New Physics Contribution to Neutral Trilinear Gauge Boson Couplings

Mamta

Department of Physics
S.G.T.B. Khalsa College
University of Delhi
Delhi, India

Collaborators

Ashok Goyal
Sukanta Dutta

LCWS10, Beijing (March 26–30, 2010)

Plan of the Talk

- Introduction
- Generic Structure
- One Loop Contribution to TNGBCs
- SM, MSSM Contribution Revisited
- Little Higgs Models
- Results and Discussion
- Summary

- Based on

- **S. Dutta, A. Goyal and Mamta**
  Eur.Phys.J. **C63** *305-315, 2009*

Introduction

- The high precision measurements of LEP, SLC and the Tevatron have firmly established that the SM based on $SU(3)_C \times SU(2)_L \times U(1)_Y$ correctly describes the electroweak and strong interactions of quarks and leptons.

- However, many important aspects of the model, besides the electroweak symmetry breaking mechanism for particle mass generation, need more investigation.

- There is a need to measure the interactions amongst gauge bosons much more precisely than it was possible at LEP and the Tevatron.

- The couplings among the electroweak gauge bosons are directly given by the structure of the gauge group. This structure can thus be directly determined by a measurement of the gauge boson interactions.

Introduction

- One of the important properties of the Standard Model is its non-Abelian character, leading to the prediction of triple-gauge-boson couplings.

- While non-zero values of these couplings are predicted for the charged ($WW\gamma$, WWZ) sector, $SU(2)_L \times U(1)_Y-$ symmetry of the Standard Model predicts the absence of such couplings in the neutral sector, namely at the ZZZ, $ZZ\gamma$ and $Z\gamma\gamma$ vertices.

- The precise values of the anamolous TNGBC's possible at the clean environment of $e^+ e^-$ collider and at high $\sqrt{s}$.

- The TNGBC's where only one of the gauge boson is off-shell may be studied at in the reactions $e^+ e^- \to Z\gamma$ and $e^+ e^- \to ZZ$ with final state consisting of two on-shell bosons.

Generic Structure

TNGBC's DO NOT exist in SM, at tree level.
Anomalous Couplings may lead to vertices ZZZ, $ZZ\gamma$ and $Z\gamma\gamma$

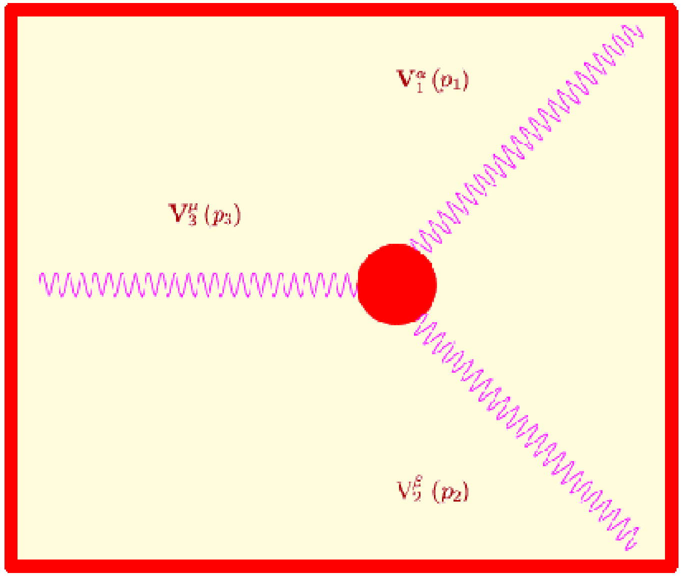

Figure 1: *The Triple Neutral Gauge Boson Vertex* $\Gamma^{V_1\,V_2\,V_3}_{\alpha\beta\mu}(p_1,\ p_2,\ p_3)$.

$$V_i = \gamma/Z \quad \text{and} \quad p_1 + p_2 + p_3 = 0$$

Comments

- Non vanishing TNGB interactions contribute to V_1V_2 production via a photon or Z in s-channel.

- Bose Symmetry and em gauge invariance restricts the form of interactions to a smaller number of couplings/parameters than in the WWV case.

- Bose-Einstein's statistics – TNGBV's vanish when all three neutral gauge bosons are on shell.

- Described by 12 parameters using the most general Lagrangian that is Lorentz invariant, $U(1)_{\rm em}$ invarinat and imposing Bose symmetry.

- h_1^γ, h_2^γ, h_1^Z, h_2^Z – describe the CP-violating contribution in $e^+e^- \to \gamma/Z \to Z\gamma$ while f_4^γ, f_4^Z in $e^+e^- \to \gamma/Z \to ZZ$.

- Parameters for CP-conserving contributions : h_3^γ, h_4^γ, h_3^Z, h_4^Z, f_5^γ and f_5^Z.

- In SM all these parameters are ZERO at tree level.

Neutral Gauge Boson Coupings

The most general CP conserving coupling of one off-shell boson $\mathrm{V} \equiv Z/\gamma$ carrying momentum Q to a pair of on-shell $Z\gamma$ and ZZ gauge bosons (all incoming) can be written as

$$\Gamma^{\mu\alpha\beta}_{\mathrm{V}Z\gamma}(Q,p_1,p_2) = i\left[\mathcal{H}^{\mathrm{V}}_3 \epsilon^{\mu\beta\alpha\eta}\, p_{2_\eta} + \frac{\mathcal{H}^{\mathrm{V}}_4}{M^2_Z}\left\{\epsilon^{\mu\beta\rho\eta}\, p_{2_\rho}\, Q_\eta\, Q^\alpha\right\}\right] \quad (1a)$$

$$\Gamma^{\mu\alpha\beta}_{\mathrm{V}ZZ}(Q,p_1,p_2) = i\left[\mathcal{F}^{\mathrm{V}}_5 \epsilon^{\mu\alpha\beta\sigma}\, (p_1 - p_2)_\eta\right] \quad (1b)$$

where

$$\mathcal{H}^{\gamma}_i = \frac{Q^2}{m^2_Z} h^{\gamma}_i,\; \mathcal{H}^{Z}_i = \frac{m^2_Z - Q^2}{m^2_Z} h^{Z}_i\; \{i = 3,\, 4\} \quad (1c)$$

$$\mathcal{F}^{\mathrm{V}}_5 = -\frac{Q^2 - m^2_{\mathrm{V}}}{m^2_Z}\, f^{V}_5 \quad (1d)$$

Neutral Gauge Boson Coupings

- In the SM all these couplings vanish at the tree level.
- Can however, be generated at the loop level.
- On account of the totally antisymmetric nature of $\epsilon^{\mu\alpha\beta\sigma}$, these couplings can never be generated by scalars and vector-bosons running in the loop.
- Only fermions running in the loop with one axial and two vector-couplings or all the three axial-couplings at the vertices can generate such couplings.
- At the one loop level,

$$\mathcal{H}_4^{\gamma} = \mathcal{H}_4^{Z} = 0 \tag{2}$$

- **The only couplings likely to appear at one-loop are $\mathcal{F}_5^{\gamma,Z}$ and $\mathcal{H}_3^{\gamma,Z}$.**
- These couplings can in general be complex quantities. However, they pick up imaginary contribution only when Q^2 crosses the threshold for fermion pair production (i.e. $Q^2 > 4m_f^2$) for timelike Q^2 or when M_Z^2 exceeds this threshold (i.e. $M_Z > 2m_f$) for spacelike Q^2.

Fermionic One Loop Contribution to CP even Couplings

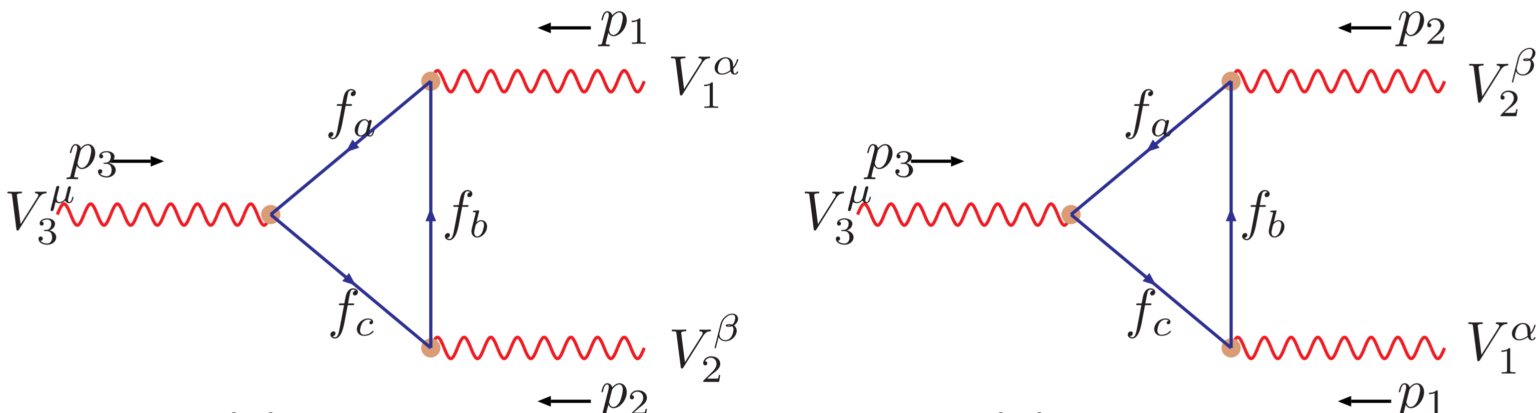

Figure 2: *Generic diagram contributing to trilinear neutral gauge boson vertices for the CP conserving case.*

Momenta p_1 and p_2 denote same kind of bosons, say V_1^α and V_2^β, and momenta p_3 denotes V_3^μ which is of the second kind of vector boson.
Denote the fermion-gauge coupling by

$$\mathcal{L} = \bar{f}_i\ \gamma_\mu\ \left[g_{L_{ij}}^{\ \mathrm{V}}\ P_L + g_{R_{ij}}^{\ \mathrm{V}}\ P_R \right] f_j\ \mathrm{V}^\mu\ . \tag{3}$$

with $P_{L,R} = (1 \mp \gamma_5)/2$. For $\mathrm{V} \equiv \gamma$, $g_{L_{ij}} = g_{R_{ij}} = e\, q_i\, \delta_{ij}$, q_i being the charge of fermion f_i.
For $\mathrm{V} = Z$, the couplings $g_{_L}$ and $g_{_R}$ are as defined in various models.
In the absence of any CP violating interactions, all these couplings are real and $g_{L,R_{ij}} = g_{L,R_{ji}}$ because of hermiticity.

Fermionic One Loop Contribution to CP even Couplings

1. The loop amplitude for $\gamma^\star \gamma Z$ is

$$\kappa \sum_{a=1}^{N_f} \sum_{b=1}^{N_f} \sum_{c=1}^{N_f} \mathcal{C}_f \left[\mathcal{A}^{\mathcal{D}}_{a,b,c}(\gamma,\, Z;\; 0,\, \sqrt{Q^2},\, m_Z) + \mathcal{A}^{\mathcal{E}}_{a,b,c}(\gamma,\, Z;\, \sqrt{Q^2},\, 0,\, m_Z) \right],$$

where N_f = total number of flavors, $\mathcal{C}_f$ = the color factor of the fermion in the loop and κ = over all loop factor.
Electromagnetic interactions at two vertices forbids any flavor mixing, therefore

$$\mathcal{H}_3^\gamma \;=\; - \left(\frac{4\,\pi \alpha_{\mathrm{em}}}{(2\, \cos\theta_W\, \sin\theta_W)\,(16\,\pi^2)} \right) \otimes \sum_{a=1}^{N_f} \mathcal{C}_f \left[\mathcal{A}^{\mathcal{D}}_{a,a,a}(\gamma,\, Z;\; 0,\, \sqrt{Q^2},\, m_Z) + \mathcal{A}^{\mathcal{E}}_{a,a,a}(\gamma,\, Z;\, \sqrt{Q^2},\, 0,\, m_Z) \right].$$

Fermionic One Loop Contribution to CP even Couplings

2. The $\gamma^\star ZZ$ vertex allows the mixing among the weak interaction eigenstates at two weak vertices giving

$$\mathcal{F}_5^\gamma = \left(\frac{4\,\pi\alpha_{\rm em}}{(2\,\cos\theta_W\,\sin\theta_W)\,(16\,\pi^2)}\right)\otimes \sum_{a=1}^{N_f}\sum_{b=1}^{N_f}\mathcal{C}_f\left[\mathcal{A}^{\mathcal{D}}_{a,b,a}(Z,\,\gamma;\,m_Z,\,m_Z,\,\sqrt{Q^2})+\mathcal{A}^{\mathcal{E}}_{a,b,a}(Z,\,\gamma;\,m_Z,\,m_Z,\,\sqrt{Q^2})\right]$$

3. The $Z^\star\gamma Z$ Vertex gives

$$\mathcal{H}_3^Z = \left(\frac{4\,\pi\alpha_{\rm em}}{(2\,\cos\theta_W\,\sin\theta_W)^2\,(16\,\pi^2)}\right)\otimes \sum_{a=1}^{N_f}\sum_{b=1}^{N_f}\mathcal{C}_f\left[\mathcal{A}^{\mathcal{D}}_{a,b,a}(Z,\,\gamma;\,m_Z,\,\sqrt{Q^2},\,0)+\mathcal{A}^{\mathcal{E}}_{a,b,a}(Z,\,\gamma;\,\sqrt{Q^2},\,m_Z,\,0)\right]$$

Fermionic One Loop Contribution to CP even Couplings

4. The loop amplitude for $Z^{\star}ZZ$ allow weak mixing at all vertices.

$$\mathcal{F}_5^Z = \left(\frac{4\,\pi\alpha_{\rm em}}{\left(2\ \sin\theta_W\ \cos\theta_W\right)^3\left(16\,\pi^2\right)} \right) \otimes$$

$$\sum_{a=1}^{N_f}\sum_{b=1}^{N_f}\sum_{c=1}^{N_f}\mathcal{C}_f\left[\mathcal{A}^{\mathcal{D}}_{a,b,c}(Z,\, Z;\ m_Z,\, m_Z,\, \sqrt{Q^2}) + \mathcal{A}^{\mathcal{E}}_{a,b,c}(Z,\, Z;\ m_Z,\, m_Z,\, \sqrt{Q^2})\right]$$

SM Contribution to TNGBC

- In SM the 1-loop contribution to TNGBC's come from three families of quarks and leptons.
- Anomaly Canccellation $\Rightarrow$ all couplings $\rightarrow 0$ for $Q^2 >>$ fermion pair production threshold.
- Of all the thresholds, largest contribution comes from the heaviest fermionic loop.
- Same fermion runs in all the three sides of the triangle loop.
- $\mathcal{F}_5^\gamma$ and $\mathcal{H}_3^{\gamma,Z}$ get contributions only from the charged fermions, whereas $\mathcal{F}_5^Z$ receives contributions from the neutrinos as well.

MSSM Contribution

- We recalculate the MSSM contribution to TNGBC's in the light of the reference point **SPS1a'** which is defined at a characteristic scale of 1 TeV with its origin in minimal super-gravity (mSUGRA) (*G. J. Gounaris, J. Layssac and F. M. Renard, Phys. Rev. D* **61**, *073013 (2000)* and *D. Choudhury et. al. Int. J. Mod. Phys. A* **16**, *4891 (2001)*).
- Parameters in this reference point SPS1a':
  - gaugino mass $M_{1/2} = 250$ GeV, universal scalar mass $M_0 = 70$ GeV,
  - trilinear coupling $A_0 = -300$ GeV,
  - $\tan\beta(\tilde{M}) = 10$ and sign$(\mu) = +1$.
- Extrapolating these parameters to $\tilde{M} = 1$ TeV generates the MSSM Lagrangian parameters.
  Higgs mixing parameter $\mu = 396$ GeV and $M_2 = 193.2$ GeV.
- chargino and Neutrilano masses :
  - $m_{\chi_1^+} = 183.7\ \mathrm{GeV}$, $m_{\chi_2^+} = 415.4\ \mathrm{GeV}$ and
  - $m_{\chi_1^0} = 94.8\ \mathrm{GeV}$, $m_{\chi_2^0} = 180.3\ \mathrm{GeV}$, $m_{\chi_3^0} = 401.9\ \mathrm{GeV}$, $m_{\chi_4^0} = 411.8\ \mathrm{GeV}$.

LH Contribution to TNGBC

- Higgs fields as pseudo Nambu-Goldstone bosons of a global symmetry which is spontaneously broken at some high scale f.

- The Higgs acquires mass through electro-weak symmetry breaking triggered by radiative corrections leading to a Coleman-Weinberg type of potential.

- Higgs is protected by approximate global symmetry therefore remains light.

- Quadratic divergent one loop contributions to its mass are canceled by the contributions of heavy gauge bosons and heavy fermionic states that are introduced in the model.

- The realization of little Higgs mechanism essentially fall into two classes:
  - **Product Group** for the gauge symmetry $\rightarrow$ The Littlest Higgs model (LH)
  - **Simple Group** structure – has an additional model parameter $\rightarrow$ SU(3) simple group model.

LH Contribution to TNGBC

The Littlest Higgs Model :

$[SU(2) \times U(1)]^2$ gauge symmetry is embedded in an $SU(5)$ global symmetry.
The gauge symmetry is broken down to the SM $SU(2) \times U(1)$ gauge symmetry by a single vacuum condensate $f \approx 1$ TeV.
The resultant top sector : a top quark t and its heavy partner T :

$$m_t = \frac{\lambda_1 \, \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \, v \tag{4}$$

$$M_T = \sqrt{\lambda_1^2 + \lambda_2^2} \, f = \frac{m_t}{\sqrt{X_L(1 - X_L)}} \frac{f}{v} \tag{5}$$

where $X_L = \lambda_1^2 / \left(\lambda_1^2 + \lambda_2^2\right)$
Lower Bound $M_T \geq 2 \, \frac{m_t}{v} \, f \approx \sqrt{2} \, f$ for $\lambda_1 = \lambda_2$.

Extra Contribution to TNGBC's comes from the triangle graph with only heavy top and also both the SM top t and heavy top T simultaneously present in the loop.

LH Contribution to TNGBC

Simple Group Model :

$SU(N) \times U(1)$ gauge symmetry is broken down to $SU(2)_L \times U(1)$, giving rise to a set of TeV-scale gauge bosons.

The two gauge couplings of $SU(N) \times U(1)$ are fixed in terms of two SM gauge couplings, leaving no free parameters in the gauge sector.

All fermionic SM representations are extended to transform as fundamental or conjugate representations of $SU(N) \rightarrow$ additional heavy fermions in all the three quark and lepton sectors.

Simplest realization: $SU(3)$ simple gauge model with anomaly-free embedding of extra fermions $SU(2)_L \rightarrow SU(3)$

- Heavy fermions associated with each $SU(2)_L$ doublet of the SM
  - New TeV-scale D and S quarks of charge $-1/3$
  - Heavy third generation quark T of charge $+2/3$
  - Electrically Neutral Heavy leptons N_i of three generations

LH Contribution to TNGBC

Masses:

$$
\begin{aligned}
M_T &= \sqrt{\lambda_1^2\, c_\beta^2 + \lambda_2^2\, s_\beta^2}\, f = \sqrt{2}\, \frac{t_\beta^2 + X_\lambda^2}{(1+t_\beta^2)\, X_\lambda}\, \frac{m_t}{v}\, f \\
M_{D,\,S} &= s_\beta\, \lambda_{D,\,S}\, f \\
M_{N_i} &= s_\beta\, \lambda_{N_i}\, f
\end{aligned}
\qquad (6)
$$

where $t_\beta \equiv \tan\beta = f_2/\, f_1$ and $X_L = \lambda_1^2/\, \left(\lambda_1^2 + \lambda_2^2\right)$. $f \equiv \sqrt{f_1^2 + f_2^2}$.

- Lower Bound

$$
M_T \geq 2\, \sqrt{2}\, s_\beta\, c_\beta\, \frac{m_t}{v}\, f \approx f\ \sin(2\,\beta)\ \ \text{for}\ \frac{\lambda_1}{\lambda_2} = \tan\beta
$$

- Contribution from mixed t and T, mixed SM and TeV range quarks of the first two generations and the mixed neutrino and TeV mass heavy neutrinos (N_i) of all the three generations in the triangle loop.

- Pure T quark loop, pure TeV mass quark loop of first two generations and TeV mass heavy neutrinos of three generations do not contribute to $\mathcal{F}_5^Z$ in the model.

LH Contribution to TNGBC

Constraints from electro-weak precision measurements $\Rightarrow$ Littlet Higgs Model: $f > 5$ TeV. can be brought down to about $2 - 3$ TeV
SU(3) Simple Model : $f > 3.9$ TeV for $t_\beta = 3$. Can only be marginally brought down by slightly different realization
Severe constraints from precision electro-weak measurements can be satisfied only by tuning the model parameters $\rightarrow$ little hierarchy problem.

Little Higgs Model with T-Parity (LHT)

- Discrete symmetry T-parity is proposed — explicitly forbids any tree-level contribution from the heavy mass states to observables involving only the SM particles.

- Forbids the interactions that impart vev to triplet Higgs, thereby generating the corrections to precision electro-weak observables only at the one loop level.

- Heavy T-odd partners of the SM gauge bosons and SM fermions called mirror fermions that couple vectorially to Z.

LH Contribution to TNGBC

- Top quark sector: two heavy T-even and T-odd top quarks in addition to the T-even SM top quark $\rightarrow$ canceling the quadratically divergent contribution of the SM top quark to the Higgs mass.
- T-parity conservation $\Rightarrow$ there is no coupling between Z and T-odd and T-even fermions.
- Thus the **mirror fermions do not contribute to the trilinear neutral gauge boson couplings**.
- The T-even partner of the top quark T_+ however, has both axial and vector couplings with Z and hence contributes to the triangle loop.
- The top quark masses in this model

$$m_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} v \left\{ 1 + \frac{v^2}{f^2} \left(-\frac{1}{3} + \frac{1}{2} X_L \left(1 - X_L\right) \right) \right\} \tag{7a}$$

$$M_T = \frac{m_t}{\sqrt{X_L \left(1 - X_L\right)}} \frac{f}{v} \left\{ 1 + \frac{v^2}{f^2} \left(\frac{1}{3} - X_L \left(1 - X_L\right) \right) \right\} \tag{7b}$$

- For the same value of f, the mass m_{T_+} is the same for the ratio $r = \frac{\lambda_1}{\lambda_2} = 1$ and $1/r$.

Results and Discussion

Some Common Features

- All couplings vanish asymptotically for large $\sqrt{Q^2}$ compared to the highest fermion mass in the theory.
- The relative importance of the real and imaginary parts of the couplings is strongly energy dependent.
- Below the $2m_t$ threshold, the imaginary parts of all the couplings are negligible. At and above this threshold the imaginary parts become comparable or even dominant in comparison to the real parts.

Results and Discussion

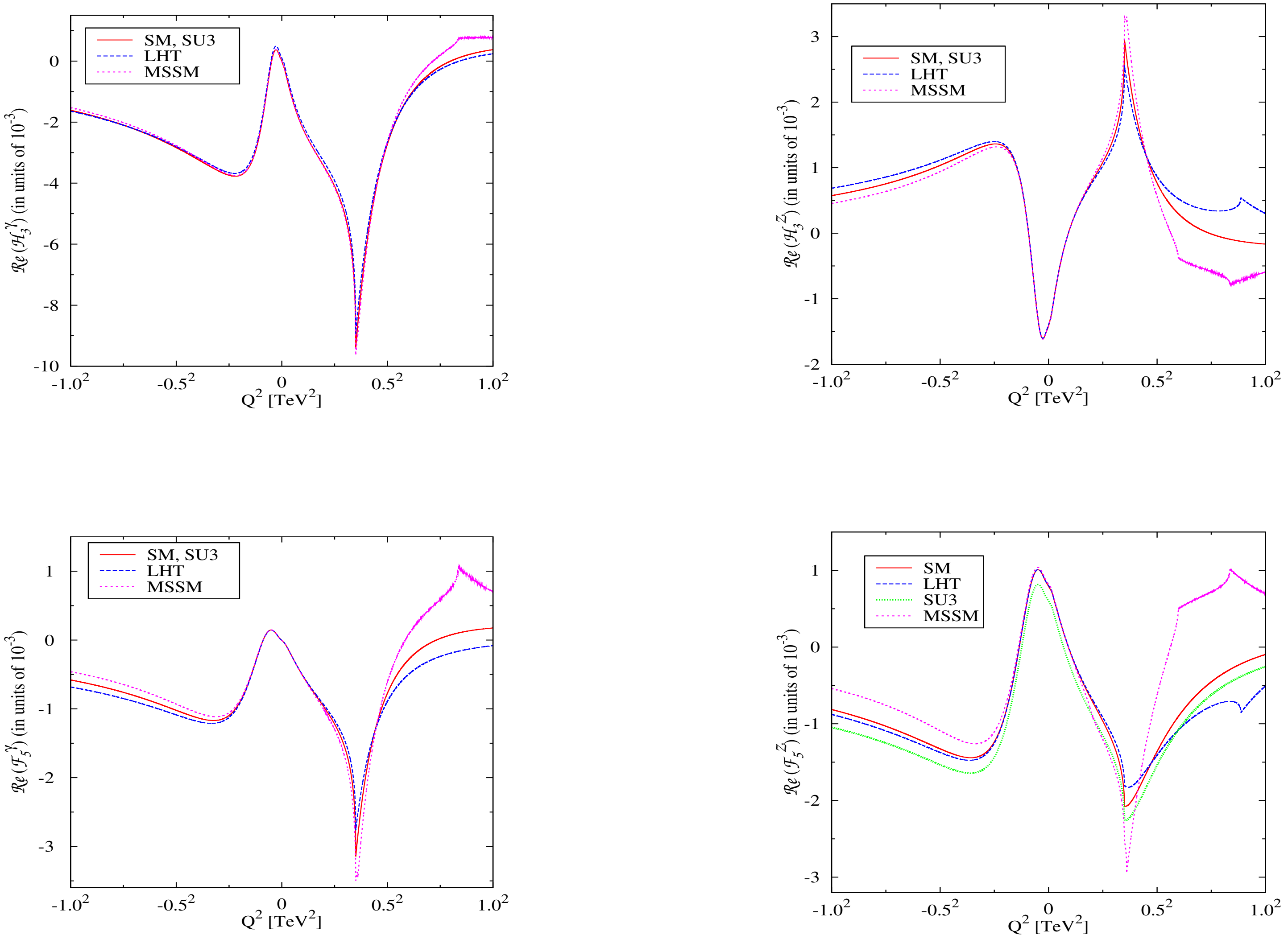

Figure 3: *Real parts [LHT: $f = 500$ GeV, $r = 1$; SU(3): $f = 3$ TeV, $t_\beta = 3$, $M_i = 2$ TeV].*

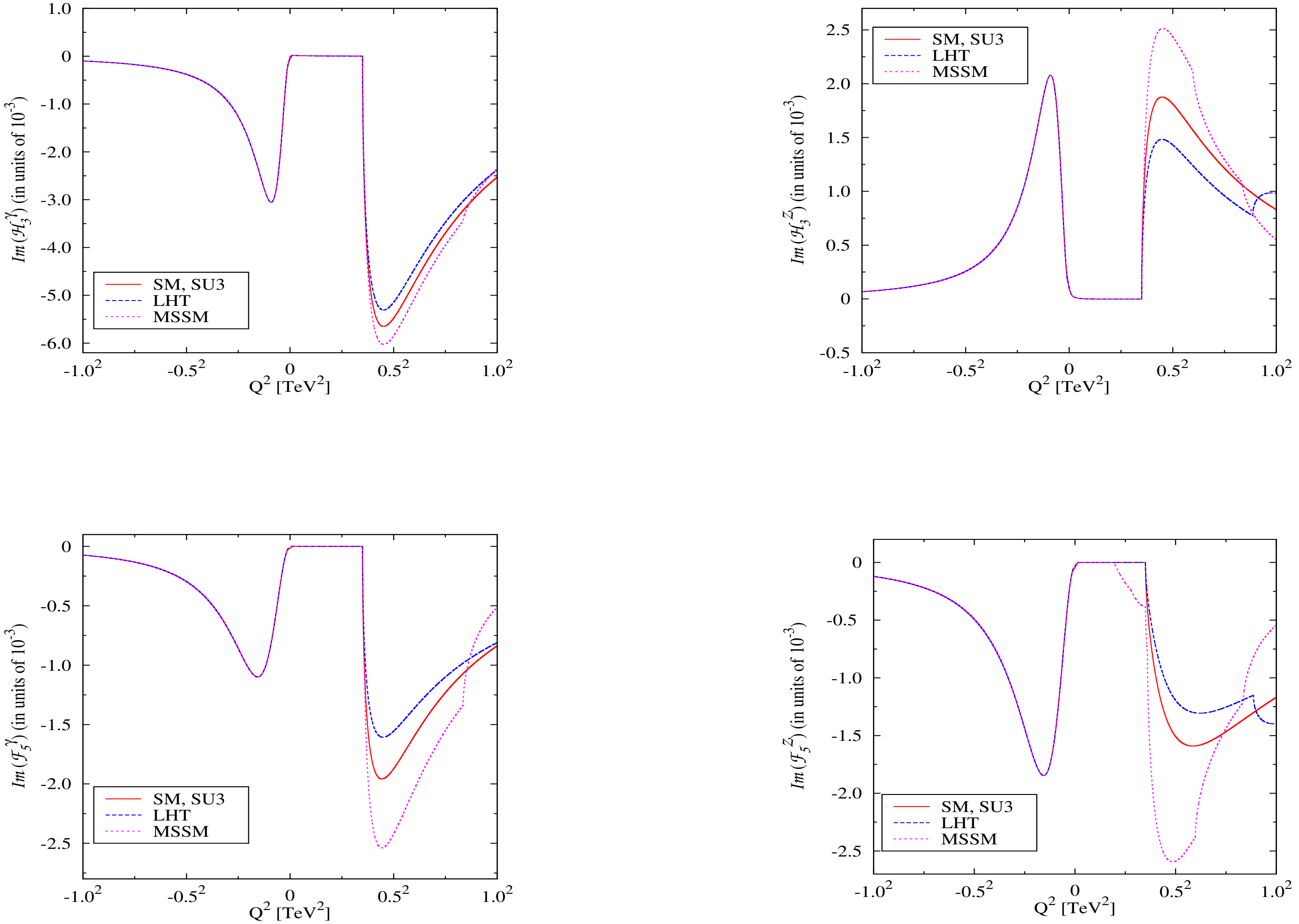

Figure 4: *Imaginary parts [LHT: $f = 500$ GeV, $r = 1$; SU(3): $f = 3$ TeV, $t_\beta = 3$ and $M_i = 2$ TeV].*

Results and Discussion

Features in Different Models

- In the **MSSM** $\sqrt{Q^2} = 2\, m_{\chi_1^+}$, which is very near to the $2\, m_t$ SM peak resulting in the enhancement of the couplings at this point.
- This effect is more pronounced in the imaginary parts of the couplings.
- In the MSSM new peaks appear at $m_{\chi_1^+} + m_{\chi_2^+} \simeq 600$ GeV and $2m_{\chi_2^+} \simeq 800$ GeV which is more pronounced in the real parts of $\mathcal{F}_5^\gamma$ and $\mathcal{F}_5^Z$. In SM and Little Higgs Model there is **no such effect** upto 1 TeV.
- The neutralinos contribute only to $\mathcal{F}_5^Z$.
- The effect of extra heavy fermions in the LHT Model is to decrease the threshold effects of the the SM whereas the particles in MSSM enhance it.
- The new threshold in the LHT at $\sqrt{Q^2} = m_t + M_{T_+}$ and in the MSSM as mentioned above are opposite to each other but the magnitudes are comparable.

Results and Discussion

- The anomaly free SU(3) simple Model does not show any appreciably different behaviour than the SM upto $\sqrt{Q^2} = 1$ TeV. At higher $\sqrt{Q^2}$, the effect of new heavy fermions shows up but the threshold values are an order of magnitude lower than that at the $2m_t$ threshold. However, at these $\sqrt{Q^2}$, the SM contribution is negligible.

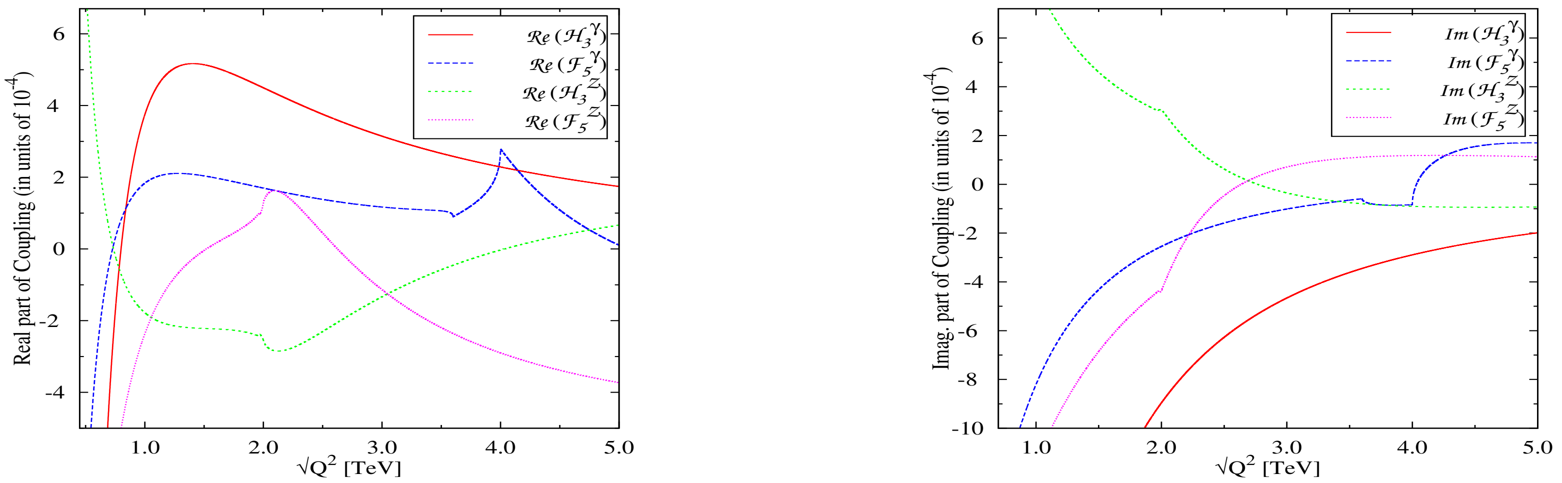

Figure 5: $\sqrt{Q^2}$*-variation in the range* $1-5$ TeV *of the 1) real and 2) imaginary parts of the couplings in SU(3) simple Little Higgs Model with anomaly free embedding for* $\tan\beta = r = 3$, $f = 3$ TeV*. With this choice of parameters the mass of heavy top* $M_T = 1.8$ TeV *while the masses of all other heavy fermions are fixed at* 2 TeV*.*

Results and Discussion

$\sqrt{Q^2}$ (in TeV)	$\mathcal{H}_3^\gamma$ (10^{-4})	$\mathcal{H}_3^Z$ (10^{-4})	$\mathcal{F}_5^\gamma$ (10^{-4})	$\mathcal{F}_5^Z$ (10^{-4})
$2m_t$	$-94.17 - \iota\, 0.0158$	$29.53 + 0\iota$	$-31.50 + \iota\, 0.0149$	$-22, .42 + \iota\, 0.0254$
$m_t + M_T$	$4.533 - \iota\, 9.136$	$-1.487 + \iota\, 3.008$	$1.757 - \iota\, 2.802$	$-0.1062 - \iota\, 4.751$
$2M_T$	$2.582 - \iota\, 3.448$	$-2.417 - \iota\, 0.0712$	$0.5535 - \iota\, 0.677$	$0.9483 + \iota\, 0.9624$
M_U	$3.146 - \iota\, 4.660$	$-2.503 + \iota\, 2.036$	$1.146 - \iota\, 1.152$	$1.699 - \iota\, 2.449$
$2M_U$	$1.372 - \iota\, 1.455$	$0.1424 - \iota\, 1.523$	$2.947 - \iota\, 0.191$	$-3.151 + \iota\, 2.236$

Table 1: *The values of various couplings (written as complex numbers) at some typical $\sqrt{Q^2}$ (where peaks are expected) in the SU(3) simple model with anomaly free embedding. All values correspond to $\tan\beta = r = 3$, scale $f = 3$TeV and $m_t = 175$GeV. At these values of parameters, the mass of heavy top is $M_T = 1.8$ TeV and masses of all other heavy fermions have been taken to be $M_i = 3$ TeV.*

Results and Discussion

Ratio	M_{T_+}(GeV)	$\sqrt{Q^2}$	$\mathcal{H}_3^\gamma\,(10^{-4})$	$\mathcal{H}_3^Z\,(10^{-4})$	$\mathcal{F}_5^\gamma\,(10^{-4})$	$\mathcal{F}_5^Z\,(10^{-4})$
		$2m_t$	$-93.40 - \iota\,0.0149$	$28.85 + 0\,\iota$	$-30.83 + 0\,\iota$	$-20.29 + 0\,\iota$
0.5	889.2	$m_t + M_{T_+}$	$4.068 - \iota\,22.963$	$-0.705 + \iota\,7.341$	$1.413 - \iota\,7.612$	$-1.601 - \iota\,10.82$
		$2M_{T_+}$	$4.595 - \iota\,10.66$	$-1.499 + \iota\,3.875$	$1.441 - \iota\,3.666$	$1.574 - \iota\,6.475$
		$2m_t$	$-89.34 - \iota\,0.0130$	$25.49 + 0\,\iota$	$-27.65 + 0\,\iota$	$-18.06 + 0\,\iota$
1.0	711.4	$m_t + M_{T_+}$	$0.9154 - \iota\,28.02$	$5.388 + \iota\,7.730$	$-1.313 - \iota\,9.229$	$-8.534 - \iota\,11.50$
		$2M_{T_+}$	$3.710 - \iota\,14.24$	$-0.3113 + \iota\,6.901$	$-0.0809 - \iota\,5.481$	$1.717 - \iota\,10.66$
		$2m_t$	$-81.80 - \iota\,0.0094$	$19.84 - \iota\,0.0054$	$22.00 + 0\,\iota$	$-14.31 + \iota\,0.0002$
2.0	889.2	$m_t + M_{T_+}$	$0.7583 - \iota\,19.59$	$14.88 + \iota\,3.813$	$-4.727 - \iota\,7.226$	$-15.09 - \iota\,8.566$
		$2M_{T_+}$	$1.116 - \iota\,9.099$	$1.424 + \iota\,9.069$	$-2.866 - \iota\,5.549$	$-5.549 - \iota\,13.21$
		$2m_t$	$-78.51 - \iota\,0.0079$	$17.58 - \iota\,0.0068$	$-19.40 + 0\,\iota$	$-12.69 + \iota\,0.0002$
3.0	1185.6	$m_t + M_{T_+}$	$0.4505 - \iota\,13.03$	$18.70 + \iota\,2.076$	$-6.278 - \iota\,6.100$	$-16.71 - \iota\,6.937$
		$2M_{T_+}$	$-0.623 - \iota\,5.49$	$1.759 + \iota\,9.726$	$-3.972 - \iota\,5.508$	$4.953 - \iota\,13.55$
		$2m_t$	$-77.05 - \iota\,0.0072$	$16.60 - \iota\,0.0074$	$-18.16 + 0\,\iota$	$-11.95 + \iota\,0.0002$
4.0	1511.7	$m_t + M_{T_+}$	$0.2019 - \iota\,9.182$	$20.18 + \iota\,1.320$	$-7.086 - \iota\,5.541$	$-17.0 - \iota\,6.011$
		$2M_{T_+}$	$-1.712 - \iota\,3.614$	$1.824 + \iota\,9.939$	$-4.461 - \iota\,5.508$	$5.479 - \iota\,13.48$

Table 2: *Ratio,* $r = \lambda_1/\lambda_2$ *in the LHT Model.* $f = 500$ *GeV and* $m_t = 175$ *GeV.*

Results and Discussion

For higher ratios, a very interesting behaviour is shown by the couplings $\mathcal{H}_3^Z$ and $\mathcal{F}_5^Z$. Not only the imaginary part becomes appreciable at high $\sqrt{Q^2}$ but also the threshold values of the couplings at $\sqrt{Q^2} = m_t + m_{T_+}$ are higher than those at $\sqrt{Q^2} = 2m_t$ and are comparable to the SM values.

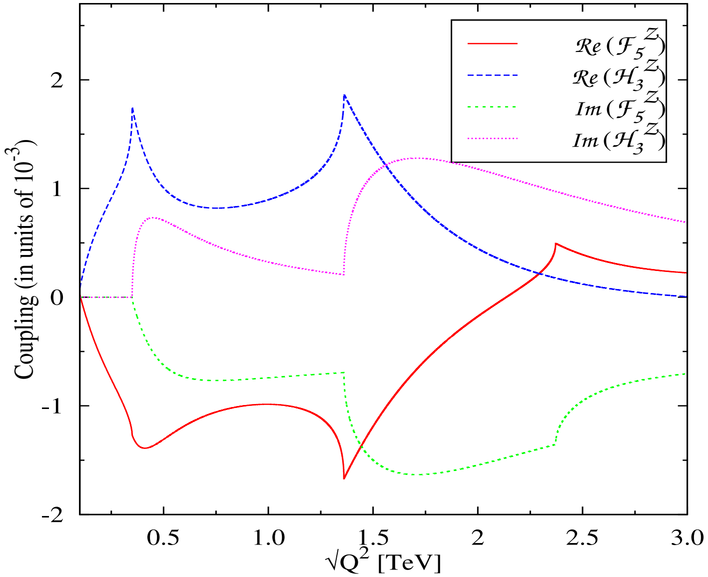

Figure 6: $\sqrt{Q^2}$*-variation of real and imaginary parts of* $\mathcal{H}_3^Z$ *and* $\mathcal{F}_5^Z$ *in the range* $0 - 3$ TeV *in Little Higgs Model with T-parity for* $r = 3$ *and* $f = 0.5$ TeV*. With this choice of parameters the mass of T-even top,* $M_{T_+} = 1.186$ TeV*.*

Summary

- We have studied the trilinear neutral gauge boson couplings $\gamma^\star \gamma$, $Z^\star Z \gamma$, $\gamma^\star Z Z$ and $Z^\star Z Z$ involving one off-shell vector boson as a function of Q^2 in SM, MSSM and Little Higgs models.

- We have made theoretical prediction of these couplings for the model parameters which are constrained by the electro-weak precision measurements.

- The large s-channel contributions in the $Z Z$ and $Z \gamma$ production at the LHC due to the anomalous triple gauge boson couplings, could be the first indirect manifestation of new physics. However, the precise measurement of the triple gauge bosn couplings will only be possible with a 10 fb^{-1} luminosity.

- Our analysis presented above allows us to confront and discriminate among various models considered here on the basis of these couplings.

- The complementary study of the one-loop contribution to the triple charge - neutral gauge boson vertices $W^+ W^- \gamma$ and $W^+ W^- Z$ in the context of various Little Higgs Models with and without T parity is in process.

- A detailed simulation required to check the sensitivity of these couplings at ILC.

Thank You!

Generic Structure of the vertex:

We study the **CP-conserving** TNGBC's where one of the gauge bosons is off shell.

$$\begin{aligned}\Gamma_{\alpha\beta\mu}(p_1,p_2;p_3) &= \epsilon_{\alpha\beta\mu\eta}[\mathcal{A}_1 p_1^\eta + \mathcal{A}_2 p_2^\eta] + \epsilon_{\alpha\beta p_1 p_2}[\mathcal{A}_3 p_{1\mu} + \mathcal{A}_4 p_{2\mu}] \\ &+ \epsilon_{\alpha\mu p_1 p_2}[\mathcal{A}_5 p_{1\beta} + \mathcal{A}_6 p_{2\beta}] \\ &+ \epsilon_{\beta\rho p_1 p_2}[\mathcal{A}_7 p_{1\alpha} + \mathcal{A}_8 p_{2\alpha}]\,, \qquad (8)\end{aligned}$$

$\mathcal{A}_i \equiv \mathcal{A}_i(p_1,p_2)$. (*D. Choudhury et. al. Int. J. Mod. Phys. A* **16**, *4891 (2001)*).

- Two of the above form factors (say $\mathcal{A}_5$ and $\mathcal{A}_8$) may be eliminated by use of Schouten's identity (nonexistencce of a totally anti-symmetric fifth rank tensor)
- Bose symmetry ($p_1 \leftrightarrow p_2$, $\alpha \leftrightarrow \beta$) relates the remaining form factors pairwise

$$\begin{aligned}\Gamma_{\alpha\beta\mu}(p_1,p_2;p_3) &= \epsilon_{\alpha\beta\mu\eta}[\mathcal{B}_1 p_1^\eta - \bar{\mathcal{B}}_1 p_2^\eta] + \epsilon_{\theta\mu p_1 p_2}[\mathcal{B}_2 p_{2\beta}\delta_\alpha^\theta - \bar{\mathcal{B}}_2 \delta_\alpha^\theta p_{1\alpha}] \\ &+ \epsilon_{\alpha\beta p_1 p_2}[\mathcal{B}_3 p_{1\mu} + \bar{\mathcal{B}}_3 p_{2\mu}] \qquad (9)\end{aligned}$$

- Gauge Invariance and/or current conservation further reduces the number of free parameters.

The loop contribution (with three distinct internal fermions f_a, f_b, f_c having masses m_a, m_b and m_c respectively), corresponding to the direct and exchange diagrams of Fig. 2(a) is proportional to $\epsilon_{\alpha\beta\mu\eta}$ and can be parameterized as *

$$
\begin{aligned}
\mathcal{A}^{\mathcal{D}}_{a,b,c}(\mathrm{V}, V_3;\, p_1,\, p_2,\, p_3) &= g_1^{\mathrm{V},\,\mathrm{V}_3}\Big[p_3^2(C_{11}+C_{21}) - p_2^2(C_{12}+C_{22}) - 2p_1\cdot p_2(C_{12}+C_{23})\Big] \\
&+ (m_a^2 g_1^{\mathrm{V},\mathrm{V}_3} + g_3^{\mathrm{V},\mathrm{V}_3} - g_2^{\mathrm{V},\mathrm{V}_3})(C_{11}+C_0) - g_4^{\mathrm{V},\mathrm{V}_3}C_{11} \\
\mathcal{A}^{\mathcal{E}}_{a,b,c}(\mathrm{V}, \mathrm{V}_3;\, p_1,\, p_2,\, p_3) &= -(m_a^2 g_1^{\mathrm{V},\mathrm{V}_3} - g_4^{\mathrm{V},\mathrm{V}_3} - g_2^{\mathrm{V},\mathrm{V}_3})(C_{11}-C_{12}) - g_3^{\mathrm{V},\mathrm{V}_3}(C_{11}+C_0-C_{12}) \\
&- g_1^{\mathrm{V},\mathrm{V}_3}\Big[B_{023}+B_{123}-2C_{24}+p_1^2(C_{12}+C_{21}) \\
&\qquad +(p_2^2+2p_1\cdot p_2).(C_{22}-2C_{23}+C_{21})\Big]
\end{aligned}
\tag{10}
$$

Here $\mathrm{V}\equiv \mathrm{V}_1/\mathrm{V}_2$ and

$$
\begin{aligned}
g_1^{\mathrm{V},\,\mathrm{V}_3} &= \left(g_{L_{ab}}^{\mathrm{V}}\; g_{L_{bc}}^{\mathrm{V}}\; g_{L_{ca}}^{\mathrm{V}_3} - g_{R_{ab}}^{\mathrm{V}}\; g_{R_{bc}}^{\mathrm{V}}\; g_{R_{ca}}^{\mathrm{V}_3}\right), \\
g_2^{\mathrm{V},\,\mathrm{V}_3} &= m_a\, m_c\; \left(g_{L_{ab}}^{\mathrm{V}}\; g_{L_{bc}}^{\mathrm{V}}\; g_{R_{ca}}^{\mathrm{V}_3} - g_{R_{ab}}^{\mathrm{V}}\; g_{R_{bc}}^{\mathrm{V}}\; g_{L_{ca}}^{\mathrm{V}_3}\right), \\
g_3^{\mathrm{V},\,\mathrm{V}_3} &= m_a\, m_b\; \left(g_{L_{ab}}^{\mathrm{V}}\; g_{R_{bc}}^{\mathrm{V}}\; g_{R_{ca}}^{\mathrm{V}_3} - g_{R_{ab}}^{\mathrm{V}}\; g_{L_{bc}}^{\mathrm{V}}\; g_{L_{ca}}^{\mathrm{V}_3}\right), \\
g_4^{\mathrm{V},\,\mathrm{V}_3} &= m_b\, m_c\; \left(g_{L_{ab}}^{\mathrm{V}}\; g_{R_{bc}}^{\mathrm{V}}\; g_{L_{ca}}^{\mathrm{V}_3} - g_{R_{ab}}^{\mathrm{V}}\; g_{L_{bc}}^{\mathrm{V}}\; g_{R_{ca}}^{\mathrm{V}_3}\right).
\end{aligned}
\tag{11}
$$

*We adopt the convention $\epsilon_{0123} = 1$.

$B_{\mu jk}$ and C_μ, $C_{\mu\,\nu}$ are respectively the 2-point and 3-point Passarino Veltman functions defined as

$$B_{i23} = B_i(p_2^2,\, m_b^2,\, m_c^2); \quad \text{with} \quad i = 0, 1$$
$$C_0;\, C_\mu;\, C_{\mu\nu} = C_0;\, C_\mu;\, C_{\mu\nu}(p_3,\, p_2,\, m_a,\; m_c,\, m_b) \qquad (12)$$

with

$$B_0; B_\mu; B_{\mu\nu}(p, m_i, m_j) = \frac{1}{\iota\,\pi^2}\int d^4k \frac{1; k_\mu; k_{\mu\nu}}{k^2 + m_i^2)((k+p)^2 + m_j^2)}$$
$$C_0; C_\mu; C_{\mu\nu}(p_1, p_2, m_i, m_j, m_k) = \frac{1}{\iota\,\pi^2}\int d^4k \frac{1; k_\mu; k_{\mu\nu}}{k^2 + m_i^2)((k+p_1)^2 + m_j^2)((k+p_1+p_2)^2 + m \ldots} \qquad (1\ldots$$

G. Passarino and M. Veltman, Nucl. Phys. **B 160**, *151 (1971)*

Vertex	LHT Model		SU(3) simple group	
	g_L	g_R	g_L	g_R
$\bar{q}_i Z q_i$ (for $i = 1-5$) i.e. all SM quarks except top	$2(T_3^i - Q^i s_w^2)$	$-2Q^i s_w^2$	$2(T_3^i - Q^i s_w^2)$	$-2Q^i s_w^2$
$\bar{l}_i Z l_i$ (for $i = 1-6$) i.e. all SM leptons	$2(T_3^i - Q^i s_w^2)$	$-2Q^i s_w^2$	$2(T_3^i - Q^i s_w^2)$	$-2Q^i s_w^2$
$\bar{t} Z t$	$1 - \frac{4}{3}s_w^2 - x_L^2 \frac{v^2}{f^2}$	$-\frac{4}{3}s_w^2$	$1 - \frac{4}{3}s_w^2 - x_L^2 \frac{v^2}{f^2}$	$-\frac{4}{3}s_w^2$
$\bar{T} Z t$	$x_L \frac{v}{f}$	0	$\frac{1}{2\sqrt{2}(r^2+t_\beta^2)} s_{2\beta}(1+t_\beta^2)(r^2-1)\frac{v}{f}$	0
$\bar{T} Z T$	$x_L^2 \frac{v^2}{f^2} - \frac{4}{3}s_w^2$	$-\frac{4}{3}s_w^2$	0	0
$\bar{D}_i Z d_i$ $i = 1, 2$	$\times\times$	$\times\times$	$\frac{1}{\sqrt{2}t_\beta}\frac{v}{f}$	0
$\bar{N}_i Z \nu_i$ $i = 1, 2, 3$	$\times\times$	$\times\times$	$-\frac{1}{\sqrt{2}t_\beta}\frac{v}{f}$	0

Table 3: *Couplings of fermions with Z-boson in units of $g/2c_w$ in the Little Higgs Models.* $x_L = \frac{1}{1+r^{-2}}$ *with* $r = \lambda_1/\lambda_2$.