Lepton Collisions in MadGraph5_aMC@NLO

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Based on 2108.10261(Frixione, Mattelaer, Zaro, XZ)

and work in process(Bertone, Cacciari, Frixione, Stagnitto, Zaro, XZ)

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MadGraph5_aMC@NLO

- Source code at: https://launchpad.net/mg5amcnlo
- Now on version 2.9.6 (legacy) and 3.2.0 (including e^+e^- features)

It can:

- Automatically compute hard scattering matrix element at (N)LO
- Perform phasespace integration for infrared-safe observables at (N)LO(i.e. fixed order)
- Generate hard events at (N)LO to be subsequently showered by either Pythia or Herwig(i.e. (N)LO+PS)
- Support BSM via a user-defined Lagrangian.

With MG5_aMC, from 1405.0301 $\rightarrow$

Process		Syntax	Cross section (pb)			
Heavy quarks and jets			LO 1 TeV		NLO 1 TeV	
i.1	$e^+e^- \to jj$	`e+ e- > j j`	$6.223 \pm 0.005 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$6.389 \pm 0.013 \cdot 10^{-1}$	$^{+0.2\%}_{-0.2\%}$
i.2	$e^+e^- \to jjj$	`e+ e- > j j j`	$3.401 \pm 0.002 \cdot 10^{-1}$	$^{+9.6\%}_{-8.0\%}$	$3.166 \pm 0.019 \cdot 10^{-1}$	$^{+0.2\%}_{-2.1\%}$
i.3	$e^+e^- \to jjjj$	`e+ e- > j j j j`	$1.047 \pm 0.001 \cdot 10^{-1}$	$^{+20.0\%}_{-15.3\%}$	$1.090 \pm 0.006 \cdot 10^{-1}$	$^{+0.0\%}_{-2.8\%}$
i.4	$e^+e^- \to jjjjj$	`e+ e- > j j j j j`	$2.211 \pm 0.006 \cdot 10^{-2}$	$^{+31.4\%}_{-22.0\%}$	$2.771 \pm 0.021 \cdot 10^{-2}$	$^{+4.4\%}_{-8.6\%}$
i.5	$e^+e^- \to t\bar{t}$	`e+ e- > t t~`	$1.662 \pm 0.002 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$1.745 \pm 0.006 \cdot 10^{-1}$	$^{+0.4\%}_{-0.4\%}$
i.6	$e^+e^- \to t\bar{t}j$	`e+ e- > t t~ j`	$4.813 \pm 0.005 \cdot 10^{-2}$	$^{+9.3\%}_{-7.8\%}$	$5.276 \pm 0.022 \cdot 10^{-2}$	$^{+1.3\%}_{-2.1\%}$
i.7*	$e^+e^- \to t\bar{t}jj$	`e+ e- > t t~ j j`	$8.614 \pm 0.009 \cdot 10^{-3}$	$^{+19.4\%}_{-15.0\%}$	$1.094 \pm 0.005 \cdot 10^{-2}$	$^{+5.0\%}_{-6.3\%}$
i.8*	$e^+e^- \to t\bar{t}jjj$	`e+ e- > t t~ j j j`	$1.044 \pm 0.002 \cdot 10^{-3}$	$^{+30.5\%}_{-21.6\%}$	$1.546 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-11.6\%}$
i.9*	$e^+e^- \to t\bar{t}t\bar{t}$	`e+ e- > t t~ t t~`	$6.456 \pm 0.016 \cdot 10^{-7}$	$^{+19.1\%}_{-14.8\%}$	$1.221 \pm 0.005 \cdot 10^{-6}$	$^{+13.2\%}_{-11.2\%}$
i.10*	$e^+e^- \to t\bar{t}t\bar{t}j$	`e+ e- > t t~ t t~ j`	$2.719 \pm 0.005 \cdot 10^{-8}$	$^{+29.9\%}_{-21.3\%}$	$5.338 \pm 0.027 \cdot 10^{-8}$	$^{+18.3\%}_{-15.4\%}$
i.11	$e^+e^- \to b\bar{b}$ (4f)	`e+ e- > b b~`	$9.198 \pm 0.004 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$9.282 \pm 0.031 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$
i.12	$e^+e^- \to b\bar{b}j$ (4f)	`e+ e- > b b~ j`	$5.029 \pm 0.003 \cdot 10^{-2}$	$^{+9.5\%}_{-8.0\%}$	$4.826 \pm 0.026 \cdot 10^{-2}$	$^{+0.5\%}_{-2.5\%}$
i.13*	$e^+e^- \to b\bar{b}jj$ (4f)	`e+ e- > b b~ j j`	$1.621 \pm 0.001 \cdot 10^{-2}$	$^{+20.0\%}_{-15.3\%}$	$1.817 \pm 0.009 \cdot 10^{-2}$	$^{+0.0\%}_{-3.1\%}$
i.14*	$e^+e^- \to b\bar{b}jjj$ (4f)	`e+ e- > b b~ j j j`	$3.641 \pm 0.009 \cdot 10^{-3}$	$^{+31.4\%}_{-22.1\%}$	$4.936 \pm 0.038 \cdot 10^{-3}$	$^{+4.8\%}_{-8.9\%}$
i.15*	$e^+e^- \to b\bar{b}b\bar{b}$ (4f)	`e+ e- > b b~ b b~`	$1.644 \pm 0.003 \cdot 10^{-4}$	$^{+19.9\%}_{-15.3\%}$	$3.601 \pm 0.017 \cdot 10^{-4}$	$^{+15.2\%}_{-12.5\%}$
i.16*	$e^+e^- \to b\bar{b}b\bar{b}j$ (4f)	`e+ e- > b b~ b b~ j`	$7.660 \pm 0.022 \cdot 10^{-5}$	$^{+31.3\%}_{-22.0\%}$	$1.537 \pm 0.011 \cdot 10^{-4}$	$^{+17.9\%}_{-15.3\%}$
i.17*	$e^+e^- \to t\bar{t}b\bar{b}$ (4f)	`e+ e- > t t~ b b~`	$1.819 \pm 0.003 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$2.923 \pm 0.011 \cdot 10^{-4}$	$^{+9.2\%}_{-8.9\%}$
i.18*	$e^+e^- \to t\bar{t}b\bar{b}j$ (4f)	`e+ e- > t t~ b b~ j`	$4.045 \pm 0.011 \cdot 10^{-5}$	$^{+30.5\%}_{-21.6\%}$	$7.049 \pm 0.052 \cdot 10^{-5}$	$^{+13.7\%}_{-13.1\%}$

Process		Syntax	Cross section (pb)			
Top quarks +bosons			LO 1 TeV		NLO 1 TeV	
j.1	$e^+e^- \to t\bar{t}H$	`e+ e- > t t~ h`	$2.018 \pm 0.003 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$1.911 \pm 0.006 \cdot 10^{-3}$	$^{+0.4\%}_{-0.5\%}$
j.2*	$e^+e^- \to t\bar{t}Hj$	`e+ e- > t t~ h j`	$2.533 \pm 0.003 \cdot 10^{-4}$	$^{+9.2\%}_{-7.8\%}$	$2.658 \pm 0.009 \cdot 10^{-4}$	$^{+0.5\%}_{-1.5\%}$
j.3*	$e^+e^- \to t\bar{t}Hjj$	`e+ e- > t t~ h j j`	$2.663 \pm 0.004 \cdot 10^{-5}$	$^{+19.3\%}_{-14.9\%}$	$3.278 \pm 0.017 \cdot 10^{-5}$	$^{+4.0\%}_{-5.7\%}$
j.4*	$e^+e^- \to t\bar{t}\gamma$	`e+ e- > t t~ a`	$1.270 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$1.335 \pm 0.004 \cdot 10^{-2}$	$^{+0.5\%}_{-0.4\%}$
j.5*	$e^+e^- \to t\bar{t}\gamma j$	`e+ e- > t t~ a j`	$2.355 \pm 0.002 \cdot 10^{-3}$	$^{+9.3\%}_{-7.9\%}$	$2.617 \pm 0.010 \cdot 10^{-3}$	$^{+1.6\%}_{-2.4\%}$
j.6*	$e^+e^- \to t\bar{t}\gamma jj$	`e+ e- > t t~ a j j`	$3.103 \pm 0.005 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$4.002 \pm 0.021 \cdot 10^{-4}$	$^{+5.4\%}_{-6.6\%}$
j.7*	$e^+e^- \to t\bar{t}Z$	`e+ e- > t t~ z`	$4.642 \pm 0.006 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$4.949 \pm 0.014 \cdot 10^{-3}$	$^{+0.6\%}_{-0.5\%}$
j.8*	$e^+e^- \to t\bar{t}Zj$	`e+ e- > t t~ z j`	$6.059 \pm 0.006 \cdot 10^{-4}$	$^{+9.3\%}_{-7.8\%}$	$6.940 \pm 0.028 \cdot 10^{-4}$	$^{+2.0\%}_{-2.6\%}$
j.9*	$e^+e^- \to t\bar{t}Zjj$	`e+ e- > t t~ z j j`	$6.351 \pm 0.028 \cdot 10^{-5}$	$^{+19.4\%}_{-15.0\%}$	$8.439 \pm 0.051 \cdot 10^{-5}$	$^{+5.8\%}_{-6.8\%}$
j.10*	$e^+e^- \to t\bar{t}W^{\pm}jj$	`e+ e- > t t~ wpm j j`	$2.400 \pm 0.004 \cdot 10^{-7}$	$^{+19.3\%}_{-14.9\%}$	$3.723 \pm 0.012 \cdot 10^{-7}$	$^{+9.6\%}_{-9.1\%}$
j.11*	$e^+e^- \to t\bar{t}HZ$	`e+ e- > t t~ h z`	$3.600 \pm 0.006 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$3.579 \pm 0.013 \cdot 10^{-5}$	$^{+0.1\%}_{-0.0\%}$
j.12*	$e^+e^- \to t\bar{t}\gamma Z$	`e+ e- > t t~ a z`	$2.212 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$2.364 \pm 0.006 \cdot 10^{-4}$	$^{+0.6\%}_{-0.5\%}$
j.13*	$e^+e^- \to t\bar{t}\gamma H$	`e+ e- > t t~ a h`	$9.756 \pm 0.016 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$9.423 \pm 0.032 \cdot 10^{-5}$	$^{+0.3\%}_{-0.4\%}$
j.14*	$e^+e^- \to t\bar{t}\gamma\gamma$	`e+ e- > t t~ a a`	$3.650 \pm 0.008 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$3.833 \pm 0.013 \cdot 10^{-4}$	$^{+0.4\%}_{-0.4\%}$
j.15*	$e^+e^- \to t\bar{t}ZZ$	`e+ e- > t t~ z z`	$3.788 \pm 0.004 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$4.007 \pm 0.013 \cdot 10^{-5}$	$^{+0.5\%}_{-0.5\%}$
j.16*	$e^+e^- \to t\bar{t}HH$	`e+ e- > t t~ h h`	$1.358 \pm 0.001 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$1.206 \pm 0.003 \cdot 10^{-5}$	$^{+0.9\%}_{-1.1\%}$
j.17*	$e^+e^- \to t\bar{t}W^+W^-$	`e+ e- > t t~ w+ w-`	$1.372 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$1.540 \pm 0.006 \cdot 10^{-4}$	$^{+1.0\%}_{-0.9\%}$

Limitations

Are we done? No:

- Only NLO in α_S, not α
- No beamstrahlung
- No ISR effects($\ln(E/m)$ resummation)

Beamstrahlung and ISR effects are included since version 3.2.0!

Full NLO EW will be included in the near future.

Collider-level cross section

Considering a process at an e^+e^- collider:

$$e^+(P_{e^+}) + e^-(P_{e^-}) \to X \tag{1}$$

The cross section can be written as:

$$\mathrm{d}\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{k,l=e^+,e^-,\gamma} \int \mathrm{d}y_+ \mathrm{d}y_- \mathcal{B}_{kl}(y_+, y_-) \mathrm{d}\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-}) \tag{2}$$

- $\mathrm{d}\Sigma_{e^+e^-}$: the collider-level cross section
- $\mathrm{d}\sigma_{kl}$: the particle-level cross section
- $\mathcal{B}_{kl}(y_+, y_-)$: describes beam dynamics(including beamstrahlung). Collider-dependent but process independent.

Particle-level cross section

The particle-level cross section is perturbatively computable, but receives $\ln(m/E)$ terms which is factorisable:

$$\mathrm{d}\sigma = \mathcal{K}(\ln \frac{m}{E}) \otimes \mathrm{d}\hat{\sigma} + \mathcal{O}(\frac{m}{E}) \quad (3)$$

- $\mathcal{K}$: universal, can be compute to all orders and resummed
- $\mathrm{d}\hat{\sigma}$: partonic cross section after subtracting $\ln \frac{m}{E}$ terms: fixed order in perturbation theory

Collinear factorisation

$$\mathrm{d}\sigma_{kl}(p_k, p_l) = \sum_{i,j=e^+,e^-,\gamma} \int \mathrm{d}z_+ \mathrm{d}z_- \Gamma_{i/k}(z_+, \mu^2) \Gamma_{j/l}(z_-, \mu^2) \quad (4)$$

$$\times\, \mathrm{d}\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) \quad (5)$$

- $k, l = e^+, e^-, \gamma$ on the lhs: the particles
- $i, j = e^+, e^-, \gamma$ on the rhs: the partons
- $\mathrm{d}\sigma_{kl}$: particle-level cross section
- $\mathrm{d}\hat{\sigma}_{ij}$: the subtracted parton-level cross section: generally with $m = 0$
- $\Gamma_{i/k}$: the PDF of parton i inside particle k
- μ: the hard scale, $m^2 \ll \mu^2 \sim s$

z-space PDFs

LL($\alpha^n \ln^n(E/m)$) available long ago:

- $0 \leq n \leq \infty$ for $z \to 1$:Gribov,Lipatov
- $0 \leq n \leq 3$ for $z < 1$:Skrzypek,Jadach;Cacciari, Deandrea, Montagna, Nicrosini; Skrypek
- Matching between these two regimes.

NLL($\alpha^n[\ln^n(E/m) + \ln^{n-1}(E/m)]$) available recently:

1909.03886,1911.12040,2105.06688

- $0 \leq k \leq \infty$ for $z \to 1$
- $0 \leq k \leq 3$ for $z < 1$, i.e. $\mathcal{O}(\alpha^3)$
- matching between these two regimes
- Both numerical and analytical

Phasespace integration: change of variables

The PDFs strongly peak at $z \to 1$:

$$\Gamma_{e^\pm/e^\pm}(z) \to (1-z)^{-1+\beta}\hat{\Gamma}_{e^\pm/e^\pm}(z), \quad z \to 1 \tag{6}$$

where $\beta \sim 0.05$ at CEPC@240GeV.
We need specific variable transformation:

$$t_\pm = \left(\frac{(1-z_\pm)}{1-z_{0\pm}}\right)^{1-\gamma}, \quad \frac{\mathrm{d}z_\pm}{\mathrm{d}t_\pm} = \frac{(1-z_{0\pm})^{1-\gamma}}{1-\gamma}(1-z_\pm)^\gamma \tag{7}$$

With $1-\beta \leq \gamma < 1$, it regularises the $z \to 1$ singularities in PDF:

$$\mathrm{d}z_\pm \Gamma_{e^\pm/e^\pm}(z_\pm) = \mathrm{d}t_\pm \hat{\Gamma}_{e^\pm/e^\pm} \frac{(1-z_{0\pm})^{1-\gamma}}{1-\gamma}(1-z_\pm)^{\gamma-1+\beta} \tag{8}$$

Phasespace integration: s-channel resonance

For an s-channel resonance with mass M and width Γ, we define $\tau = z_+ z_-, \tau_M = M^2/s, \tau_\Gamma = \Gamma^2/s$, and introduce:

$$f_{\mathrm{res}} = \frac{1}{(\tau - \tau_M)^2 + \tau_M \tau_\Gamma}, \quad f_{\mathrm{nr}} = \frac{1}{(1-\tau)^{1-2\beta}} \tag{9}$$

where f_{res} is corresponding to Breit-Wigner function, while f_{nr} is the $\tau \to 1$ behavior of effective luminosity. Split into two channels:

$$1 = F_{\mathrm{res}} + F_{\mathrm{nr}}, \quad F_{\mathrm{res}} = \frac{f_{\mathrm{res}}}{f_{\mathrm{res}} + f_{\mathrm{nr}}}, \quad F_{\mathrm{nr}} = \frac{f_{\mathrm{nr}}}{f_{\mathrm{res}} + f_{\mathrm{nr}}} \tag{10}$$

- F_{nr}: non-resonance channel, proceeding as before
- F_{res}: resonance channel
  - first changing variables $(z_+, z_-) \to (\tau, z_i)$,where $z_i = z_+$ or z_- randomly.
  - perform variable transformation on z_i as before
  - perform variable transformation on τ by inverse of the integral of f_{res}

Beamstrahlung in MG5_aMC

$$\mathcal{B}_{kl}(y_+, y_-) \approx \sum_{n=1}^{N} b_{n,kl}^{(e^+)}(y_+) b_{n,kl}^{(e^-)}(y_-) \tag{11}$$

Currently:

$$N = 4, \quad k = e^+, \quad l = e^- \tag{12}$$

And (factorise leading behavior of $y \to 1$):

$$\begin{aligned}
\mathcal{B}_{e^+e^-}(y_+, y_-) =& \hat{f}_{11}\delta(1 - y_+)\delta(1 - y_-) && (13)\\
&+ (1 - y_+)^{\kappa} f_{01}(y_+)\delta(1 - y_-) && (14)\\
&+ \delta(1 - y_+)(1 - y_-)^{\kappa} f_{10}(y_-) && (15)\\
&+ (1 - y_+)^{\kappa} f_{00+}(y_+)(1 - y_-)^{k} f_{00-}(y_-) && (16)
\end{aligned}$$

where $\kappa = -2/3$

Fitting procedure

Currently fitting MC simulation:

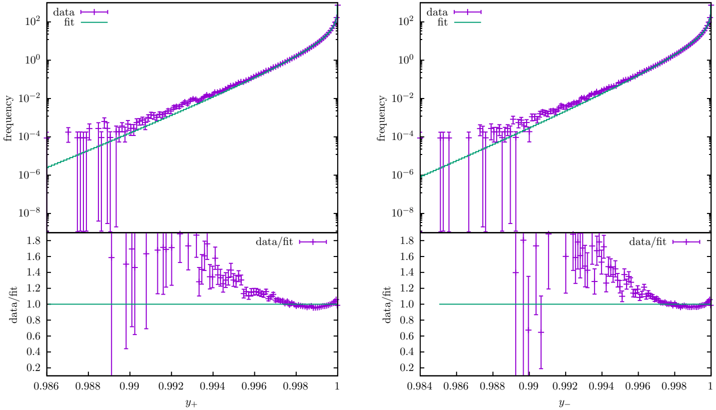
- For any given collider, run GuineaPig with very high statistics
- Choose functional form for $f_\alpha(y)$ functions(possibly collider-specfic)
- Fit $f_\alpha(y)$ on GuineaPig results

In future it may be determined from experiments.

CEPC240 results

For CEPC@240GeV:

$$f(y;p,q) = e^{p(1-y)} e^{q\sqrt{1-y}} \tag{17}$$



Convolution beamstrahlung and ISR PDFs

The collider-level cross section can be written as:

$$d\Sigma_{e^+e^-} = \sum_{n=1}^{N} \sum_{ijkl} \int dx_+ dx_- \phi^{(e^+)}_{i/k,n,kl}(x_+, \mu^2) \phi^{(e^-)}_{j/l,n,kl}(x_-, \mu^2) \tag{18}$$

$$\times\, d\hat{\sigma}(x_+ P_{e^+}, x_- P_{e^-}, \mu^2) \tag{19}$$

with

$$\phi^{(e^\pm)}_{i,k,n,kl}(x, \mu^2) = \int dy dz \delta(x - yz) b^{(e^\pm)}_{n,kl}(y) \Gamma_{i/k}(z, \mu^2) \tag{20}$$

Process independent, hence once for all. The code for this convolution is included within MG5_aMC.
The convoluted PDF has similar behavior as Γ, e.g.

$$\phi^{(e^+)}_{e^+,e^+,n,e^+e^-}(x, \mu^2) = (1 - x)^{-1+\bar{\beta}} \hat{\phi}^{(e^+)}_{e^+,e^+,n,e^+e^-}(x, \mu^2) \tag{21}$$

where $\bar{\beta} = \beta$ or $\bar{\beta} = 1 + \kappa + \beta$

Run MG5_aMC 3.2.0 with lepton-collision specific features

- Generating process using the NLO template: LO only:

  `>generate e+ e- > j j [LOonly=QCD]`

  With QCD NLO:

  `>generate e+ e- > j j [QCD]`

- Modifying the run_card.dat:

```
#*********************************************************************
# Collider type and energy
#*********************************************************************
  -3|    = lpp1 ! beam 1 type (0 = no PDF)
  3|= lpp2 ! beam 2 type (0 = no PDF)
  120.0|= ebeam1 ! beam 1 energy in GeV
  120.0|= ebeam2 ! beam 2 energy in GeV
#*********************************************************************
# PDF choice: this automatically fixes also alpha_s(MZ) a
#*********************************************************************
  cepc240ll = pdlabel ! PDF set
  244600|    = lhaid ! If pdlabel=lhapdf, this is the lhap
```

Conclusion

At e^+e^- colliders, MadGraph_aMC@NLO currently can:

- QCD NLO+PS available long ago
- beamstrahlung(since 3.2.0)
- QED ISR(LL since 3.2.0)

In the near future:

- QED ISR at NLL
- EW NLO

Backup slides

CEPC240 fit results

$$f(y;p,q) = e^{p(1-y)} e^{q\sqrt{1-y}} \tag{22}$$

$$f_{01}(y) = \hat{f}_{01} f(y;p_{01},q_{01}) \tag{23}$$

$$f_{10}(y) = \hat{f}_{10} f(y;p_{10},q_{10}) \tag{24}$$

$$f_{00+}(y) = \hat{f}_{00+} f(y;p_{00+},q_{00+}) \tag{25}$$

$$f_{00-}(y) = \hat{f}_{00-} f(y;p_{00-},q_{00-}) \tag{26}$$

i	$\hat{f}_i$	p_i	q_i	Integral
11	0.8698			
01	0.2863	-901.2	-19.30	0.06234
10	0.2853	-916.5	-18.70	0.06230
00+	0.3308	-899.2	-18.03	0.07308
00-	0.3303	-918.7	-17.48	0.07306
Sum				0.9998

Why γ

At LL:

$$\beta = \frac{\alpha(\mu)}{\pi}(\ln \frac{\mu^2}{m^2} - 1) \tag{27}$$

It depends on the hard scale μ both implicitly through $\alpha(\mu)$ and explicitly. For fixed scale choice such as $\mu = \sqrt{s}$, β is fixed and we can choose $\gamma = 1 - \beta$, but it is impossible for dynamic choice(e.g. $\mu = \sqrt{sz_+z_-}$, as μ is determined after determining the phasespace point.

Current LL+LO in MG5_aMC

$$\Gamma_{e^\pm/e^\pm} = \frac{e^{3\beta/4-\gamma_E\beta}}{\Gamma(1+\beta)}\beta(1-z)^{\beta-1} - \frac{\beta}{2}h_1(z) - \frac{\beta^2}{8}h_2(z) \quad (28)$$

$$h_1(z) = 1 + z \quad (29)$$

$$h_2(z) = \frac{1+3z^2}{1-z}\ln(z) + 4(1+z)\ln(1-z) + 5 + z \quad (30)$$

$$\beta = \frac{\alpha}{\pi}(\ln\frac{\mu^2}{m^2} - 1) \quad (31)$$