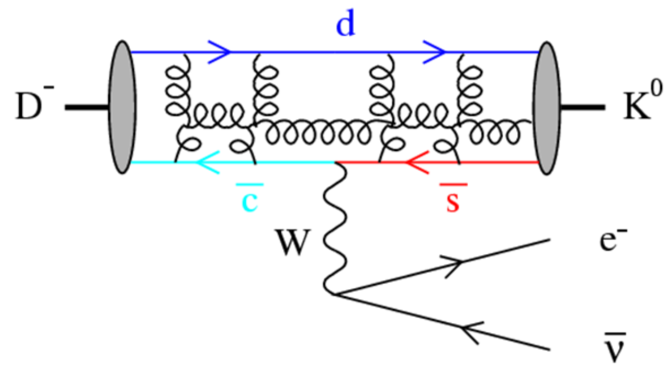
Decay constants: $\langle 0|A_\mu|H_q(p)\rangle = i f_{H_q} p_\mu$

Which can be derived from the two-point function,

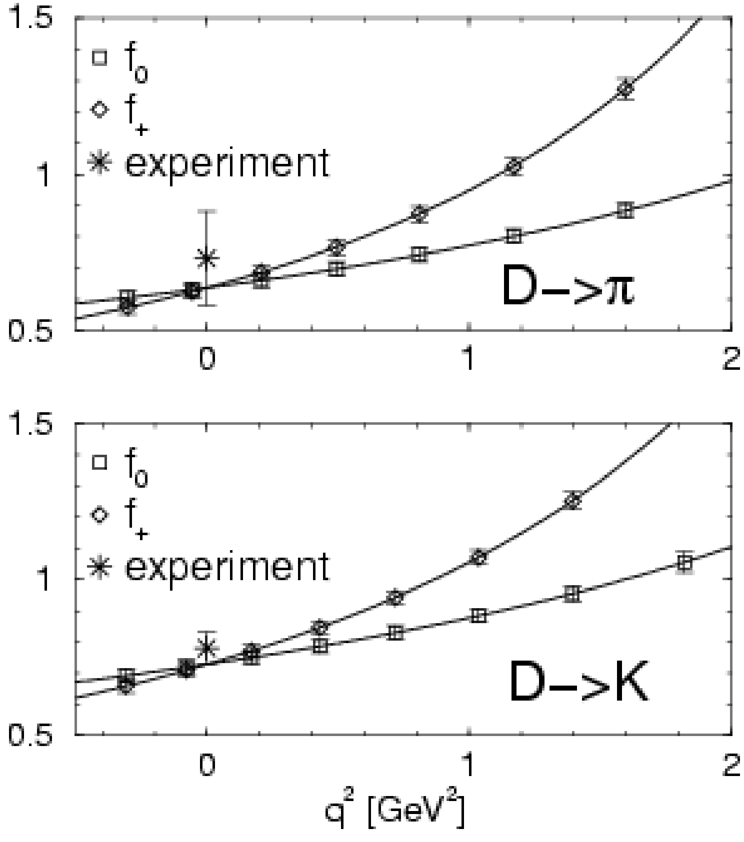
$$C_{A_4}(t) = \langle A_4(t) O^\dagger_{H_q}(0)\rangle$$

Form factors

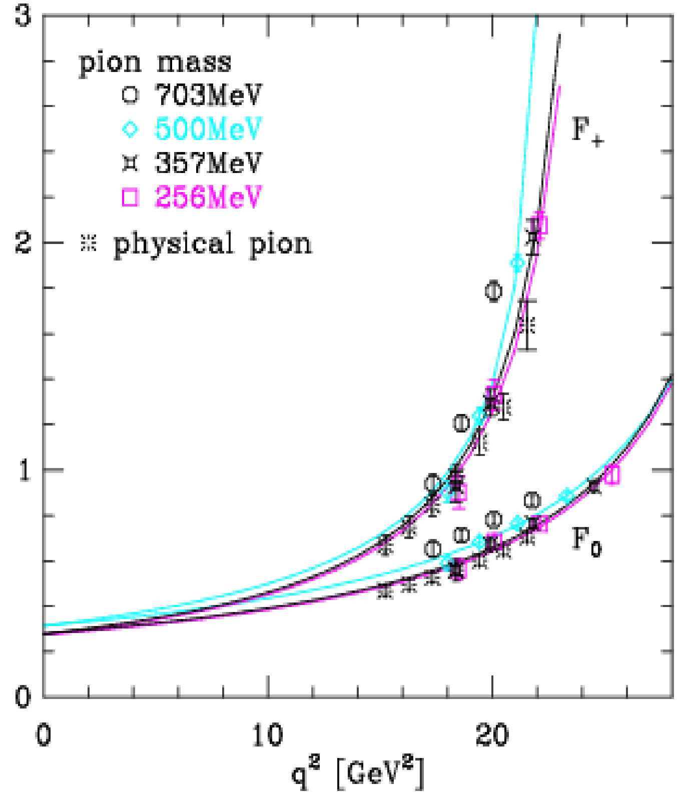




$$
\begin{aligned}
C_{3,\mu}(t_x, \mathbf{p}, t_y; \mathbf{q}) &= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+qy)} \langle 0 | T \left[K^{+} V_{\mu} D^{0} \right] | 0 \rangle \\
&= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+qy)} \langle 0 | \bar{s}(0)\gamma_5 u(0) \bar{c}(y)\gamma_\mu s(y) \bar{u}(x)\gamma_5 c(x) | 0 \rangle \\
&\sim \frac{\exp\left(E_K(\mathbf{p}+\mathbf{q})t_y - E_D(\mathbf{p})(t_x - t_y)\right)}{4E_K(\mathbf{p}+\mathbf{q})E_D(\mathbf{q})} \\
&\times \langle 0 | \bar{s}\gamma_5 u | K(\mathbf{p}+\mathbf{q}) \rangle \langle K(\mathbf{p}+\mathbf{q}) | \bar{c}\gamma_\mu s | D(\mathbf{p}) \rangle \langle D(\mathbf{p} | \bar{u}\gamma_5 c | 0 \rangle
\end{aligned}
$$



Aubin et al 2005

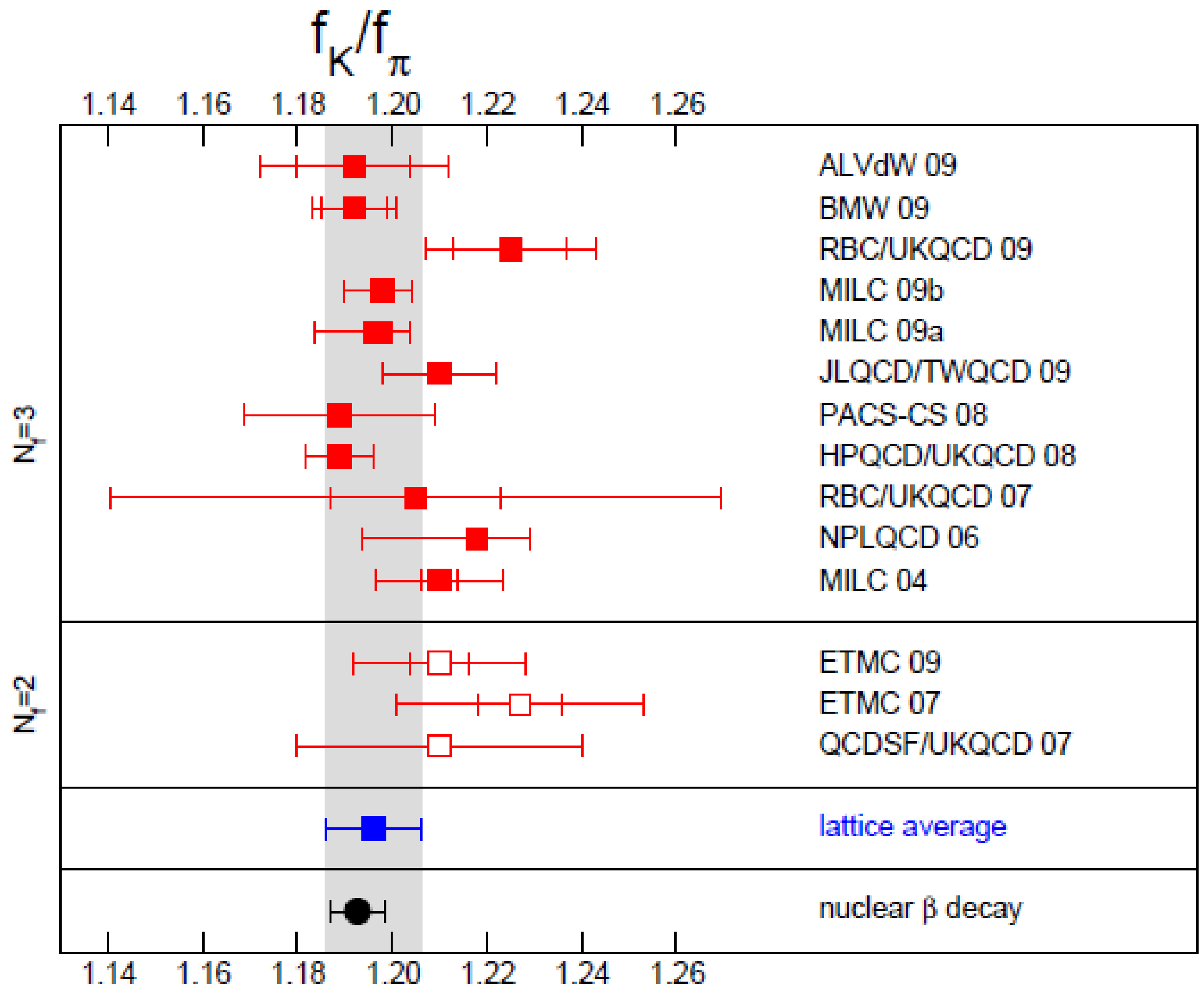


Shigemitsu *et al* 2005

a) Latest results of f_pi and f_K

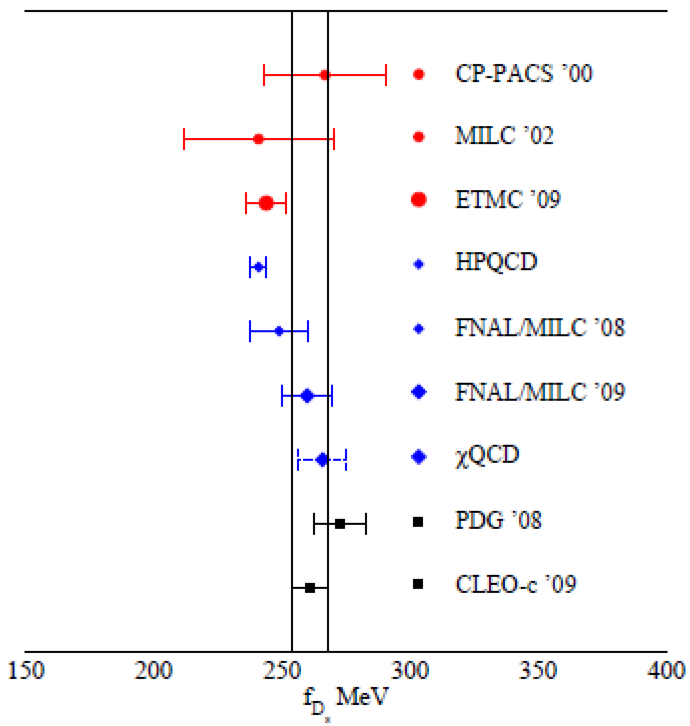
E. E. Scholz, PoS(LAT2009)005

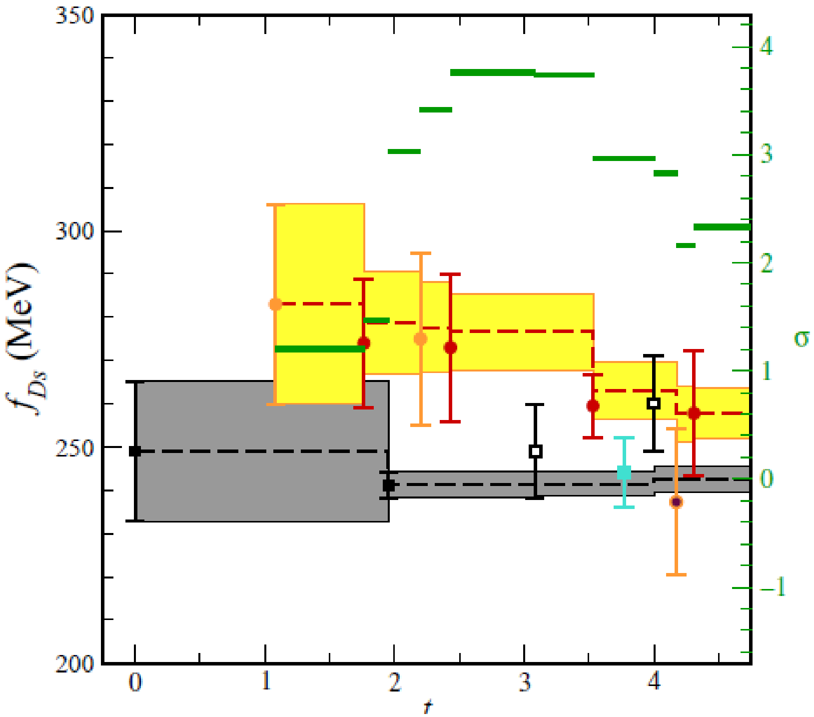
	f_π [MeV]	f_K [MeV]	f [MeV]	f_0 [MeV]
ETMC	input	158.1(0.8)(2.0)(1.1)	*121.57(70)*	
JLQCD (2)	$119.6(3.0)(1.0)(^{+6.4}_{-0.0})$		$111.7(3.5)(1.0)(^{+6.0}_{-0.0})$	
JLQCD (2+1)	input	*157.3(5.5)*	*121(14)*	*79(20)*
RBC-UKQCD	*122.2(3.4)(7.3)*	*149.7(3.8)(2.0)*	*113.0(3.8)(6.8)*	93.5(7.3)
PACS-CS	134.0(4.3)	159.4(3.1)	126.4(4.7)	118.5(9.0)
MILC	*128.0(.3)(2.9)*	*153.8(0.3)(3.9)*	*122.8(.3)(.5)*	*111.0(2.0)(4.1)*
Aubin et al.	*131.1(1.3)(2.2)*	*156.3(1.3)(2.0)*		
PDG	130.4(.04)(.2)	155.5(.2)(.8)(.2)	–	–



f_K/f_π
1.14
1.16
1.18
1.20
1.22
1.24
1.26
ALVdW 09
BMW 09
RBC/UKQCD 09
MILC 09b
MILC 09a
JLQCD/TWQCD 09
PACS-CS 08
HPQCD/UKQCD 08
RBC/UKQCD 07
NPLQCD 06
MILC 04
ETMC 09
ETMC 07
QCDSF/UKQCD 07
lattice average
nuclear β decay
$N_f=3$
$N_f=2$

b) Latest results of f_D and f_Ds





Grey bands indicate the HPQCD results (Kronfeld 2010)

c) Latest results of f_B and f_Bs

2+1 flavor

Group	f_B (MeV)	f_{B_s} (MeV)	f_{B_s}/f_B	ξ
Fermilab/MILC	195(11)*	243(11)*	1.245(43)*	1.205(50)
HPQCD I	190(13)	231(15)	1.226(26)	1.258(33)
RBC/UKQCD (APE)			1.20(14)	1.187(112)
RBC/UKQCD (HYP2)			1.19(16)	1.18(19)

2 flavor

Group	f_B	f_{B_s}	f_{B_s}/f_B	ξ
ETMC	203(17)	247(16)	1.22(6)	
Burch *et al* (0.16 fm)			1.108(29)	
Burch *et al* (0.11 fm)			1.089(41)	

d) Other lattice inputs to CKM

Current status of lattice inputs to the global fit of the CKM unitarity triangle. These are obtained by averaging all available 2+1 flavor results documented in proceedings and publications that contain Complete error budgets, and account forcorrelations between different calculations in a conservative manner.

Quantity	Error
$\hat{B}_K = 0.725 \pm 0.026$	$\sim 4\%$
$f_{B_s}\sqrt{\hat{B}_{B_s}} = 275 \pm 13$	$\sim 5\%$
$\xi = 1.243 \pm 0.028$	$\sim 2\%$
$\lvert V_{ub}\rvert_{\text{excl.}} = (3.42 \pm 0.37) \times 10^{-3}$	$\sim 11\%$
$\lvert V_{cb}\rvert_{\text{excl.}} = (38.6 \pm 1.2) \times 10^{-3}$	$\sim 3\%$
$\kappa_\varepsilon = 0.92 \pm 0.01$	$\sim 1\%$
$f_K = 155.8 \pm 1.7$ MeV	$\sim 1\%$

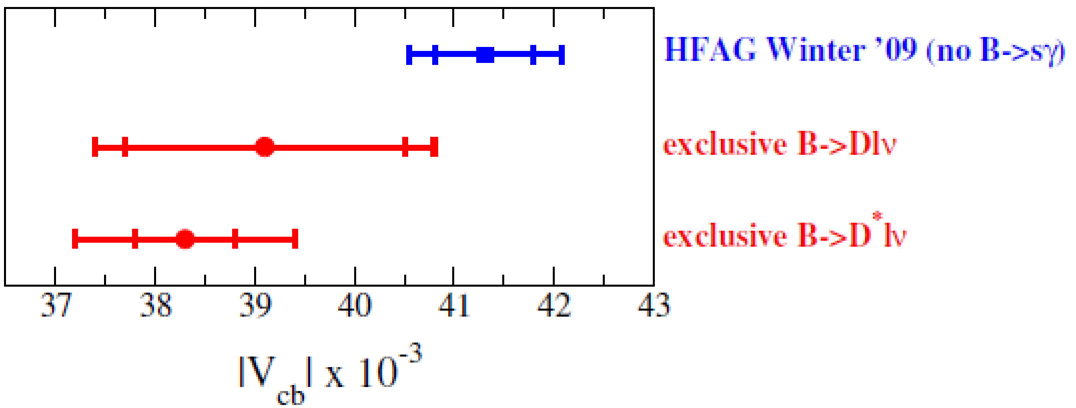


Figure 5: Tension between inclusive (blue square) and exclusive (red circles) determinations of $|V_{cb}|$. The inner and outer errors bars on the inclusive value denote the errors from the moment fit and from other systematics, respectively. The inner and outer errors on the exclusive values are experimental and theoretical, respectively. The inclusive determination [15] disagrees with the exclusive determination from $B \to D\ell\nu$ by 1.2σ and from $B \to D^*\ell\nu$ by 2.3σ.

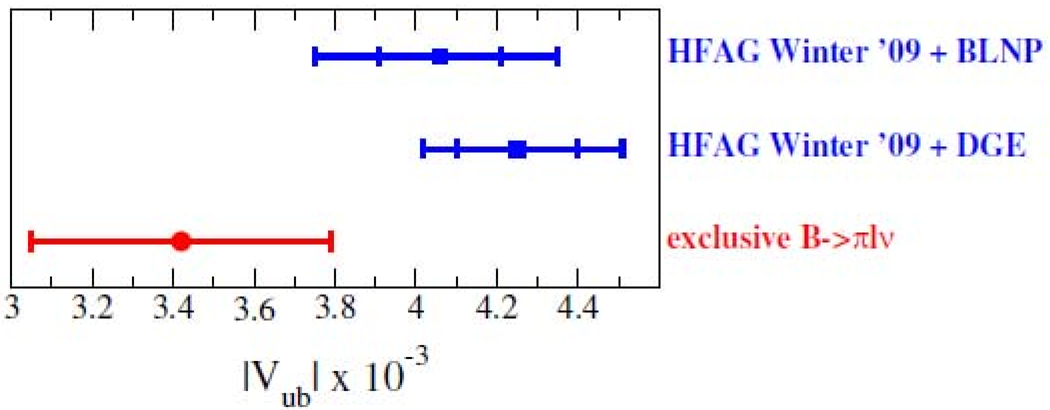
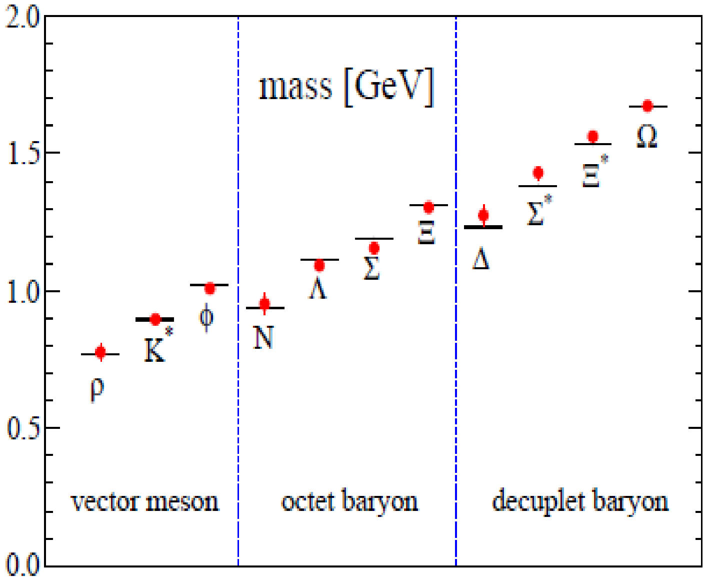


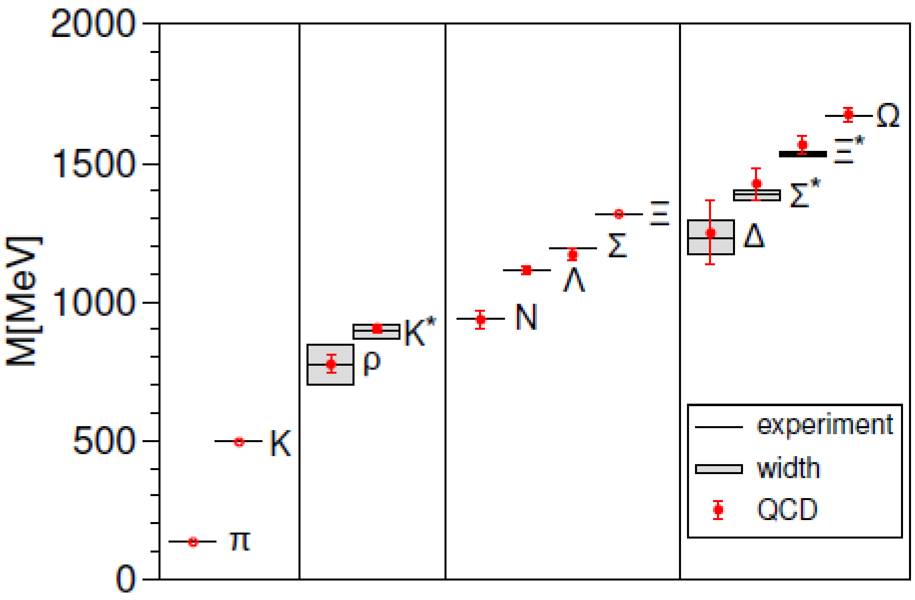
Figure 6: Tension between inclusive (blue squares) and exclusive (red circle) determinations of $|V_{ub}|$. The inner and outer errors on the inclusive values are experimental and theoretical, respectively. The error bar on the exclusive value denotes the total statistical plus systematic error added in quadrature. The exclusive determination disagrees with the inclusive determinations [15] by 1–2σ depending upon the theoretical framework used to obtain the inclusive value.

5. Hadron spectroscopy

- Light hadron spectrum

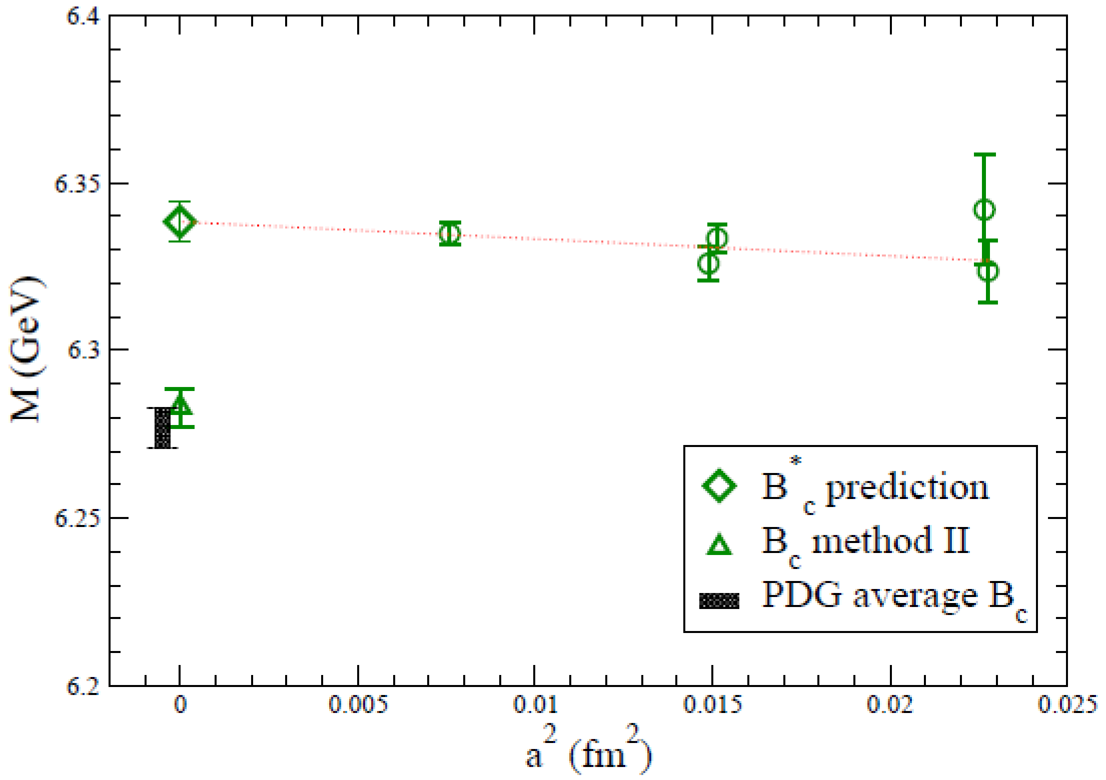


S. Aoki et al. (PACS-CS Collab.)
Phys. Rev. D 79, 034503 (2009).



S.Durr et al.(BMW Collab.),
Science 322, 1224 (2008).

- **Lattice predictions of heavy flavored hadrons**



- **Rho resonance** $\rho \to \pi + \pi$

On the finite lattice,

$$W = 2\sqrt{m_\pi^2 + k^2}, \sim\sim\sim \text{ or } \sim\sim\sim \left(2\sinh\frac{W}{4}\right)^2 = M_\pi^2 + k^2$$

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2} q}{Z_{00}(1, q^2)}\right\} \bmod \pi, q = \frac{kL}{2\pi}$$

$$\tan\delta_{11}(k) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{W(M_\rho^2 - W^2)} \Rightarrow g_{\rho\pi\pi}^2, M_\rho$$

$$\Gamma_\rho = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k_\rho^3}{M_\rho^2} \qquad k_\rho = \frac{1}{2}\sqrt{M_\rho^2 - 4M_\pi^2}$$

Pion-pion I=1, L=1 phase shift and BW fit
(G. Shierholz, arXiv:0810.5337(hep-lat))

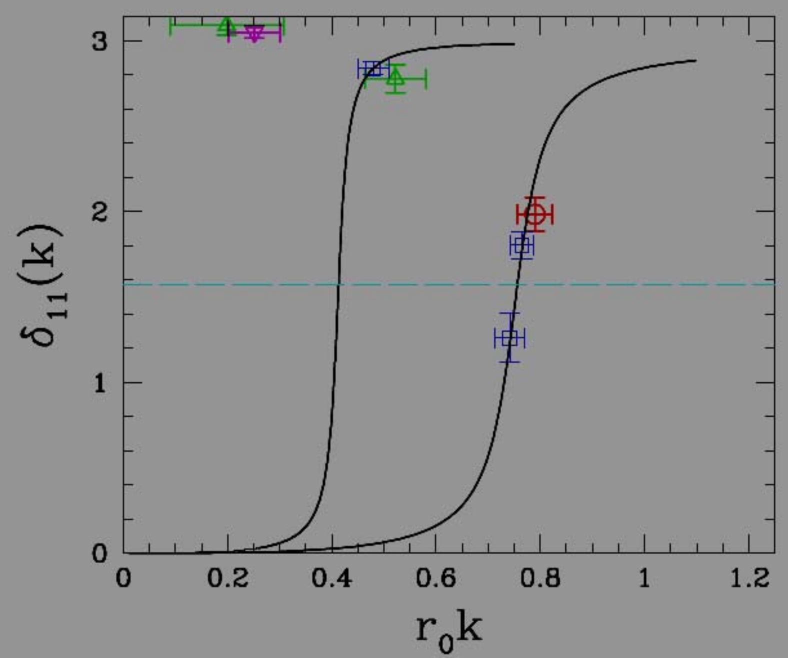


Figure 4: The phase shift $\delta_{11}(k)$ as a function of r_0k, together with the effective range fit from Fig. 5. The symbols indicate the different couplings: $\beta = 5.25$ (▽), $\beta = 5.29$ (□), $\beta = 5.30$ (△), and $\beta = 5.40$ (○). The $\beta = 5.30$ data points have been inferred from [5]. The curves refer to pion masses of $m_\pi = 250$ (right) and 390 MeV (left), respectively.

$$m_\rho = 770(111)\ \text{MeV}$$

$$\Gamma_\rho = 200^{+130}_{-100}\ \text{MeV}$$

6. Summary

- High precision for standard model parameters.
- The spectrum of quite a few light hadrons can be reproduced.
- Further efforts desired for most of hadron resonances.
- Lattice calculation at the physical point is expected in the following ten years.

Thank You!