SN neutrino-nucleosynthesis, Laboratory for fundamental physics (1)

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1. Neutrino properties
2. Neutrino oscillations in vacuum, constant electron density, and stars

Reference on neutrino oscillation: M. Fukugita & T. Yanagida
``Physics of Neutrinos and Applications to Astrophysics”

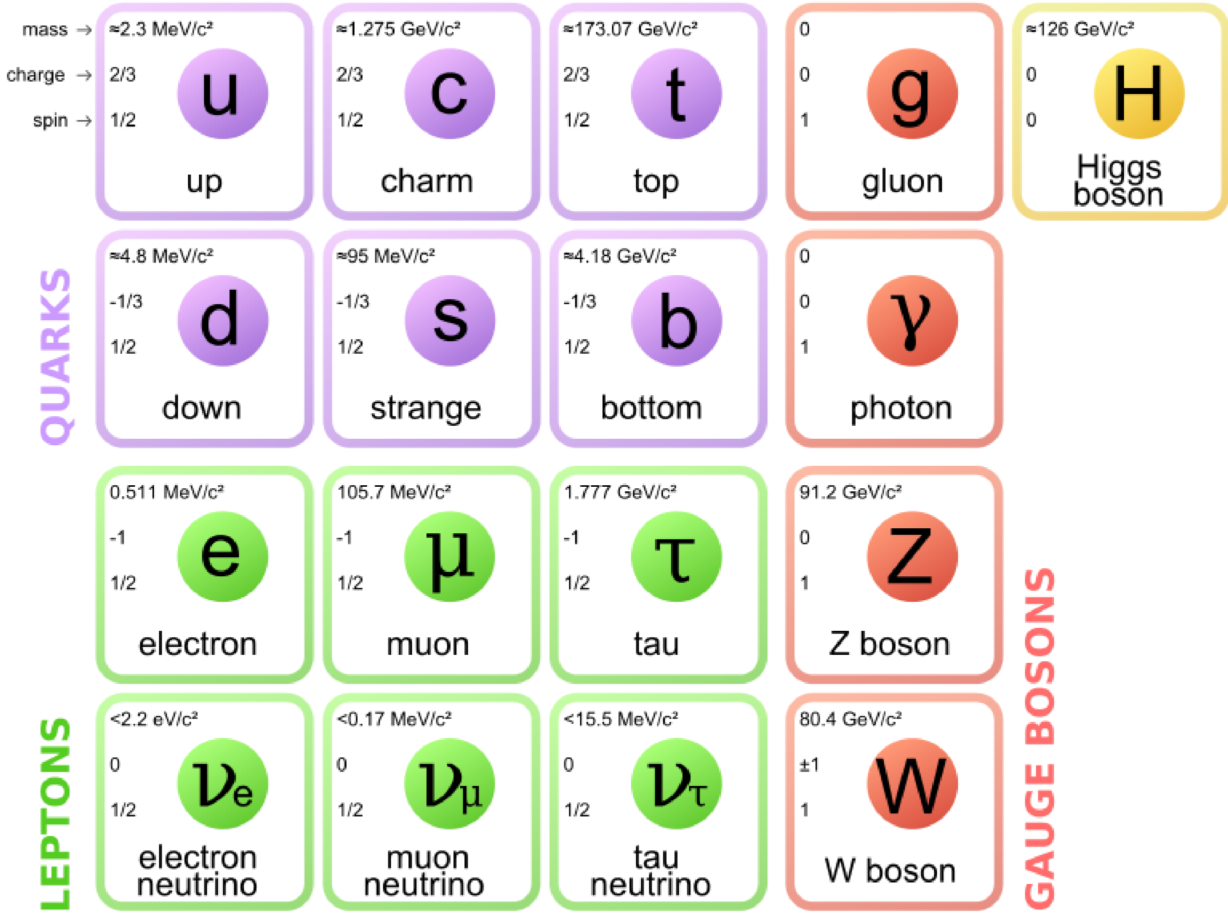
The 16th Nuclei in Cosmos School

2021/9/16

1. Neutrino properties

1. Standard model of particle physics

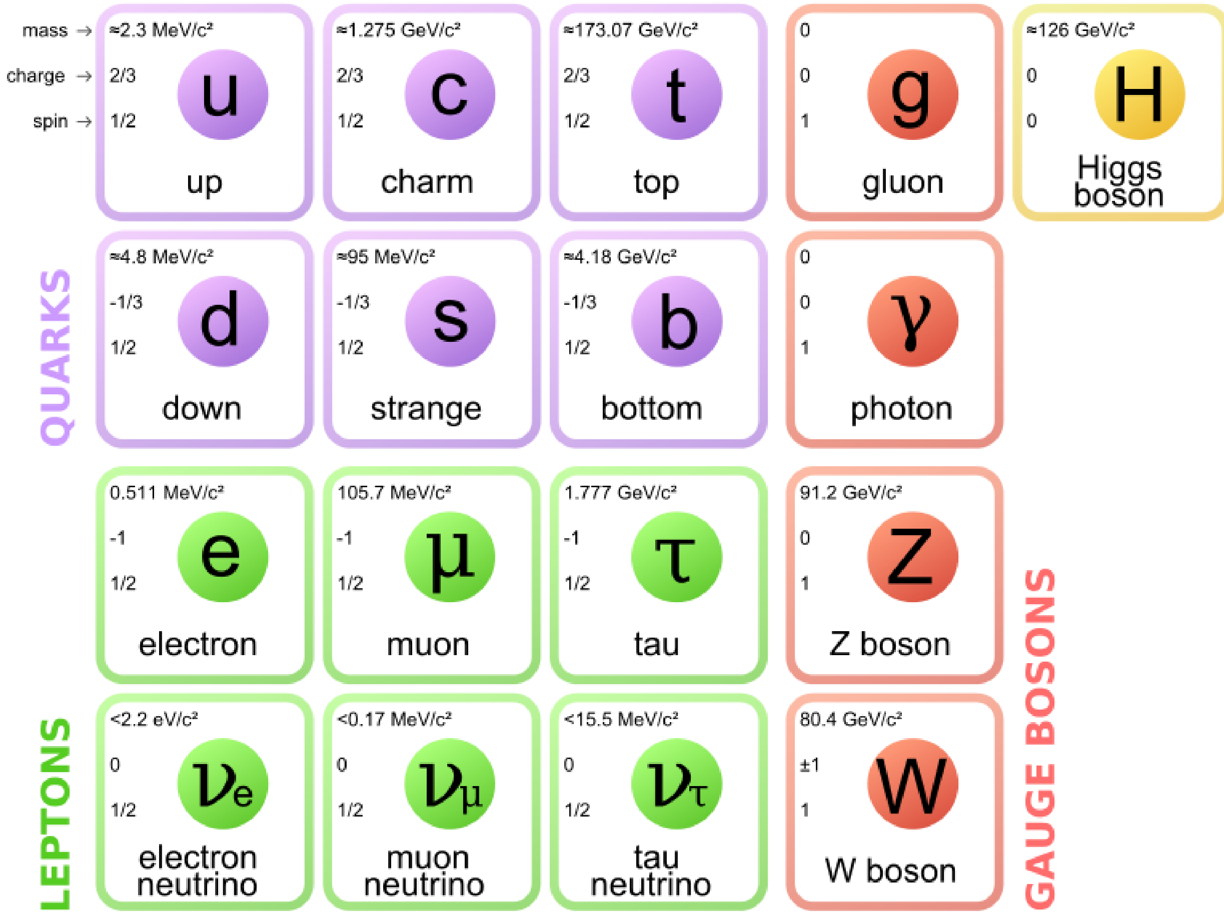
- Excepting the very weak gravity, 3 forces operate to elementary particles.



Wikipedia contributors. Standard Model [Internet]. Wikipedia, The Free Encyclopedia; 2014 Jan 27, 14:33 UTC [cited 2014 Feb 18]. Available from: http://en.wikipedia.org/w/index.php?title=Standard_Model&oldid=592640485.

2. neutrino

- Elementary particle, spin 1/2
- Three flavors, **only left-handed ν's and right-handed $\bar{\nu}$'s**
- Weak and gravitational interactions



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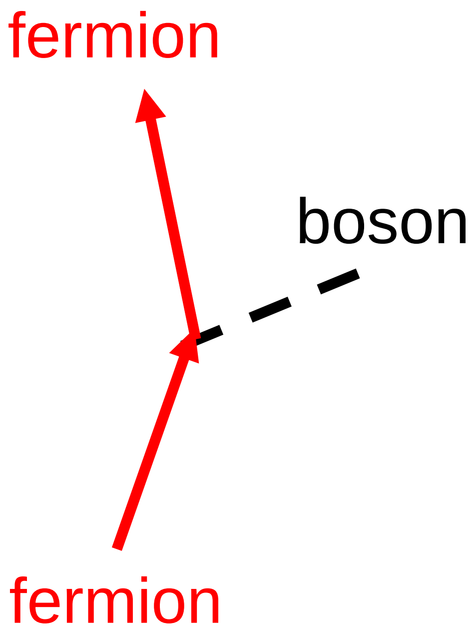
3. Gauge theory

- Interaction of fermions (spin ½) and a mediator boson (spin 1).
- Multiple groups that include gauge theories
  - ✓ $SU(2)_L \times U(1)_Y$: electro-weak interaction
  - ✓ SU(3): strong interaction



four interactions in the universe

interaction	boson		Phenomena
Electromagnetic	Photon	γ	Almost all phenomena around us including radiative nuclear reactions
Weak	Weak bosons	$W^{\pm}$, Z^0	β-decay, ν scattering, …
Strong	Gluon	g	Operating only to quarks
Gravitational	Graviton	G	Attractions between masses

4. Weak interaction (1)

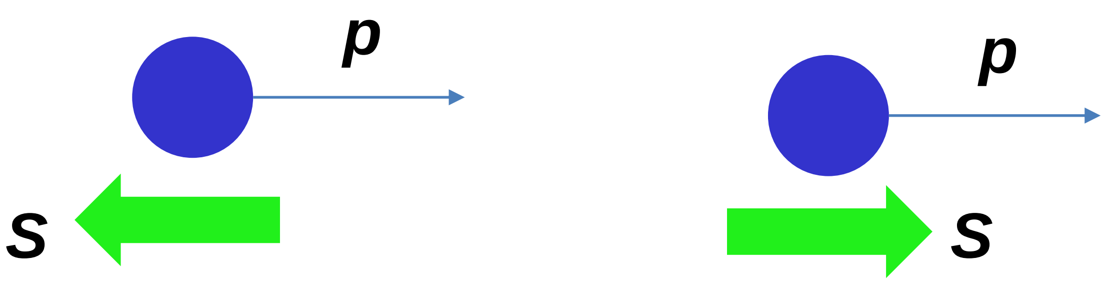
- β-decay, scattering
- V-A interaction; Mediated by weak bosons ($W^{\pm}$, Z^0)
- Masses: m_W =80.4 GeV/c^2, m_Z =91.2 GeV/c^2
- Range is very short $\sim 1/m_W$

- if $E < m_W$, interaction is weak

  (scattering cross section $\propto G_F^2 = 1/m_W^4$)
- if $E > m_W$, interaction is as strong as electromagnetic interaction

  (scattering cross section $\propto g^4$)

- Only <u>left-handed</u> particles & <u>right-handed</u> antiparticles interact

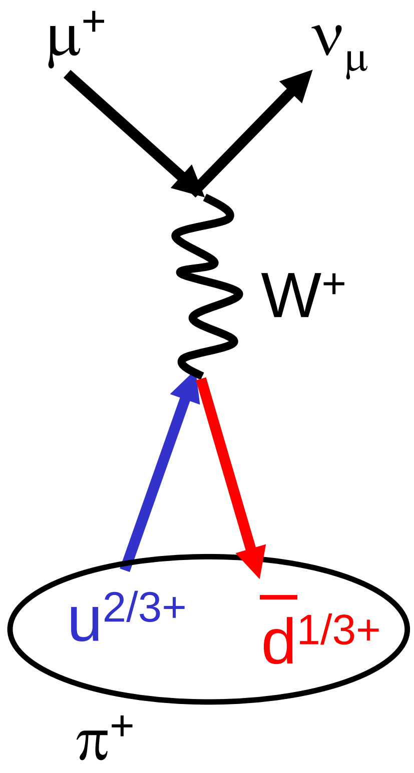


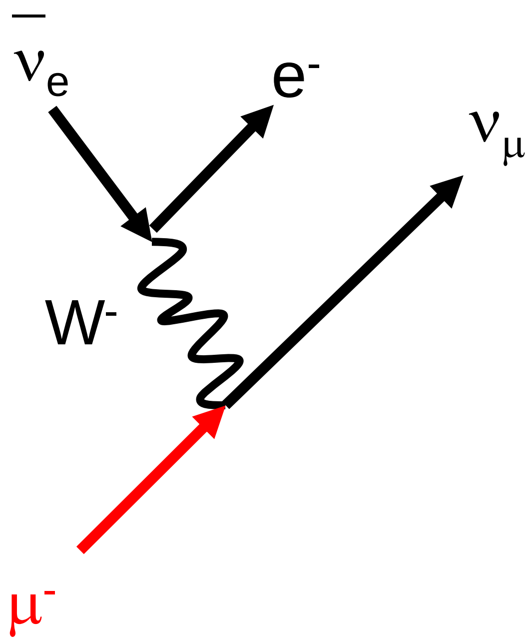
4. Weak interaction (2)

- Mediator particle: $W^{\pm}$ and Z^0 bosons
- Coupling to weak charge (C_V, C_A)
- Coupling constant $= g = G_F^{1/2}$
  $G_F^2 = 1.166 \times 10^{-5}\ GeV^{-2}$

- Short-range force: $V \sim \exp(-m_W r)/r$

- Example:
  - ✓ Pion decay: $\pi^+ \rightarrow \mu^+ + \nu_\mu$
  - ✓ Muon decay: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$



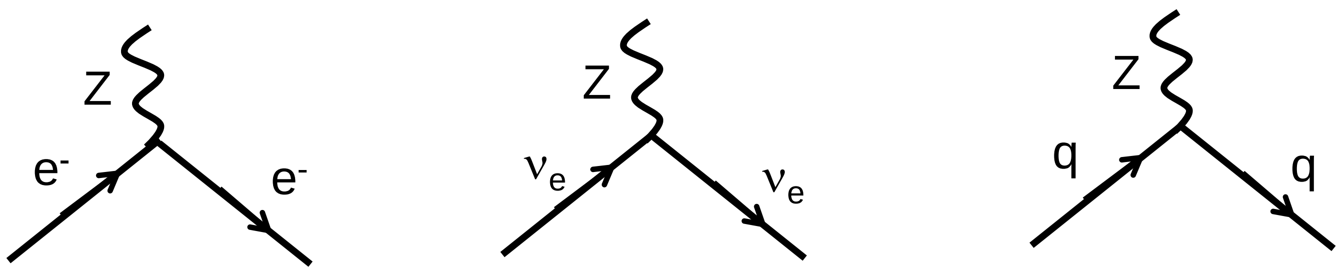


5. neutrino reactions

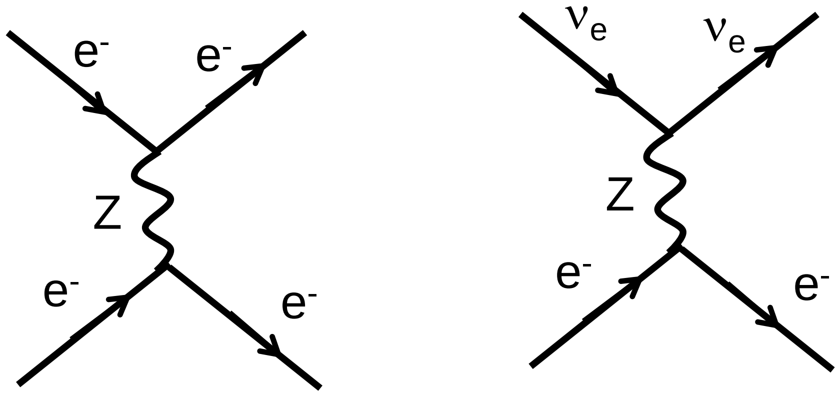
- 2 gauge bosons ($W^{\pm}$ & Z^0)
  - ✓ Neutral current: no flavor change (u ↔ u , e ↔ e)
  - ✓ Charged current: flavors change (u ↔ d , e ↔ ν_e)

(i) Neutral current

Z^0 decay

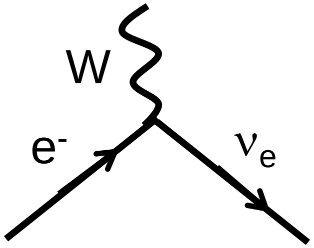


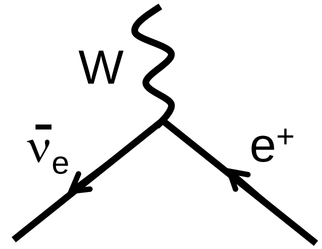
Scattering

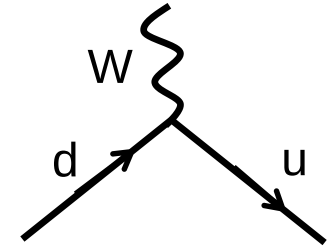


(ii) Charged current

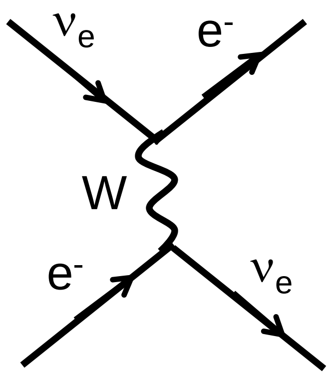
$W^{\pm}$ decay

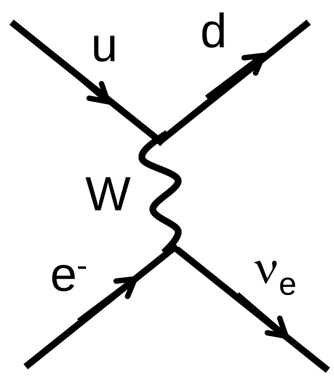


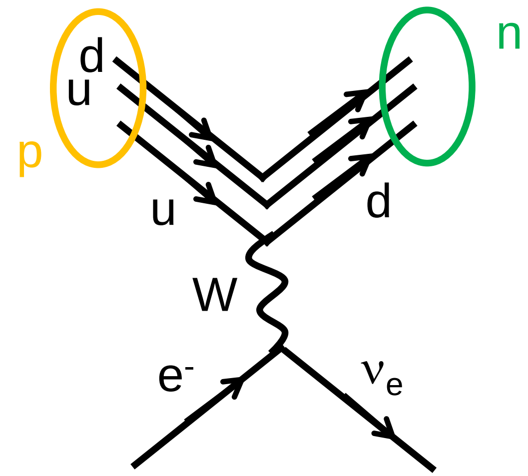




Scattering







$\nu_e + e^- \rightarrow \nu_e + e^-$

$u^{2/3+} + e^- \leftrightarrow d^{1/3-} + \nu_e$

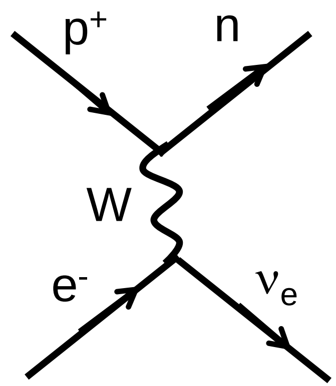
Flavors are different between initial and final states

$p^+ + e^- \leftrightarrow n + \nu_e$

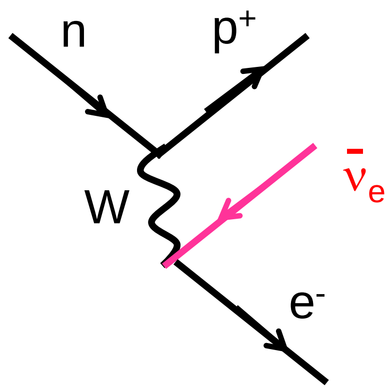
Weak reaction for Conversion of $p \leftrightarrow n$

6. Weak reactions of nuclei

- Electron capture of proton
- Neutron β-decay
- Inverse reactions



$p^+ + e^- \leftrightarrow n + \nu_e$



$n \leftrightarrow p^+ + e^- + \bar{\nu}_e$

- General weak reactions of nuclei

$(Z,A) + e^- \rightarrow (Z-1,A) + \nu_e$	e-capture
$(Z,A) + e^+ \rightarrow (Z+1,A) + \bar{\nu}_e$	positron capture
$(Z,A) \rightarrow (Z+1,A) + e^- + \bar{\nu}_e$	β-decay
$(Z,A) \rightarrow (Z-1,A) + e^+ + \nu_e$	β^+-decay
$\nu_e + (Z-1,A) \rightarrow e^- + (Z,A)$	CC ν reaction
$\bar{\nu}_e + (Z+1,A) \rightarrow e^+ + (Z,A)$	CC $\bar{\nu}$ reaction

7. ν-mass

- **Extremely light**
- ✓ Cosmological bound: $\sum m_i < 0.152$ eV (2019 Review of Particle Physics Particle Data Group)
- ✓ ν-oscillation experiments:

$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m_{32}^2 = (-2.55 \pm 0.04) \times 10^{-3}\ \mathrm{eV}^2 \text{ (for inverted hierarchy), or}$$

$$\Delta m_{32}^2 = (2.444 \pm 0.034) \times 10^{-3}\ \mathrm{eV}^2 \text{ (for normal hierarchy)}$$

	Quarks			Leptons		
Generation	Name	Symbol	M (MeV/c^2)	Name	Symbol	M (MeV/c^2)
1	up	u	$2.16^{+0.49}_{-0.26}$	Electron	e^-	0.511
	down	d	$4.67^{+0.48}_{-0.17}$	Electron neutrino	ν_e	$< 2 \times 10^{-6}$
2	charm	c	1270 ± 20	Muon	μ^-	105.7
	strange	s	93^{+11}_{-5}	Muon neutrino	ν_μ	< 0.19
3	top	t	173,100 $\pm$600	Tau	τ^-	1,776.86 $\pm$0.12
	bottom	b	$4{,}180^{+30}_{-20}$	Tau neutrino	ν_τ	< 18.2

8. History of neutrinos

- No electromagnetic and strong interaction

  → hard to detect

- postulated by Pauli (1930) for energy conservation at the beta decay
- Taken into account in the beta decay theory (Fermi 1934)

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

- Cowan, Reines, Harrison, Kruse, & McGuire (1956) found the neutrino

$$\bar{\nu}_e + p^+ \rightarrow n + e^+$$

$$e^+ + e^- \rightarrow \underline{\gamma + \gamma}$$

$$^{108}Cd + n \rightarrow {}^{109}Cd + \underline{\gamma}$$

Detection of photons

- Homestake experiment (late 1960s) detected ν_e emitted from nuclear fusions in the Sun

$$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$$

→ ν_e flux is smaller by a factor of ~3 than theoretical estimate

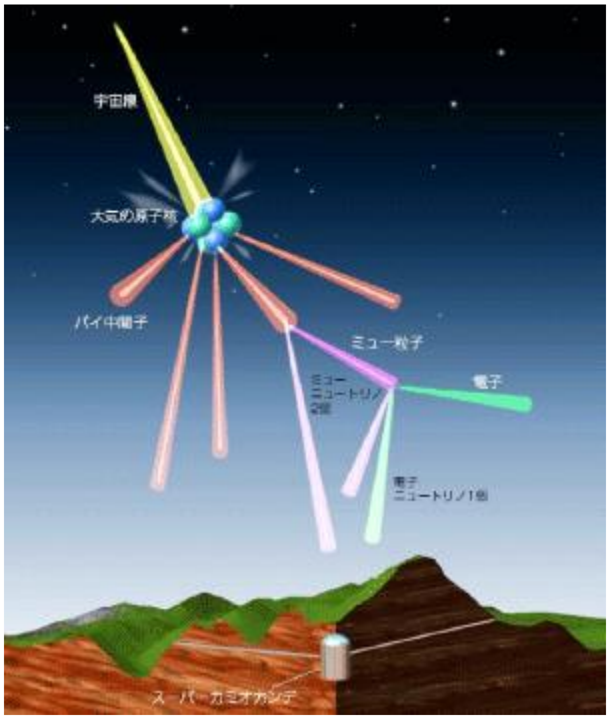
→ Solar ν problem

8. History of neutrinos

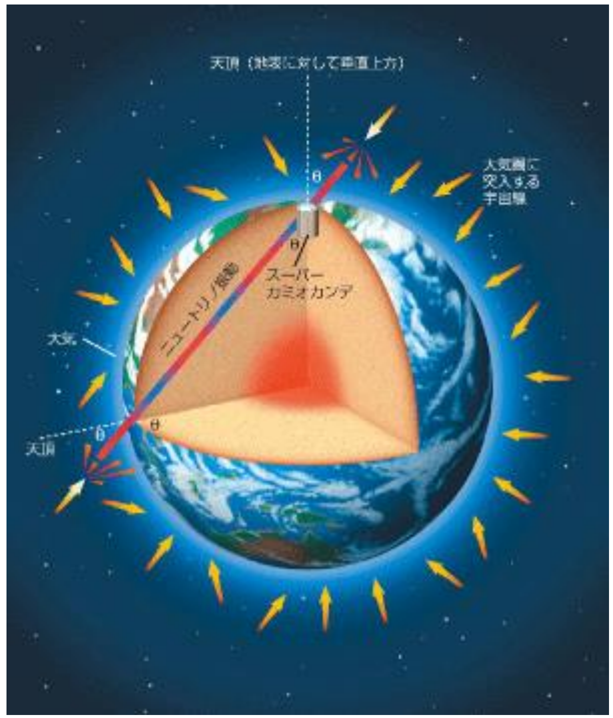
➢Neutrinos oscillate →flavor changes

→Solar ν problem was solved

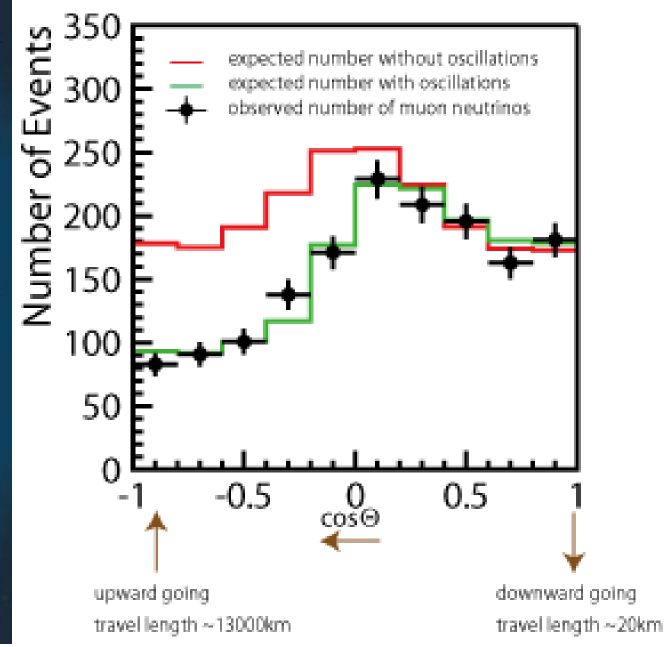
(SK web page: http://www-sk.icrr.u-tokyo.ac.jp/sk/physics/atmnu-e.html)



Cosmic ray reactions in atmosphere generates ν"s



ν's from different directions ←different travel lengths



ν_μ oscillates into another type

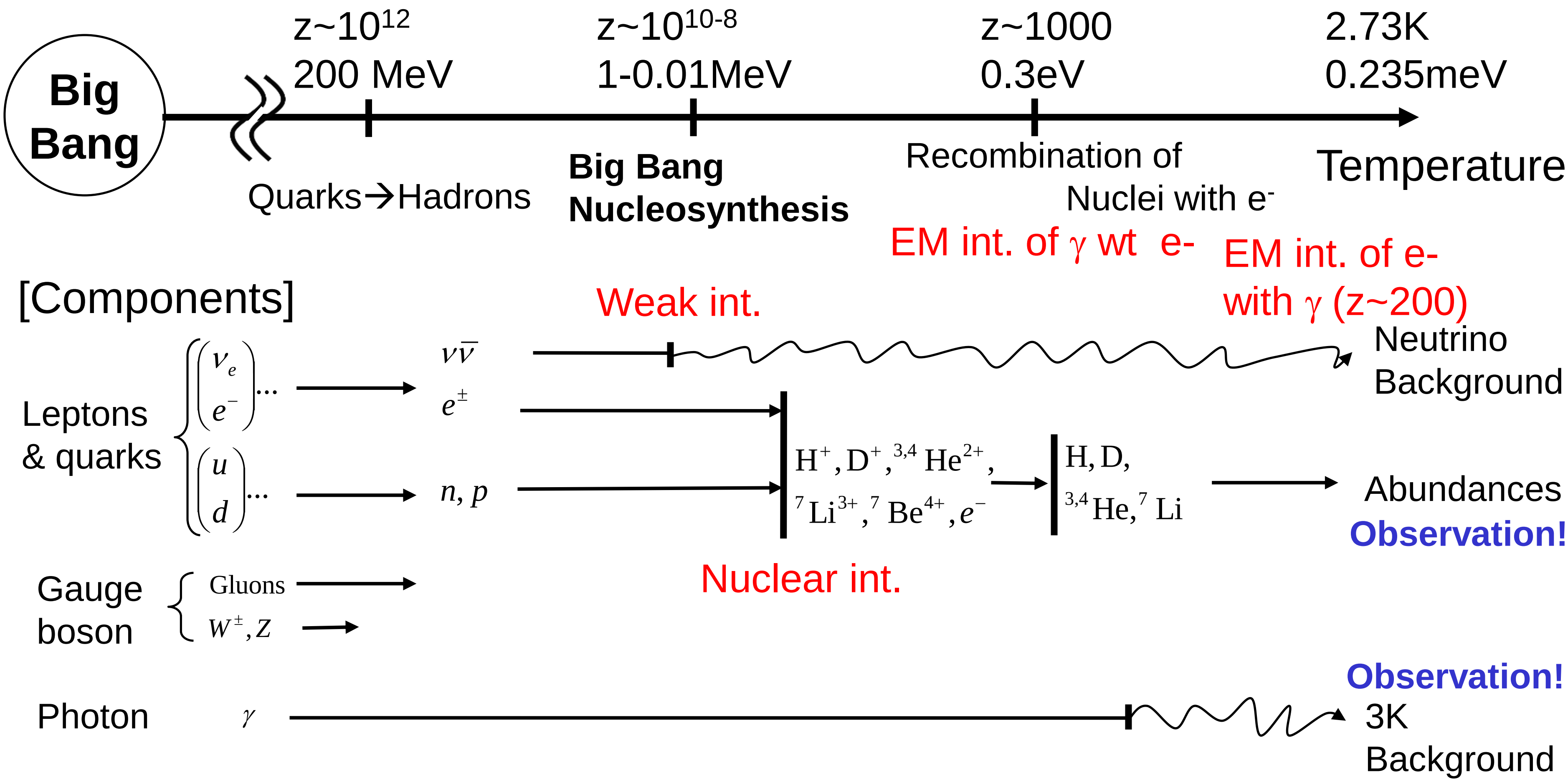
9. Importance of weak interactions

- Weak interaction is weaker than electromagnetic and strong interactions
- In the early universe, the weak interaction becomes unimportant firstly.

Thermal history of the universe

$\hbar = c = k_B \equiv 1$

$1\text{MeV} = 1.1605 \times 10^{10}\ \text{K}$



10. Big bang nucleosynthesis (BBN)

➢Proposal: elements originate from a big bang

➢Various nuclei are formed from reactions of neutrons

(Alpha, Bethe, Gamow, PR 73, 803, 1948)

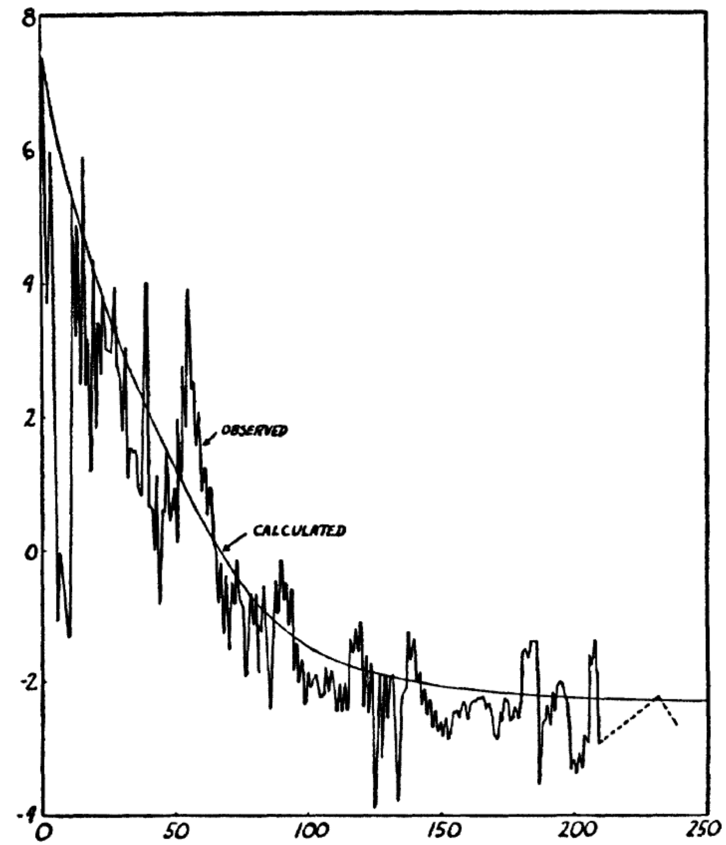


FIG. 1.

Log of relative abundance

Atomic weight

➢BBN starts from n and p

➢n/p ratio is controlled via the weak int.

(Hayashi, PTP 5, 224, 1950)

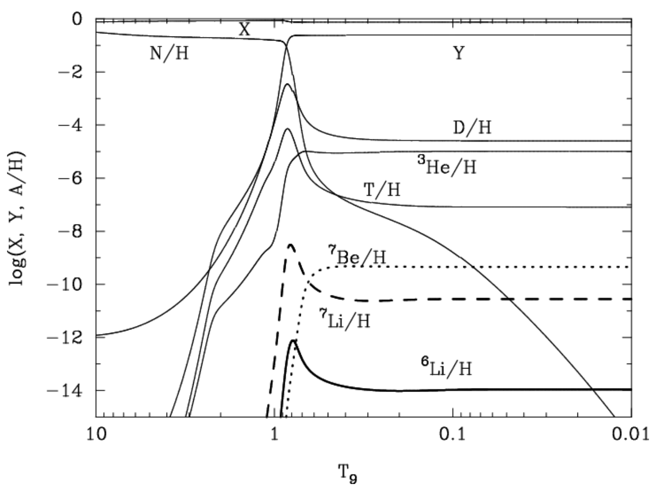
$$n \leftrightarrow p + e^- + \bar{\nu}_e,$$

$$\nu_e + n \leftrightarrow p + e^-,$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e.$$

$$n/p = \exp\left(-\frac{Q}{T}\right)$$

$Q \equiv m_n - m_p = 1.293$ MeV



➢Only H & He are produced
(+ trace amounts of ^{7}Be, ^{7}Li)

11. Astrophysical sites of neutrino production

1) Cosmic background ν's

→very abundant

- ✓ Here is cosmic microwave background radiation, a remnant of hot big bang

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 \quad \Rightarrow \quad n_\gamma = 410.7 \text{ cm}^{-3}$$

$\hbar = c = k_B \equiv 1$ $\qquad$ $T_0 = 2.7255$ K (Fixsen 2009)

- ✓ Similarly, background ν's exist

$$n_\nu = 3\frac{4}{11}\frac{3}{4} n_\gamma = \frac{9}{11} n_\gamma \quad \Rightarrow \quad n_\nu = 336.0 \text{ cm}^{-3}$$

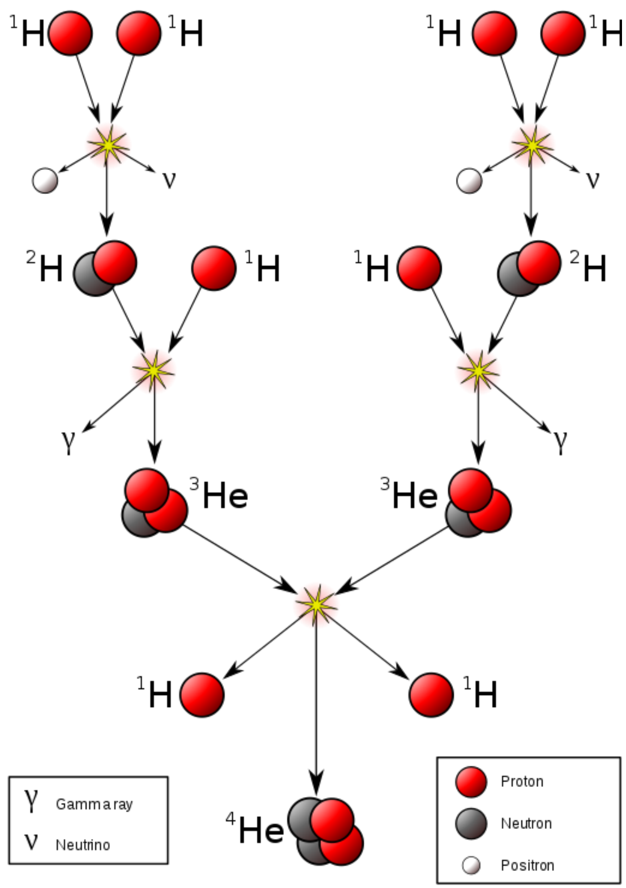
- ✓ The cosmic average baryon # density is about 6×10^{-10} times as much as n_γ
- ✓ Note that the baryon # density in astronomical bodies such as the Galaxy, the Sun and the Earth is enhanced from the average value

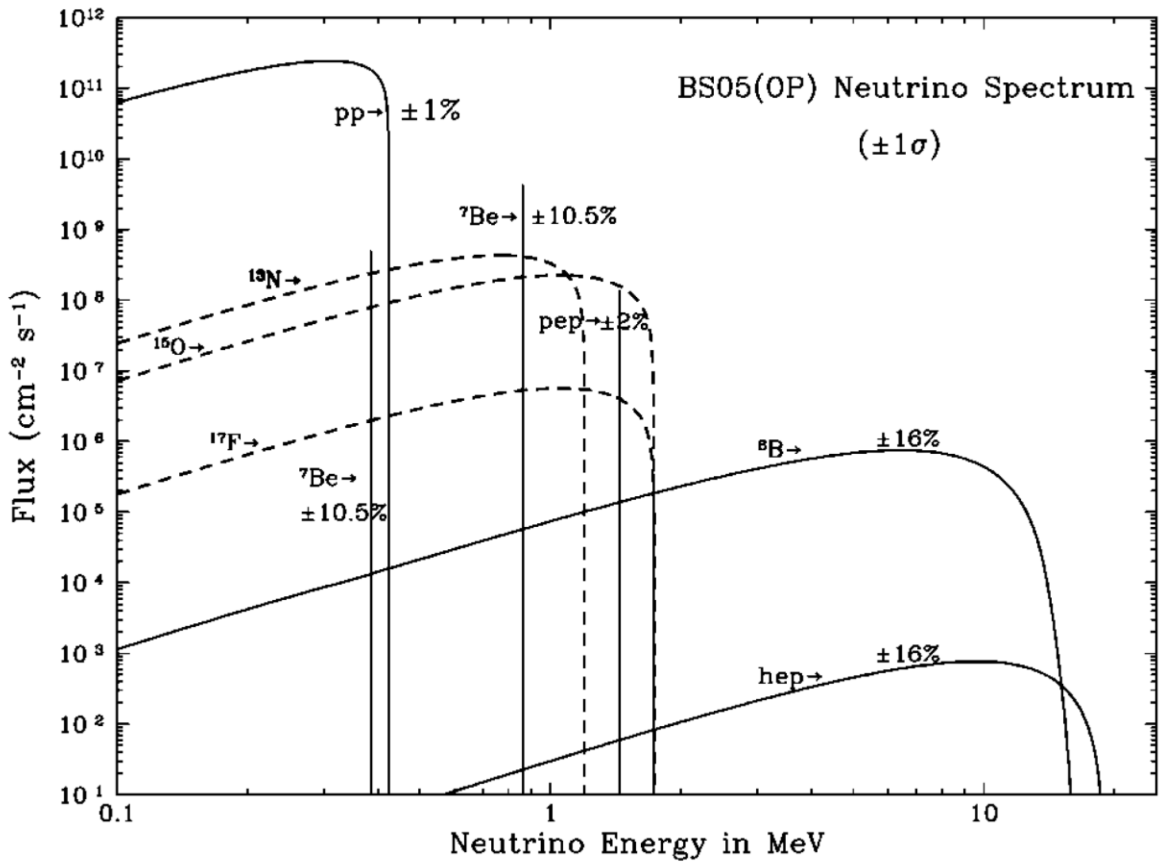
11. Astrophysical sites of neutrino production

2) Atmosphere

3) Sun (and other starts)

Bahcall et al., ApJL, 621, L85 (2005)





Wikipedia, ID: 845279810

https://en.wikipedia.org/w/index.php?title=Proton%E2%80%93proton_chain_reaction&oldid=845279810

11. Astrophysical sites of neutrino production

4) Core collapse supernovae

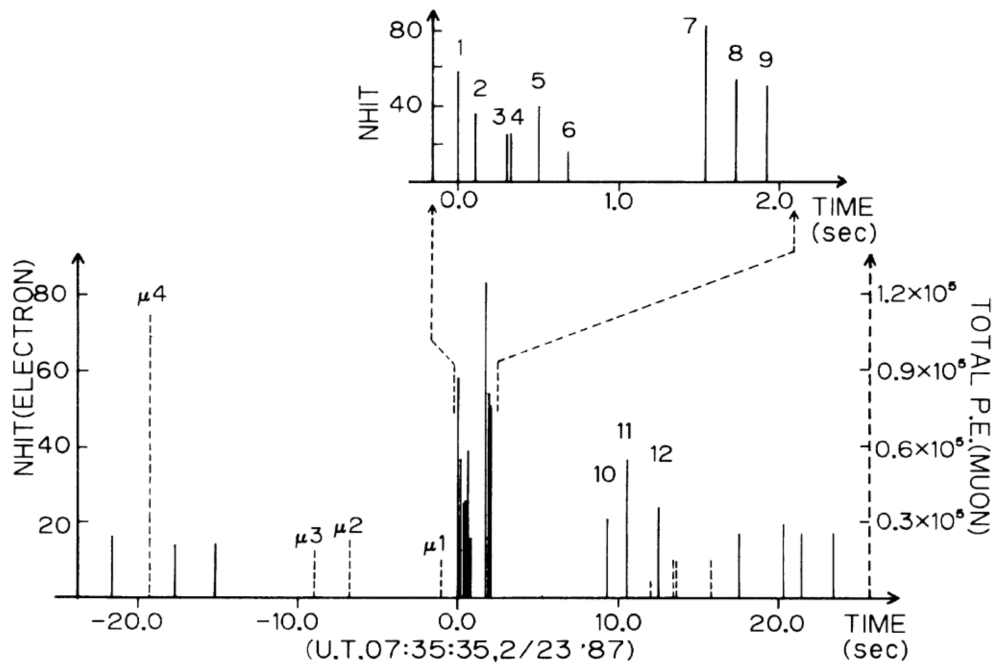
Weaver et al., "The Dawn of a New Era for Supernova 1987a" NASA, 25 Feb. 2017.
<https://www.nasa.gov/feature/goddard/2017/the-dawn-of-a-new-era-for-supernova-1987a>.

11. Astrophysical sites of neutrino production

4) Core collapse supernovae

➢ **Neutrino burst was detected at the time of SN1987A**

Hirata et al., PRL, 58, 1490 (1987)

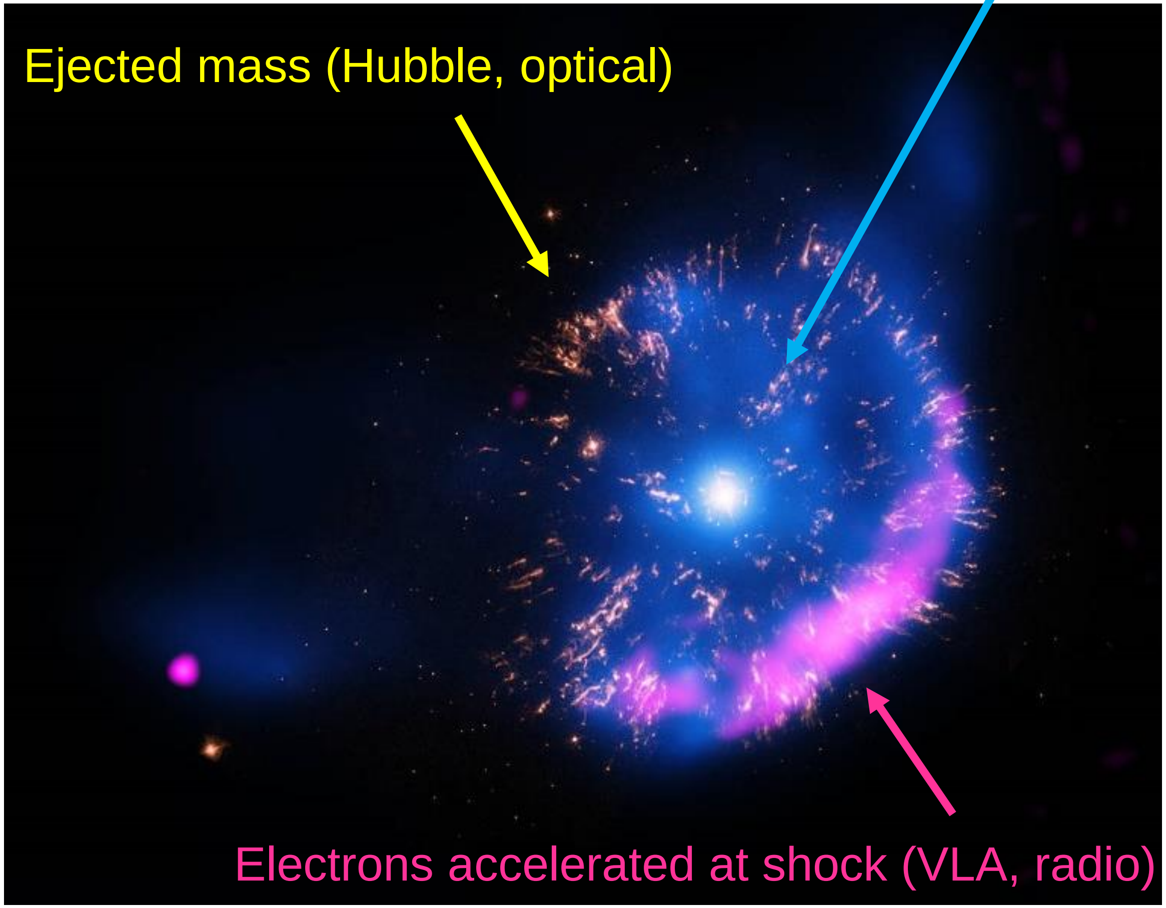


Weaver et al., "The Dawn of a New Era for Supernova 1987a" NASA, 25 Feb. 2017. <https://www.nasa.gov/feature/goddard/2017/the-dawn-of-a-new-era-for-supernova-1987a>.

12. Supernovae (SNe)

- Nova: transient luminous object ($L < O(10^5)\ L_\odot \sim 10^{38}$ erg/s)
- Supernova: much brighter than novae

  $L \sim 10^{43}$ erg/s



`"Mini Supernova" Explosion Could Have Big Impact' NASA, Mar. 13, 2015.
<https://www.nasa.gov/mission_pages/chandra/mini-supernova-explosion-could-have-big-impact.html>.

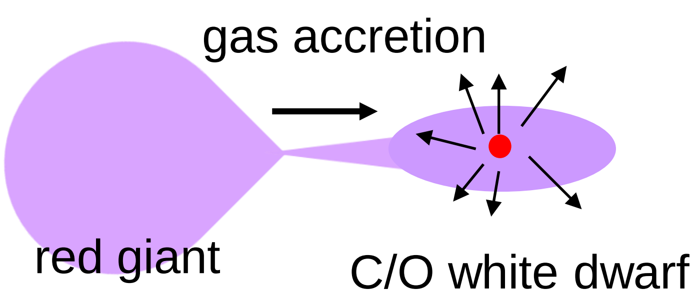
13. SN classification

- Type I: No absorption lines of H in the spectrum
  - →subclass (spectral types)
    - ✓ Ia: Si II absorption line ← Thermonuclear explosion
    - ✓ Ib: He I absorption line
    - ✓ Ic: no Si & He absorption lines

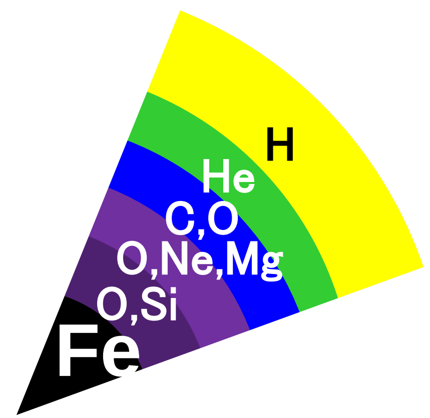
- Type II: Absorption lines of H (more frequent than Type I)
  - →subclass (light curves or luminosity evolution)
    - ✓ IIP: plateau light curves
    - ✓ IIL: linear decrease of magnitude
    - …

Ib, Ic, IIP, IIL, …: Core collapse

Thermonuclear explosion



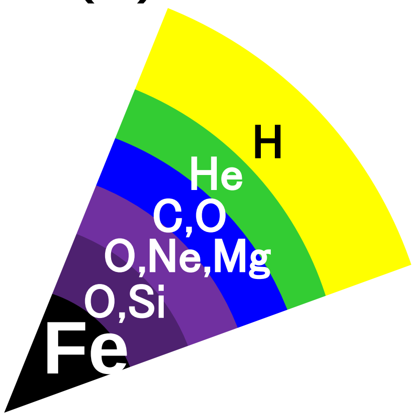
Core collapse SN



14. ν emission in core-collapse SNe (1)

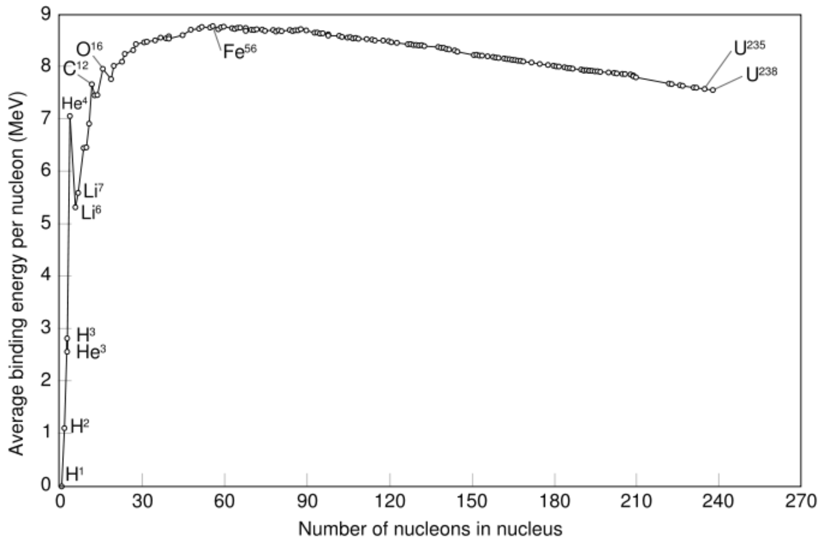
Gravitational collapse of massive star ($M_{ZAMS}>10M_\odot$)

- H, He, C, O, Si burnings
  - → Fe core: the most strongly bound nuclide



- $M_{Fe} \sim M_{ch} \sim 1.4M_\odot$: collapse
- Photodisintegration of ^{56}Fe at $T_9=T/(10^9K) > 5$
- Nuclear matter density → collapse stops
- Bounce → shock propagates outward
- ν energy is absorbed by gas → SN

(Wikipedia)



- ν-process on nuclei is important:
  - ^{7}Li, ^{11}B, ^{19}F, ^{138}La, ^{180}Ta, …
  - (Woosley et al. 1990)

14. ν emission in core-collapse SNe (2)

- **Core-collapse SNe are important producers of neutrinos**

- ✓ Fe core with the mass M~1.4$M_{\odot}$ collapses to a neutron star (NS)

- ✓ Gravitational potential energy (3×10^{53} erg ~ 2×10^{59} MeV) is released, and escape from the NS as neutrinos.

1 erg =624.151 GeV

- ✓ The SN is composed of n, p, nuclei, $e^{\pm}$, ν
  dense and hot → frequent scatterings
  but ν's can the most easily escape because of weakest interaction.

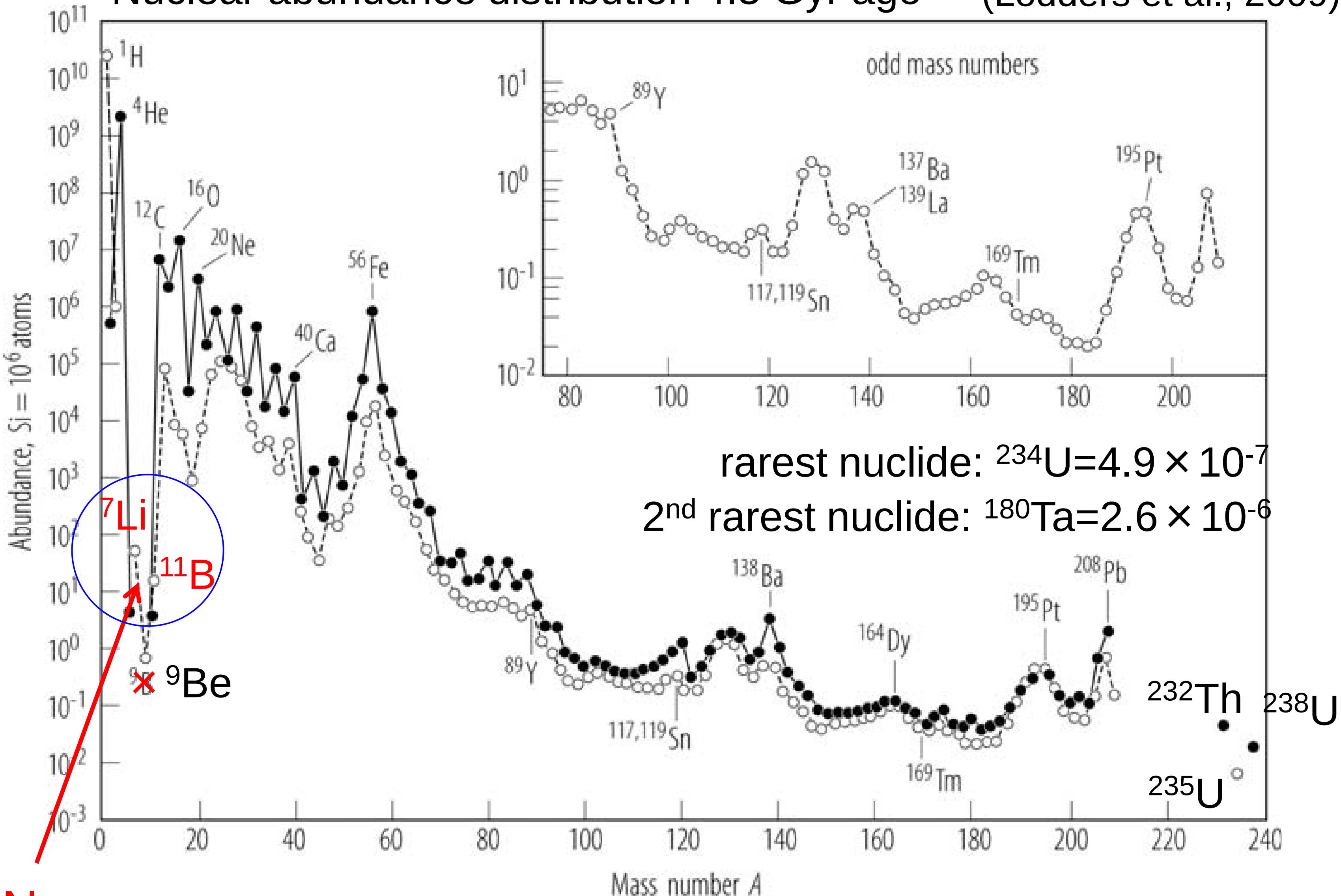
$$N_{\nu} \sim \frac{2\times10^{59}\ \mathrm{MeV}}{10\ \mathrm{MeV}} = 10^{58}$$

- ✓ Huge number of neutrinos are emitted per 1 SN.

- ✓ These neutrinos scatter with matters → explosion occurs.

15. Solar abundance

Nuclear abundance distribution 4.5 Gyr ago (Lodders et al., 2009)



SN ν-process

Rare p-nuclides (^{138}La, ^{180}Ta)

Short-lived (^{92}Nb, ^{98}Tc)

2. Neutrino oscillations

1. Introduction

- Mass eigenstates are different from eigenstates of weak interaction (flavors)
- Wave functions propagates in mass eigenstates.
  → flavor can change during propagation.

- Cabibbo-Kobayashi-Maskawa (CKM) matrix (1963, 1973, Nobel prize in 2008)

s quark as well as d couples to u.

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}.$$

Weak eigenstates Mass eigenstates

- Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix (1962)

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

Weak eigenstates Mass eigenstates

2. ν oscillation in vacuum (1)

- Oscillation of 2-flavor neutrinos in vacuum
  - ✓ Neutrino with flavor α is produced at t =0

$$|\nu^{\alpha}\rangle_{t=0} = \sum_i U_{\alpha i}\,|\nu'^{i}\rangle \qquad |\nu^{e}\rangle_{t=0} = \cos\theta\,|\nu'^{1}\rangle + \sin\theta\,|\nu'^{2}\rangle$$

  - ✓ It propagates, and at time t it becomes

$$|\nu^{\alpha}\rangle_{t} = \sum_i U_{\alpha i}\, e^{-iE_i t}|\nu'^{i}\rangle \quad ① \quad |\nu^{e}\rangle_{t} = \cos\theta\, e^{-iE_1 t}|\nu'^{1}\rangle + \sin\theta\, e^{-iE_2 t}|\nu'^{2}\rangle$$

  - ✓ Amplitude for transition to the state $|\nu^{\beta}\rangle$

$$\langle\nu^{\beta}|\nu^{\alpha}\rangle_{t} = \sum_i U_{\alpha i}(U^{\dagger})_{i\beta}\, e^{-iE_i t}\langle\nu'^{i}|\nu'^{i}\rangle = \sum_i U_{\alpha i}(U^{\dagger})_{i\beta} e^{-iE_i t}$$

  - ✓ Using the ultra-relativistic approximation

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

  - ✓ ① is transformed

$$|\nu^{\alpha}\rangle_{t} = \sum_i U_{\alpha i}\, e^{-iE_i t}(U^{\dagger})_{i\beta}|\nu^{\beta}\rangle \simeq e^{-ipt} U \begin{bmatrix} e^{-im_1^2 t/2E} & 0 \\ 0 & e^{-im_2^2 t/2E} \end{bmatrix} U^{\dagger}|\nu^{\beta}\rangle \quad ②$$

2. ν oscillation in vacuum (2)

➢Oscillation of 2-flavor neutrinos in vacuum

$$|\nu^\alpha\rangle_t = \sum_i U_{\alpha i}\, e^{-iE_it}(U^\dagger)_{i\beta}\left|\nu^\beta\right\rangle \simeq e^{-ipt} U \begin{bmatrix} e^{-im_1^2 t/2E} & 0 \\ 0 & e^{-im_2^2 t/2E} \end{bmatrix} U^\dagger\left|\nu^\beta\right\rangle \qquad ②$$

✓ Using a mass matrix satisfying

$$m = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = U^* \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U^\dagger$$

$$m^2_{\mathrm{diag}} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} = U^\dagger m^\dagger m U$$

✓ The neutrino wave function is

$$|\nu^\alpha\rangle_t \simeq e^{-ipt}[e^{-im^\dagger m t/2E}]_{\alpha\beta}\left|\nu^\beta\right\rangle \qquad ③$$

✓ Then, $|\nu^\alpha\rangle_t$ is a solution of the Schrödinger Eq.

$$i\frac{d}{dt}|\nu^\alpha\rangle_t \simeq \frac{m^\dagger m}{2E}|\nu^\alpha\rangle \qquad ④$$

✓ Here the common phase factor e^{-ipt} was dropped.

2. ν oscillation in vacuum (3)

- Oscillation of 2-flavor neutrinos in vacuum
  - ✓ Schrödinger Eq. for ν

$$i\frac{d}{dt}|\nu^{\alpha}\rangle_t \simeq \frac{m^{\dagger}m}{2E}|\nu^{\alpha}\rangle \qquad ④$$

  - ✓ We can take the unitary matrix for mixing

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$m^{\dagger}m = Um^2_{\mathrm{diag}}U^{\dagger} = \frac{m_1^2+m_2^2}{2} + \frac{\Delta m^2}{2}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

  - ✓ We use

$$|\nu^{\alpha}\rangle_t \simeq e^{-ipt}[e^{-im^{\dagger}mt/2E}]_{\alpha\beta}|\nu^{\beta}\rangle \qquad ③$$

→

$$|\nu^{\alpha}\rangle_t = \begin{pmatrix} \cos\frac{\Delta m^2}{4E}t - i\sin\frac{\Delta m^2}{4E}t\cos 2\theta & -i\sin\frac{\Delta m^2}{4E}t\sin 2\theta \\ -i\sin\frac{\Delta m^2}{4E}t\sin 2\theta & \cos\frac{\Delta m^2}{4E}t + i\sin\frac{\Delta m^2}{4E}t\cos 2\theta \end{pmatrix}|\nu^{\beta}\rangle \qquad ⑤$$

Here the common phase factor $e^{-i(p+\frac{m_1^2+m_2^2}{4E})t}$ was dropped.

2. ν oscillation in vacuum (4)

- Oscillation of 2-flavor neutrinos in vacuum
  - ✓ ν at time t

$$|\nu^{\alpha}\rangle_t = \begin{pmatrix} \cos\frac{\Delta m^2}{4E}t + i\sin\frac{\Delta m^2}{4E}t\cos 2\theta & -i\sin\frac{\Delta m^2}{4E}t\sin 2\theta \\ -i\sin\frac{\Delta m^2}{4E}t\sin 2\theta & \cos\frac{\Delta m^2}{4E}t - i\sin\frac{\Delta m^2}{4E}t\cos 2\theta \end{pmatrix}|\nu^{\beta}\rangle \quad ⑤$$

  - ✓ Transition from $|\nu^e\rangle=\begin{pmatrix}1\\0\end{pmatrix}$ to $|\nu^e\rangle_t$ is then derived

$$\langle\nu^e|\nu^e\rangle_t = \cos\frac{\Delta m^2}{4E}t + i\sin\frac{\Delta m^2}{4E}t\cos 2\theta$$

  - ✓ The transition probability is

$$P_{\nu_e\to\nu_e} = |\langle\nu^e|\nu^e\rangle_t|^2 = 1 - \sin^2 2\theta\sin^2\frac{\Delta m^2}{4E}t$$

Same for

$\nu_\mu\to\nu_\mu$, $\bar{\nu}_e\to\bar{\nu}_e$, $\bar{\nu}_\mu\to\bar{\nu}_\mu$

Oscillation of ν propagating in a vacuum (Gribov & Pontecorvo)

  - ✓ Transition from $|\nu^e\rangle=\begin{pmatrix}1\\0\end{pmatrix}$ to $|\nu^\mu\rangle=\begin{pmatrix}0\\1\end{pmatrix}$

$$P_{\nu_e\to\nu_\mu} = |\langle\nu^\mu|\nu^e\rangle_t|^2 = \sin^2 2\theta\sin^2\frac{\Delta m^2}{4E}t$$

Same for

$\nu_\mu\to\nu_e$, $\bar{\nu}_e\to\bar{\nu}_\mu$, $\bar{\nu}_\mu\to\bar{\nu}_e$

2. ν oscillation in vacuum (5)

➢ Oscillation length

$$\frac{\Delta m^2}{4E} l_0 = \pi$$

- ✓ Transition probabilities measured at a distance L=t

$$P_{\nu_e \to \nu_e} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} t$$
$$= 1 - \sin^2 2\theta \sin^2 \left(\pi \frac{L}{l_0}\right)$$

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} t$$
$$= \sin^2 2\theta \sin^2 \left(\pi \frac{L}{l_0}\right)$$

- ✓ The signal for oscillation disappears if L >> l_0, or

$$\Delta m^2 \gg 4\pi E / L$$

→t-dependent sine factor oscillates rapidly, and it reduces to

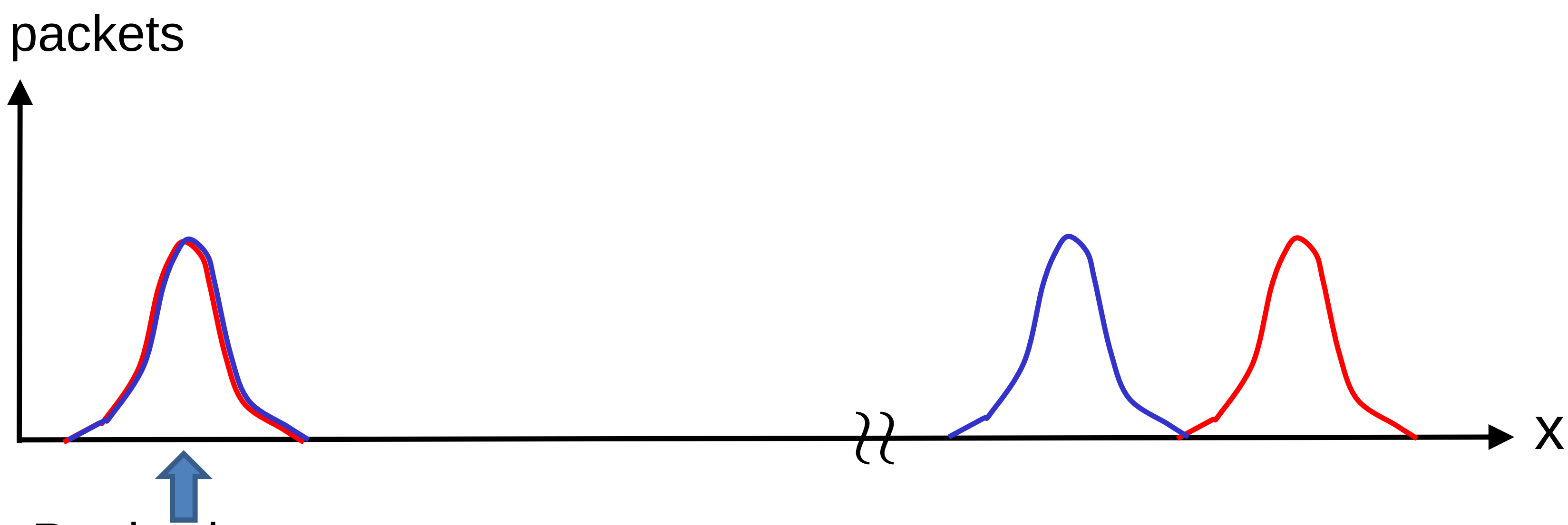
$$P_{\nu_e \to \nu_e} = 1 - \frac{1}{2} \sin^2 2\theta$$

→`time averaged oscillation'

- oscillation information has been lost
- we can only measure the mixing angle.

3. End of ν oscillation (concept of wave packets)

- ν oscillation stops when waves of ν_1 and ν_2 separate



$|\nu^e\rangle_0 = \cos\theta\,|\boldsymbol{\nu'^1}\rangle + \sin\theta\,|\boldsymbol{\nu'^2}\rangle$

- $m_1 > m_2 \rightarrow v_1 < v_2$ (difference in velocity) →waves of 1 & 2 separate

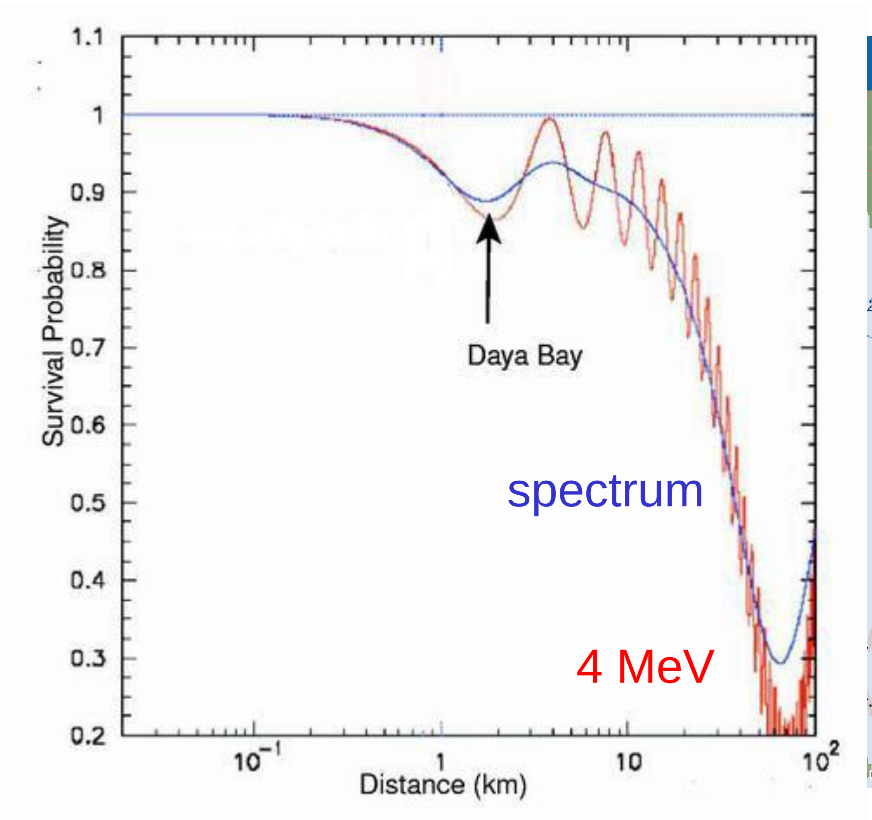
$$v_i = p_i/E = \frac{\sqrt{E^2 - m_i^2}}{E} \approx 1 - \frac{m_i^2}{2E^2}$$

$$\Delta v = v_2 - v_1 \approx \frac{\Delta m^2}{2E^2}$$

- The interference between waves of 1 & 2 disappears →oscillation stops

Appendix 1. Neutrino facility in China

- Daya Bay
  - ✓ east side of the Dapeng peninsula, on the west coast of Daya Bay
  - ✓ Measurement of θ_{13} using $\bar{\nu}_e$ produced in reactors



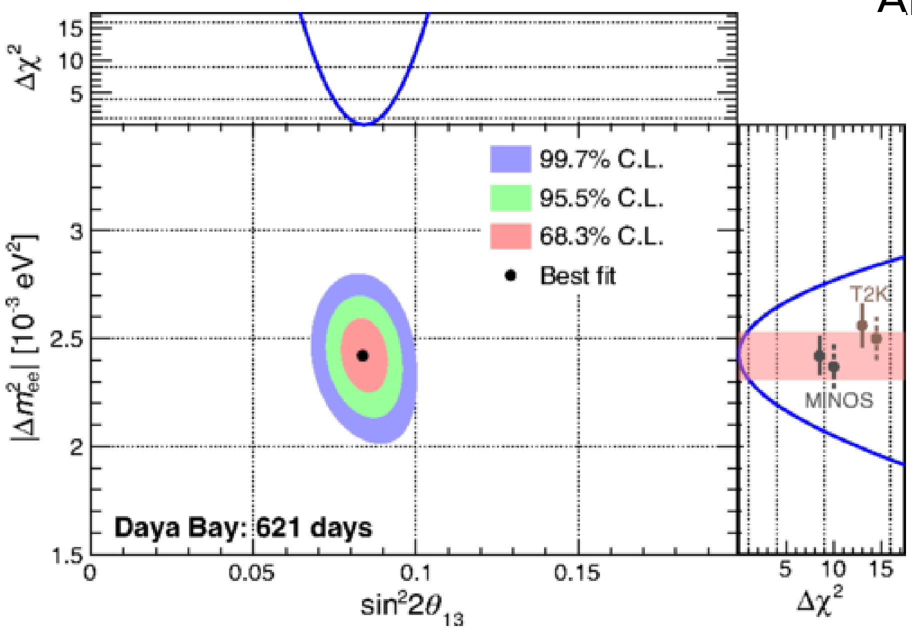
Survival of $\bar{\nu}_e$
$\Delta m^2_{13} = 2.5 \times 10^{-3}$ eV^2

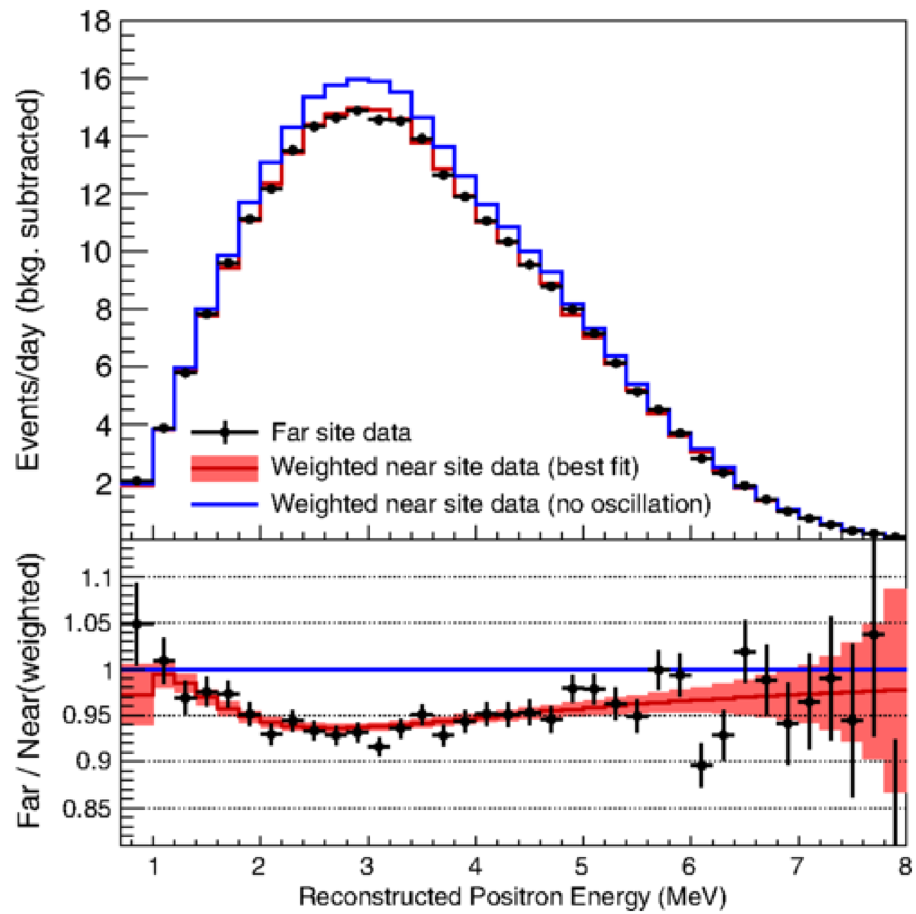
http://dayawane.ihep.ac.cn/twiki/bin/view/Public/

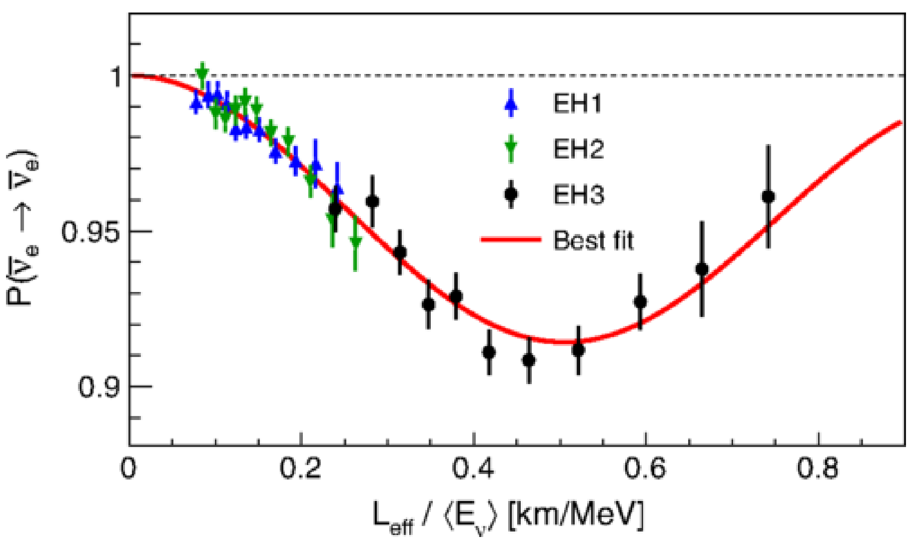
Appendix 1. Neutrino facility in China

- **$\sin^2 2\theta_{13} = 0.084 \pm 0.005$**, $|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$

An et al. Phys. Rev. Lett. **115**, 111802 (2015)

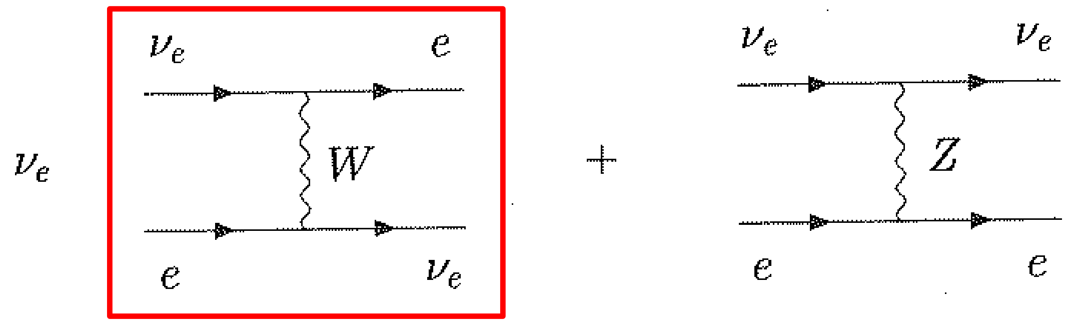






4. ν oscillation in matter

- In stars and neutron stars, temperature is low [T ≤O(10) MeV]
  →abundant $e^{\pm}$
  no $\mu^{\pm}$ and $\tau^{\pm}$
  →charged current interaction of ν_e



$$H_{\rm eff} = \frac{G_F}{\sqrt{2}}\ \bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e\ \bar{e}\gamma_\mu(1-\gamma_5)e$$

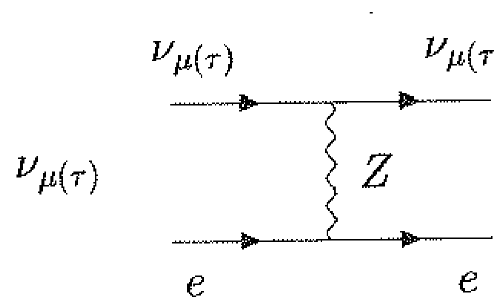


Fig. 8.8. Coherent scattering of ν_e and $\nu_\mu(\nu_\tau)$ in matter.

- Maximal conversion of $\nu_e \rightarrow \nu_\mu$ even for small θ

Mikheyev-Smirnov-Wolfenstein (MSW) mechanism

$$i\frac{d}{dr}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E_\nu}\cos 2\theta + \boxed{\sqrt{2}G_F n_e} & \frac{\Delta m^2}{4E_\nu}\sin 2\theta \\ \frac{\Delta m^2}{4E_\nu}\sin 2\theta & \frac{\Delta m^2}{4E_\nu}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

5. Effective masses and mixing angle in matter (1)

- Using

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = U \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}$$

~: quantities in matter

the Hamiltonian is diagonalized

$$i\frac{d}{dr}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E_\nu}\cos 2\theta + \boxed{\sqrt{2}G_F n_e} & \frac{\Delta m^2}{4E_\nu}\sin 2\theta \\ \frac{\Delta m^2}{4E_\nu}\sin 2\theta & \frac{\Delta m^2}{4E_\nu}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

⇒

$$\cos 2\tilde{\theta} = \frac{-A/\Delta m^2 + \cos 2\theta}{\sqrt{(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}}$$

$$\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}}$$

$$\tilde{m}^2_{\frac{1}{2}} = \frac{A}{2} \mp \frac{1}{2}\sqrt{(A - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2)^2 \sin^2 2\theta},$$

$$\begin{aligned} A &= 2\sqrt{2}E_\nu G_F n_e \\ &= 7.633 \times 10^{-5}\ \mathrm{eV}^2 \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right)\left(\frac{\rho}{10^3 \mathrm{g\ cm}^{-3}}\right)\left(\frac{Y_e}{0.5}\right). \end{aligned}$$

5. Effective masses and mixing angle in matter (2)

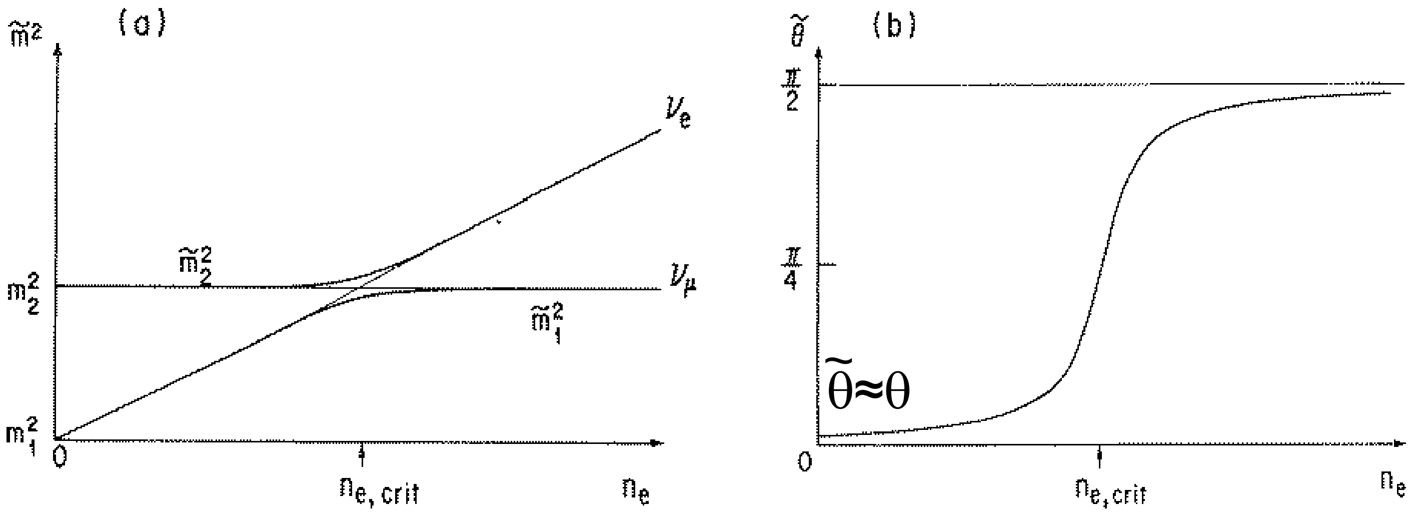


Fig. 8.9. (a) Effective neutrino mass squared in the medium with electron density n_e. $n_{e,crit}$ is the crossing point defined by (8.107). This figure corresponds to $\sin^2 2\theta = 0.01$. (b) Mixing angle $\tilde{\theta}$ in the medium as a function of n_e, corresponding to the case shown in (a).

$$\tilde{m}^2_{\frac{1}{2}} = \frac{A}{2} \mp \frac{1}{2}\sqrt{(A - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2)^2 \sin^2 2\theta},$$

$$\tilde{\theta} = \frac{1}{2}\tan^{-1}\left(\frac{\sin 2\theta}{-A/\Delta m^2 + \cos 2\theta}\right)$$

- Level crossing at $: n_{e,\mathrm{crit}} \equiv \frac{1}{2\sqrt{2}G_F}\frac{\Delta m^2}{E}\cos 2\theta$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\tilde{\theta} & \sin\tilde{\theta}\\ -\sin\tilde{\theta} & \cos\tilde{\theta}\end{pmatrix}\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{pmatrix}$$

- For $n_e \ll n_{e,crit}$, $\tilde{\theta}$ and $\tilde{l}$ approach the values in vacuum.
- For $n_e \gg n_{e,crit}$, $\tilde{\theta} \rightarrow \pi/2$ and …

6. Effective mixing length

- Oscillation length in matter is

$$\tilde{l} = l_0 \frac{1}{\left[(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta\right]^{1/2}}.$$

$$\begin{aligned} l_0 &= \pi \frac{4E_\nu}{\Delta m^2} \\ &= 24.80 \text{ km} \left(\frac{E_\nu}{1 \text{ MeV}}\right)\left(\frac{\Delta m^2}{10^{-4} \text{ eV}^2}\right)^{-1}. \end{aligned}$$

→much shorter than that in a vacuum

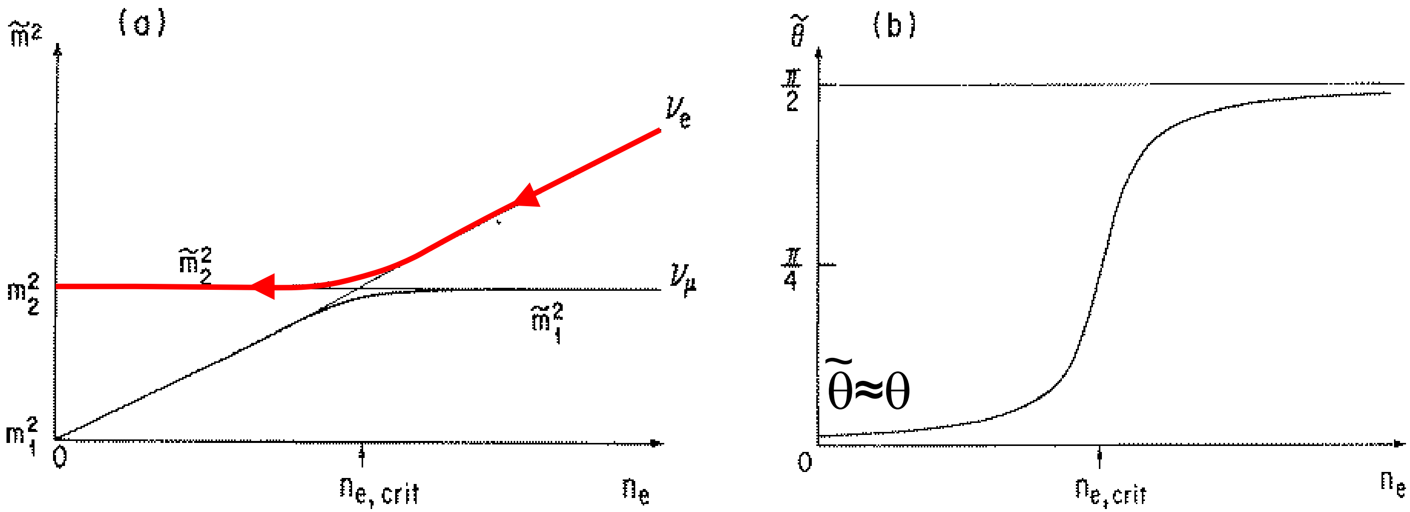
- Maximal mixing ($\tilde{\theta} = \pi/4$) occurs at $n_e = n_{e,crit}$

$$n_{e,\mathrm{crit}} \equiv \frac{1}{2\sqrt{2}G_F} \frac{\Delta m^2}{E} \cos 2\theta$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}$$

7. ν propagation in stars

- Assume: ν_e produced in a region $n_e > n_{e,crit}$ propagates into a region $n_e < n_{e,crit}$
- If the density gradient $d(\ln n_e)/dr$ is small, ν conversion occurs adiabatically



→full conversion: $\nu_e \to \nu_\mu$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}$$

- Probability for $\nu_e \to \nu_e$

$$P_{\nu_e \to \nu_e} = \underbrace{\cos^2\tilde{\theta}\cos^2\theta}_{\nu_1} + \underbrace{\sin^2\tilde{\theta}\sin^2\theta}_{\nu_2}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

8. Adiabatic condition

- Adiabatic condition: (energy gap) × (transition time) >> ħ

$$\delta E = (\tilde{m}_+^2 - \tilde{m}_-^2)_{\min}/2E = \frac{1}{2E}(\Delta m^2)\sin 2\theta \qquad \delta t = \delta r/c = \left(\frac{1}{n_e}\frac{dn_e}{dr}\right)^{-1}\frac{\delta n_e}{n_e}$$

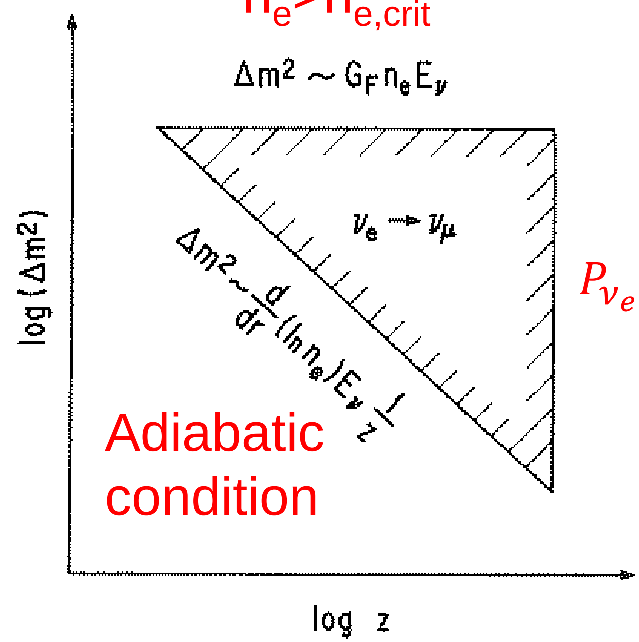
- The MSW resonance occurs at $A = \Delta m^2 \cos 2\theta$
  → its width is $\delta A \sim \Delta m^2 \sin 2\theta$

- The condition →

$$\frac{1}{n_e}\frac{dn_e}{dr} \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}$$

MSW triangle
($z = \sin^2 2\theta/\cos 2\theta$)

$n_e > n_{e,crit}$



$P_{\nu_e \to \nu_e} \le \frac{1}{2}$

9. Numerical solutions

9-1. 2-flavor oscillation in a constant n_e

- Schrödinger equation:

$$i\frac{d}{dr}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E_\nu}\cos 2\theta + \sqrt{2}G_F n_e & \frac{\Delta m^2}{4E_\nu}\sin 2\theta \\ \frac{\Delta m^2}{4E_\nu}\sin 2\theta & \frac{\Delta m^2}{4E_\nu}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

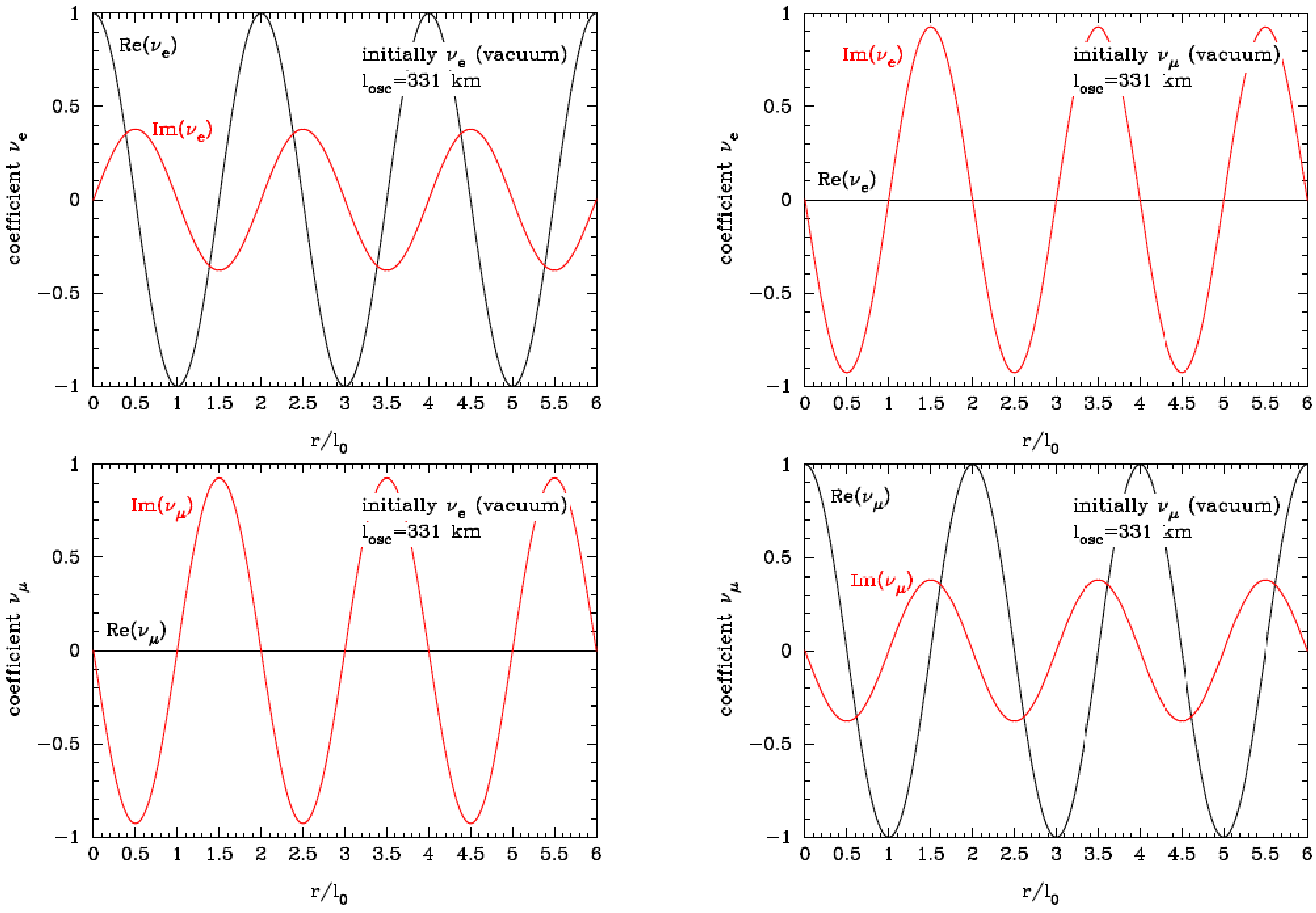
- The squared mass of energy eigenvalues of the above Hamiltonian are

$$\tilde{m}^2_{\frac{1}{2}} = \frac{A}{2} \mp \frac{1}{2}\sqrt{\left(A - \Delta m^2 \cos 2\theta\right)^2 + \left(\Delta m^2\right)^2 \sin^2 2\theta},$$

$$\begin{aligned} A &= 2\sqrt{2}E_\nu G_F n_e \\ &= 7.633 \times 10^{-5}\ \mathrm{eV}^2 \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right)\left(\frac{\rho}{10^3 \mathrm{g\ cm}^{-3}}\right)\left(\frac{Y_e}{0.5}\right) \end{aligned}$$

9-1a. 2-flavor oscillation in vacuum (1)

Coefficients of wave functions in vacuum



- Wave functions oscillates with the wavelength l_0

9-1a. 2-flavor oscillation in vacuum (2)

Transition probabilities in vacuum

- Analytic solution

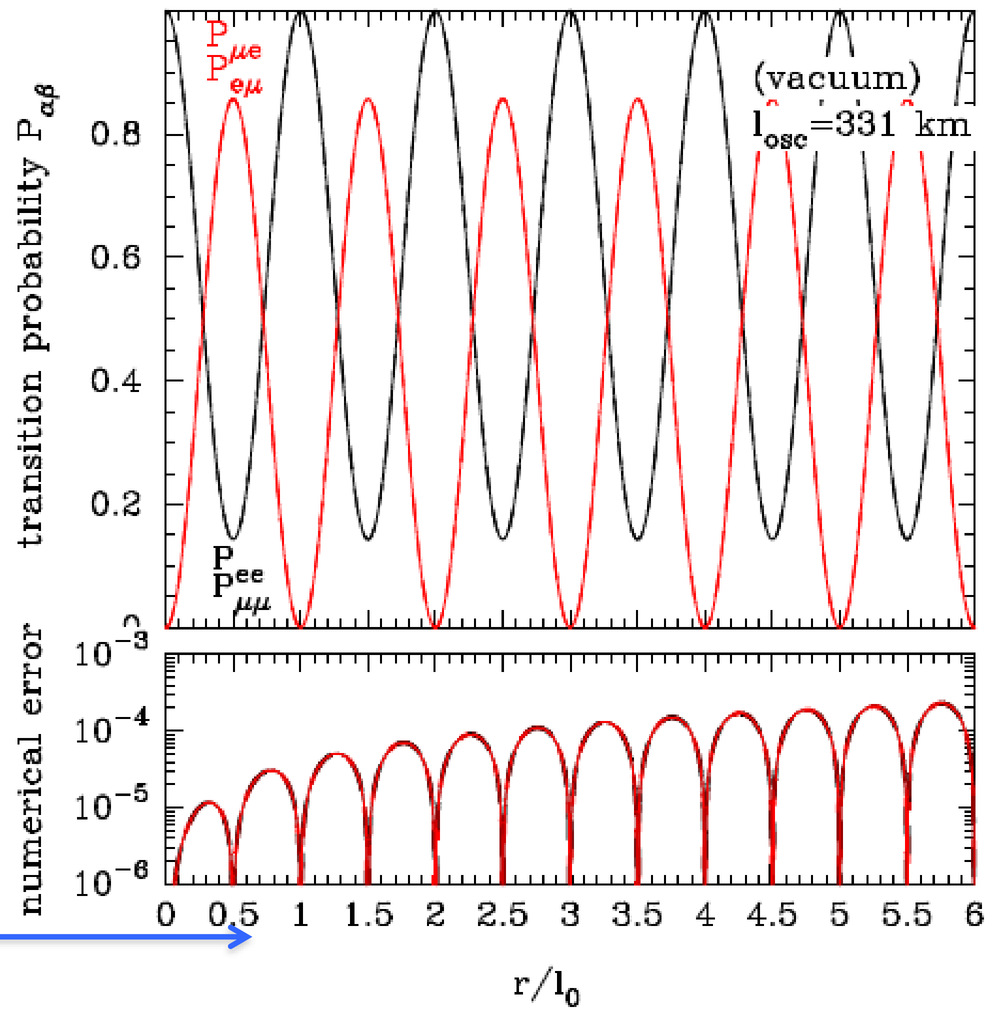
$$P_{ee} = P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2 (\pi L/l_0)$$
$$P_{e\mu} = P_{\mu e} = \sin^2 2\theta \sin^2 (\pi L/l_0)\,,$$

- oscillation length

$$l_0 = \pi \frac{4E_\nu}{\Delta m^2} = 24.80\ \mathrm{km} \left(\frac{E_\nu}{1\ \mathrm{MeV}}\right) \left(\frac{\Delta m^2}{10^{-4}\ \mathrm{eV}^2}\right)^{-1}$$

→only one characteristic radial scale

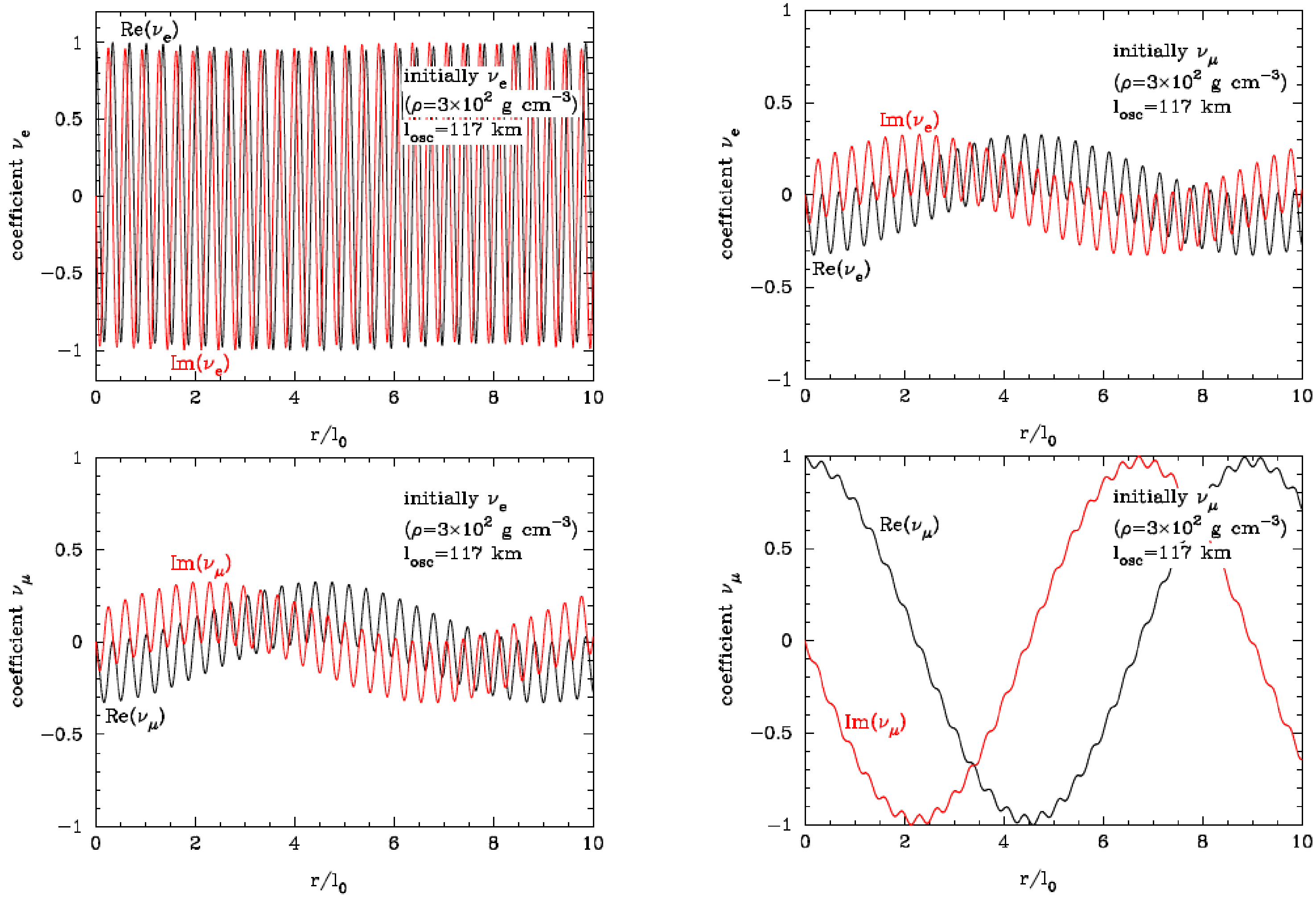
Solid lines: P_{ee}, $P_{e\mu}$
Dashed lines: $P_{\mu e}$, $P_{\mu\mu}$



$\sin^2 2\theta$=0.857
Δm^2=7.50×10^{-5} eV^2
E_ν=10 MeV
ρ_e=0
→l_0=331 km

9-1b. 2-flavor oscillation in matter (1)

Coefficients of wave functions ($\rho=3\times10^2$ g cm^{-3}, $Y_e=0.5$)



- Superposition of two sinusoids of different lengths.

$l_{1,2} = 4\pi E_\nu / \tilde{m}^2_{1,2}$ $=2.97\times10^3$ and 112 km

9-1b. 2-flavor oscillation in matter (2)

Transition probabilities (ρ=3×10^2 g cm^{-3}, Y_e=0.5)

- Analytic solution

$$P_{ee} = P_{\mu\mu} = 1 - \sin^2 2\tilde{\theta} \sin^2\left(\pi L/\tilde{l}\right)$$
$$P_{e\mu} = P_{\mu e} = \sin^2 2\tilde{\theta} \sin^2\left(\pi L/\tilde{l}\right),$$

- mixing angle

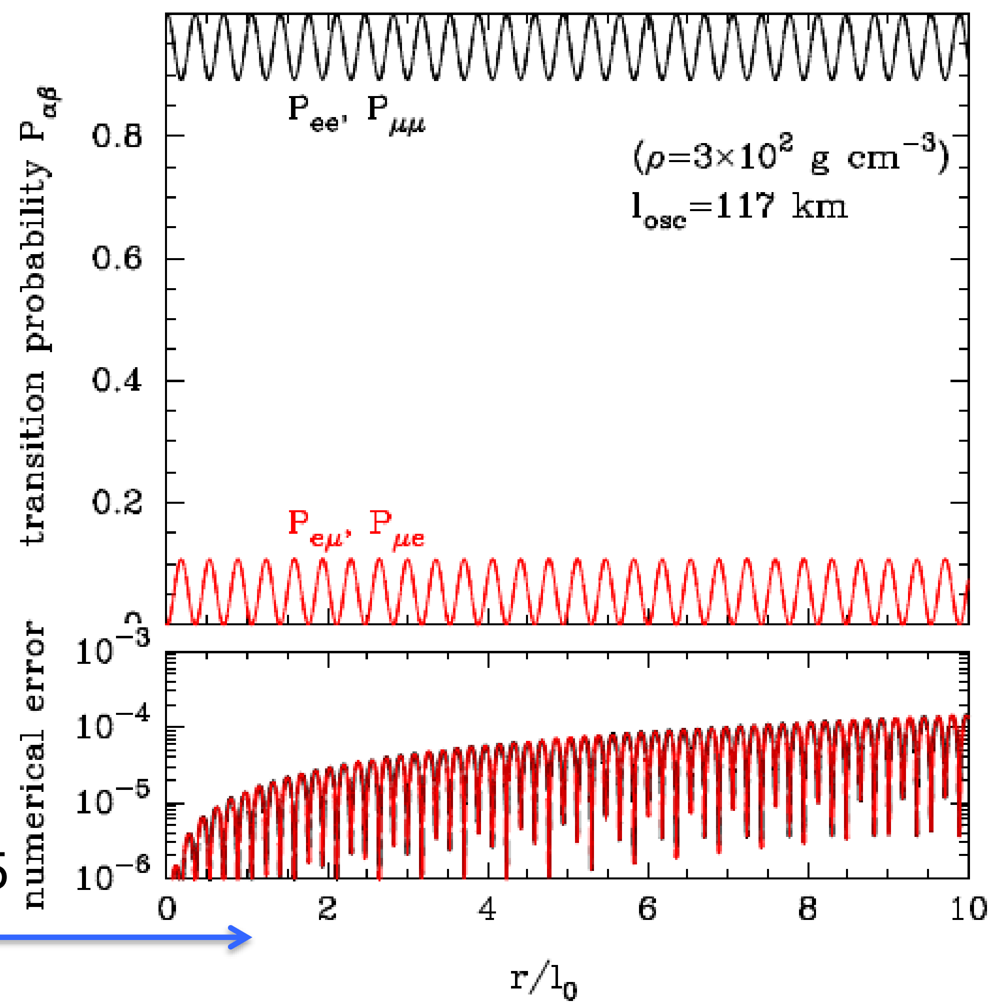
$$\tilde{\theta} = \frac{1}{2}\tan^{-1}\left(\frac{\sin 2\theta}{-A/\Delta m^2 + \cos 2\theta}\right)$$

- oscillation length

$$\tilde{l} = l_0 \frac{1}{\left[(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta\right]^{1/2}}.$$

$\sin^2 2\theta$=0.857
Δm^2=7.50×10^{-5} eV^2
E_ν=10 MeV
ρ=3×10^2 g cm^{-3}, Y_e=0.5
→$\tilde{l}$ =117 km

Solid lines: P_{ee}, $P_{e\mu}$
Dashed lines: $P_{\mu e}$, $P_{\mu\mu}$



9-2. 3-flavor oscillation in general $n_e(r)$

➢ 3-flavor oscillation

$$i\frac{d}{dr}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0 & 0 & 0\\ 0 & \frac{\Delta m^2_{21}}{2E_\nu} & 0\\ 0 & 0 & \frac{\Delta m^2_{31}}{2E_\nu}\end{pmatrix}U^\dagger + \begin{pmatrix}\pm\sqrt{2}G_{\rm F}n_e(r) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}, \qquad (1)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13}\end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}, \quad c_{ij} = \cos\theta_{ij}$$

ν parameters (Particle Data Group, as of May 25, 2020)

$\sin^2(\theta_{12}) = 0.307 \pm 0.013$ $\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}$ eV^2

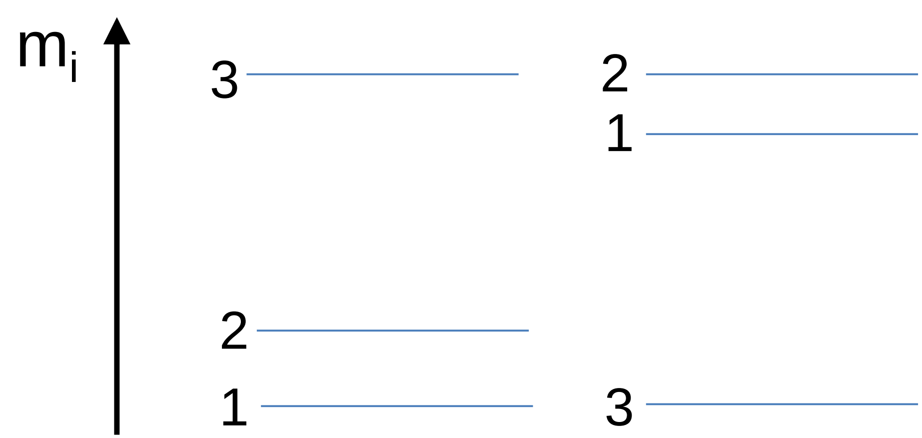
$\sin^2(\theta_{23}) = 0.536^{+0.023}_{-0.028}$ $\Delta m^2_{32} = (2.444 \pm 0.034) \times 10^{-3}$ eV^2 (NH)

$\sin^2(\theta_{13}) = 0.0218 \pm 0.0007$ $(-2.55 \pm 0.04) \times 10^{-3}$ eV^2 (IH)

$\delta/\pi = 1.37^{+0.18}_{-0.16}$

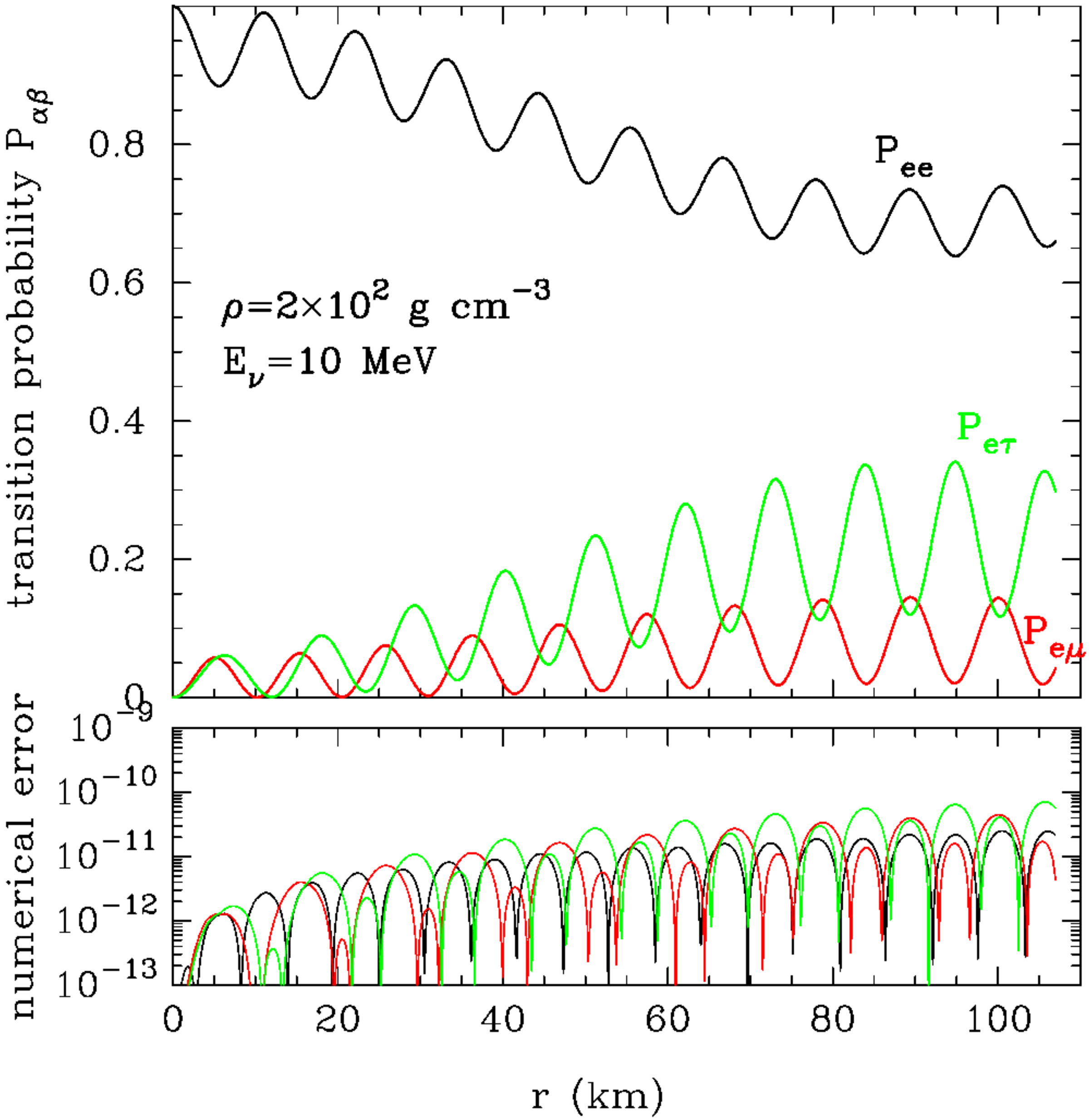
normal vs. inverted

➢ Mass hierarchy still undetermined



9-2a. 3-flavor oscillation in a constant n_e

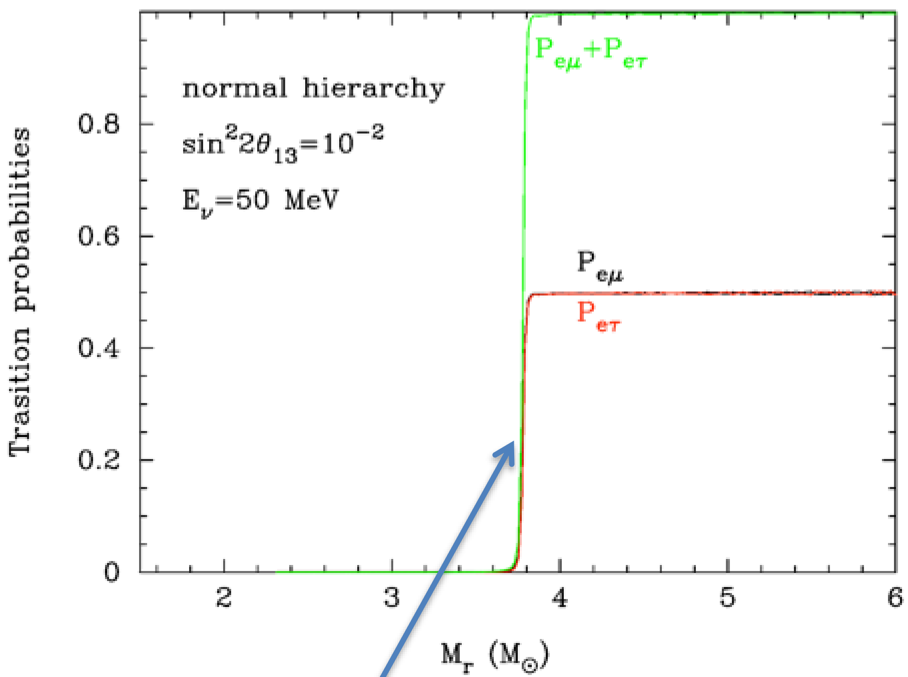
2 flavors
→3 flavors

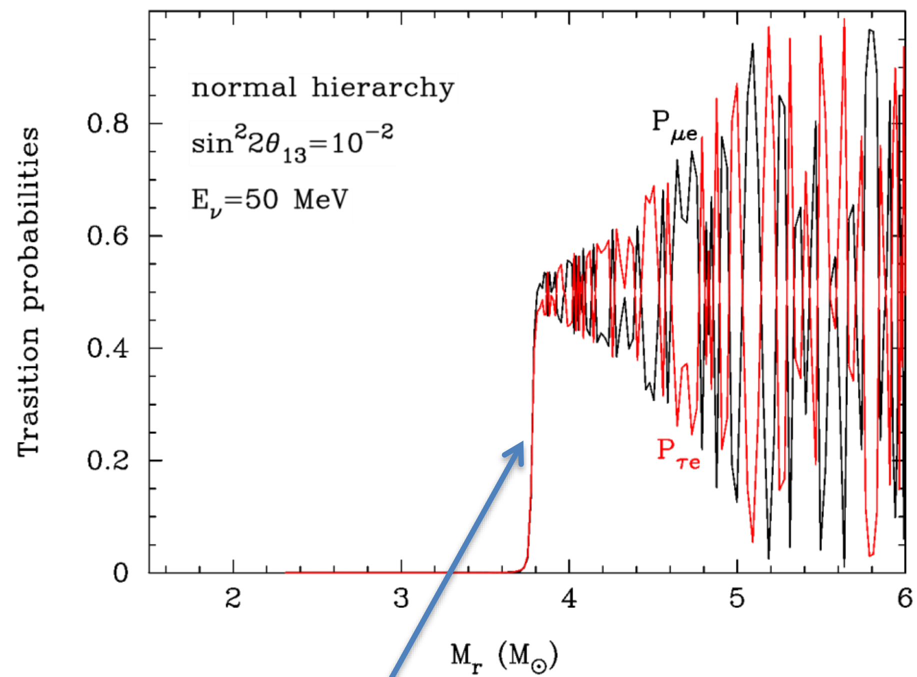


9-2b. 3-flavor oscillation in a star (1)

ν-oscillation in a model of SN1987A

(Shigeyama & Nomoto, ApJ 360, 242, 1990)

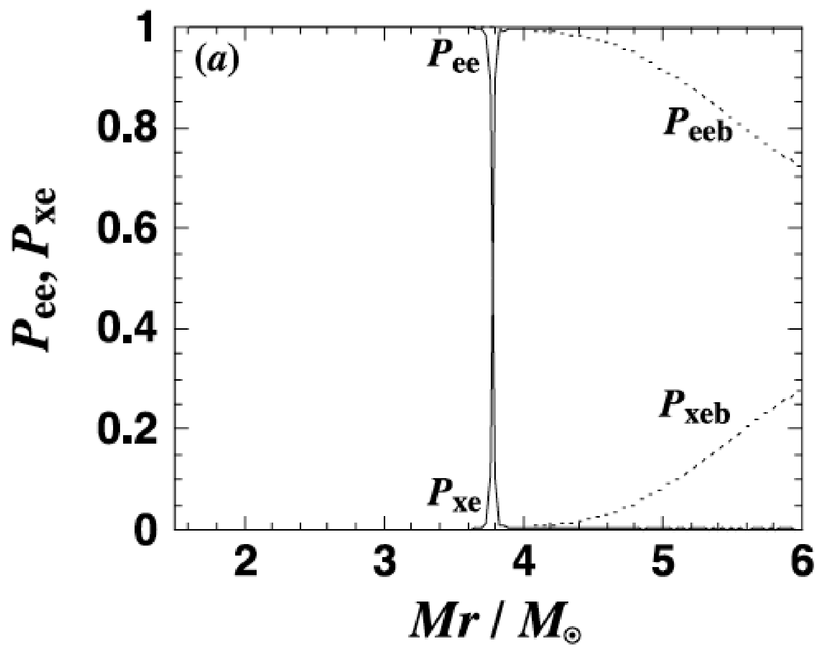


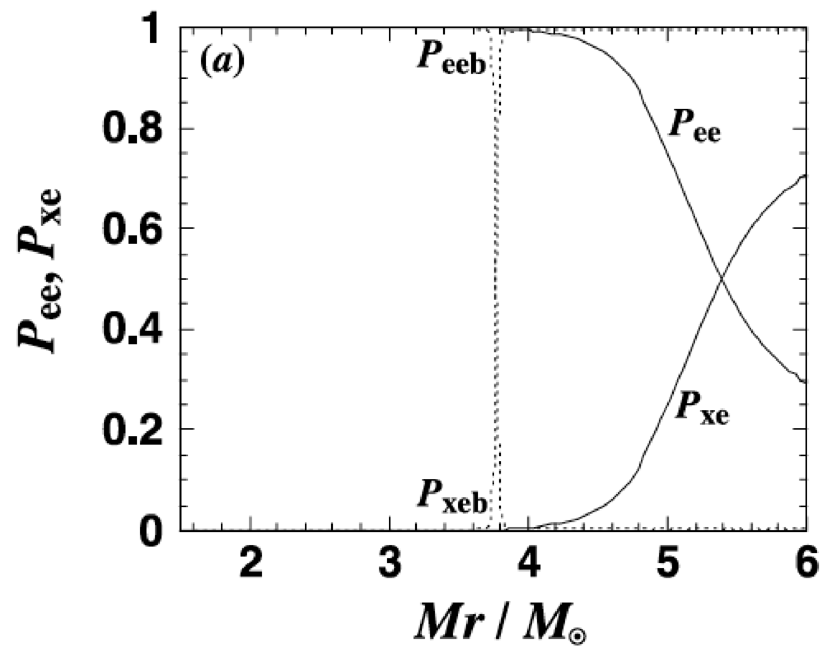


MSW H-resonance:
Nearly complete transition of e→μ+τ

$P_{\mu e}$ and $P_{\tau e}$ increases,
and then oscillate around 0.5

9-2b. 3-flavor oscillation in a star (2)





(Yoshida et al., ApJ 649, 319, 2006)

Summary

1. Neutrinos interact only weakly
    - Neutral and charged current reactions operate in stars and SNe
    - Neutrinos are very light, and produced in various astrophysical sites.
    - 10^{58} ν's are produced at one core-collapse SN.
    - Some stable nuclei are produced by SN ν-process.

2. Neutrino flavors change
    - Mass eigenstates and flavor eigenstates of ν's are different.
    - Then, ν-flavors change during propagation
    - The ν-oscillation occurs due to interference of different mass eigenstates.
    - The oscillation ends when the wave packets separate.
    - In dense matter, the mixing angle and oscillation length of ν's are different from those in vacuum.
    - In SNe, flavor changes of ν's occur through MSW resonances.

Backup

Appendix 1: Sterile neutrino decay

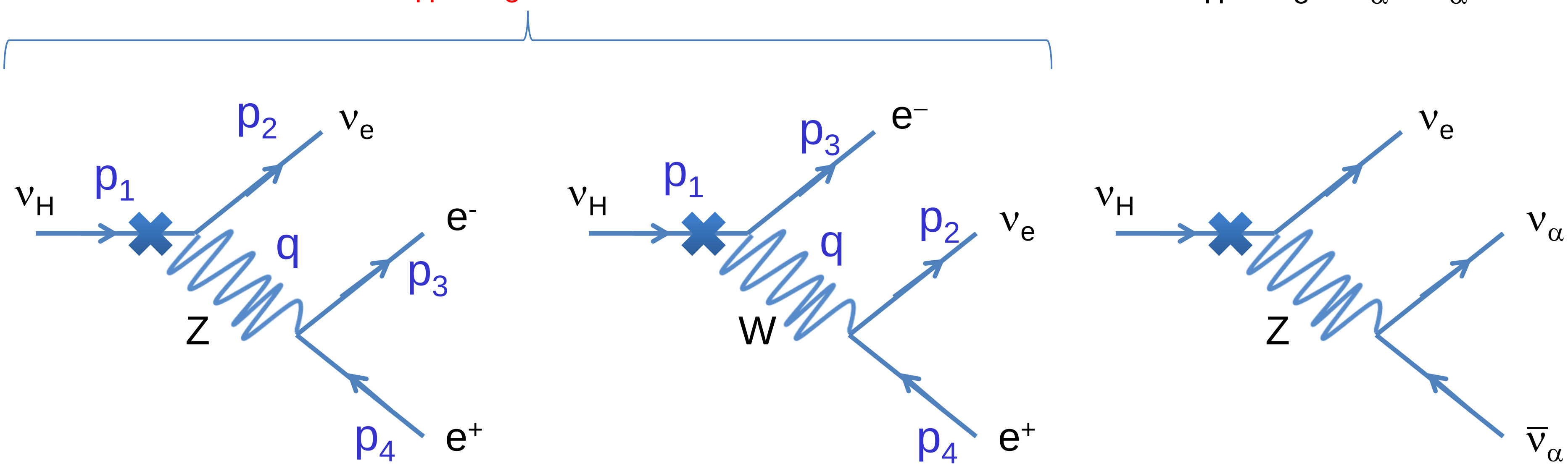
- sterile neutrino ν_H, mass $M_{\nu H}$, active-sterile mixing $\Theta \ll 1$
- Lagrangian

$$L_{\text{int}} = \frac{g_2}{2\cos\theta_w}\bar{\nu}_{L\alpha}\gamma^{\mu}Z^0_{\mu}\nu_{L\alpha} + \frac{g_2}{\sqrt{2}}\left(\bar{l}^-_{\alpha}\gamma^{\mu}W^-_{\mu}\nu_{L\alpha} + \bar{\nu}_{L\alpha}\gamma^{\mu}W^+_{\mu}l^-_{\alpha}\right)$$

$$\begin{pmatrix} \nu_{L\alpha} \\ \nu_{R\alpha} \end{pmatrix}_l = \begin{pmatrix} U_{\alpha i} & \Theta_{\alpha} \\ \Theta'_{\alpha} & 1 \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_H \end{pmatrix}_m$$

$\nu_H \rightarrow \nu_e + e^+ + e^-$

$\nu_H \rightarrow \nu_e + \nu_\alpha + \bar{\nu}_\alpha$



7. Sterile neutrino interactions

- Lagrangian related to the weak interaction of ν_H:

$$\overline{\nu}_{\alpha L} \qquad\qquad \nu_{\alpha L}$$

$$\begin{aligned}
L_{\mathrm{int}} = & \underline{\left[\bar{\nu}_i (U_{\alpha i})^\dagger + \bar{\nu}_{\mathrm{H}} \Theta \delta_{e\alpha}\right]} \left(\frac{g}{2\cos\theta_{\mathrm{w}}} Z_\mu\right) \underline{\left(U_{\alpha i}\nu_i + \Theta\delta_{e\alpha}\nu_{\mathrm{H}}\right)} \\
& + \frac{g}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu W_\mu^- \underline{\left(U_{\alpha i}\nu_i + \Theta\delta_{e\alpha}\nu_{\mathrm{H}}\right)} + \frac{g}{\sqrt{2}} \underline{\left[\bar{\nu}_i (U_{\alpha i})^\dagger + \bar{\nu}_{\mathrm{H}} \Theta \delta_{e\alpha}\right]} \gamma^\mu W_\mu^+ l_\alpha \\
& \ni \bar{\nu}_i (U_{\alpha i})^\dagger \left(\frac{g}{2\cos\theta_{\mathrm{w}}} Z_\mu\right) \Theta\delta_{e\alpha}\nu_{\mathrm{H}} + \bar{\nu}_{\mathrm{H}} \Theta\delta_{e\alpha} \left(\frac{g}{2\cos\theta_{\mathrm{w}}} Z_\mu\right) U_{\alpha i}\nu_i \\
& + \frac{g}{\sqrt{2}} \bar{l}_\alpha \gamma^\mu W_\mu^- \Theta\delta_{e\alpha}\nu_{\mathrm{H}} + \frac{g}{\sqrt{2}} \bar{\nu}_{\mathrm{H}} \Theta\delta_{e\alpha}\gamma^\mu W_\mu^+ l_\alpha .
\end{aligned}$$

- Feynman rule for the weak theory →matrix

$$\begin{aligned}
|\mathcal{M}|^2 = & 32 G_{\mathrm{F}}^2 \Theta^2 [A(p_1 \cdot p_3)(p_2 \cdot p_4) + B(p_1 \cdot p_4)(p_2 \cdot p_3) \\
& + C m_e^2 (p_1 \cdot p_2)], \qquad \text{(A1)}
\end{aligned}$$

- Decay rate is

$$\begin{aligned}
\Gamma(\nu_{\mathrm{H}} \to \nu_\alpha e^- e^+) = & \frac{1}{2M_{\nu_{\mathrm{H}}}} \int \frac{d^3p_2}{(2\pi)^3 2E_{\nu_\alpha}} \int \frac{d^3p_3}{(2\pi)^3 2E_{e^-}} \int \frac{d^3p_4}{(2\pi)^3 2E_{e^+}} \\
& \times \left(\frac{1}{2}|\mathcal{M}|^2\right) (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - p_4).
\end{aligned}$$

9. Sterile neutrino decay rates

- Massive sterile neutrino can decay.
- If the mass is larger than $2m_e$, the decay products can include an e^+e^- pair.

(Gorbunov & Shaposhnikov, JHEP 10 2007, 015)

$$\Gamma\left(N \to \sum_{\alpha,\beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot \sum_\alpha |U_\alpha|^2 ,$$

$$\Gamma\left(N \to l^-_{\alpha\neq\beta} l^+_\beta \nu_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot |U_\alpha|^2 \left(1 - 8x_l^2 + 8x_l^6 - x_l^8 - 12x_l^4 \log x_l^2\right) ,$$

$$x_l = \frac{\max\left[M_{l_\alpha},\ M_{l_\beta}\right]}{M_N} ,$$

$$\Gamma\left(N \to \nu_\alpha l^+_\beta l^-_\beta\right) = \frac{G_F^2 M_N^5}{192\pi^3} \cdot |U_\alpha|^2 \cdot \Big[\left(C_1 \cdot (1-\delta_{\alpha\beta}) + C_3 \cdot \delta_{\alpha\beta}\right) \times$$
$$\times \left(\left(1 - 14x_l^2 - 2x_l^4 - 12x_l^6\right)\sqrt{1-4x_l^2} + 12x_l^4\left(x_l^4 - 1\right)L\right)$$
$$+ 4\left(C_2 \cdot (1-\delta_{\alpha\beta}) + C_4 \cdot \delta_{\alpha\beta}\right)\left(x_l^2\left(2 + 10x_l^2 - 12x_l^4\right)\sqrt{1-4x_l^2}\right.$$
$$\left. + 6x_l^4\left(1 - 2x_l^2 + 2x_l^4\right)L\right)\Big] ,$$

$$L = \log\left[\frac{1 - 3x_l^2 - (1 - x_l^2)\sqrt{1-4x_l^2}}{x_l^2\left(1 + \sqrt{1-4x_l^2}\right)}\right] , \quad x_l \equiv \frac{M_l}{M_N} ,$$

$$C_1 = \frac{1}{4}\left(1 - 4\sin^2\theta_w + 8\sin^4\theta_w\right) , \qquad C_2 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w - 1\right) ,$$

$$C_3 = \frac{1}{4}\left(1 + 4\sin^2\theta_w + 8\sin^4\theta_w\right) , \qquad C_4 = \frac{1}{2}\sin^2\theta_w\left(2\sin^2\theta_w + 1\right) .$$

e :	0.5110 MeV
μ :	105.6 MeV
τ :	1784 MeV
$W^\pm$:	80.2 GeV
Z^0 :	91.19 GeV
p :	938.3 MeV
n :	939.6 MeV
$\pi^\pm$:	139.6 MeV
π^0 :	135.0 MeV

Particle masses

10. Differential decay rates (1)

- Amplitude of the matrix element squared for the decay $\nu_H \rightarrow \nu_\alpha + e^+ + e^-$

$$|\mathcal{M}|^2 = 32 G_F^2 \Theta^2 [A(p_1 \cdot p_3)(p_2 \cdot p_4) + B(p_1 \cdot p_4)(p_2 \cdot p_3) + C m_e^2 (p_1 \cdot p_2)], \quad \text{(A1)}$$

G_F: Fermi constant
$\Theta \ll 1$: mixing angle
p_i momentum
i =1 (ν_H), 2 (ν_α), 3 (e^-), 4(e^+)

$$A = (c_V + c_A)^2,$$

$$B = (c_V - c_A)^2 + 4\delta_{e\alpha} + 4(c_V + c_A)\delta_{e\alpha},$$

$$C = (c_V^2 - c_A^2) + 2(c_V - c_A)\delta_{e\alpha},$$

Z_0 exchange
$W^\pm$ exchange
interference

$c_V = -1/2 + 2\sin^2\theta_W$ Vector coupling of charged leptons to Z^0
$c_A = -1/2$ Axial coupling of charged leptons to Z^0
$\sin^2\theta_W = 0.23$ Weak mixing angle

- Differential decay rate vs. energies of e^+ & e^-

$$\frac{d^2\Gamma}{dx_3 dx_4} = \frac{G_F^2 \Theta^2 M_{\nu_H}^5}{64\pi^3} [A x_3(1 - x_3) + B x_4(1 - x_4) + 2C x_m^2 (2 - x_3 - x_4)],$$

(Ishida, MK, Okada, PRD 90, 083519, 2014)

$$x_m = m_e / M_{\nu_H}$$

$$x_i = 2E_i / M_{\nu_H}$$

10. Differential decay rates (2)

- Differential decay rates vs. x_3 & x_4 (Johnson, et al. PRD 56, 2970, 1997)

$$\frac{d\Gamma}{dx_3} = \frac{G_F^2 \Theta^2 M_{\nu_H}^5}{64\pi^3} \left\{ A x_3 (1 - x_3) x_{f3} + B \left(\frac{x_{f3}^2}{2} - \frac{x_{f3}^3}{3} \right) + 2 C x_m^2 \left[(2 - x_3) x_{f3} - \frac{x_{f3}^2}{2} \right] \right\}_{x_{f3} = x_{f3,-}}^{x_{f3,+}}, \quad \text{(A6)}$$

$$\frac{d\Gamma}{dx_4} = \frac{G_F^2 \Theta^2 M_{\nu_H}^5}{64\pi^3} \left\{ A \left(\frac{x_{f4}^2}{2} - \frac{x_{f4}^3}{3} \right) + B x_4 (1 - x_4) x_{f4} + 2 C x_m^2 \left[(2 - x_4) x_{f4} - \frac{x_{f4}^2}{2} \right] \right\}_{x_{f4} = x_{f4,-}}^{x_{f4,+}}, \quad \text{(A7)}$$

Range of parameters $2x_m \le x_i \le 1$

$$x_{fi,\pm} = \frac{(2 - x_i)(1 + 2x_m^2 - x_i) \pm (1 - x_i)\sqrt{x_i^2 - 4x_m^2}}{2(1 + x_m^2 - x_i)}.$$

- Total decay rate

$$\Gamma(\nu_H \to \nu_\alpha e^+ e^-) = \frac{G_F^2 \Theta^2 M_{\nu_H}^5}{192\pi^3} \{ C_1 [(1 - 14x_m^2 - 2x_m^4 - 12x_m^6)\sqrt{1 - 4x_m^2} - 12x_m^4(1 - x_m^4)L] + 4C_2[x_m^2(2 + 10x_m^2 - 12x_m^4)\sqrt{1 - 4x_m^2} + 6x_m^4(1 - 2x_m^2 + 2x_m^4)L] \},$$

$$C_1 = \frac{A + B}{4},$$

$$C_2 = \frac{C}{4},$$

$$L = \ln\left[\frac{1 - 3x_m^2 - (1 - x_m^2)\sqrt{1 - 4x_m^2}}{x_m^2(1 + \sqrt{1 - 4x_m^2})} \right]$$

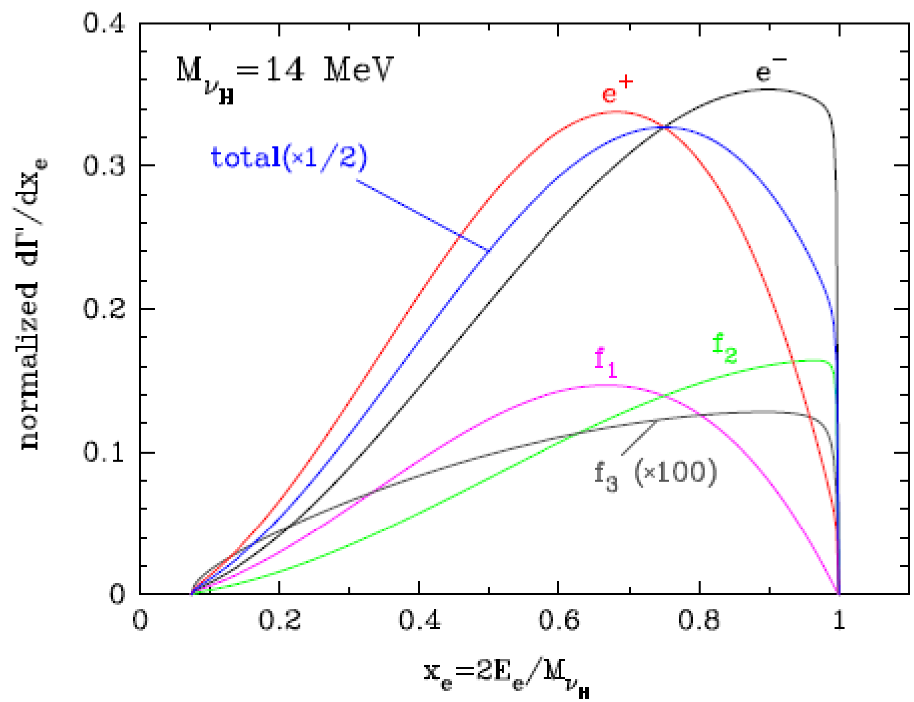
11. Spectra of decay products

➢Number spectra of e^- and e^+ emitted at the decay of $\nu_H \rightarrow \nu_\alpha + e^+ + e^-$

(Ishida, MK, Okada, PRD 90, 083519, 2014)

$$P_{e^-}(x_3) = \frac{1}{\Gamma}\frac{d\Gamma}{dx_3}, \qquad P_{e^+}(x_4) = \frac{1}{\Gamma}\frac{d\Gamma}{dx_4}.$$

$$\begin{aligned} P_e(x) &= P_{e^-}(x) + P_{e^+}(x) \\ &= \frac{1}{\Gamma}\frac{G_F^2\Theta^2 M_{\nu_H}^5}{64\pi^3}\left\{(A+B)\left[\frac{x_f^2}{2} - \frac{x_f^3}{3} + x_f x(1-x)\right]\right. \\ &\quad \left. + 2Cx_m^2 x_f(4-2x-x_f)\right\}_{x_f=x_{f,-}}^{x_{f,+}}. \qquad \text{(A21)} \end{aligned}$$



Broad total spectrum →$e^\pm$ with various energies are produced

→nonthermal $e^\pm$ spectrum is also broad

12. Average energies of e^+ & e^-

- Dependence of energy fractions given to ν_α, e^- and e^+

(Ishida, MK, Okada, PRD 90, 083519, 2014)

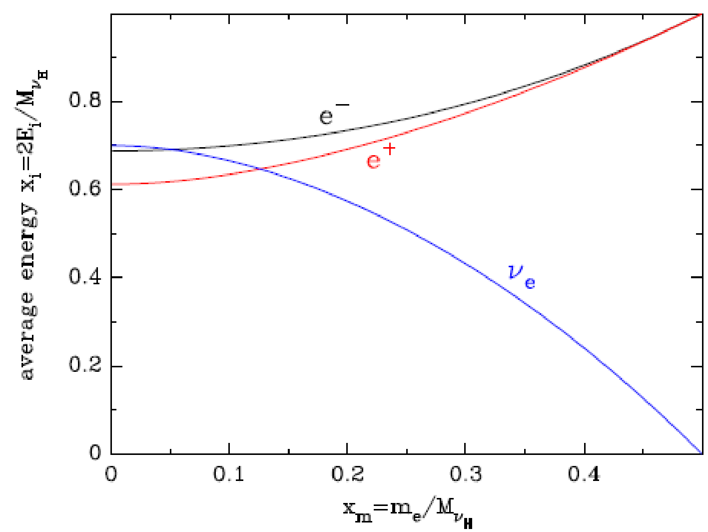
$$\bar{x}_3 = \frac{1}{\Gamma}\int_{2x_m}^{1} x_3 \frac{d\Gamma}{dx_3} dx_3 = \frac{1}{\Gamma}\frac{G_F^2\Theta^2 M_{\nu_H}^5}{64\pi^3} f_E(A,B,C,x_m),$$

$$\bar{x}_4 = \frac{1}{\Gamma}\int_{2x_m}^{1} x_4 \frac{d\Gamma}{dx_4} dx_4 = \frac{1}{\Gamma}\frac{G_F^2\Theta^2 M_{\nu_H}^5}{64\pi^3} f_E(B,A,C,x_m),$$

$$L_1 = \ln\left[\frac{1-3x_m^2-(1-x_m^2)\sqrt{1-4x_m^2}}{2x_m^3}\right],$$

$$L_2 = \ln\left[\frac{1+\sqrt{1-4x_m^2}}{2x_m}\right],$$

$$\begin{aligned}
f_E(A,B,C,x_m) = {} & A\left\{\frac{1}{60}\sqrt{1-4x_m^2}(3-29x_m^2+48x_m^4-70x_m^6-60x_m^8)\right.\\
& \left. - x_m^4[(1+x_m^2)(1-x_m^4)L_1+(3-x_m^2+x_m^4+x_m^6)L_2]\right\}\\
& - B\left\{\frac{1}{60}\sqrt{1-4x_m^2}(9-52x_m^2+14x_m^4+80x_m^6+120x_m^8)\right.\\
& \left. + x_m^4[(1-x_m^4-2x_m^6)L_1+(3+x_m^4+2x_m^6)L_2]\right\}\\
& + C\left\{\frac{1}{6}x_m^2\sqrt{1-4x_m^2}(5-12x_m^2+10x_m^4-12x_m^6)\right.\\
& \left. + 2x_m^4[(1-x_m^2)(1-x_m^4)L_1+(1+x_m^2+x_m^4-x_m^6)L_2]\right\}\\
& + (B-2Cx_m^2)\left\{\frac{1}{24}\sqrt{1-4x_m^2}(5-38x_m^2+6x_m^4+36x_m^6)\right.\\
& \left. -\frac{1}{2}x_m^4(1+3x_m^4)(L_1-L_2)\right\},
\end{aligned}$$



Heavy $M_{\nu H}$: almost equal partition

Light $M_{\nu H}$~m_e: mass energy of $e^\pm$ is significant

3. Effects of sterile ν mixing to active ν

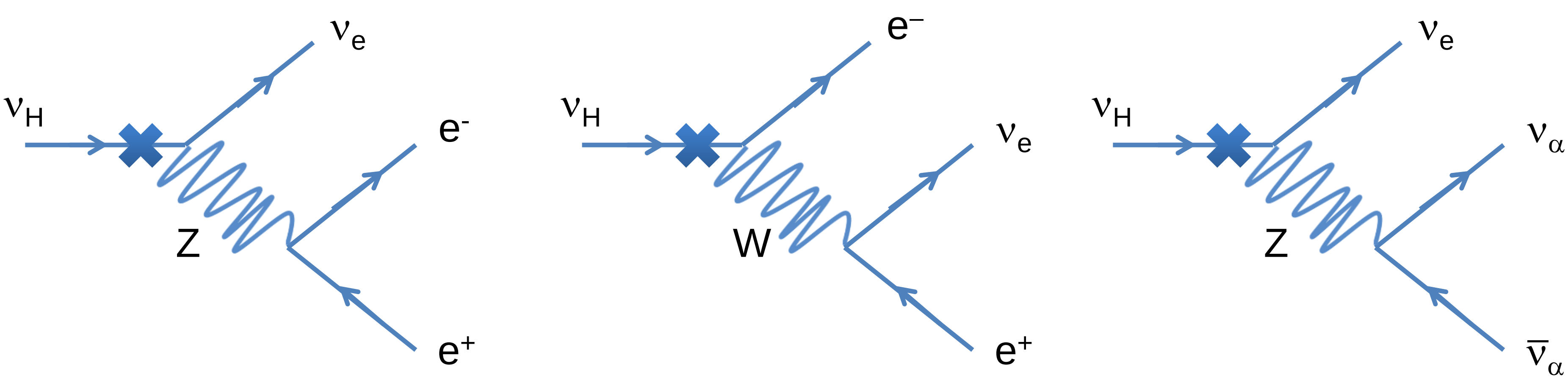
(Ishida, MK, Okada, PRD 90, 083519, 2014)

3-1. model

- Dirac sterile neutrino, mass $M_{\nu H}$=O(10) MeV, active-sterile mixing Θ
- Lagrangian

$$L_{\rm int} = \frac{g_2}{2\cos\theta_W}\bar{\nu}_{L\alpha}\gamma^\mu Z^0_\mu \nu_{L\alpha} + \frac{g_2}{\sqrt{2}}\left(\bar{l}^-_\alpha \gamma^\mu W^-_\mu \nu_{L\alpha} + \bar{\nu}_{L\alpha}\gamma^\mu W^+_\mu l^-_\alpha\right)$$

$$\begin{pmatrix} \nu_{L\alpha} \\ \nu_{R\alpha} \end{pmatrix}_f = \begin{pmatrix} U_{\alpha i} & \Theta_\alpha \\ \Theta'_\alpha & 1 \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_H \end{pmatrix}_m$$



3-1. model

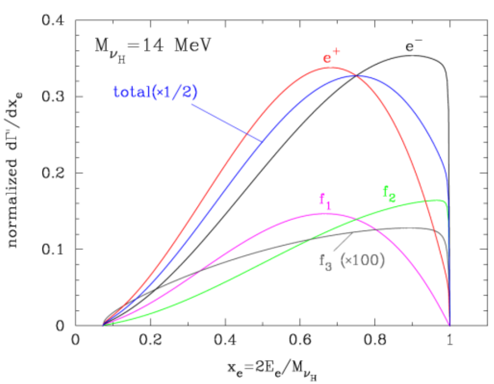
- ν_H decay → injection of energetic $e^{\pm}$ and ν
- ν freely streams in the universe after BBN
  - →nonthermal ν production →ν energy density is increased

$$\rho = \rho_\gamma + (\rho_{e^-} + \rho_{e^+}) + \rho_{\nu,\mathrm{th}} + \rho_b + \rho_{\nu_\mathrm{H}} + \rho_{\nu,\mathrm{nt}}$$

- energies of $e^{\pm}$ are transferred to background γ via interactions with background γ and e^-
  - →background γ is heated
  - →baryons and thermal background neutrino are diluted
  - →η is decreased, and ν energy density is decreased

- γ injection spectrum

$$p_\gamma(E_\gamma; T, M_{\nu_\mathrm{H}}) = \frac{1}{\Gamma}\int_{m_e}^{M_{\nu_\mathrm{H}}/2} \frac{d\Gamma}{dE_e}(M_{\nu_\mathrm{H}})\, dE_e \int_0^{E_{\gamma 0,\mathrm{max}}} P_{\mathrm{iC}}(E_e, E_{\gamma 0}; T)\, p_{\gamma,\mathrm{EC}}(E_\gamma, E_{\gamma 0}; T)\, dE_{\gamma 0},$$



Diff. decay rate

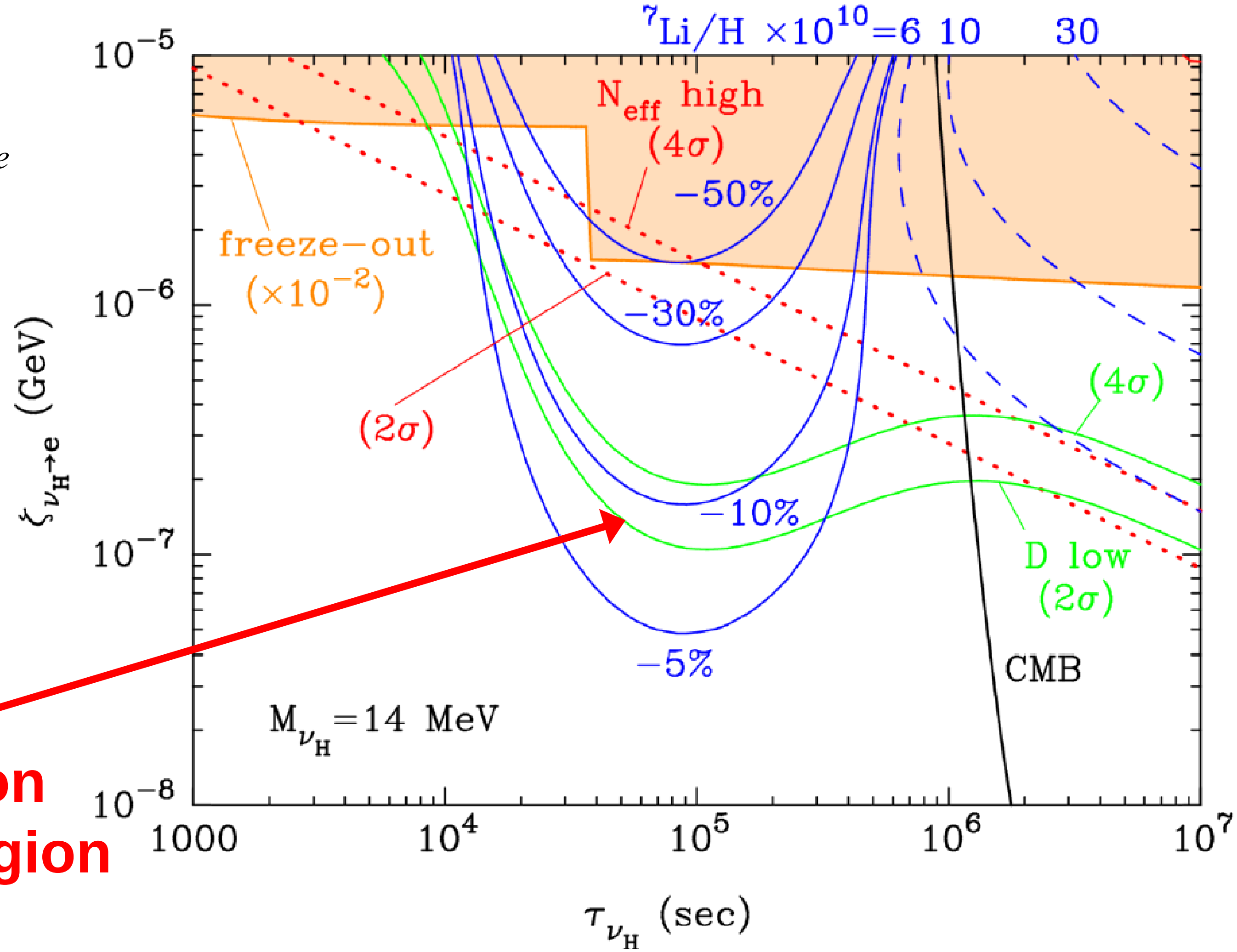
Energy spectrum of primary γ produced via inverse Compton scatterings of $e^{\pm}$ (E_e)

Energy spectrum of γ produced in the electromagnetic cascade showers of primary γ ($E_{\gamma 0}$)

4. Results: decay into $e^{\pm}$ and ν

4-1. Derived constraints

$$z_{nH \to e} = \left(\frac{n^0_{nH}}{n^0_g} \right) E_{nH \to e}$$



$^7Be(\gamma, \alpha)^3He$

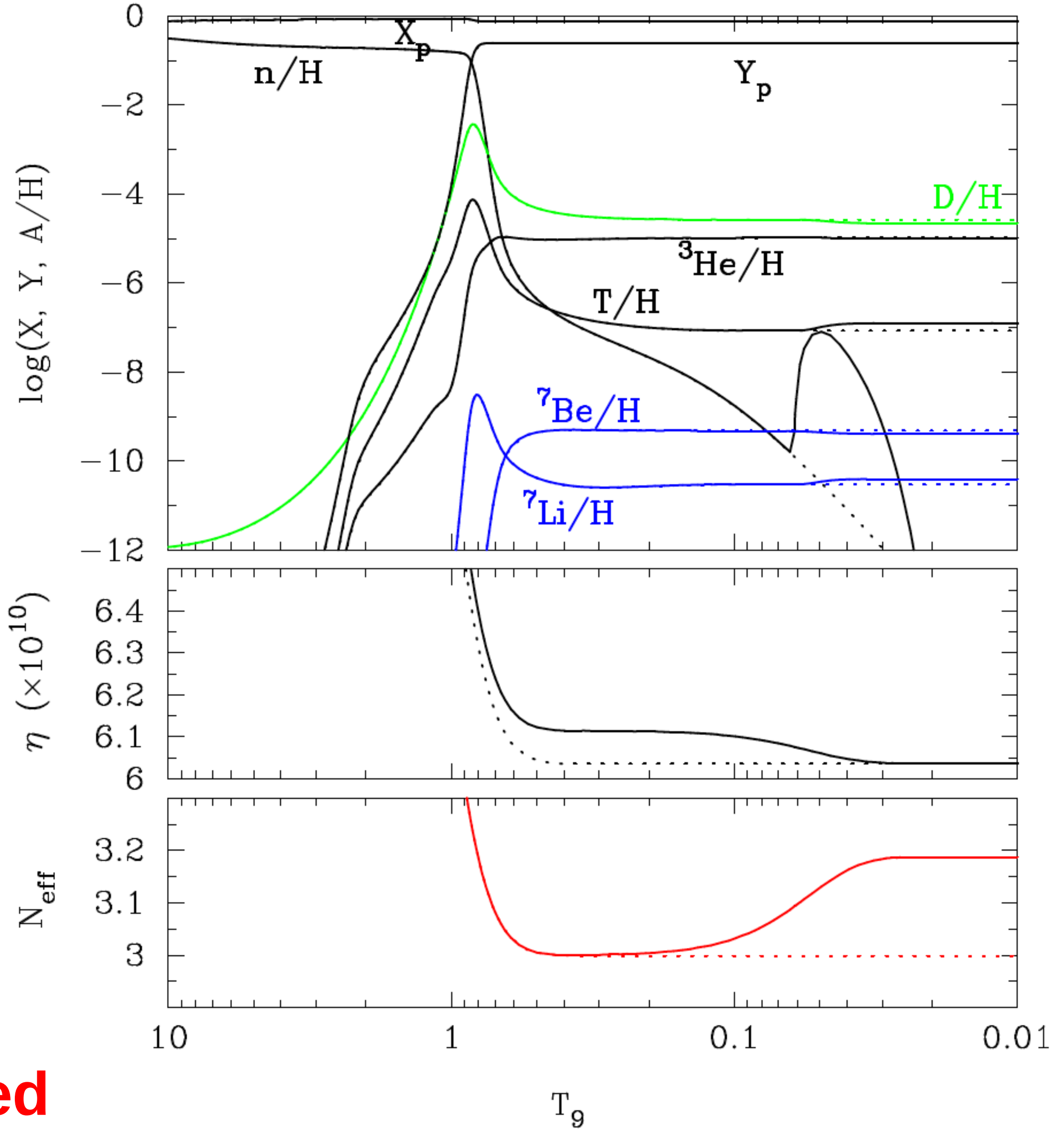
weak ^{7}Li reduction in the allowed region

- The ν_H decay alone cannot be a solution to the Li abundance of MPSs
- It could be a solution if the stellar Li abundances are depleted by a factor ∼2
- This model can be tested with future measurements of N_{eff}

4. Results: decay into $e^{\pm}$ and ν

4-2. Time evolution of quantities

$M_{\nu H}$=14 MeV
$\tau_{\nu H}=4\times10^{4}$ s
$\zeta_{\nu H\to e}=3\times10^{-7}$ GeV



$$N_{\rm eff} = \frac{\rho_\nu}{\frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T^4}$$

N_{eff} value is increased