Lectures on NRQCD Factorization for Quarkonium Production and Decay

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I. Nonrelativistic QCD
II. Annihilation decays
III. Inclusive hard production

NRQCD Factorization for Quarkonium Production and Decay

I. Nonrelativistic QCD
II. Annihilation decays
III. Inclusive hard production
    - Introduction
    - Color-singlet Model, Color Evaporation model
    - NRQCD Factorization
    - Applications
    - Polarization
    - Conclusion

Quarkonium Production

References

Bodwin @ KITPC

- Geoff Bodwin

  Quarkonium Production and Decay:
  NRQCD Confronts Experiment
  Program on Effective Field theories
  in Particle and Nuclear Physics
  KITPC, August 2009

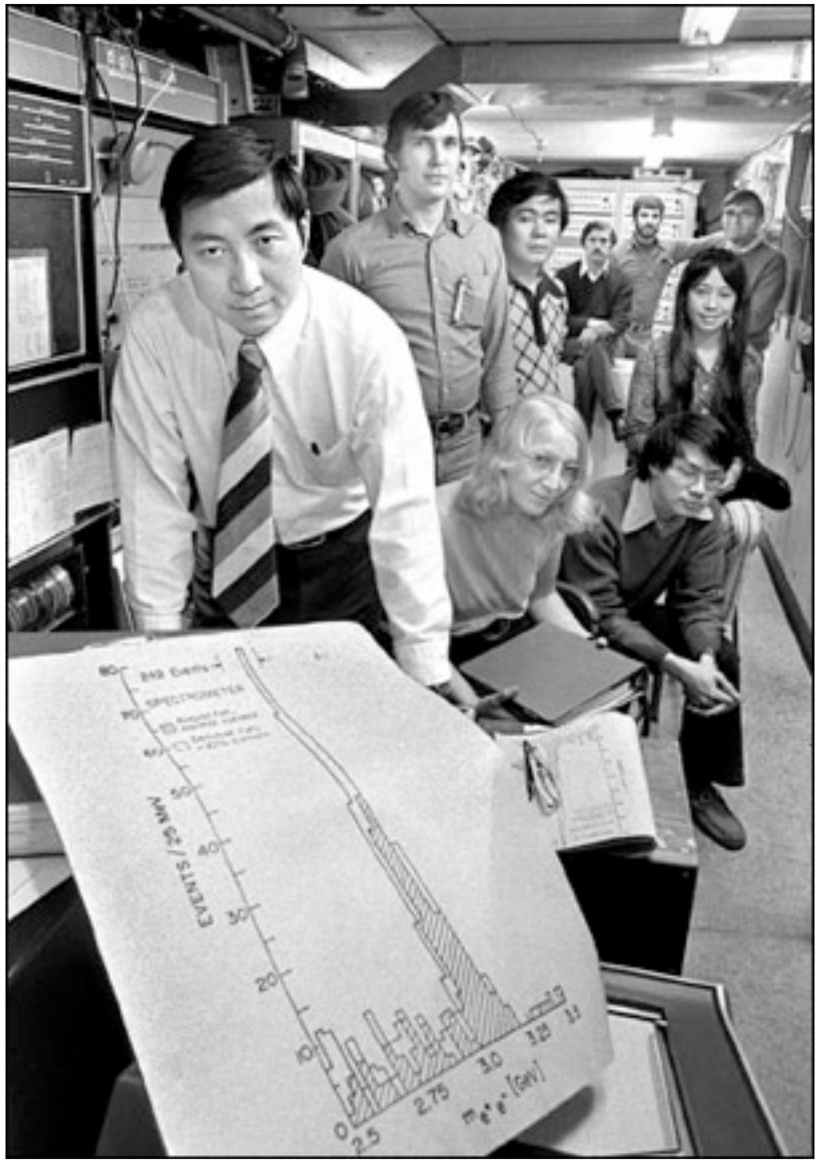
- Pierre Artoisenet

  Status of Theory Calculations & LHC Predictions
  Workshop on Quarkonium Production
  CERN, February 2010

Discovery of Charmonium

November 1974

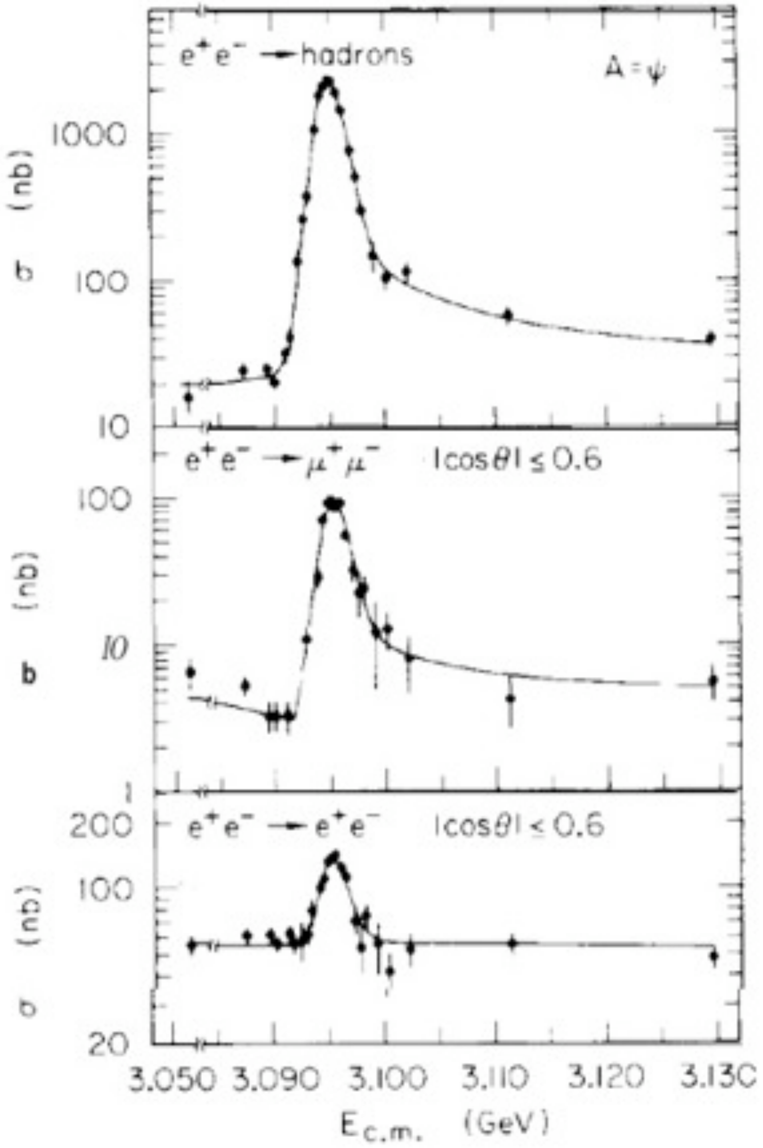
p on Be target at Brookhaven



$p\,N \rightarrow J/\psi + X,$

$J/\psi \rightarrow e^+e^-$

e^+e^- annihilation at SLAC



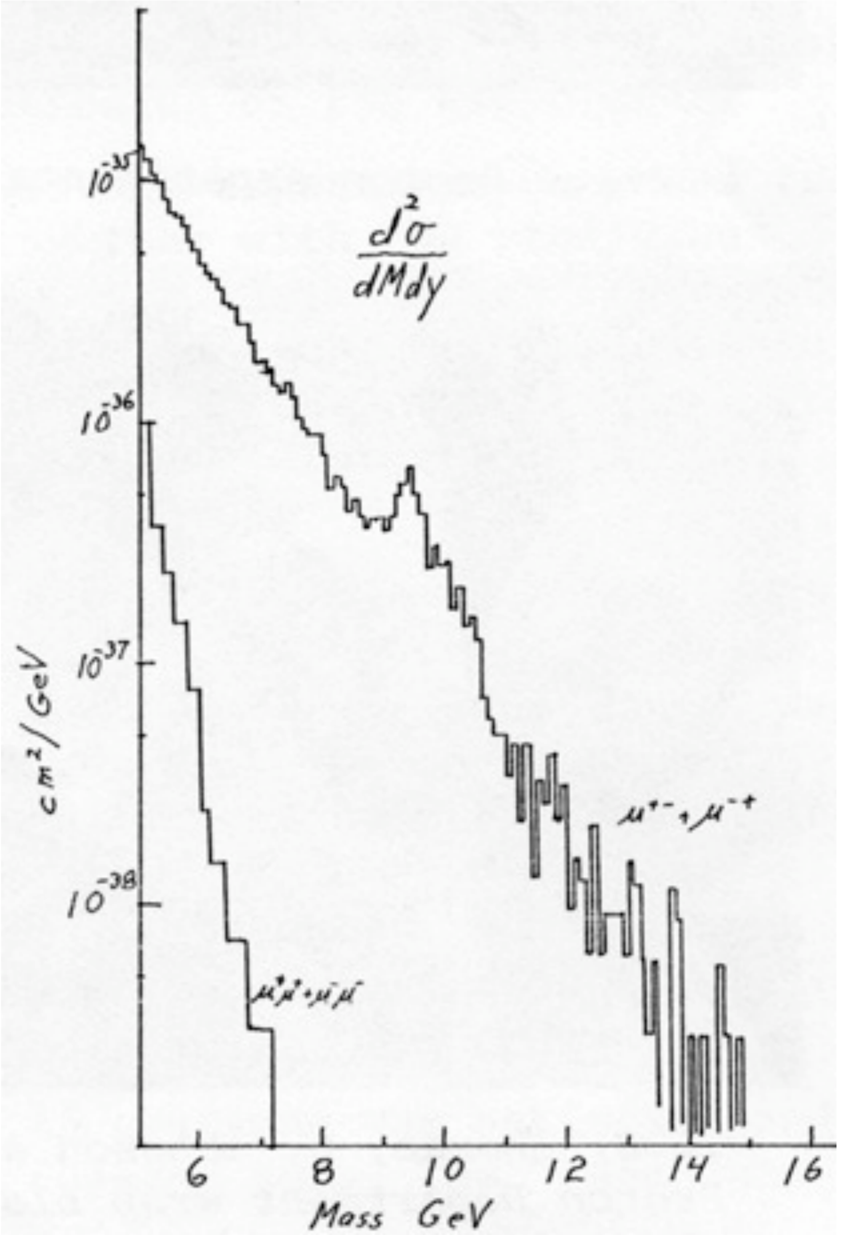
$e^+e^- \rightarrow J/\psi,$

$J/\psi \rightarrow$ hadrons

Discovery of Bottomonium

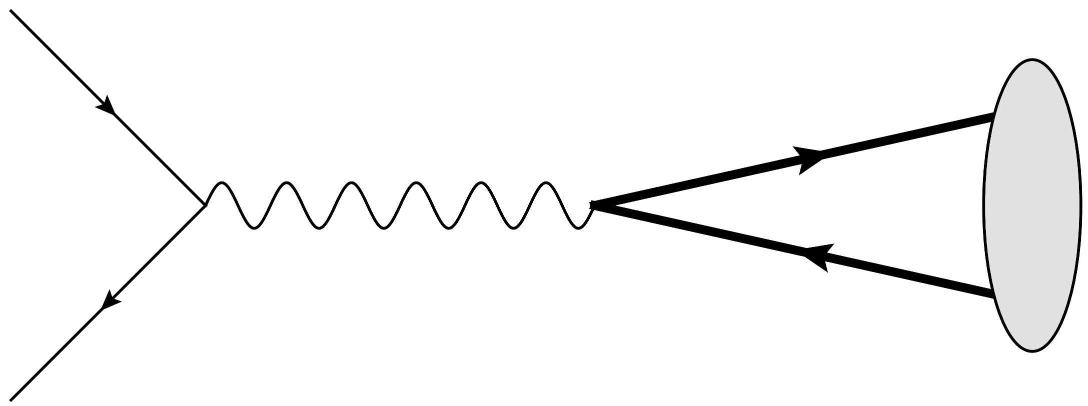
August 1977

p on Be target at Fermilab



$p\,N \rightarrow \Upsilon + X,\ \Upsilon \rightarrow \mu^+\mu^-$

e^+e^- annihilation into J/ψ



- $c\,\bar{c}$ created by virtual photon
- rate determined by one constant f_ψ

$$\langle J/\psi|\bar{c}\gamma^\mu c|0\rangle = f_\psi\ \epsilon^\mu_\psi$$

- can be estimated using potential models: $f_\psi \propto R(0)$
- can be calculated using lattice QCD

Production of Charmonium using Hadrons

- How is the $c\,\bar{c}$ pair created?
  What are the relevant parton processes?
  Can they be calculated using perturbative QCD?

- How does the $c\,\bar{c}$ pair bind to form charmonium?
  Can effects of binding be reduced to a few constants?
  Can they be calculated using lattice QCD?

- Proposed answers: Color-singlet model (1976?)
  Color evaporation model (1977)
  NRQCD factorization (1995)

Color-singlet Model

Ellis, Einhorn, Quigg 1976; Carlson and Suaya 1976; Kuhn 1980; Degrand, Toussaint 1980;
Kuhn, Nussinov, Ruckl 1980; Wise 1980; Chang 1980; Baier, Ruckl 1981; Berger, Jones 1981

- $c\,\bar{c}$ is created by parton collisions
  with negligible relative momentum

- $c\,\bar{c}$ can bind into charmonium only if it is created
  in same color/angular momentum state as charmonium
  $\underline{1}\ {}^3S_1$ for J/ψ
  $\underline{1}\ {}^3P_J$ for χ_{cJ}

- probability that $c\,\bar{c}$ binds into charmonium
  is determined by wavefunction near origin
  $\propto R(0)$ for $J/\psi, \eta_c$
  $\propto R'(0)$ for χ_{cJ}, h_c

one constant for each multiplet
can be determined from annihilation decays: $J/\psi \to e^+ e^-$
$\chi_{c0} \to \gamma\,\gamma$

Color Evaporation Model

Fritzsch 1977; Halzen 1977

- $c\,\bar{c}$ pair is created by parton collisions
  with invariant mass below $D\,\bar{D}$ threshold
  (between $2m_c$ and $2m_D$)

- $c\,\bar{c}$ pair can bind into charmonium regardless
  of its color/angular momentum state

- probability that $c\,\bar{c}$ binds to form charmonium H
  is single constant f_H for each multiplet
  the fractions f_H should add up to 1:

$$\sum_H f_H = 1$$

Color-singlet Model vs Color Evaporation Model

- Applicability

  CSM: annihilation decays

  exclusive and inclusive production

  definite predictions for polarization

  CEM: only sufficiently inclusive production

  assumes no polarization

- Predictive power

  CSM: one constant for each multiplet

  determined by annihilation decays

  CEM: one constant for each multiplet

  adjustable parameter

Color-singlet Model vs Color Evaporation Model

- consistency

  CSM: infrared divergences for P-waves

  CEM: no infrared divergences

$$\chi_{cJ} \to q\,\bar{q}\,g$$
$$b \to \chi_{cJ} + s + g$$

- perturbative corrections

  CSM: separate NLO calculation for each process

  CEM: can use NLO calculation for inclusive $Q\bar{Q}$

  Nason, Dawson, Ellis 1988

- Dominant theoretical prejudice in early 1990's

  CSM: can probably be extended
  to a theory based on QCD

  CEM: purely phenomenological model

Color-singlet Model vs Color Evaporation Model

Experimental status in early 1990's

- Fixed target experiments (pN, πN, γN)

  feeddown to J/ψ from $\psi(2S)$, χ_{cJ} decays

  contributions from small p_T ⇒ nonperturbative?

  large experimental errors

  roughly compatible with CSM or CEM

- $p\,\bar{p}$ collisions at the Tevatron

  feeddown to J/ψ from $\psi(2S)$, χ_{cJ} decays

  feeddown from B decays

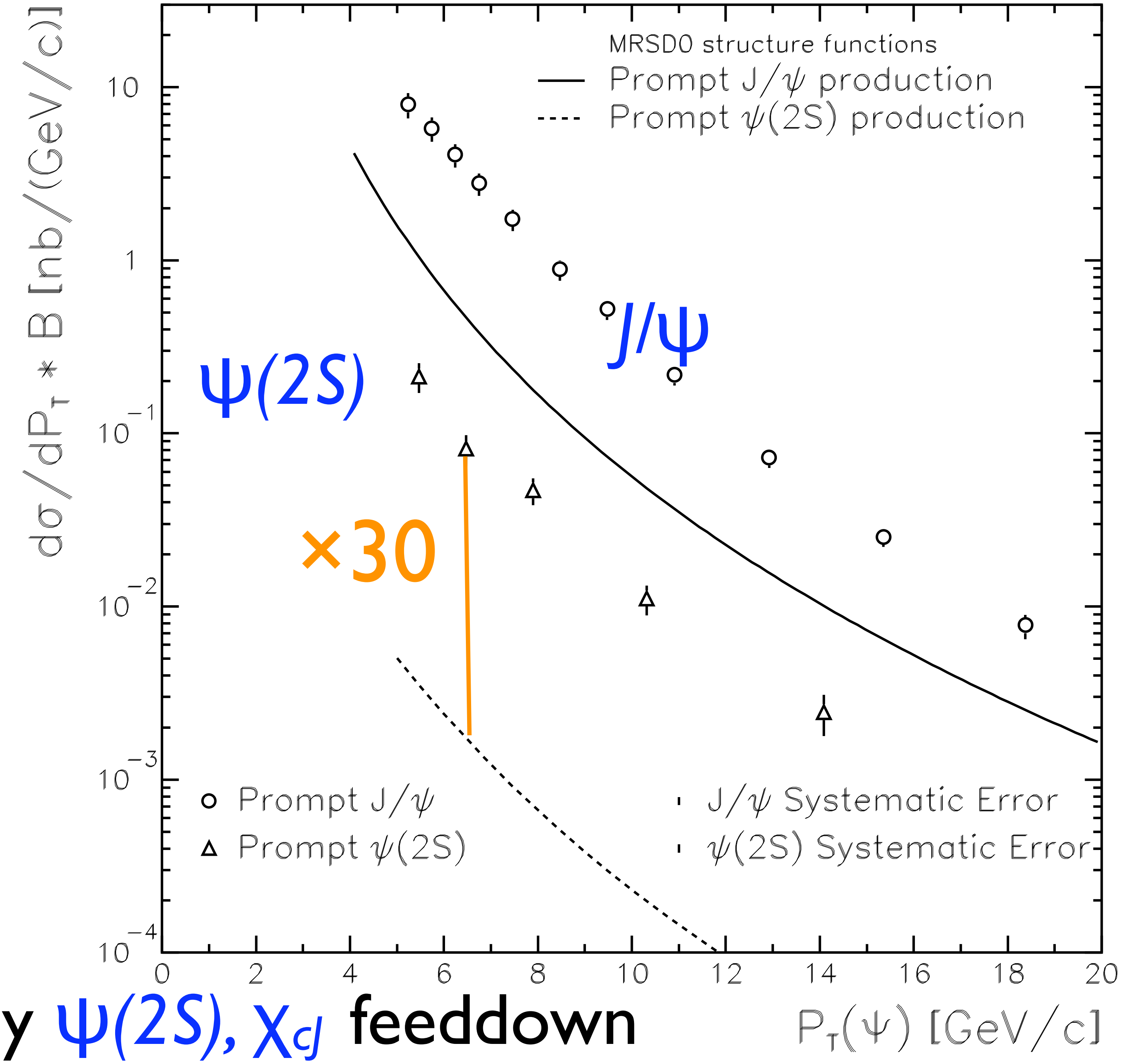
  $p_T > 5$ GeV ⇒ perturbative?

  production rates much larger than predicted by CSM?

Demise of Color-singlet Model

CDF collaboration 1997

- use vertex detector to remove *B* feeddown



- prompt *J/ψ*: complicated by *ψ(2S)*, χ_{cJ} feeddown
- prompt *ψ(2S)*: 30 times larger than CSM prediction
  (in retrospect, compatible with CEM)

NonRelativistic QCD

Caswell and Lepage 1986

- effective field theory for $Q\bar{Q}$ sector of QCD
  at energies << M from $Q\bar{Q}$ threshold

- in quarkonium, small velocity v is generated dynamically
  by balance between potential energy
  and kinetic energy
  charmonium: $v^2 \approx 1/3$
  bottomonium: $v^2 \approx 1/10$

- nonperturbative effects
  can be organized according to their scaling with v

Nonrelativistic QCD

Soft gluon emission

- electric transitions: $\Delta L = 1, \Delta S = 0$ amplitude ~ v
- magnetic transitions: $\Delta L = 0, \Delta S = 1$ amplitude ~ v^2

Fock state expansion for quarkonium
can be organized in powers of v

$$|J/\psi\rangle = \mathcal{O}(1)\ |c\bar{c}(\underline{1}\ {}^3S_1)\rangle + \mathcal{O}(v)\ |c\bar{c}(\underline{8}\ {}^3P_J) + g\rangle + \mathcal{O}(v^2)$$

Lattice NRQCD Lepage et al. 1992

calculate properties of quarkonium nonperturbatively

NRQCD Collaboration,
HPQCD Collaboration, ...

NRQCD Factorization

Bodwin, Braaten and Lepage 1995

- apply NRQCD to
  annihilation decays/hard inclusive production of quarkonium

- motivation:
  infrared divergences for P-waves in CSM
  decays $\chi_{cJ} \rightarrow q\,\bar{q}\,g$
  production $b \rightarrow \chi_{cJ} + s + g$

- use effective field theory NRQCD
  to separate hard momentum scales (M and larger)
  from soft momentum scales ($M v$ and smaller)

- annihilation/creation of $Q\overline{Q}$ pair: hard

- evolution/formation of quarkonium: soft

NRQCD Factorization Formula

for annihilation decay rate of charmonium *H*

$$\Gamma[H] = \sum_n \hat{\Gamma}[c\bar{c}(n)] \; \langle H | \mathcal{O}_n | H \rangle$$

- sum over color/angular momentum channels

  $\underline{1}$ or $\underline{8}$ $\quad {}^1S_0, {}^3S_1, {}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2, \ldots$

- hard factors: annihilation rate into partons for $c\bar{c}$ at threshold

  expand in powers of $\alpha_s(m_c)$

- soft factors: NRQCD matrix element $\langle H \mid O_n \mid H \rangle$

  probability density at the origin

  for Fock state with $c\bar{c}$ in state n

  scales as definite power of v

- <u>rigorous</u> factorization formula

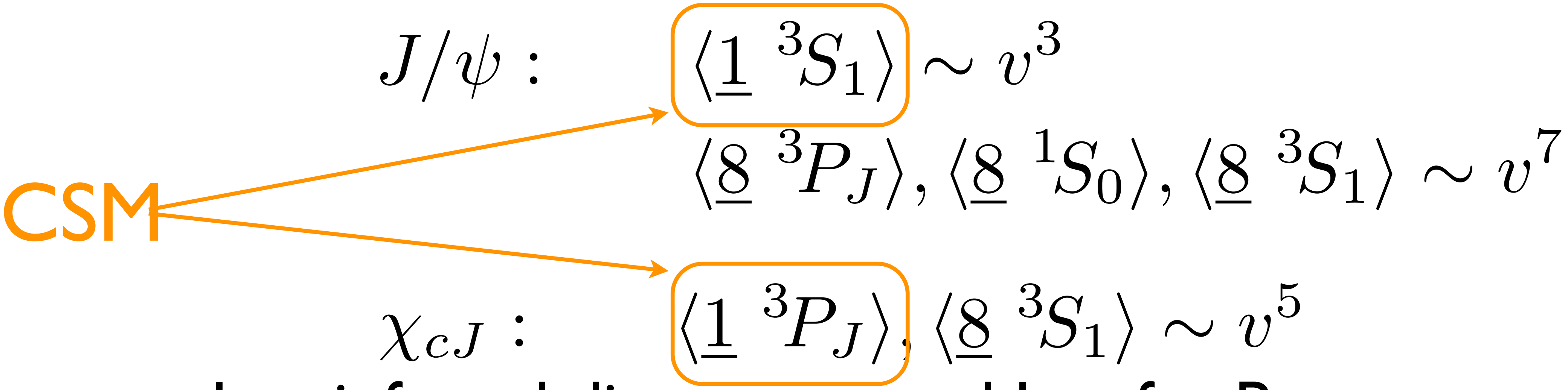
  double expansion in $\alpha_s(m_c)$ and v

NRQCD Factorization Formula

Annihilation decay rate of charmonium *H*

$$\Gamma[H] = \sum_n \hat{\Gamma}[c\bar{c}(n)]\ \langle H|\mathcal{O}_n|H\rangle$$

- velocity scaling of NRQCD matrix elements



CSM

$$J/\psi : \quad \boxed{\langle \underline{1}\ {}^3S_1\rangle} \sim v^3$$

$$\langle \underline{8}\ {}^3P_J\rangle, \langle \underline{8}\ {}^1S_0\rangle, \langle \underline{8}\ {}^3S_1\rangle \sim v^7$$

$$\chi_{cJ} : \quad \boxed{\langle \underline{1}\ {}^3P_J\rangle}, \langle \underline{8}\ {}^3S_1\rangle \sim v^5$$

- solves infrared divergence problem for P-waves

- spin symmetry relates *J/ψ*, η_c

  χ_{c0}, χ_{c1}, χ_{c2}, h_c

NRQCD Factorization

for inclusive hard production of charmonium H

- Inclusive: cross section for $H + X$
  summed over all possible hadronic states X

- hard: involves large momentum transfer
  (creation of $c\bar{c}$ pair is not be enough)

Hadron collisions:

forward (diffractive) production can create cc pair
without large momentum transfer
NRQCD factorization will not apply

production with large transverse momentum p_T
should be sufficient to use NRQCD factorization
but how large does p_T need to be?
$p_T >> \Lambda_{QCD}$? $p_T >> M v$? $p_T >> M$?

NRQCD Factorization Formula

Inclusive differential cross section for charmonium *H*

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- conjectured factorization formula
  motivated by perturbative QCD factorization theorems
- sum over color/angular momentum channels
  1 or 8 $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$, ...
- hard factors: parton cross sections for creating $c\bar{c}$
  expand in powers of $\alpha_s(m_c)$
- soft factors: NRQCD matrix element $\langle O_n^H \rangle \equiv \langle n \rangle$
  probability density at origin for $c\bar{c}$ pair in state *n*
  to bind into charmonium *H* plus soft gluons
  scale as definite powers of *v*

NRQCD Factorization

Inclusive production of charmonium *H*

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- velocity scaling of NRQCD matrix elements

CSM

$$J/\psi : \qquad \boxed{\langle \underline{1}\ {}^3S_1 \rangle} \sim v^3$$

$$\chi_{cJ} : \qquad \boxed{\langle \underline{1}\ {}^3P_J \rangle},\ \langle \underline{8}\ {}^3S_1 \rangle \sim v^5$$

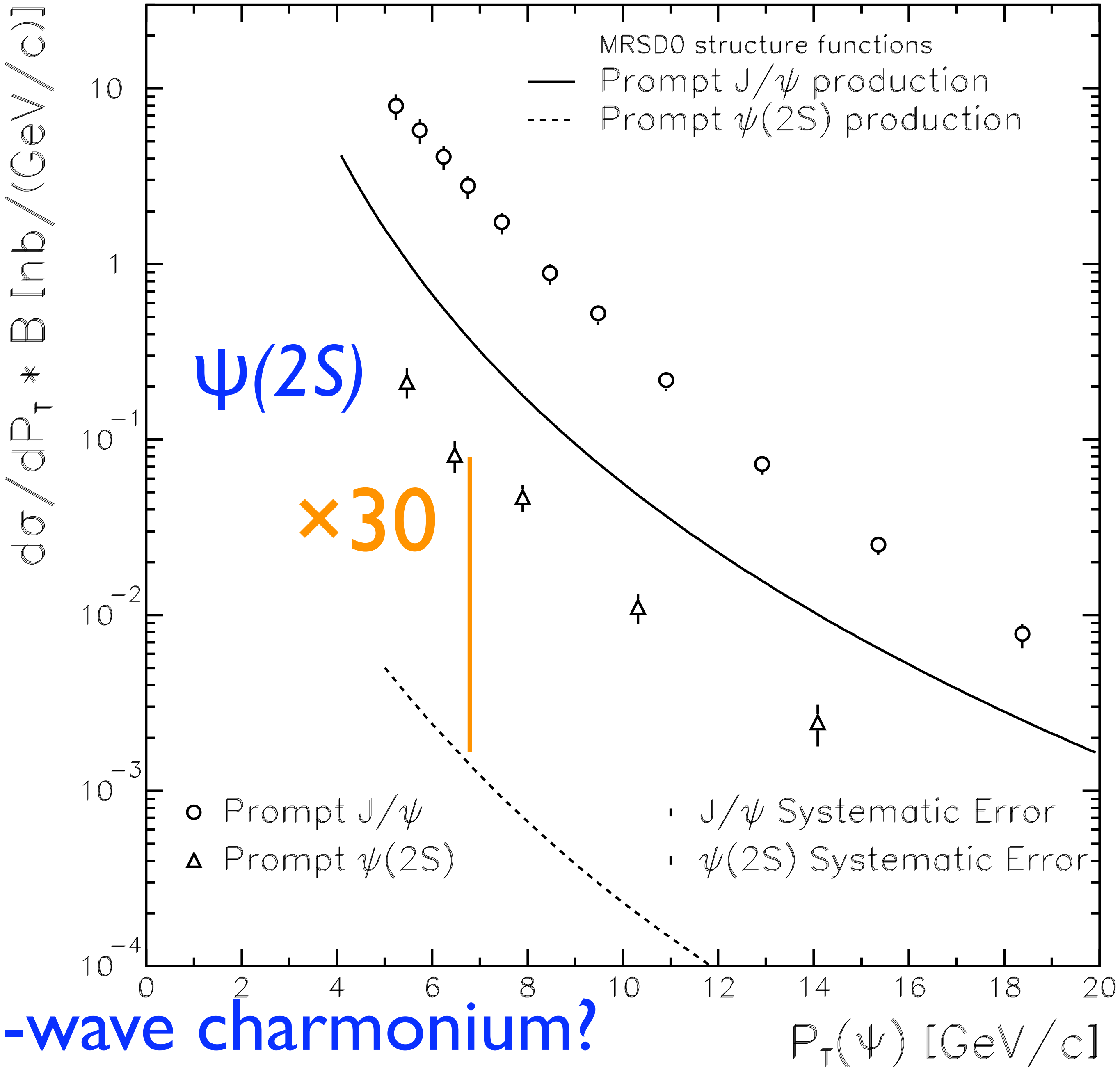
- solves infrared divergence problem for P-waves

- vacuum saturation approximation
  relates CSM matrix elements for production
  to those for annihilation decays

$\psi(2S)$ Surplus at the Tevatron

CDF collaboration

- prompt ψ(2S) is 30 times larger than CSM prediction



feeddown from P-wave or D-wave charmonium?
charmonium hybrids?

Color evaporation model?

$\psi(2S)$ Surplus at the Tevatron

- NRQCD Factorization predicts
  both color-singlet and color-octet production mechanisms

$$\psi(2S): \qquad \langle \underline{1}\ {}^3S_1 \rangle \sim v^3$$

$$\langle \underline{8}\ {}^3P_J \rangle, \langle \underline{8}\ {}^1S_0 \rangle, \langle \underline{8}\ {}^3S_1 \rangle \sim v^7$$

- at large p_T,
  CSM term $\langle \underline{1}\ {}^3S_1 \rangle$ is suppressed by α_s^2

  color-octet terms $\langle \underline{8}\ {}^1S_0 \rangle$, $\langle \underline{8}\ {}^3P_J \rangle$ are suppressed by $\alpha_s\, v^4$

  color-octet term $\langle \underline{8}\ {}^3S_1 \rangle$ is suppressed only by v^4

- proposed solution to $\psi(2S)$ surplus:
  prompt $\psi(2S)$ at large p_T at the Tevatron
  is dominated by $\langle \underline{8}\ {}^3S_1 \rangle$ term (color-octet mechanism)

Braaten and Fleming 1995

NRQCD Factorization Model

Inclusive production of charmonium *H*

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- for S-waves, truncate after order v^7

$$J/\psi: \qquad \langle \underline{1}\ {}^3S_1 \rangle \sim v^3$$

$$\langle \underline{8}\ {}^3P_J \rangle, \langle \underline{8}\ {}^1S_0 \rangle, \langle \underline{8}\ {}^3S_1 \rangle \sim v^7$$

⇒ 4 universal constants for J/ψ, η_c

(1 determined by $J/\psi \to e^+e^-$)

- for P-waves, truncate after order v^5

$$\chi_{cJ}: \qquad \langle \underline{1}\ {}^3P_J \rangle, \langle \underline{8}\ {}^3S_1 \rangle \sim v^5$$

⇒ 2 universal constants for χ_{c0}, χ_{c1}, χ_{c2}, h_c

(1 determined by $\chi_{c0} \to \gamma\gamma$)

NRQCD Factorization Model

Inclusive production of charmonium H

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- for S-waves, truncate after order v^7

$$J/\psi : \qquad \boxed{\langle \underline{1}\ {}^3S_1 \rangle} \sim v^3$$

$$\langle \underline{8}\ {}^3P_J \rangle, \boxed{\langle \underline{8}\ {}^1S_0 \rangle, \langle \underline{8}\ {}^3S_1 \rangle} \sim v^7$$

CSM

⇒ 4 universal constants for J/ψ, η_c

(1 determined by J/ψ → e^+e^-)

CEM

- for P-waves, truncate after order v^5

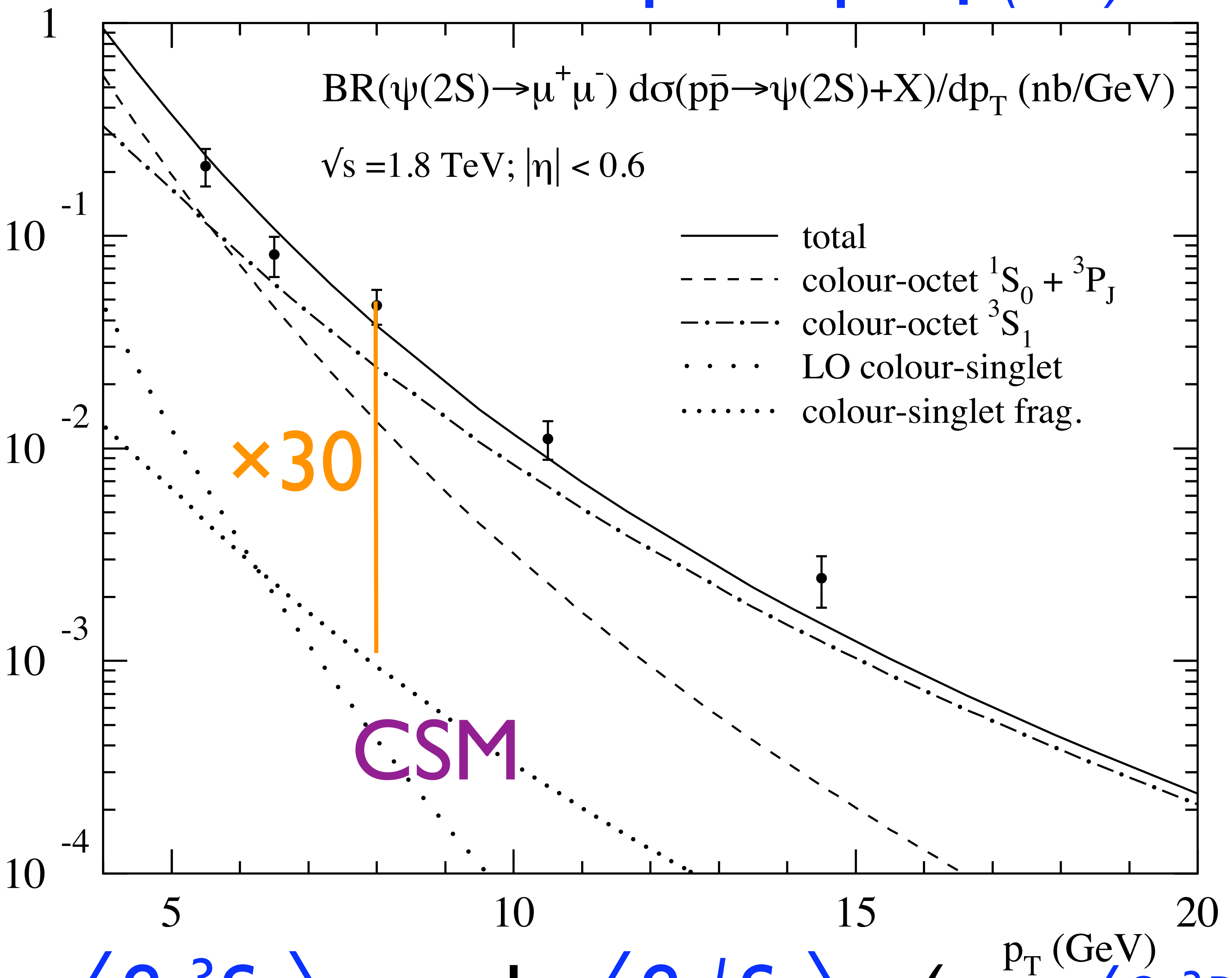
$$\chi_{cJ} : \qquad \boxed{\langle \underline{1}\ {}^3P_J \rangle}, \boxed{\langle \underline{8}\ {}^3S_1 \rangle} \sim v^5$$

⇒ 2 universal constants for χ_{c0}, χ_{c1}, χ_{c2}, h_c

(1 determined by $\chi_{c2} \to \gamma\gamma$)

NRQCD Factorization Model

CDF data on prompt ψ(2S)



fit $\langle \underline{8}\ {}^3S_1\rangle$ and $\langle \underline{8}\ {}^1S_0\rangle$ (or $\langle \underline{8}\ {}^3P_J\rangle$)

Kramer 2001

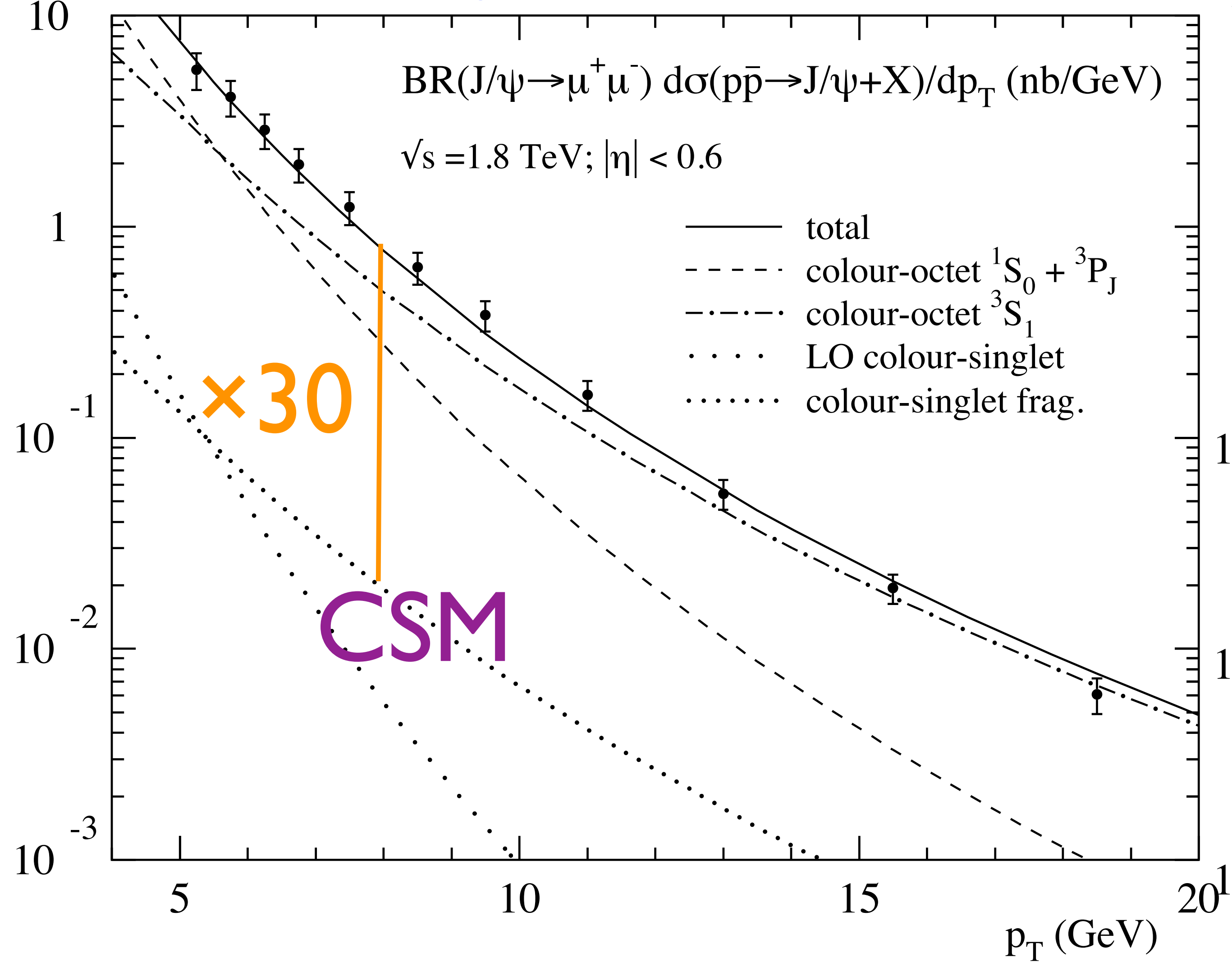
NRQCD factorization can accomodate the Tevatron data

natural explanation with sound theoretical basis

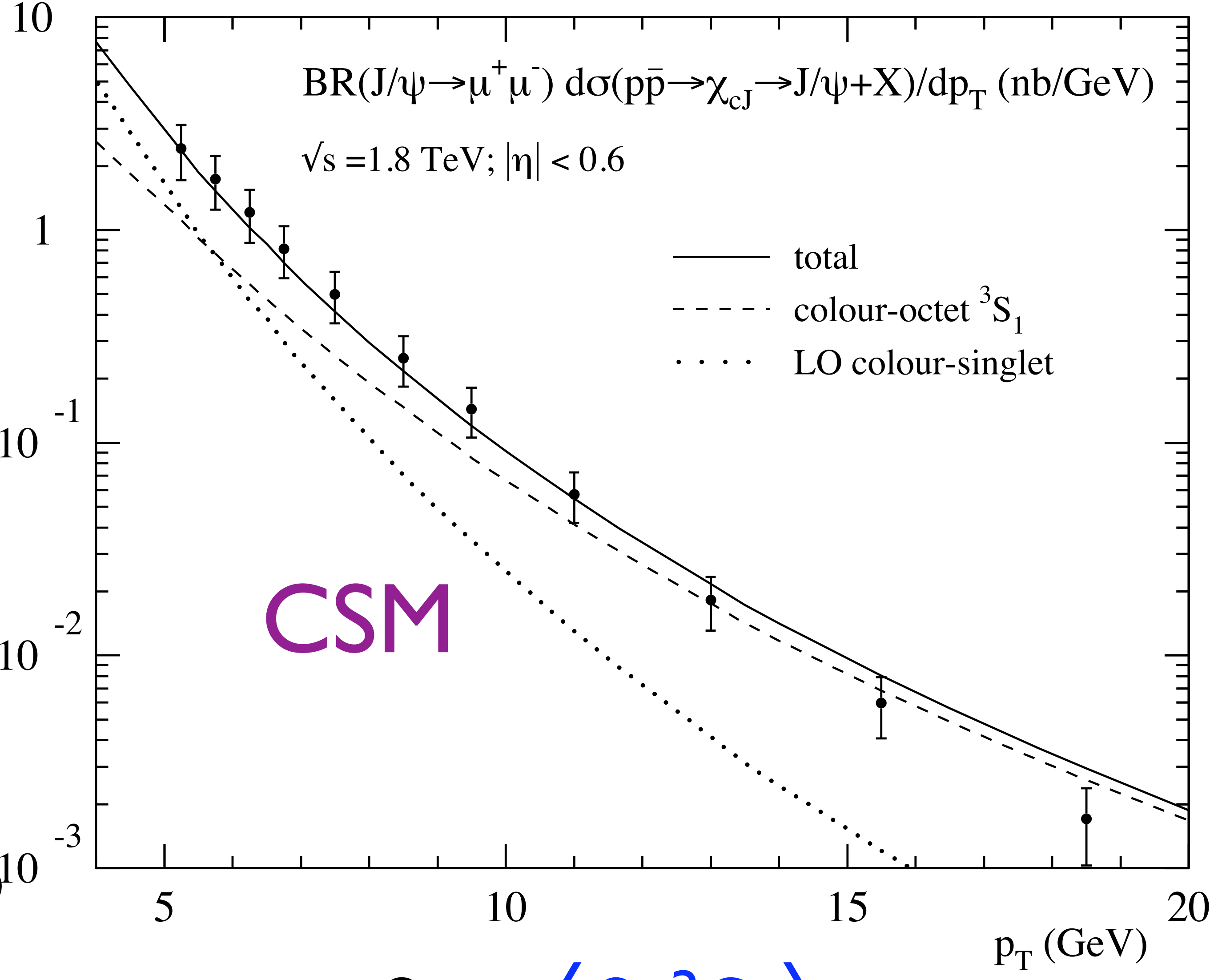
NRQCD Factorization Model

CDF data

direct J/ψ



prompt χc



fit $\langle \underline{8}\ {}^3S_1 \rangle$ and $\langle \underline{8}\ {}^1S_0 \rangle$

fit $\langle \underline{8}\ {}^3S_1 \rangle$

Kramer 2001

NRQCD factorization can accomodate the Tevatron data

natural explanation with sound theoretical basis

NRQCD Factorization:
theory of quarkonium production?

- proof of NRQCD factorization formula?

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- NRQCD matrix elements

  are they universal?

  can they be calculated using lattice QCD?

  is truncation of NRQCD factorization model adequate

- parton cross sections

  can higher orders in α_s be calculated?

Proof of NRQCD Factorization?

- NRQCD factorization formulas are conjectured
  motivated by perturbative QCD factorization theorems
  must be proven to all orders in α_s

- exclusive production

  $e^+ e^-$ → quarkonium + quarkonium
  B → (light meson) + quarkonium

  proof of factorization to all orders in α_s

  Bodwin, Lee, Tormo 2008, 2010

- inclusive production in hadron collisions

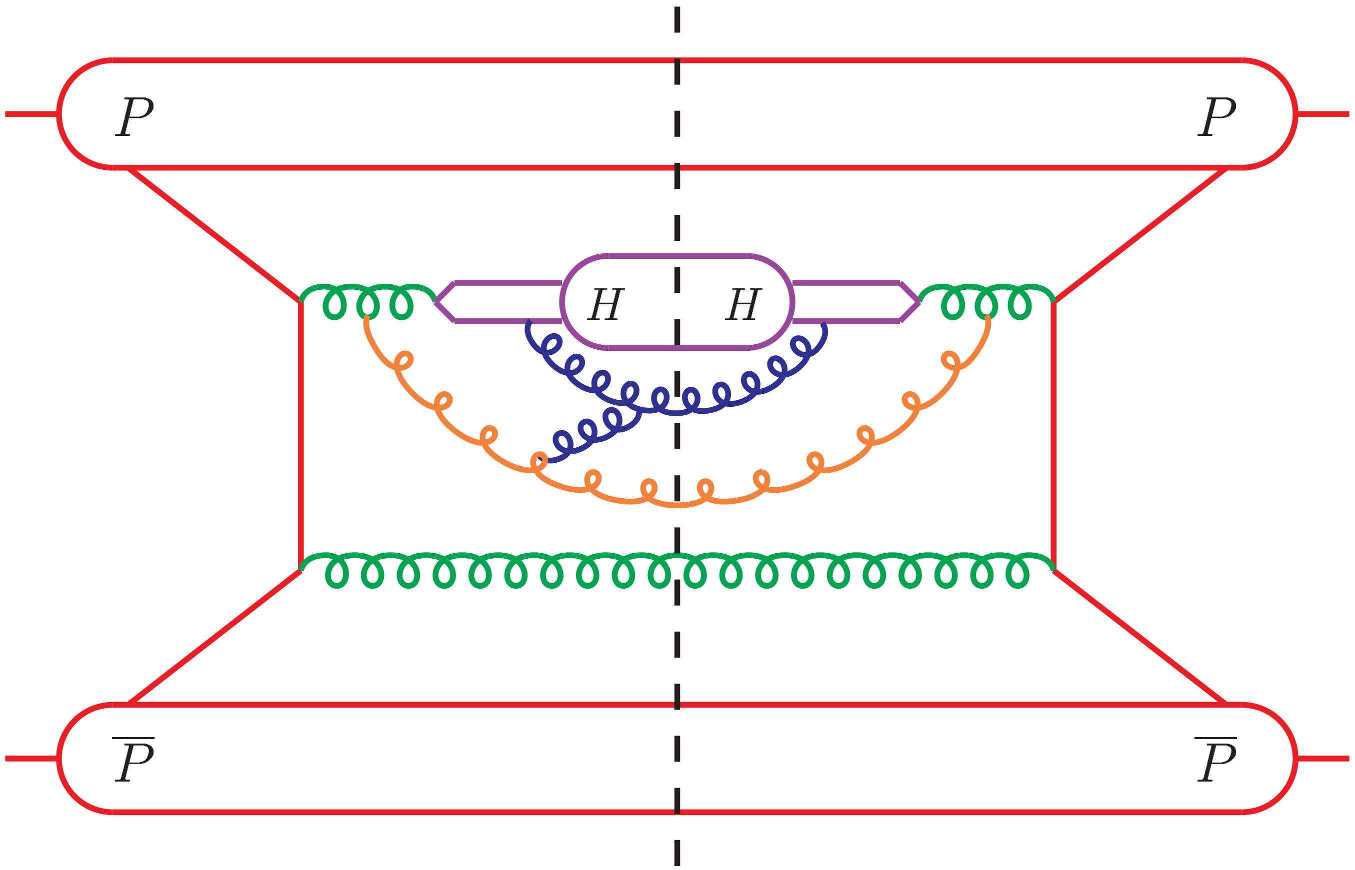
  hadron + hadron → quarkonium + *X*

  verification of factorization to N^2LO in α_s
  eikonal line required in color-octet matrix elements

  Nayak, Qiu, Sterman 2005, 2006

Proof of NRQCD Factorization? (cont.)

- Nayak, Qiu, Sterman (2005, 2006): A key difficulty in proving factorization to all orders is the treatment of gluons with momenta of order m_c in the quarkonium rest frame.



- If the orange gluon has momentum of order m_c, it can't be absorbed into the NRQCD matrix element as a quarkonium constituent.
- But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.
- The orange gluon can be treated as the eikonal-line part of the NRQCD matrix element, provided that the answer does not depend on the direction of the eikonal line (universality of the matrix elements).

- Nayak, Qiu, Sterman (2005, 2006): At two-loop order, the eikonal lines contribute but a "miracle" occurs: The dependence on the direction of the eikonal line cancels.

- In general, factorization of the inclusive cross section beyond two-loop order is still an open question.
- An all-orders proof is essential because the α_s associated with soft gluons is not small.

Proof of NRQCD Factorization? (cont.)

new development at QWG workshop in May 2010
proof of factorization to all orders in α_s at large p_T !
Jian-Wei Qiu and collaborators

- separate very hard scale $p_T >> M$ from hard scale M
  by expanding in powers of M^2/p_T^2

- at leading power: factorization (parton fragmentation)
  at order M^2/p_T^2: factorization ($Q\overline{Q}$ fragmentation)
  at higher orders: no factorization?

- parton fragmentation functions
  and $Q\overline{Q}$ fragmentation functions involve scale M
  and softer scales $\lesssim Mv$
  use NRQCD factorization to express them
  in terms of NRQCD matrix elements?

NRQCD matrix elements

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\,\boxed{\langle \mathcal{O}_n^H \rangle}$$

- are they universal?

  in absence of proof, use phenomenology

- can they be calculated using lattice QCD?

  CSM matrix elements: YES, up to $O(v^4)$

  color-octet matrix elements: NO

- truncation of NRQCD factorization model

  S-waves: $\langle \underline{1}\,{}^3S_1\rangle$, $\langle \underline{8}\,{}^3S_1\rangle$, $\langle \underline{8}\,{}^1S_0\rangle$, $\langle \underline{8}\,{}^3P_J\rangle$

  P-waves: $\langle \underline{1}\,{}^3P_J\rangle$, $\langle \underline{8}\,{}^3S_1\rangle$

  is this sufficiently accurate for charmonium? maybe

  for bottomonium? maybe

Parton cross sections

$$d\sigma[H] = \sum_n \boxed{d\hat{\sigma}[c\bar{c}(n)]} \langle \mathcal{O}_n^H \rangle$$

accurate predictions require at least NLO in α_s

for charmonium, $\alpha_s(m_c) \approx 0.25$

for bottomonium, $\alpha_s(m_b) \approx 0.18$

- photoproduction
  Kramer, Zunft, Steegborn, Zerwas 1995; Kramer 1996
  Artoisenet, Campbell, Maltoni, Tramontano 2009
  Chang, Li, Wang 2009; Li, Chao 2009
  Butenschoen, Kniehl 2009
- $\gamma\gamma$ collisions
  Klasen, Kniehl, Mihaila, Steinhauser 2005
- $e^+ e^-$ → double charmonium
  Zhang, Gao, Chao 2005; Zhang, Ma, Chao 2008
  Gong, Wang 2008
- $e^+ e^-$ → charmonium + X
  Zhang, Chao 2006; Ma, Zhang, Chao 2008
  Gong, Wang 2008, 2009
  Zhang, Ma, Wang, Chao 2009
- hadron collisions
  Petrelli, Cacciari, Greco, Maltoni, Mangano 1988
  Campbell, Maltoni, Tramontano 2008; Artoisenet, Lansberg, Maltoni, 2008
  Li, Wang 2008; Gong, Wang 2008; Gong, Li, Wang 2009

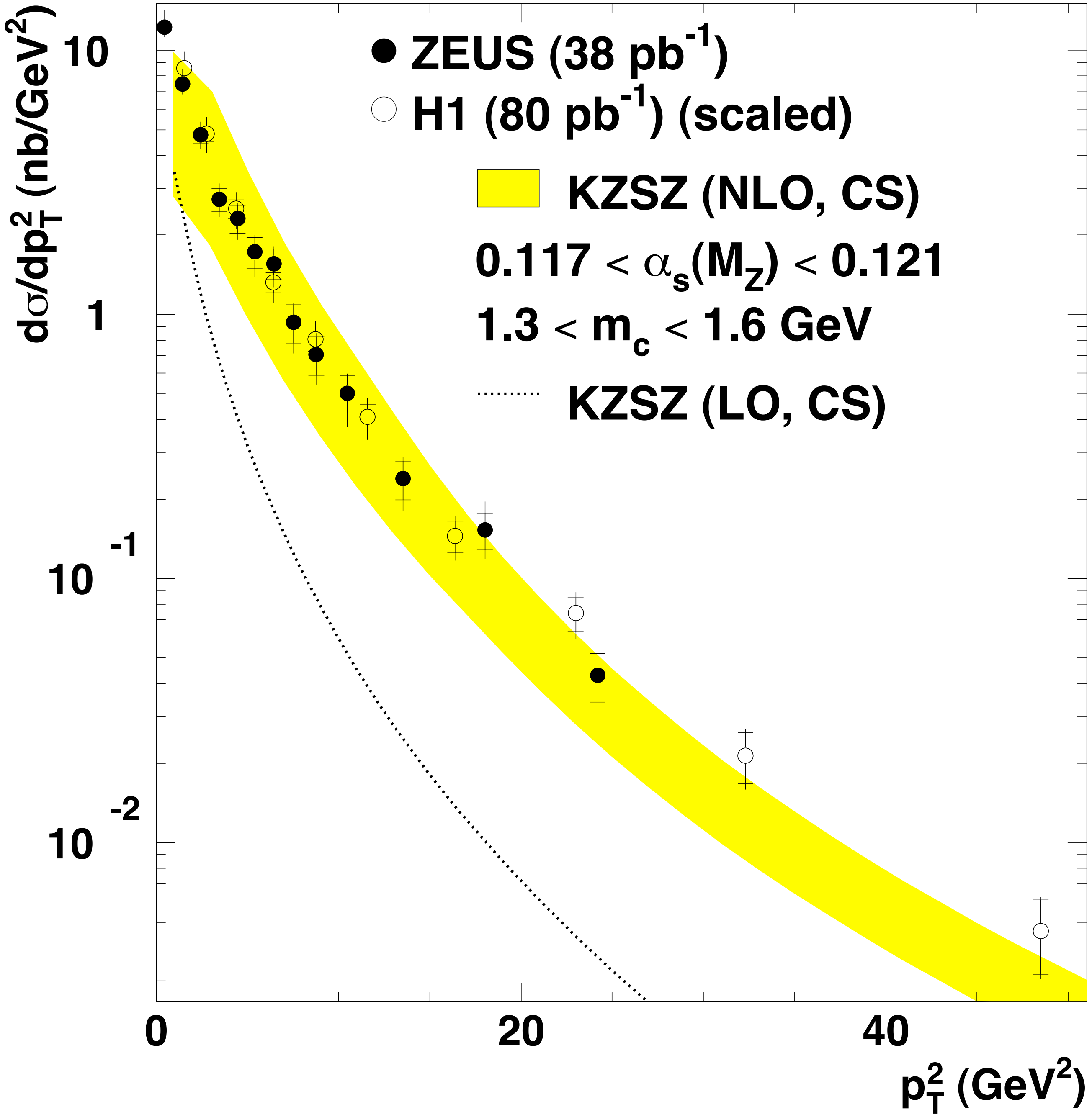
Phenomenological Status of NRQCD Factorization

- photoproduction
- $e^+ e^- \rightarrow$ double charmonium
- $e^+ e^- \rightarrow$ charmonium + X
- γγ collisions
- hadron collisions

Photoproduction

Inelastic J/ψ Photoproduction Cross Section at HERA

- It had been believed that NLO color-singlet calculations leave little room for a color-octet contribution.

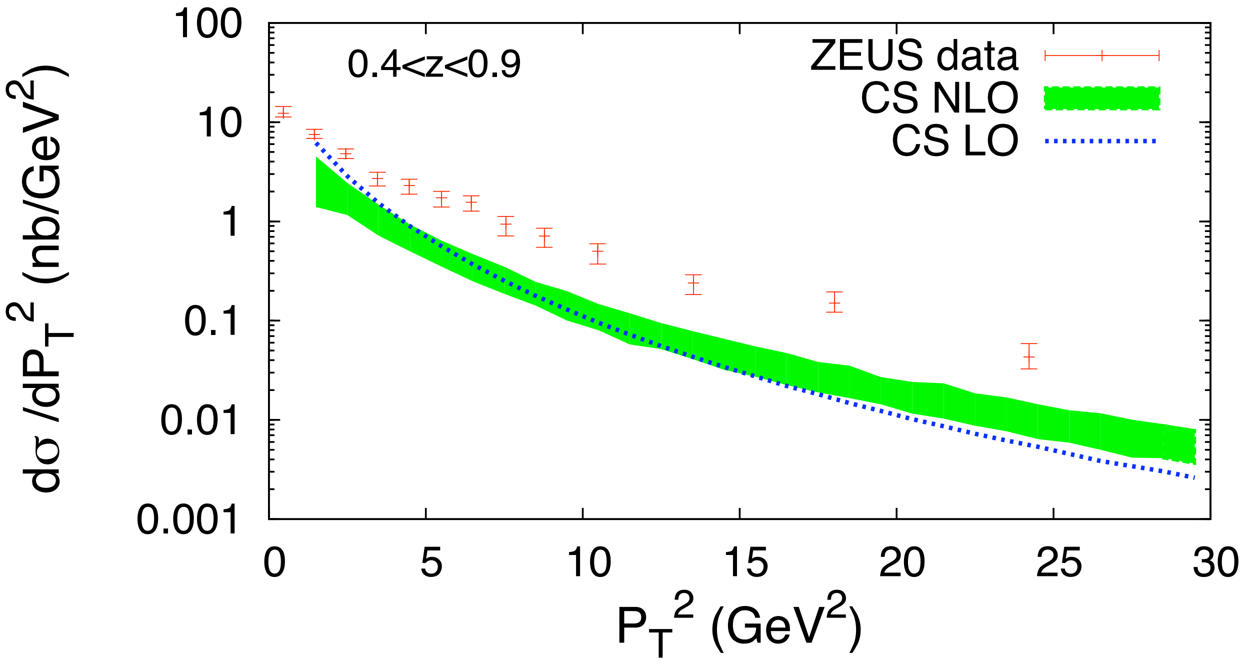


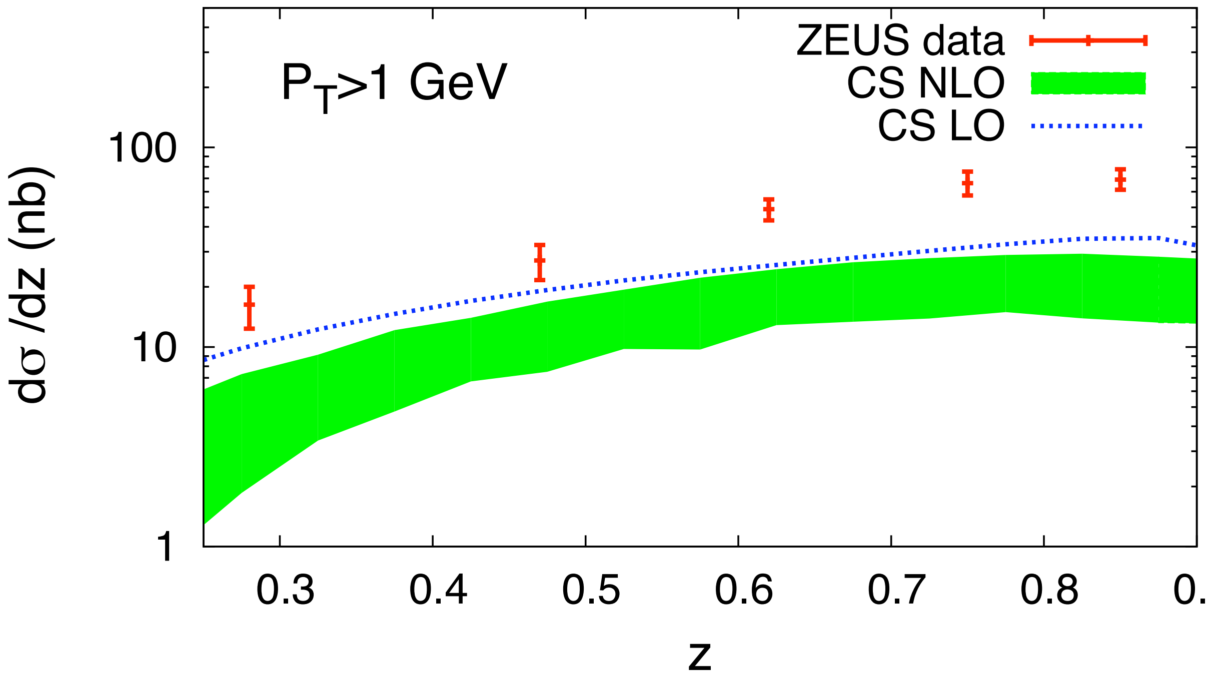
- NLO corrections increase the color-singlet contribution substantially. (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))
- NLO corrections include $\gamma + g \rightarrow (c\bar{c}) + gg$, which is dominated by t-channel gluon exchange.
- For large p_T, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.

Photoproduction (cont.)

Recent Theoretical Developments

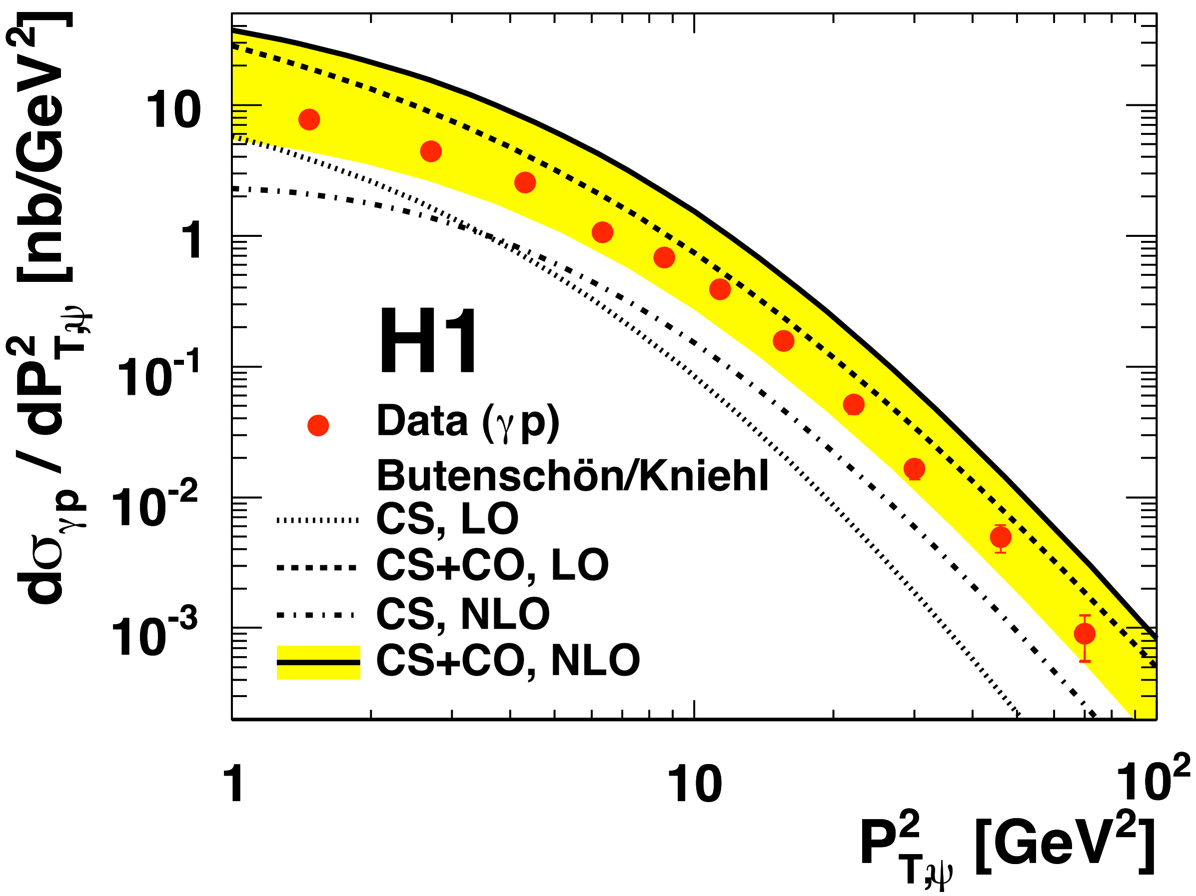
- Artoisenet, Campbell, Maltoni, Tramontano (2009): A new calculation of NLO color-singlet contribution
  - Confirms the analytic results of previous calculations.
  - But a more reasonable choice of renormalization/factorization scale ($\sqrt{4m_c^2 + p_T^2}$ instead of $m_c/\sqrt{2}$) yields much smaller numerical results for cross sections.

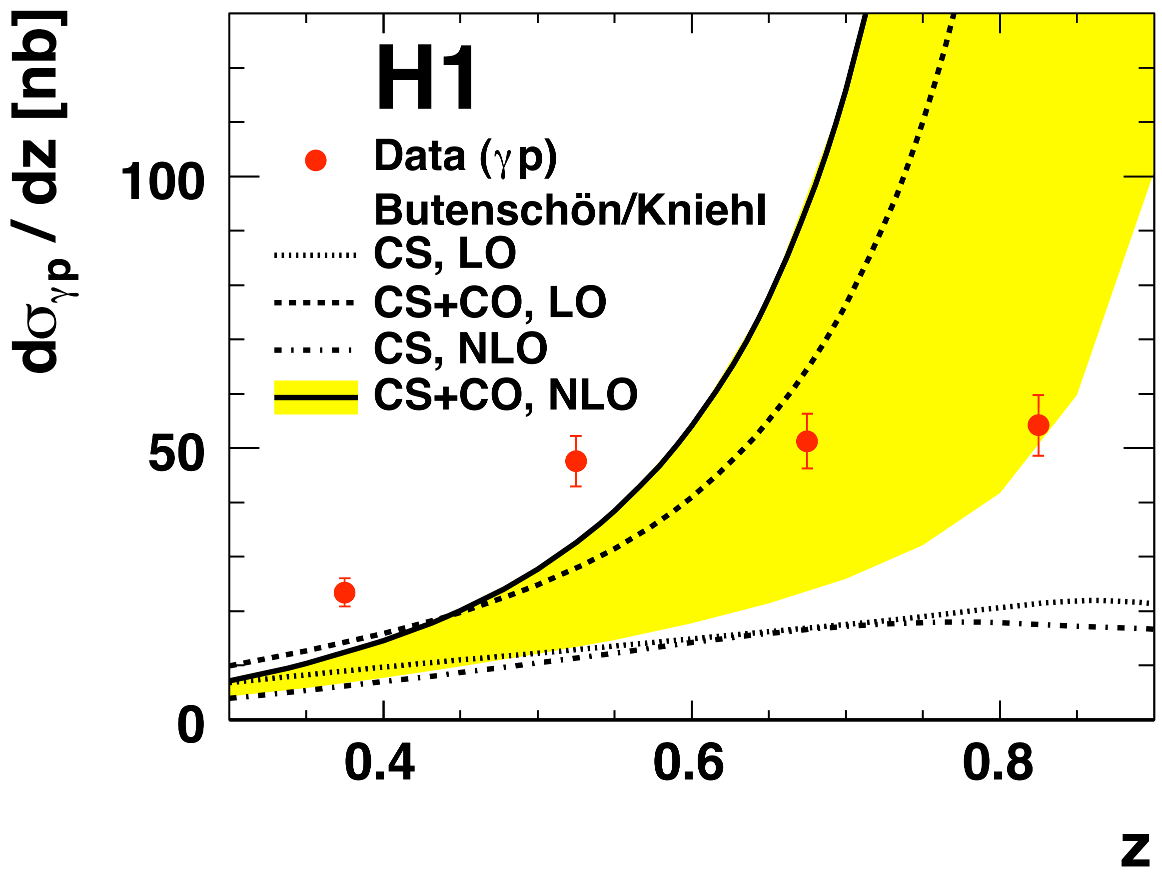




- Leaves room for a color-octet contribution.
- There is no longer an obvious conflict between the NRQCD prediction and the HERA data.

- Butenschön and Kniehl (2009) have carried out a complete NLO calculation of photoproduction in NRQCD factorization, including both color-octet and color-singlet contributions.





- The comparison of the H1 (2009) data with the Butenschön and Kniehl calculation strongly favors NRQCD factorization over the color-singlet model.

Double charmonium from e^+e^-

Exclusive Double-Charmonium Production at Belle and BABAR

$$e^+e^- \to J/\psi + \eta_c$$

- Experiment

  Belle (2004): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.

  BABAR (2005): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

larger by ≈5

- NRQCD at LO in α_s and v

  Braaten, Lee (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26$ fb.

  Liu, He, Chao (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5$ fb.

  The two calculations employ different choices of m_c, NRQCD matrix elements, and α_s.

  Braaten and Lee include QED effects.

  Confirmed by Brodsky, Ji, and Lee in light-front QCD in the quarkonium nonrelativistic limit.

- Exclusive process: the color-octet contribution is suppressed as v^4.

- The LO color-singlet matrix elements are determined from $\eta_c \to \gamma\gamma$ and $J/\psi \to e^+e^-$.

Double charmonium from e^+e^-

α_s Corrections to $e^+e^- \to J/\psi + \eta_c$

- An important step in resolving the discrepancy:
  Zhang, Gao, Chao (2005) found that corrections at NLO in α_s yield a K factor of about 1.96.
- Confirmed by Gong and Wang (2007).
- Not enough to bring theory into agreement with experiment.

relativistic corrections:

×1.5?

- Theory and experiment agree within uncertainties:
  - Theory: $\sigma[e^+e^- \to J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7}\ \mathrm{fb}$
  - Belle: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  - BABAR: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.
- *Caveat:* $B_{>2}$ is not known.
  - Could be as small as 0.5–0.6.
  - Even so, the error bars of theory and the BABAR experiment overlap.
- Zhang, Ma, Chao (2008): In the cases of $\sigma[e^+e^- \to J/\psi(\psi(2S)) + \chi_{c0}]$, large K factors (~ 2.8) may bring theory into agreement with experiment.

Inclusive Double $c\bar{c}$ Production at Belle

- Belle (2002):

$$\boxed{\frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]} = 0.59^{+0.15}_{-0.13} \pm 0.12}$$

- pQCD plus color-singlet model (Cho, Leibovich (1996); Baek, Ko, Lee, Song (1997); Yuan, Qiao, Chao (1997)):

$$\boxed{\frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]} \approx 0.1}$$

- There is a significant disagreement between experiment and the LO color-singlet model.

Inclusive J/ψ + charm from e^+e^- (cont.)

NLO corrections: Zhang , Chao 2007; Gong, Wang 2009
Ma, Zhang, and Chao 2008; Gong, Wang 2009

Effect of NLO calculations on the ratio

- NLO calculations significantly reduce the discrepancy between theory and experiment for the ratio of cross sections:

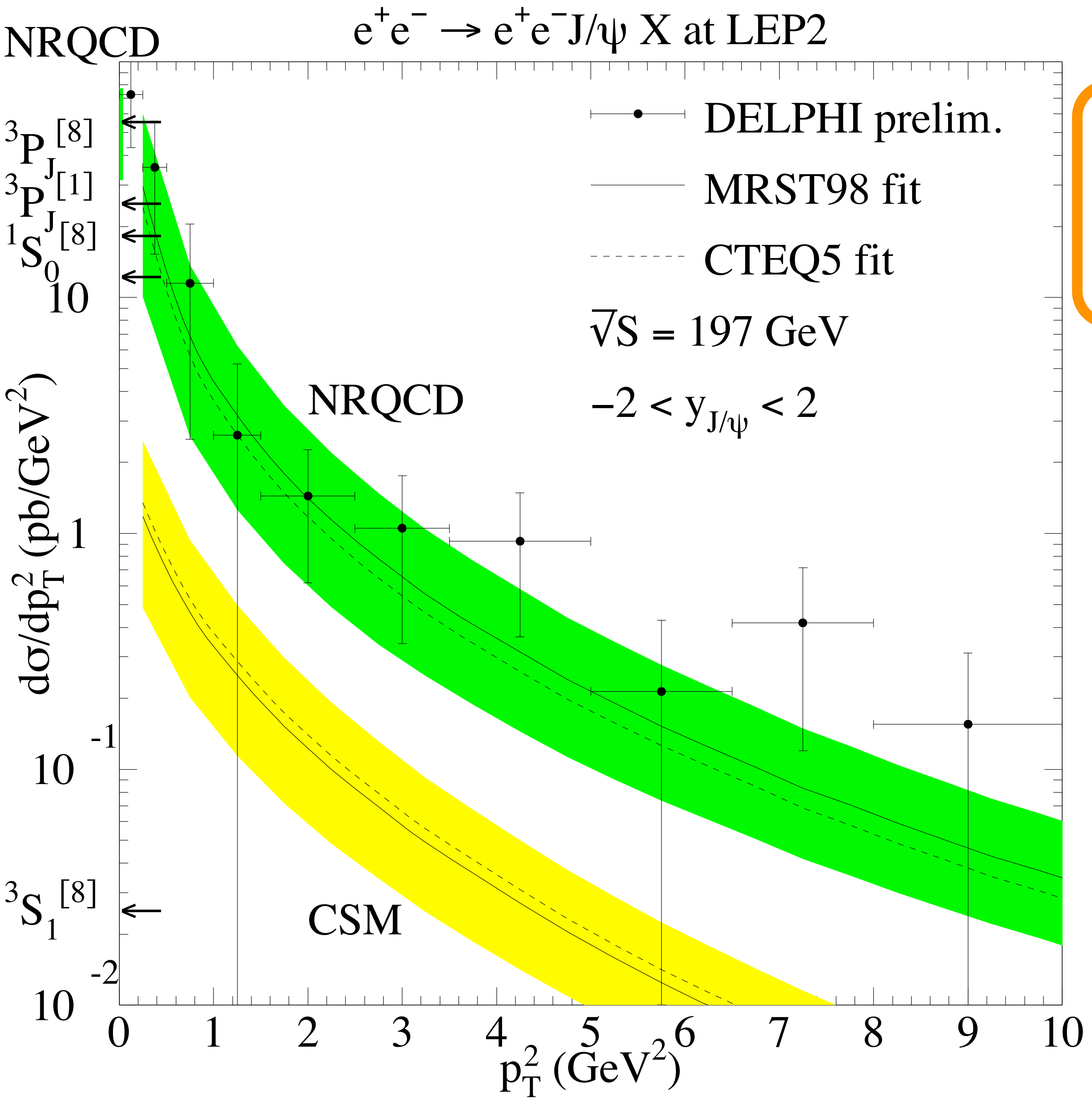
$$\frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]} \approx 0.5$$

  - Only color-singlet contributions are included.
- No longer an apparent disagreement between experiment and color-singlet theory.
- It would be good to have a detailed error analysis for the theoretical prediction.
- It is important for BABAR to check the Belle results for inclusive double-$c\bar{c}$ production.

Belle: $$\frac{\sigma[e^+e^- \to J/\psi + c\bar{c} + X]}{\sigma[e^+e^- \to J/\psi + X]} = 0.59^{+0.15}_{-0.13} \pm 0.12$$

Inclusive J/ψ from $\gamma\gamma$

$\gamma\gamma \to J/\psi + X$ at LEP



- Comparison of theory (Klasen, Kniehl, Mihaila, Steinhauser) with Delphi data clearly favors NRQCD over the color-singlet model.
- Theory uses Braaten-Kniehl-Lee matrix elements from Tevatron data and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.

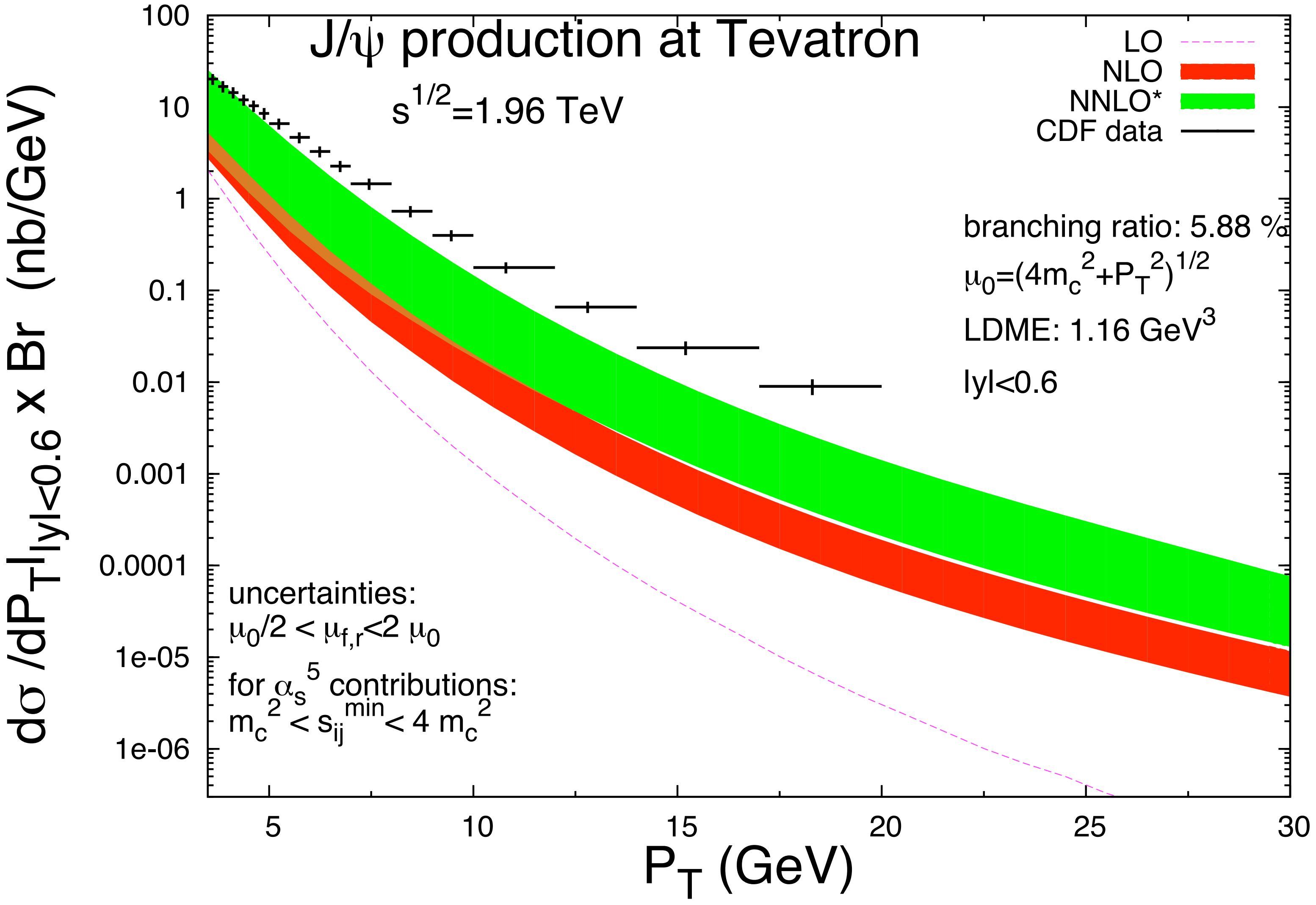
J/ψ Production in DIS at HERA

- Note that NLO calculations are not yet available for this process.
- The NRQCD (Kniehl, Zwirner (2001)) prediction uses Braaten-Kniehl-Lee (1999) matrix elements extracted from the Tevatron data and MRST98LO and CTEQ5L PDF's.
- Theoretical uncertainties from
  - PDF's,
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.

Inclusive quarkonium at Tevatron

New Results for J/ψ Production

- Color-singlet contribution:

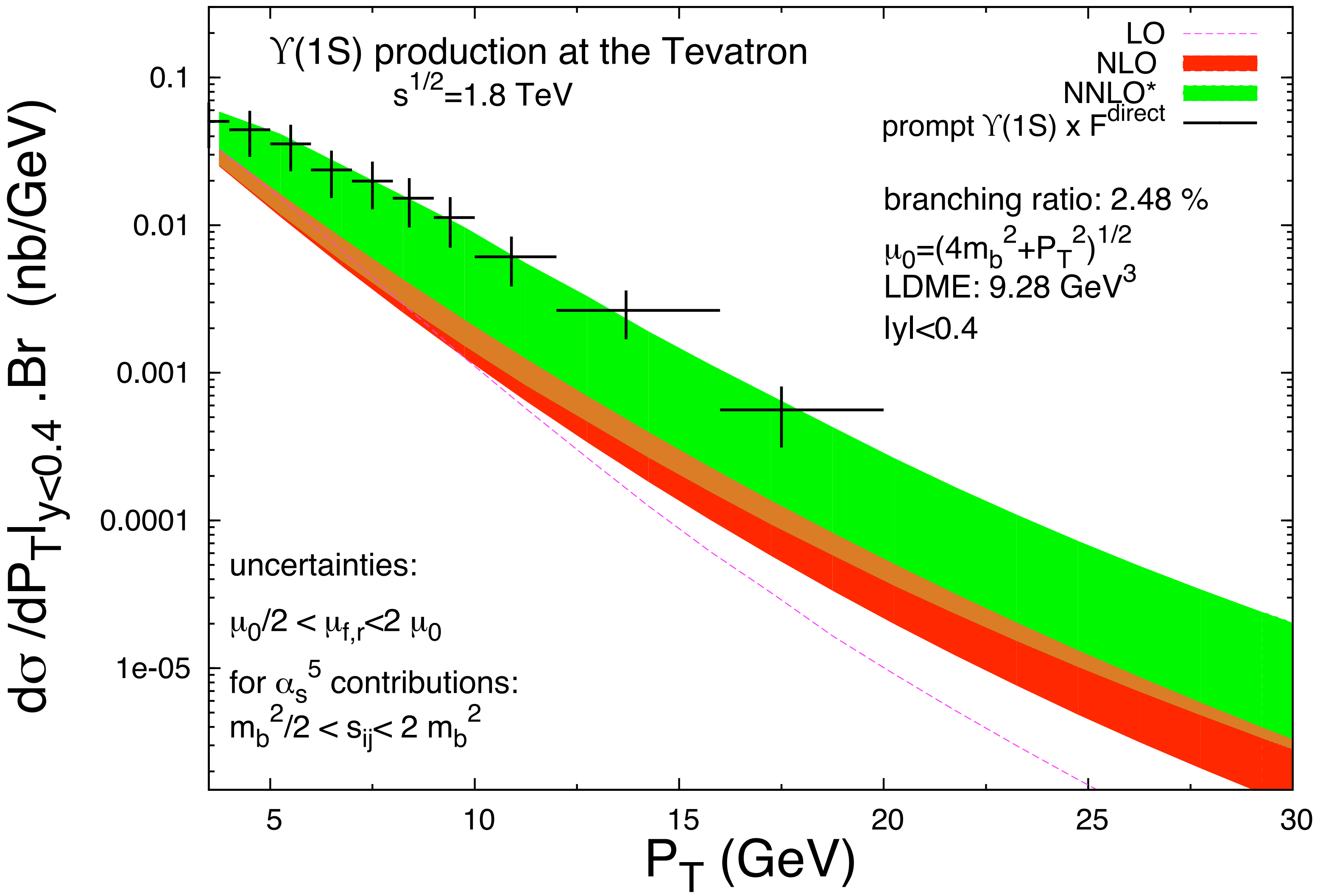


- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (in progress)
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution, but its size may be reduced from previous estimates. Affects the matrix elements used to compute all other processes.

- Color-octet contribution:
  NLO corrections are about 14% (Gong, Li, and Wang (2008)).

Inclusive quarkonium at Tevatron

New Results for Color-Singlet Υ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).

- The data could be explained by color-singlet production alone.
- There is still room for a substantial amount of color-octet production.
- Color-octet production is suppressed as v^4.
  Should be smaller for Υ $(v^2 \approx 0.1)$ than for J/ψ $(v^2 \approx 0.3)$.

Inclusive quarkonium at Tevatron

Are perturbative QCD calculations beyond NLO necessary?

Are NLO corrections to color-octet processes
sufficient to reproduce data? see talk by Yin-Qing Ma

new development at QWG workshop in May 2010!
large-p_T factorization Jian-Wei Qiu et al.

- expand in powers of M^2/p_T^2
  to separate scales M and p_T

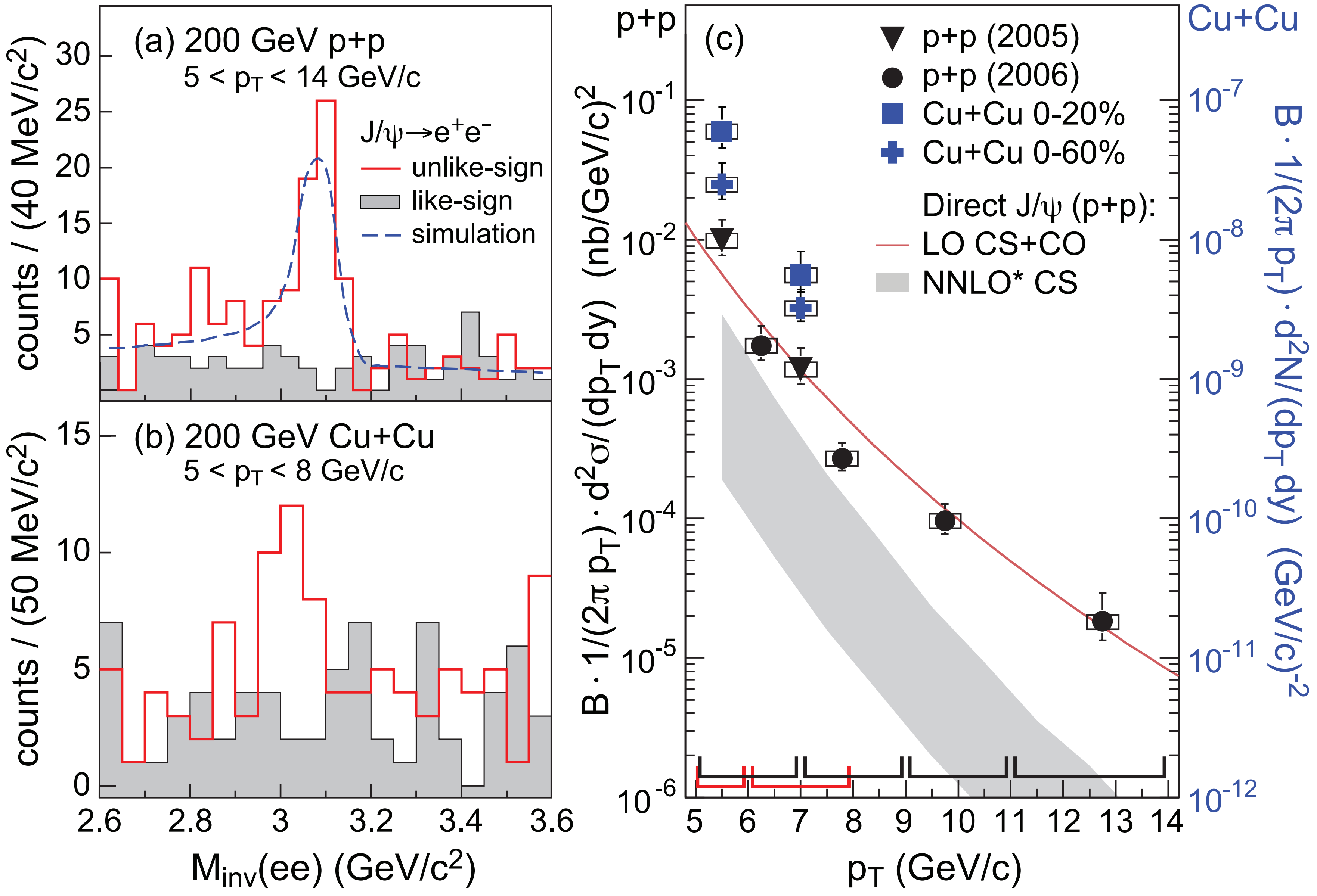
- at leading power: parton fragmentation
  at order M^2/p_T^2: $Q\bar{Q}$ fragmentation

- Will predictions including $Q\bar{Q}$ fragmentation at LO
  agree with data?
  Will they improve the convergence of perturbation theory?

Inclusive charmonium at RHIC

J/ψ Production at RHIC

- The STAR Collaboration has measured the J/ψ p_T distributions in $p+p$ and Cu+Cu collisions:



- The LO color-singlet plus color-octet calculation (Nayak, Liu, Cooper (2003)) fits the data well.
  - Does not include feeddown from $\psi(2S)$, χ_c, or B decays. (Estimated to be a factor 1.5.)

Polarization

NRQCD factorization

predicts the polarization of quarkonium
with no additional parameters

dramatic qualitative prediction for hadron collisions:

direct J/ψ, Υ transversely polarised at large p_T

Cho, Wise 1995

- at sufficiently large p_T, charmonium production is dominated by gluon fragmentation

$$g + g \to g^* + g$$

- at LO in α_s, gluon fragments into color-octet $c\bar{c}$ pair that inherits transverse polarization of gluon

$$g^*\Uparrow \to c\bar{c}(\underline{8}\;{}^3S_1)\Uparrow$$

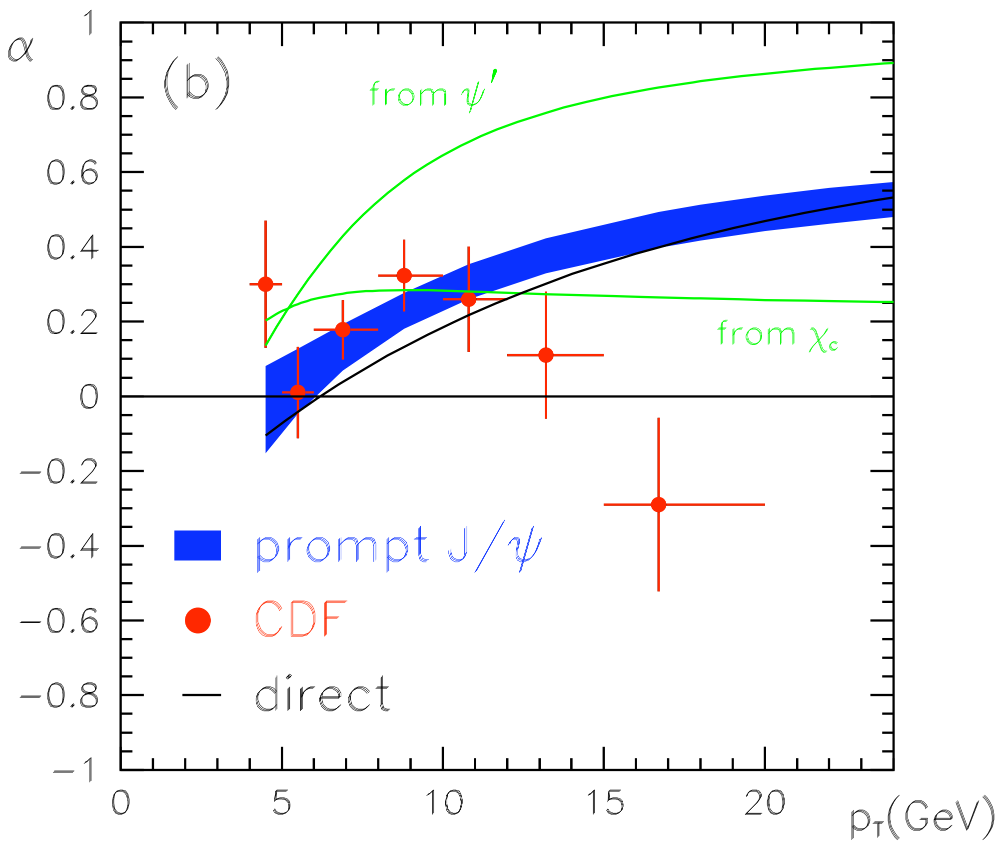
- at LO in v, hadronization into ${}^3S_1$ charmonium preserves transverse polarization of $c\bar{c}$ pair

$$\underline{cc}(\underline{8}\;{}^3S_1)\Uparrow \to J/\psi\Uparrow + X$$

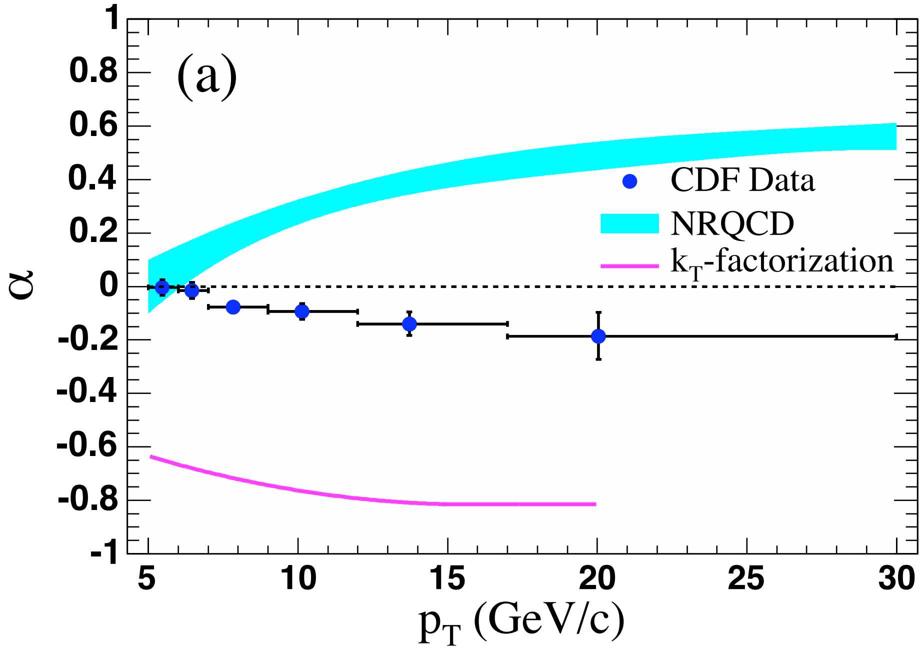
Polarization (cont.)

J/ψ Polarization

Run I:



Run II:

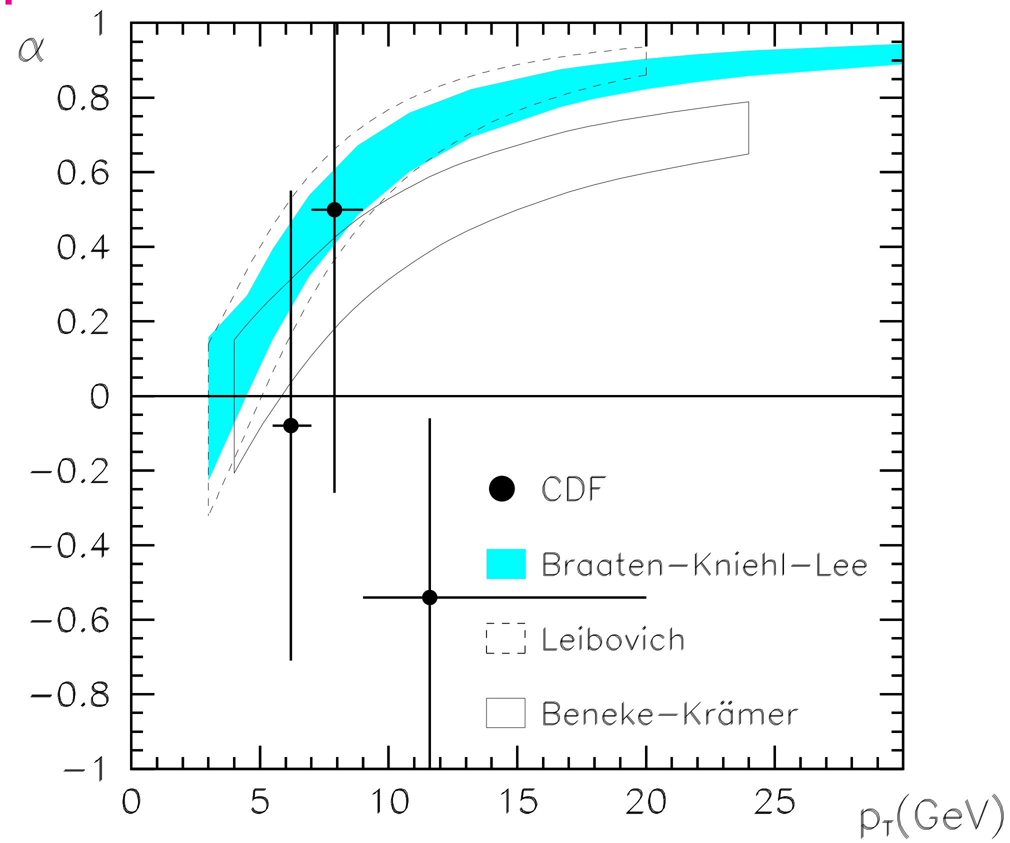


- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feedown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also, inconsistent with Run I results. CDF was unable to track down the source of the Run I-Run II discrepancy.

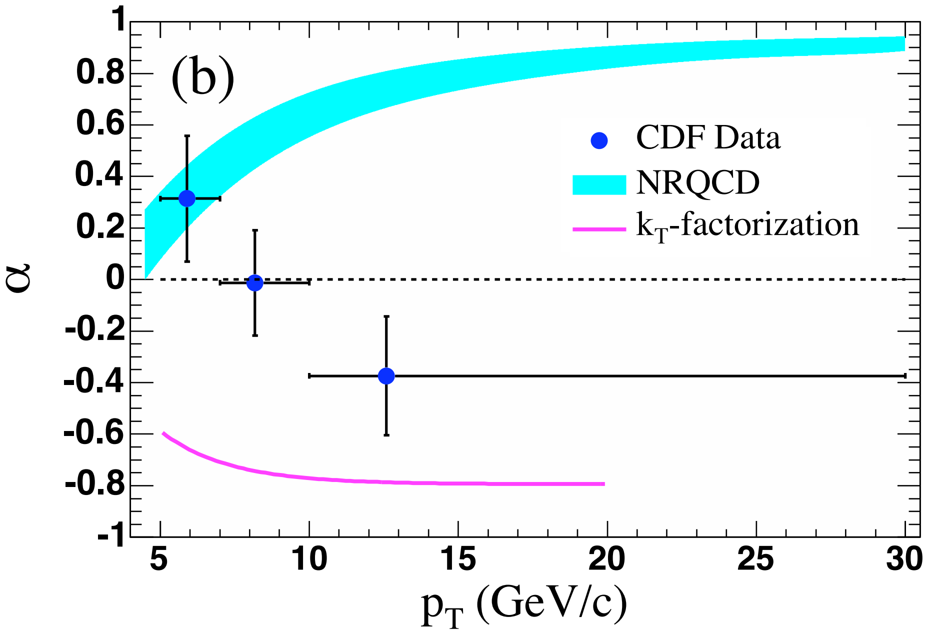
Polarization (cont.)

$\psi(2S)$ Polarization

Run: I



Run: II

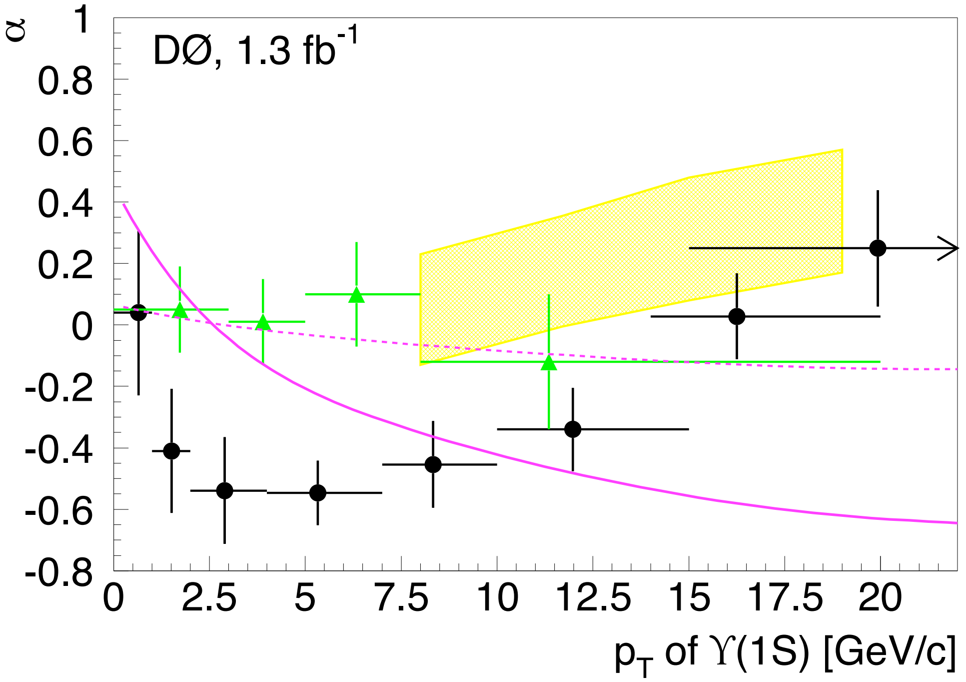


- The Run II data are incompatible with the NRQCD prediction.

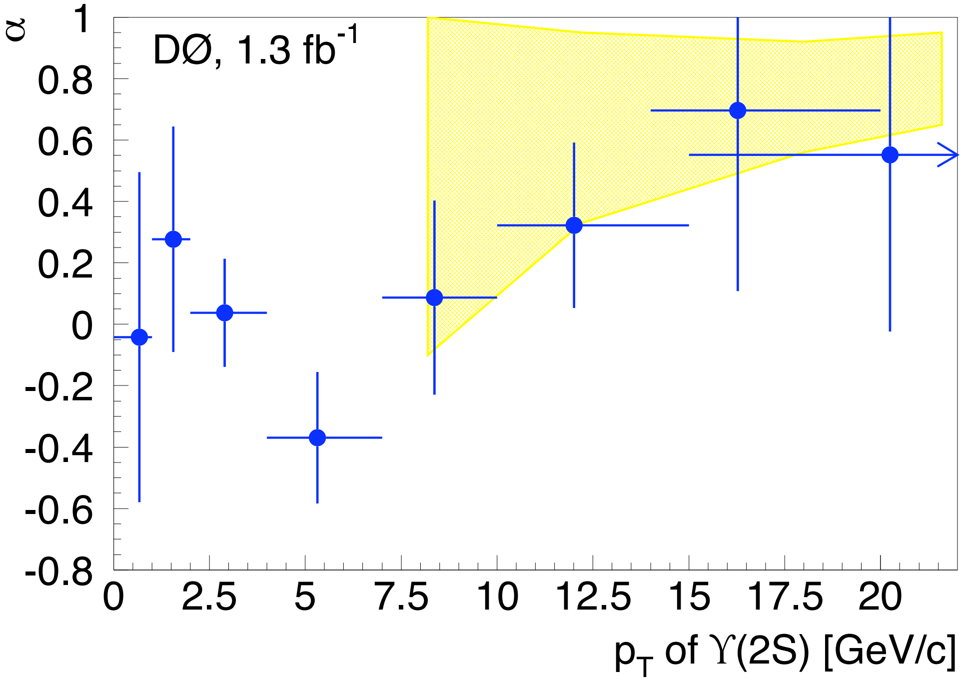
Polarization (cont.)

Υ Polarization

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:



- In the $\Upsilon(1S)$ case, the D0 results (black) are incompatible with the CDF results (green).
- The CDF results are compatible with the NRQCD prediction (yellow).
- The D0 results are marginally incompatible with the NRQCD prediction.
- The curves are the limiting cases of the k_T-factorization prediction.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

Polarization (cont.)

Does NRQCD factorization fail for polarization?

Are NLO corrections to color-octet processes sufficient
to give observed polarization? see talk by Yin-Qing Ma

new development at QWG workshop in May 2010!
large-p_T factorization Jian-Wei Qiu et al.

- expand in powers of M^2/p_T^2
  to separate scales M and p_T

- at leading power: parton fragmentation ⇒ T

  at order M^2/p_T^2: $Q\overline{Q}$ fragmentation ⇒ L

- Will predictions including $Q\overline{Q}$ fragmentation at LO
  give observed polarization?

Stumbling towards a Theory of Quarkonium Production

Color-singlet model (1976-1995)
Color evaporation model (1977-?)

NRQCD factorization

- still a viable theory of quarkonium production!
- exclusive quarkonium: proven to all orders
- inclusive quarkonium: verified to NNLO
- can it be combined with large-p_T factorization?

NRQCD factorization model

S-wave multiplets: 3 color-octet parameters
P-wave multiplets: 1 color-octet parameter

- still a viable model of charmonium production
  bottomonium production

Stumbling towards a Theory of Quarkonium Production

NLO perturbative QCD corrections

- have removed most dramatic discrepancies between NRQCD factorization and experiment
- polarization remains a question

Large-p_T factorization Jian-Wei Qiu et al.

- separates hardest scale p_T from scales $\lesssim M$
- introduces new mechanism: $Q\overline{Q}$ fragmentation
- fators involving scales $\lesssim M$ are fragmentation functions
- still requires NRQCD factorization to reduce fragmentation functions to a few constants

Experimental outlook

- final results from B factories (Belle, Babar)
  DESY (H1, Zeus)
  Tevatron (CDF, D0)

- additional results from RHIC

- first results from LHC experiments
  extend charmonium out to fragmentation region
  high statistics measurements of bottomonium

- additional results from RHIC

- future results from super-B factories

Will NRQCD factorization
remain a viable theory of quarkonium production?