Structure of low-lying excitations of the nucleon and Delta

Langtian Liu liulangtian@nju.edu.cn

In collaboration with Jorge Segovia, Chen Chen and Craig D. Roberts

Nanjing University

October 26, 2021

Workshop on Hadron Structure at High-Energy, High-Luminosity Facilities, Nanjing, China, 2021

Introduction

- ♣ The excited states can tell us more details about the interactions of a theory.

Introduction

- ♣ The excited states can tell us more details about the interactions of a theory.
- ♣ The parity partner can be used to test the effects of dynamical chiral symmetry breaking — emergence of hadron mass.

Introduction

- ♣ The excited states can tell us more details about the interactions of a theory.
- ♣ The parity partner can be used to test the effects of dynamical chiral symmetry breaking — emergence of hadron mass.
- ♣ The presence of diquark correlations owns largely to the mechanisms responsible for the emergence of hadron mass.

Introduction

- ♣ The excited states can tell us more details about the interactions of a theory.
- ♣ The parity partner can be used to test the effects of dynamical chiral symmetry breaking — emergence of hadron mass.
- ♣ The presence of diquark correlations owns largely to the mechanisms responsible for the emergence of hadron mass.
- ♣ Unlike the isoscalar-scalar and isovector-pseudovector diquarks in the nucleon case, for the parity partner, one should also include the isoscalar-pseudoscalar, isoscalar-vector and isovector vector diquarks.

Introduction

- ♣ The excited states can tell us more details about the interactions of a theory.
- ♣ The parity partner can be used to test the effects of dynamical chiral symmetry breaking — emergence of hadron mass.
- ♣ The presence of diquark correlations owns largely to the mechanisms responsible for the emergence of hadron mass.
- ♣ Unlike the isoscalar-scalar and isovector-pseudovector diquarks in the nucleon case, for the parity partner, one should also include the isoscalar-pseudoscalar, isoscalar-vector and isovector vector diquarks.
- ♣ We firstly solve the 16 low-lying excitations of nucleon and Delta ($N(1/2^{+-})$, $N(3/2^{+-})$, $\Delta(3/2^{+-})$ and $\Delta(1/2^{+-})$) with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation and explore their internal structures.

Faddeeev equations

Faddeev equation for the baryons:

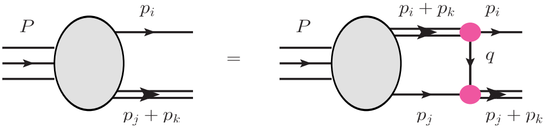
$$\lambda(P^2)\Phi^a(p;P) = \int \frac{d^4k}{(2\pi)^4} K^{ab}(p,k;P)\Psi^b(k;P)\,,$$

$\Phi^a(p,P)$: quark-diquark-baryon vertex; $\Psi^b(k,P)$: quark-diquark wave function

$$\Psi^b(k;P) = S(k+\eta P)D^{bc}((1-\eta)P-k)\Phi^c(k;P)\,,$$

$\lambda(P^2) = 1$ correspond to the mass shell. The quark exchange kernel :

$$K^{ab}(p,k;P) = t^a\Gamma^a(p_1,p_1^d)S^T(q)t^b\bar{\Gamma}^b(p_2,p_2^d)$$



Diquark and orbital angular momentum content of the baryons

☆ Diquark content

- Mass fraction from each diquark.

diquark 1, diquark 1 + diquark 2, $\cdots$

- Amplitude fraction from each diquark.

$$\mathcal{N}_i = \int \frac{d^4 p}{(2\pi)^4} |A_i(p^2, p\cdot P)|^2 , \quad W = \sum_{i=1}^{n} \mathcal{N}_i , \quad Q_t = W^{-1} \sum_{i\in t} \mathcal{N}_i ,$$

t is the diquark channel and n is the total scalar wave functions.

☆ Orbital angular momentum content

- Mass fraction from each orbital angular momentum.

L1, L1 + L2, $\cdots$

- Wave function fraction from each orbital angular momentum.

$$\mathcal{L}_i = \int \frac{d^4 p}{(2\pi)^4} |F_i(p^2, p\cdot P)|^2 , \quad W = \sum_{i=1}^{n} \mathcal{L}_i , \quad Q_t = W^{-1} \sum_{i\in t} \mathcal{L}_i ,$$

t is the orbital angular momentum channel and n is the total scalar wave functions.

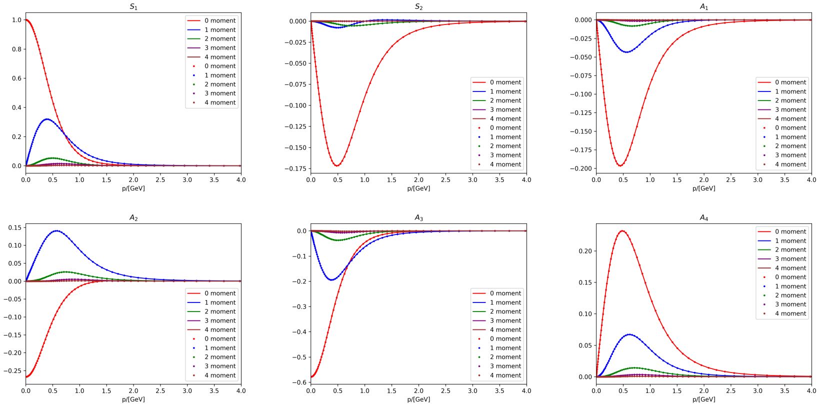
$(I = 1/2, J^P = 1/2^{+-})$ baryons check

- The Faddeev solution of nucleon have been tested tens of years.
- We take it as the benchmark of our program.

$(I = 1/2, J^P = 1/2^{+-})$ baryons check

- The Faddeev solution of nucleon have been tested tens of years.
- We take it as the benchmark of our program.

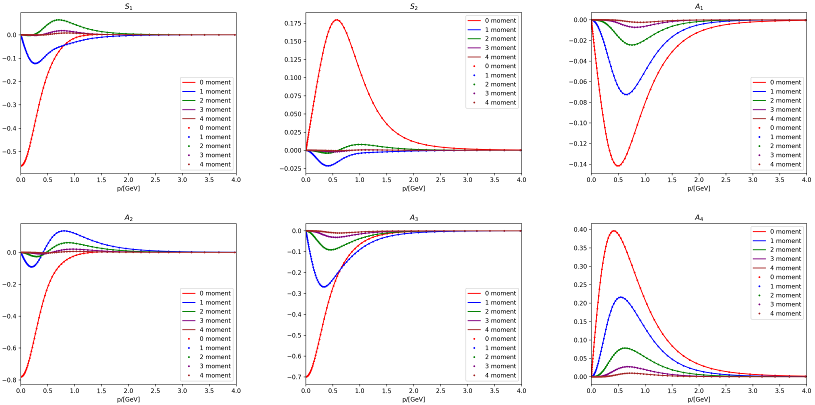
$(I = 1/2, J^P = 1/2^{+})$ Solid lines are my results and dotted lines are adapted from Chen et al. Phys. Rev., D97, 034016 .



They are exactly the same.

$(I = 1/2, J^P = 1/2^{+-})$ baryons check

$(I = 1/2, J^P = 1/2^{-})$ Solid lines are my results and dotted lines are adapted from Chen et al. Phys. Rev., D97, 034016 .



They are exactly the same.

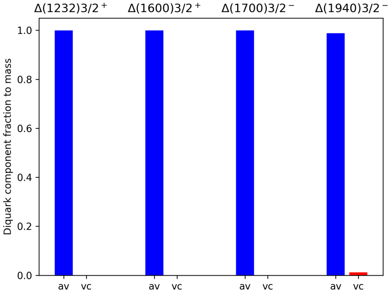
Structure of ($I = 3/2, J^P = 3/2^{+-}$) baryons

The ($I = 3/2, J^P = 3/2^{+-}$) baryons consist of 2 type of diquarks: isovector-pseudovector (av) and isovector vector (vc) diquarks.

Structure of $(I = 3/2, J^P = 3/2^{+-})$ baryons

The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 2 type of diquarks: isovector-pseudovector (av) and isovector vector (vc) diquarks.

Diquark fraction to mass

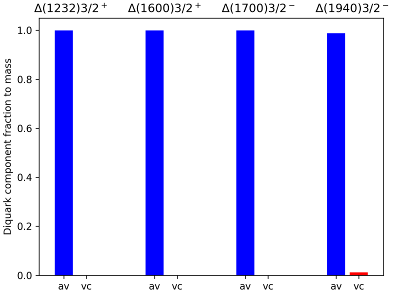


The isovector-pseudovector diquark overwhelmingly dominates.

Structure of $(I = 3/2, J^P = 3/2^{+-})$ baryons

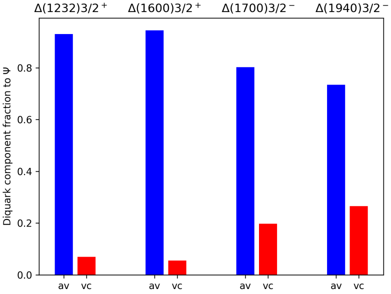
The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 2 type of diquarks: isovector-pseudovector (av) and isovector vector (vc) diquarks.

Diquark fraction to mass



The isovector-pseudovector diquark overwhelmingly dominates.

Diquark fraction to amplitudes



The isovector-pseudovector diquark dominates, and the isovector-vector diquark contributes larger in the parity partners.

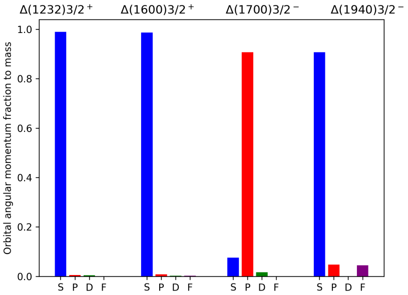
Structure of $(I = 3/2, J^P = 3/2^{+-})$ baryons

The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Structure of ($I = 3/2, J^P = 3/2^{+-}$) baryons

The ($I = 3/2, J^P = 3/2^{+-}$) baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Orbital angular momentum fraction to mass

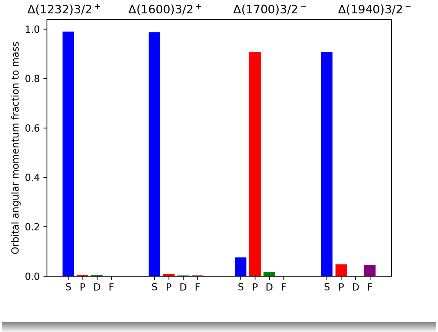


The S wave dominates.

Structure of $(I = 3/2, J^P = 3/2^{+-})$ baryons

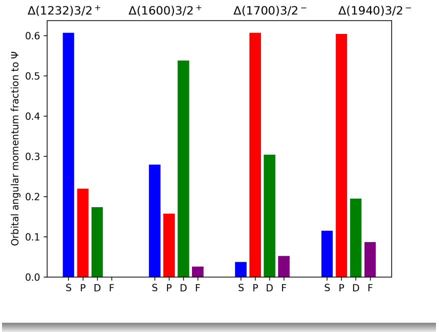
The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Orbital angular momentum fraction to mass



The S wave dominates.

Orbital angular momentum fraction to wave function

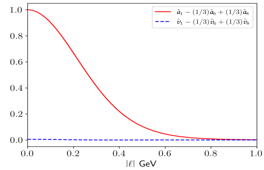


The S wave dominate ground state, and P wave dominate parity partners.

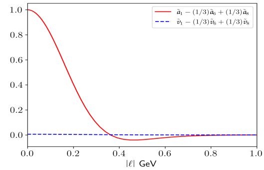
The structures of wave function do not show in the structure of mass.

Comparison of S wave functions

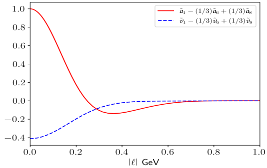
0 Chebyshev moment of S wave $(\Delta(1232))$



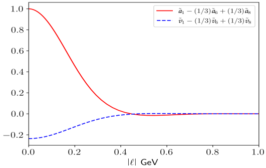
0 Chebyshev moment of S wave $(\Delta(1600))$



0 Chebyshev moment of S wave $(\Delta(1700))$



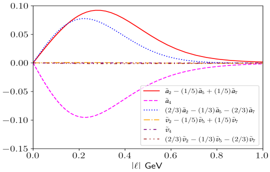
0 Chebyshev moment of S wave $(\Delta(1940))$



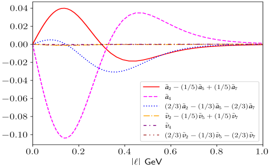
- The excitations states have zero.
- The wave function from vector diquark is getting larger for negative parity states.

Comparison of P wave functions

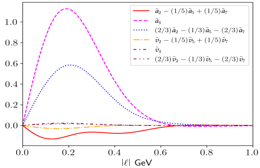
0 Chebyshev moment of P wave ($\Delta(1232)$)



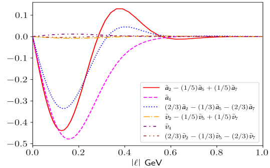
0 Chebyshev moment of P wave ($\Delta(1600)$)



0 Chebyshev moment of P wave ($\Delta(1700)$)



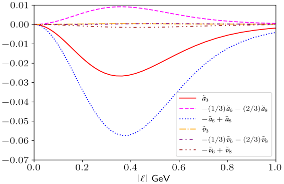
0 Chebyshev moment of P wave ($\Delta(1940)$)



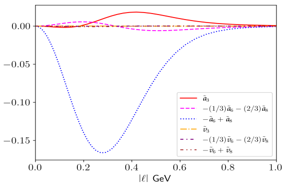
- The higher partial wave is getting important.
- The wave function from vector diquark is getting larger for negative parity states.

Comparison of D wave functions

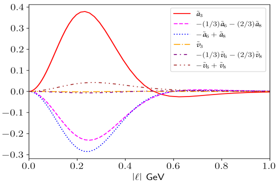
0 Chebyshev moment of D wave ($\Delta(1232)$)



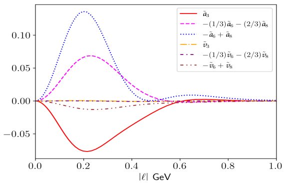
0 Chebyshev moment of D wave ($\Delta(1600)$)



0 Chebyshev moment of D wave ($\Delta(1700)$)

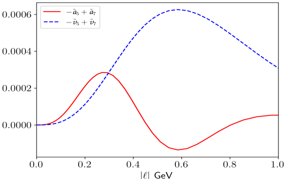


0 Chebyshev moment of D wave ($\Delta(1940)$)

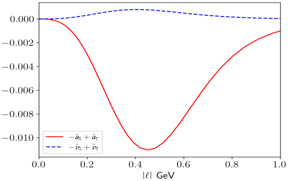


Comparison of F wave functions

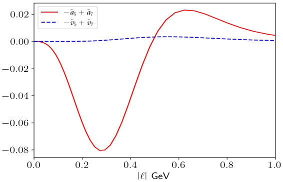
0 Chebyshev moment of F wave ($\Delta(1232)$)



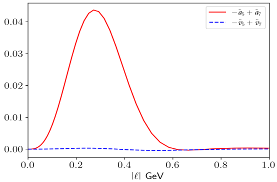
0 Chebyshev moment of F wave ($\Delta(1600)$)



0 Chebyshev moment of F wave ($\Delta(1700)$)



0 Chebyshev moment of F wave ($\Delta(1940)$)



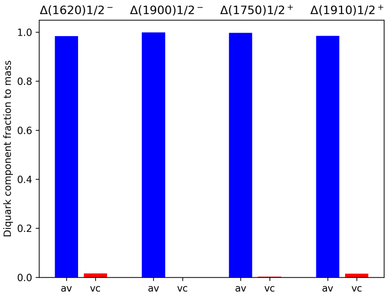
Structure of $I = 3/2$, $J^P = 1/2^{+-}$ baryons

The $I = 3/2, J^P = 1/2^{+-}$ baryons consist of isovector-pseudovector (av) and isovector vector (vc) diquarks.

Structure of $I = 3/2$, $J^P = 1/2^{+-}$ baryons

The $I = 3/2, J^P = 1/2^{+-}$ baryons consist of isovector-pseudovector (av) and isovector vector (vc) diquarks.

Diquark fraction to mass

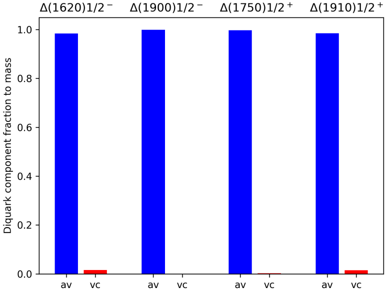


The isovector-pseudovector diquark overwhelmingly dominates.

Structure of $I = 3/2, J^P = 1/2^{+-}$ baryons

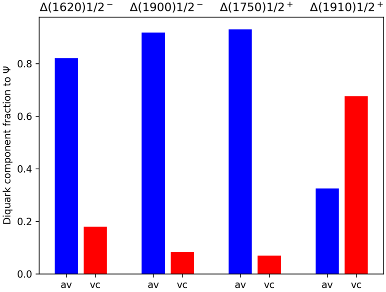
The $I = 3/2, J^P = 1/2^{+-}$ baryons consist of isovector-pseudovector (av) and isovector vector (vc) diquarks.

Diquark fraction to mass



The isovector-pseudovector diquark overwhelmingly dominates.

Diquark fraction to amplitude



The isovector-pseudovector diquark dominates in the first 3 Δ states, but the isovector-vector diquark contributes larger in the $\Delta(1910)$.

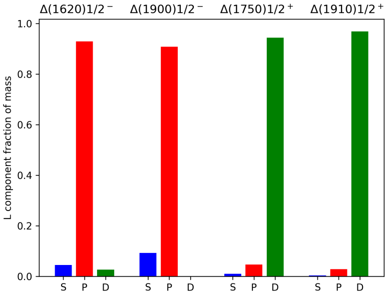
Structure of $I = 3/2$, $J^P = 1/2^{+-}$ baryons

The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 3 type of orbital angular momentum: S, P and D.

Structure of $I = 3/2$, $J^P = 1/2^{+-}$ baryons

The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 3 type of orbital angular momentum: S, P and D.

Orbital angular momentum fraction to mass

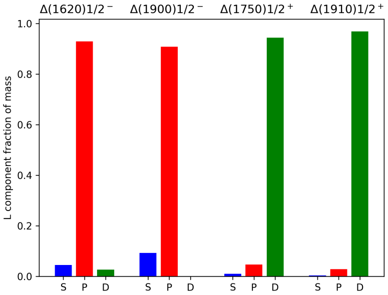


The P wave dominates for minus parity states and D wave dominates for positive parity states.

Structure of $I = 3/2,\ J^P = 1/2^{+-}$ baryons

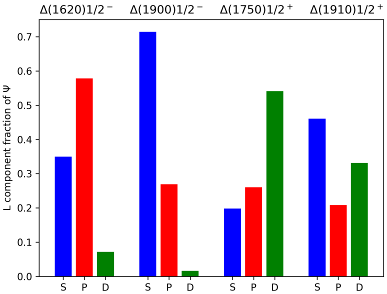
The $(I = 3/2, J^P = 3/2^{+-})$ baryons consist of 3 type of orbital angular momentum: S, P and D.

Orbital angular momentum fraction to mass



The P wave dominates for minus parity states and D wave dominates for positive parity states.

Orbital angular momentum fraction to wave function

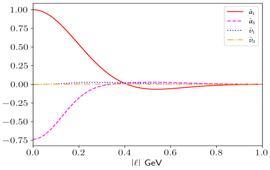


S,P dominate the negative parity states and for positive parity states, they are mixed.

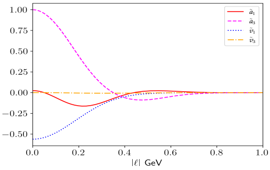
The wave function structure is more complex than the mass structure.

Comparison of S wave functions

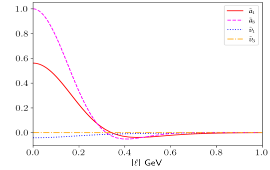
0 Chebyshev moment of S wave ($\Delta(1750)$)



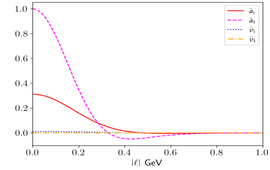
0 Chebyshev moment of S wave ($\Delta(1910)$)



0 Chebyshev moment of S wave ($\Delta(1620)$)



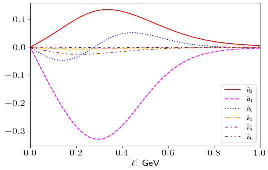
0 Chebyshev moment of S wave ($\Delta(1900)$)



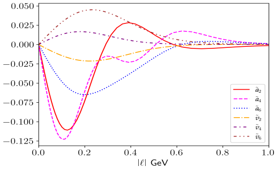
- They all have zero.
- The wave function from vector diquark is getting larger for $\Delta(1910)$.

Comparison of P wave functions

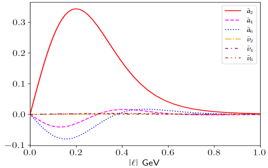
0 Chebyshev moment of P wave ($\Delta(1750)$)



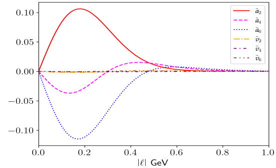
0 Chebyshev moment of P wave ($\Delta(1910)$)



0 Chebyshev moment of P wave ($\Delta(1620)$)



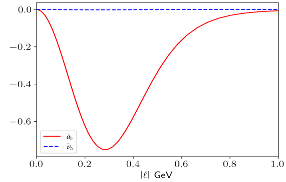
0 Chebyshev moment of P wave ($\Delta(1900)$)



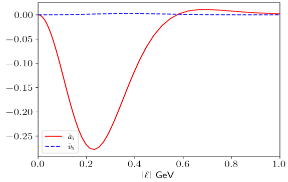
- The wave function from vector diquark is getting larger for excited states.
- The higher partial wave is getting important for them.

Comparison of D wave functions

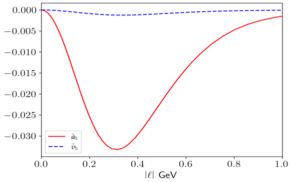
0 Chebyshev moment of D wave ($\Delta(1750)$)



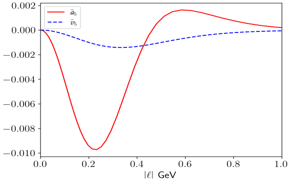
0 Chebyshev moment of D wave ($\Delta(1910)$)



0 Chebyshev moment of D wave ($\Delta(1620)$)



0 Chebyshev moment of D wave ($\Delta(1900)$)



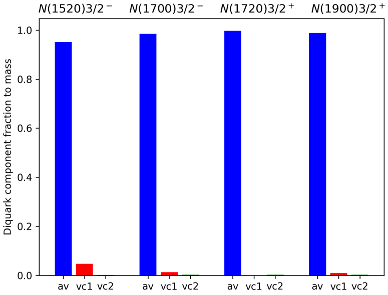
Structure of $(I = 1/2, J^P = 3/2^{+-})$ baryons

The $I = 1/2, J^P = 3/2^{+-}$ baryons consist of isovector-pseudovector (av), isoscalar-vector (vc1) and isovector-vector (vc2) diquarks.

Structure of ($I = 1/2, J^P = 3/2^{+-}$) baryons

The $I = 1/2, J^P = 3/2^{+-}$ baryons consist of isovector-pseudovector (av), isoscalar-vector (vc1) and isovector-vector (vc2) diquarks.

Diquark fraction to mass

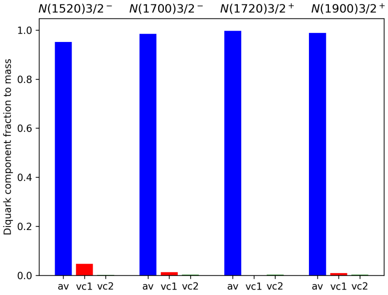


The isovector-pseudovector diquark overwhelmingly dominates.

Structure of ($I = 1/2, J^P = 3/2^{+-}$) baryons

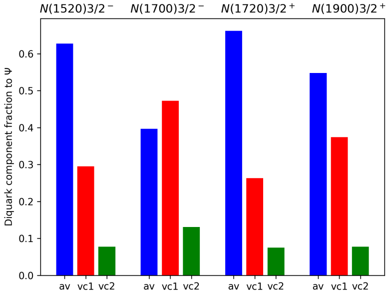
The $I = 1/2, J^P = 3/2^{+-}$ baryons consist of isovector-pseudovector (av), isoscalar-vector (vc1) and isovector-vector (vc2) diquarks.

Diquark fraction to mass



The isovector-pseudovector diquark overwhelmingly dominates.

Diquark fraction to amplitude



The pseudovector and vector diquarks are mixed.

Although the vector diquark is umimportant in mass structure, it is important in amplitude.

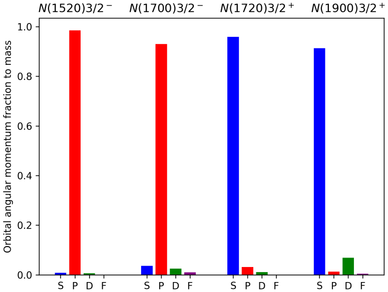
Structure of $I = 1/2, J^P = 3/2^{+-}$ baryons

The $(I = 1/2, J^P = 3/2^{+-})$ baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Structure of $I = 1/2$, $J^P = 3/2^{+-}$ baryons

The ($I = 1/2, J^P = 3/2^{+-}$) baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Orbital angular momentum fraction to mass

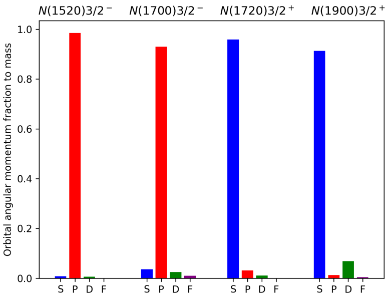


The P wave dominates for minus parity states and S wave dominates for positive parity states.

Structure of $I = 1/2, J^P = 3/2^{+-}$ baryons

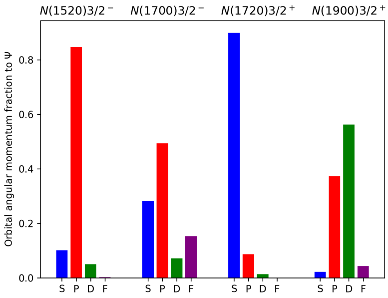
The $(I = 1/2, J^P = 3/2^{+-})$ baryons consist of 4 type of orbital angular momentum: S, P, D and F.

Orbital angular momentum fraction to mass



The P wave dominates for minus parity states and S wave dominates for positive parity states.

Orbital angular momentum fraction to wave function

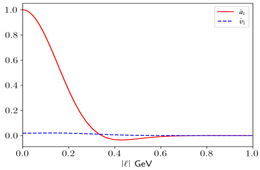


P wave dominates $N(1520)$, S wave dominates $N(1720)$. For $N(1700)$ SP mix, For $N(1900)$, PD mix.

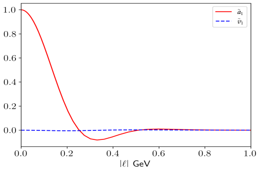
The dominance of $N(1900)$ is different in mass and wave function fraction.

Comparison of S wave functions

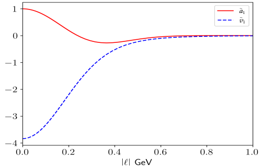
0 Chebyshev moment of S wave ($N(1720)$)



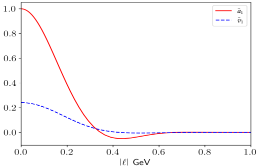
0 Chebyshev moment of S wave ($N(1900)$)



0 Chebyshev moment of S wave ($N(1520)$)



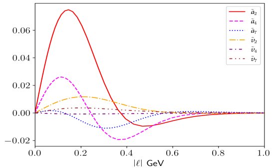
0 Chebyshev moment of S wave ($N(1700)$)



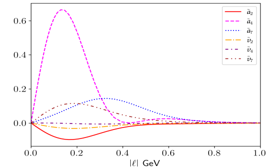
- They all have zero.
- The wave function from vector diquark is getting important for negative parity states.

Comparison of P wave functions

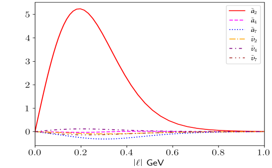
0 Chebyshev moment of P wave ($N(1720)$)



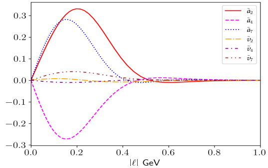
0 Chebyshev moment of P wave ($N(1900)$)



0 Chebyshev moment of P wave ($N(1520)$)



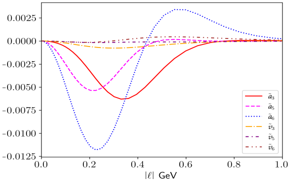
0 Chebyshev moment of P wave ($N(1700)$)



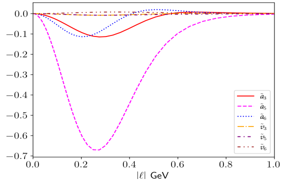
- The wave function from vector diquark is getting important for them.
- The higher partial wave contributes.

Comparison of D wave functions

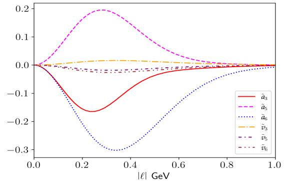
0 Chebyshev moment of D wave ($N(1720)$)



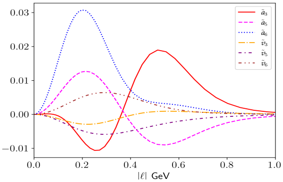
0 Chebyshev moment of D wave ($N(1900)$)



0 Chebyshev moment of D wave ($N(1520)$)

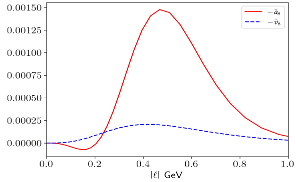


0 Chebyshev moment of D wave ($N(1700)$)

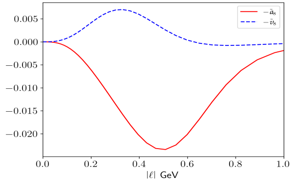


Comparison of F wave functions

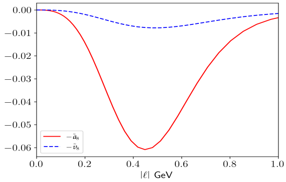
0 Chebyshev moment of F wave ($N(1720)$)



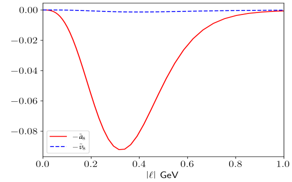
0 Chebyshev moment of F wave ($N(1900)$)



0 Chebyshev moment of F wave ($N(1520)$)



0 Chebyshev moment of F wave ($N(1700)$)



Faddeev results of low-lying excitations of nucleon and Δ

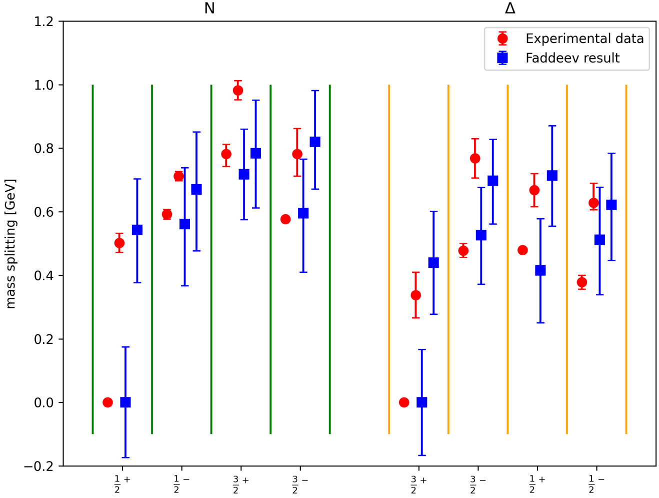
Baryon	Status	JP	Lower Bound	Center Value	Upper Bound
N0940	****	1/2+	1.107	1.195	1.281
N1440	****	1/2+	1.658	1.738	1.810
N1535	****	1/2-	1.648	1.756	1.845
N1650	****	1/2-	1.758	1.865	1.958
N1720	****	3/2+	1.856	1.913	1.967
N1900	****	3/2+	1.893	1.979	2.058
N1520	****	3/2-	1.691	1.790	1.872
N1700	***	3/2-	1.952	2.015	2.088
D1232	****	3/2+	1.263	1.346	1.430
D1600	****	3/2+	1.707	1.786	1.864
D1700	****	3/2-	1.802	1.872	1.939
D1940	**	3/2-	1.991	2.043	2.091
D1750	*	1/2+	1.680	1.761	1.841
D1910	****	1/2+	1.984	2.060	2.133
D1620	****	1/2-	1.769	1.858	1.940
D1900	***	1/2-	1.877	1.968	2.047

Comparison of the dominated wave with quark model

Baryon	Status	JP	Faddeev	Quark Model
N0940	****	1/2+	S	S
N1440	****	1/2+	S	S
N1535	****	1/2-	P	P
N1650	****	1/2-	P	P
N1720	****	3/2+	S	D
N1900	****	3/2+	D	D
N1520	****	3/2-	P	P
N1700	***	3/2-	P	P
D1232	****	3/2+	S	S
D1600	****	3/2+	D	S
D1700	****	3/2-	P	P
D1940	**	3/2-	P	P
D1750	*	1/2+	D	S
D1910	****	1/2+	S	D
D1620	****	1/2-	P	P
D1900	***	1/2-	S	P

- The dominated wave for $N(1/2^{+-})$ are the same.
- The dominated wave for $N(1900)$, $N(3/2^{-})$ are the same.
- The dominated wave for $\Delta(1232), \Delta(3/2-)$ are the same.
- The dominated wave for $\Delta(1620)$ are the same.
- The most frequent difference is the S and D wave of excited states.

Mass splittings of low-lying excitations of nucleon and Δ



Conclusion

In our Works, We

- ☆ We firstly solve the $N(I=1/2, J^P=1/2^{+-}), N(I=1/2, J^P=3/2^{+-}), \Delta(I=3/2, J^P=3/2^{+-}), \Delta(I=3/2, J^P=1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I=1/2, J^P=1/2^{+-}), N(I=1/2, J^P=3/2^{+-}), \Delta(I=3/2, J^P=3/2^{+-}), \Delta(I=3/2, J^P=1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).
- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I = 1/2, J^P = 1/2^{+-}), N(I = 1/2, J^P = 3/2^{+-}), \Delta(I = 3/2, J^P = 3/2^{+-}), \Delta(I = 3/2, J^P = 1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).
- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.
  - ♣ The negative parity diquarks and higher partial waves can not be ignored in the excitations of nucleon and Delta.

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I = 1/2, J^P = 1/2^{+-}), N(I = 1/2, J^P = 3/2^{+-}), \Delta(I = 3/2, J^P = 3/2^{+-}), \Delta(I = 3/2, J^P = 1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).
- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.
  - ♣ The negative parity diquarks and higher partial waves can not be ignored in the excitations of nucleon and Delta.
  - ♣ Sometimes their contribution to the mass and amplitude (wave function) are different. The wave function structure is more complex than mass structure. We need further observable (such as transition form factor) to judge the solutions.

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I=1/2, J^P=1/2^{+-})$, $N(I=1/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).
- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.
  - ♣ The negative parity diquarks and higher partial waves can not be ignored in the excitations of nucleon and Delta.
  - ♣ Sometimes their contribution to the mass and amplitude (wave function) are different. The wave function structure is more complex than mass structure. We need further observable (such as transition form factor) to judge the solutions.
- ☆ Analyze the mass splittings of low-lying excited states of nucleon and Delta to their corresponding ground states.

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I=1/2, J^P=1/2^{+-})$, $N(I=1/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).

- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.
  - ♣ The negative parity diquarks and higher partial waves can not be ignored in the excitations of nucleon and Delta.
  - ♣ Sometimes their contribution to the mass and amplitude (wave function) are different. The wave function structure is more complex than mass structure. We need further observable (such as transition form factor) to judge the solutions.

- ☆ Analyze the mass splittings of low-lying excited states of nucleon and Delta to their corresponding ground states.
  - ♣ Our results are consistent with the experimental results within the error bar.

Conclusion

In our Works, We

- ☆ We firstly solve the $N(I=1/2, J^P=1/2^{+-})$, $N(I=1/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=3/2^{+-})$, $\Delta(I=3/2, J^P=1/2^{+-})$ baryons with the symmetry preserving Poincaré covariant quark + diquark Faddeev equation approach. We not only use the positive parity diquarks (scalar and pseudovector diquark) but also use the negative parity diquarks (pseudoscalar and vector diquark).
- ☆ Analyze their internal structure from two perspectives: diquark content and orbital angular momentum content.
  - ♣ The negative parity diquarks and higher partial waves can not be ignored in the excitations of nucleon and Delta.
  - ♣ Sometimes their contribution to the mass and amplitude (wave function) are different. The wave function structure is more complex than mass structure. We need further observable (such as transition form factor) to judge the solutions.
- ☆ Analyze the mass splittings of low-lying excited states of nucleon and Delta to their corresponding ground states.
  - ♣ Our results are consistent with the experimental results within the error bar.
  - ♣ The quark-diquark picture and the Faddeev equation we used at present are valid for these low-lying excitations of nucleon and Delta.

Thank you!

Diquark amplitudes

$$\Gamma^{sc}(p,p^d) = g_s i\gamma_5\, C\mathcal{F}(p^2/\omega_{sc}^2)\,,$$
$$\Gamma^{av}_\mu(p,p^d) = g_a i\gamma_\mu\, C\mathcal{F}(p^2/\omega_{av}^2)\,,$$
$$\Gamma^{ps}(p,p^d) = g_p\, 1\; C\mathcal{F}(p^2/\omega_{ps}^2)\,,$$
$$\Gamma^{vc}_\mu(p,p^d) = g_v\gamma_\mu\gamma_5\, C\mathcal{F}(p^2/\omega_{sc}^2)\,,$$
$$\vec{\Gamma}^{vc}_\mu(p,p^d) = g_v\left[\gamma_\mu, \frac{i\gamma\cdot p^d}{m_{vc}}\right]\gamma_5\, C\mathcal{F}(p^2/\omega_{sc}^2)\,,$$

$$\bar{\Gamma}^{sc}(p,p^d) = C\Gamma^{sc}(-p,-p^d)^T C^{-1} = g_s i C^{-1}\gamma_5\mathcal{F}(p^2/\omega_{sc}^2)\,,$$
$$\bar{\Gamma}^{av}_\mu(p,p^d) = -C\Gamma^{av}(-p,-p^d)^T C{-1} = g_a i C^{-1}\gamma_\mu\mathcal{F}(p^2/\omega_{av}^2)\,,$$
$$\bar{\Gamma}^{ps}(p,p^d) = C\Gamma^{ps}(-p,-p^d)^T C^{-1} = g_p\, C^{-1}\, 1\mathcal{F}(p^2/\omega_{ps}^2)\,,$$
$$\bar{\Gamma}^{vc}_\mu(p,p^d) = -C\Gamma^{vc}(-p,-p^d)^T C^{-1} = g_v C^{-1}\gamma_5\gamma_\mu\mathcal{F}(p^2/\omega_{sc}^2)\,,$$
$$\bar{\vec{\Gamma}}^{vc}_\mu(p,p^d) = -C\Gamma^{vc}(-p,-p^d)^T C^{-1} = g_v C^{-1}\gamma_5\left[\frac{i\gamma\cdot p^d}{m_{vc}}, \gamma_\mu\right]\mathcal{F}(p^2/\omega_{sc}^2)\,.$$

$$\mathcal{F}(x) = \frac{1-e^{-x}}{x}\,.$$

Numerical calculation procedures

arXiv:hep-ph/0109285

We use first kind of Chebyshev polynomial to expand the scalar functions of Faddeev amplitudes and wave functions

$$F(p^2, z) = \sum_{n=0}^{N} i^n F^n(p^2)\, T_n(z)\,, \quad F^n(p^2) = (-i)^n \frac{2}{N+1} \sum_{k=1}^{N+1} T_n(z_k) F(p^2, z_k)\,.$$

Then the numerical procedure follows:

- ★ Generate the grids.
- ★ Compute the propagator matrix and exchange kernel matrix.
- ★ Compute eigenvalues and eigenvectors of full kernel.
- ★ Search the mass shell and corresponding Faddeev amplitudes.

Numerical calculation parameters

n_p	n_y	m_{max}	n_{max}	η	g_{DB}	
100	30	20	20	$1/3$	1.0	
M_s	M_a	M_p	M_v			
0.8	0.9	1.22	1.32			
λ	$\bar{m}$	b_0	b_1	b_2	b_3	ε
0.566	0.00897	0.131	2.90	0.603	0.185	0.0001

Adapted from Chen's paper (Chen et al. Phys. Rev., D97, 034016).

$N(J^P = 1/2^+)$ Decomposition

$$\Phi^5(k,P) = \sum_{i=1}^{2} S_i(k^2, k\cdot P)\mathcal{S}_i(k,P)$$

$$\Phi^\mu(k,P) = \sum_{i=1}^{6} A_i(k^2, k\cdot P)\gamma_5\mathcal{A}_i^\mu(k,P)$$

$$\Phi(k,P) = \sum_{i=1}^{2} P_i(k^2, k\cdot P)\mathcal{P}_i(k,P)$$

$$\Phi^{5\mu}(k,P) = \sum_{i=1}^{6} V_i(k^2, k\cdot P)\gamma_5\mathcal{V}_i^\mu(k,P)$$

$$\begin{aligned}
\mathcal{S}_1(k,P) &= \Lambda^+ \\
\mathcal{S}_2(k,P) &= -\frac{i}{k}\not{k}_\perp\Lambda^+ \\
\mathcal{A}_1^\mu(k,P) &= -\frac{i}{k}\hat{P}^\mu\not{k}_\perp\Lambda^+ \\
\mathcal{A}_2^\mu(k,P) &= \hat{P}^\mu\Lambda^+ \\
\mathcal{A}_3^\mu(k,P) &= \hat{k}_\perp^\mu\hat{\not{k}}_\perp\Lambda^+ \\
\mathcal{A}_4^\mu(k,P) &= \frac{i}{k}k_\perp^\mu\Lambda^+ \\
\mathcal{A}_5^\mu(k,P) &= \gamma_\perp^\mu\Lambda^+ - \mathcal{A}_3^\mu(k,P) \\
\mathcal{A}_6^\mu(k,P) &= \frac{i}{k}\gamma_\perp^\mu\not{k}_\perp\Lambda^+ - \mathcal{A}_4^\mu(k,P)
\end{aligned}$$

$$\mathcal{P}_i(k,P) = i\gamma_5\mathcal{S}_i(k,P)\,, \qquad \mathcal{V}_i^\mu(k,P) = i\gamma_5\mathcal{A}_i^\mu(k,P)\,.$$

Adapted from Martin Oettel's paper (Phys.Rev.,C58,2459).
For $N(J^P = 1/2^-)$ baryon, One should multiply all the bases by $i\gamma_5$.

$N(J^P = 3/2^+)$ Decomposition

$$\Phi^{\mu\beta}(k;P) = \sum_{i=1}^{8} A_i(k,P)\mathcal{A}_i^{\mu\beta}(k,P) + \sum_{i=1}^{8} V_i(k,P)\mathcal{V}_i^{\mu\beta}(k,P) + \sum_{i=1}^{8} \vec{V}_i(k,P)\vec{\mathcal{V}}_i^{\mu\beta}(k,P)\,,$$

$$\mathcal{A}_1^{\mu\beta}(k,P) = \delta^{\mu\rho}\Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_2^{\mu\beta}(k,P) = \frac{1}{\sqrt{5}}\left(2\gamma_\perp^\mu q^\rho - 3\delta^{\mu\rho}\not{q}\right)\Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_3^{\mu\beta}(k,P) = -\sqrt{3}\hat{P}^\mu q^\rho \not{q}\Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_4^{\mu\beta}(k,P) = \sqrt{3}\hat{P}^\mu q^\rho \Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_5^{\mu\beta}(k,P) = -\gamma_\perp^\mu q^\rho \not{q}\Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_6^{\mu\beta}(k,P) = \left(\gamma_\perp^\mu q^\rho \not{q} - \delta^{\mu\rho} - 3q^\mu q^\rho\right)\Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_7^{\mu\beta}(k,P) = -\gamma_\perp^\mu q^\rho \Lambda^+\Lambda^{\rho\beta}\,,$$

$$\mathcal{A}_8^{\mu\beta}(k,P) = \frac{1}{\sqrt{5}}\left(\delta^{\mu\rho}\not{q} + \gamma_\perp^\mu q^\rho + 5q^\mu q^\rho \not{q}\right)\Lambda^+\Lambda^{\rho\beta}\,,$$

$$q^\mu = i\frac{\left(\delta^{\mu\nu} - \hat{P}^\mu\hat{P}^\nu\right)\hat{k}_\nu}{\sqrt{1-(\hat{k}\cdot\hat{P})^2}}\,,\quad \mathcal{V}_i^{\mu\beta}(k,P) = \vec{\mathcal{V}}_i^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_i^{\mu\beta}(k,P)\,.$$

$$\Lambda^+\Lambda_{\nu\beta} = u_\nu(P)\bar{u}_\beta(P) = \Lambda^+(P)\left(\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3}\hat{P}_\nu\hat{P}_\beta + \frac{1}{3}\left(\hat{P}_\nu\gamma_\beta - \hat{P}_\beta\gamma_\nu\right)\right)$$

Adapted from Gernot Eichmann's thesis (arXiv:0909.0703).
For $N(J^P = 3/2^-)$ baryon, One should multiply all the bases by $i\gamma_5$.

$\Delta(J^P = 3/2^+)$ Decomposition

$$\Phi^{\mu\beta}(k;P) = \sum_{i=1}^{8} D_i(k,P)\mathcal{D}_i^{\mu\beta}(k,P) + \sum_{i=1}^{8} E_i(k,P)\mathcal{E}_i^{\mu\beta}(k,P)\,,$$

$$\mathcal{E}_i^{\mu\beta}(k,P) = i\gamma_5\mathcal{D}_i^{\mu\beta}(k,P)\,.$$

$$\mathcal{D}_1^{\mu\beta}(k,P) = \mathcal{S}_1\Lambda^+\Lambda^{\mu\beta}\,,$$
$$\mathcal{D}_2^{\mu\beta}(k,P) = \mathcal{S}_2\Lambda^+\Lambda^{\mu\beta}\,,$$
$$\mathcal{D}_3^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_1^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$
$$\mathcal{D}_4^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_2^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$
$$\mathcal{D}_5^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_3^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$
$$\mathcal{D}_6^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_4^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$
$$\mathcal{D}_7^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_5^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$
$$\mathcal{D}_8^{\mu\beta}(k,P) = i\gamma_5\mathcal{A}_6^{\mu}(k,P)\Lambda^+\hat{k}_\perp^\nu\Lambda_{\nu\beta}\,,$$

$$\Lambda^+\Lambda_{\nu\beta} = u_\nu(P)\bar{u}_\beta(P) = \Lambda^+(P)\left(\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3}\hat{P}_\nu\hat{P}_\beta + \frac{1}{3}\left(\hat{P}_\nu\gamma_\beta - \hat{P}_\beta\gamma_\nu\right)\right)$$

Adapted from Martin Oettel's paper (Phys.Rev.,C58,2459).
For $\Delta(J^P = 3/2^-)$ baryon, One should multiply all the bases by $i\gamma_5$.

$\Delta(J^P = 1/2^+)$ Decomposition

$$\Phi^{\mu}(k;P) = \Phi^{\mu}(k;P) + \Phi^{5\mu}(k;P) \,,$$

$$\Phi^{\mu}(k,P) = \sum_{i=1}^{6} A_i(k^2, k\cdot P)\gamma_5 \mathcal{A}_i^{\mu}(k,P) \,,$$

$$\Phi^{5\mu}(k,P) = \sum_{i=1}^{6} V_i(k^2, k\cdot P)\gamma_5 \mathcal{V}_i^{\mu}(k,P) \,,$$

$$q^{\mu} = i\frac{\left(\delta^{\mu\nu} - \hat{P}^{\mu}\hat{P}^{\nu}\right)\hat{k}_{\nu}}{\sqrt{1-(\hat{k}\cdot\hat{P})^2}}$$

$$\mathcal{V}_i^{\mu}(k,P) = i\gamma_5 \mathcal{A}_i^{\mu}(k,P) \,.$$

$$\Lambda^{+} = u(P)\bar{u}(P) = \Lambda^{+}(P) = \frac{1+\hat{P}}{2} \,.$$

$$\mathcal{A}_1^{\mu}(k,P) = \frac{1}{\sqrt{3}}\gamma_{\perp}^{\mu}\Lambda^{+}(P) \,,$$

$$\mathcal{A}_2^{\mu}(k,P) = \frac{1}{\sqrt{3}}\gamma_{\perp}^{\mu}\not{q}\Lambda^{+}(P) \,,$$

$$\mathcal{A}_3^{\mu}(k,P) = \hat{P}^{\mu}\Lambda^{+}(P) \,,$$

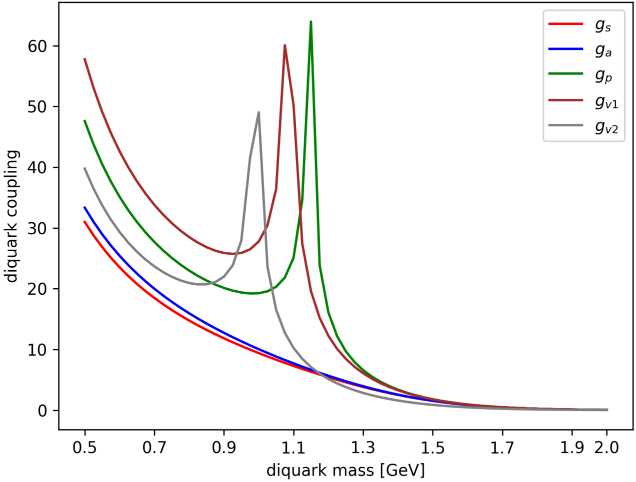
$$\mathcal{A}_4^{\mu}(k,P) = \hat{P}^{\mu}\not{q}\Lambda^{+}(P) \,,$$

$$\mathcal{A}_5^{\mu}(k,P) = \frac{1}{\sqrt{6}}\left(\gamma_{\perp}^{\mu} + 3q^{\mu}\not{q}\right)\Lambda^{+}(P) \,,$$

$$\mathcal{A}_6^{\mu}(k,P) = \frac{1}{\sqrt{6}}\left(\gamma_{\perp}^{\mu}\not{q} - 3q^{\mu}\right)\Lambda^{+}(P) \,,$$

Adapted from Gernot Eichmann's thesis (arXiv:0909.0703).
For $\Delta(J^P = 1/2^-)$ baryon, One should multiply all the bases by $i\gamma_5$.

The running of diquark coupling with diquark mass



Numerical calculation parameters for mass splittings error bar

n_p	n_y	m_{max}	n_{max}	η	g_{DB}	
100	30	20	20	1/3	1.0	
M_s	M_a	M_p	M_v			
0.8	0.9	1.3	1.4			
λ	$\bar{m}$	b_0	b_1	b_2	b_3	ε
0.566	0.00897	0.131	2.90	0.603	0.185	0.0001

and obtain the errors by varying the diquark masses with 5%.