Storage Rings to detect Gravitational Waves

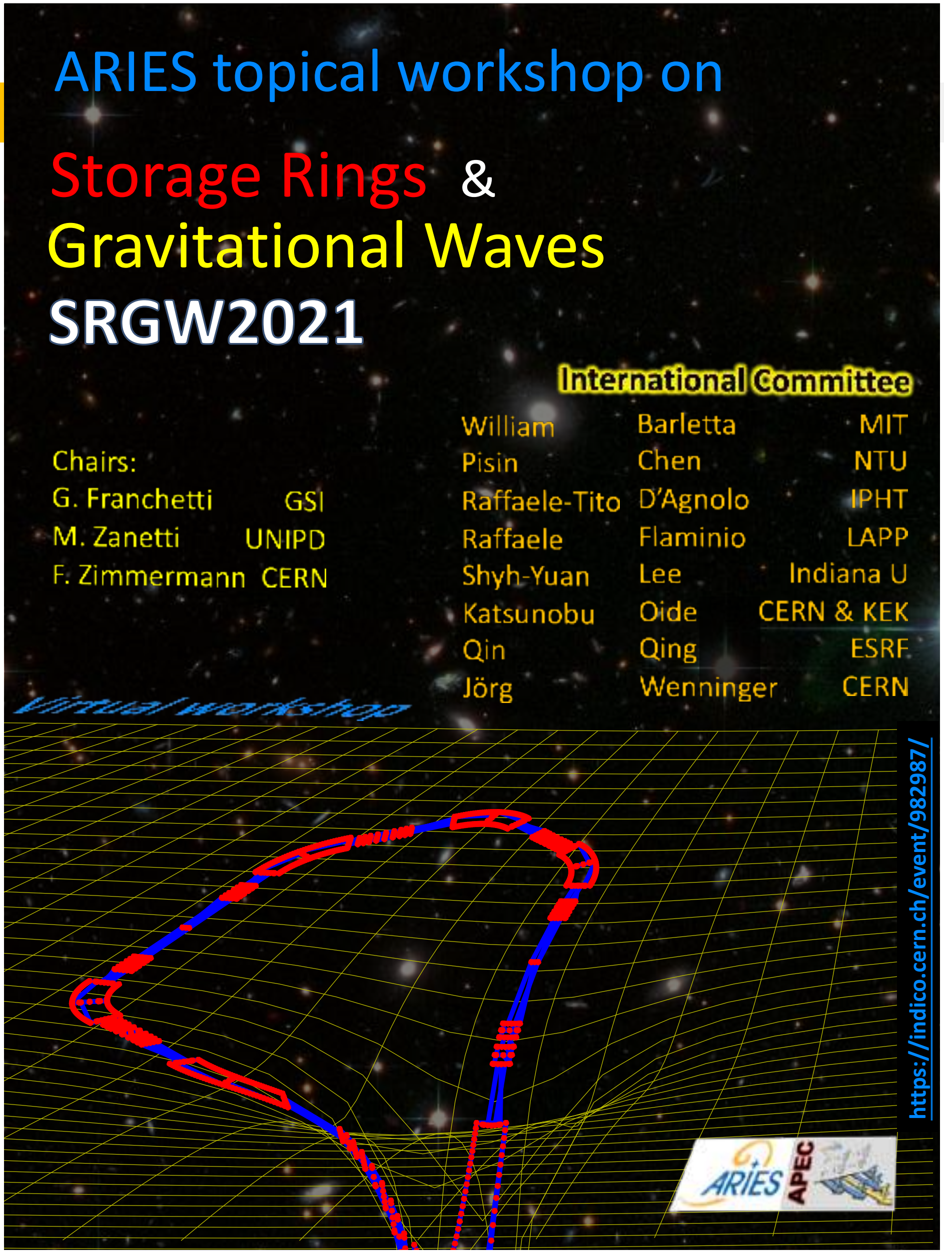
G. Franchetti*, F. Zimmermann

*GSI & Goethe University Frankfurt

CEPC Workshop

Contents

Workshop by: F. Zimmermann, M. Zanetti, G. Franchetti



ARIES topical workshop on

Storage Rings & Gravitational Waves

SRGW2021

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Virtual workshop

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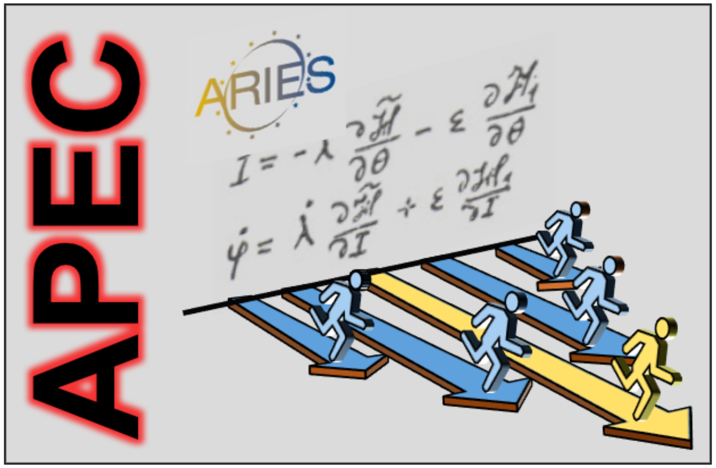


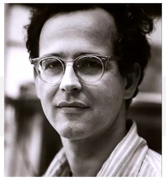
Material from
Jorge Cervantes

1919
Equation of gravitational waves

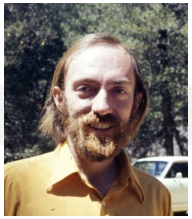
1936
Einstein denies GW

1949
Babson

1957
Chapel Hill Meeting
"Sticky bead" argument by Feynmann

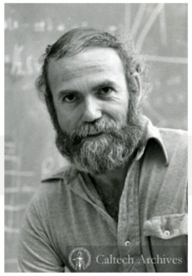


1967
Reiner Weiss



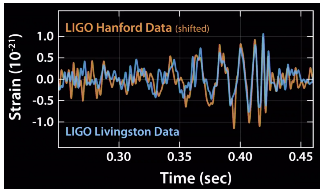
1975
kip Thorne

Interferometers. →
1960 →

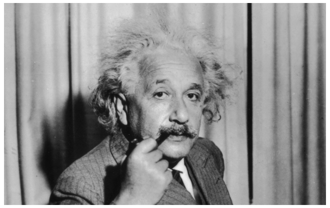


1994
LIGO
Barry Barish

2015
GW detection



1915
General theory of relativity



1922
Eddington

1939-1945
WWII

1956
Pirani Felix

1958
Weber GW antenna

1974-1979
Pulsar
Binary pulsar
Evidence of
Existence of GW

1995
VIRGO construction

2021
Gravitational wave
& Storage rings

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Jörg Wenninger	CERN

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LIGO
VIRGO
KAGRA
LIGO-India

Einstein
Telescope

Cosmic
Explorer

LISA

Space-time geometry

Event $(t, \vec{x})$

Distance between two events

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Metrics (dimensionless)

$g_{\mu\nu}$ Defines the geometric properties of space-time

Einstein field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Stress-energy tensor

Einstein tensor: This is a nonlinear differential function of $g_{\mu\nu}$

(see Appendix)

In absence of gravity

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For weak gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + \underset{\substack{\uparrow \\ \text{perturbation}}}{h_{\mu\nu}}$$

Einstein equation
for weak field

$$\Box \bar{h}^{\mu\nu} = -16\pi \frac{G}{c^4} T^{\mu\nu}$$

(see Appendix)

Waves in $g_{\mu\nu}$

$$\left(-\frac{\partial^2}{\partial^2}+\nabla^2\right)\overline{h}^{\mu\nu}=-16\pi\frac{G}{c^4}T^{\mu\nu}$$

← Source

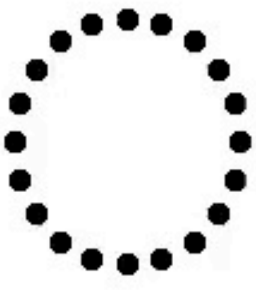
Far field solution: amplitude

$$h_{jk}=\frac{G}{c^4}\frac{2}{r}\frac{d^2Q_{jk}}{dt^2}$$

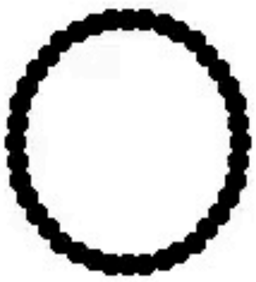
$$Q_{jk}=\int_{source}\rho x_j x_k dx^3$$

Quadrupolar momentum

GW polarization



plus +



cross ×

Amplitude estimates of *h*

$$Q_{jk} = \int_{source} \rho x_j x_k dx^3 \quad \Rightarrow \quad \frac{d^2}{dt^2} Q_{jk} \sim M v^2$$

Non-spherical

Maximum amplitude @ distance r

$$h \lesssim \frac{G}{c^4} \frac{2}{r} M v^2$$

Effect of the metric perturbation on distances

$$\frac{\Delta l}{l} \simeq h$$

Example of GW source

Extreme Amusement Park Attraction

$$\begin{array}{lcl} R & \sim & 10\,\mathrm{m} \\ N_{persons} & \sim & 28 \\ weight & \sim & 100\,\mathrm{Kg} \\ Structureweight & \sim & 6000\,\mathrm{Kg} \\ v_{max} & \sim & \underline{20\,\mathrm{m/s}} \end{array}$$

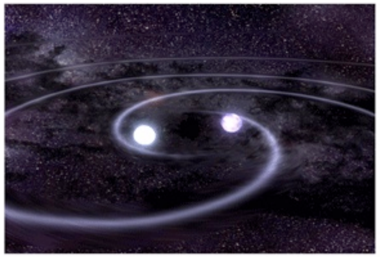
$$f = 0.3\,\mathrm{Hz}$$

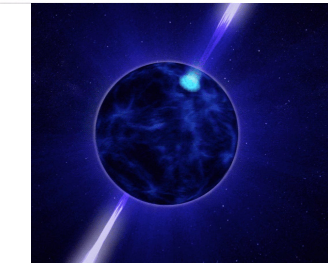
At distance of one wavelength

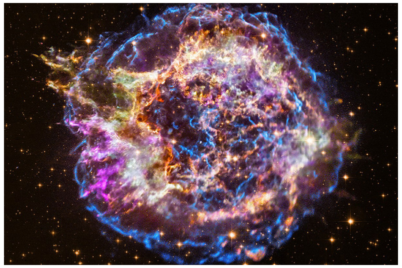
$$r = \lambda = 9.4 \times 10^8\,\mathrm{m}$$

$$h \sim 6 \times 10^{-47}$$

GW Sources







Characteristics of GW sources:
f → frequency
h → strain

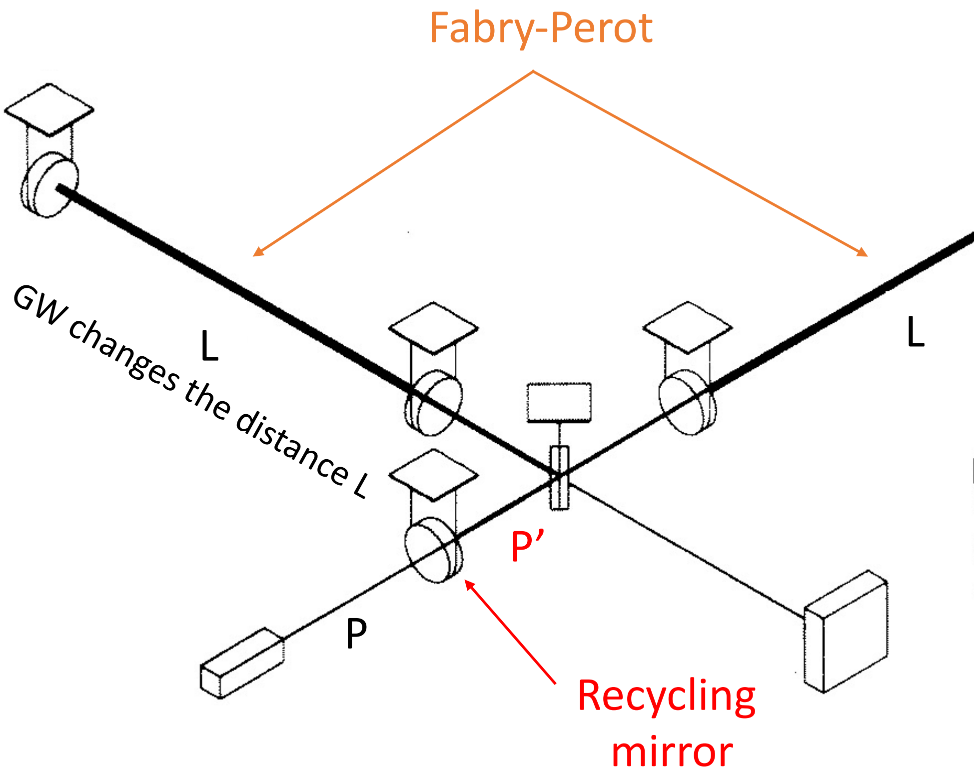
- **Binaries**
have a large and varying quadrupole moment

- **Continuous sources**
A spinning source can emit gravitational waves at a single frequency for a long time. → Neutron stars

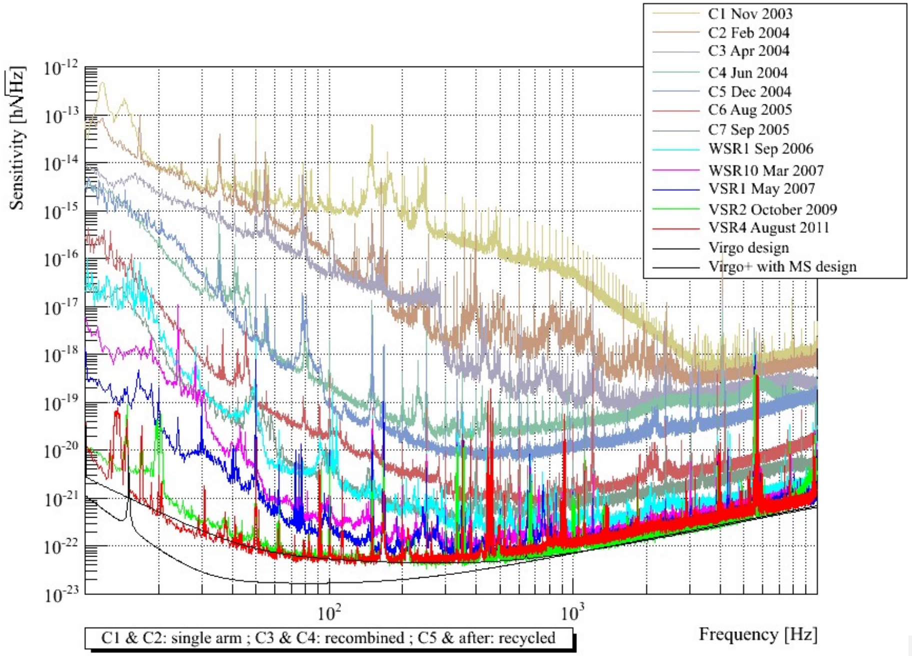
- **Bursts**
Events of very limited duration without any special periodicity.
An example → core-collapse supernova.

- **Stochastic sources**
Broad bands of frequency with many sources. Examples: huge foreground of double white dwarf binaries in our Galaxy, or possibly a background from the very early universe.

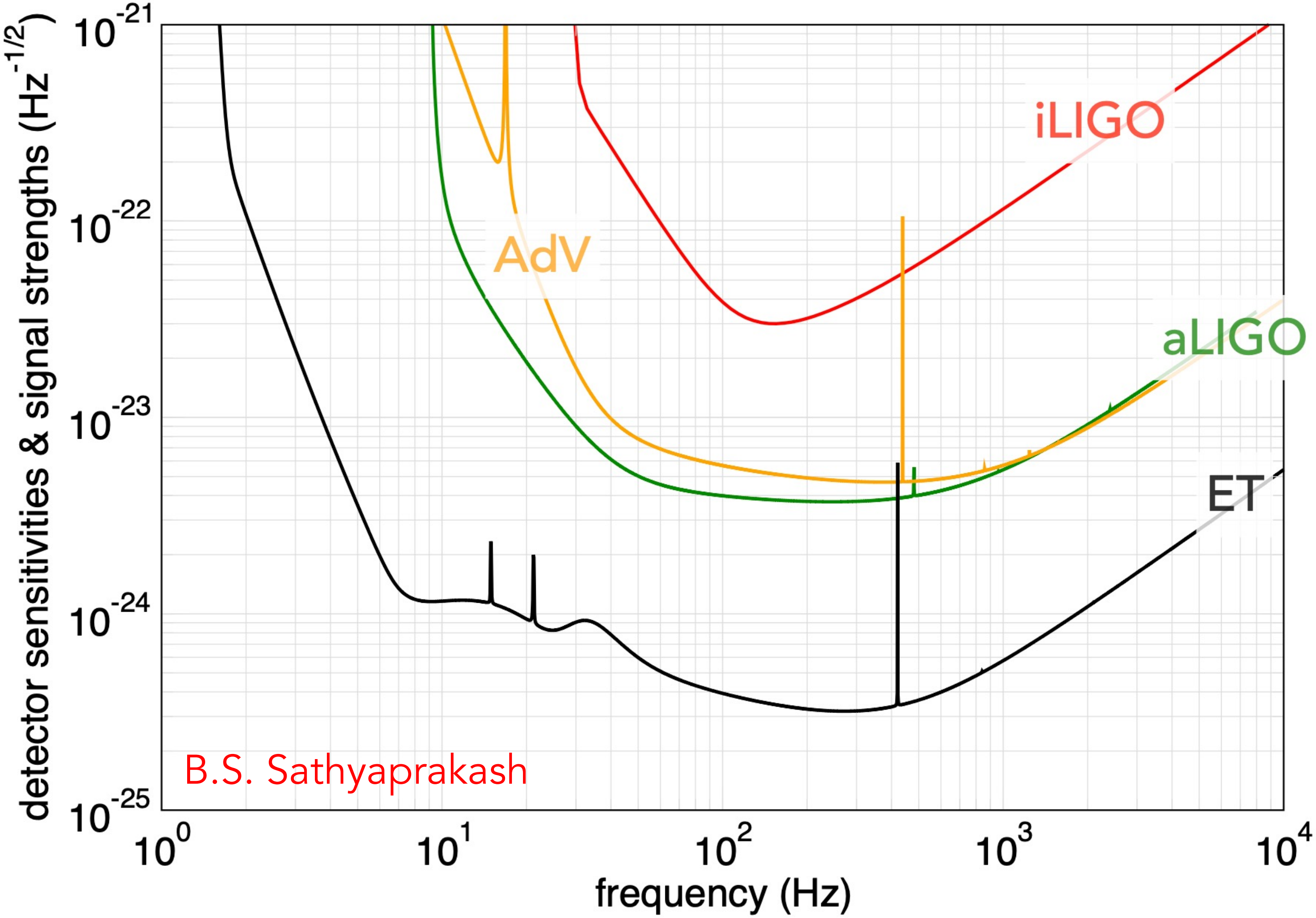
Ground-based LASER interferometry



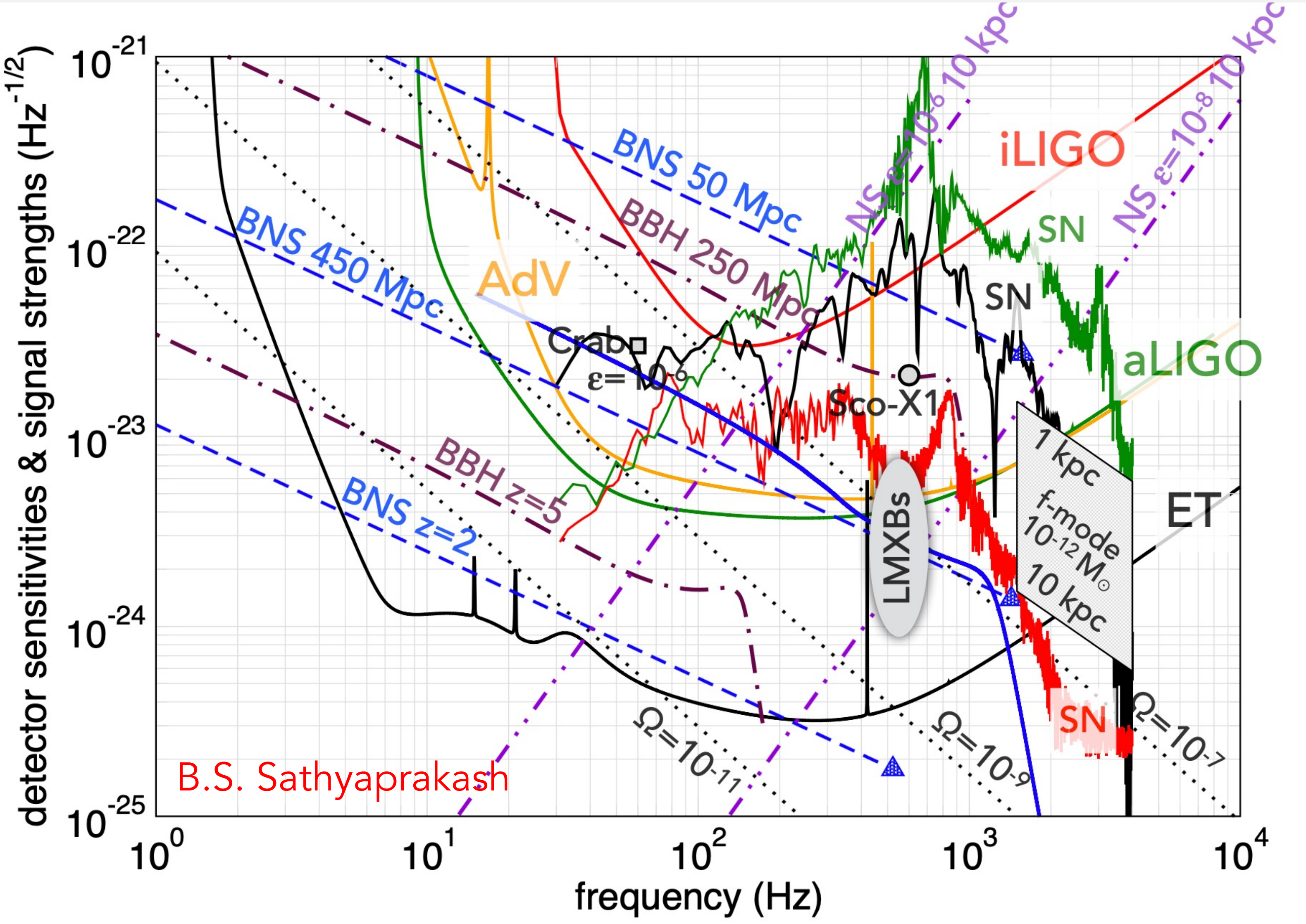
Raffaele Flaminio



LASER interferometers sensitivity

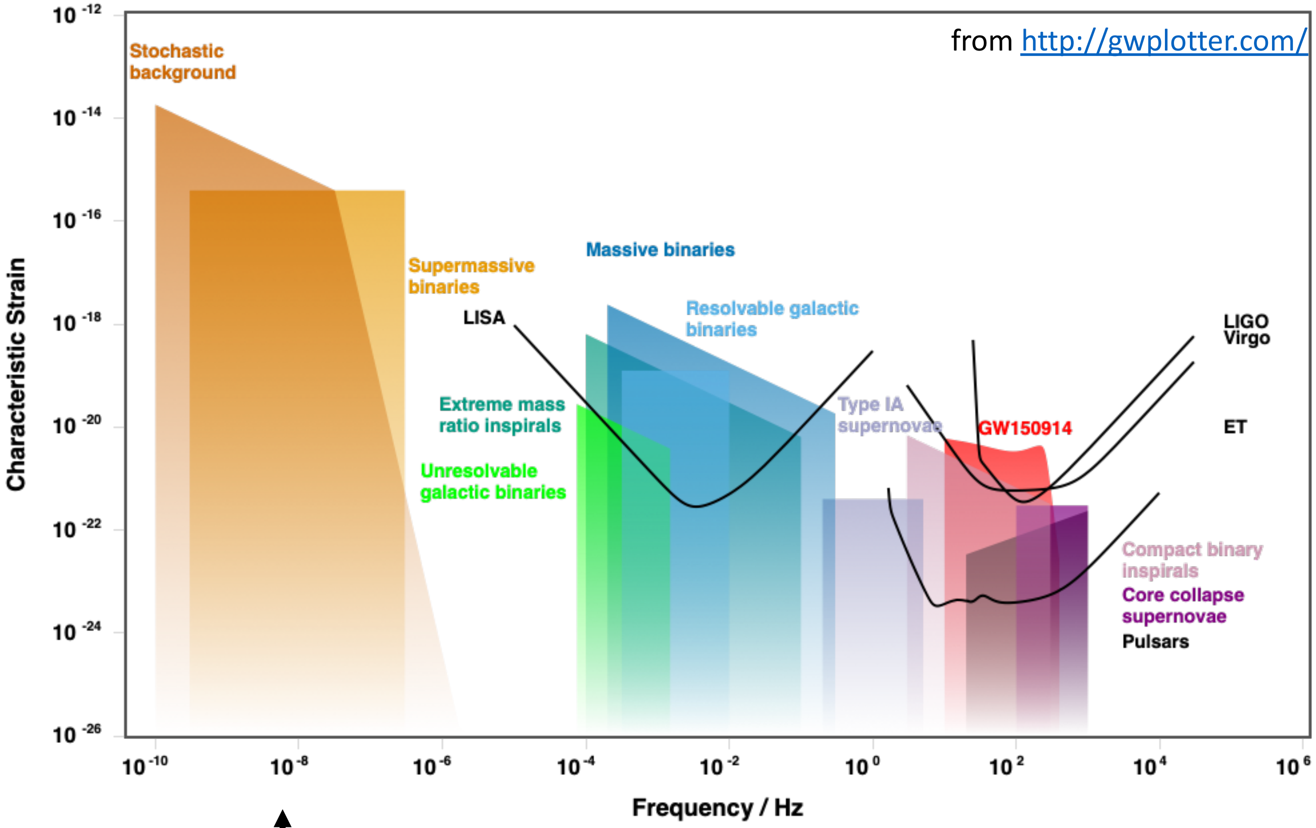


Detector sensitivity and GW sources



B.S. Sathyaprakash

GW sources and antenna sensitivity



from http://gwplotter.com/

↑

Periodicity of GW 3 yrs

GW detection typical figures

1967 Reiner Weiss

1975 kip Thorne

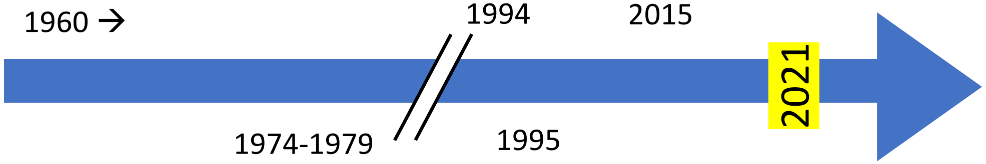
Interferometers. →

1960 →

LIGO

1994 Barry Barish

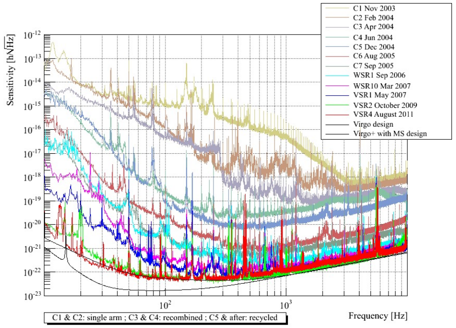
2015 GW detection



1974-1979

Pulsar
Binary pulsar
Evidence of
Existence of GW

1995 VIRGO construction



$$h \sim 10^{-21}$$

$$\Delta x \sim 3 \times 10^{-18}\mathrm{m}$$

Power variation: 2 x 10^{-8} W

Fight against

1. Readout noises
2. Displacement noises

20 years of improvement…

New ideas for GW detection

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Gravitational Waves

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Storage rings and GW

Observables

- Storage ring circumference changes
- Revolution time around the storage ring changes
- Enhanced beam oscillation by GW

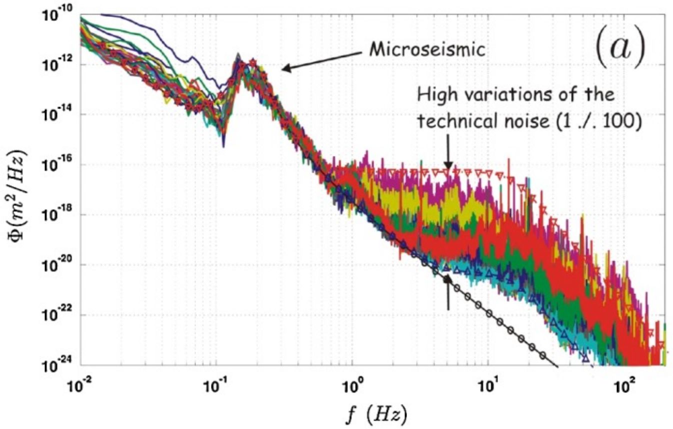
Issues

- Level of vibrations in storage rings, and beam response to vibrations, earthquakes, tides
- Can a GW be a driving term for a ring **resonance**?
- Extreme disturbances

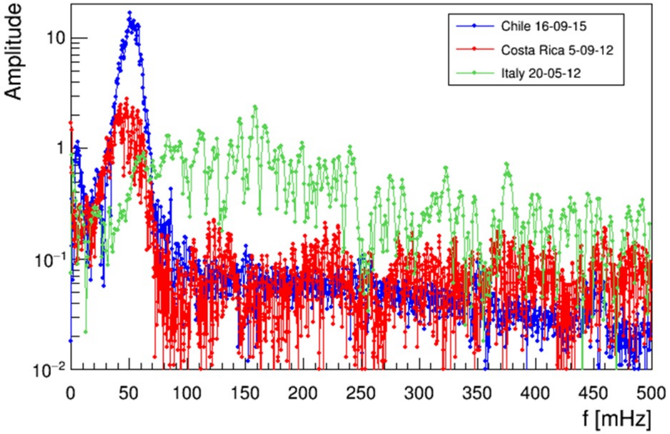
Vibrations and noise in the LHC

J. Wenninger

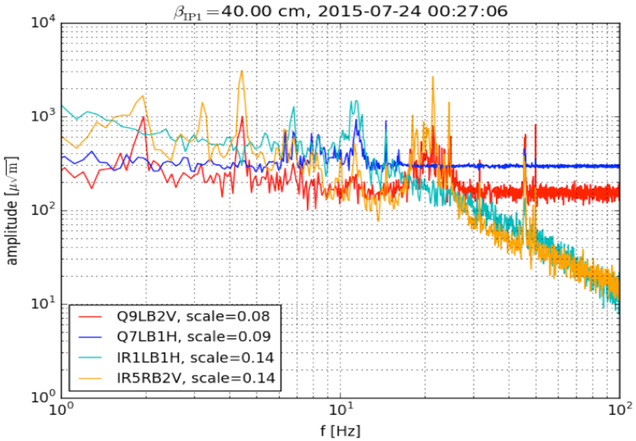
Noise on the tunnel



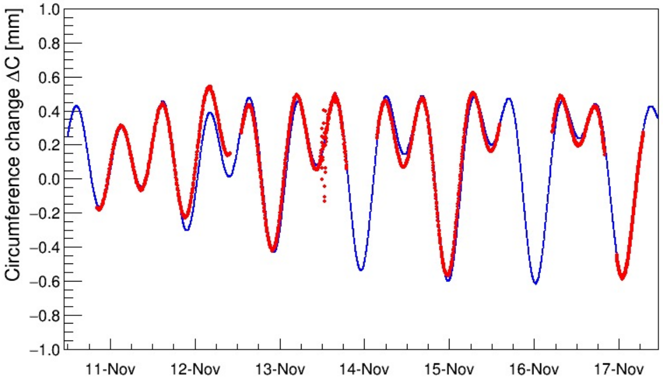
Earthquakes observation at LHC



Beam oscillations



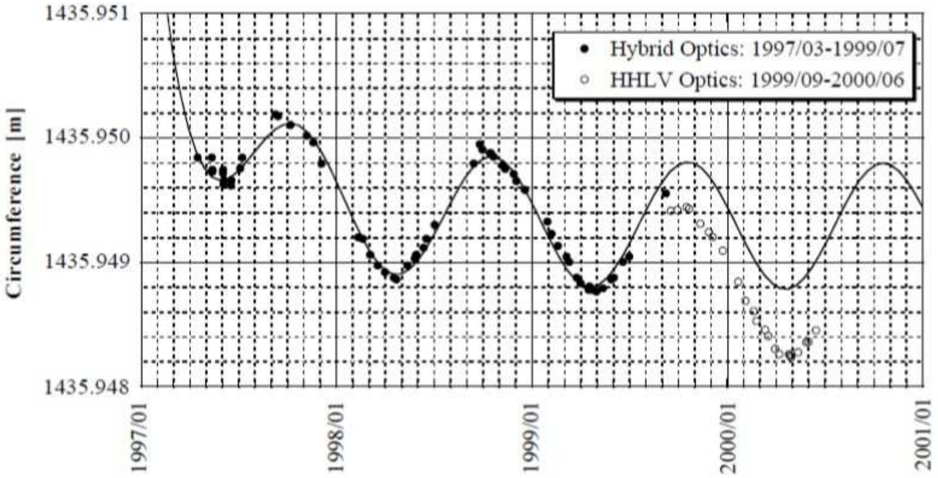
Tides @ LHC



frequency range ≈ 0.1-1 Hz provides the **highest sensitivity at the level of ≈10^{-12} for relative circumference changes**.

Interpretation of SPring-8 observations

Spring-8 seasonal variations of machine Circumference and damping



Gravitational strain on Earth

$$\frac{\Delta C}{\Delta t} = \left(\frac{\Delta C}{\Delta t}\right)_{\text{tid.}} + \left(\frac{\Delta C}{\Delta t}\right)_{\text{seas.}} + \left(\frac{\Delta C}{\Delta t}\right)_{\text{gw}}$$

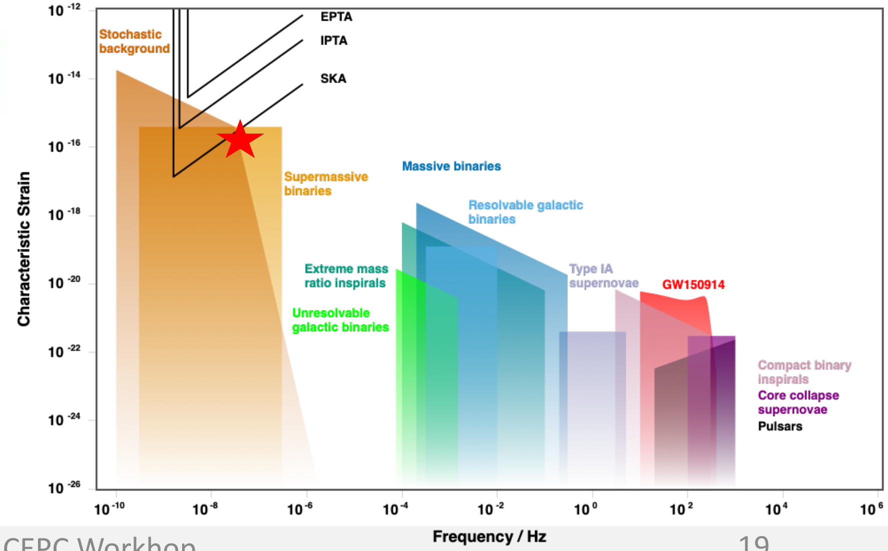
Andrey Ivanov

relic gravitational-wave background

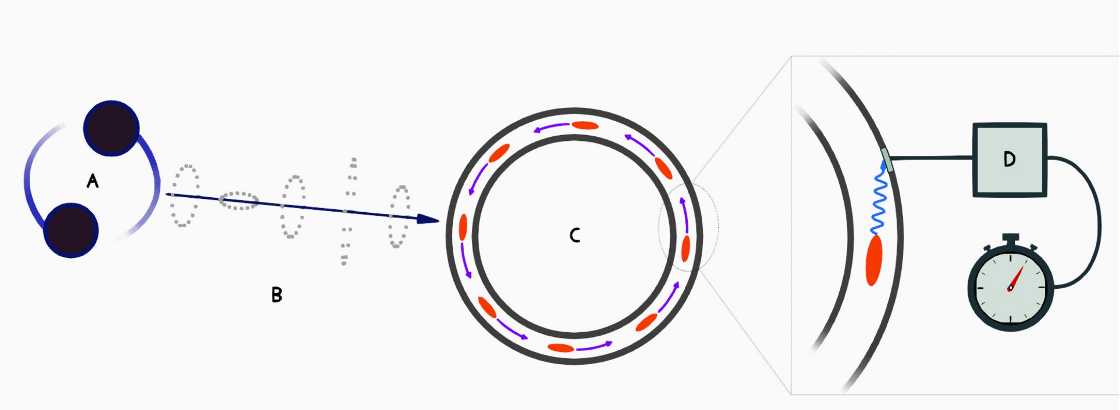
$$f \sim 10^{-7}\,\mathrm{Hz} \quad h \sim 10^{-16}$$

$$\frac{\Delta C_{\text{gw}}(t)}{\Delta t} = -2 \times 10^{-4}\,\mathrm{m/yr}$$

consistent with Spring-8 data



Travel time on LHC



$$\Delta T_{GW} = \frac{1}{2}\left(1-\frac{v_0^2}{2c^2}\right)\int_0^T h_+(t).F_+(t)dt$$

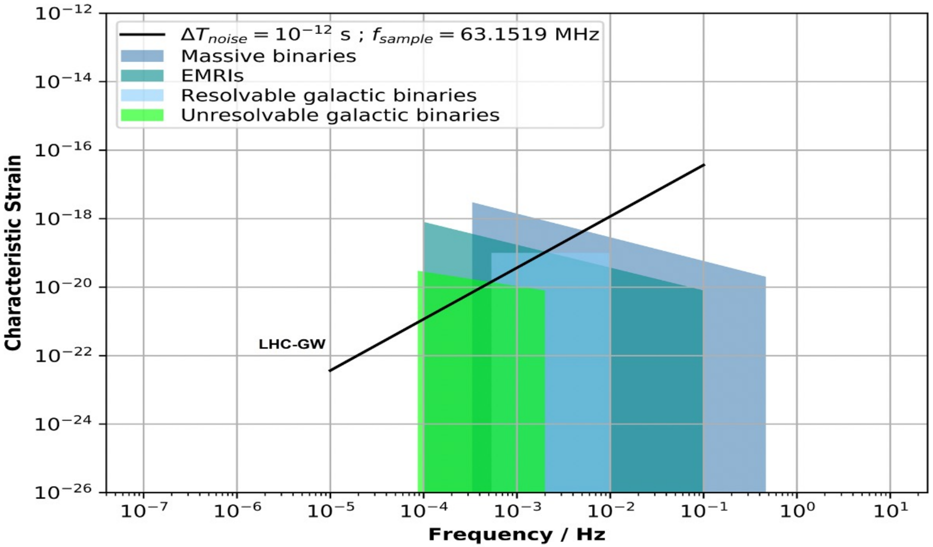
$$F_+ = \sin^2\theta\cos 2\psi$$

$$\Delta T_{\rm GW} \sim a\frac{h_0}{f_{\rm GW}},$$

Change in travel time due to change in test mass velocities, not orbit distortion!

Travel time orbit distortion → h^2

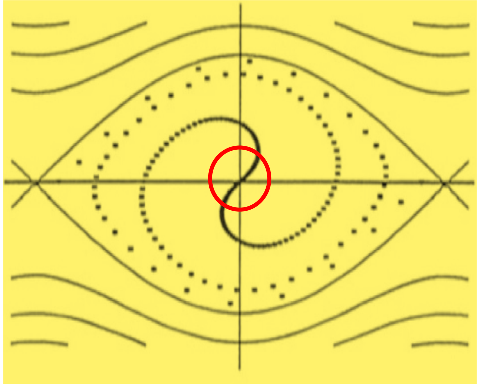
Travel time velocity change → h



Suvrat Rao

Oscillations on the longitudinal plane

R. Tito d'Agnolo



$$\ddot{\delta}_l + \frac{\dot{\delta}_l}{\tau_l} + \omega_l^2 \delta_l = \omega_g^2 f(\omega_g, t)$$

$$f(\omega_g, t) \simeq h \times L \times \cos(\omega_g t + \phi)$$

On the resonance

$$\boxed{\delta_t = \frac{\delta_l}{c} \simeq (hT)(\omega_l \tau_l)}$$

From phase measurement in an RF cavity the experimental → $\Delta T/T = 10^{-7}$

$$\omega_l \sim 10 \;\; H_z \qquad \tau_l = 1 \;\; hour$$

$h > 10^{-11}$

Slower proton →

3 orders of magnitude better

Exploiting transverse ring resonances

K. Oide

$$\frac{\mathrm{d}^2 X_\mu}{\mathrm{d}t^2} = \frac{1}{2}\ddot{h}_{\mu\nu} X^\nu ,$$

$$\frac{\mathrm{d}p_x}{\mathrm{d}s} = -\frac{k^2}{2} hR \cos 2\theta \cos(\omega_{\mathrm{GR}} t)$$

$$x(s) = \int_0^s ds' R_{12}(s,s') \frac{\mathrm{d}p_x}{\mathrm{d}s}(s')$$

$$= \int_0^s ds' \sqrt{\beta_x(s)\beta_x(s')} \sin\left(\psi_x(s) - \psi_x(s')\right) \frac{\mathrm{d}p_x}{\mathrm{d}s}(s') ,$$

Accumulation per turn

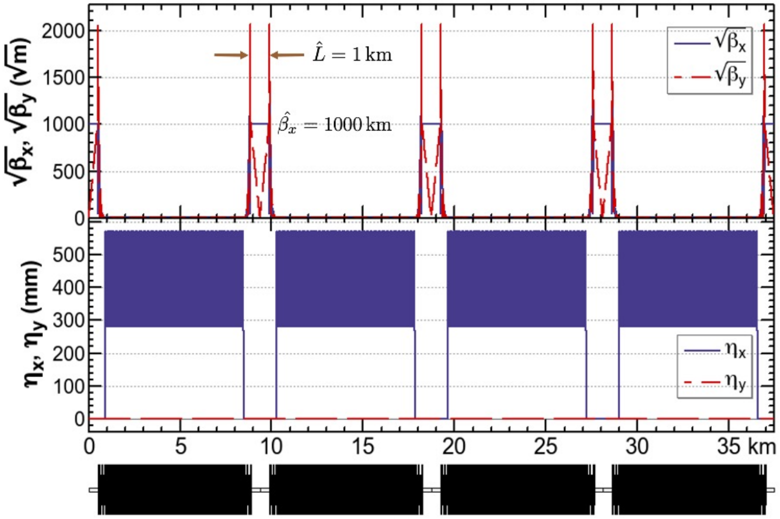
$$\Delta\hat{x} \approx -\frac{k^2 R\hat{L}}{2}\hat{\beta}_x h$$

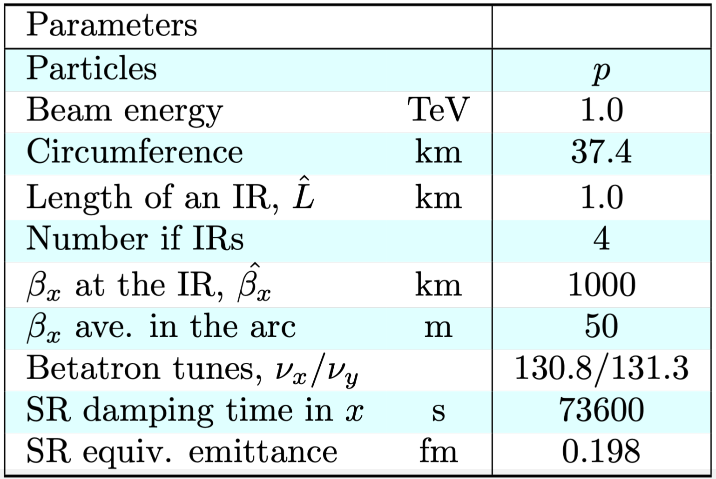
$$\omega_{GR} \sim 1\,\mathrm{MHz}$$

$$h \sim 10^{-22}$$

$$\Delta x \sim 10^{-13}\, m$$

Amplification of the resonance via a design that makes sector with large beta functions





Parameters		
Particles		p
Beam energy	TeV	1.0
Circumference	km	37.4
Length of an IR, $\hat{L}$	km	1.0
Number if IRs		4
β_x at the IR, $\hat{\beta}_x$	km	1000
β_x ave. in the arc	m	50
Betatron tunes, ν_x/ν_y		130.8/131.3
SR damping time in x	s	73600
SR equiv. emittance	fm	0.198

Sensitivity / Noise sources

Estimate sensitivity scaling $fh \sim 10^{-14} \times (\mathrm{BPM\,resol.\,in\,nm})$

* Noise due to the thermal vibration of quadrupoles

$$x_Q \sim 6\,\mathrm{pm}$$
$$x_n \sim 3 \times 10^4 \Delta x_Q = 0.17\,\mu\mathrm{m}$$

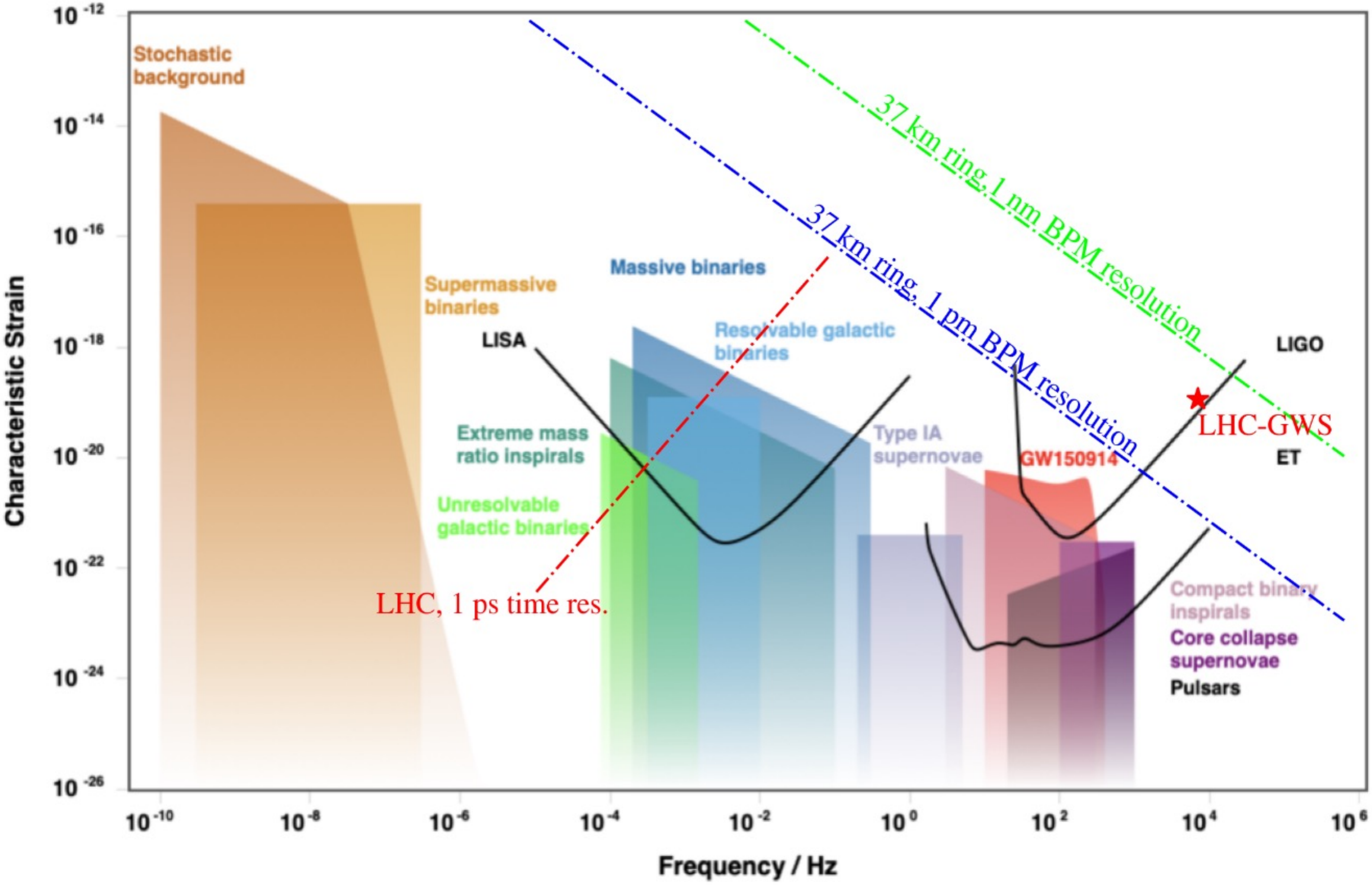
Center of mass fluctuation due to finite number of particle

$$\Delta \hat{x}_s = \sqrt{\frac{\hat{\beta}_x \epsilon_x}{N_p}} = 0.19,\ \mu m$$

K. Oide

after the damping time in LHC → emittance ~ 0.2 fm → Δx_s = 40 pm

* Beam fluctuation due to synchrotron radiation, acceleration



Cern courier
July/August 2021

Concepts to detect GW

- gravitational wave (GW) detection by resonant betatron oscillations, for the 10 kHz range;
- GW detection through the change in revolution period, but using a ``low-energy'' coasting ion beam without a longitudinally focusing radiofrequency system – Can the sensitivity be down to 0.01 mHz?

Storage Rings for GW

- Heterodyne detection using superconducting radiofrequency cavities, with a sensitivity possibly up to 10 MHz.
- Possibility of using an LHC access shaft to house a 100 m atom interferometer targeting the 1 to 0.01 Hz range.

Novel Methods interferometry

- GW generated by the beam, and the orbital frequency ~10 kHz for LHC, at the LHC and FCC-hh bunch frequency of 40 MHz, or, with a Gertsenshtein signal in the 10 THz range – combined with a high-frequency detector concept. By **Pisin Chen (NTU Taiwan)**

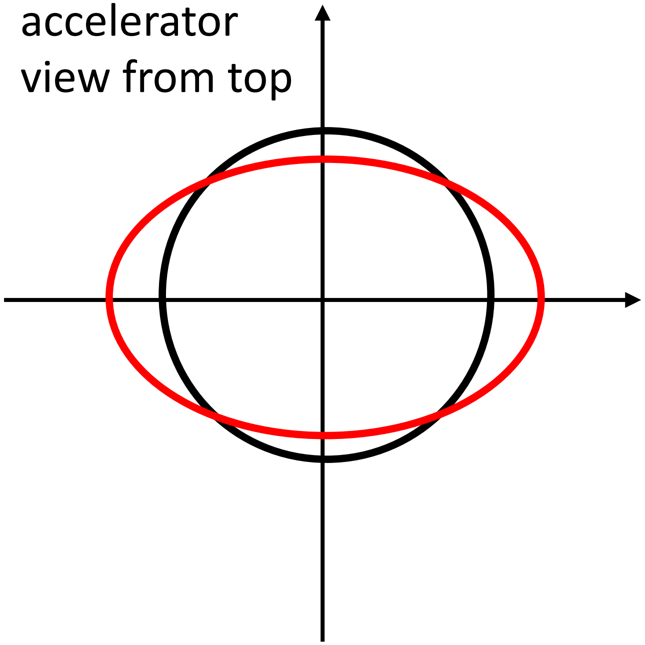
Advanced Concepts

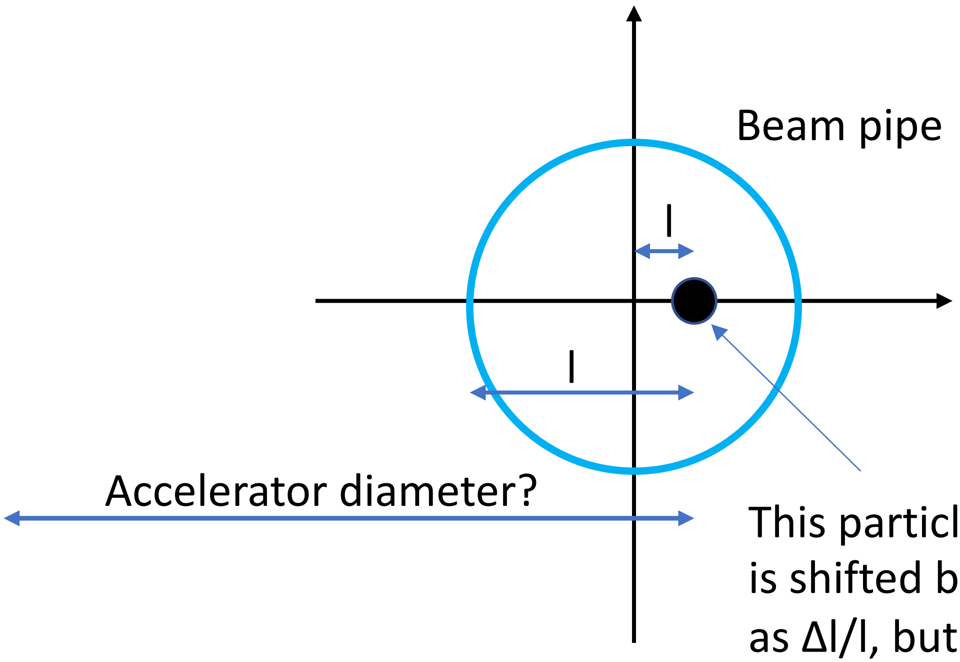
Confusions

$$\frac{\Delta l}{l} \simeq h$$

What is it the effect of GW on a beam particle?

accelerator
view from top





This particle
is shifted by GW
as Δl/l, but from
where?

'I had nothing to offer anybody except my own confusion.' - Jack Kerouac

Important Subtleties ….

General Relativity community

Accelerator community

$$\ddot{x}^\mu + \Gamma^\mu_{\lambda\nu}\dot{x}^\lambda\dot{x}^\nu = \frac{q}{m}F^\mu{}_\nu\dot{x}^\nu$$

$$\begin{aligned}\frac{dx}{ds^2} + k_x(s)x(s) &= F_x(x,y)\\ \frac{dy}{ds^2} + k_y(s)y(s) &= F_y(x,y)\end{aligned}$$

Force

Force by GW on a beam particle?

Summary/Outlook

- Detection of gravitational waves has a strong interest for testing GR and understanding cosmo

  - Proposed methods: use detection of
    1. Travel time
    2. Frequency shifts
    3. GW resonate with beam oscillation

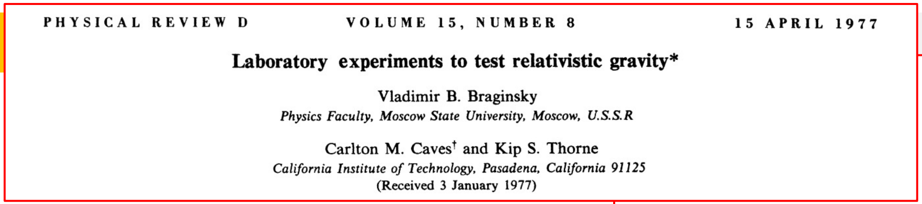
- Sensitivity of the methods discussed

- There are still difficulties to formulate the particle accelerator beam dynamics under the influence of GW

- Noise influence on the beam is presently large with respect to amplitudes to be measured

- New approaches: atomic interferometry. Existing ideas are revived → SRF cavity

Renaissance: exploring storage rings for detecting GW

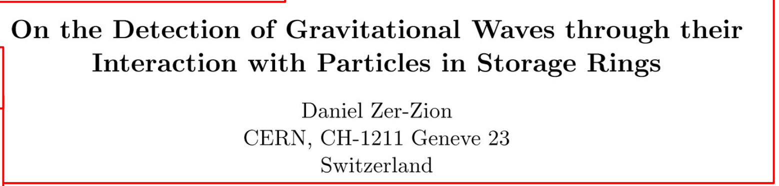


PHYSICAL REVIEW D VOLUME 15, NUMBER 8 15 APRIL 1977

Laboratory experiments to test relativistic gravity*

Vladimir B. Braginsky
Physics Faculty, Moscow State University, Moscow, U.S.S.R

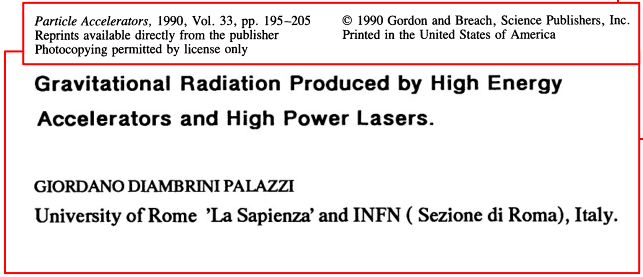
Carlton M. Caves[†] and Kip S. Thorne
California Institute of Technology, Pasadena, California 91125
(Received 3 January 1977)



CERN-EP/98-63
March 25, 1998

On the Detection of Gravitational Waves through their Interaction with Particles in Storage Rings

Daniel Zer-Zion
CERN, CH-1211 Geneve 23
Switzerland



Particle Accelerators, 1990, Vol. 33, pp. 195–205
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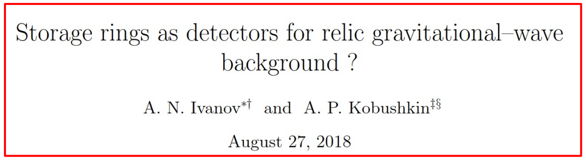
Gravitational Radiation Produced by High Energy Accelerators and High Power Lasers.

GIORDANO DIAMBRINI PALAZZI
University of Rome 'La Sapienza' and INFN (Sezione di Roma), Italy.



Storage rings as detectors for relic gravitational–wave background ?

A. N. Ivanov*† and A. P. Kobushkin‡§

August 27, 2018

ARIES topical workshop on
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SRGW2021

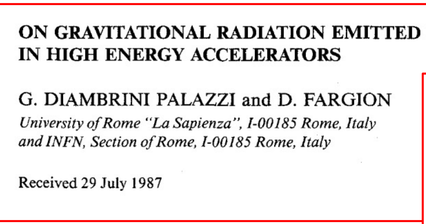
Chairs: G. Franchetti GSI, M. Zanetti UNIPD, F. Zimmermann CERN

International Committee
William Barletta MIT
Pisin Chen NTU
Raffaele Tito D'Agnolo IPHT
Shyh-Yuan Lee Indiana U
Katsunobu Oide CERN & KEK

Virtual workshop
https://indico.cern.ch/event/982987/

1977 1990 1998 2018 2021

1987 1994 1999 2020



ON GRAVITATIONAL RADIATION EMITTED BY CIRCULATING PARTICLES IN HIGH ENERGY ACCELERATORS

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RESONANT PHOTON-GRAVITON CONVERSION IN EM FIELDS: FROM EARTH TO HEAVEN*

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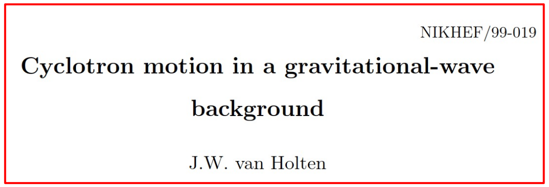
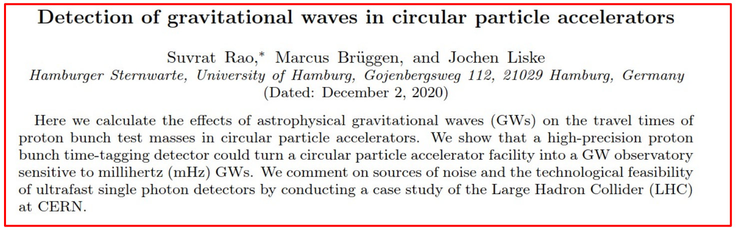


NIKHEF/99-019

Cyclotron motion in a gravitational-wave background

J.W. van Holten



Detection of gravitational waves in circular particle accelerators

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(Dated: December 2, 2020)

Here we calculate the effects of astrophysical gravitational waves (GWs) on the travel times of proton bunch test masses in circular particle accelerators. We show that a high-precision proton bunch time-tagging detector could turn a circular particle accelerator facility into a GW observatory sensitive to millihertz (mHz) GWs. We comment on sources of noise and the technological feasibility of ultrafast single photon detectors by conducting a case study of the Large Hadron Collider (LHC) at CERN.

Thank you for the attention

Appendix

Einstein tensor. → $$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

$$h = \eta^{\alpha\beta} h_{\alpha\beta} \qquad \overline{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h$$

For weak gravity the Einstein tensor reads

$$G_{\alpha\beta} = -\frac{1}{2}[\bar{h}_{\alpha\beta,\mu}{}^{,\mu} + \eta_{\alpha\beta}\bar{h}_{\mu\nu}{}^{,\mu\nu} - \bar{h}_{\alpha\mu,\beta}{}^{,\mu} - \bar{h}_{\beta\mu,\alpha}{}^{,\mu} + 0(h^2_{\alpha\beta})].$$