Solenoid modeling for the FCC-ee IR

2T solenoid, strongest magnet in the lattice arc bends 0.016 T at Z-energies

average transverse component 0.015 × 2T = **0.03 T** **2 ×** stronger

peak fringe fields depending on geometry details ~**10 ×** stronger than main bends

Goal : solenoid perfectly compensated by anti-solenoid, well approximated by “nosol” lattices

Accurate treament **non-trivial** :

synchrotron radiation, fringes, 15 mrad (half) crossing angle,

and possible quadrupole overlap **difficult** for lattice programs like MAD-X

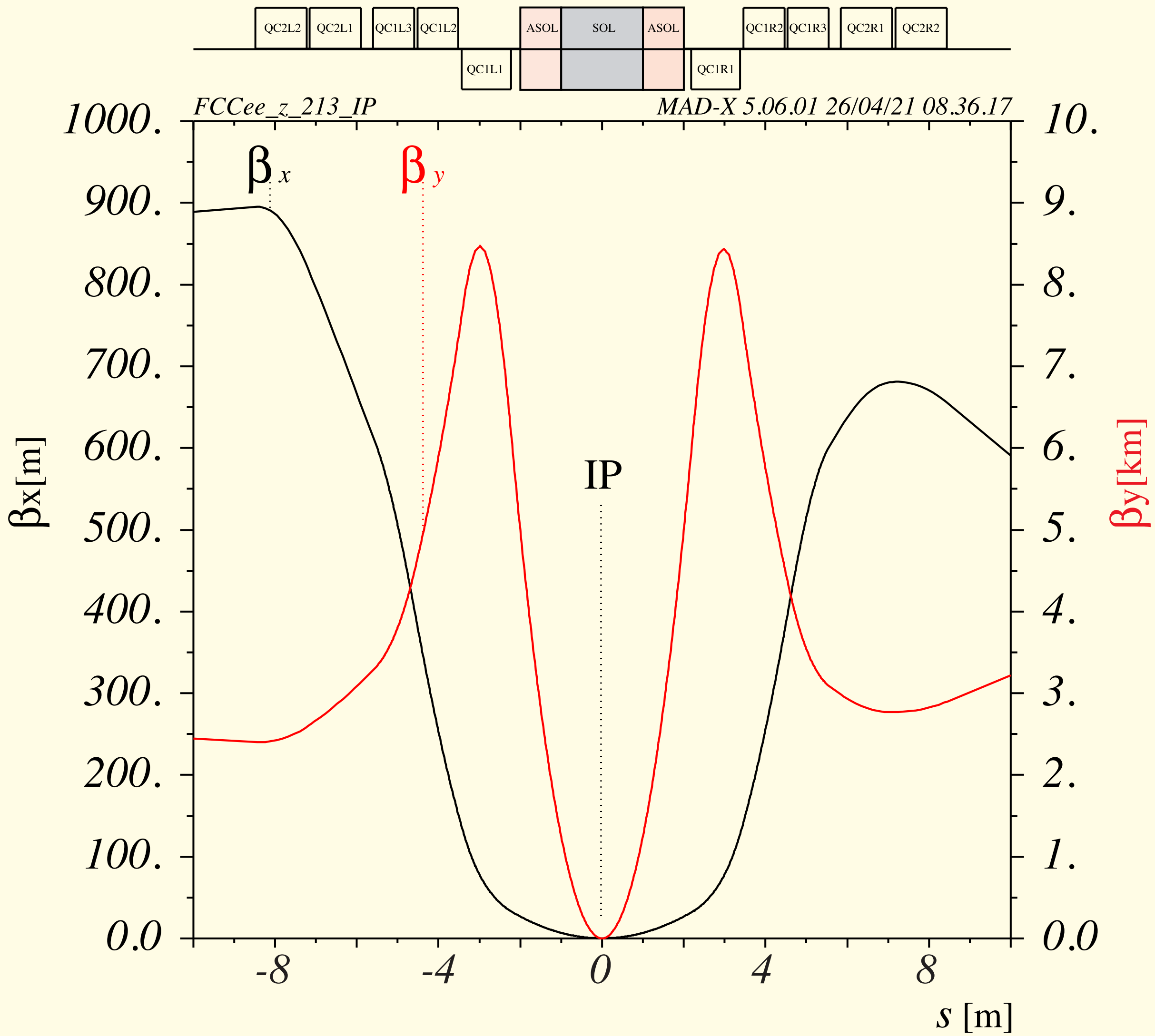
Model :

numercial methods - tracking in small steps using measured (for the moment analytical) fields

+ IR-lattice using small magnet **slices** with parameters adjusted to reproduce the tracking

Acknowledgement : many fruitful discussions with CERN/FCC colleagues including
Rogelio Tomas, Tobias Persson, Leon van Riesen-Haupt, Riccardo de Maria, Laurent Deniau, Katsunobu Oide,
Frank Zimmermann, Anton Bogomyagkov, Dmitry Shatilov, Kyrre Sjobaek, Manuela Boscolo, Barbara Dalena

FCC-IR



βy max = 8471m

βy min = 0.8 mm

107 variation**

IP region : strongly varying twiss parameters

here illustrated using MAD-X with many slices to get smooth curves

Analytical solenoid

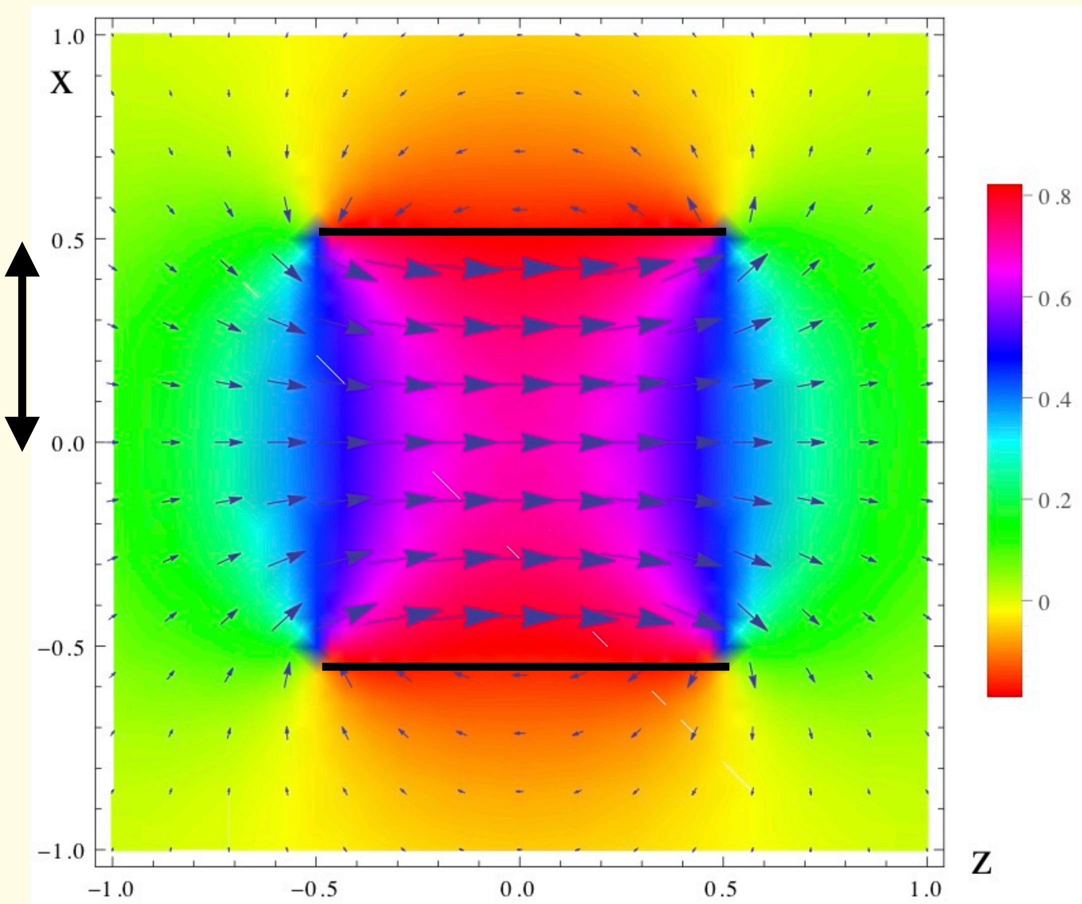
Analytic expression of solenoid field

in terms of complete elliptic integrals

of 1st, 2nd, 3rd kind K, E, Π

available in Mathematica, SymPi and C++17

R = 0.5



L = 1

Default units :

length in meter

fields in Tesla

energy in GeV

Vector potential

cylindrical coordinates z, ρ - distance from axis

$$A_\phi(\rho, z) = \frac{\mu_0\, I}{2\, L} \sqrt{\frac{a}{\rho}}\, \frac{k}{2\,\pi} \left[\zeta \left(\frac{k^2 + h^2 - h^2 k^2}{h^2 k^2} K(k^2) - \frac{1}{k^2} E(k^2) + \frac{h^2 - 1}{h^2} \Pi(h^2, k^2) \right) \right]_{z=\zeta-L/2}^{z=\zeta+L/2}$$

$$h^2 = \frac{4R\rho}{(R+\rho)^2} \qquad k^2 = \frac{4R\rho}{(R+\rho)^2 + \zeta^2}$$

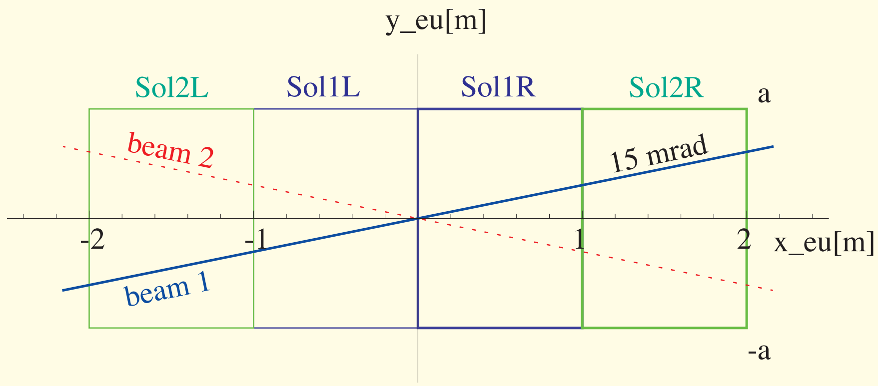
K.F. Müller, Berechnung der Induktivität von Spulen
01/05/1926, doi = [10.1007/BF01655986](https://doi.org/10.1007/BF01655986)

IR transverse fields, horizontal 15 mrad angle

four 1m solenoid

anti-solenoid

pieces, 2T field

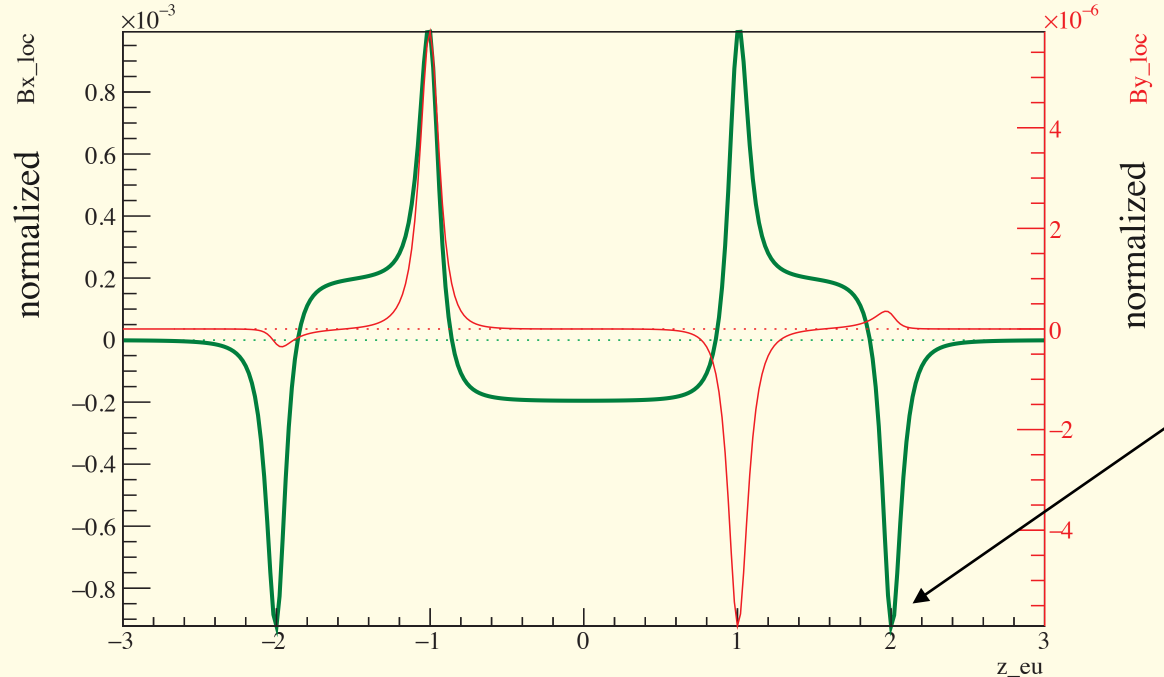


fields in

local system

main field Bx

vertical bump



peaks and SR

by fringe increases

when R, here 0.1 m

is reduced

	Min	Max		
Bx/Brho	-0.9221e-3	0.99422e-3		kick y
By/Brho	-0.0059e-3	0.00591e-3	170× smaller	kick -x

Bρ = 152.1 Tm max Bx = 0.15 T nearly 10× the 0.016 T of arc bends

at IP ~ 0.015 × 2 T = 0.03 T 2× arc bends

Solenoid in lattice programs

A. Dragt + students, G. Ripken et al.

Maps from **Hamiltonian in presence of fields** $-\sqrt{p_t^2/c^2 - m^2c^2 - (p_x - q\,A_x)^2 - (p_y - qA_y)^2} - qA_z$

symplectic for canonical coordinates derived from the Hamiltonian V. Arnold, H. Goldstein, Landau-Lifschitz

Standard machine magnets have **A** = (0, 0, Az) (ideal quadrupoles, multipoles…)

Canonical coordinates correspond here to real space momenta (with a bit of normalization)

ok to use slicing to extrapolate into the magnets

The **solenoid** instead has nonzero **Ax, Ay** depending on position,

such that there is no trivial correspondence to real space coordinates

Transformations based on real space coordinates like Talman's solenoid with separate edges

$$R_{\text{sol}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -K & 0 \\ 0 & 0 & 1 & 0 \\ K & 0 & 0 & 1 \end{pmatrix}}_{\text{edge1}} \underbrace{\begin{pmatrix} 1 & \frac{SC}{K} & 0 & \frac{S^2}{K} \\ 0 & C^2 - S^2 & 0 & 2CS \\ 0 & -\frac{S^2}{K} & 1 & \frac{SC}{K} \\ 0 & -2CS & 0 & C^2 - S^2 \end{pmatrix}}_{\text{solenoid body}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & K & 0 \\ 0 & 0 & 1 & 0 \\ -K & 0 & 0 & 1 \end{pmatrix}}_{\text{edge2}}$$

are not symplectic inside the solenoid where Ax, Ay ≠ 0, ok outside

Dragt ebook, in chapter 16.2 : *It is therefore highly desirable, in the case of a solenoid, to treat fringe-field effects with care (which must be done numerically) using realistic profiles b[n](z).*

MAD-X Solenoid, thick, thin

```
label: SOLENOID, L=length, KS=ksval;                          ! thick version
label: SOLENOID, L=0, Lrad=length, KS=ksval, KSI:=ksval*Lrad;  ! thin version
```

more recently also possible to add permanent dtheta tilt

Note : currently in MAD-X no information about solenoid radius — extend of fringe fields

fringe field effect taken into account as seen from outside

Example, FCC-ee solenoid pieces, Bsol = 2 T beam pc = 45.6 GeV

```
Thick  L = 1 m ,   ksval := Bsol / beam->brho = 0.0131488
```

Slicing, MAD-X module MAKETHIN

```
Thin  solenoid, lrad:= length / nslices , ks:=ksval , ksi:= ksval * lrad;
```

Literature :

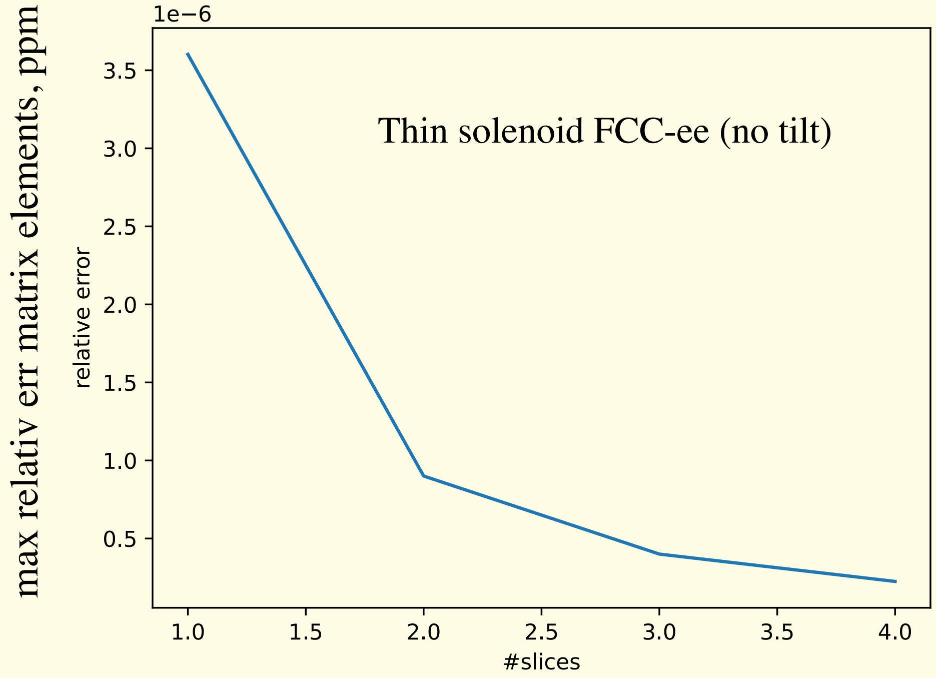
MAD8 Physics Guide

On The Implementation Of Experimental Solenoids In MAD-X And Their Effect On Coupling In The LHC, A. Koschik, H. Burkhardt, T. Risselada, F. Schmidt EPAC 2006

Upgrade of slicing and tracking in MAD-X, H. Burkhardt, L. Deniau, A. Latina, IPAC2014

MAD-X Thin solenoid

Thin solenoid
converges well to reproduce the thick solenoid



The MAD-X thin solenoid transfer matrix can be written as product of a rotation about the s-axis and "a thin quadrupole" focusing in both x, y

$$
\begin{bmatrix}
\cos(\psi) & 0 & \sin(\psi) & 0 & 0 & 0 \\
0 & \cos(\psi) & 0 & \sin(\psi) & 0 & 0 \\
-\sin(\psi) & 0 & \cos(\psi) & 0 & 0 & 0 \\
0 & -\sin(\psi) & 0 & \cos(\psi) & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{f} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{f} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$\psi = \mathrm{L\,KS}/2 = \mathrm{KSI}/2$ 6.5744 mrad

$\frac{1}{f} = \frac{\mathrm{KS}}{2}\frac{\mathrm{KSI}}{2}$ 4.32e-5 / m

for the 1m long solenoid piece

Numerical solution, FieldStep tracking

numerical solution of equations of motion:

$$\mathbf{x}'' = \mathbf{x}' \times \mathbf{B_n}$$
$$\mathbf{x}' + = \Delta s\, \mathbf{x}''$$
$$\mathbf{x} + = \Delta s\, \mathbf{x}'$$

determine design passage by tracking

converges quickly, excellent agreement GEANT4 and (my) FieldStep

fast for small lattice sections like ± 6 m around IP

input : magnet field as function of position, analytic of measured solendoid + quad ..

output :

track coordinates at every step

optionally track design particle and 6 particles with small offsets in all coordinates

(multi-treading, ~ same speed as single particle)

transfer matrices in real space coordinates from numerical Jacobian, symplectic when Ax = Ay = 0

outside : reproducing the Solenoid map (for small solenoid radius)

inside : real space positions and angles x, x’

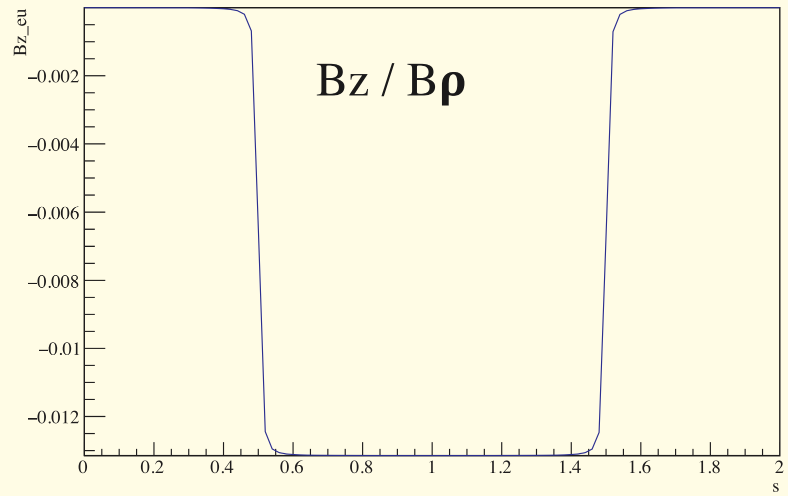
Numerical transfer matrix in real space coordinates

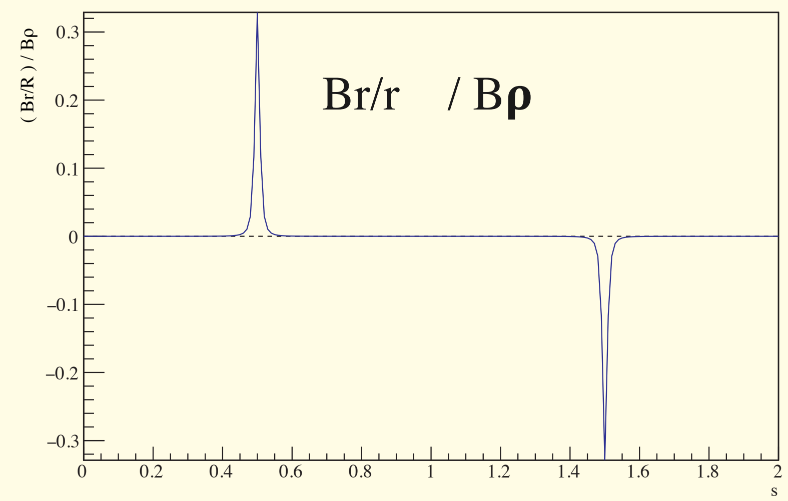
Example **no tilt single L = 1m solenoid**, R = 0.1 m, 2 T,
Jacobian from tracking with small offsets
done in 6D, numbers here just for the 4D part

```
s=0.5  solenoid entry
             1          -0.5   -3.2196e-05    1.5952e-05
   -1.5004e-07             1      -0.00327     0.0016026
    3.2196e-05   -1.5952e-05             1          -0.5
       0.00327    -0.0016026   -1.5004e-07             1

s=1   solenoid center
       0.99999    1.7235e-06    -0.0032864   -5.8978e-07
   -4.2867e-05             1    -0.0065724    -0.0032869
     0.0032864    5.8978e-07       0.99999    1.7235e-06
     0.0065724     0.0032869   -4.2867e-05             1

s=2   0.5 meter from solenoid    back symplectic
       0.99994       0.99998    -0.0065725    -0.0065751
   -4.2547e-05       0.99998     2.794e-07    -0.0065751
     0.0065725     0.0065751       0.99994       0.99998
    -2.794e-07     0.0065751   -4.2547e-05       0.99998
always det = 1
```





possible to read these into MAD-X and use with option, sympl=false;   consequences with SR and emit would need reconsideration, maybe not such a good idea for an overall small effect ?

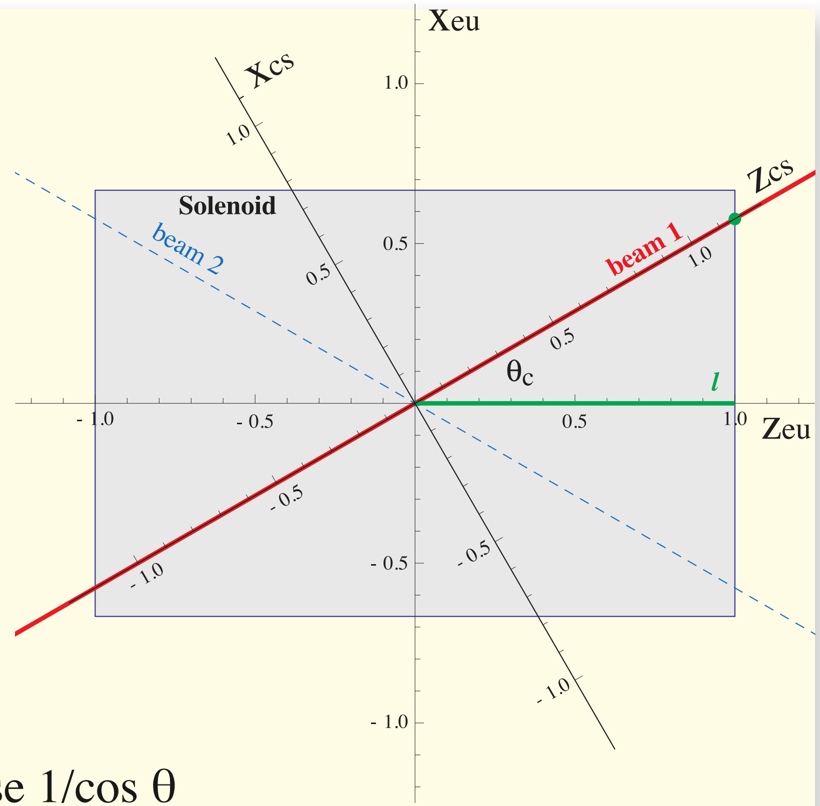
Of some interest for other applications where SR not essential like solenoid focusing at low energy ?

Geometry of tilted solenoid

Illustrated for $\theta_c = \pi/6 = 30^o$

$$\mathrm{Rot}_{y,3D}(\theta_c) = \begin{bmatrix} \cos(\theta c) & 0 & \sin(\theta c) \\ 0 & 1 & 0 \\ -\sin(\theta c) & 0 & \cos(\theta c) \end{bmatrix}$$

$$\mathbf{x}_{cs} = \begin{bmatrix} x_{cs} \\ y_{cs} \\ z_{cs} \end{bmatrix} \qquad \mathbf{x}_{eu} = \mathrm{Rot}_{y,3D}(\theta_c)\,\mathbf{x}_{cs} = \begin{bmatrix} x_{cs}\cos(\theta c) + z_{cs}\sin(\theta c) \\ y_{cs} \\ -x_{cs}\sin(\theta c) + z_{cs}\cos(\theta c) \end{bmatrix}$$



eu : detector system, solenoid axis z_{eu}

cs : beam reference system, solenoid rotated, length increase 1/cos θ

FCC : **beam divergence ~ 40 μrad small compared to (half) crossing angle θc = 15 mrad**

fields seen and main effects of tilted solenoid similar for all beam particles

possible to approximate well using MAD-X with

1) orbit correctors reproducing the design particle trajectory determined with tracking

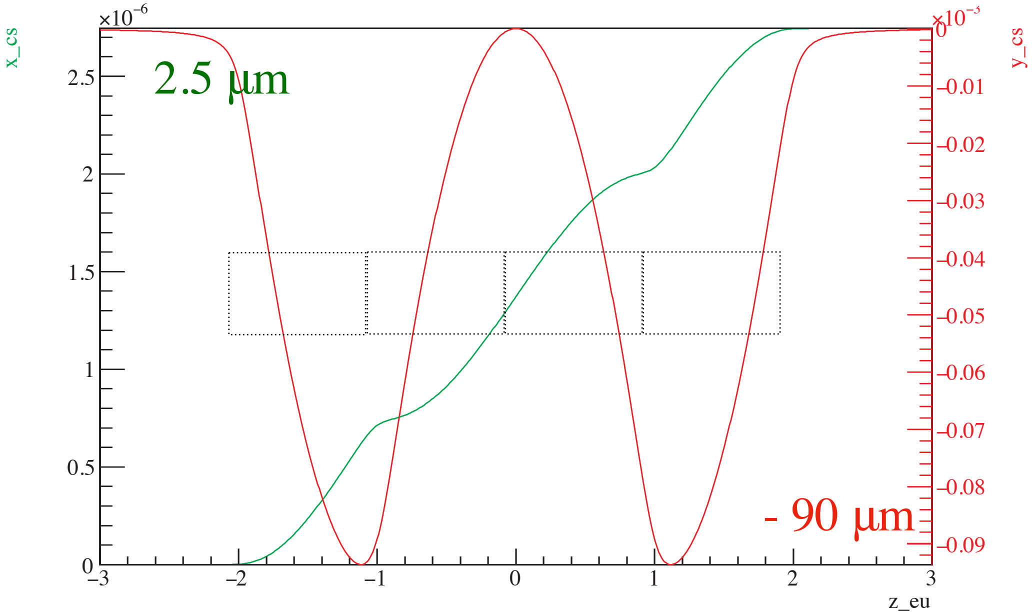
2) slices of on axis-solenoid aligned to design particle track (work in progress)

Comparison, FieldStep, MAD-X just using orbit correctors slices

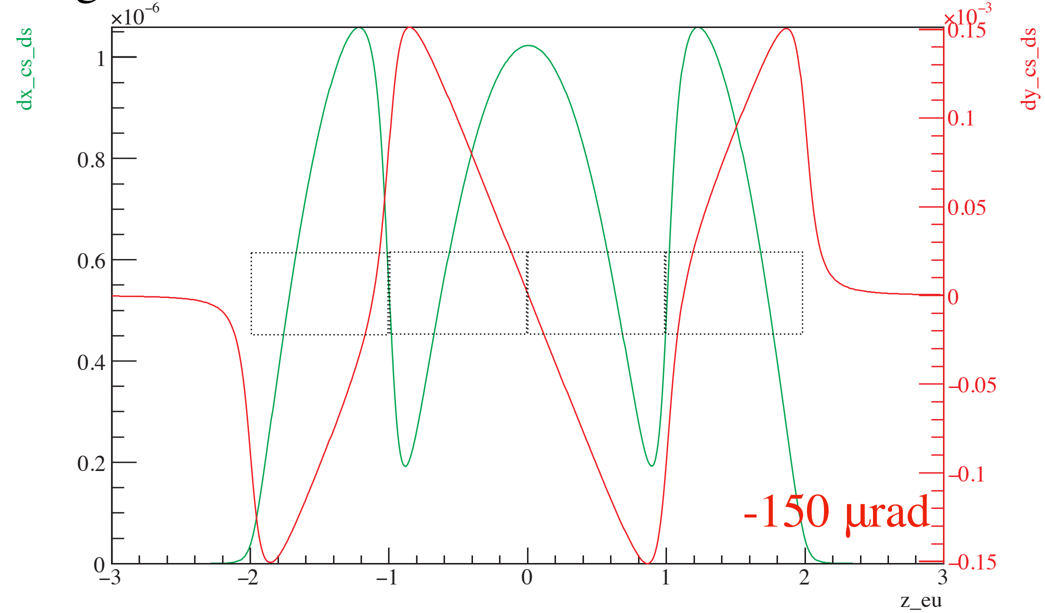
±15 mrad crossing angle, 2 T, 45.6 GeV

Tracking

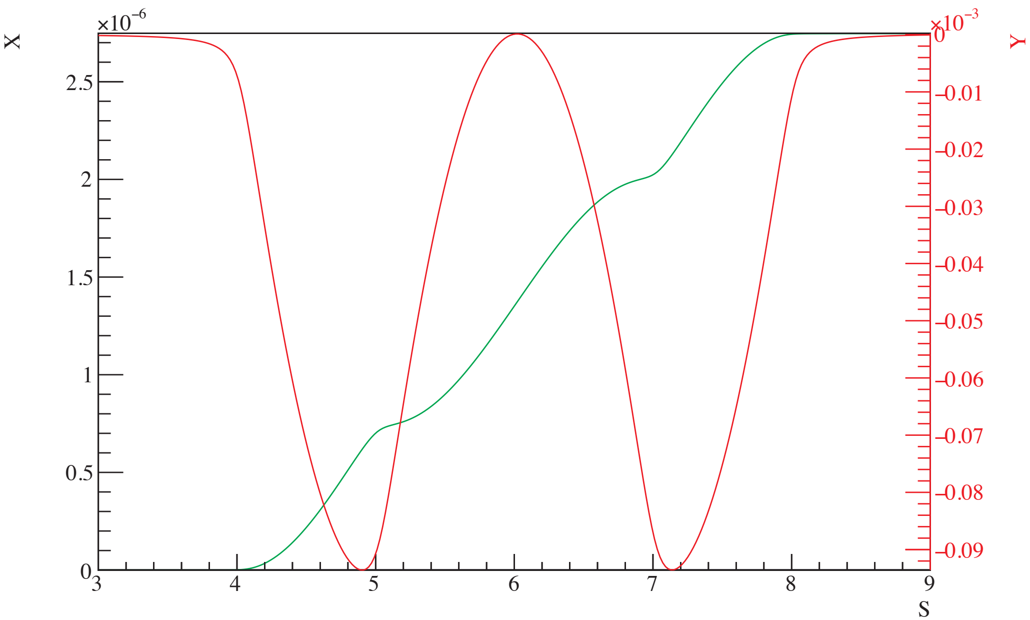
position

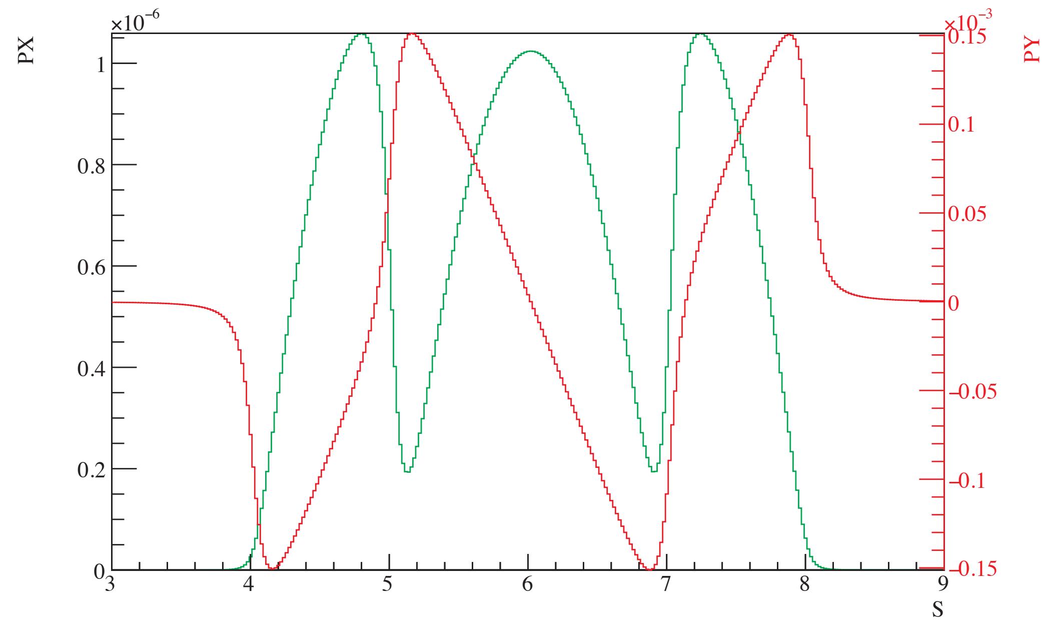


direction —> orbit “kicks”



well reproduced with MAD-X, just using slices of orbit correctors



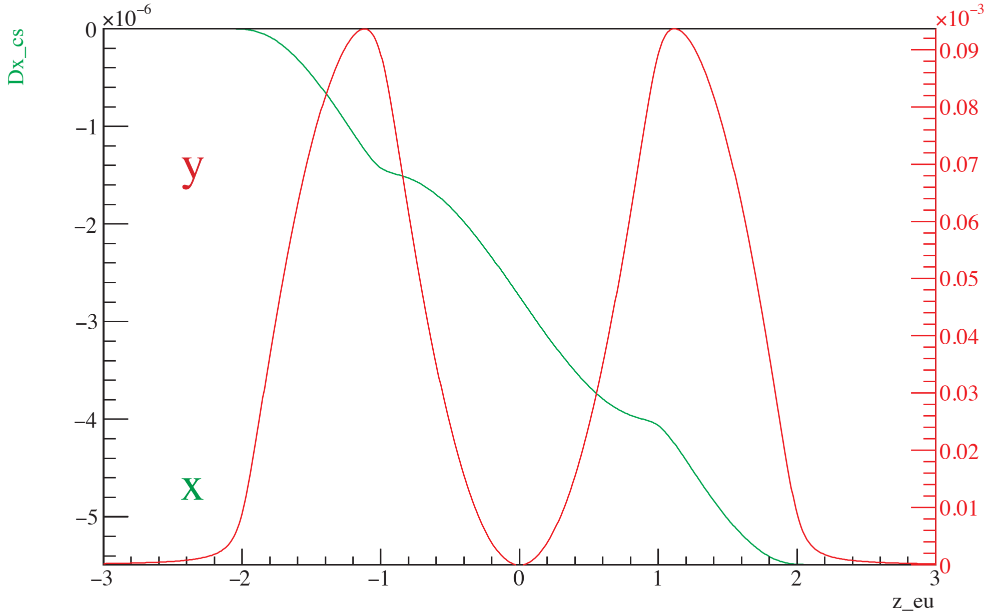


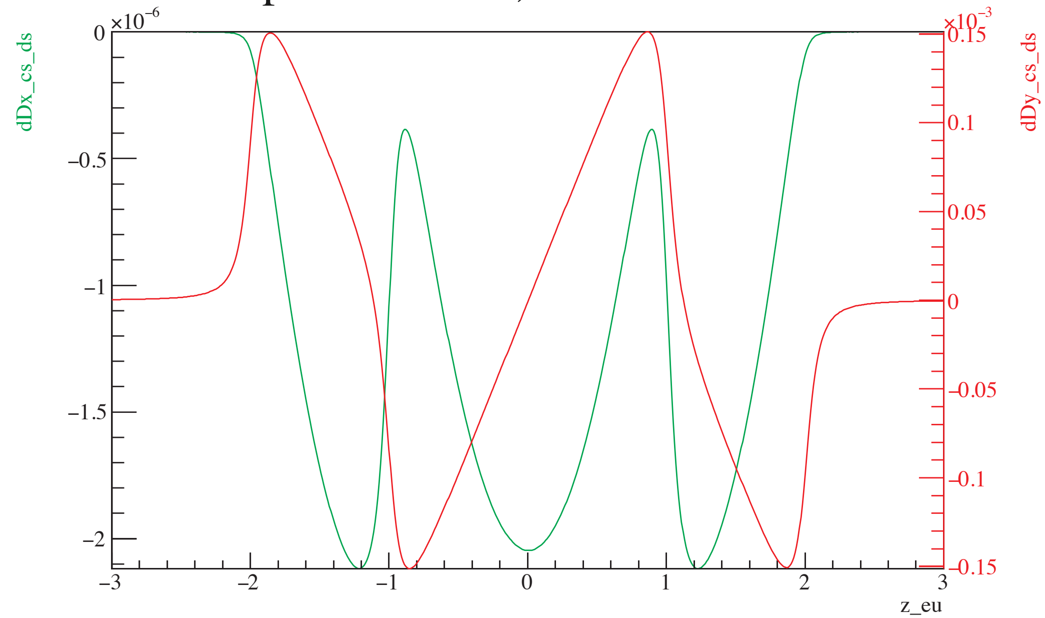
Comparison, FieldStep, MAD-X just using orbit correctors slices

Dispersion DX, DY

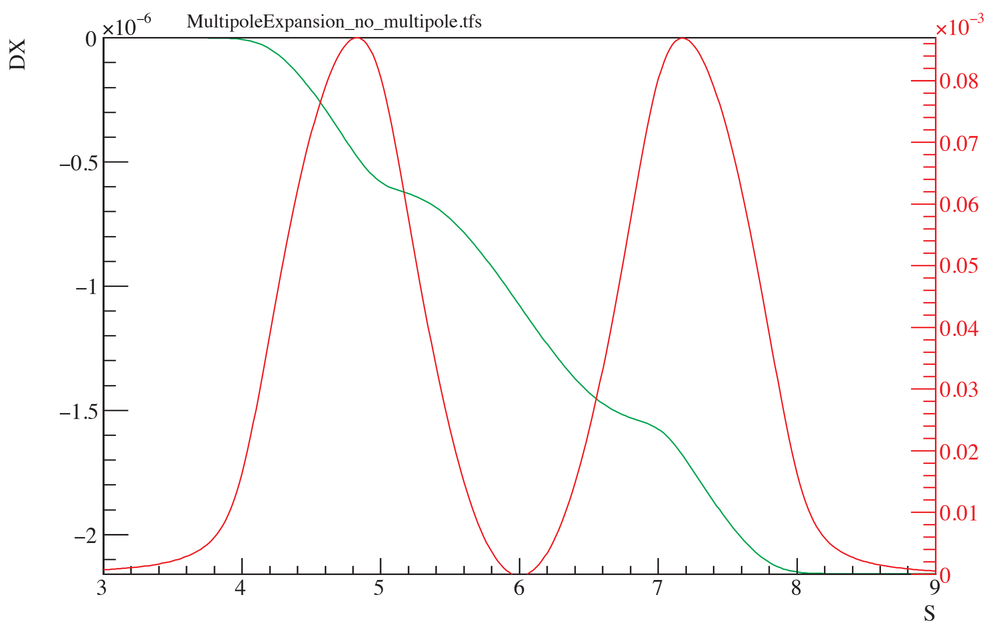
Tracking

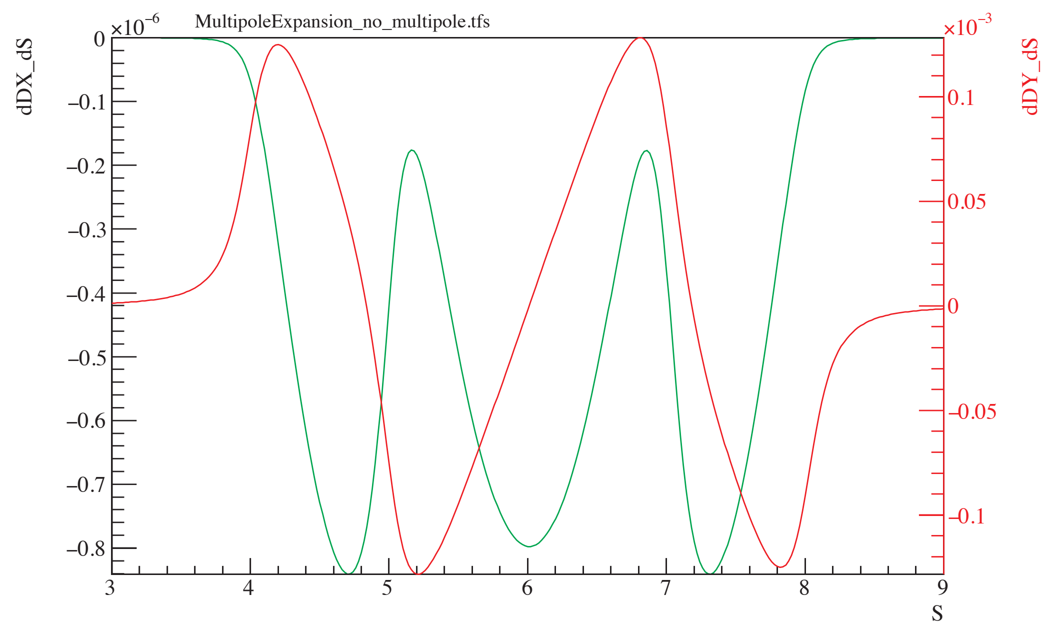
Dispersion DPX, DPY





MAD-X good agreement in Dy   really small, but not closed in x





SR from solenoids power estimate

estimated using two different methods   FCC-ee CDR design parameters

1) "full simulation"   GEANT4 tracking 1000 e+ particles through IR region
   using the FCC-CDR MDISim generated geometry (in gdml format) + fields
   1095 photons generated,
   Emean = 24.17 +/- 1.531 keV
   E_rms = 50.66 keV   Power 37 kW / IP

2) MAD-X 500 slices with just orbit correctors, U0_IR = 3.2e-5 GeV, 1134 photons
   Emean = 28.2 keV   Power 44.5 kW / IP

or in both cases roughly **40 kW per IP**

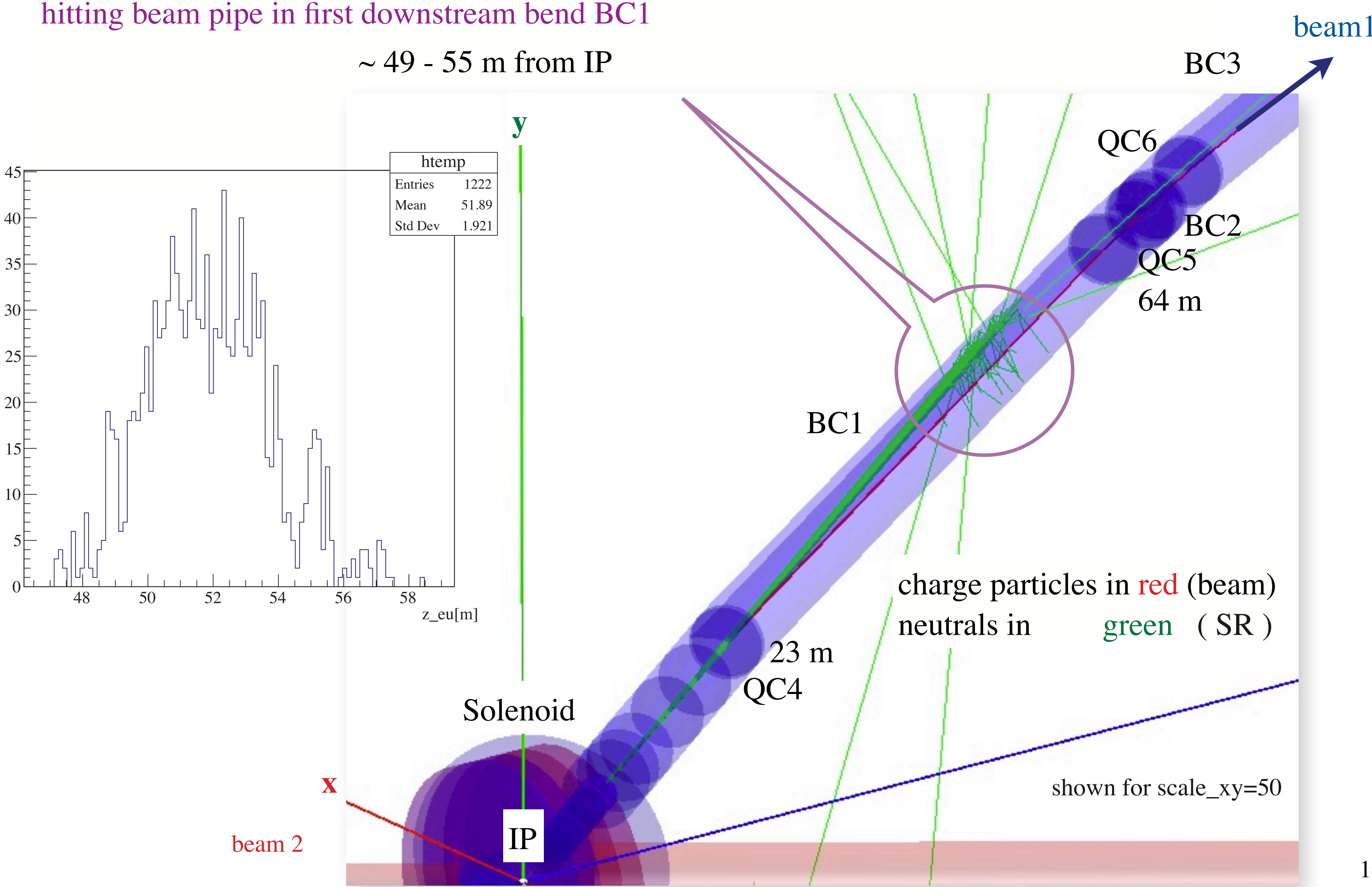
and likely more with more realistic (shorter, narrower) anti-solenoids and offsets in quadrupoles

Loss location of the solenoid generated photons

Using MDISim (GEANT4, geometry automatically generated from MAD-X, display with ROOT)

hitting beam pipe in first downstream bend BC1

~ 49 - 55 m from IP



Concluding remarks

The solenoid effects for beam dynamics are rather small

generating closed bumps in y and vertical dispersion of ~ 100 µm, peaks at ± 1 m from the IP

and still ~ 17x smaller in but not closed in x

The synchrotron radiation photon energies and radiated power ~ 40 kW are significant

radiated mostly in vertical direction at angles up to ~150 µrad

hitting the beam pipe far (~ 46 m) from the IP

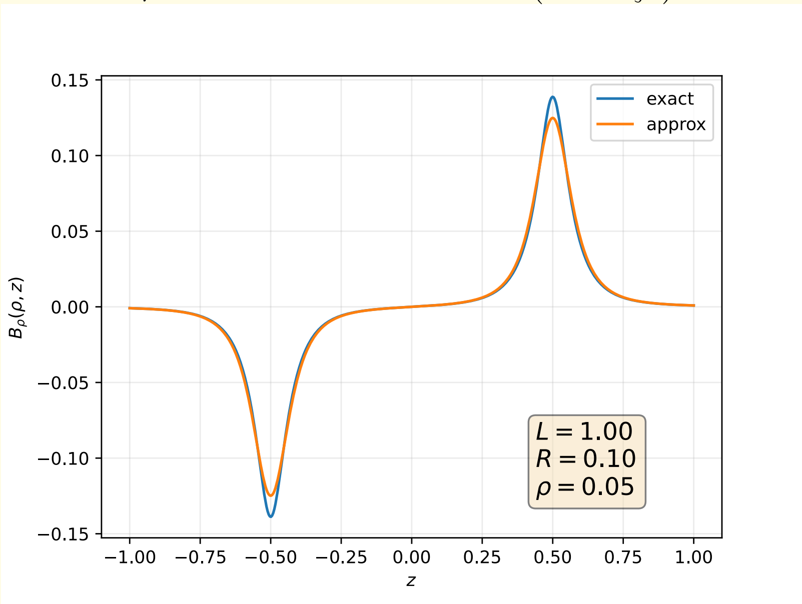
further follow up in progress with the vacuum teams (Roberto Kersevan et al.)

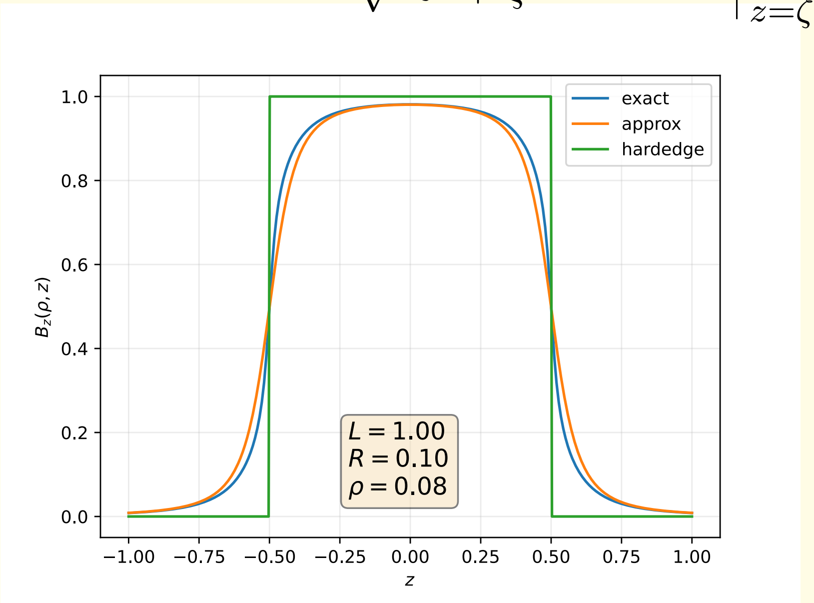
Backup

Fields close to axis, fringe field kick

$$\frac{B_\rho(\zeta,\rho)}{\rho} = B_{\rho\_\mathrm{over}\_\rho}(\zeta) \approx -\frac{R^2}{4\,(R^2+\zeta^2)^{3/2}}$$

$$B_z(\zeta) \approx \frac{\zeta}{2\sqrt{R^2+\zeta^2}} \Bigg|_{z=\zeta-L/2}^{z=\zeta+L/2}$$





related by $$\frac{dB_z(\rho,\zeta)}{d\zeta} = \frac{R^2}{2\,(R^2+\zeta^2)^{\frac{3}{2}}} = -2\,B_{\rho\_\mathrm{over}\_\rho}$$

$$B_\rho = -\frac{\partial A_\varphi}{\partial z}$$
$$B_\phi = 0$$
$$B_z = \frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial \rho}$$

as expected for only non-zero Aϕ

$$\int_{-\infty}^{\infty} B_\rho(\zeta,\rho)\; dz = \mp\frac{2}{\rho}$$ fringe field kick close to axis

cartesian close to axis $$\mathbf{B}(x,y,z) = (x\,B_{\rho\_\mathrm{over}\_\rho},\, y\,B_{\rho\_\mathrm{over}\_\rho},\, B_z)_{z-L/2}^{z+L/2}$$

good for insight and solving equations of motion in real space coordinates

not needed for numerical evaluation, very fast to evaluate exact formulas or later to use measured field map