

Precise measurement of the Z resonance parameters at CEPC

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IHEP

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Outline

- Motivation
- Theoretical framework
- Statistical and systematic uncertainties
- Data-taking strategy
- Summary & Outlook

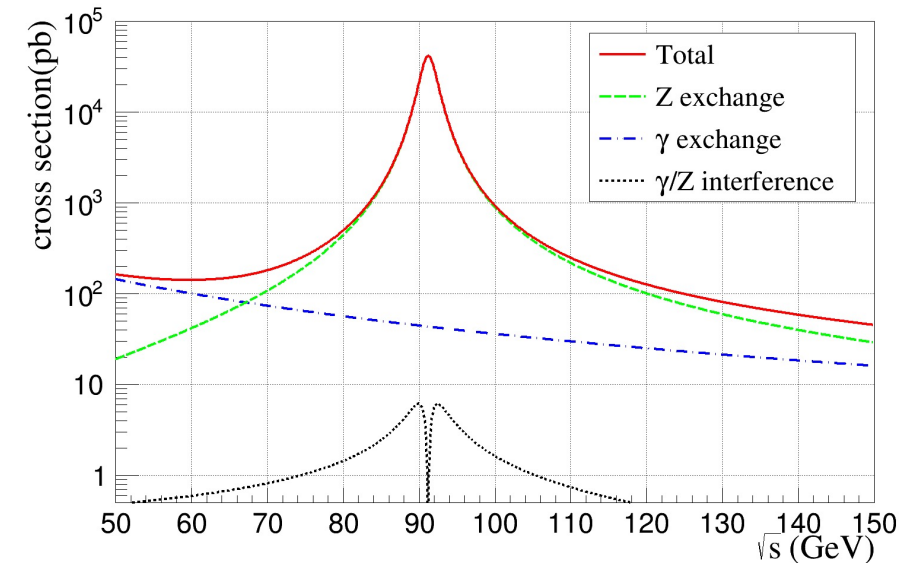
Motivation

- Properties of Z boson are fundamental parameters and could provide stringent tests of the SM.
- Possible new physics beyond the SM might be revealed through the subtle changes of measured results for example R_b , $A_{\text{FB}}^{0,b}$, ...

	measured value	SM prediction
R_b^0	0.21629 ± 0.00066	0.21578 ± 0.00011
$A_{\text{FB}}^{0,b}$	0.0992 ± 0.0016	0.1032 ± 0.0004
\mathcal{A}_b	0.923 ± 0.020	0.93463 ± 0.00004

Gfitter, EPJC 74, 3046

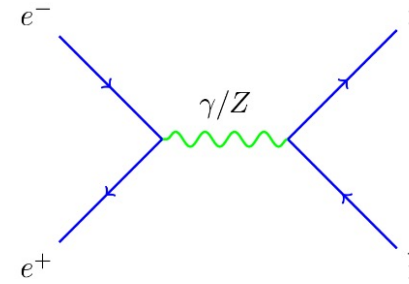
- Develop data-taking strategy to maximize the performance & to optimize the design.



Theoretical framework

Theoretical framework

- Main process: $e^+e^- \rightarrow f\bar{f}$ around Z pole
- In lowest order & neglecting the fermion masses :



$$\frac{2s}{\pi N_c} \frac{d\sigma_{f\bar{f}}}{d\cos\theta} = \alpha^2 Q_f^2 (1 + \cos^2\theta) + 8 \operatorname{Re} \left\{ \alpha Q_f \chi^*(s) \left[C_{\gamma Z}^s (1 + \cos^2\theta) + 2C_{\gamma Z}^a \cos\theta \right] \right\} + 16 |\chi(s)|^2 \left[C_{ZZ}^s (1 + \cos^2\theta) + 8C_{ZZ}^a \cos\theta \right],$$

with

$$\chi(s) = \frac{G_F M_Z^2}{8\pi\sqrt{2}} \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z},$$

$$C_{\gamma Z}^s = g_{Ve}g_{Vf},$$

$$C_{\gamma Z}^a = g_{Ae}g_{Af},$$

[Ref: Eur.Phys.J.C 19 \(2001\) 587-651](#)

$$C_{ZZ}^s = (g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2),$$

$$C_{ZZ}^a = g_{Ve}g_{Ae}g_{Vf}g_{Af}.$$

Theoretical framework

- Integrated over the angular space

Z-exchange term :

$$\sigma_{\bar{f}f}^Z = \sigma_f^0 \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2},$$

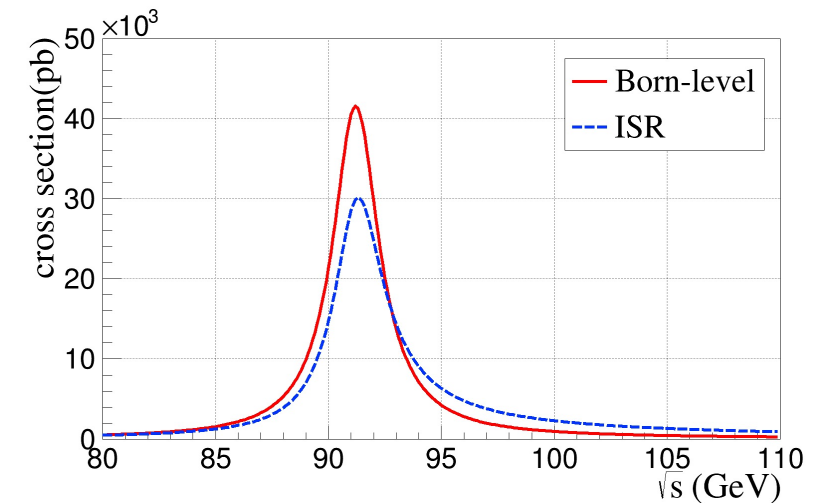
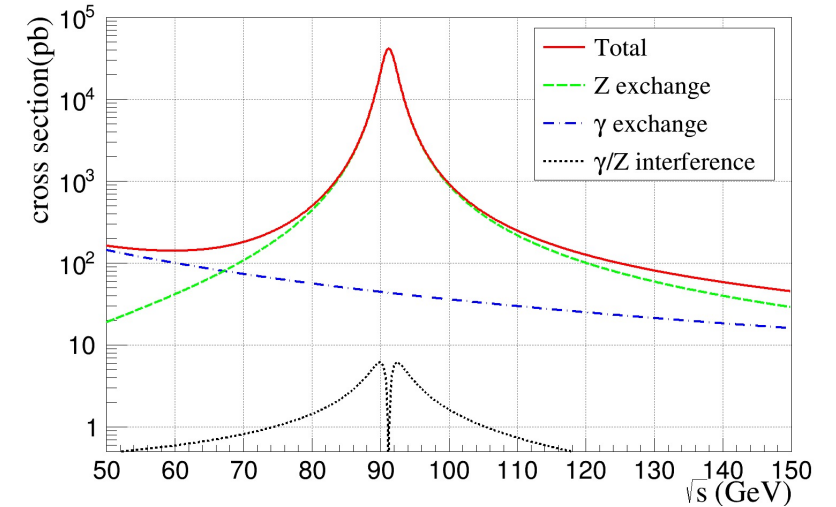
$$\sigma_f^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2} = \frac{C_{ZZ}^s N_c}{6\pi} \left(\frac{M_Z^2 G_F}{\Gamma_Z} \right)^2$$

- Forward-backward asymmetry A_{FB} :

$$A_{\text{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

- ISR

$$\sigma_{\bar{f}f}^{\text{obs}}(s) = \int_0^{1-s_m/s} dx \sigma(s(1-x)) F(x, s)$$



Theoretical framework

- $\sigma_{\text{ff}} = \sigma_{\text{ff}}(M_Z, \Gamma_Z, \sigma_{\text{had}}^0, C_{ZZ}^s, C_{\gamma Z}^s)$, $A_{\text{FB}}^{ll} = A_{\text{FB}}(M_Z, \Gamma_Z, C_{ZZ}^s, C_{ZZ}^a, C_{\gamma Z}^s, C_{\gamma Z}^a)$
- Parameter set: M_Z Γ_Z σ_{had}^0 C_{ZZ}^s C_{ZZ}^a $C_{\gamma Z}^s$ $C_{\gamma Z}^a$ [Ref: Eur.Phys.J.C 19 \(2001\) 587-651](#)
- Parameters can be obtained by fitting to the $N_{\text{obs}}(\sigma_{\text{ff}}, A_{\text{FB}}^{ll})$
- Use toy MC to generate N_{obs} and perform χ^2 fits to evaluate the uncertainties

Statistical and systematic uncertainties

Statistical and systematic uncertainties

- **Statistical uncertainties**

- For single data point:

- Cross-section :

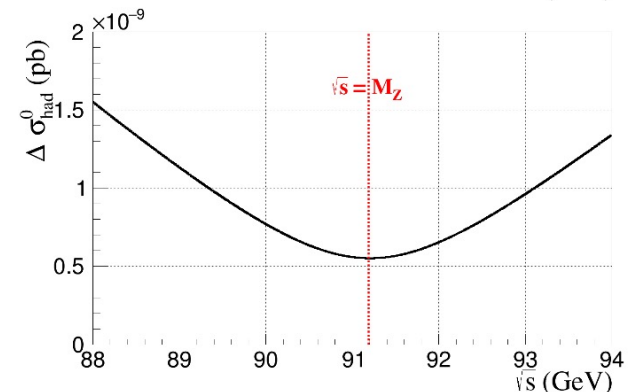
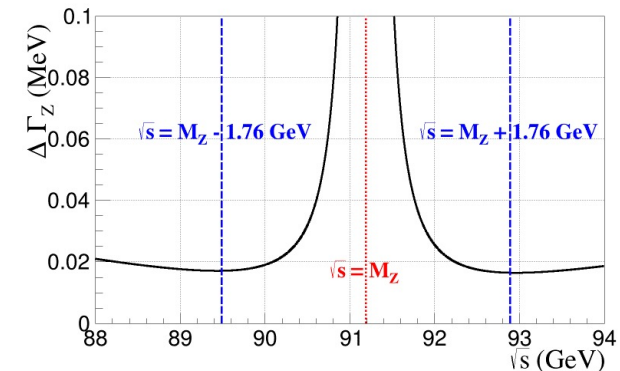
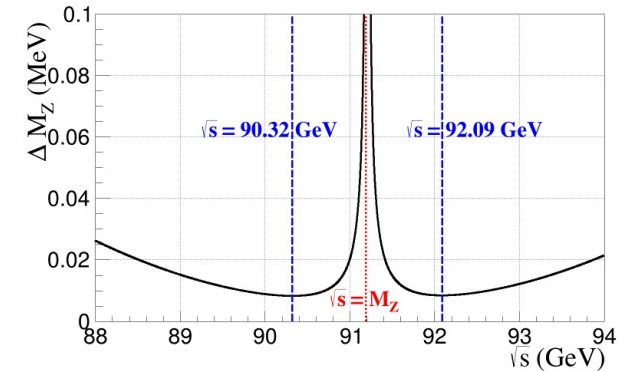
$$\sigma_{\text{meas}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\epsilon \mathcal{L}} = \frac{N_{\text{meas}}}{\epsilon \mathcal{L}}$$

- N_{meas} subject to normal distribution

$$\Delta \sigma_{\text{meas}}(\text{stat}) = \frac{\sqrt{N_{\text{meas}}}}{\epsilon \mathcal{L}}$$

- Statistical uncertainty of a parameter

$$\Delta P(\text{stat}) = \left| \frac{\partial P}{\partial \sigma_{\text{meas}}} \right| \Delta \sigma_{\text{meas}}(\text{stat}) = \left| \frac{\partial \sigma_{\text{meas}}}{\partial P} \right|^{-1} \Delta \sigma_{\text{meas}}(\text{stat})$$



Statistical and systematic uncertainties

- **Statistical uncertainties**

- For two data points or more:
- The χ^2 defined as ($\xi = N$ or A_{FB}):

$$\chi^2 = \sum_i \frac{(\xi_{\text{meas}}^i - \xi_{\text{th}}^i)^2}{\delta_i^2}$$

- Covariance matrix calculated :

$$C = \begin{bmatrix} \frac{\partial^2 \chi^2}{\partial M_Z^2} & \frac{\partial^2 \chi^2}{\partial M_Z \partial \Gamma_Z} & \frac{\partial^2 \chi^2}{\partial M_Z \partial \sigma_{\text{had}}^0} & \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial \Gamma_Z} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z^2} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial \sigma_{\text{had}}^0} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial \sigma_{\text{had}}^0} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial \sigma_{\text{had}}^0} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0{}^2} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{ZZ}^s} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s{}^2} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{ZZ}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^a{}^2} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^a \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^a \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^a \partial C_{\gamma Z}^a} & \frac{\partial^2 \chi^2}{\partial C_{\gamma Z}^a{}^2} & \frac{\partial^2 \chi^2}{\partial C_{\gamma Z}^a \partial C_{\gamma Z}^s} \\ \frac{\partial^2 \chi^2}{\partial M_Z \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial \Gamma_Z \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial \sigma_{\text{had}}^0 \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^s \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial C_{ZZ}^a \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial C_{\gamma Z}^a \partial C_{\gamma Z}^s} & \frac{\partial^2 \chi^2}{\partial C_{\gamma Z}^s{}^2} \end{bmatrix}^{-1}$$

Statistical and systematic uncertainties

- **Systematic uncertainties**

- Sources of systematic uncertainties that we have studied:

$$\sigma_E, \Delta E, \Delta\sigma_E, \Delta L$$

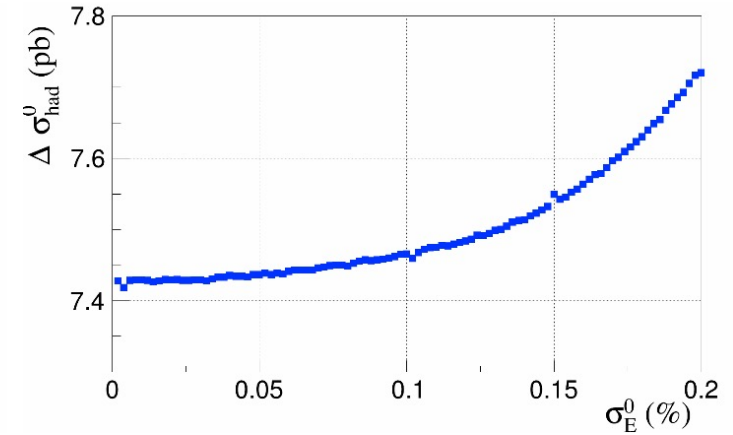
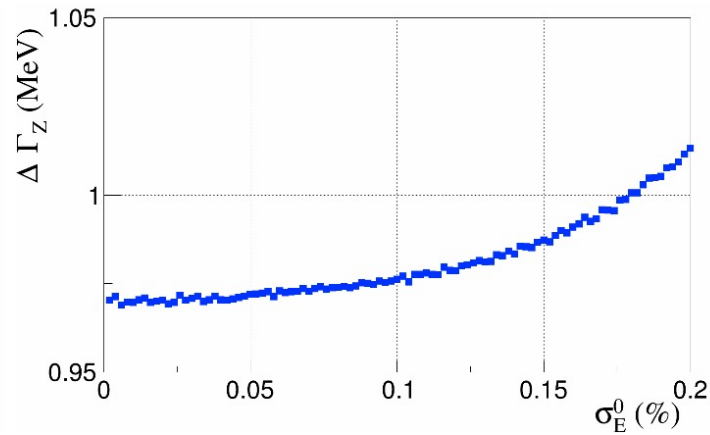
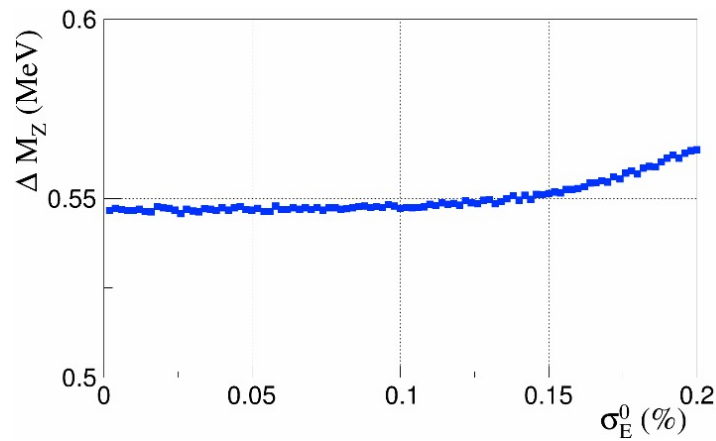
- σ_E beam energy spread :

$$\begin{aligned}\sigma_{\bar{f}\bar{f}}(E_0, \sigma_E^0) &= \int \sigma_{\bar{f}\bar{f}}(E') G(E_0, \sigma_E^0) dE' \\ &= \int \sigma_{\bar{f}\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E^0} e^{-\frac{(E_0-E')^2}{2\sigma_E^0{}^2}} dE'\end{aligned}$$

Statistical and systematic uncertainties

- Systematic uncertainties

➤ σ_E beam energy spread :



Statistical and systematic uncertainties

Systematic uncertainties

- Assume that E , σ_E subject to normal distribution:

$$E = G(E_0, \Delta E), \quad \sigma_E = G(\sigma_E^0, \Delta\sigma_E)$$

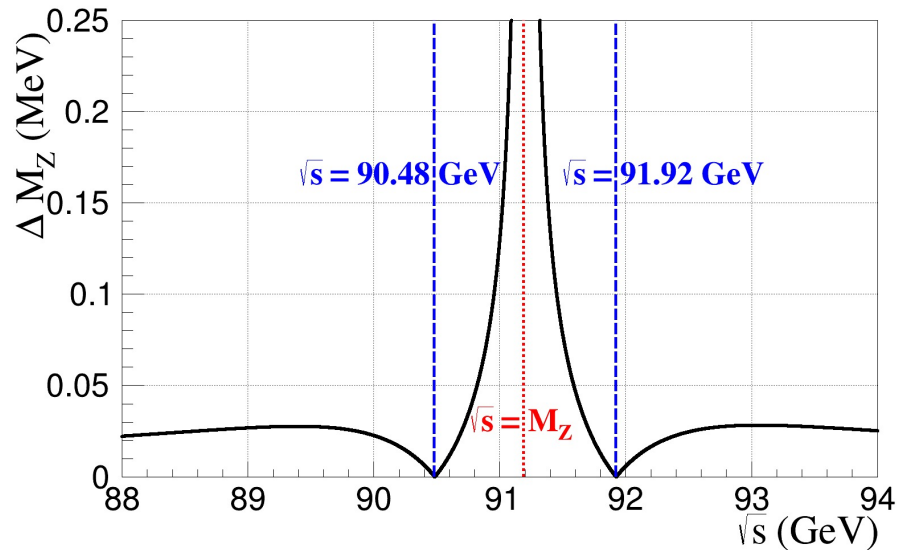
- Effect of ΔE , $\Delta\sigma_E$ can be considered by

$$\begin{aligned} \sigma_{\bar{f}\bar{f}}(E_0, \sigma_E^0) &= \int \sigma_{\bar{f}\bar{f}}(E') G(E_0, \sigma_E^0) dE' \\ &= \int \sigma_{\bar{f}\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E^0} e^{-\frac{(E_0 - E')^2}{2\sigma_E^0{}^2}} dE' \end{aligned} \quad \longrightarrow \quad \begin{aligned} \sigma_{\bar{f}\bar{f}}(E, \sigma_E) &= \int \sigma_{\bar{f}\bar{f}}(E') \cdot \frac{1}{\sqrt{2\pi}\sigma_E} e^{-\frac{(E - E')^2}{2\sigma_E^2}} dE' \end{aligned}$$

Statistical and systematic uncertainties

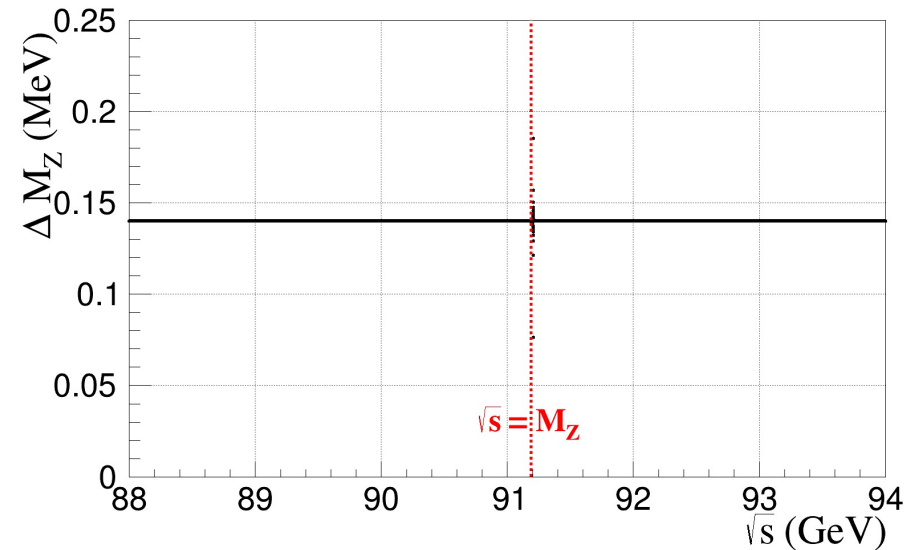
Systematic uncertainties

- Effect of ΔE , $\Delta\sigma_E$ on ΔM_Z



$$\Delta E = 0.14 \text{ MeV}$$

Uncertainty of energy scale



$$\Delta\sigma_E = 0.57 \text{ MeV}$$

Uncertainty of energy spread

Statistical and systematic uncertainties

Systematic uncertainties

- ΔL : Uncertainty of integrated luminosity
- $\Delta\sigma$ due to ΔL

$$\sigma_{\text{meas}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\epsilon\mathcal{L}} = \frac{N_{\text{meas}}}{\epsilon\mathcal{L}}$$

$$\Delta\sigma(\Delta\mathcal{L}) = \sigma_{\text{meas}}\Delta\mathcal{L}$$

- The uncertainty of a parameter P :

$$\Delta P(\Delta\mathcal{L}) = \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \right| \Delta\sigma(\Delta\mathcal{L}) = \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \right| \cdot \sigma_{\text{meas}} \Delta\mathcal{L}$$

- In the simulation (uncorrelated)

$$\sigma_{\text{meas}} = G(\sigma_{\text{meas}}^0, \sigma_{\text{meas}}^0 \cdot \Delta\mathcal{L})$$

Data-taking strategy

Data-taking strategy

- An optimization problem, a single purpose optimization is favored
- Global Determinant Parameter (GDP)
- For n parameters, GDP defined as the determinant of the covariance matrix raised to the $\frac{1}{2n}$ power,

$$\text{GDP} \equiv \sqrt[2n]{\det \text{Cov}}$$

- Here, GDP serve as an object parameter for optimization

[Ref: JHEP 2017, 14 \(2017\).](#)

- To focus on $M_Z \Gamma_Z \sigma_{\text{had}}^0$, a transformation of weight performed

$$C = VC_0V^T \quad V = \text{diag}(5, 5, 3, 1, 1, 1, 1)$$

Data-taking strategy

- Energy points selection

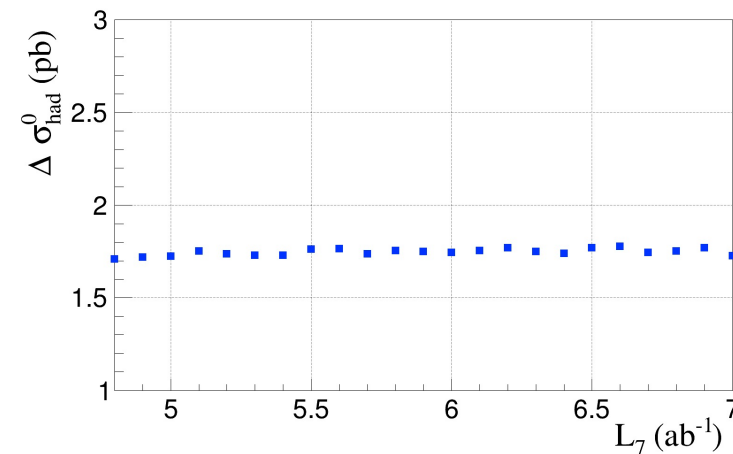
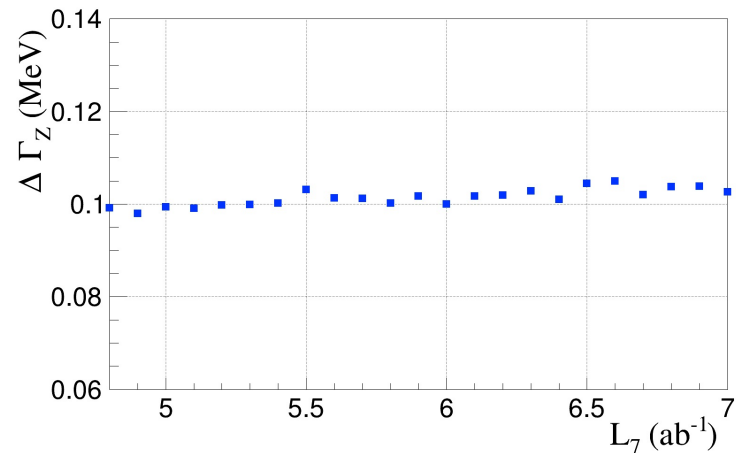
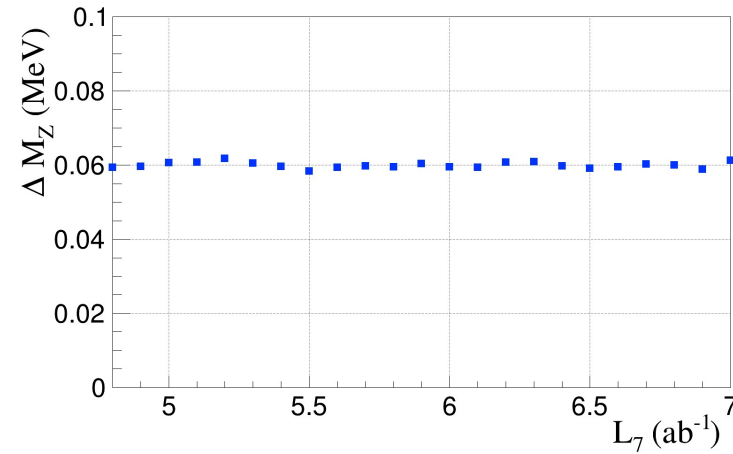
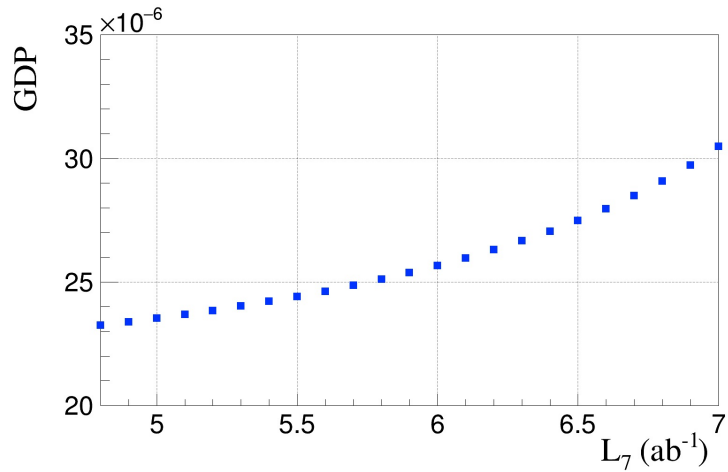
\sqrt{s} (GeV)	\mathcal{L}	\sqrt{s} (GeV)	\mathcal{L}	\sqrt{s} (GeV)	\mathcal{L}
$E_1 = 84.6$	\mathcal{L}_1	$E_6 = 90.4$	\mathcal{L}_6	$E_{10} = 93.2$	\mathcal{L}_{10}
$E_2 = 85.6$	\mathcal{L}_2	$E_7 = 91.2$	\mathcal{L}_7	$E_{11} = 94.3$	\mathcal{L}_{11}
$E_3 = 87.9$	\mathcal{L}_3	$E_8 = 92.0$	\mathcal{L}_8	$E_{12} = 95.3$	\mathcal{L}_{12}
$E_4 = 88.7$	\mathcal{L}_4	$E_9 = 92.5$	\mathcal{L}_9	$E_{13} = 96.2$	\mathcal{L}_{13}
$E_5 = 89.9$	\mathcal{L}_5				

- Luminosity allocation

- ➔ Total integrated luminosity $\mathcal{L} = 8 \text{ ab}^{-1}$
- ➔ At $E_7 = 91.2 \text{ GeV}$ (Z pole) $\mathcal{L}_7 \geq 60\% \cdot \mathcal{L}$

Data-taking strategy

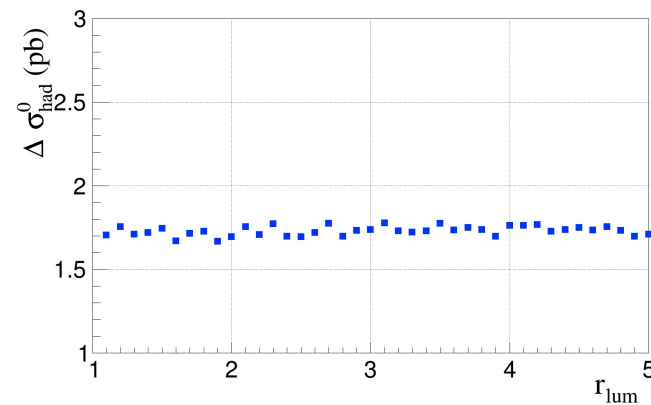
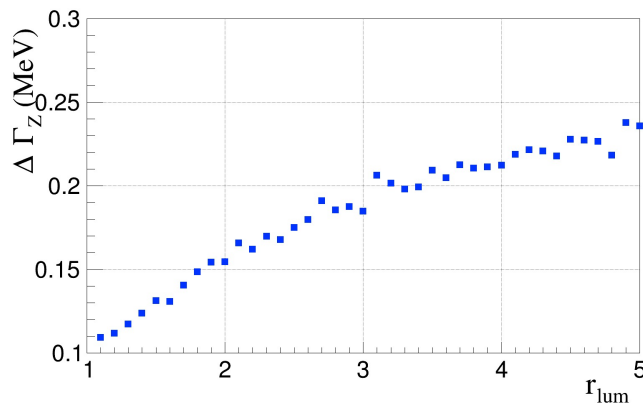
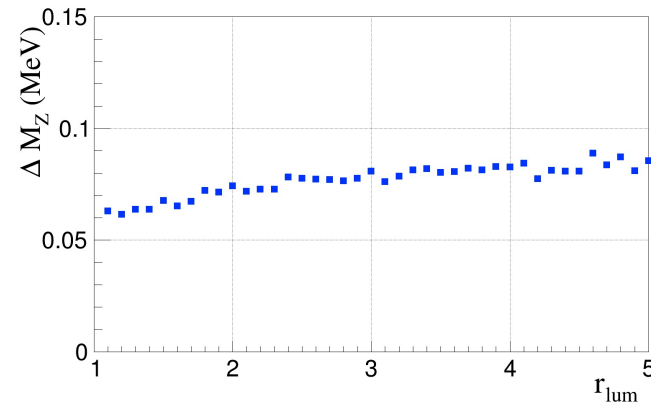
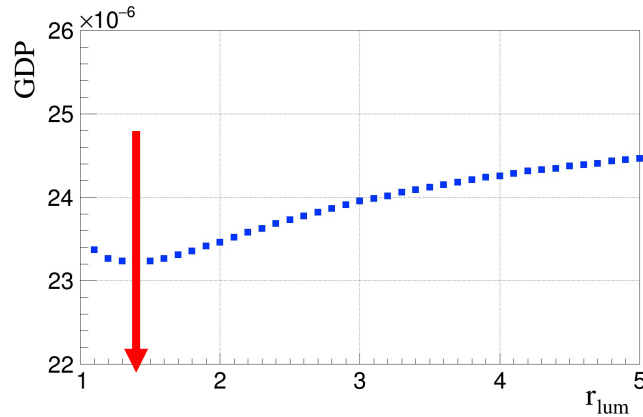
- Luminosity scan on E_7 (Z pole) : smaller fraction favored



Data-taking strategy

- Luminosity scan on off-peak energy points
- Luminosity distribution is in the form of geometric series with common ratio r_{lum}

$$\mathcal{L}_1 : \mathcal{L}_2 : \mathcal{L}_3 : \mathcal{L}_4 : \mathcal{L}_5 : \mathcal{L}_6 = r_{\text{lum}}^0 : r_{\text{lum}}^1 : r_{\text{lum}}^2 : r_{\text{lum}}^3 : r_{\text{lum}}^4 : r_{\text{lum}}^5$$



Data-taking strategy

- A preliminary data-taking scheme :

\sqrt{s} (GeV)	\mathcal{L} (ab ⁻¹)	\sqrt{s} (GeV)	\mathcal{L} (ab ⁻¹)	\sqrt{s} (GeV)	\mathcal{L} (ab ⁻¹)
$E_1 = 84.6$	$\mathcal{L}_1 = 0.09$	$E_6 = 90.4$	$\mathcal{L}_6 = 0.50$	$E_{10} = 93.2$	$\mathcal{L}_{10} = 0.25$
$E_2 = 85.6$	$\mathcal{L}_2 = 0.13$	$E_7 = 91.2$	$\mathcal{L}_7 = 5.00$	$E_{11} = 94.3$	$\mathcal{L}_{11} = 0.18$
$E_3 = 87.9$	$\mathcal{L}_3 = 0.18$	$E_8 = 92.0$	$\mathcal{L}_8 = 0.50$	$E_{12} = 95.3$	$\mathcal{L}_{12} = 0.13$
$E_4 = 88.7$	$\mathcal{L}_4 = 0.25$	$E_9 = 92.5$	$\mathcal{L}_9 = 0.35$	$E_{13} = 96.2$	$\mathcal{L}_{13} = 0.09$
$E_5 = 89.9$	$\mathcal{L}_5 = 0.35$				

- Uncertainties

Parameter	δ_{stat}	δ_{total}
M_Z (KeV)	7	66
Γ_Z (KeV)	13	126
σ_{had}^0 (pb)	0.09	1.73

(ISR effect not considered due to technical problems)

Summary & Outlook

Summary & Outlook

- **Summary:**

- The precise measurement of Z boson resonance parameters studied with toy MC
- Statistical and some important systematic uncertainties investigated
- A very preliminary data-taking scheme is provided

- **Outlook:**

- Take ISR effect & other radiative corrections into account
- A more comprehensive study on various sources of uncertainties
- New parameter scheme with less members
- More efficient way to perform optimization, genetic method ...

Backup

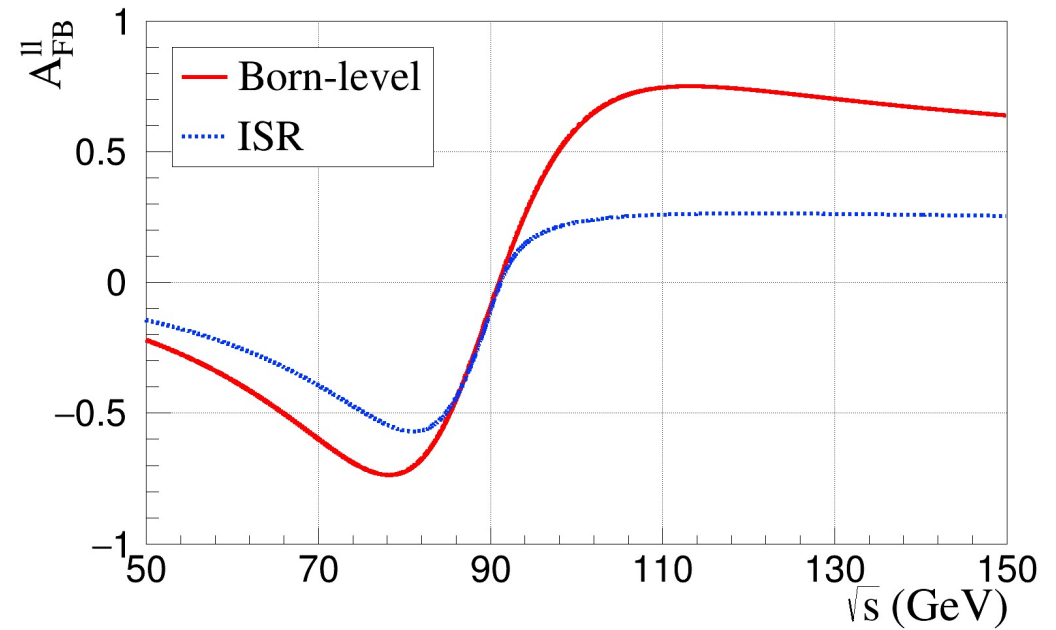
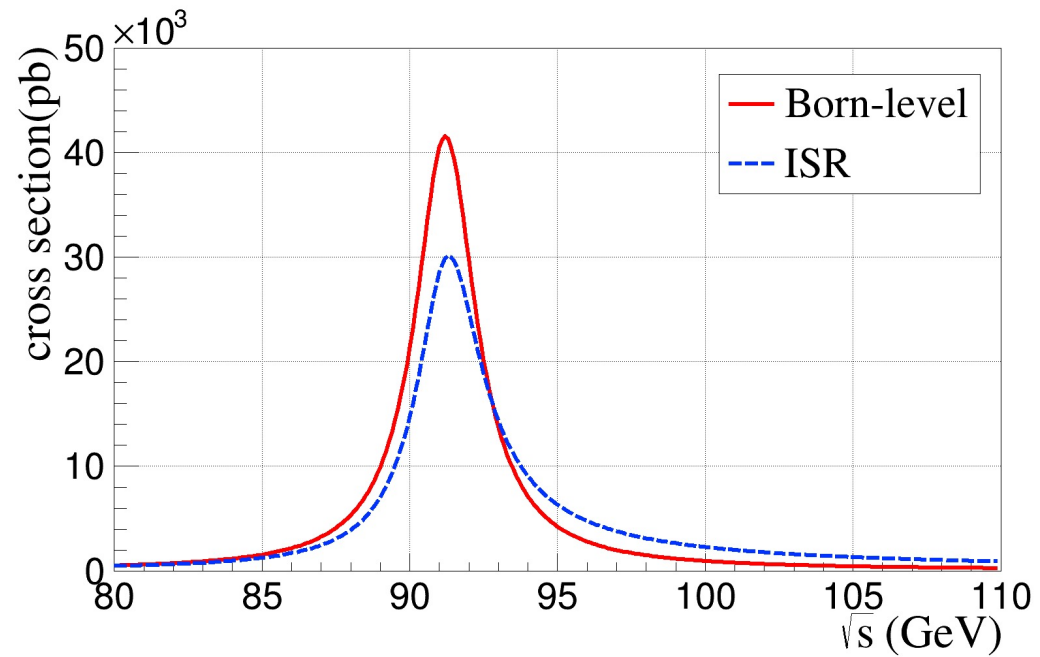
Backup

- Configurations:

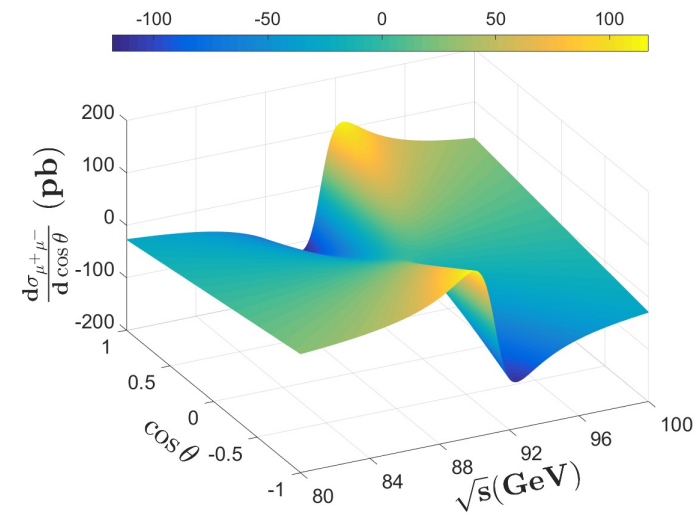
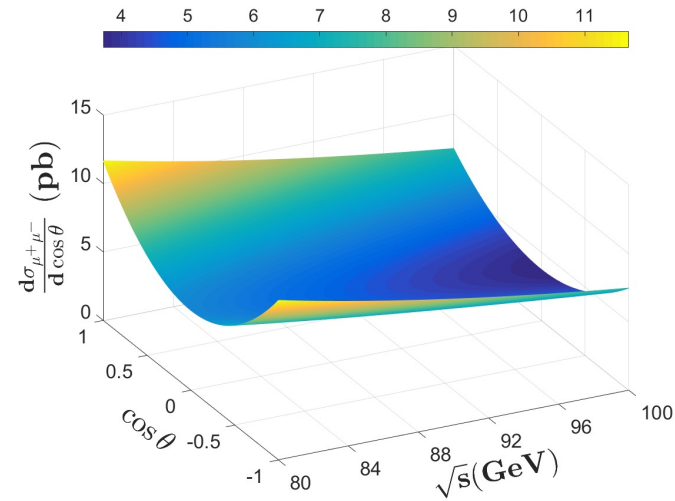
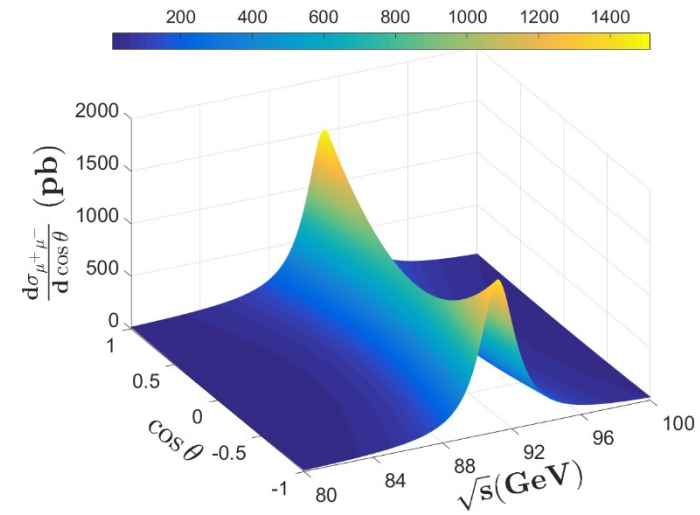
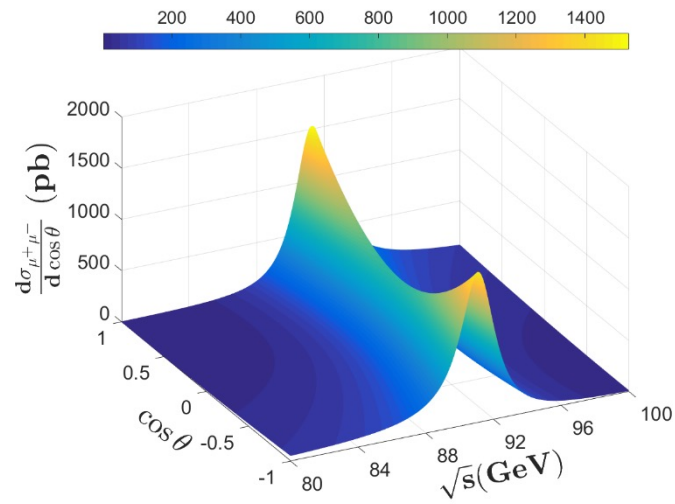
Parameter	Value
M_Z (GeV)	91.1876 ± 0.0021
Γ_Z (GeV)	2.4952 ± 0.0023
BR_{ee} (%)	(3.3632 ± 0.0042)
BR_{had} (%)	(69.911 ± 0.056)
$\sin^2 \theta_W$	0.23122(4)
\mathcal{L} (ab^{-1})	8
ϵ	0.9
σ_E (%)	0.08
ΔE (MeV)	0.1
$\Delta\sigma_E$ (MeV)	0.4
$\Delta\mathcal{L}$ (%)	0.05

Backup

- ISR:



Backup



Backup

- Analytical way to consider the effect of σ_E , ΔE , $\Delta\sigma_E$
- Taylor expansion of cross-section

$$\begin{aligned}\sigma_{\text{ff}}(E) &= \sigma_{\text{ff}}(E_0) + \frac{d\sigma_{\text{ff}}}{dE}(E - E_0) \\ &+ \frac{1}{2} \frac{d^2\sigma_{\text{ff}}}{dE^2}(E - E_0)^2 + \dots \\ &+ \frac{1}{n} \frac{d^n\sigma_{\text{ff}}}{dE^n}(E - E_0)^n + \dots\end{aligned}$$

- Perform the convolution

$$\sigma_{\text{ff}}(E_0, \sigma_E^0) = \int \sigma_{\text{ff}}(E') G(E_0, \sigma_E^0) dE'$$

$$\begin{aligned}\sigma_{\text{ff}}(E_0, \sigma_E^0) &= \sigma_{\text{ff}}(E_0) + \frac{1}{2} \frac{d^2\sigma_{\text{ff}}}{dE^2} \sigma_E^{0,2} \\ &+ \frac{1}{8} \frac{d^4\sigma_{\text{ff}}}{dE^4} \sigma_E^{0,4} + \dots\end{aligned}$$

- Uncertainty caused by σ_E^0

$$\Delta\sigma_{\text{ff}}(E_0, \sigma_E^0) = \frac{1}{2} \frac{d^2\sigma_{\text{ff}}}{dE^2} \sigma_E^{0,2}$$

$$\begin{aligned}\Delta P(\sigma_E^0) &= \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \Delta\sigma_{\text{ff}}(E_0, \sigma_E^0) \right| \\ &= \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \cdot \frac{1}{2} \frac{d^2\sigma_{\text{ff}}}{dE^2} \sigma_E^{0,2} \right|\end{aligned}$$

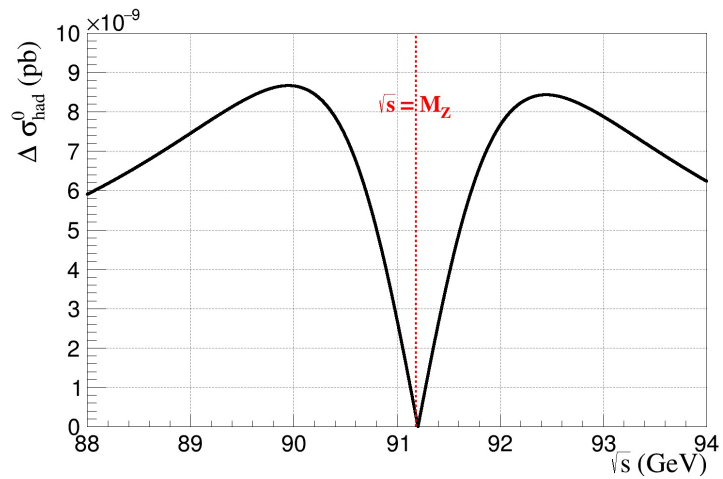
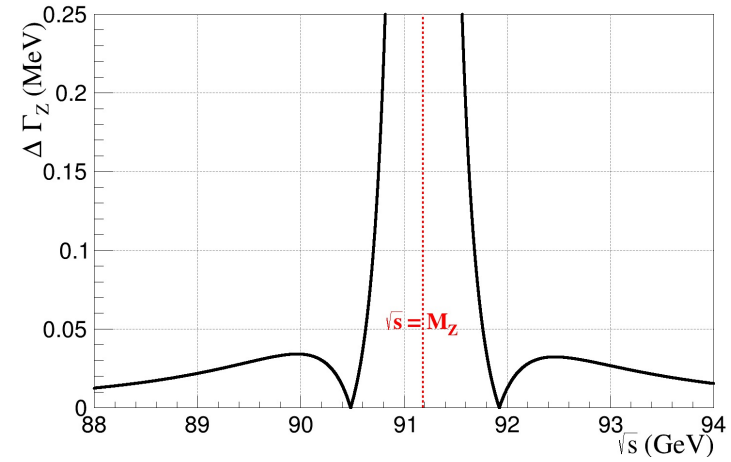
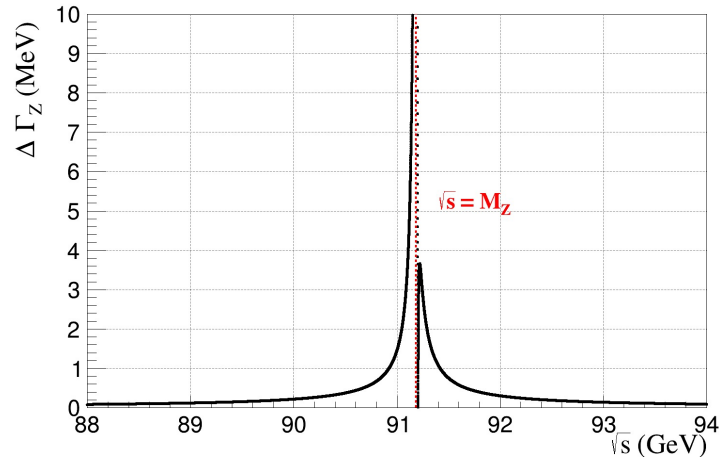
- Uncertainty caused by ΔE

$$\Delta P(\Delta E) = \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \cdot \frac{\partial \sigma_{\text{ff}}}{\partial E} \right| \Delta E$$

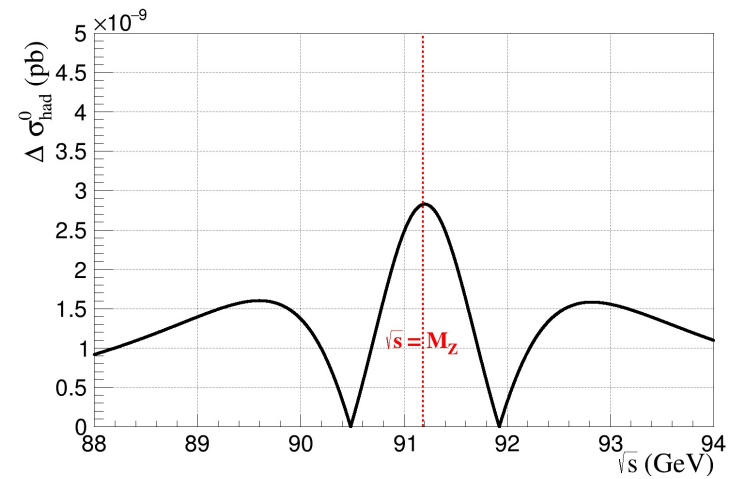
- Uncertainty caused by $\Delta\sigma_E$

$$\begin{aligned}\Delta P(\Delta\sigma_E) &= \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \cdot \frac{\partial \sigma_{\text{ff}}}{\partial \sigma_E} \right| \Delta\sigma_E \\ &= \left| \frac{\partial P}{\partial \sigma_{\text{ff}}} \cdot \frac{d^2\sigma_{\text{ff}}}{dE^2} \sigma_E^0 \right| \Delta\sigma_E\end{aligned}$$

Backup



$$\Delta E = 0.14 \text{ MeV}$$



$$\Delta \sigma_E = 0.57 \text{ MeV}$$

Backup

$$M_Z \quad \Gamma_Z \quad \sigma_{\text{had}}^0 \quad C_{ZZ}^s \quad C_{ZZ}^a \quad C_{\gamma Z}^s \quad C_{\gamma Z}^a$$

PARAMETER NO.	GLOBAL	1	2	3	4	5	6	7
1	0.29059	1.000	-0.080	0.028	-0.094	0.141	0.001	-0.226
2	0.85195	-0.080	1.000	-0.349	0.846	0.025	0.151	0.005
3	0.34935	0.028	-0.349	1.000	-0.295	-0.009	-0.053	-0.002
4	0.84771	-0.094	0.846	-0.295	1.000	0.028	0.179	-0.003
5	0.17363	0.141	0.025	-0.009	0.028	1.000	-0.024	0.054
6	0.19692	0.001	0.151	-0.053	0.179	-0.024	1.000	-0.079
7	0.25487	-0.226	0.005	-0.002	-0.003	0.054	-0.079	1.000