Analysis of coherent beam-beam instability with longitudinal impedance at CEPC

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Simulation procedures

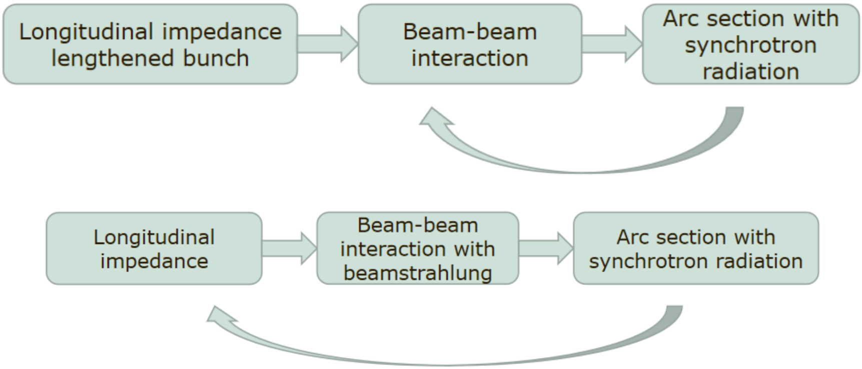

Figure: Conventional v.s. self-consistent beam-beam simultion with ZL

Simulation results

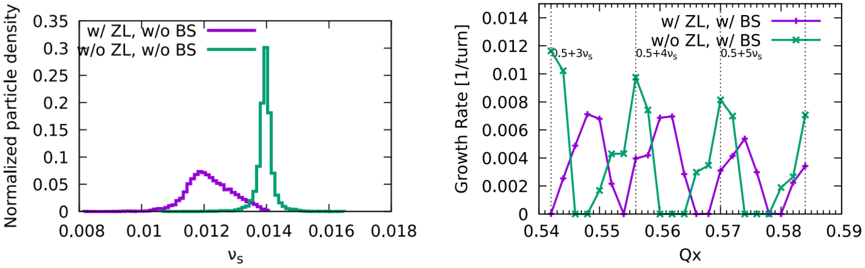

Figure: Simulation results w/ and w/O ZL

1. The shift of stable tune area.
2. The squeeze of stable tune area.
3. Decrease of growth rate.

Cross-wake force model

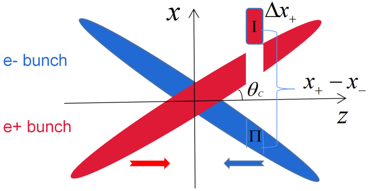

Figure: Illustration of the evaluation of cross wake force.

The "cross-wake force"[1] has been introduced to explain the coherent beam-beam instability with a large Piwinski angle without ZL.

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} W_x^{(-)}\left(z - z'\right) \rho^{(+)}(z') \cdot x^{(+)}(z') dz' \tag{1}$$

where $W_x^{(-)}(z)$ is cross-wake function induced by beam-beam interaction.

[1] **K.Ohmi et al.**, **Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle**, **PRL (2017)**.

σ mode and π mode

$\rho_x(z) \equiv \rho(z)x(z)$

Assuming σ mode $\rho_x^{(+)}(z) = \rho_x^{(-)}(z)$ and π mode $\rho_x^{(+)}(z) = -\rho_x^{(-)}(z)$, the beam-beam kick can be expressed by usual formula of a normal wake force for single beam.

$$\Delta p_x(z) = \mp \int_{-\infty}^{\infty} W_x^{(\pm)}\left(z - z'\right) \rho_x\left(z'\right) dz' \tag{2}$$

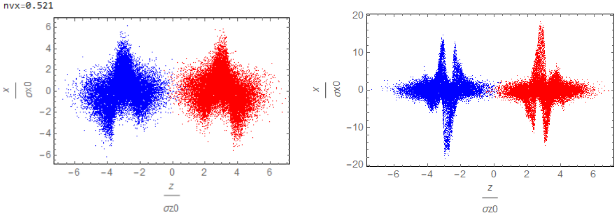

Figure: σ mode v.s. π mode

Single bunch instability theory

Beam-beam kick,

$$\Delta p_x(z) = \mp \int_{-\infty}^{\infty} W_x^{(\pm)} \left(z - z'\right) \rho_x \left(z'\right) dz' \quad (3)$$

1. Ordinary transverse mode coupling instability(TMCI) theory[2].
2. Transverse mode coupling method for localized wake force.
   For example,
   - Transverse mode coupling method for localized impedance structures by Ruggiero[3]
   - Discretize the longitudinal phase space (J, ϕ)[4]
   - Dipole modes expaned by azimuthal and radial modes[5]
   - ...
3. Longitudinal action discretization proposed by Oide[6] which includes the incoherent synchrotron tune shift due to the potential well distortion(longitudinal instability).

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[2] **A. Chao**, **Physics of Collective Beam Instabilities in High Energy Accelerators** , **New York**, **1993**.

[3] **F. Ruggiero**, **Transverse mode coupling instability due to localized structure**, **Part. Accel. 20**, **45 (1986)**.

[4] **K.Ohmi et al.**, **Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle**, **PRL (2017)**.

[5] **K. Nami et al. PhysRevAccelBeams.21.031002**.

[6] **K. Oide and K. Yokoya**, **KEK Report No. 90-10**, **1990.**.

Longitudinal beam dynamics

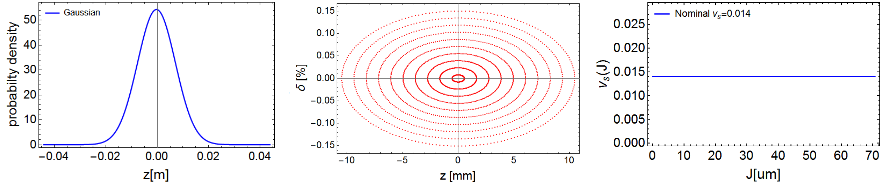


Figure: Longitudinal beam dynamics without longitudinal impedance

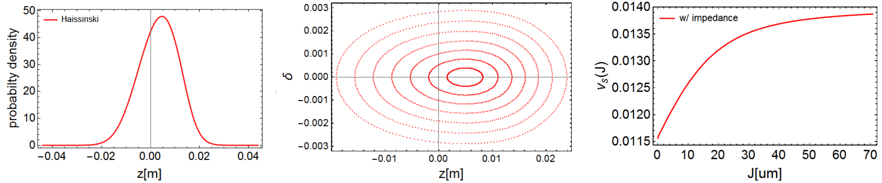


Figure: Longitudinal beam dynmics with longitudinal impedance

Action discretization method

Consider the dipole amplitude $x(J,\phi), p_x(J,\phi)$ in longitudinal phase, we expand them as Fourier series and truncate l at $\pm l_{max}$, and discretize J at $J_1, J_2, ..., J_{n_J}.$,

$$x(J_i,\phi) = \sum_{l=-l_{max}}^{l_{max}} x_l(J_i)e^{il\phi}, \quad p_x(J_i,\phi) = \sum_{l=-l_{max}}^{l_{max}} p_l(J_i)e^{il\phi}$$

We consider the transformation of the discreted dipole moment vector $(x_l(J_i), p_l(J_i))$.

In the arc section,

$$\begin{pmatrix} x_l\left(J_i\right) \\ p_l\left(J_i\right) \end{pmatrix} = e^{-2\pi i l \nu_s(J_i)} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \begin{pmatrix} x_l\left(J_i\right) \\ p_l\left(J_i\right) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l\left(J_i\right) \\ p_l\left(J_i\right) \end{pmatrix}$$

Action discretization method

At the IP

$$\Delta p_l\left(J_i\right) = \mp\frac{\beta_x}{2\pi}\sum_{l'}\sum_{i'}\Delta J_{i'}W_{ll'}\left(J_i, J_{i'}\right)\psi\left(J_{i'}\right)x_{l'}\left(J_{i'}\right) \equiv \beta_x M_{lil'i'}x_{l'}\left(J_{i'}\right) \tag{4}$$

or in a more concise form

$$M_W = \begin{pmatrix} 1 & 0 \\ \beta_x M_{lil'i'} & 1 \end{pmatrix} \tag{5}$$

Finally, the stability of the colliding beams is determined by the eigenvalues $\lambda's$ of the revolution matrix $M_0 M_W$.

Cross-check without ZL

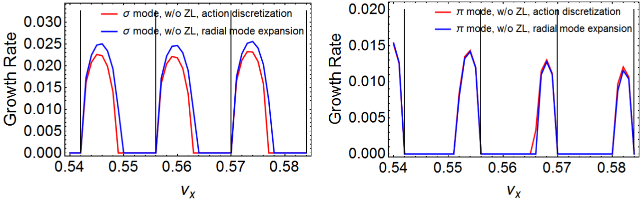


Figure: Comparision of growth rate v.s. ν_x without ZL.

The agreement between the two methods is satisfying. This could be a crosscheck for our formalism.

Eigenvector without ZL

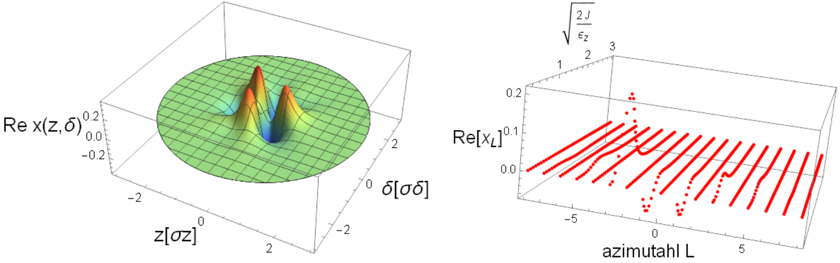


Figure: Real part of eigenvector $x_l(J_i)$ with fastest growth rate, where $\nu_x = 0.546$

The excited modes exist only in l with same parity, that is, $l = \pm 1, \pm 3, \pm 5....$ and no for $l = 0, \pm 2, \pm 4....$. That is to say, the unstable eigenmode is induced by the coupling of modes with the same parity, and there is no mode-mixing with different parities

Growth rate versus tune with ZL

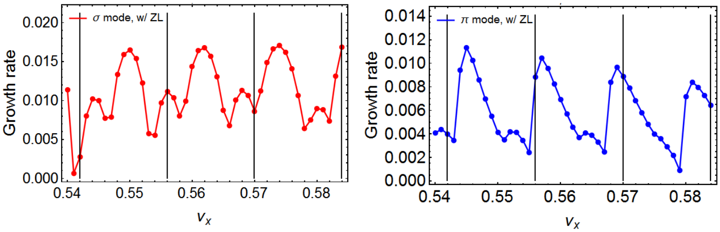


Figure: Growth rate v.s. ν_x considering the longitudinal wake. Red is for σ mode, and blue is for π mode.

The gap $\Delta\nu$ is reduced from 0.014 to 0.011. Noted that $\nu_s(J) \approx 0.011$ is the synchrotron tune of small amplitude particles. The original stable areas become unstable for both σ mode and π mode.

Eigenvector with ZL

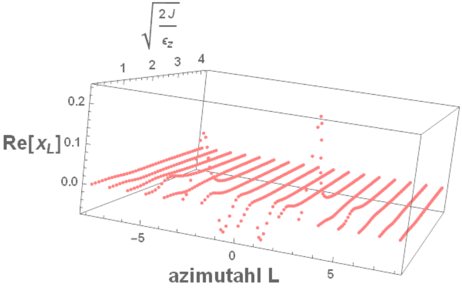
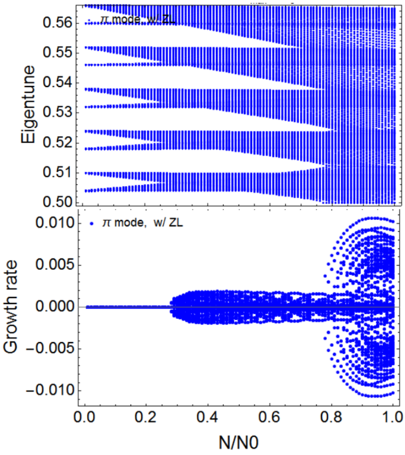


Figure: Real part of eigenmode with fastest growth rate, where $\nu_x = 0.546$

As shown before, without considering the effects of impedance, only the same parity modes would couple with each other. But now, with the distortion of particle's trajectory,all modes are excited and could be mixing.

Bunch population scanning



Eigenvalues as function of bunch population for π mode with longitudinal impedance, where $\nu_x = 0.546$. There are two instability thresholds.

Coherent beam-beam instability considering quadrupole force

The total cross-wake force induced by beam-beam interaction is composed of two parts: dipole and quadrupole terms (beam-beam tune-shift term),

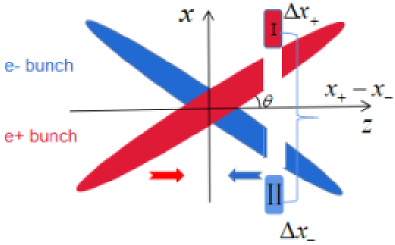


Figure: Illustration of the evaluation of cross wake force.

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} W_x^{(-)}\left(z - z'\right)\rho^{(+)}(z') \cdot x^{(+)}(z')dz' + \int_{-\infty}^{\infty} W_x^{(-)}\left(z - z'\right)\rho^{(+)}(z')dz' \cdot x^{(-)}(z) \qquad (6)$$

With beam-beam tune-shift

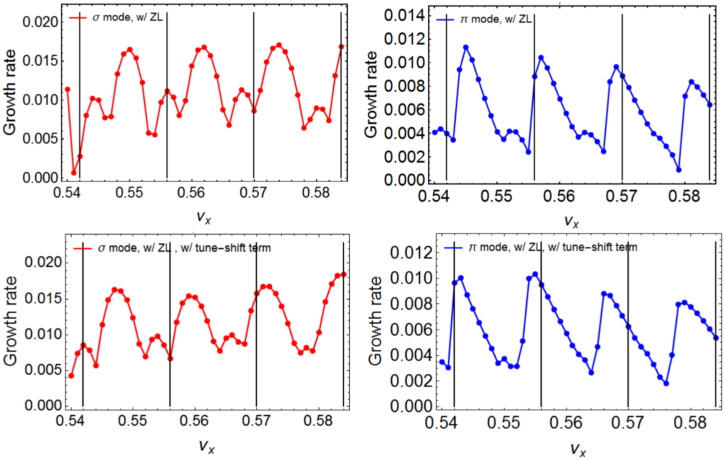


Figure: Growth rate v.s. ν_x w/ and w/o beam-beam tune-shift term. Red is for σ mode, and blue is for π mode.

Summary

The beam-beam coherent head-tail instability in collision with a large crossing angle is strongly dependent on the longitudinal beam dynamics. The longitudinal impedance would distort the distribution and introduce an incoherent synchrotron tune shift.

1. The shift of stable tune area.
2. The squeeze of stable tune area.
3. Modes with different parities could be coupled with each other.

Thank you for your attention!