$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x} \langle 0| T\{ j_\mu(x)\, j_\nu(0)\} |0\rangle$$

show: $2\,\mathrm{Im}\,\Pi_{\mu\nu}(q) = \Sigma \langle 0| j_\mu(0) |x\rangle \langle x| j_\nu(0) |0\rangle\, d\tau_x\, (2\pi)^4 \delta^4(q-P_x)$

$$d\tau_x \sim \int \frac{d^3P_x}{(2\pi)^3}\frac{1}{2E_x}$$

Proof:

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x} \langle 0| \theta(x_0)\, j_\mu(x)\, j_\nu(0) + \theta(-x_0)\, j_\nu(0)\, j_\mu(x) |0\rangle$$

$$= i\sum_x d\tau_x \int d^4x\, e^{iq\cdot x} \Big[\langle 0| \theta(x_0)\, j_\mu(x) |x\rangle\langle x| j_\nu(0) |0\rangle + \theta(-x_0) \langle 0| j_\nu(0) |x\rangle \langle x| j_\mu(x) |0\rangle \Big]$$

$$= i\sum_x d\tau_x \Big[\int d^4x\, e^{iq\cdot x - iP_x\cdot x}\, \theta(x_0) \langle 0| j_\mu(0) |x\rangle\langle x| j_\nu(0) |0\rangle$$

$$+ \underbrace{\int d^4x\, e^{iq\cdot x + iP_x\cdot x}}_{\to 0}\, \theta(-x_0) \langle 0| j_\nu(0) |x\rangle\langle x| j_\mu(0) |0\rangle$$

$\vec{P}_x + \vec{q} = 0$ impossible

$$= i\sum_x d\tau_x \frac{-1}{i(q_0 - P_{x0} + i\varepsilon)} \langle 0| j_\mu |x\rangle\langle x| j_\nu |0\rangle (2\pi)^3 \delta^3(\vec{q} - \vec{P}_x)$$

$$= \sum_x d\tau_x \frac{-1}{q_0 - P_{x0} + i\varepsilon} \langle 0| j_\mu |x\rangle\langle x| j_\nu |0\rangle (2\pi)^3 \delta^3(\vec{q} - \vec{P}_x)$$

$$\to \mathcal{P}\frac{-1}{q_0 - P_{x0}} + i\pi\,\delta(q_0 - P_{x0})$$

Then

$$2\,\mathrm{Im}\,\Pi_{\mu\nu}(q) = \sum_x d\tau_x \langle 0| j_\mu |x\rangle\langle x| j_\nu |0\rangle (2\pi)^4 \delta^4(q - P_x)$$

$$\psi(x) = \psi(0) + x_\alpha \vec{D}^\alpha \psi(0) + \frac{1}{2} x_\alpha x_\beta \vec{D}^\alpha \vec{D}^\beta \psi(0) + \frac{1}{3!} x_\alpha x_\beta x_\rho \vec{D}^\alpha \vec{D}^\beta \vec{D}^\rho \psi(0)$$

$$+ \frac{1}{4!} x_\alpha x_\beta x_\rho x_\sigma \vec{D}^\alpha \vec{D}^\beta \vec{D}^\rho \vec{D}^\sigma \psi(0) + \cdots\cdots$$

$$\bar{\psi}(x) = \bar{\psi}(0) + \bar{\psi}(0) \overleftarrow{D}^\alpha x_\alpha + \frac{1}{2} \bar{\psi}(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta x_\alpha x_\beta + \frac{1}{3!} \bar{\psi}(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overleftarrow{D}^\rho x_\alpha x_\beta x_\rho$$

$$+ \frac{1}{4!} \bar{\psi}(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overleftarrow{D}^\rho \overleftarrow{D}^\sigma x_\alpha x_\beta x_\rho x_\sigma + \cdots\cdots$$

$$D^\alpha = \partial^\alpha - i g T^a A^{a\alpha}, \qquad [D^\alpha, D^\beta] = -i g T^a G^{a\alpha\beta}$$

$$\langle 0 | \bar{\psi}^a_i(x) \psi^b_j(0) | 0 \rangle = \langle 0 | \bar{\psi}^a_i(0) \psi^b_j(0) | 0 \rangle + x_\alpha \langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \psi^b_j(0) | 0 \rangle$$

$$+ \frac{1}{2} x_\alpha x_\beta \langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \psi^b_j(0) | 0 \rangle + \frac{1}{3!} x_\alpha x_\beta x_\rho \langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overleftarrow{D}^\rho \psi^b_j(0) | 0 \rangle$$

$$+ \frac{1}{4!} x_\alpha x_\beta x_\rho x_\sigma \langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overleftarrow{D}^\rho \overleftarrow{D}^\sigma \psi^b_j(0) | 0 \rangle + \cdots\cdots$$

① $$\langle 0 | \bar{\psi}^a_i(0) \psi^b_j(0) | 0 \rangle = C \delta_{ab} \delta_{ij} = \frac{1}{12} \langle 0 | \bar{\psi} \psi | 0 \rangle \delta_{ab} \delta_{ij}$$

② $$\langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \psi^b_j(0) | 0 \rangle = C \delta_{ab} (\gamma^\alpha)_{ji}$$

$$\langle 0 | \bar{\psi} \overleftarrow{\not{D}} \psi | 0 \rangle = C \cdot 3 \cdot \mathrm{Tr}[\gamma^\alpha \gamma_\alpha] = 48 C$$

$$\left.\begin{array}{l} (i\vec{\not{D}} - m)\psi = 0 \\ \bar{\psi}(i\overleftarrow{\not{D}} + m) = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \vec{\not{D}}\psi = -im\psi \\ \bar{\psi}\overleftarrow{\not{D}} = im\bar{\psi} \end{array}\right.$$

$$C = \frac{1}{48} \langle 0 | \bar{\psi} \overleftarrow{\not{D}} \psi | 0 \rangle = \frac{im}{48} \langle 0 | \bar{\psi} \psi | 0 \rangle$$

$$\langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \psi^b_j(0) | 0 \rangle = \frac{im}{48} \langle 0 | \bar{\psi} \psi | 0 \rangle \delta_{ab} (\gamma^\alpha)_{ji}$$

③ $$\langle 0 | \bar{\psi}^a_i(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \psi^b_j(0) | 0 \rangle = A g^{\alpha\beta} \delta_{ab} \delta_{ij} + B (\sigma^{\alpha\beta})_{ji} \delta_{ab}$$

$$\langle 0 | \bar{\psi}(0) \overleftarrow{D}^\alpha \overleftarrow{D}^\beta g_{\alpha\beta} \psi(0) | 0 \rangle = 48 A$$

a, b — color index

i, j — spinor index

$$A = \frac{1}{48} \langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} g_{\alpha\beta} \psi(0) | 0 \rangle$$

$$= \frac{1}{48} \langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} \frac{1}{2} (\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha) \psi(0) | 0 \rangle$$

$$= \frac{1}{96} \Big[\langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle + \langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} \gamma_\beta \gamma_\alpha \psi(0) | 0 \rangle \Big]$$

$$\langle 0 | \bar{\Psi}(0) \left([\overleftarrow{D}^{\alpha}, \overleftarrow{D}^{\beta}] + \overleftarrow{D}^{\beta} \overleftarrow{D}^{\alpha} \right) \gamma_\beta \gamma_\alpha \psi(0) | 0 \rangle$$

$$= \langle 0 | \bar{\Psi}(0) [\overleftarrow{D}^{\alpha}, \overleftarrow{D}^{\beta}] \gamma_\beta \gamma_\alpha \psi(0) | 0 \rangle + \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle$$

$$= -ig \langle 0 | \bar{\Psi}(0) T^a G^{a\alpha\beta} \gamma_\beta \gamma_\alpha \psi(0) | 0 \rangle + \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle$$

$$= g \langle 0 | \bar{\Psi}(0) T^a G^{a\alpha\beta} \sigma_{\alpha\beta} \psi(0) | 0 \rangle + \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle$$

$$= \frac{1}{48} \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle + \frac{1}{96} g \langle 0 | \bar{\Psi}(0) T^a G^{a\mu\nu} \sigma_{\mu\nu} \psi(0) | 0 \rangle$$

$$= -\frac{m^2}{48} \langle 0 | \bar{\Psi}(0) \psi(0) | 0 \rangle + \frac{1}{96} g \langle 0 | \bar{\Psi}(0) T^a G^{a\mu\nu} \sigma_{\mu\nu} \psi(0) | 0 \rangle$$

$$\langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} \sigma_{\alpha\beta} \psi(0) | 0 \rangle = B \cdot 3 \, \mathrm{Tr} [\sigma^{\alpha\beta} \sigma_{\alpha\beta}]$$

$$\mathrm{Tr}[\sigma^{\alpha\beta} \sigma_{\alpha\beta}] = -\frac{1}{4} \mathrm{Tr} [(\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha)(\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha)]$$

$$= -\frac{1}{4} \mathrm{Tr} [\gamma^\alpha \gamma^\beta \gamma_\alpha \gamma_\beta - \gamma^\alpha \gamma^\beta \gamma_\beta \gamma_\alpha - \gamma^\beta \gamma^\alpha \gamma_\alpha \gamma_\beta + \gamma^\beta \gamma^\alpha \gamma_\beta \gamma_\alpha]$$

$$= -\frac{1}{4} \mathrm{Tr} [-2\gamma^\beta \gamma_\beta - 4\gamma^\alpha \gamma_\alpha - 4\gamma^\beta \gamma_\beta - 2\gamma^\alpha \gamma_\alpha]$$

$$= 48$$

$$\langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} \sigma_{\alpha\beta} \psi(0) | 0 \rangle = \frac{i}{2} \langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \psi(0) | 0 \rangle$$

$$= \frac{i}{2} \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle - \frac{i}{2} \langle 0 | \bar{\Psi}(0) \overleftarrow{D}^{\alpha} \overleftarrow{D}^{\beta} \gamma_\beta \gamma_\alpha \psi(0) | 0 \rangle$$

$$= \frac{i}{2} \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle - \frac{i}{2} \Big[g \langle 0 | \bar{\Psi}(0) T^a G^{a\alpha\beta} \sigma_{\alpha\beta} \psi(0) | 0 \rangle + \langle 0 | \bar{\Psi}(0) \overleftarrow{\not{D}} \overleftarrow{\not{D}} \psi(0) | 0 \rangle \Big]$$

$$\langle 0|\bar{\psi}(0)\overleftarrow{D}^{\alpha}\overleftarrow{D}^{\beta}\sigma_{\alpha\beta}\psi(0)|0\rangle = -\frac{i}{2}g\langle 0|\bar{\psi}(0)T^{a}G^{a\alpha\beta}\sigma_{\alpha\beta}\psi(0)|0\rangle$$

$$B = -\frac{i}{288}g\langle 0|\bar{\psi}(0)T^{a}G^{a\alpha\beta}\sigma_{\alpha\beta}\psi(0)|0\rangle$$

$$\begin{cases} A = -\frac{m^2}{48}\langle 0|\bar{\psi}\psi|0\rangle + \frac{1}{96}g\langle 0|\bar{\psi}(0)T^{a}G^{a\mu\nu}\sigma_{\mu\nu}\psi|0\rangle \\ B = -\frac{i}{288}g\langle 0|\bar{\psi}T^{a}G^{a\mu\nu}\sigma_{\mu\nu}\psi(0)|0\rangle \end{cases}$$

To the order x^2

$$\langle 0|\bar{\psi}_i^a(x)\psi_j^b(0)|0\rangle = \delta_{ab}\Big[\frac{1}{12}\langle 0|\bar{\psi}\psi|0\rangle\delta_{ij} + \frac{im}{48}\langle 0|\bar{\psi}\psi|0\rangle \not{x}_{ji}$$

$$+\frac{1}{2}x_\alpha x_\beta g^{\alpha\beta}\Big(-\frac{m^2}{48}\langle 0|\bar{\psi}\psi|0\rangle + \frac{1}{96}g\langle 0|\bar{\psi}(0)T^{a}G^{a\mu\nu}\sigma_{\mu\nu}\psi|0\rangle\Big)\delta_{ij}+\cdots\Big]$$

$$= \delta_{ab}\Big[\frac{1}{12}\langle 0|\bar{\psi}\psi|0\rangle\delta_{ij} + \frac{im}{48}\langle 0|\bar{\psi}\psi|0\rangle\not{x}_{ji} - \frac{m^2}{96}\langle 0|\bar{\psi}\psi|0\rangle x^2\delta_{ij}$$

$$+\frac{1}{192}x^2 g\langle 0|\bar{\psi}T^{a}G^{a\mu\nu}\sigma_{\mu\nu}\psi|0\rangle\delta_{ij}+\cdots\Big]$$

$$\langle 0|\bar{\psi}_i^a(x)\psi_j^b(0)|0\rangle = \delta_{ab}\Big[\frac{1}{12}\langle 0|\bar{\psi}\psi|0\rangle\delta_{ij} + \frac{im}{48}\langle 0|\bar{\psi}\psi|0\rangle(\not{x}-\not{y})_{ji}$$

$$-\frac{m^2}{96}\langle 0|\bar{\psi}\psi|0\rangle(x-y)^2\delta_{ij} + \frac{1}{192}(x-y)^2 g\langle 0|\bar{\psi}T^{a}G^{a\mu\nu}\sigma_{\mu\nu}\psi|0\rangle\delta_{ij}$$

$$+\cdots\Big]$$

$\langle 0|\bar{\Psi}^a_i(x)\,\Psi^b_j(0)|0\rangle$ up to order x^3

$D_\mu = \partial_\mu - igT^a A^a_\mu$, adjoint $(\hat{D}_\mu)_{ab} = \partial_\mu \delta_{ab} - gf^{abc}A^a_\mu$

$\hat{D}_\rho G^c_{\mu\nu} = (\hat{D}^{cd}_\rho G^d_{\mu\nu})$

$\langle 0|\bar{\Psi}^a_i \Psi^b_j \hat{D}_\rho G^c_{\mu\nu}|0\rangle = A(g_{\rho\nu}\gamma_\mu - g_{\rho\mu}\gamma_\nu)_{ji}(T^c)_{ba}$

multiplying $g^{\rho\mu}\gamma^\nu_{ij}(T^c)_{ab}$, and use $(D_\mu G^{\mu\nu})^a = -g\bar{\Psi}\gamma^\nu T^a\Psi$

$$A = \frac{g}{3\cdot 2^6}\langle 0|\bar{\Psi}\gamma_\mu T^a\Psi\,\bar{\Psi}\gamma^\mu T^a\Psi|0\rangle = -\frac{g}{3^3\cdot 2^4}\langle 0|\bar{\Psi}\Psi|0\rangle^2$$

$$\langle 0|\bar{\Psi}^a_i(0)\overleftarrow{D}^\alpha\overleftarrow{D}^\beta\overleftarrow{D}^\rho\Psi^b_j(0)|0\rangle = \delta^{ab}(B\gamma^\alpha g^{\beta\rho} + C\gamma^\beta g^{\alpha\rho} + D\gamma^\rho g^{\alpha\beta})_{ji}$$

① $(\gamma_\alpha)_{ij}\, g_{\beta\rho}\,\delta_{ab}$:

$$\langle 0|\bar{\Psi}\overleftarrow{\not{D}}\overleftarrow{D}^\beta\overleftarrow{D}^\rho g_{\beta\rho}\Psi|0\rangle = 3\times 4(B\times 4^2 + C g^\beta{}_\alpha g^{\alpha\rho}g_{\beta\rho} + D g^\rho{}_\alpha g^{\alpha\beta}g_{\beta\rho})$$

$$= 3\times 4(4^2B + 4C + 4D)$$

$$= 48(4B + C + D)$$

$$im\underbrace{\langle 0|\bar{\Psi}\overleftarrow{D}^\beta\overleftarrow{D}^\rho g_{\beta\rho}\Psi|0\rangle}_{-m^2\langle 0|\bar{\Psi}\Psi|0\rangle + \frac{1}{2}g\langle 0|\bar{\Psi}T^aG^{a\mu\nu}\sigma_{\mu\nu}\Psi|0\rangle} = 48(4B + C + D)$$

$\langle 0|\bar{\Psi}\Psi|0\rangle \to O_3$, $\langle 0|\bar{\Psi}T^aG^{a\mu\nu}\sigma_{\mu\nu}\Psi|0\rangle \to O_5$

$$-im^3 O_3 + \frac{i}{2}mg\,O_5 = 48(4B + C + D)$$

② $(\gamma_\beta)_{ij}\, g_{\alpha\rho}\,\delta_{ab}$:

$$\langle 0|\bar{\Psi}\overleftarrow{D}^\alpha\overleftarrow{\not{D}}\overleftarrow{D}^\rho g_{\alpha\rho}\Psi|0\rangle = 3\times 4(4B + 16C + 4D) = 48(B + 4C + D)$$

$$D^\alpha\not{D} = [D^\alpha, \not{D}] + \not{D}D^\alpha$$

$$= -igT^aG^{a\alpha\beta}\gamma_\beta + \not{D}D^\alpha$$

$$\langle 0|\bar{\Psi}\overleftarrow{\not{D}}\overleftarrow{D}^\alpha\overleftarrow{D}^\rho g_{\alpha\rho}\Psi|0\rangle - ig\langle 0|\bar{\Psi}T^aG^{a\alpha\beta}\gamma_\beta\overleftarrow{D}^\rho g_{\alpha\rho}\Psi|0\rangle$$

$$= im\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\rho g_{\alpha\rho}\psi|0\rangle - ig\langle 0|\bar\Psi T^a G^{a\rho\beta}\gamma_\beta \overleftarrow{D}_\rho \psi|0\rangle$$

$$\boxed{[D_\rho, T^a G^{a\rho\beta}] = T^a(\hat{D}_\rho G^{\rho\beta})^a}$$ $[T^a G^a_{\mu\nu}, \overleftarrow{D}_\rho] = T^a(\hat{D}_\rho G_{\mu\nu})^a$

$$= im\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\rho g_{\alpha\rho}\psi|0\rangle - ig\Big[\langle 0|(G^{\rho\beta}\hat{\overleftarrow{D}}_\rho)^a \bar\Psi T^a\gamma_\beta \psi|0\rangle$$

$$+ \langle 0|\bar\Psi \overleftarrow{D}_\rho T^a G^{a\rho\beta}\gamma_\beta \psi|0\rangle\Big]$$

On the other hand

$$\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{\not D}\, \overleftarrow{D}^\rho g_{\alpha\rho}\psi|0\rangle$$

$$= \langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{\not D}\, \overleftarrow{D}_\alpha \psi|0\rangle = \langle 0|\bar\Psi \overleftarrow{D}^\alpha \underbrace{\overrightarrow{\not D}\, \overrightarrow{D}_\alpha}\psi|0\rangle$$

$$= \langle 0|\bar\Psi \overleftarrow{D}^\alpha \overrightarrow{D}_\alpha \overrightarrow{\not D}\psi|0\rangle + \langle 0|\bar\Psi \overleftarrow{D}^\alpha [\overrightarrow{\not D}, \overrightarrow{D}_\alpha]\psi|0\rangle$$

$$= -im\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overrightarrow{D}_\alpha \psi|0\rangle - ig\langle 0|\bar\Psi \overleftarrow{D}^\alpha T^a G^a_{\beta\alpha}\gamma^\beta \psi|0\rangle$$

$$= im\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}_\alpha \psi|0\rangle + ig\langle 0|\bar\Psi \overleftarrow{D}^\alpha T^a G^a_{\alpha\beta}\gamma^\beta \psi|0\rangle$$

$$\Downarrow \langle 0|\bar\Psi \overleftarrow{D}_\rho T^a G^{a\rho\beta}\gamma_\beta \psi|0\rangle = -\frac{1}{2}\langle 0|(G^{\rho\beta}\overleftarrow{D}_\rho)^a \bar\Psi T^a \gamma_\beta \psi|0\rangle$$

$$\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{\not D}\, \overleftarrow{D}^\rho g_{\alpha\rho}\psi|0\rangle = im\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\rho g_{\alpha\rho}\psi|0\rangle - \frac{i}{2}g\langle 0|(G^{\rho\beta}\hat{\overleftarrow{D}}_\rho)^a \bar\Psi T^a\gamma_\beta\psi|$$

$$\boxed{(\hat{\overrightarrow{D}}_\mu G^{\mu\nu})^a = -g\bar\Psi\gamma^\nu T^a\psi}$$

$$= -im^3 O_3 + \frac{i}{2}mg\,O_5 + \frac{i}{2}g^2\underbrace{\langle 0|\bar\Psi\gamma^\beta T^a\psi\,\bar\Psi T^a\gamma_\beta\psi|0\rangle}$$

$$\downarrow$$

$$\frac{1}{12^2}\left(-\mathrm{Tr}[\gamma^\beta T^a T^a\gamma_\beta]\right) = -\frac{16\times 4}{12\times 12} = -\frac{4}{3}$$

$$= -im^3 O_3 + \frac{i}{2}mg\,O_5 - \frac{2}{9}ig^2\langle\bar\Psi\psi\rangle^2$$

$$-im^3 O_3 + \frac{i}{2}mg\,O_5 - \frac{2}{9}ig^2\langle\bar\Psi\psi\rangle^2 = 48(B+4C+D)$$

③ $(\gamma_\rho)_{ij}\, g_{\alpha\beta}\, \delta_{ab}$

$$\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overleftarrow{\not D}\, g_{\alpha\beta}\Psi|0\rangle = 3\times 4\,(4B + 4C + 4^2 D)$$

$$= 48\,(B + C + 4D)$$

$\downarrow$

$$-\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\beta \overrightarrow{\not D}\, g_{\alpha\beta}\Psi|0\rangle = im \underbrace{\langle 0|\bar\Psi \overleftarrow{D}^\alpha \overleftarrow{D}^\beta g_{\alpha\beta}\Psi|0\rangle}_{-m^2 \underset{\downarrow\, O_3}{\langle 0|\bar\Psi\Psi|0\rangle} + \frac{1}{2} g \underset{\downarrow\, O_5}{\langle 0|\bar\Psi t^a G^{a\mu\nu}\sigma_{\mu\nu}\Psi|0\rangle}}$$

$$= -im^3 O_3 + \frac{i}{2} mg O_5$$

$$-im^3 O_3 + \frac{1}{2} img O_5 = 48(B + C + 4D)$$

$$\begin{cases} -im^3 O_3 + \frac{1}{2} img O_5 = 48(4B + C + D) & ① \\ -im^3 O_3 + \frac{1}{2} img O_5 - \frac{2}{9} ig^2 \langle\bar\Psi\Psi\rangle^2 = 48(B + 4C + D) & ② \\ -im^3 O_3 + \frac{1}{2} img O_5 = 48(B + C + 4D) & ③ \end{cases}$$

①−③ $\quad 0 = 48(3B - 3D) \quad \Rightarrow B = D$

$$\begin{cases} -im^3 O_3 + \frac{1}{2} img O_5 = 48(5B + C) \\ -im^3 O_3 + \frac{1}{2} img O_5 - \frac{2}{9} ig^2 \langle\bar\Psi\Psi\rangle^2 = 48(2B + 4C) \end{cases}$$

$$3\left(-im^3 O_3 + \frac{1}{2} img O_5\right) + \frac{2}{9} ig^2 \langle\bar\Psi\Psi\rangle^2 = 48\times 18\, B$$

$$\frac{1}{3^2\times 48\times 9} = \frac{1}{3^5\times 2^4}$$

$$\begin{cases} B = \frac{1}{48\times 6}\left(-im^3 O_3 + \frac{i}{2} mg O_5\right) + \frac{ig^2}{3^5\times 2^4}\langle\bar\Psi\Psi\rangle^2 \\ B = D \\ C = \frac{1}{48}\left(-im^3 O_3 + \frac{i}{2} mg O_5\right) - 5B \end{cases}$$

$$48\times 6 = 3^2\times 2^5$$

$$= \frac{1}{48\times 6}\left(-im^3 O_3 + \frac{i}{2} mg O_5\right) - \frac{5ig^2}{3^5\times 2^4}\langle\bar\Psi\Psi\rangle^2$$

$$\frac{1}{3!} x_\alpha x_\beta x_\rho \langle 0 | \bar{\psi}_i^a \overleftarrow{D}^\alpha \overleftarrow{D}^\rho \overleftarrow{D}^\rho \psi_j^b | 0 \rangle$$

$$= \frac{1}{3!} x_\alpha x_\beta x_\rho \, \delta^{ab} \left(B \gamma^\alpha g^{\beta\rho} + C \gamma^\beta g^{\alpha\rho} + D \gamma^\rho g^{\alpha\beta} \right)_{ji}$$

$$= \frac{1}{3!} \delta^{ab} \left(B \not{x} x^2 + C \not{x} x^2 + D \not{x} x^2 \right)_{ji} = \frac{1}{3!} \delta^{ab} (B+C+D) x^2 \not{x}_{ji}$$

$$\boxed{B+C+D = \frac{1}{48 \times 2} \left(-i m^3 O_3 + \frac{i}{2} m g O_5 \right) - \frac{i g^2}{3^4 \times 2^4} \langle \bar{\psi}\psi \rangle^2}$$

$$= \frac{1}{3!} \delta_{ab} \left[\frac{1}{48 \times 2} \left(-i m^3 O_3 + \frac{i}{2} m g O_5 \right) - \frac{i g^2}{3^4 \times 2^4} \langle \bar{\psi}\psi \rangle^2 \right] x^2 \not{x}_{ji}$$

$$48 \times 2 = 3 \times 2^5$$

$$= \frac{i}{3!} \delta_{ab} \left[\frac{1}{96} \left(- m^3 \langle \bar{\psi}\psi \rangle + \frac{1}{2} m g \langle \bar{\psi} T G \sigma \psi \rangle \right) - \frac{g^2}{3^4 \times 2^4} \langle \bar{\psi}\psi \rangle^2 \right] x^2 \not{x}_{ji}$$

up to the order of x^3

$$\langle 0 | \bar{\psi}_i^a(x) \psi_j^b(0) | 0 \rangle = \delta_{ab} \Big[\frac{1}{12} \langle \bar{\psi}\psi \rangle \delta_{ij} + \frac{im}{48} \langle \bar{\psi}\psi \rangle \not{x}_{ji} - \frac{m^2}{96} \langle 0 | \bar{\psi}\psi | 0 \rangle x^2 \delta_{ij}$$

$$+ \frac{1}{192} x^2 g \langle \bar{\psi} T G \sigma \psi \rangle \delta_{ij} + \frac{i}{3!} \left(\frac{1}{96} \left(- m^3 \langle \bar{\psi}\psi \rangle + \frac{1}{2} m g \langle \bar{\psi} T G \sigma \psi \rangle \right) - \frac{g^2}{3^4 \times 2^4} \langle \bar{\psi}\psi \rangle^2 \right)$$

$$\cdot x^2 \not{x}_{ji} + \cdots \Big]$$

$$\langle 0 | \bar{\psi}_i^a(x) \psi_j^b(y) | 0 \rangle = \delta_{ab} \Big[\frac{1}{12} \langle \bar{\psi}\psi \rangle \delta_{ij} - \frac{im}{48} \langle \bar{\psi}\psi \rangle (\not{x} - \not{y})_{ji} - \frac{m^2}{96} \langle \bar{\psi}\psi \rangle (x-y)^2 \delta_{ji}$$

$$+ \frac{1}{192} (x-y)^2 g \langle \bar{\psi} T G \sigma \psi \rangle \delta_{ji} + \frac{i}{3!} \left(\frac{1}{96} \left(- m^3 \langle \bar{\psi}\psi \rangle + \frac{1}{2} m g \langle \bar{\psi} T G \sigma \psi \rangle \right) - \frac{g^2}{3^4 \times 2^4} \langle \bar{\psi}\psi \rangle^2 \right)$$

$$\cdot (x-y)^2 (\not{x} - \not{y})_{ji} + \cdots \Big]$$

$$\langle 0|\bar{\Psi}^i_\alpha(x)\,\psi^j_\beta(0)\,G^a_{\mu\nu}|0\rangle$$

$$= \langle 0|\bar{\Psi}^i_\alpha(0)\,\psi^j_\beta(0)\,G^a_{\mu\nu}|0\rangle + x^\rho\langle 0|\bar{\Psi}^i_\alpha\,\overleftarrow{D}_\rho\,\psi^j_\beta\,G^a_{\mu\nu}|0\rangle + \cdots$$

$$\langle 0|\bar{\Psi}^i_\alpha\,\psi^j_\beta\,G^a_{\mu\nu}|0\rangle = C(\sigma_{\mu\nu})_{\beta\alpha}\,T^a_{ji}$$

$$\langle 0|\bar{\Psi}\,\sigma_{\mu\nu}\,T^a\,G^{a\mu\nu}\,\psi|0\rangle = C\,\mathrm{Tr}[\sigma_{\mu\nu}\sigma^{\mu\nu}]\,\mathrm{Tr}[T^aT^a]$$

$$= 48\times 4\,C$$

So: $C = \frac{1}{48\times 4}\langle 0|\bar{\Psi}\sigma T G\psi|0\rangle$

$$\langle 0|\bar{\Psi}^i_\alpha\,\overleftarrow{D}_\rho\,\psi^j_\beta\,G^a_{\mu\nu}|0\rangle = \left[A(g_{\rho\mu}\gamma_\nu - g_{\rho\nu}\gamma_\mu) + iB\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^\sigma\right]_{\beta\alpha}T^a_{ji}$$

$$\boxed{\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^\rho\gamma^\sigma = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)}$$

① $\gamma^\rho\sigma^{\mu\nu}T^a$:

$$\langle 0|\bar{\Psi}\overleftarrow{\not{D}}\sigma T G\psi|0\rangle = 4\,\mathrm{Tr}\Big[A(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu)\sigma^{\mu\nu}$$

$$+ Bi\,\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^\sigma\gamma^\rho\sigma^{\mu\nu}\Big]$$

$$= 4\,\mathrm{Tr}\Big[A\tfrac{i}{2}(+48) - iB\,i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\sigma^{\mu\nu}\Big]$$

$$= 4\,\mathrm{Tr}\Big[+\tfrac{i}{2}48A + B\tfrac{i}{2}(-48)\Big]$$

$$= 96\times 4\,i(A-B)$$

$$\bar{\Psi}\overleftarrow{\not{D}} = im\bar{\Psi}$$

So: $m\langle\bar{\Psi}\sigma T G\psi\rangle = 96\times 4(A-B)$

② $g^{\rho\mu}\gamma^{\nu}T^{a}_{ij}$:

$$\langle 0|\bar\Psi \overleftarrow{D}^{\mu} T^{a}\gamma^{\nu}\Psi G^{a}_{\mu\nu}|0\rangle = 4\,\mathrm{Tr}\left[A(4\times 4-4)\right] = 48\times 4A$$

from previous pages: $[D_\rho, T^a G^a_{\mu\nu}] = T^a (D_\rho G_{\mu\nu})^a$

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$$\langle 0|\bar\Psi \overleftarrow{D}^{\mu} T^{a}\gamma^{\nu}\Psi G^{a}_{\mu\nu}|0\rangle = -\frac{1}{2}\langle 0|(G_{\mu\nu}\hat{D}^{\mu})^{a}\,\bar\Psi T^{a}\gamma^{\nu}\Psi|0\rangle$$

$$= \frac{1}{2}g\langle 0|\bar\Psi T^{a}\gamma_{\nu}\Psi\;\bar\Psi T^{a}\gamma^{\nu}\Psi|0\rangle$$

$$= \frac{1}{2}g\,\frac{1}{12^2}\left(-\mathrm{Tr}[T^a T^a \gamma_\nu\gamma^\nu]\right) = \frac{1}{2}g\,\frac{-4\times 16}{12^2}$$

$$= -\frac{2}{9}g\,\langle\bar\Psi\Psi\rangle^2$$

$$A = -\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2$$

$$B = -\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2 - \frac{m}{96\times 4}\langle\bar\Psi\sigma T G\Psi\rangle$$

$$\langle 0|\bar\Psi^{i}_{\alpha}(x)\Psi^{j}_{\beta}(0)\,G^{a}_{\mu\nu}|0\rangle$$

$$= \frac{1}{192}\langle 0|\bar\Psi\sigma T G\Psi|0\rangle(\sigma_{\mu\nu})_{\beta\alpha}T^{a}_{ji} + x^{\rho}\Big[-\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2(g_{\rho\mu}\gamma_\nu - g_{\rho\nu}\gamma_\mu)$$

$$+ i\Big(-\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2 - \frac{m}{96\times 4}\langle\bar\Psi\sigma T G\Psi\rangle\Big)\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^\sigma\Big]_{\beta\alpha}T^{a}_{ji}$$

~~$\langle 0|\bar\Psi^{i}_{\alpha}(0)\Psi^{j}_{\beta}(x)\,G^{a}_{\mu\nu}|0\rangle$~~

~~$= \frac{1}{192}\langle 0|\bar\Psi\sigma T G\Psi|0\rangle(\sigma_{\mu\nu})_{\beta\alpha}T^{a}_{ji} + x^{\rho}\Big[-\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2(g_{\rho\mu}\gamma_\nu - g_{\rho\nu}\gamma_\mu)$~~

~~$+ i\Big(\frac{g}{96\times 9}\langle\bar\Psi\Psi\rangle^2 + \frac{m}{96\times 4}\langle\bar\Psi\sigma T G\Psi\rangle\Big)\varepsilon_{\rho\mu\nu\sigma}\gamma_5\gamma^\sigma\Big]_{\beta\alpha}T^{a}_{ji}$~~