Transverse polarization measurement by Compton polarimeter for CEPC

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The CEPC PhysDet plenary meeting

Outline

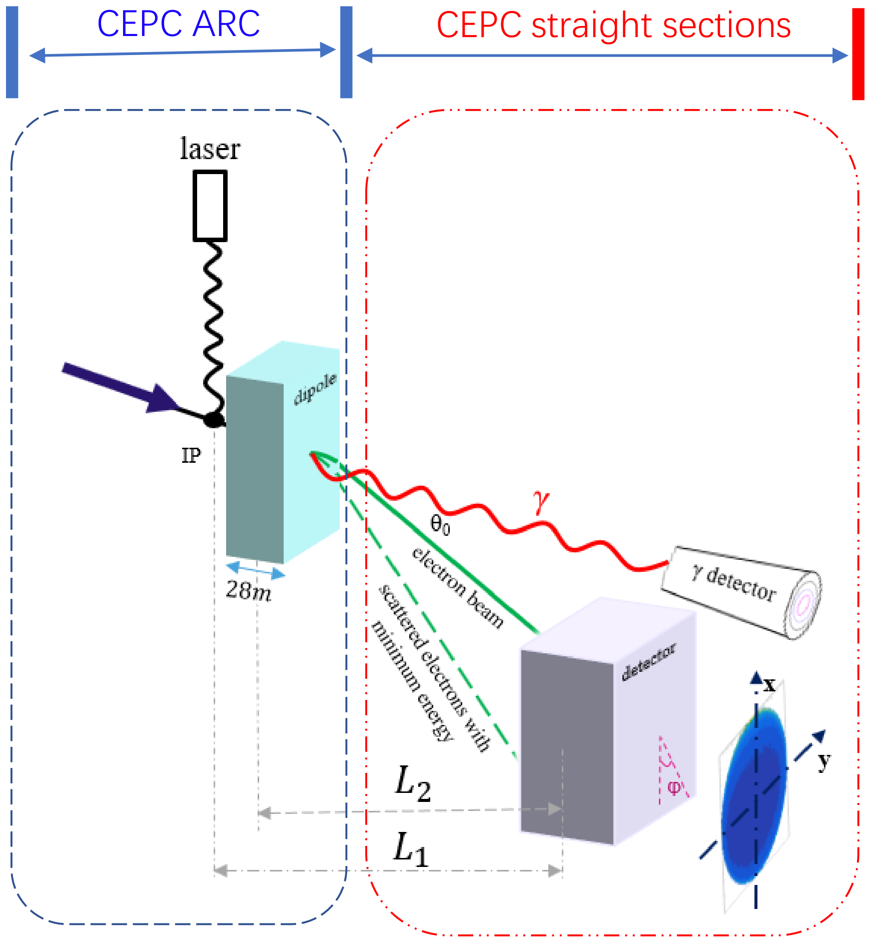
- **Compton Polarimeter for CEPC@ CEPC PhysDet plenary meeting 2021-05-12**
  - Motivation of CEPC Z-pole polarized beam program
  - Layout of Compton Polarimeter for CEPC
  - The measurements of beam transverse polarization(Z pole)
  - The toy MC simulations and The fit results of transverse polarization

- Today the outline is:
  - The layout of Compton polarimeter for CEPC operated on Z pole
  - Update the design of detector and MC samples
  - The measurement result
  - The statistical and systematic error
  - Summary

Compton Polarimeter at CEPC(Z pole)

- Discuss the system layout(for transverse polarization)

△ Compton Polarimeter



- **Prospects for the Compton polarimeter system:**
  - Our device is in straight sections on the ring.
  - The last dipole in the ARC can be used as our bending magnet of Compton polarimeter.

CEPC Z pole:	E = 45.5 GeV
Laser :	$\omega = 1.24eV$
Dipole parameter(CDR)	$\begin{cases} B = 70.7904Gs \\ l = 28.686m \end{cases} \rightarrow \theta_0 = 1.3389mrad$
Beam vacuum tube	31mm(Outer radius)
Drift distance	• $L_1 = 60m$; $L_2 = 40m$ • Meaning that after dipole the scattered particles will drift $40m - 28.686/2 = 25.657m$
Beam angle (at IP of laser and electron beam)(王毅伟)	$\sigma'_x = 1.2197$ [μrad] $\sigma'_y = 0.5164$ [μrad]

Physics of Compton Polarimeter

- **Compton Polarimeter involves two physical process: Compton scattering and magnetic deflection**

Case 1:
Compton scattering process

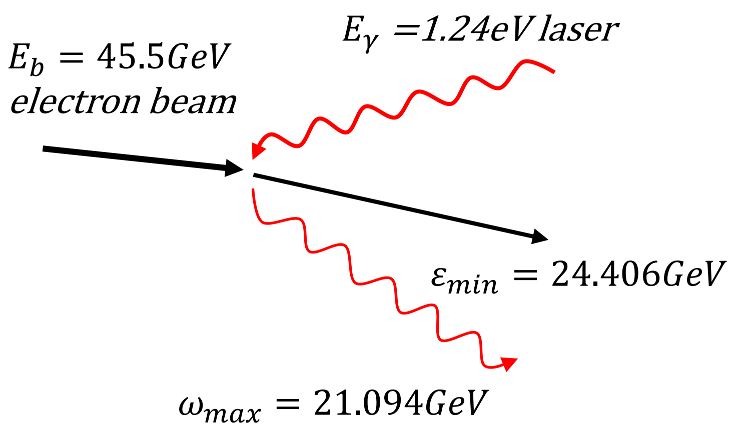
Case 2:
Compton scattering process
&
Magnetic deflection

The relationship between scattered energy & detector azimuthal angle **(u,ψ)** and the position distribution of scattered electrons (X_e, Y_e)

(X_e, Y_e) ⇨ $(\boldsymbol{u}^+, \boldsymbol{\varphi}^+)$ or $(\boldsymbol{u}^-, \boldsymbol{\varphi}^-)$

Physics of Compton Polarimeter

Case 1: Compton scattering process



$$\boldsymbol{u} = \frac{\omega}{\varepsilon}; \quad u \in [0, \kappa] \qquad \boldsymbol{\kappa} = \frac{4E_\gamma E_b}{m^2}$$

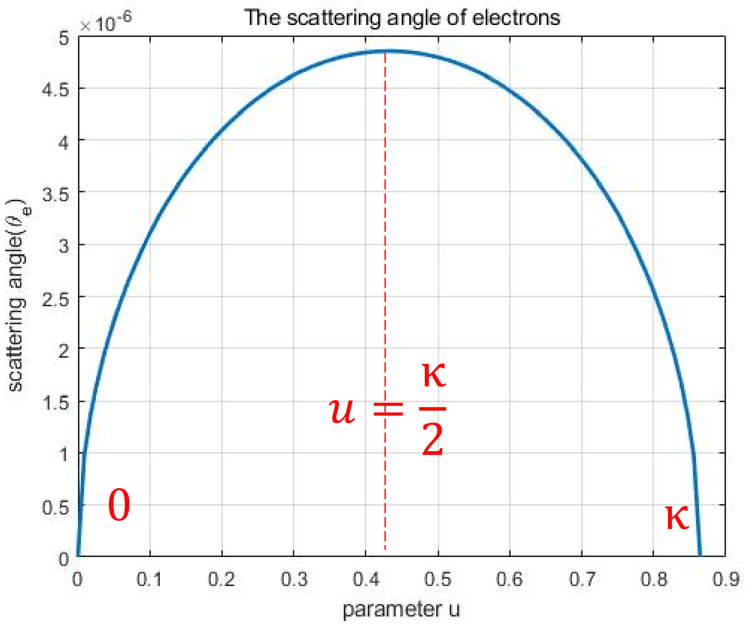
u is ratio of scattered energy of photons to electrons

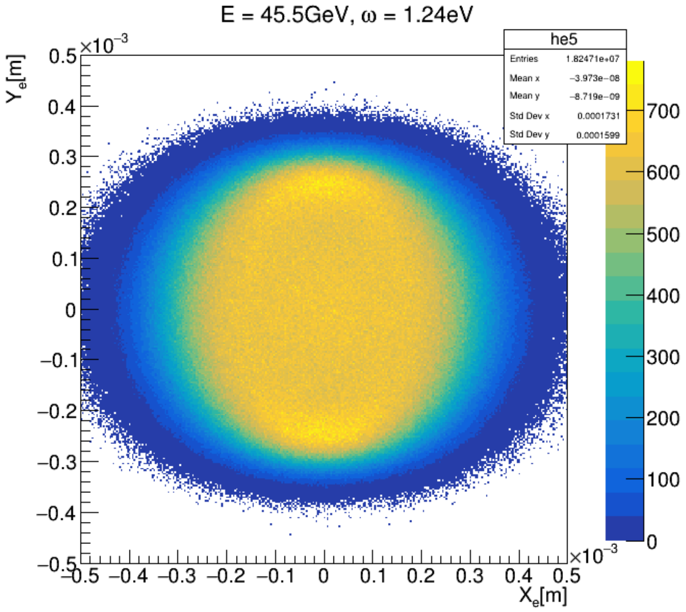
- Scattering angle of electron θ_e $\qquad \theta_e = \frac{1}{\gamma}\sqrt{u(\kappa - u)}$

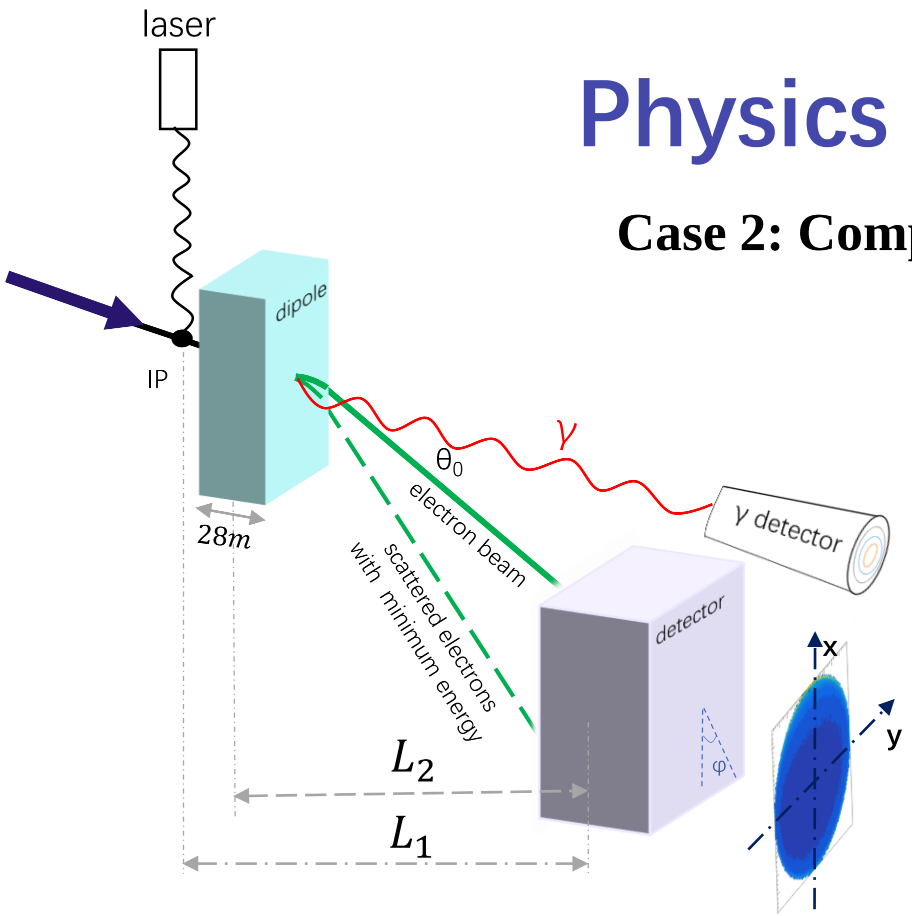
When $u = \frac{\kappa}{2}$: $\theta_e = \theta_{e\_max} = \frac{1}{\gamma}\frac{\kappa}{2}$

- Detector is perpendicular to the beam (φ is the detector's azimuthal angle):

$$\begin{cases} X_e = L_1\theta_e\cos\varphi \\ Y_e = L_1\theta_e\sin\varphi \end{cases} \Rightarrow \begin{cases} u\epsilon[0, \frac{\kappa}{2}], \quad \varphi\epsilon[0, 2\pi] \\ u\epsilon[\frac{\kappa}{2}, \kappa], \quad \varphi\epsilon[0, 2\pi] \end{cases}$$







Physics of Compton Polarimeter

Case 2: Compton scattering process + Magnetic deflection

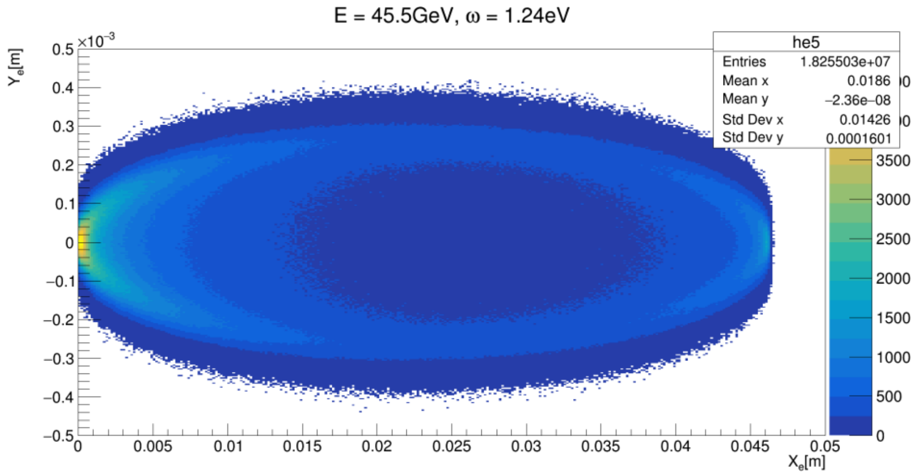
- ✓ Gathering two physical process together:

$$\begin{cases}\theta_x = \theta_e \cos\varphi + u\theta_0 \\ \theta_y = \theta_e \sin\varphi\end{cases}$$

$u\theta_0$ is the magnetic deflection angle caused by energy loss in Compton back-scattering process.

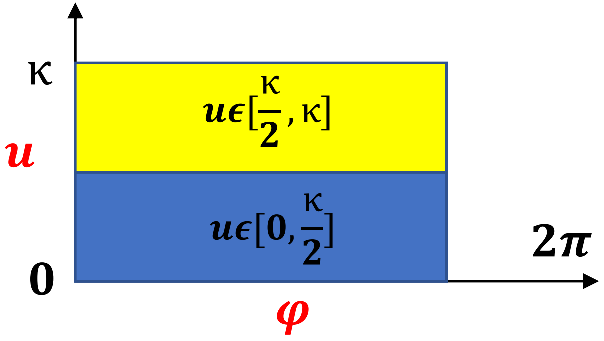
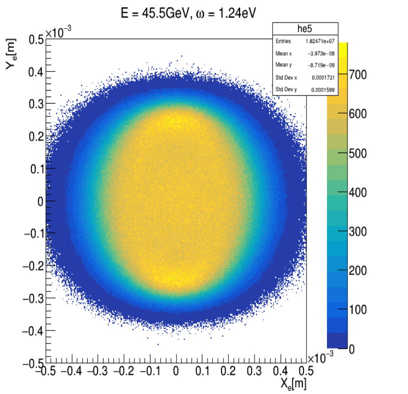
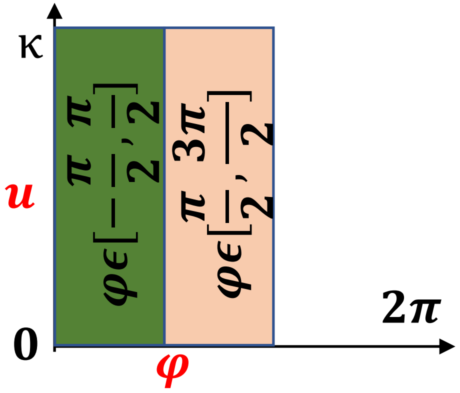
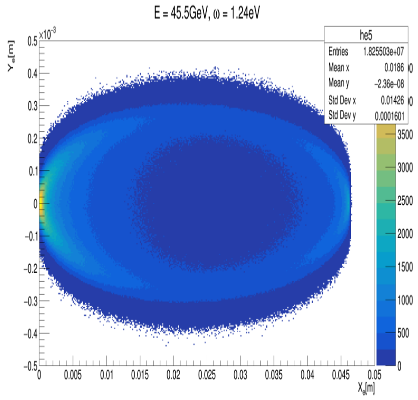
- ✓ The position of scattered electrons in detector plane is:

$$\begin{cases}X_e = L_1\theta_e \cos\varphi + u\theta_0 L_2 \\ Y_e = L_1\theta_e \sin\varphi\end{cases} \Rightarrow \begin{cases}\varphi\epsilon[-\frac{\pi}{2},\frac{\pi}{2}], \quad u\epsilon[0,\kappa] \\ \varphi\epsilon[\frac{\pi}{2},\frac{3\pi}{2}], \quad u\epsilon[0,\kappa]\end{cases}$$



Physics of Compton Polarimeter

Summary of the relationship between (u,ψ) and (X_e, Y_e)

Case 1: Compton scattering process	$u\epsilon[0,\frac{\kappa}{2}]$ $u\epsilon[\frac{\kappa}{2},\kappa]$	$\varphi\epsilon[0,2\pi]$		
Case 2: Compton scattering & Magnetic process	$u\epsilon[0,\kappa]$	$\varphi\epsilon[-\frac{\pi}{2},\frac{\pi}{2}]$ $\varphi\epsilon[\frac{\pi}{2},\frac{3\pi}{2}]$		

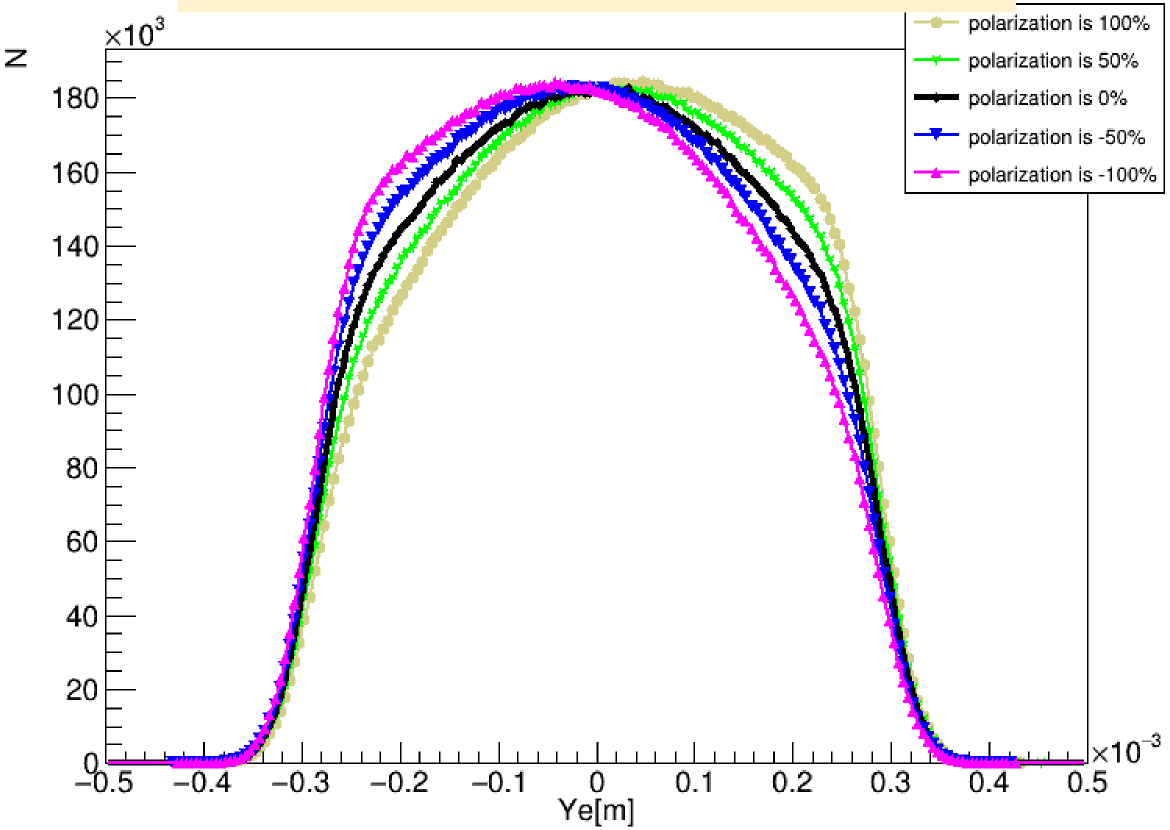
✓ The 2d distribution of scattered electrons in detector **(X_e, Y_e) correspond to** (u^+, φ^+) or (u^-, φ^-)

The distribution of scattered electrons

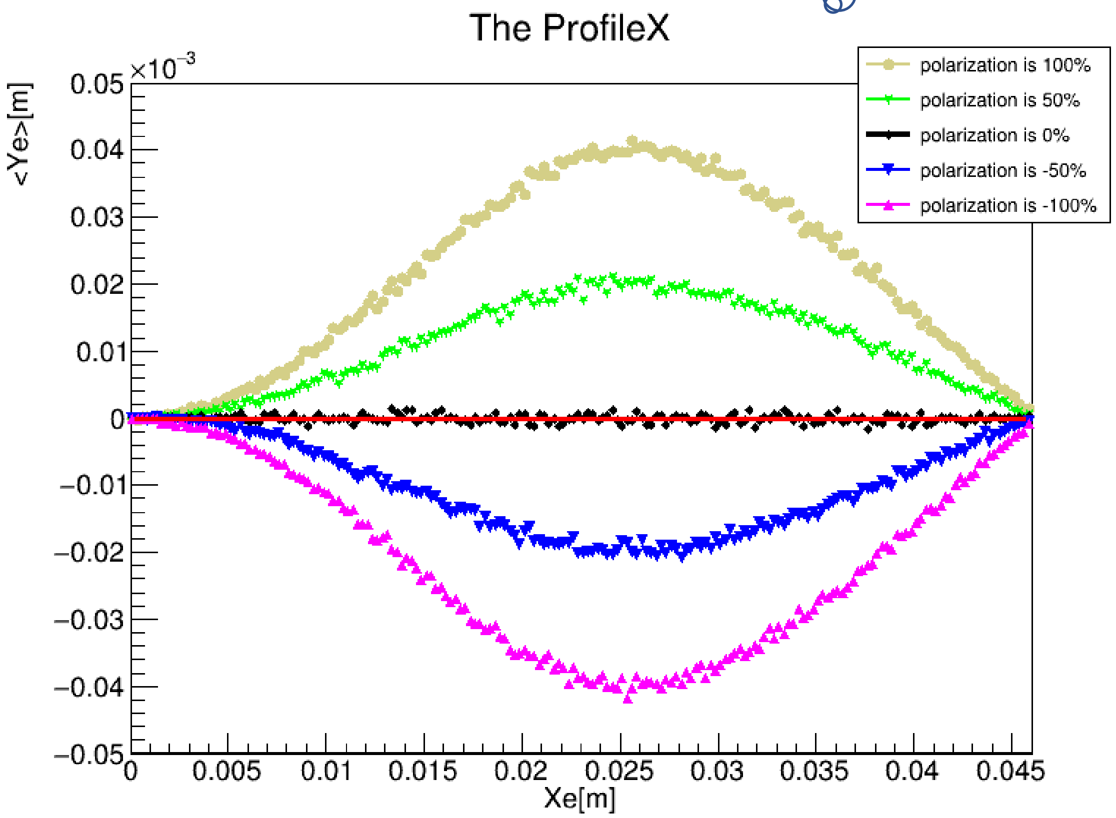
Analyzing power: The asymmetry for 100% beam polarization.

- ➢ **The different polarization**

Projection Y of 2D distribution of scattered electrons



The Y mean shift per X bin



✓ The transverse polarization is sensitive to the distribution of scattered electrons, especially in the Y axis.

The measurement

➢ The MC samples

- **The beam angle**
  $\sigma'_x = 1.2197$ [μrad]
  $\sigma'_y = 0.5164$ [μrad]
- **The design of detector**
  X*Y = 5cm*2cm;
  Pixel size: 50μm*25μm
  Smear:14.434μm*7.217μm
- **For 2D distribution:**
  - Profile X
  - Y mean shift with per X bin $\overline{Y_e}|_{X_e}$

➢ The fit method

$$\overline{Y_e}|_{X_e} = P_\perp \Pi(X_e)$$

- $P_\perp = \xi_\frown \zeta_\perp$
- $\xi_\frown$ is laser circular polarization
- $\zeta_\perp$ is electron beam transverse polarization
- $\overline{Y_e}|_{X_e}$ is 1d distribution of scattered electrons the mean of Y per X bin
- $\Pi(X_e)$ is analyzing power: its value for 100% polarization

analyzing power: $\Pi(X_e)$

$$\begin{cases} \Pi(X_e), u = u^+ \\ \Pi(X_e), u = u^- \\ \Pi(X_e), u = \bar{u} = \dfrac{u^+ + u^-}{2} \end{cases}$$

numerical solution

analytical solution

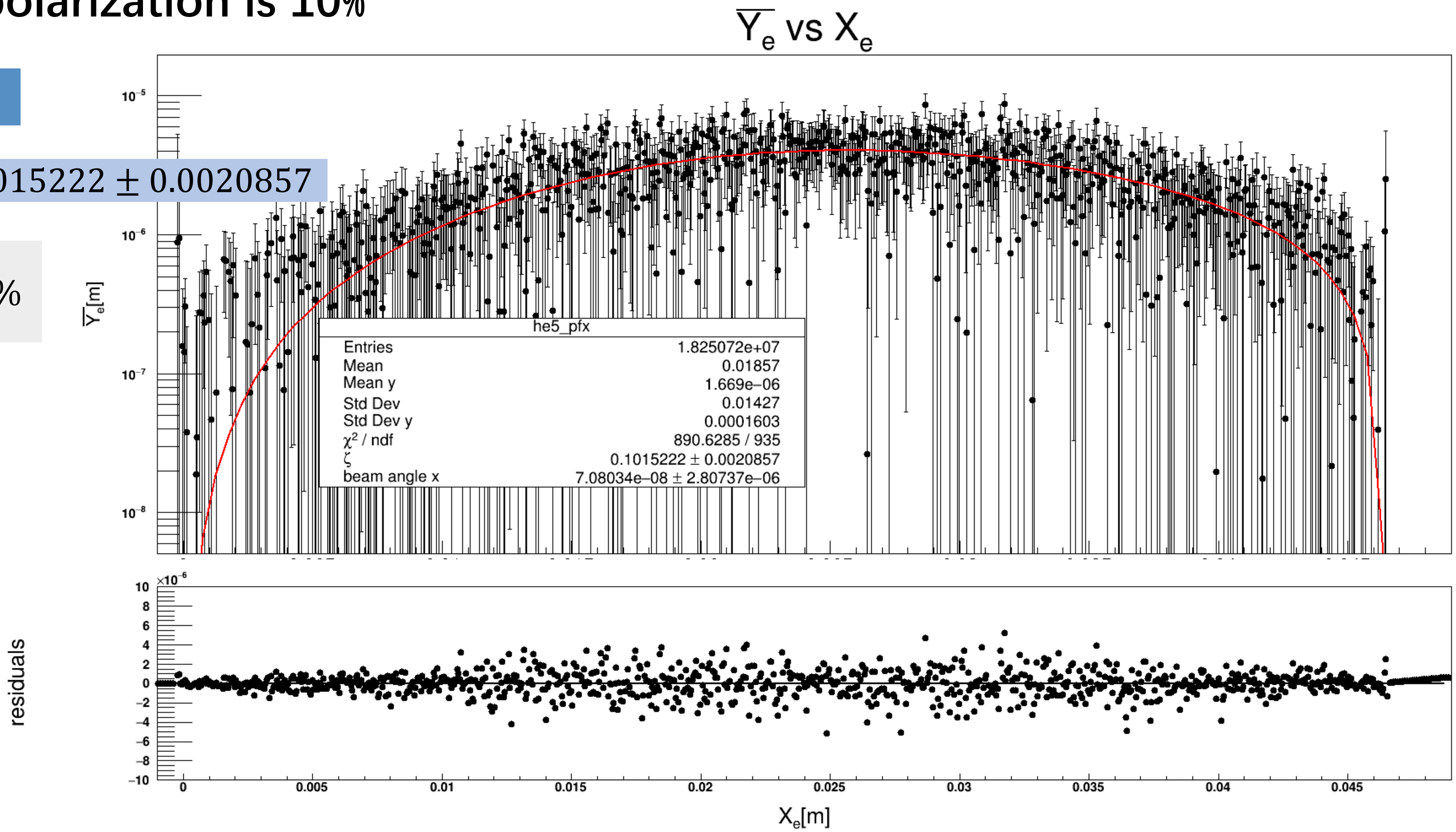
The fit result (1)

- **The polarization is 10%**

$u = \bar{u}$

$P_{\perp} = 0.1015222 \pm 0.0020857$

$\frac{\Delta P_{\perp}}{P_{\perp}} \approx 2\%$

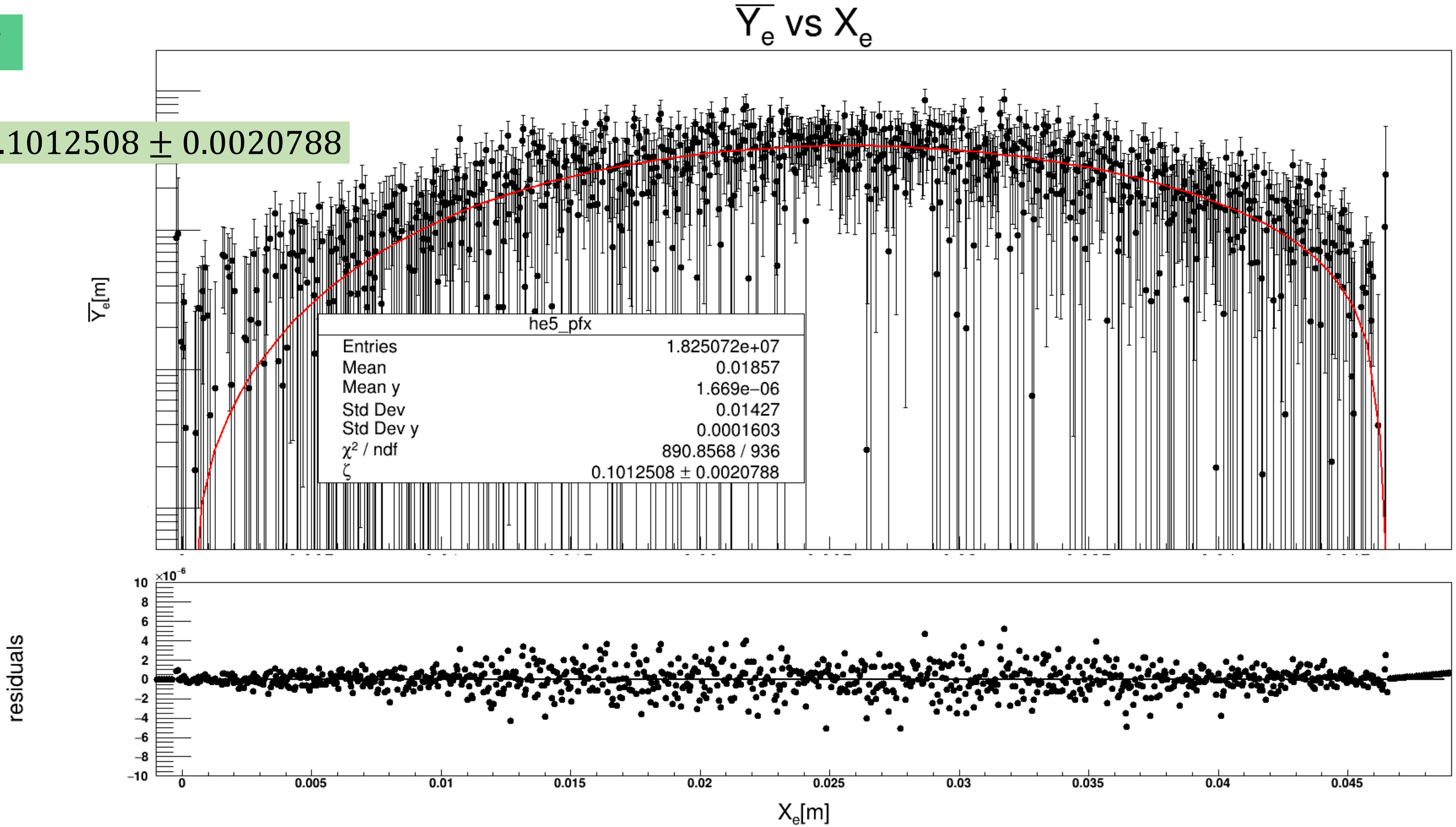


The fit result(2)

- The polarization is 10%

$u = u^{+}$

$P_{\perp} = 0.1012508 \pm 0.0020788$

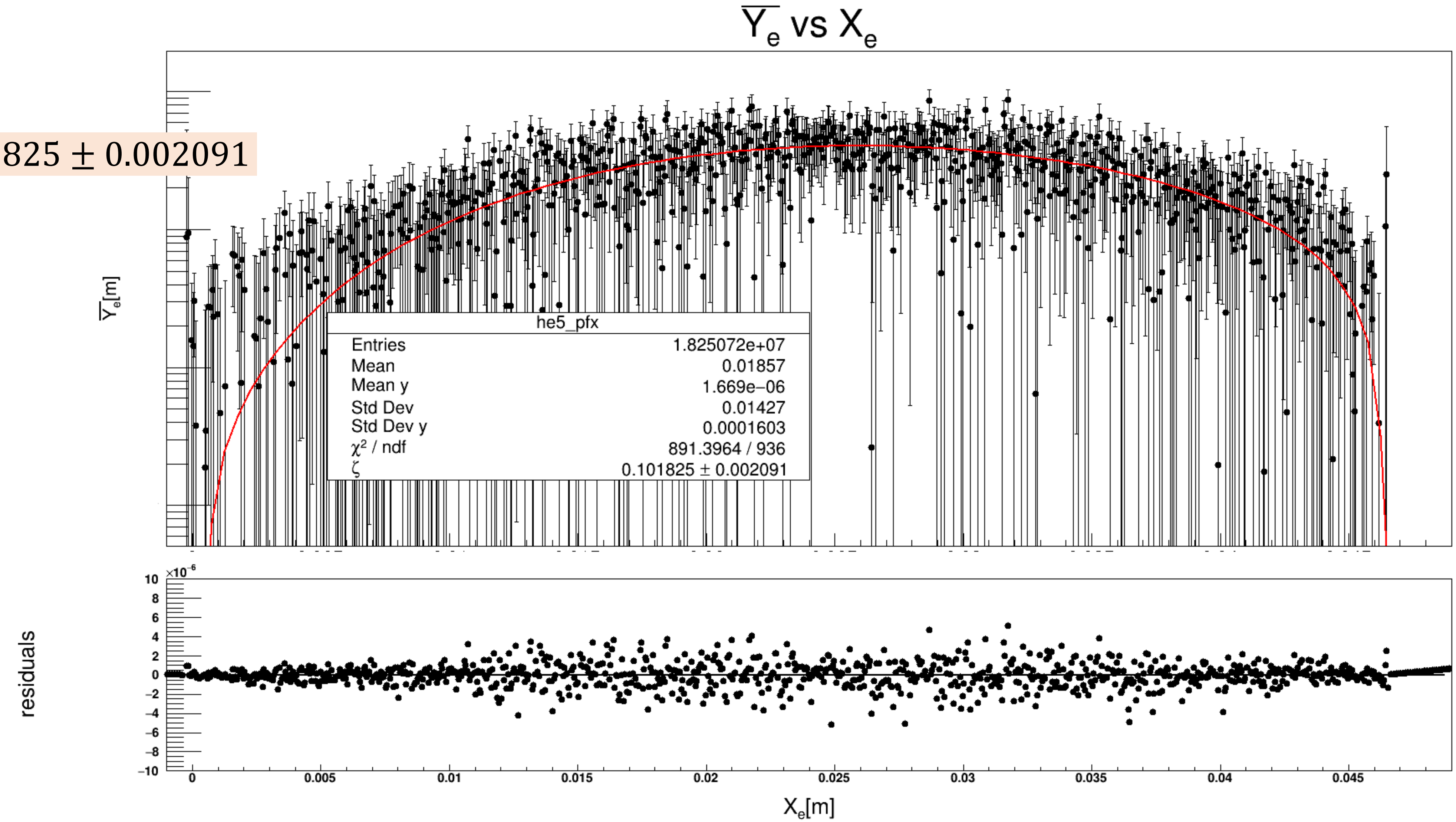


The fit result(3)

- The polarization is 10%

$u = u^-$

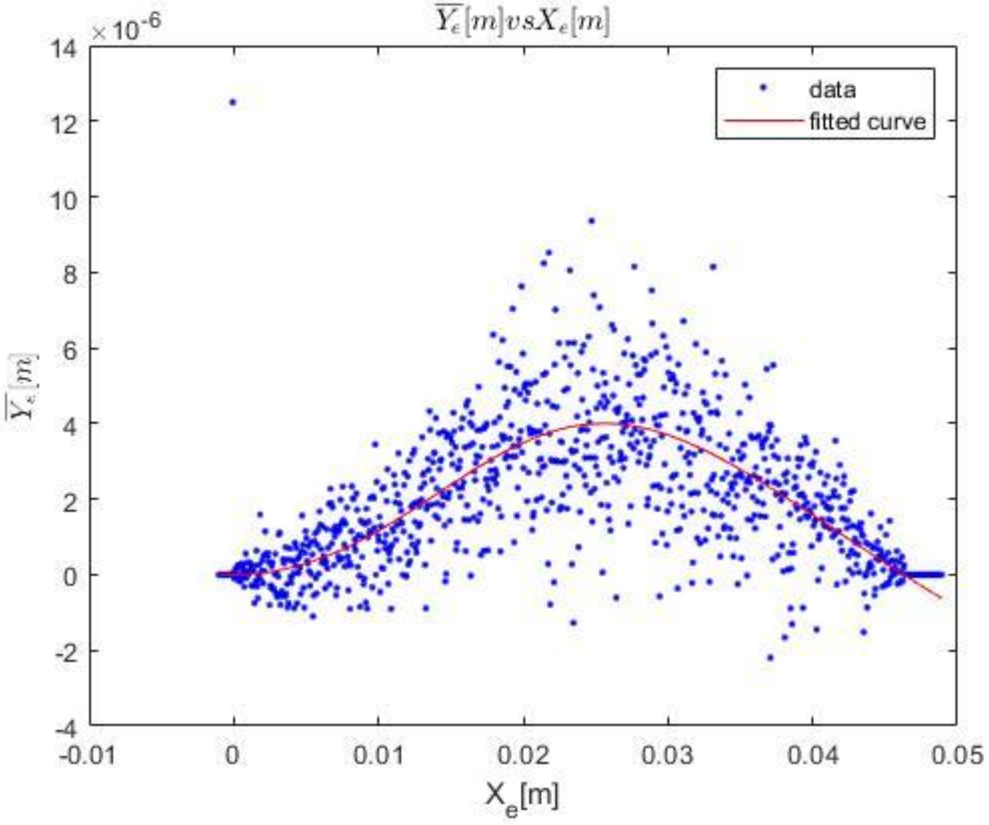
$P_\perp = 0.101825 \pm 0.002091$

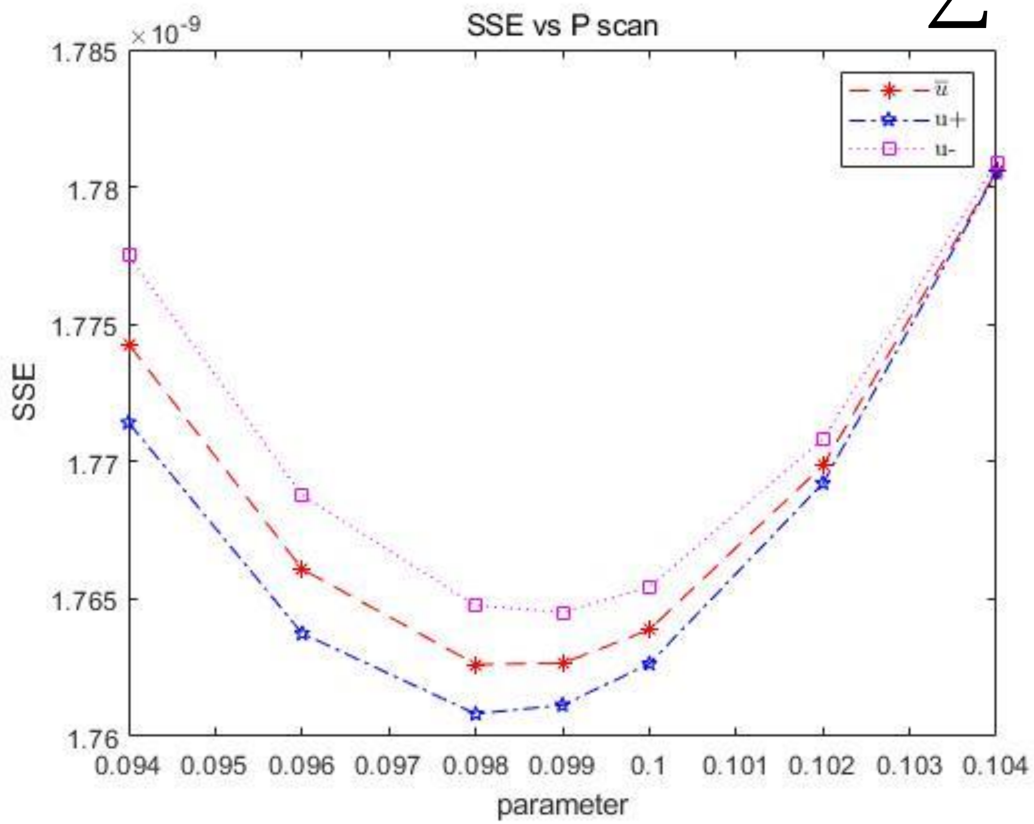


The MATLAB fit result

➢ Polarization = 10%

$$SSE = \sum w_i (y_i - \hat{y}_i)^2$$





Fit function	Fit result
$\Pi(X_e), u = u^+$	$P_\perp = 0.0980 \pm 0.0017$
$\Pi(X_e), u = u^-$	$P_\perp = 0.0990 \pm 0.0017$
$\Pi(X_e), u = \bar{u}$	$P_\perp = 0.0980 \pm 0.0017$

The fit result

- **The polarization is 10%**

	Fit function	Fit result
• ROOT fit	$\Pi(\boldsymbol{X_e}), \boldsymbol{u} = \boldsymbol{u}^+$	$P_\perp = 0.101508 \pm 0.0020788$
	$\Pi(X_e), u = u^-$	$P_\perp = 0.101825 \pm 0.002091$
	$\Pi(X_e), u = \bar{u}$	$P_\perp = 0.1015222 \pm 0.002057$
• MATLAB fit (fit function is numerical solution)	$\Pi(X_e), u = u^+$	$P_\perp = 0.0980 \pm 0.0017$
	$\Pi(\boldsymbol{X_e}), \boldsymbol{u} = \boldsymbol{u}^-$	$P_\perp = 0.0990 \pm 0.0017$
	$\Pi(X_e), u = \bar{u}$	$P_\perp = 0.0980 \pm 0.0017$

- $\Delta P_\perp / P_\perp \approx 2\%$

The statistical error

➢ The luminosity of Compton polarimeter

LC-M-2007-014

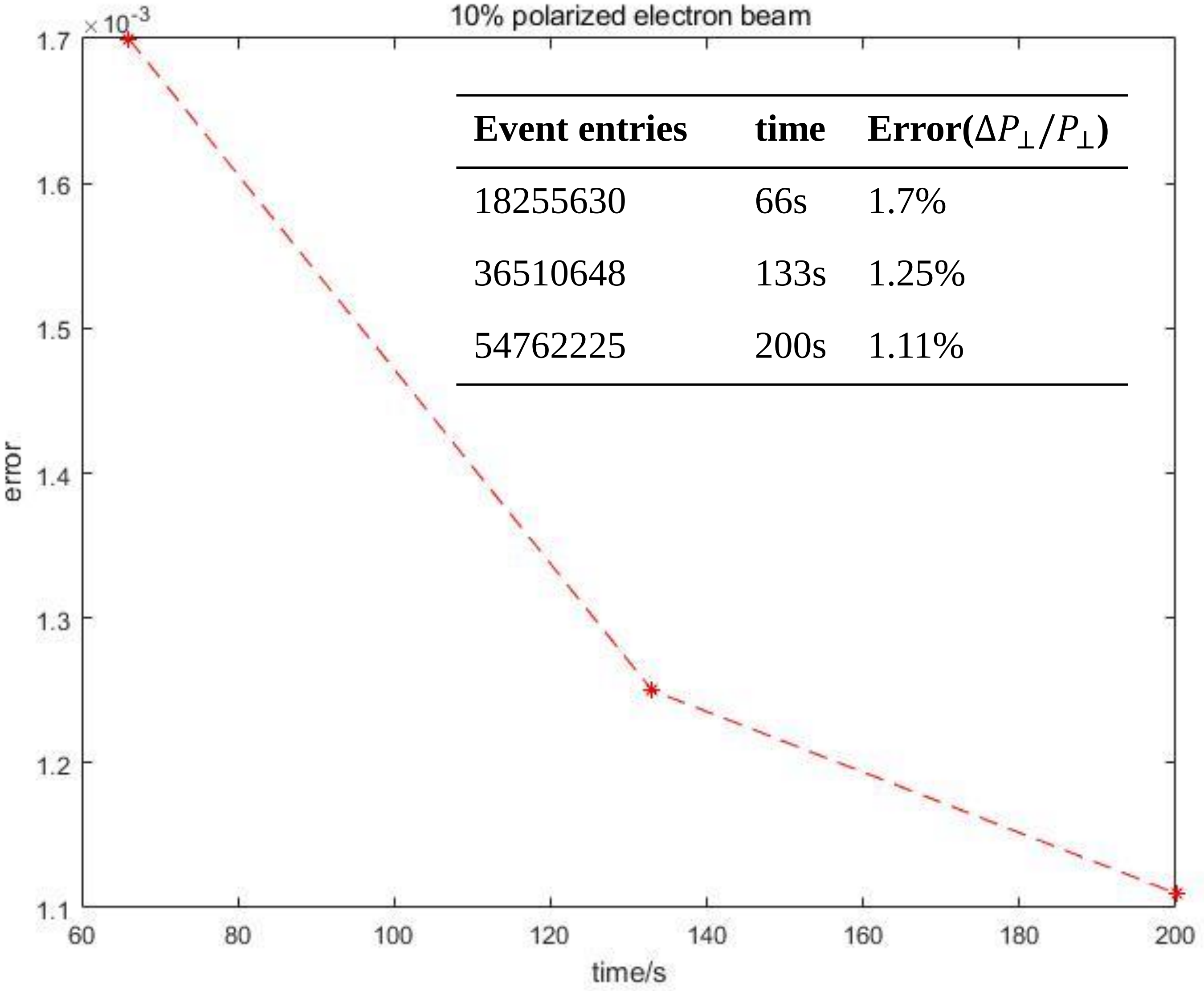
- The luminosity for a pulsed laser $$\mathfrak{L} = \frac{f_b N_e N_\gamma}{2\pi\sigma_{x\gamma}\sigma_{y\gamma}\sqrt{1 + (\frac{1}{2}\theta_0\frac{\sigma_{z\gamma}}{\sigma_{y\gamma}})^2}}$$

Par.	Unit	Physics meaning	CEPC(Z pole)
f_b		Number of bunch crossing per second	1(12000)
N_e		Number of electrons per bunch	8×10^{10}
P_L	W=J/s	power of the laser	0.1GW
λ	m	Wavelength of laser	1.002μm(1.24eV)
N_γ		Number of photons per laser pulse	1.4×10^{16}
$\sigma_{x\gamma}/\sigma_{y\gamma}$	m	Rms beam size	$\sigma_\gamma = 160\mu m$
$\mathfrak{L}$	$cm^{-2}s^{-1}$	luminosity	$6.9676 \times 10^{33} m^{-2} \cdot s^{-1}$
σ	barn	Cross section	393.5mb
Max. rate	s^{-1}	Compton scattering event rate	$\mathbf{2.742 \times 10^5} pulse^{-1}$

- The laser is 1HZ. Compton rate = $\mathbf{2.742 \times 10^5} s^{-1}$

The statistic error(2)

- The measurement error vs time



Event entries	time	Error($\Delta P_\perp / P_\perp$)
18255630	66s	1.7%
36510648	133s	1.25%
54762225	200s	1.11%

The systematic uncertainty

Table 2: The systematic uncertainty of Compton polarimeter

Sources of systematic uncertainties	Δ	$\left\|\frac{\Delta P}{P}\right\| \%$
Dipole strength	$7.07904 \times 10^{-7} T$	0.00239%
L1 (Ip to detector)	1mm	0.01653%
L2 (Dipole to detector)	1mm	0.017929%
Δα deviation of the detector	1μrad	ignored
Δβ deviation of the detector	1μrad	ignored
Detector placement		ignored
Total		0.036849%

Discussion & Summary

- Compton Polarimeter is the clear technique of choice for electron polarization at CEPC
  - 2% statistical error has been achieved in 66s time; 1% statistic error in 200s.
  - 0.037% systematic uncertainty are obtained
  - About the detector linearity and electronic noise effect are not considered.

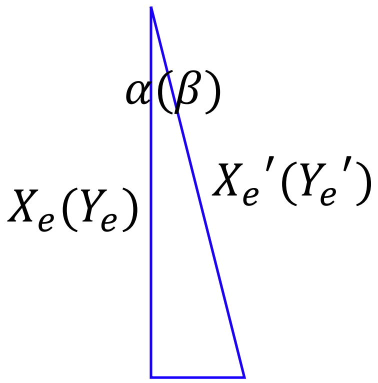
Additional slides

The systematic uncertainty(2)

Detector orientation deviation

$$\begin{cases} X_e = L_1\theta_e\cos\varphi + u\theta_0 L_2 \\ Y_e = L_1\theta_e\sin\varphi \end{cases}$$

$$\begin{cases} X_e' = X_e/\cos\alpha \\ Y_e' = Y_e/\cos\beta \end{cases}$$



Δα deviation of the detector	Δα=0.1°	$\Delta X_e = X_e' - X_e = 1.5215e-06X_e$	Detector resolution x = 14.43μm
Δβ deviation of the detector	Δβ=0.1°	$\Delta Y_e = Y_e' - Y_e = 1.5215e-06Y_e$	Detector resolution y = 7.22μm

- The systematic error by deviation of detector can be ignored.

The systematic uncertainty(1)

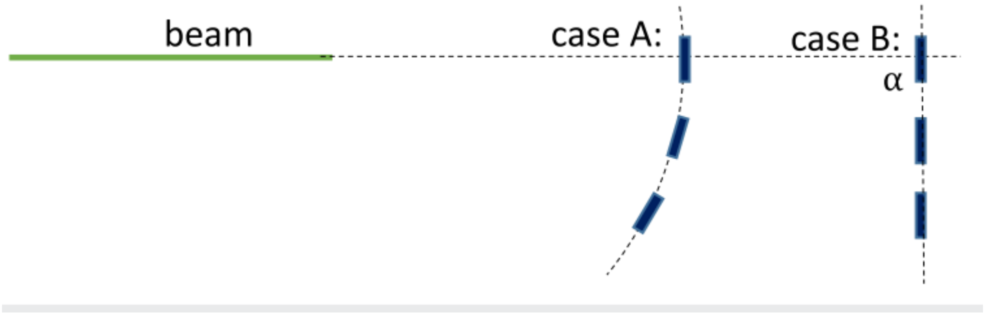
$$\left(\frac{\Delta P_{\perp}}{P_{\perp}}\right)^2 = \left(\frac{\Delta \bar{Y}_e}{\bar{Y}_e}\right)^2 + \underbrace{\left(\frac{\Delta \Pi}{\Pi}\right)^2}_{\text{Systematic error}}$$

$$\frac{\Delta P_{\perp}}{P_{\perp}} = \frac{\Delta \Pi}{\Pi} = \frac{\sum \frac{\partial \Pi(X_e)}{\partial \theta_0} \sum \frac{n_{xi}}{N} \delta \theta_0}{\Pi} + \frac{\sum \frac{\partial \Pi(X_e)}{\partial L_1} \sum \frac{n_{xi}}{N} \delta L_1}{\Pi} + \frac{\sum \frac{\partial \Pi(X_e)}{\partial L_2} \sum \frac{n_{xi}}{N} \delta L_2}{\Pi}$$

Sources of systematic uncertainties	Δ	$\left\lvert \frac{\Delta \boldsymbol{P}}{\boldsymbol{P}} \right\rvert$ %
Dipole strength	$7.07904 \times 10^{-7} T$	0.00239%
L1 (Ip to detector)	1mm	0.01653%
L2 (Dipole to detector)	1mm	0.017929%
Total		0.036849%

The systematic uncertainty(3)

Detector placement



$$X_A = L_1\theta_e\cos\varphi + u\theta_0 L_2$$

$$X_B = L_1 tan\theta\cos\varphi + uL_2 tan\theta$$

$$\theta_{e_{max}} = \frac{2\omega_0}{m_e} = 4.853\times10^{-6}rad$$

$$\theta_{0_{max}} = \kappa\theta_0 = 0.00116rad$$

$$\tan(0.00116) = 0.001160000520299$$

- The systematic error by detector placement can be ignored.

About bending angle

- 推导偏转角和偏移量

Higgs mode

Table 4.3.3.5: Parameters of the dual aperture dipole.

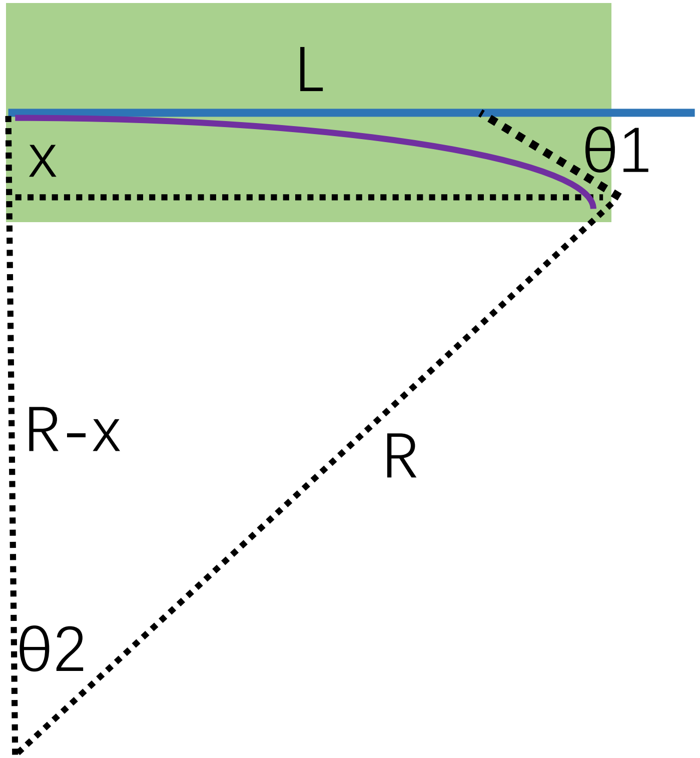
Beam center separation [mm]	350
Magnetic length [m]	28.686
Magnetic strength [Gs]	373.4
Gap [mm]	70

The magnetic strength of the dipole located in last is $\frac{1}{2}B = \frac{373.4Gs}{2} = 0.01867T$

Z pole

$$\frac{120GeV}{45.5GeV} = \frac{373.4Gs}{B_{z\_center}}$$

$$B_{z\_end} = \frac{1}{2}B_{z\_center} = 70.7904Gs$$



- 偏转角等于圆心角

$$bending\ angle: \theta 1 = \theta 2 = \frac{l}{R}$$

- 偏移量x: $R^2 = L^2 + (R - x)^2$
- $Bqv = mv^2/R$

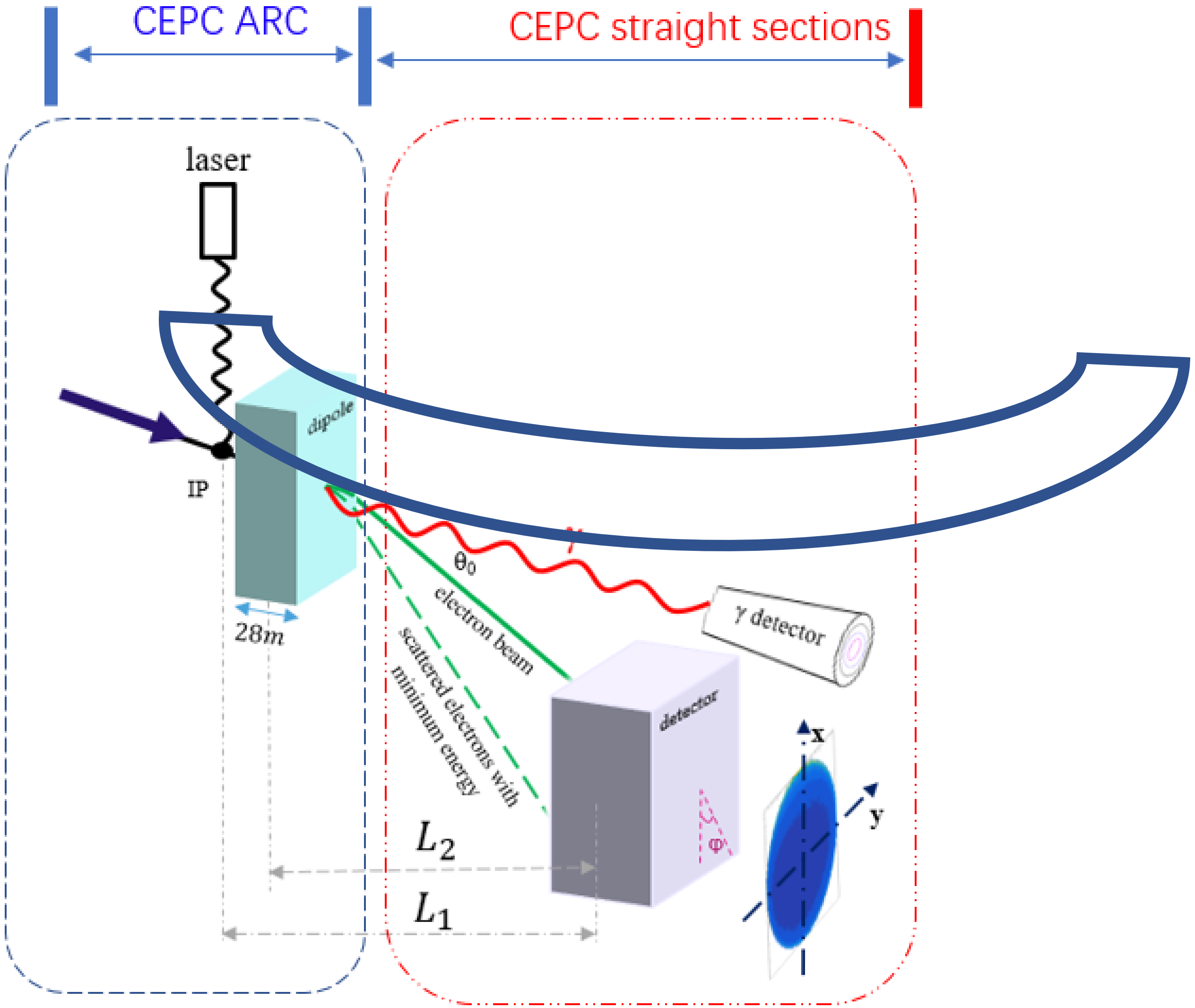
$$E = mc^2 \rightarrow mc = E/c$$

$$R = \frac{mv^2}{Bqv} = \frac{mv}{Be} = \frac{E}{Bec}$$

- $\theta = \frac{l}{R} = 0.0013389rad$

discussion

△ Compton Polarimeter



CEPC 束流管 外半径 = 31mm

由B铁参数计算 $\theta_0 = 0.0013389\text{rad}$

$$\Delta x = L * \theta_0$$

$$L > \frac{31mm}{0.0013389\text{rad}} = 23.15m$$

问题1：L从那里算起？B铁的中心？还是B铁的右边界？

B铁的中心

问题2：散射电子在探测器上的分布：

$$\begin{cases} X_e = L_1\theta_e\cos\varphi + u\theta_0 L_2 \\ Y_e = L_1\theta_e\sin\varphi \end{cases}$$

不用减去 radius of tube吧？

不用

Calculation of luminosity

➢ The luminosity of pulsed laser and electron bunch

- 1 pulse laser with 1 electron bunch $$\mathfrak{L} = \frac{N_e N_\gamma}{2\pi\sigma_{x\gamma}\sigma_{y\gamma}}$$

- N_e (Number of electrons per bunch) = 8×10^{10}

- About N_γ
  - $N_\gamma = \frac{E_{laser}}{\omega_{photon}}$

  electron bunch length =8.5mm;

  Pulse electron bunch = electron bunch length /c = $\frac{8.5mm}{3\times10^8 m/s} = 28\text{ps}$

  Considering the not destroy the mirrow: the laser power is 0.1GW

  $$E_{laser} = \text{P} * \text{t} = 0.1\text{GW} * 28\text{ps} = 2.8 \times 10^{-3} J = 2.8mJ$$

  The number of 1 pulsed laser is:

  $$N_\gamma = \frac{E_{laser}}{\omega_{photon}} = \frac{2.8 \times 10^{-3} J}{1.24 \times 1.6 \times 10^{-19} J} = 1.4 \times 10^{16}$$

- The luminosity of 1 pulse laser with 1 electron bunch:

$$\mathfrak{L} = \frac{N_e N_\gamma}{2\pi\sigma_{x\gamma}\sigma_{y\gamma}} = \frac{8 \times 10^{10} \times 1.4 \times 10^{16}}{2\pi \times (160\mu m \times 160\mu m)} = 6.967 \times 10^{33} m^{-2} \cdot s^{-1}$$

Calculation of scattering rate

➢ The maximum rate of pulsed laser and electron bunch

- The luminosity of 1 pulse laser with 1 electron bunch:

$$\mathfrak{L} = \frac{N_e N_\gamma}{2\pi\sigma_{x\gamma}\sigma_{y\gamma}} = \frac{8\times10^{10}\times1.4\times10^{16}}{2\pi\times(160\mu m\times160\mu m)} = 6.967\times10^{33}m^{-2}\cdot s^{-1}$$

- The ICS cross section is :

$$\sigma(\kappa) = \frac{2\pi r_e^2}{\kappa}\left[\left(1-\frac{4}{\kappa}-\frac{8}{\kappa^2}\right)\log(1+\kappa)+\frac{1}{2}\left(1-\frac{1}{(1+\kappa)^2}\right)+\frac{8}{\kappa}\right] = 393.5mb$$

- Compton scattering event rate:

$$N = \mathfrak{L}\sigma = 6.967\times10^{33}m^{-2}\cdot s^{-1}\times393.5mb = 2.742\times10^5\ \text{pulse}^{-1}$$

Note that: The laser is 1HZ. 段哲

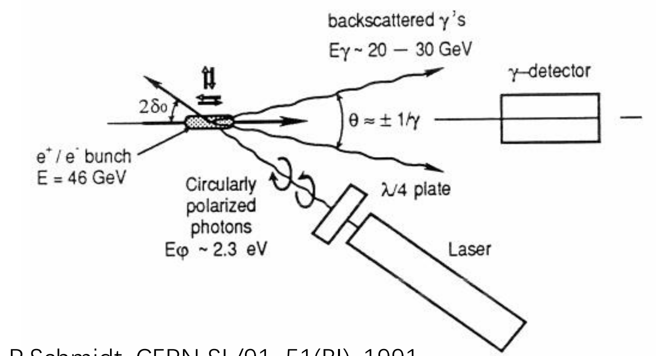
IP： 1 bunch 1 second v $= \frac{3\times10^8 m/s}{100km} = 3000$次

1s内 electron 共有12000(CEPC CDR bunch nember)*3000个束团经过IP点，但是Laser无法匹配那么高的频率，设置 laser 的频率为1Hz, 则 1s内 仅仅发生一次 pulsed laser collider with 1 electron bunch

- ➢ timing system 可以给laser一个合适的trigger，保证laser同指定的一个bunch相互作用, timing system 里面有每个bunch的时间戳
- ➢ 正常情况下，polarization 演化的时间尺度在小时以上，甚至到几十小时，1min内的变化可以忽略。 如果是进行共振退极化实验，可以对一个指定束团，扫描一次depolarizer的频率，即进行一次resonant depolarization run，然后测量一下该束团极化度的情况，主要是看扫描depolarizer频率前后，该束团极化度有没有变化，不关心测量过程中的极化度变化

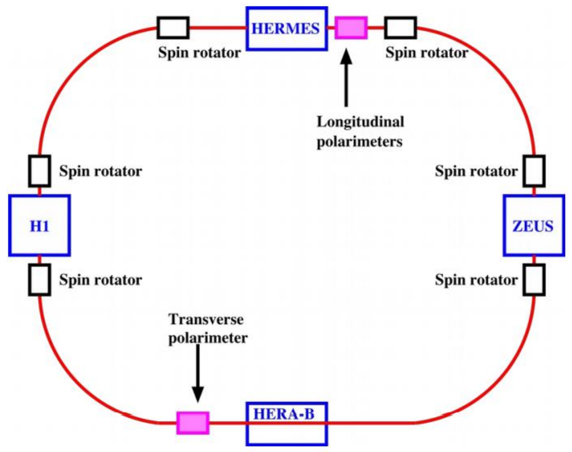
Compton polarimeter

CERN LEP 46GeV



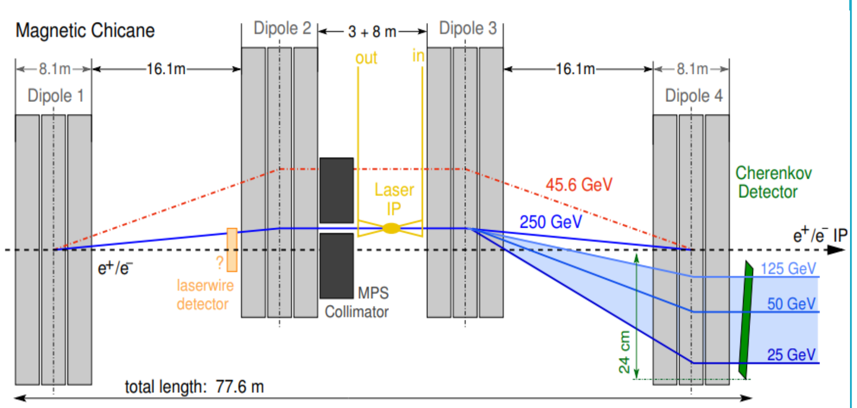
R.Schmidt. CERN SL/91-51(BI), 1991.

HERA 27GeV



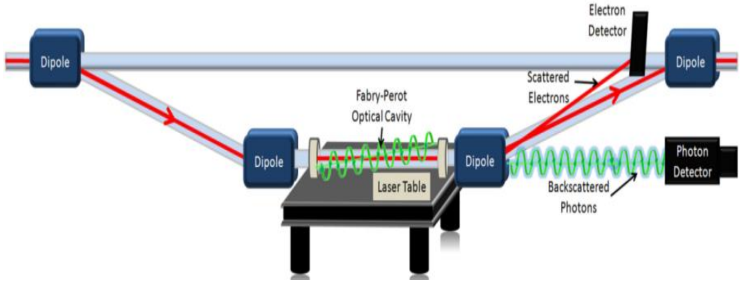
S.Schmitt, HERA polarimeter review

ILC 45.6GeV



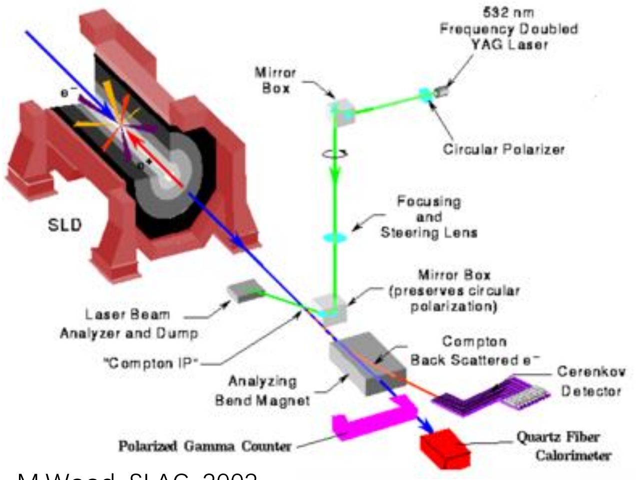
Jenny List, ILC Polarimetry,2020

JLab Hall C 1.16GeV



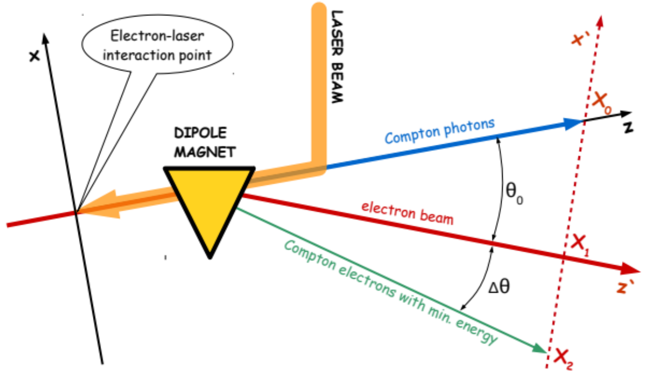
Donald Jones, Hall C Compton Polarimetry, PSTP 2013.

SLD at SLAC 45.6GeV



M.Wood. SLAC, 2003.

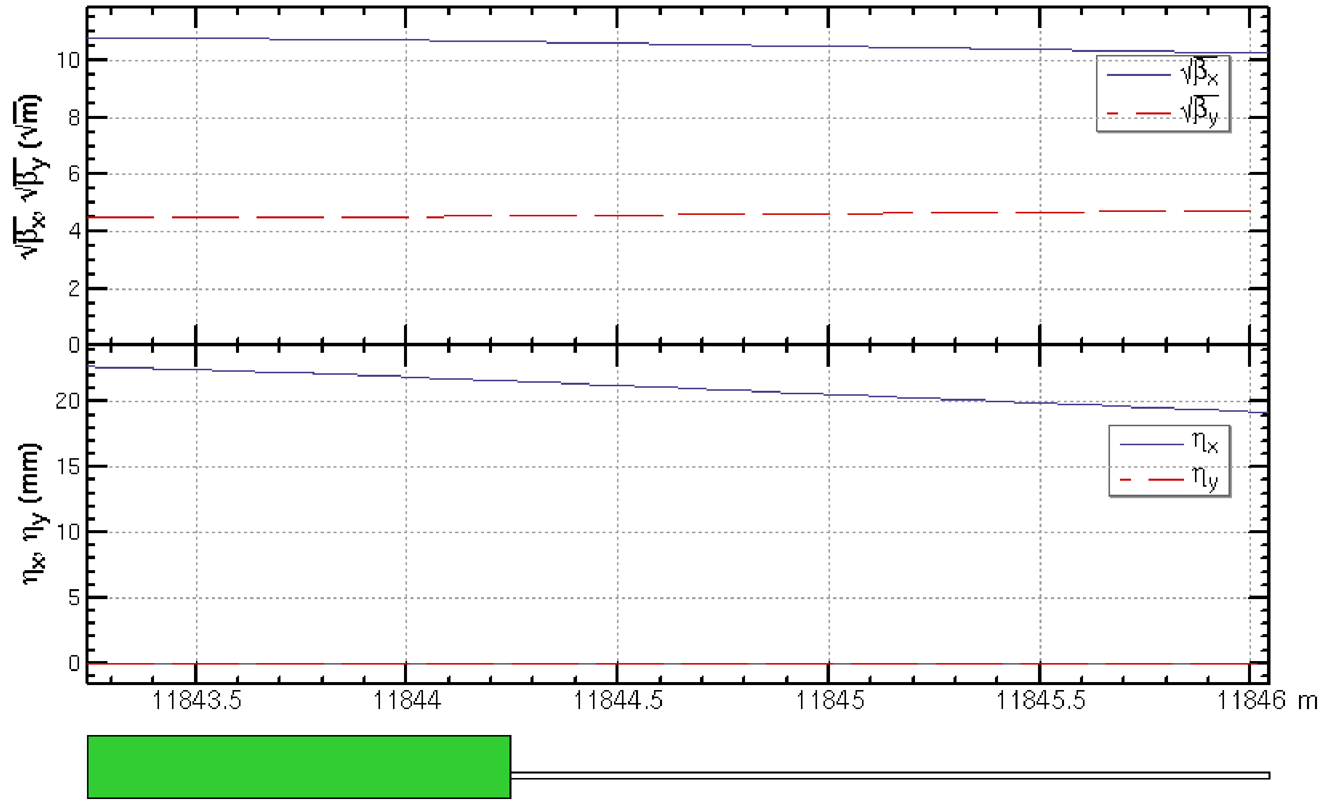
FCC-ee 45.6GeV



Nickolai Muchnoi, 2018

β function

$\beta_x = 121m$
$\beta_y = 25m$



王毅伟

$\Pi(Xe)$

$$\Pi(x) = \frac{\int y \frac{d\sigma_{\perp}}{dxdy} dy}{\int \frac{d\sigma_0}{dxdy} dy}$$

$we\ use\ the\ y = \sqrt{1-x^2} sin\theta, \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\int y \frac{d\sigma_{\perp}}{dxdy} dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{r_e^2}{(1+u)^3\sqrt{1-x^2-y^2}} uy^2 dy = \frac{ur_e^2}{(1+u)^3} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{y^2}{\sqrt{1-x^2-y^2}} dy = \frac{(1-x^2)ur_e^2}{(1+u)^3} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin\theta^2 dy = \frac{\pi(1-x^2)ur_e^2}{2(1+u)^3}$$

$$\int \frac{d\sigma_0}{dxdy} dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{r_e^2}{(1+u)^3\sqrt{1-x^2-y^2}} \left(1+(1+u)^2 - 4\frac{u}{\kappa}(1+u)(1-\frac{u}{\kappa})\right) dy = \frac{\pi r_e^2 \left(1+(1+u)^2 - 4\frac{u}{\kappa}(1+u)(1-\frac{u}{\kappa})\right)}{(1+u)^3}$$

- $\Pi(x) = \frac{(1-x^2)u}{2\left(1+(1+u)^2-4\frac{u}{\kappa}(1+u)(1-\frac{u}{\kappa})\right)}$
- $x = \frac{(X_e-\sigma_x' L_1)-\frac{\kappa}{2}\theta_0 L_2}{\frac{\kappa}{2}\sqrt{(\frac{L_1}{\gamma})^2+(\theta_0 L_2)^2}}$
- $y = \frac{Y_e-\sigma_y' L_1}{\frac{L_1}{\gamma}\frac{\kappa}{2}}$
- $u(Xe) = \frac{2(X_e-\sigma_x' L_1)\theta_0 L_2+\kappa(\frac{L_1}{\gamma})^2}{2((\frac{L_1}{\gamma})^2+(\theta_0 L_2)^2)}$

$$\Pi(Xe) = \frac{\boldsymbol{L_1}}{\boldsymbol{\gamma}}\frac{\boldsymbol{\kappa}}{\boldsymbol{2}} \frac{\left(1-\left(\frac{(X_e-\sigma_x' L_1)-\frac{\kappa}{2}\theta_0 L_2}{\frac{\kappa}{2}\sqrt{(\frac{L_1}{\gamma})^2+(\theta_0 L_2)^2}}\right)^2\right) u(Xe)}{2\left(1+(1+u)^2-4\frac{u}{\kappa}(1+u)(1-\frac{u}{\kappa})\right)}$$

The statistic error

Time[s]	entries	p	error
33.284	9126534	1.01084e-01	2.29169e-03
49.928	13690340	1.02237e-01	1.87336e-03
66.566	18252308	1.01303e-01	1.62269e-03
83.210	22816114	1.01953e-01	1.45199e-03

Polarization = 10%

MATLAB fit result

		Original	0.094	0.096	0.098	0.099	0.100	0.102	0.104
u^+	fit value	0.094 (0.09061, 0.09739)	0.096 (0.09262, 0.09938)	0.098 (0.09462, 0.1014)	0.099 (0.09562, 0.1024)	0.1 (0.09662, 0.1034)	0.102 (0.09861, 0.1054)	0.104 (0.1006, 0.1074)
	SSE_{min}	1.771422866716164e-09	1.763747313661280e-09	1.760820089152675e-09	1.761137100103227e-09	1.762641193190349e-09	1.769210625774302e-09	1.780528386904534e-09
u^-	fit value	0.094 (0.09058, 0.09742)	0.096 (0.09259, 0.09941)	0.098 (0.0946, 0.1014)	0.099 (0.0956, 0.1024)	0.1 (0.0966, 0.1034)	0.102 (0.09859, 0.1054)	0.104 (0.1006, 0.1074)
	SSE_{min}	1.777505906254044e-09	1.768784347898592e-09	1.764762141769681e-09	1.764513295790178e-09	1.765439287867310e-09	1.770815786191480e-09	1.780891636742190e-09
$\bar{u}$	fit value	0.094 (0.0906, 0.0974)	0.096 (0.0926, 0.0994)	0.098 (0.09461, 0.1014)	0.099 (0.09561, 0.1024)	0.1 (0.09661, 0.1034)	0.102 (0.0986, 0.1054)	0.104 (0.1006, 0.1074)
	SSE_{min}	1.774281698874271e-09	1.766089452127655e-09	1.762621470597824e-09	1.762659079289202e-09	1.763877754284777e-09	1.769858303188516e-09	1.780563117309038e-09

- Coefficients (with 95% confidence bounds)
- $SSE = \sum w_i (y_i - \hat{y}_i)^2$