An Odyssey in NLO CGC

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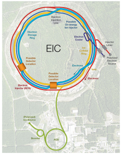
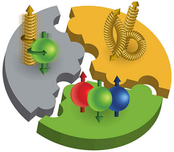
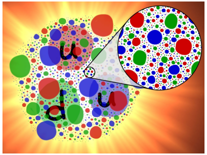
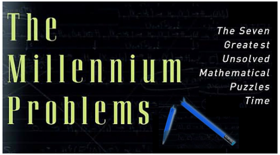
Workshop on Hadron Structure at High-Energy, High-Luminosity Facilities

Ultimate Questions and Challenges in QCD

To understand our physical world, we have to understand QCD!





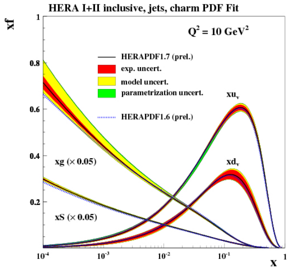
Three pillars of EIC Physics:

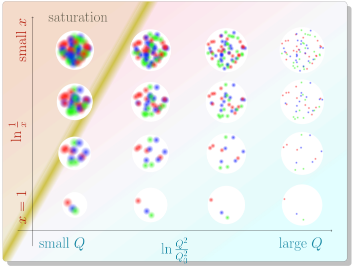
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

EICs: keys to unlocking these mysteries! Many opportunities will be in front of us!

Saturation Physics (Color Glass Condensate)

QCD matter at extremely high gluon density





- Gluon density grows rapidly as x gets small.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK/JIMWLK) $\Rightarrow$ ultra-dense gluonic matter
- Multiple Scattering (MV model) $+$ Small-x (high energy) evolution

A Tale of Two Gluon Distributions

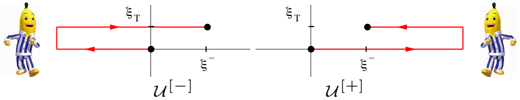
Two gauge invariant TMD operator def. *[Bomhof, Mulders and Pijlman, 06]* Link *[Dominguez, Marquet, Xiao and Yuan, 11]* Link

I. Weizsäcker Williams distribution: conventional density

$$xG_{\rm WW}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$

II. Color Dipole gluon distributions:

$$xG_{\rm DP}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$



- Modified Universality for Gluon Distributions:

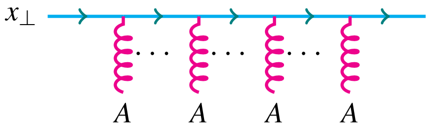
	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
$xG_{\rm WW}$	×	×	✓	×	✓
$xG_{\rm DP}$	✓	✓	×	✓	✓

✓ ⇒ Apppear. × ⇒ Do Not Appear.

Wilson Lines in Color Glass Condensate Formalism

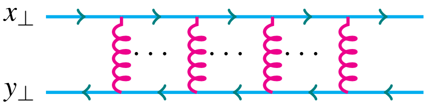
Wilson line ⇒ multiple scatterings between fast moving quark and target dense gluons.

$$U(x_\perp) = \mathcal{P}\exp\left(-ig\int dz^+ A^-(x_\perp, z^+)\right)$$



The Wilson loop (color dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$



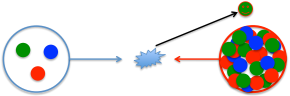
- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle.$$

Forward hadron production in pA collisions

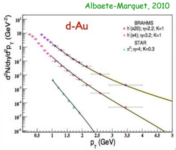
[Dumitru, Jalilian-Marian, 02] Inclusive forward hadron production in pA collisions

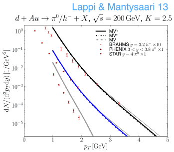
$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}}e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}}e^{-y} \ll 1$ gluon

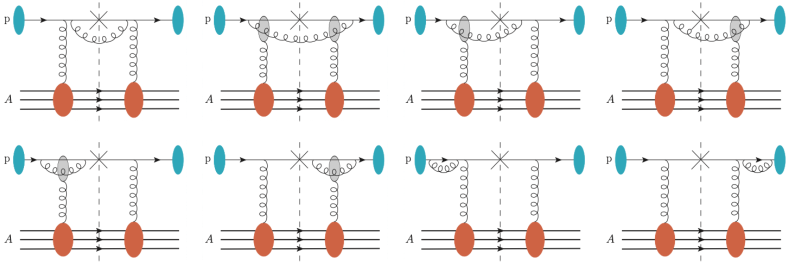




- Need NLO correction! IR cutoff: *[Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]*; Full NLO *[Chirilli, BX and Yuan, 12]*

NLO diagrams in the $q \to q$ channel

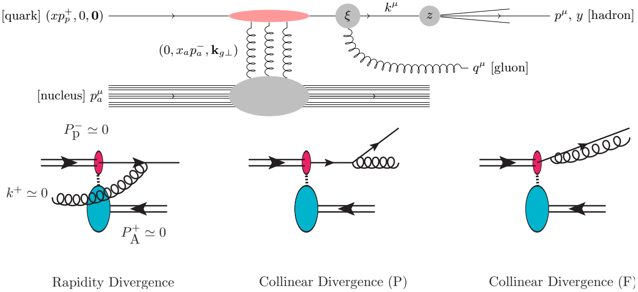
[Chirilli, BX and Yuan, 12]



- Take into account real (top) and virtual (bottom) diagrams together!
- Multiple interactions inside the grey blobs!
- Integrate over gluon phase space ⇒Divergences!.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process *[Chirilli, BX and Yuan, 12]*



- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to the target nucleus; $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

Hard Factor of the $q \to q$ channel

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right.$$
$$\left.+\int\frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}$$

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right)-3C_F\delta(1-\xi)\ln\frac{c_0^2}{r_\perp^2k_\perp^2}e^{-ik_\perp\cdot r_\perp}$$
$$-(2C_F-N_c)e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{\left(1+\xi^2\right)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right.$$
$$\left.-\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\},$$

where $$\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\right]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}.$$

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

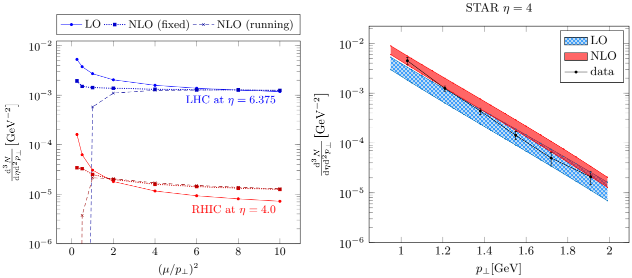
$$d\sigma = \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by *[Albacete, Dumitru, Fujii, Nara, 12]*
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution *[Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]*. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*

Numerical implementation of the NLO result

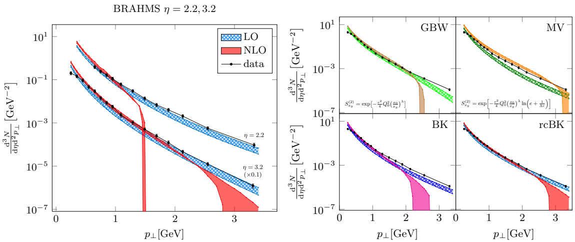
Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*



- Agree with data for $p_T < Q_s(y)$, and reduced scale dependence, no K factor.
- For more forward rapidity, the agreement gets better and better.

Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). *[Stasto, Xiao, Zaslavsky, 13]*



- The abrupt drop at NLO when $p_T > Q_s$ was surprising and puzzling.
- Fixed order calculation in field theories is not guaranteed to be positive.

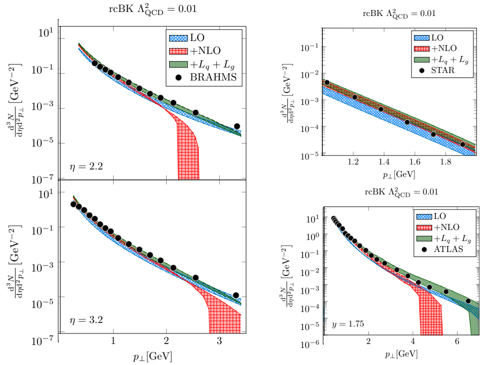
Extending the applicability of CGC calculation

Some thoughts:

- Towards a more complete framework. *[Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20;]*
- To solve this problem, needs to find a solution within our current factorization to extend the applicability of CGC.
- More than just negativity problem. Need to work reliably (describe data) from RHIC to LHC, low p_T to high p_T.
- Additional consideration: solution needs to be easy to be implemented numerically due to limited computing resources.
- A lot of logs occur in pQCD loop-calculations: DGLAP, small-x, threshold, Sudakov.
- Breakdown of pQCD expansion often happens due to the appearance of logs in certain phase spaces.

NLO hadron productions in pA collisions: An Odyssey

[Watanabe, Xiao, Yuan, Zaslavsky, 15] Rapidity subtraction!



- Including the kinematical constraints. (Originally assume the limit $s \to \infty$)

$$\int_0^{1-\frac{q_\perp^2}{x_p s}} \frac{d\xi}{1-\xi} = \underbrace{\ln\frac{1}{x_g}}_{1-\xi \subset \left[\frac{q_\perp^2}{x_p s}, \frac{q_\perp^2}{k_\perp^2}\right]} + \underbrace{\ln\frac{k_\perp^2}{q_\perp^2}}_{\text{missed earlier}} \Rightarrow$$

New terms: $L_q + L_g$ from $q_\perp^2 \leq (1-\xi)k_\perp^2$.

Related to threshold double logs!

- Negative when $x_p \to 1$ and $p_T \gg Q_s$!
- Approach threshold at high $k_\perp$. Threshold resummation (Sudakov)!

Threshold resummation in the saturation formalism

Threshold resummation: Sudakov soft gluon part and plus-function part.

- $\ln(1-x_p)$ and $\ln k_\perp^2/\Lambda^2$ in the large $k_\perp$ region ($k_\perp \gg Q_s$) near threshold

$$\int_x^1 \frac{d\xi}{(1-\xi)_+} f(\xi) = \int_x^1 d\xi \frac{f(\xi)-f(1)}{1-\xi} + f(1)\ln(1-x)$$

- Remarkable similarities between the threshold resummation in CGC formalism (fixed k_T) and that in SCET*[Becher, Neubert, 06]*.
- The forward threshold jet function $\Delta(\mu^2, \Lambda^2, z)$ satisfies

$$\begin{aligned}\frac{d\Delta(\mu^2,\Lambda^2,z)}{d\ln\mu} &= -\frac{2\alpha_s N_c}{\pi}\left[\ln z + \beta_0\right]\Delta(\mu^2,\Lambda^2,z) \\ &\quad + \frac{2\alpha_s N_c}{\pi}\int_0^z dz' \frac{\Delta(\mu^2,\Lambda^2,z)-\Delta(\mu^2,\Lambda^2,z')}{z-z'},\end{aligned}$$

Solution: $$\Delta(\mu^2,\Lambda^2,z=\ln\frac{x}{\tau}) = \frac{e^{(\beta_0-\gamma_E)\gamma_{\mu,\Lambda}}}{\Gamma[\gamma_{\mu,\Lambda}]} z^{\gamma_{\mu,\Lambda}-1}.$$

Numerical challenges

[Watanabe, Xiao, Yuan, Zaslavsky, 15; Shi, Wang, Wei, Xiao, in preparation]

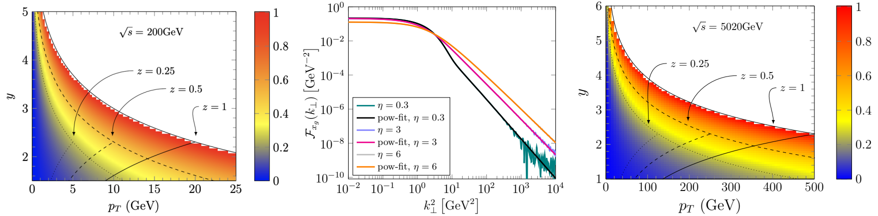
- Numerical integration (8-d in total) is notoriously hard in $x_\perp$ space. Go to $k_\perp$ space.
- A couple of identities in Fouier transformations

$$\int \frac{d^2x_\perp}{(2\pi)^2} S(x_\perp) \ln \frac{c_0^2}{x_\perp^2\mu^2} e^{-ik_\perp \cdot x_\perp} = \int \frac{d^2l_\perp}{\pi l_\perp^2} \left[F(k_\perp + l_\perp) - J_0(\frac{c_0}{\mu} l_\perp) F(k_\perp) \right]$$
$$= \frac{1}{\pi} \int \frac{d^2l_\perp}{(l_\perp - k_\perp)^2} \left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2 + (l_\perp - k_\perp)^2} F(k_\perp) \right] + F(k_\perp) \ln \frac{\Lambda^2}{\mu^2}.$$

- Introduce a semi-hard scale $\Lambda^2 = \max[(1-\xi)k_\perp^2, Q_s^2]$ which is analogous to the intermediate scale μ_i^2 in SCET *[Becher, Neubert, 06]*. (Sudakov soft part!)
- μ^2 and Λ^2 dependences cancel order by order in terms of α_s^n! At fixed order, need to choose the "natural" values for them.

Applicability of CGC and Initial Condition

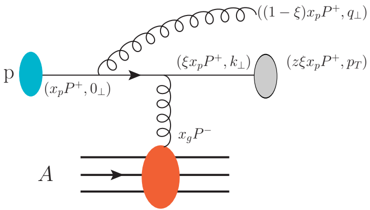
Kinematics: $\tau = \frac{p_\perp e^y}{\sqrt{s}} \leq 1$ and $x_g \equiv \frac{p_\perp e^{-y}}{z\sqrt{s}} < 10^{-2}$

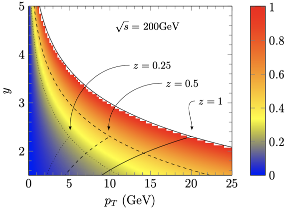


- Small-x gluon: *[Albacete, Armesto, Milhano, Quiroga-Arias and Salgado, 11]* Link
- Initial condition set by data $x_g \equiv \frac{p_\perp e^{-y}}{z\sqrt{s}} \leq 10^{-2}$ + running coupling BK evolution.
- Kinematic constraint $\tau/z = \frac{p_T e^y}{z\sqrt{s}} \leq 1$ and CGC constraint $x_g \equiv \frac{p_T e^{-y}}{z\sqrt{s}} \leq 10^{-2}$.
- Applicability of CGC: rapidity y sufficiently large and $p_T = k_\perp z$ not too large.

Gluon Radiation and Phase Space

At threshold: radiated gluon has to be soft! $\tau = \frac{p_T e^{y}}{\sqrt{s}} = x_p \xi z \leq 1$ and $x_g \equiv \frac{p_T e^{-y}}{z\sqrt{s}} < 10^{-2}$





- Introduce an auxiliary semi-hard scale $\Lambda^2 \sim (1-\xi)k_\perp^2 \sim (1-\tau)p_T^2$.
- Saddle point approximation yields the same result for fixed coupling.
- For running coupling, $\Lambda^2 = \Lambda_{QCD}^2 \left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}$. Akin to CSS and Catani *et al.*

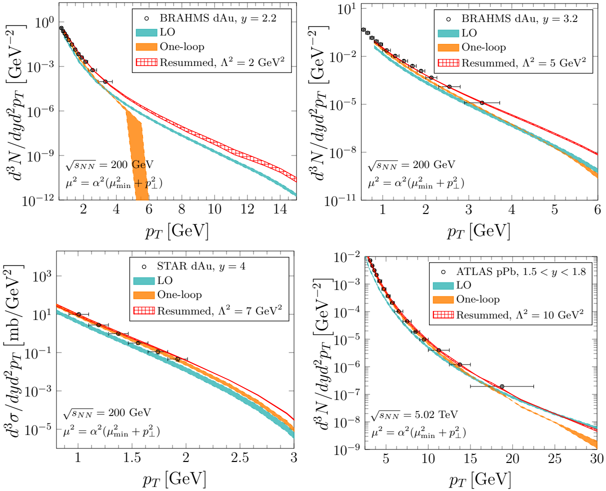
Preliminary Results

[Xiao, Yuan, 18; Shi, Wang, Wei, Xiao, in preparation]

$$\begin{aligned}
d\sigma &= \int xf_a(x,\mu) \otimes D_a(z,\mu) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \otimes \Delta(\mu,\Lambda) \otimes S_{\text{Sud}}(\mu,\Lambda) \\
&\quad + \frac{\alpha_s}{2\pi} \int xf_a(x,\ \mu) \otimes D_b(z,\ \mu) \otimes \mathcal{F}^{x_g}_{(N)ab} \otimes \mathcal{H}^{(1)}_{ab}(\mu,\Lambda), \\
&= \int xf_a(x,\ \Lambda) \otimes D_a(z,\ \Lambda) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \otimes S_{\text{Sud}}(\mu,\Lambda) \qquad \leftarrow \mu = \mu_b \ \text{TMD} \\
&\quad + \frac{\alpha_s}{2\pi} \int xf_a(x,\ \mu) \otimes D_b(z,\ \mu) \otimes \mathcal{F}^{x_g}_{(N)ab} \otimes \mathcal{H}^{(1)}_{ab}(\mu,\Lambda).
\end{aligned}$$

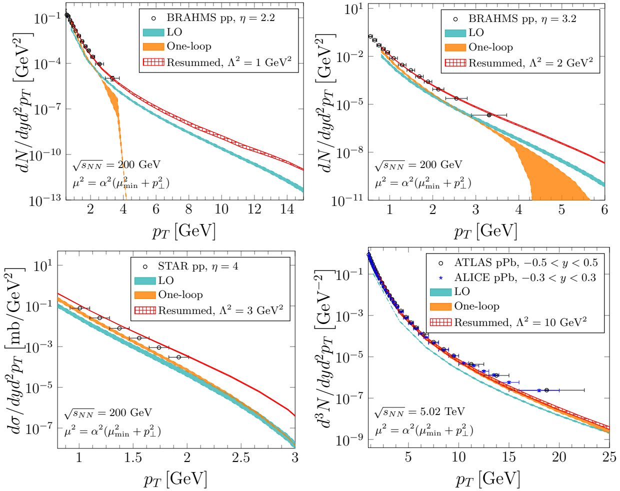
- Natural choice of Λ^2: Competition between saturation and Sudakov $\Lambda \sim c_0/r_\perp$.
- Two implementation methods give similar numerical results.
- $\Delta(\mu,\Lambda)$ and $S_{\text{Sud}}(\mu,\Lambda)$ satisfy collinear and Sudakov (soft) RGEs.
- $\Delta(\mu,\Lambda)$ represents backwards DGLAP evolution. $\Delta(\mu,\mu) = 1$

Preliminary Results for pA spectra



- Set $\mu^2 = \alpha^2(\mu^2_{\min} + p_T^2)$ with $\alpha = 2 \to 4$
- $\mu \sim Q \geq 2k_\perp$ ($\alpha > 2$) in the high p_T region. $2 \to 2$ hard scattering.
- Resummation increases σ at the threshold.
- Extraordinary agreement with data across many orders of magnitudes for different energies and p_T ranges!

Preliminary Results for middle rapidity pA and pp spectra



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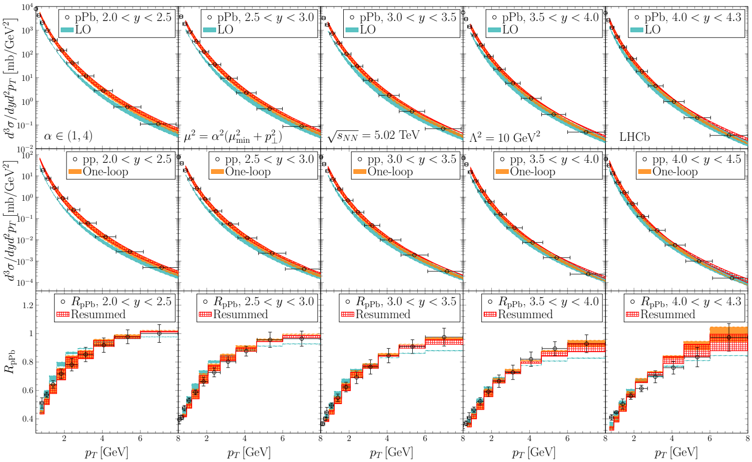
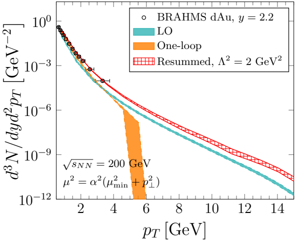
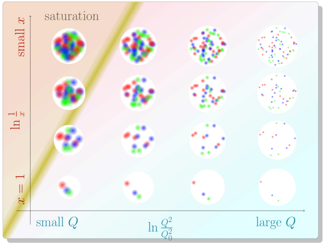
Preliminary Results

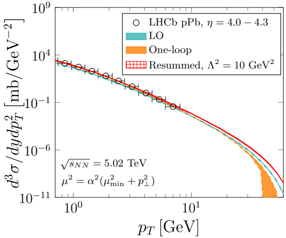


- New LHCb data: LHCb-PAPER-2021-015
- Links to preliminary data: DIS2021
- Proceedings EPS-HEP21 and Montpellier21
- $\mu = (1 \to 4)p_T$ with proper choice of Λ^2

Summary







- Odyssey in NLO hadron productions in pA collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC.
- Extension to larger $k_\perp$ region and QCD threshold resummation.
  Low-$k_\perp$ $\Leftrightarrow$ saturation; High-$k_\perp$ $\Leftrightarrow$ pQCD + Resummation.
- Gluon saturation could be the next discovery at the LHC and future EIC.