Continuum QCD for DAs and DFs

pion/kaon as an example

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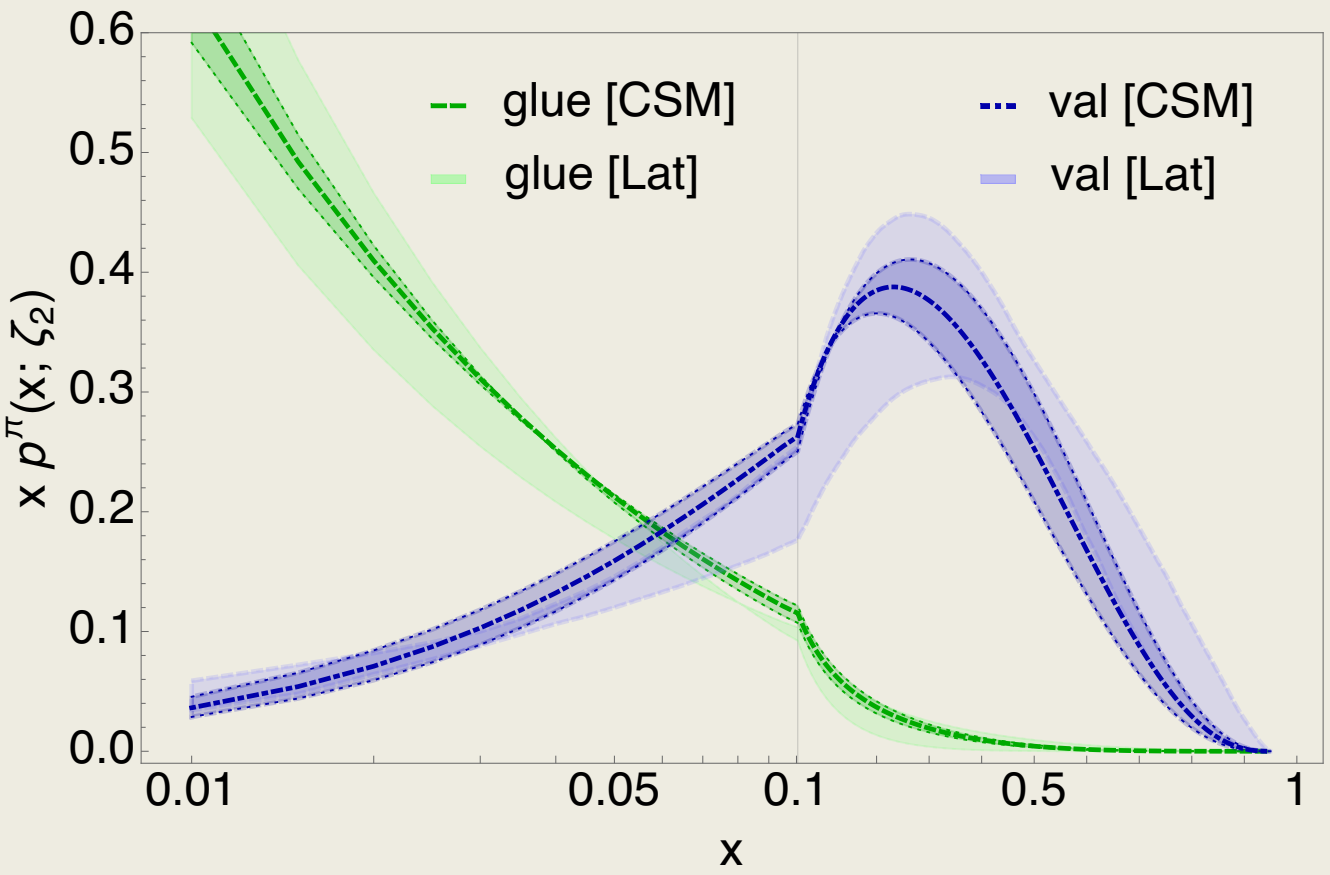
Workshop on Hadron Structure at High-Energy, High-Luminosity Facilities
2021/10/27

Breaking News!



- Within uncertainties, there is pointwise agreement between the two results on the entire depicted domains

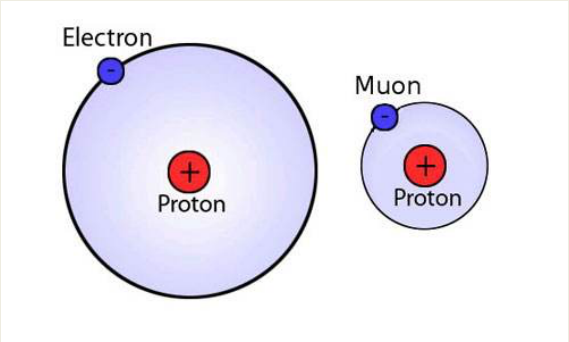
- Val[Lat] Sufian *et al.*, arXiv: 1901.03921
(valence DF: using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space)
- Glue/5[Lat] Fan *et al.*, arXiv: 2104.06372
(Glue DF: using pseudo-PDF approach(Balitsky, Morris and Radyushkin,arXiv:1910.13963))
- CSM see short review: LC and C.D.Roberts, *Chin.Phys.Lett.38(2021)081101.*

- Lattice methods: moments(…)
LaMET(Ji)
good lattice cross section(Qiu…)
pseudo-PDF(Radyushkin…)

Continuum QCD approach
A long story from 2013
I will focus on "HOW"

Describe quark-antiquark bound-state

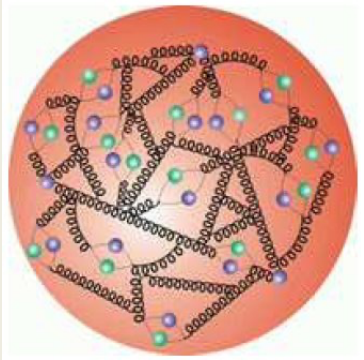
$$\Theta_0 = \beta(\alpha)\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}\,.$$



QED

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

Field theory not Successful yet:

- Growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure in hadron

Describe quark-antiquark bound-state(incomplete)

A relativistic equation for Bound-State problem
Salpeter and Bethe, PR84(1951)1232

Munczek, PRD52(1995)4736

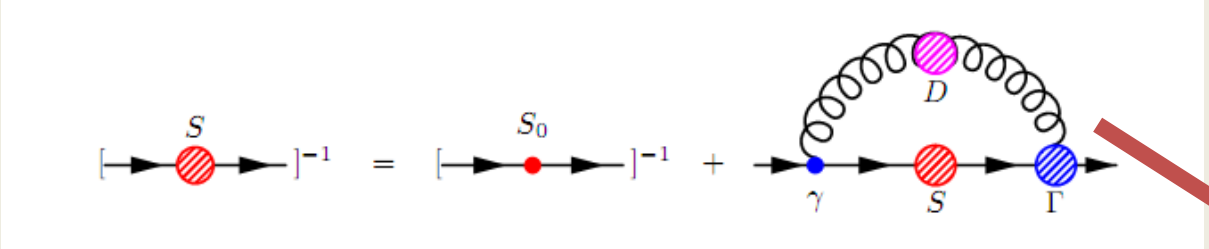
Sketching the Bethe-Salpeter kernel
Chang and Roberts, PRL103(2009)081601

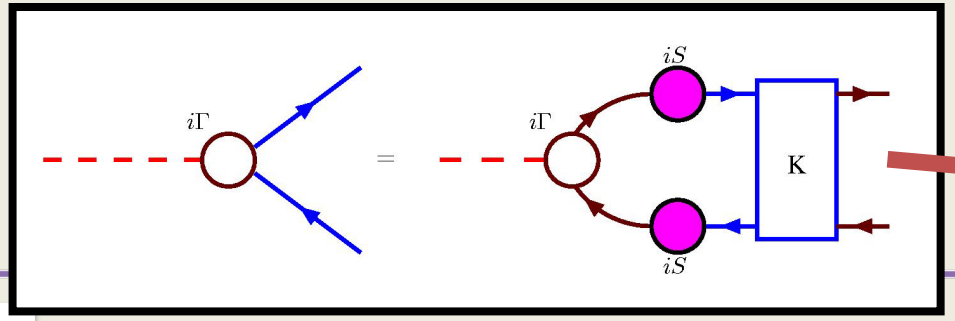
1951 1974/1976 1995 1998 2009 ……

Lane, PRD10(1974)2605
Politzer, NPB117(1976)397; Pagels…

Pion mass and decay constant
Maris, Roberts and Tandy, PLB420(1998)267

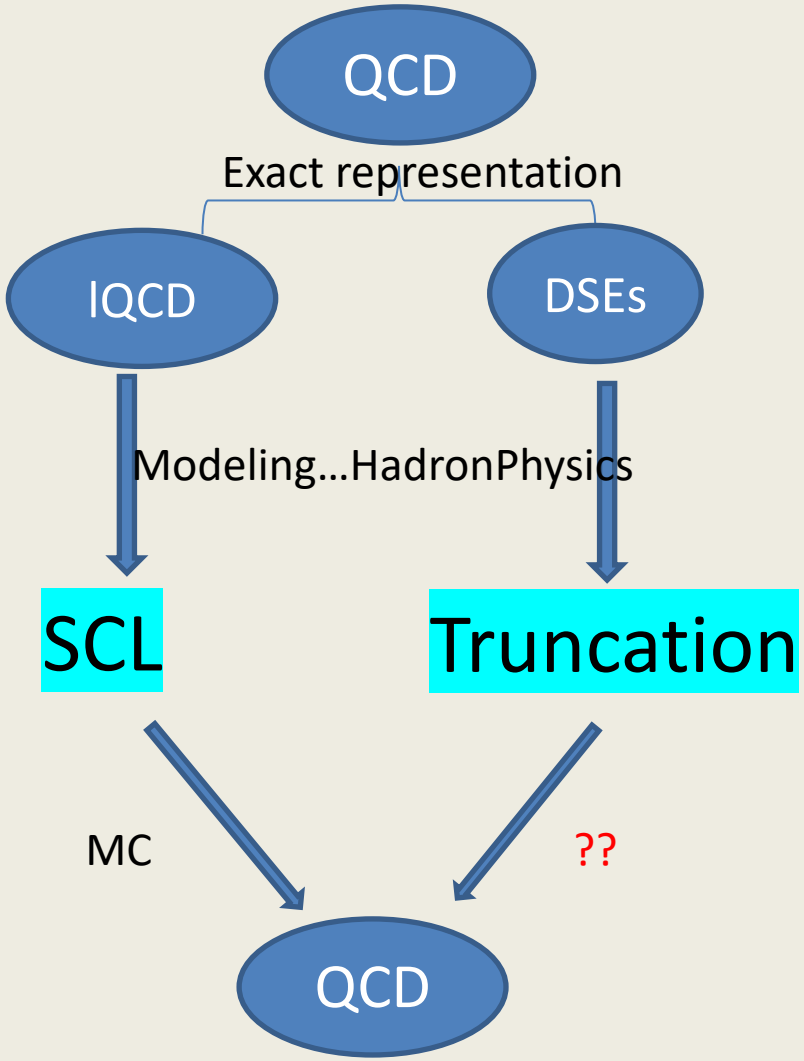
from QCD



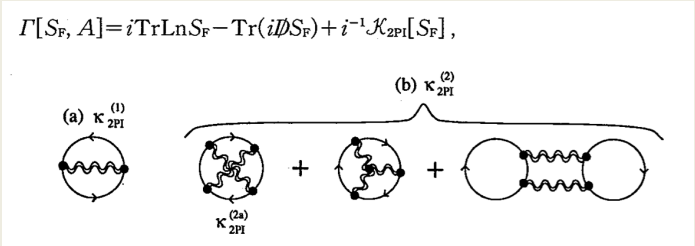


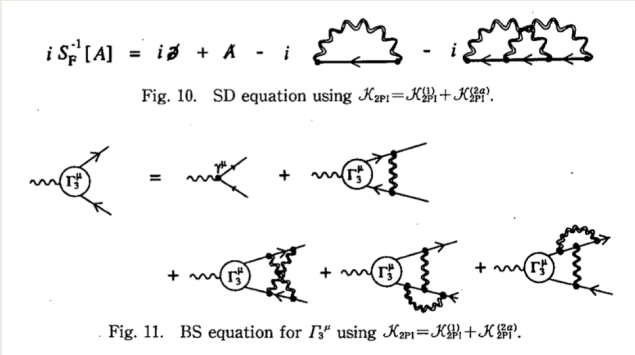
$$\mathcal{K} = \frac{\delta\Sigma}{\delta S}$$

Truncation(Put Physics at Right Place)



✓One Way: CJT approach->2PI, 3PI,…

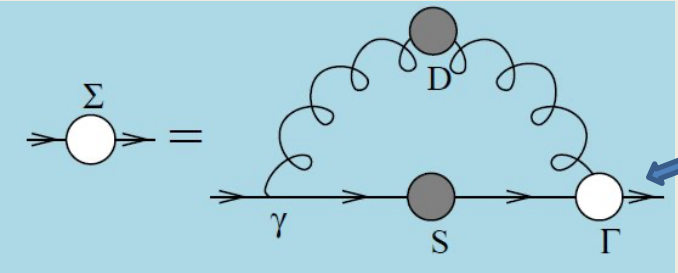




✓Our Way: Minding the quark-gluon vertex

How to construct quark-gluon vertex nonperturbatively?

Symmetry!



Quark-gluon vertex(Abelian-like)

- Gauge invariance

Color singlet vertex

Vertex BS equation is closed!

$$\Gamma_\upsilon(p+k,p) = \frac{\partial}{\partial p_\upsilon}\int_0^1 d\alpha S^{-1}(p+\alpha k) \longrightarrow \frac{\delta\Gamma_\upsilon}{\delta S} = \Lambda_\upsilon^\alpha = \frac{\partial}{\partial p_\upsilon}\int_0^1 \mathrm{d}\alpha \Gamma^\alpha(p+k-\alpha q, p-\alpha q)$$

$$\mathcal{K} = \frac{\delta\Sigma}{\delta S} \propto \frac{\delta[\gamma_\mu S\Gamma_\nu]}{\delta S}$$

- Gauge derivation(H.Haberzettl, PRD99(2019)016022)

Solution-1

$$\Gamma_\mu(p',p) = \left\{\frac{i\gamma\cdot p' A(p'^2) + A(p^2) i\gamma\cdot p}{2} + B(p^2)\right\}^S_\mu = \gamma_\mu \frac{A(p'^2)+A(p^2)}{2} - i(p'+p)_\mu\left\{\frac{1}{2}i\gamma\cdot(p'+p)\Delta_A + \Delta_B\right\}($$

Solution-2

$$\Gamma_\mu(p',p) = \left\{\frac{i\gamma\cdot p' A(p'^2) + A(p^2) i\gamma\cdot p}{2} + B(p^2)\right\}^D_\mu = \gamma_\mu \frac{A(p'^2)+A(p^2)}{2} - i(p'+p)_\mu\left\{\frac{1}{2}i\gamma\cdot(p'+p)\Delta_A + \Delta_B\right\} + \sigma_{\mu\nu}k_\nu\Delta_B + (p'_\mu\gamma\cdot p - p_\mu\gamma\cdot p' + i\gamma_\mu\sigma_{\rho\delta}p'_\rho p_\nu)\Delta_A \quad (1$$

$\mathcal{F}$ contral the strength of transverse vertex

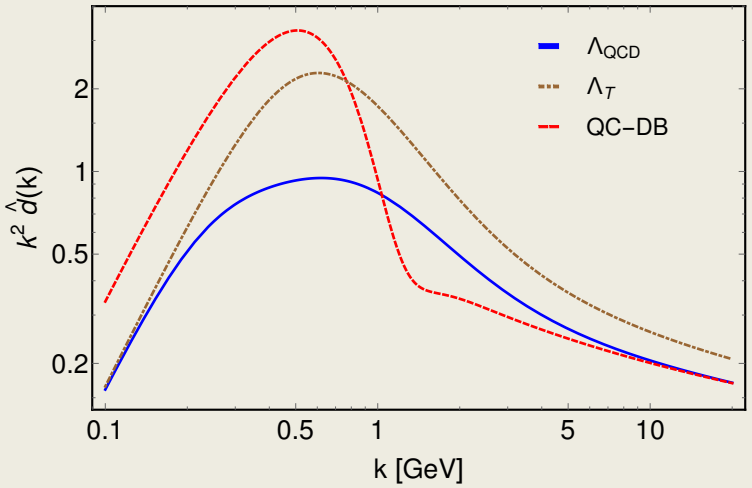
$$(1-\mathcal{F})D^S + \mathcal{F}D^D$$

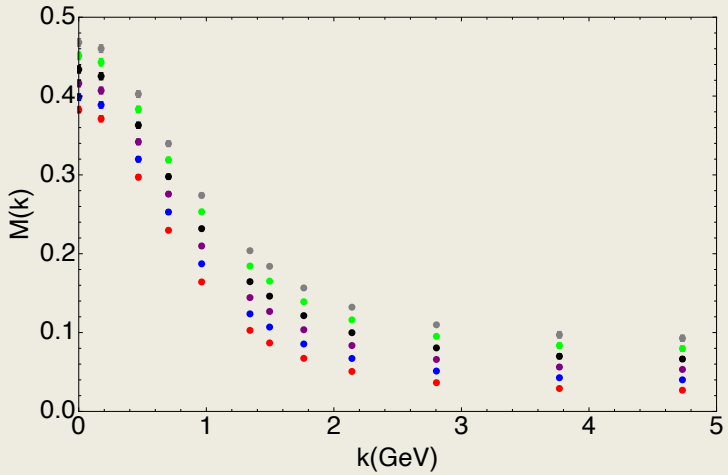
- BC term guarantee proper current mass evolution;
- ACM term has been presented directly;
- There are many good reason for including ACM;
- Λ term can be written similarly;
- Any bound-state BS wave function can be solved consistently!

LC, Yu-xin Liu and C.D.Roberts, PRL106(2011)072001
LC, et.al., arXiv:2105.06596

Effective interaction of QCD

$$\Sigma_f(p) = \frac{4}{3} Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2) T_{\mu\nu}(k) \gamma_\mu S_f(q) \widehat{\Gamma}^f_\nu(p,q)$$



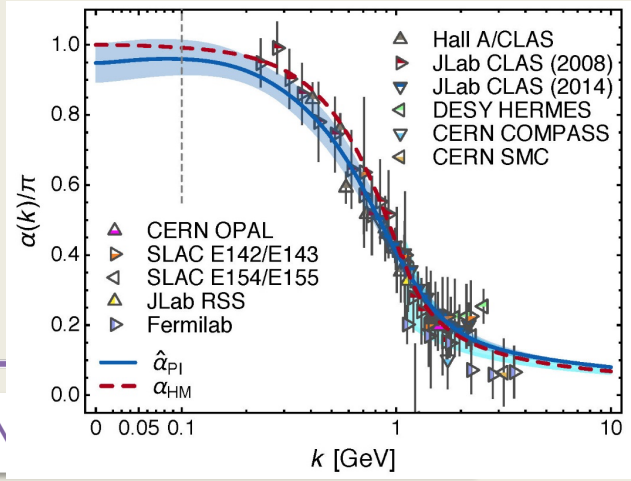


$$\widehat{d}(k^2) = \widehat{\alpha}(k^2)\mathcal{D}(k^2)$$

Need…

$$\widehat{d}(k^2) = \widetilde{\alpha}(k^2)\mathcal{D}(k^2)$$

$\mathcal{D}$ is a RGI function behaving in both the far-infrared and -ultraviolet as the propagator of a free massive boson.



$$\widetilde{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}\,, \tag{10}$$

where n_f accounts for the number of active quark flavors (within the UV domain) and $\gamma_m = 4/\beta_0$, $\beta_0 = 11 - (2/3)n_f$; and the interpolation function

$$\mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y} \tag{11}$$

- The running coupling alters at m_G so that modes with $k^2<m^2$ are screened from interactions and theory enters a practically conformal domain.

Effective interaction of QCD

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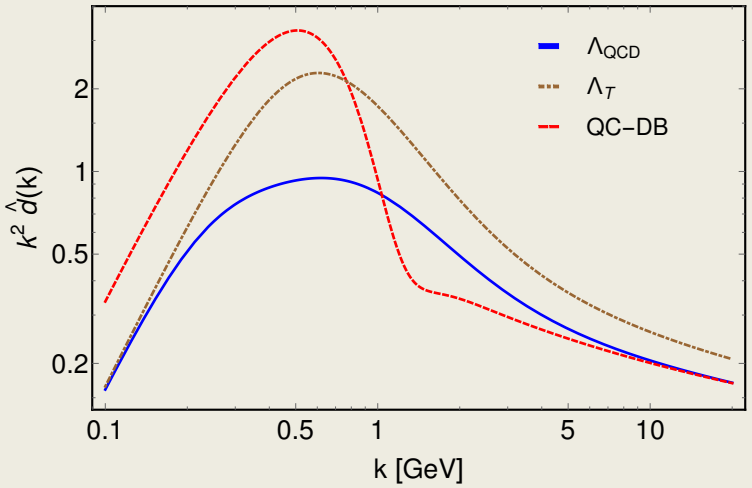

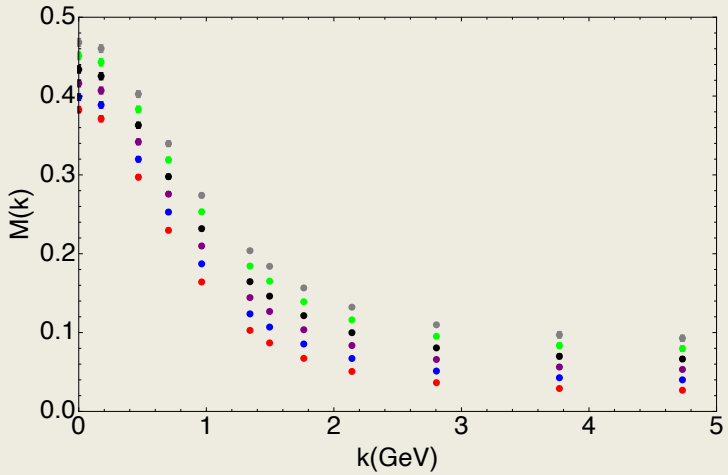

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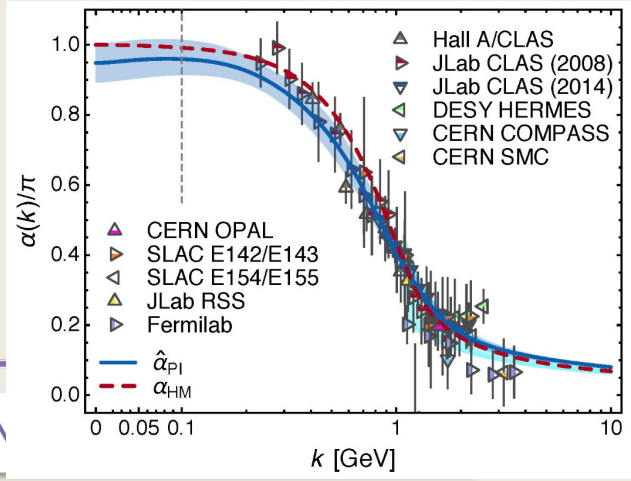

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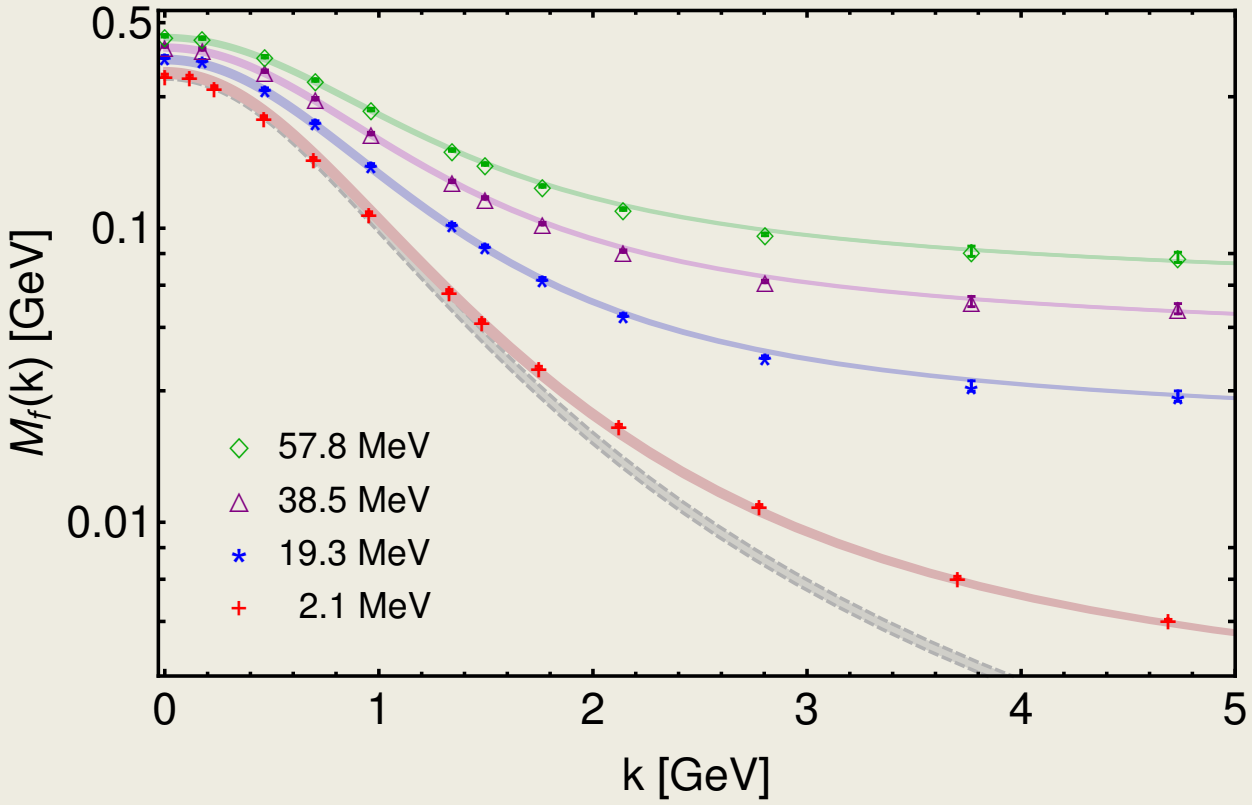
- Normalized the equations at m_G !
- m_G is the natrual hadronic scale

“Constituent” quarks



- In the chiral limit, the perturbative massless quark obtain a large infrared mass through the interactions of gluon;
- M_0 is about $m_p/3$ and runs as a logarithm-corrected $1/k^2$ power-law in the ultraviolet region;
- The strong interaction of a quark with its (gluon) surrounding gives rise to a “constituent” quark with effective mass M_0;
- Note that $M_0 \sim m_G$!
- This consistuent quark has the finite size(B. Povh and J. Hufner, PLB245(1990)653) and finite magnetic moment;

Dressed-Quark Anomalous Magnetic Moments

Lei Chang, Yu-Xin Liu, and Craig D. Roberts

Phys. Rev. Lett. **106**, 072001 (2011) - Published 16 February 2011

General properties of pion BS amplitude

Maris, Roberts and Tandy, **Phys. Lett. B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator

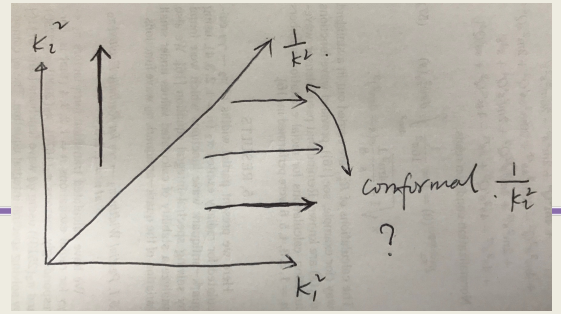
$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$\boxed{f_\pi E(k;P|P^2=0) = B(k^2)} + (k\cdot P)^2 \frac{d^2B(k^2)}{d^2k^2} + \ldots$$

- Expansion $k\cdot P$ series is crucial for the shape of DAs and DFs

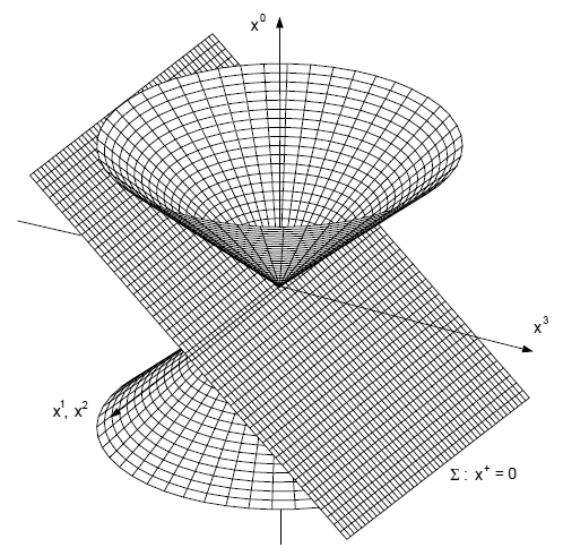
BSA ∝



The Bethe-Salpeter ampltidue of Pion decrease as

$$\Gamma(k_1^2;k_2^2) \to \frac{1}{k_1^2}$$

as $k_1^2 \to \infty$ with k_2^2 fixed finite, or with $\frac{k_1^2}{k_2^2}$ fixed.



Distribution Amplitude(truncation independent)

$$f_\pi \, \varphi_\pi(x;\mu) = Z_2 \, \mathrm{tr}_{\mathrm{CD}} \int_{dk}^{\Lambda} \delta(n\cdot k - x\, n\cdot P)\, \gamma_5 \gamma\cdot n\, \chi_\pi(k;P)\,,$$

Calculate moments;
Restruct DA from moments!

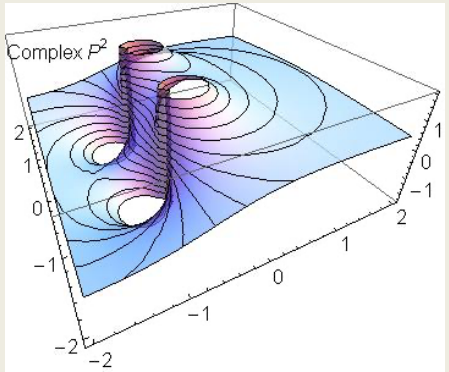
$$\langle x^m \rangle := \int_0^1 dx\, x^m \varphi_\pi(x)$$

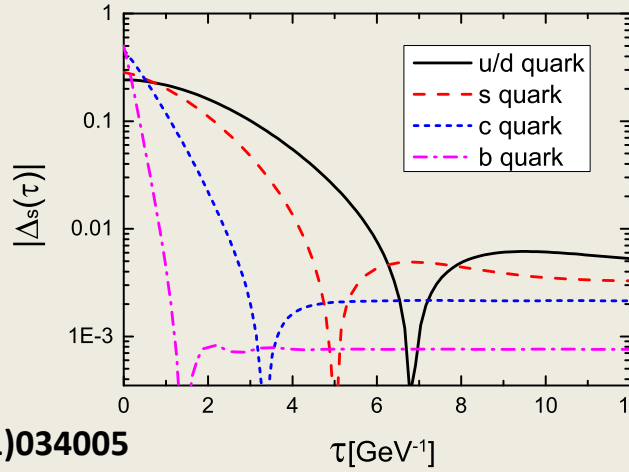
$$\langle x^m \rangle = \frac{N_c\, Z_2}{f_\pi (n\cdot P)^{m+1}} \mathrm{tr}_{\mathrm{D}} \int_{dk}^{\Lambda} (n\cdot k)^m\, \gamma_5 \gamma\cdot n\, \chi_\pi(k;P)\,.$$

Arbitrary many moments is necessary!

DA moments-----Method-1: Nakanishi-type representation

Imaging dynamical chiral symmetry breaking: pion wave function on the light front. LC, *et al.,* PRL110(2013)132001





Zehao Zhu, *et al.,* PRD103(2021)034005

✓Quark propagator

$$S(p) = \sum_{j=1}^{n_p} \left(\frac{z_j}{i\not{p} + m_j} + \frac{z_j^\star}{i\not{p} + m_j^\star} \right)$$

✓Bethe-Salpeter amplitude

$$\mathcal{F}_\sigma(q;P) = \int_{-1}^{1} d\alpha \int_0^\infty d\beta \sum_\gamma^{n_t} \frac{\hat{\rho}_\gamma(\alpha,\beta)}{(q^2 + \alpha q\cdot P + \beta_0 + \beta)^{n_\gamma}}$$

$$\hat{\rho}_\gamma(\alpha,\beta) = \rho_\gamma(\alpha)\, \delta(\beta + \beta_0 - \Lambda_\gamma^2)$$

✓Standard Feynman integrals familiar from perturbation theory

Brute force+SMP extrapolation

Leading-twist parton distribution amplitudes of S-wave heavy-qukaonia. Minghui Ding, *et al., PLB753*(2016)330; Symmetry, symmetry breaking, and pion parton distributions. Minghui Ding, , *et al.,* PRD101(2020)054014.

$$d(k^2r^2) = 1/(1+k^2r^2)^{m/2}$$

$$M_S(z) = \frac{a_0 + a_1 z + a_2 z^2}{a_0 + b_1 z + b_2 z^2 + b_3 z^3},$$

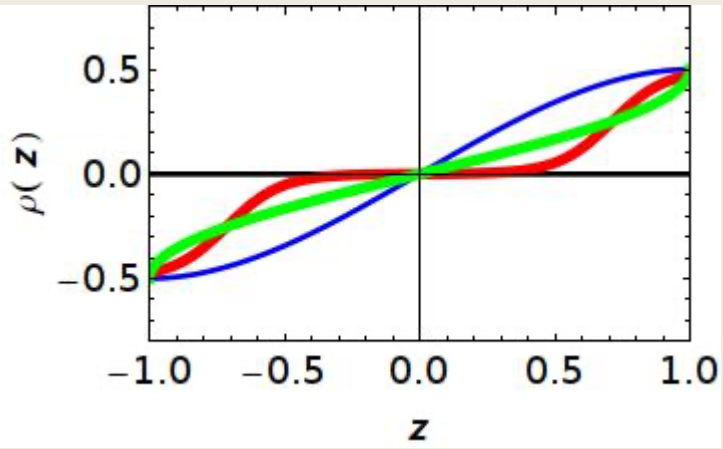
Maximum Entropy Method

Bayesian extraction of PDA from BS wave function. Fei Gao, *et al., PLB770*(2016)551.

basis is Bayes' theorem in probability theory [12], which states the probability of an event "A", given that a condition "B" is satisfied:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \quad (4)$$

DA shape-----Minding $\frac{d\rho(z)}{d\alpha} = \rho_\gamma(z)$

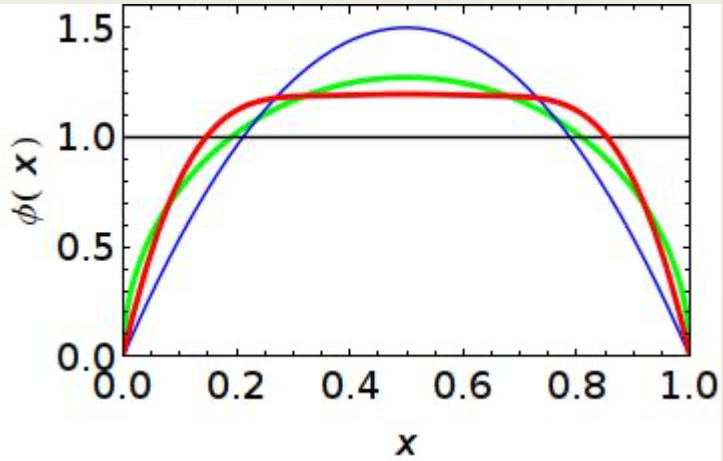


Blue line $\frac{\mathrm{d}\rho^{\text{up limit}}(z)}{\mathrm{d}z} = \frac{3}{4}(1-z^2)$

~~Black line~~ $\frac{\mathrm{d}\rho^{\text{down limit}}(z)}{\mathrm{d}z} = \frac{1}{2}(\delta(1+z)+\delta(1-z))$

Red Line $\rho_I(z) = \frac{1}{2}\left(\frac{1}{1+\mathrm{e}^{\frac{-z+z_0}{t}}} - \frac{1}{1+\mathrm{e}^{\frac{z+z_0}{t}}}\right)$

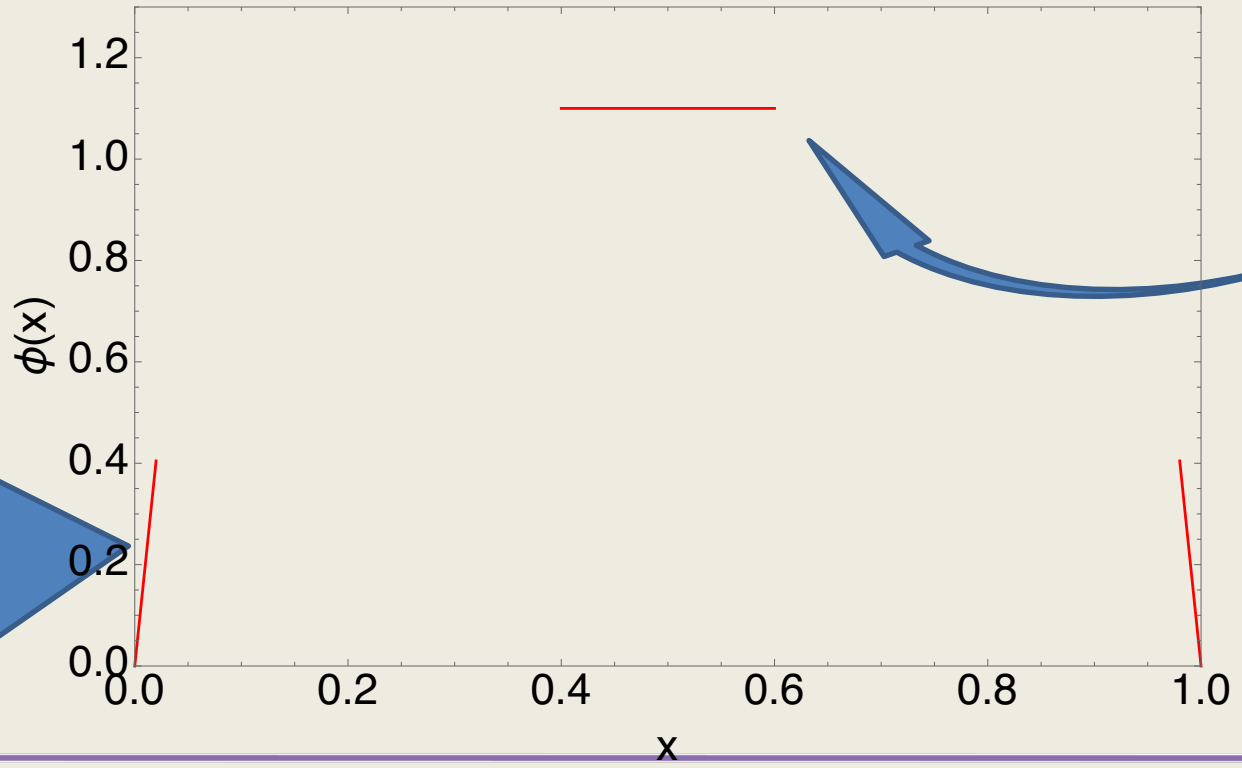
Green Line $\rho_{II}(z) = \mathrm{ArcSin}[z]$ ~~An integrable singularity at endpoints~~



1. Imaging dynamical chiral symmetry breaking: pion wave function on the light front. LC, *et al.*, PRL110(2013)132001;
2. Pion valence-quark parton distribution. LC, A.W.Thomas, PLB749(2015)547;
3. A perspective on Dyson-Schwinger equation: toy model of Pion. LC, EPJ Web Conf.113(2016)05001;
4. Pion and kaon valence-quark parton quasidistribution. Shu-Sheng Xu, *et al.*, PRD97(2018) 094014;
5. Revealing pion and kaon structure via generalised parton distributions. Khepani Raya, *et al.*, arXiv:2109.11686.

Imagine Pion global picture

- The gluon has been hidden in the constituent quarks;
- At hadronic scale, the pion is constructed by two constituent quarks which are overlapped largely;
- Valence DA(x) is symmetric function under $x \to 1 - x$
- The screening of interaction below the hadronic scale indicates the valence DA is flat on the middle of x domain
- The QCD interaction in the ultraviolet region $1/k^2$ guarantee $(1\text{-}x)^{\text{beta}>1}$ behavior near the endpoints



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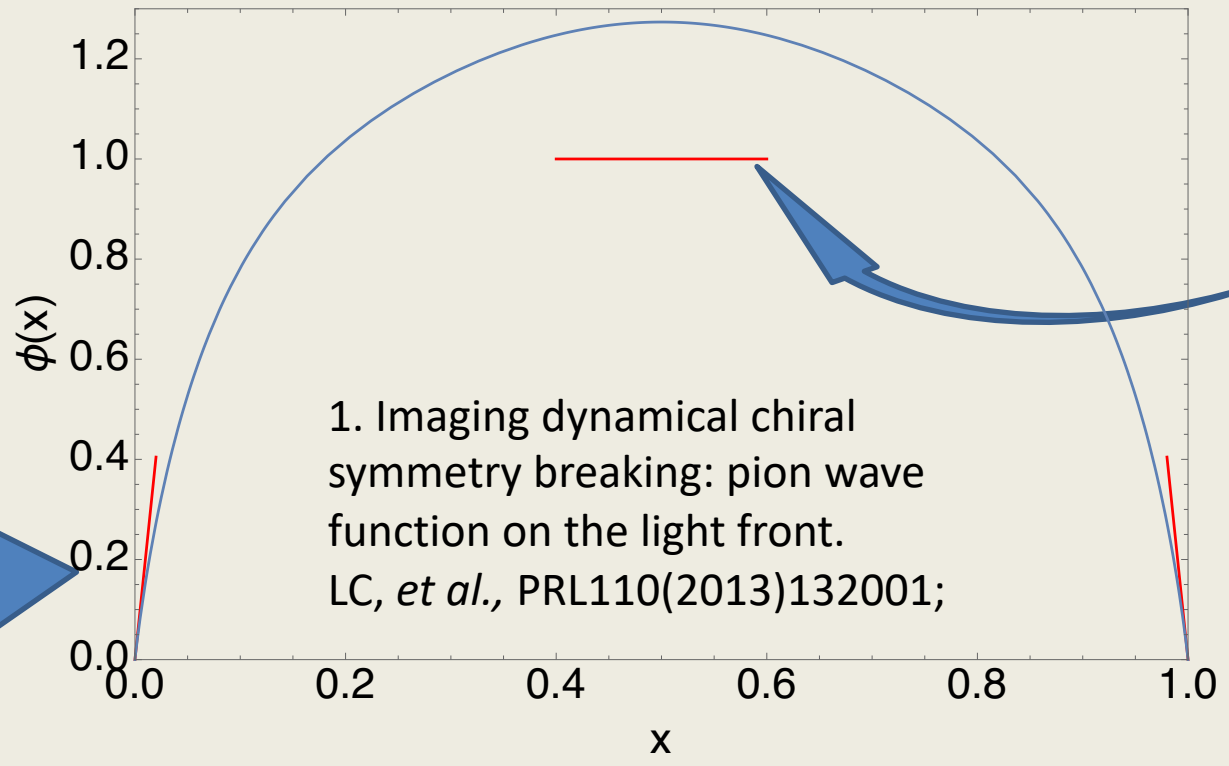


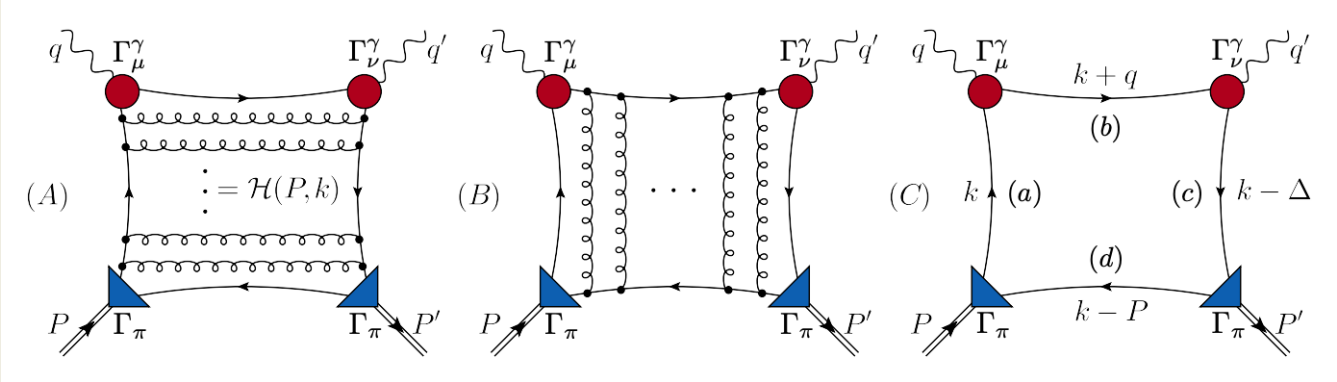
1. Imaging dynamical chiral symmetry breaking: pion wave function on the light front.
LC, *et al.*, PRL110(2013)132001;

A practical way to calculate DF

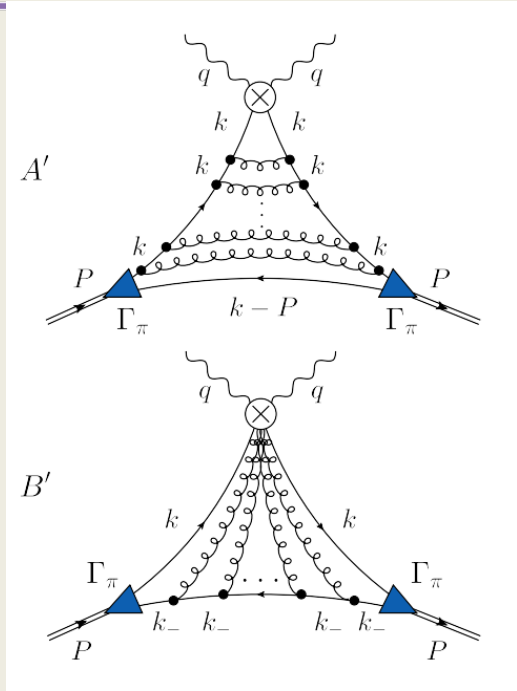
LC, et al., PLB737(2014)23, arXiv: 1406.5450

- Consider pion Compton scattering Amplitude $\mathcal{H}$

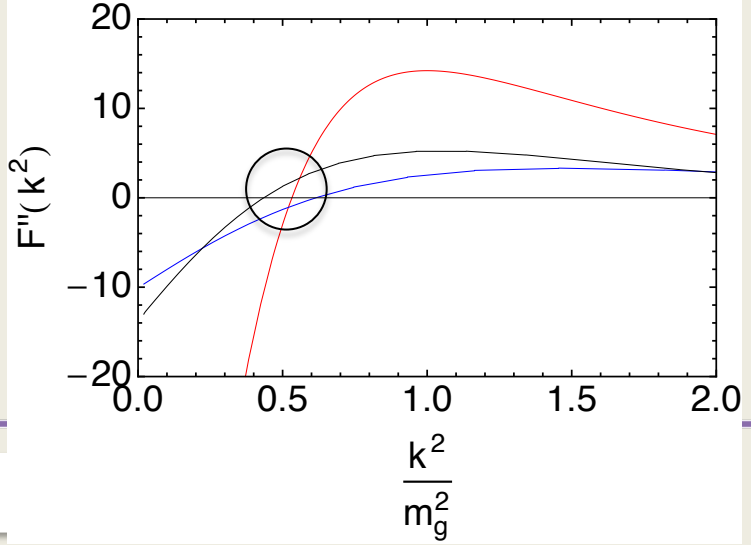
$$q_A^\pi(x;\zeta_H) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta)\, n\cdot\gamma\, \mathcal{H}_\pi(P,k)\,,$$



RL truncation



- Beyond Rainbow-Ladder truncation???



- Inflection points
- Red line: running gluon propagagor
- Blue line: vector part of propagator
- Black line: BSW function

$$\leq \frac{1}{\sqrt{2}} m_g \sim m_G \sim \zeta_H$$

A practical way to calculate DF

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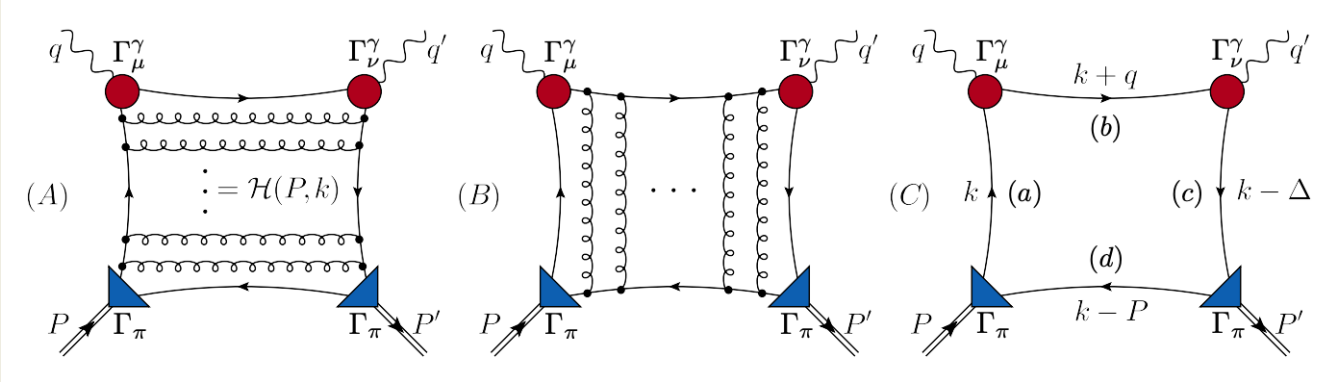

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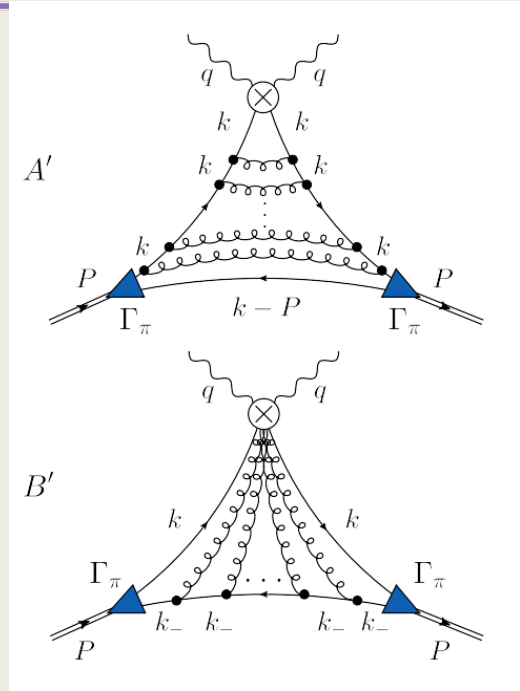
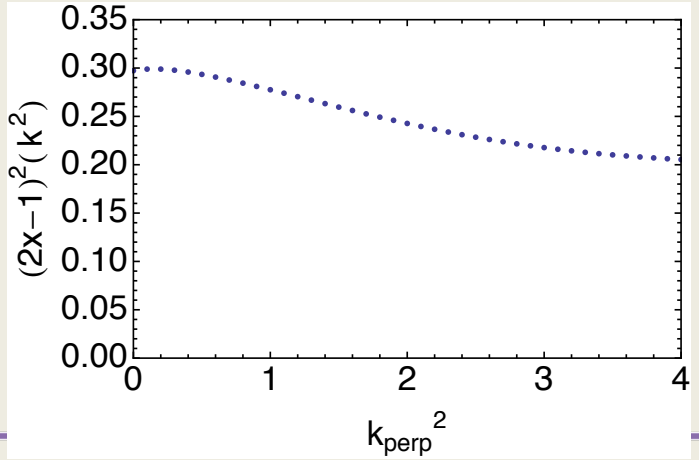


- Beyond Rainbow-Ladder truncation???

K_{perp} dependence of the second moment of LFWF



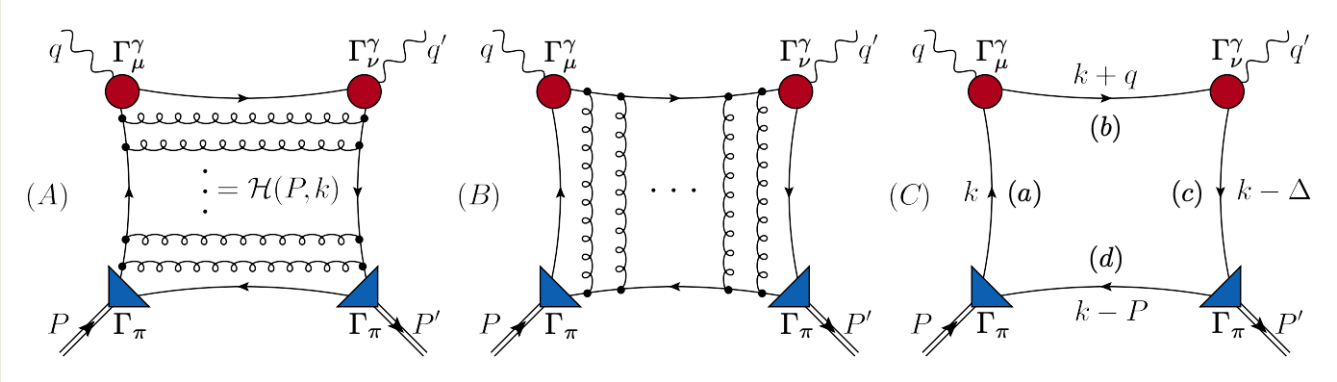
$$\leq \frac{1}{\sqrt{2}} m_g \sim m_G \sim \zeta_H$$

A practical way to calculate DF

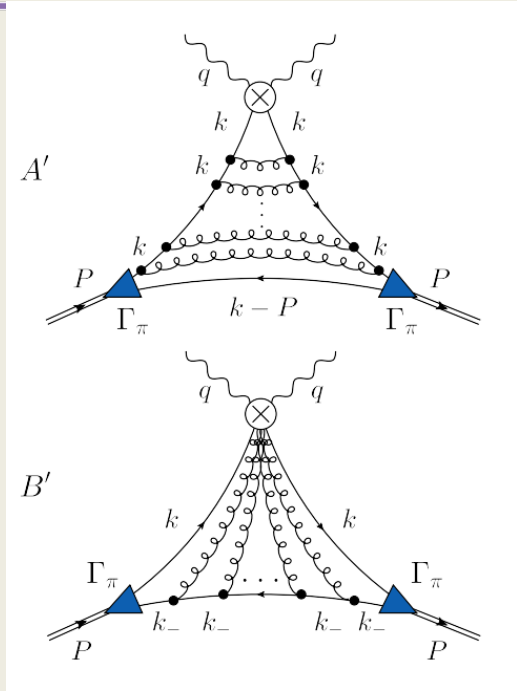
LC, et al., PLB737(2014)23, arXiv: 1406.5450

- Consider pion Compton scattering Amplitude $\mathcal{H}$

$$q_A^\pi(x;\zeta_H) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta)\, n\cdot\gamma\, \mathcal{H}_\pi(P,k)\,,$$



RL truncation



- Beyond Rainbow-Ladder truncation

$$\varphi_H(x;\zeta) \propto \int^\zeta d^2k_\perp\, \psi_H(x,\mathbf{k}_\perp;P),$$

$$q^H(x;\zeta) \propto \int^\zeta d^2k_\perp\, |\psi_H(x,\mathbf{k}_\perp;P)|^2,$$

Factorization

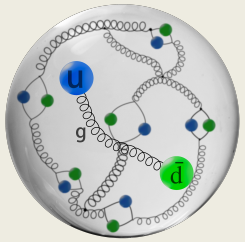
$\zeta \to \zeta_H$

at Hadronic Scale

DF(x) = DA(x)^2 !

Evolution and Probing glue

- The gluon has been hidden in the constituent quarks;
- At hadronic scale, the pion is constructed by two constituent quarks which are overlapped largely;
- Let gluon show up!



DGLAP with the effective charge

$$\frac{\partial q^{NS}}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS} ,$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f\, P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix} ,$$

$$\widetilde{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]} ,$$

$$\mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

with $\{a_0,\ a_1,\ b_0\} = \{0.104(1), 0.0975, 0.121(1)\}$

The sea quarks can arise from gluon splitting, xS(x) is expected to follow the trend of xg(x).

The scale dependence of momentum fractions

Pure valence(no gluon no sea) at hadronic scale picture

u $\bar{d}$

1, Valence quarks carry all the momentum
2, the scale dependence of momentum fractions does not depend on the details of valence distribution at hadronic scale
3, a closed equation can be derived

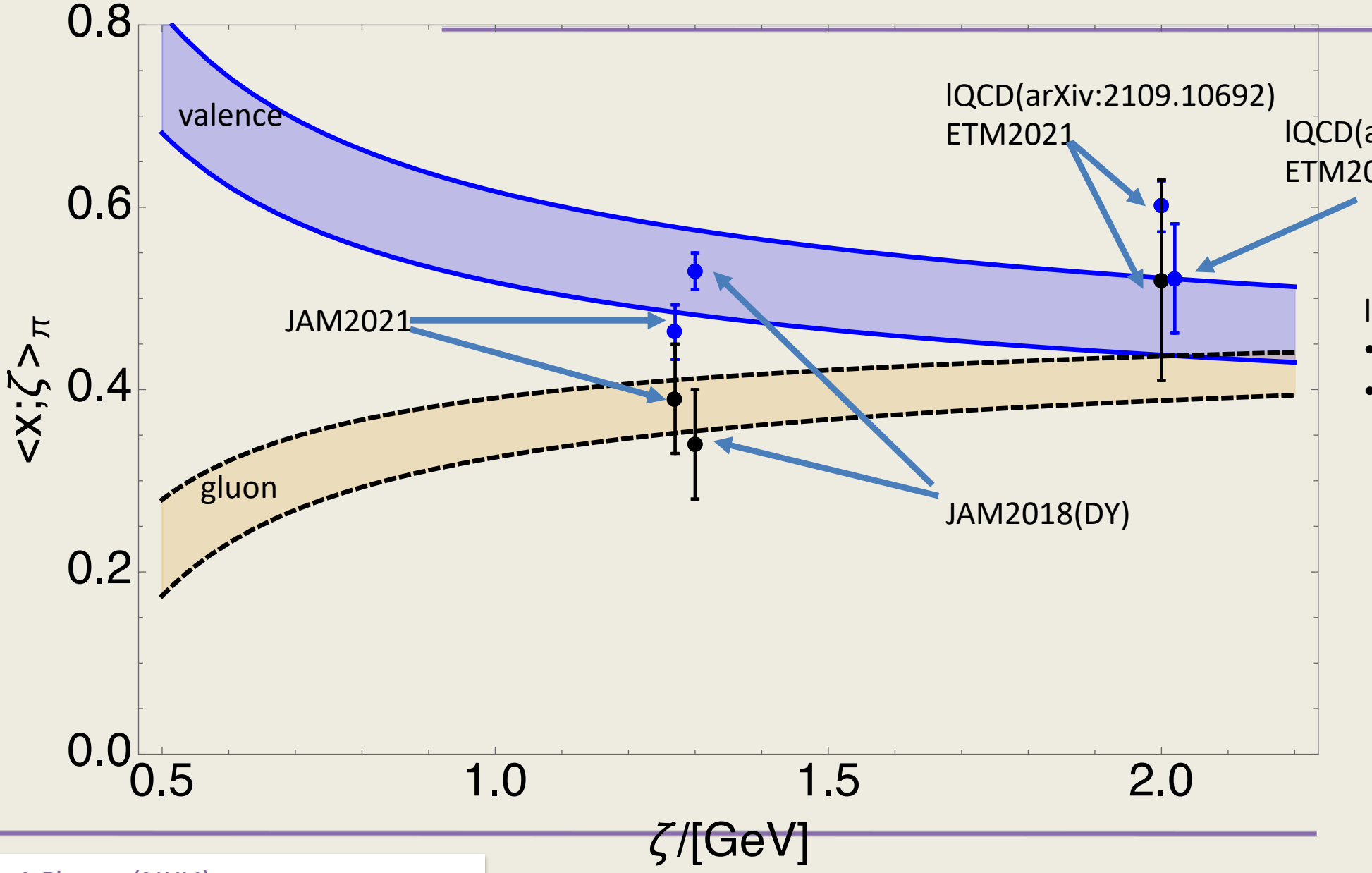
$$\langle 2x(\zeta_{\rm ex})\rangle_q = \exp\left(-\frac{8}{9\pi} S\left(\zeta_H, \zeta_{\rm ex}\right)\right),$$
$$\langle x(\zeta_{\rm ex})\rangle_{\rm sea} = \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_{\rm ex})\rangle_q^{7/4} - \langle 2x(\zeta_{\rm ex})\rangle_q\,,$$
$$\langle x(\zeta_{\rm ex})\rangle_{\rm glue} = \frac{4}{7}\left(1 - \langle 2x(\zeta_{\rm ex})\rangle_q^{7/4}\right)\,,$$

with

$$S\left(\zeta_H, \zeta_{\rm ex}\right) = \int_{t(\zeta_H)}^{t(\zeta_{\rm ex})} dt(\zeta)\,\widetilde{\alpha}(t(\zeta))$$

and $t(\zeta) = \ln\left(\zeta^2/\Lambda_{\rm QCD}^2\right)$.

Momentum evolution(valence quarks and gluon)

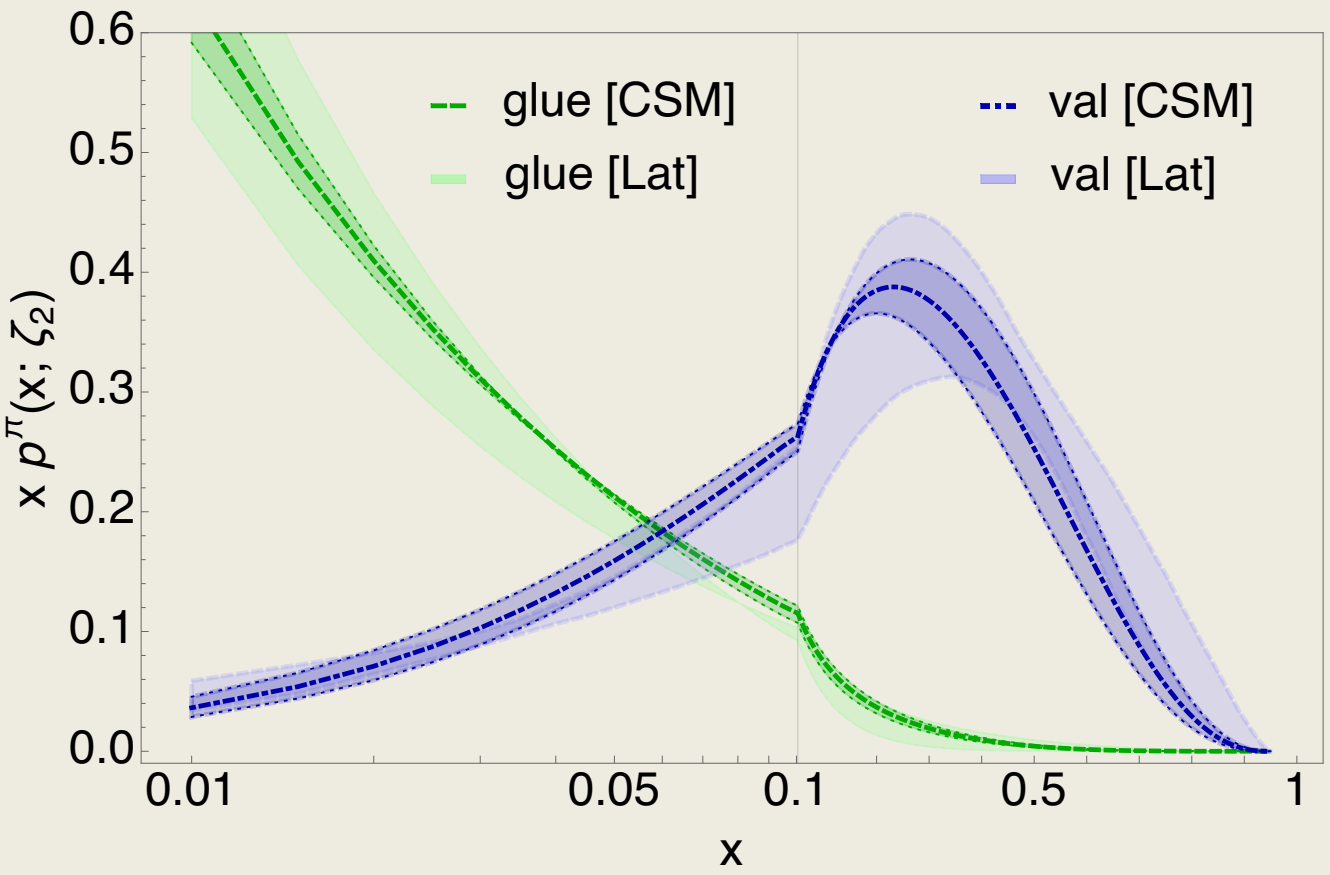


lQCD: ETM Collaboration

- Just valence
- Valence+glue+sea(momentum 1.2…need more consideration)

JAM: 2018->2021
Varius thrreshold resummation move 5% of the momentum from the valence quark to the gluon.

Breaking News!



- Within uncertainties, there is pointwise agreement between the two results on the entire depicted domains

- Val[Lat] Sufian *et al.*, arXiv: 1901.03921
(valence DF: using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space)
- Glue/5[Lat] Fan *et al.*, arXiv: 2104.06372
(Glue DF: using pseudo-PDF approach(Balitsky, Morris and Radyushkin,arXiv:1910.13963))
- CSM see short review: LC and C.D.Roberts, *Chin.Phys.Lett.38(2021)081101.*

- Lattice methods: moments(…)
  LaMET(Ji)
  good lattice cross section(Qiu)
  pseudo-PDF(Radyushkin)

Continuum QCD approach
A long story from 2013
I will focus on "HOW"