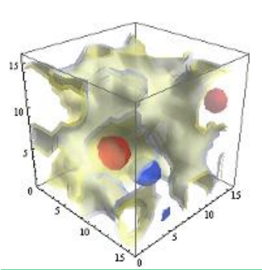
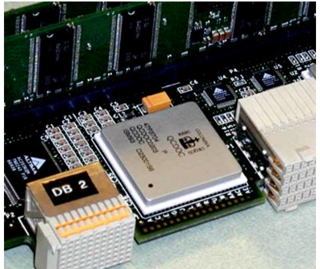
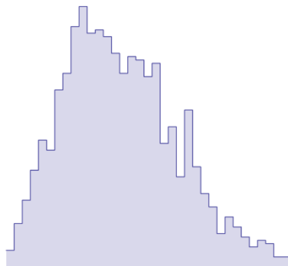
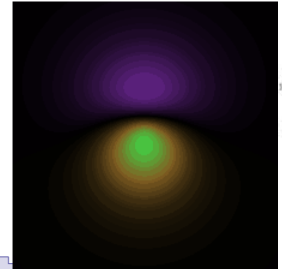
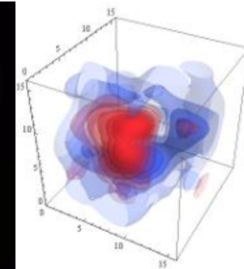
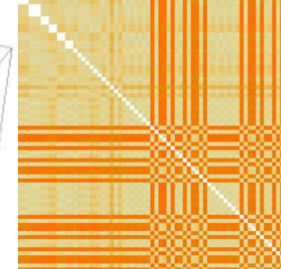
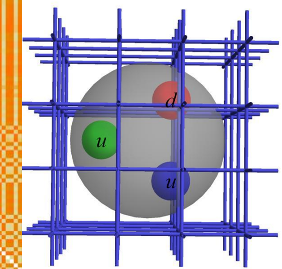
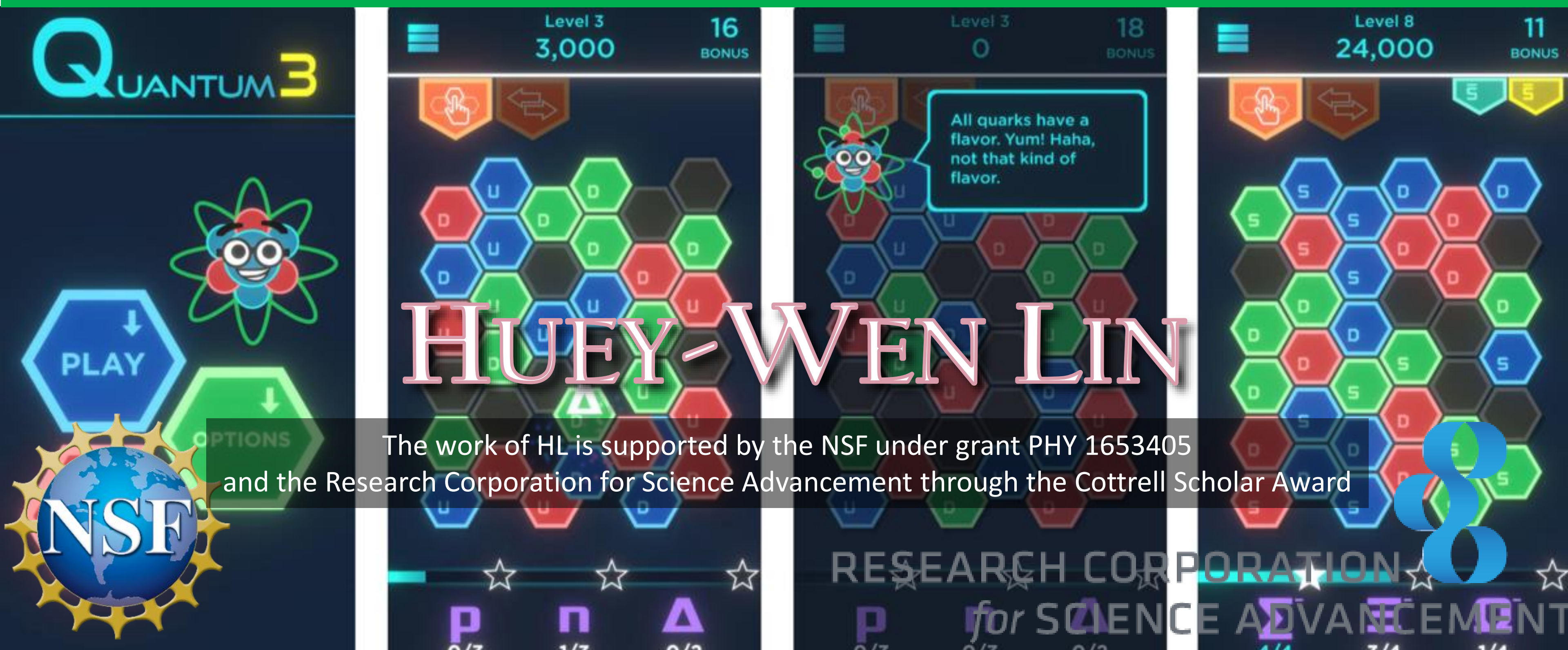



Lattice-QCD Calculations of Parton Distributions



HUEY-WEN LIN

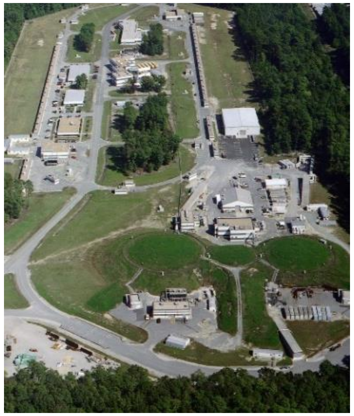
The work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

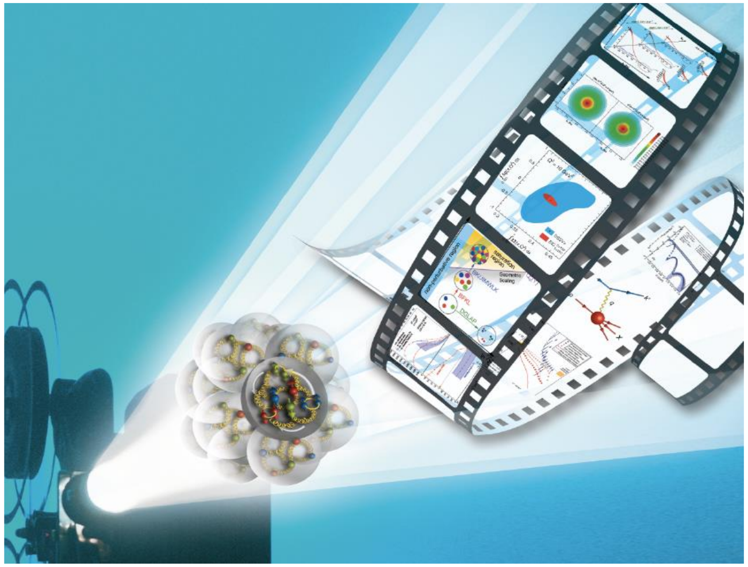
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of hadron

- Many ongoing/planned experiments
  BNL, JLab, J-PARC, COMPASS, GSI, EIC, EIcC, LHeC, …



**Electron Ion Collider:
The Next QCD Frontier**



Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

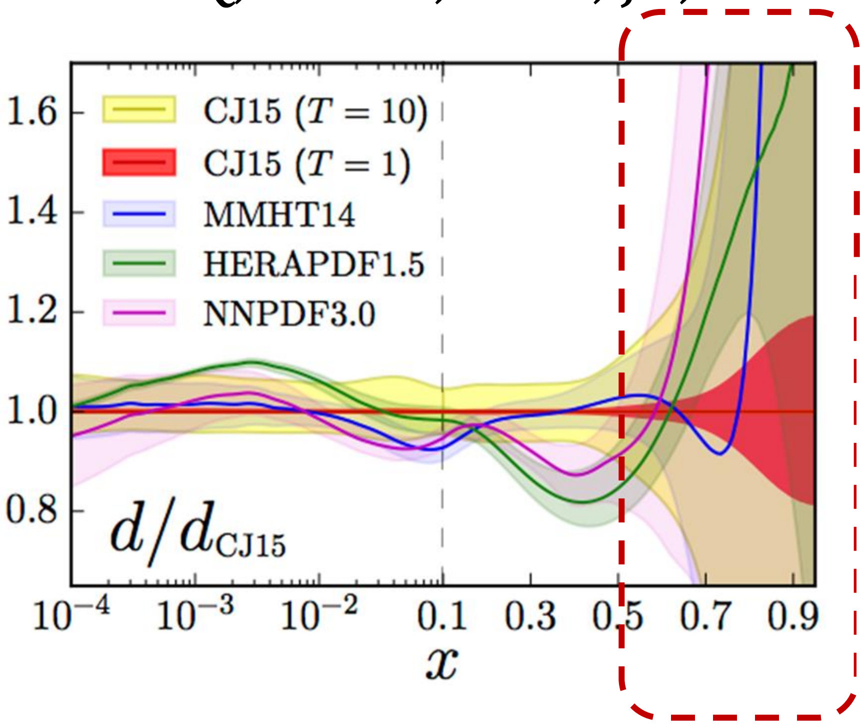
- Assumptions imposed

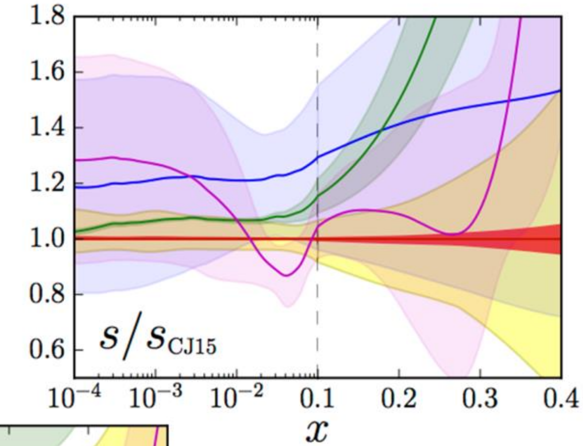
  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

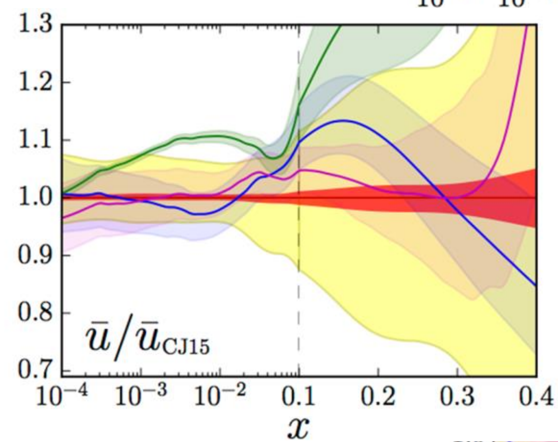
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

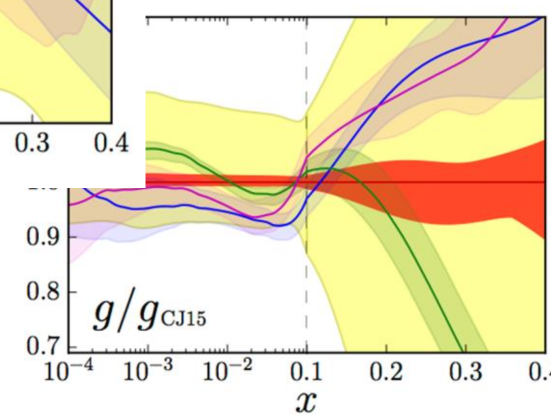
Global Analysis

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.









CTEQ-JLAB
https://www.jlab.org/theory/cj/

Outline

§ Consumer's Guide to Lattice Hadron Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)
- PDF Moments

§ *x*-dependent PDFs of Nucleon/Pion/Kaon

- Recent selected new calculations:

gluon, strange, GPDs, …

Biased selected results, highlighting work done by MSU students/postdocs

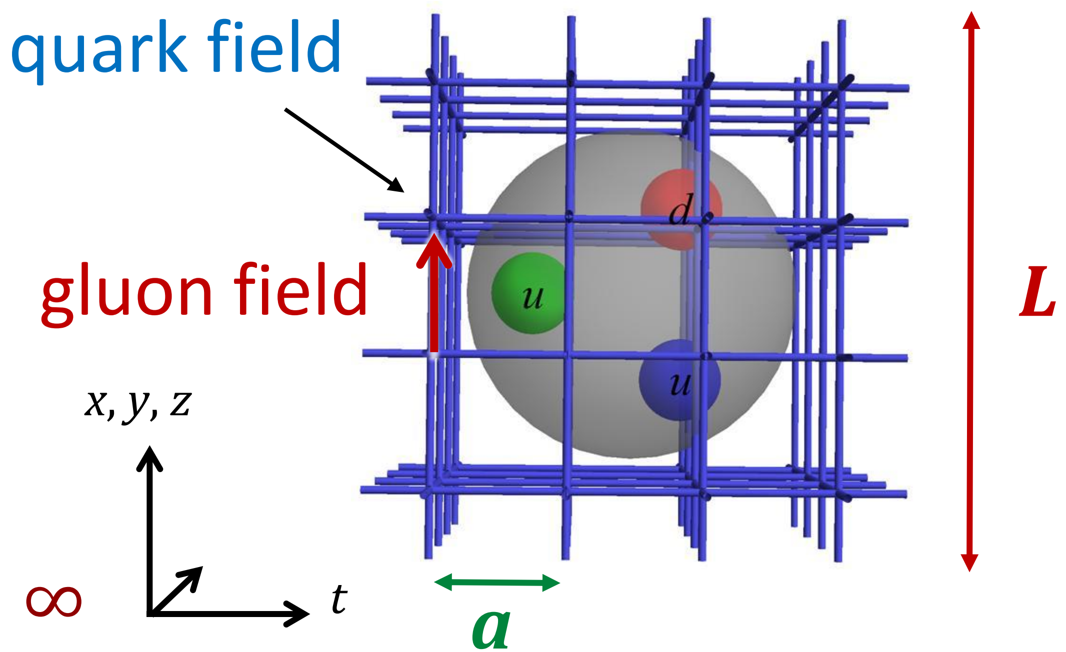
What is Lattice QCD?

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}\,O\left(\bar{\psi},\psi,A\right)$$

  in **Euclidean** space

  - Quark mass parameter (described by m_π)
  - Impose a UV cutoff
    discretize spacetime
  - Impose an infrared cutoff
    finite volume



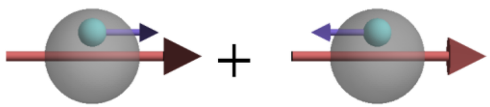
- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

PDFs on the Lattice

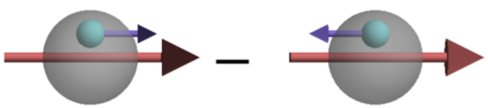
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



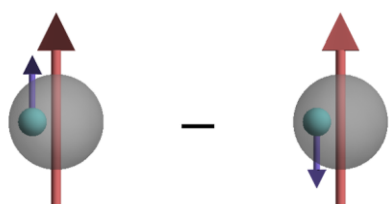
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

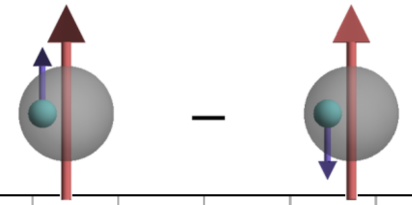
very poorly known

§ True distribution can only be recovered with all moments

Moments of PDFs

§ Only lowest few moments

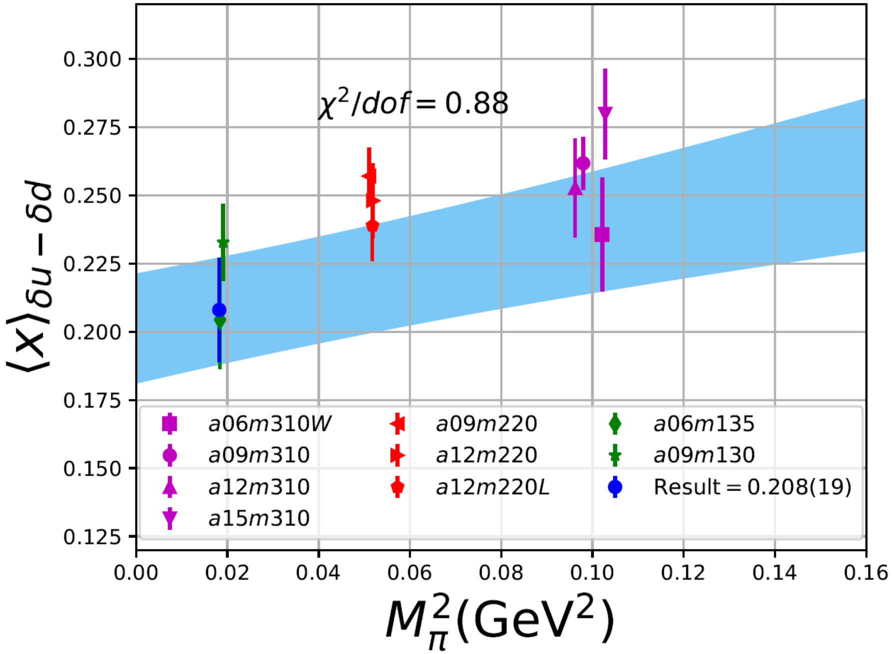
$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

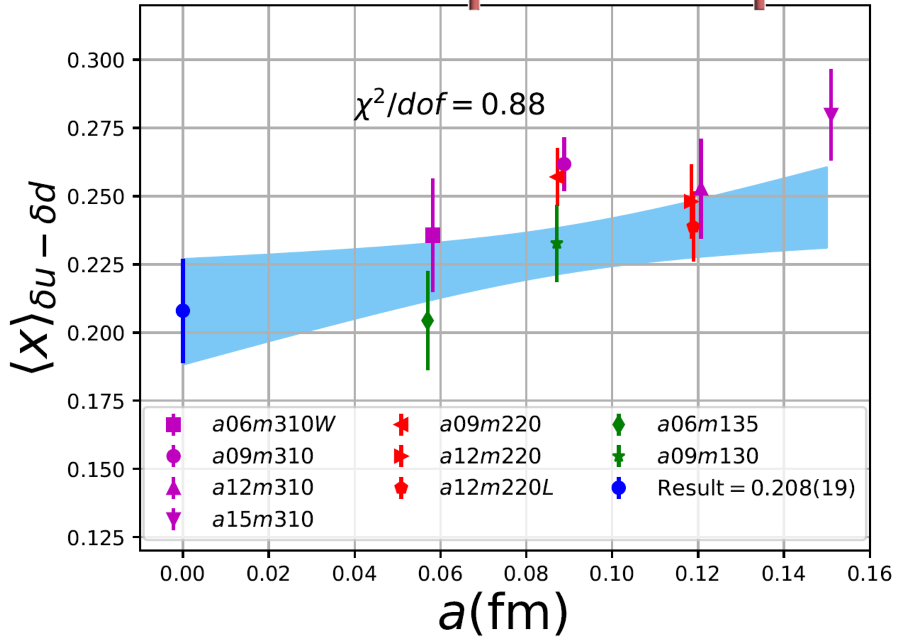


§ State-of-the-art example

- Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





§ Usually more than one LQCD calculation

- Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	0.10 — 1.1
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	-0.14 — 0.91
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	-0.97 — 0.47
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	N/A
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

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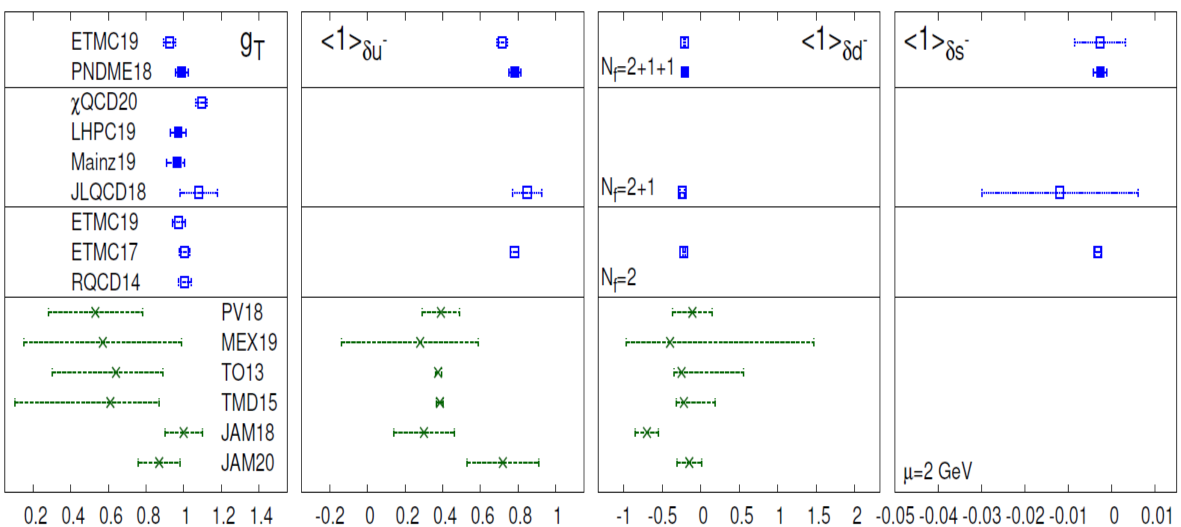
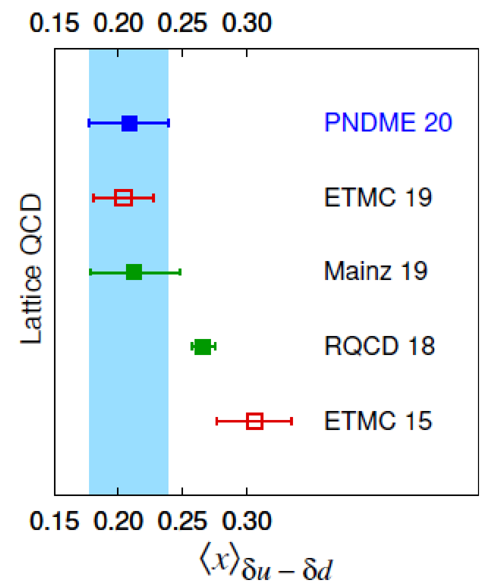


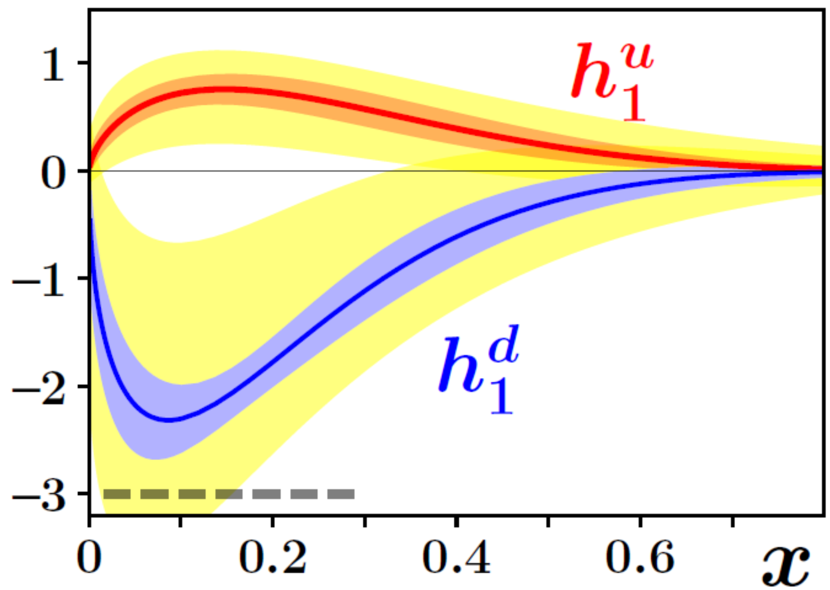


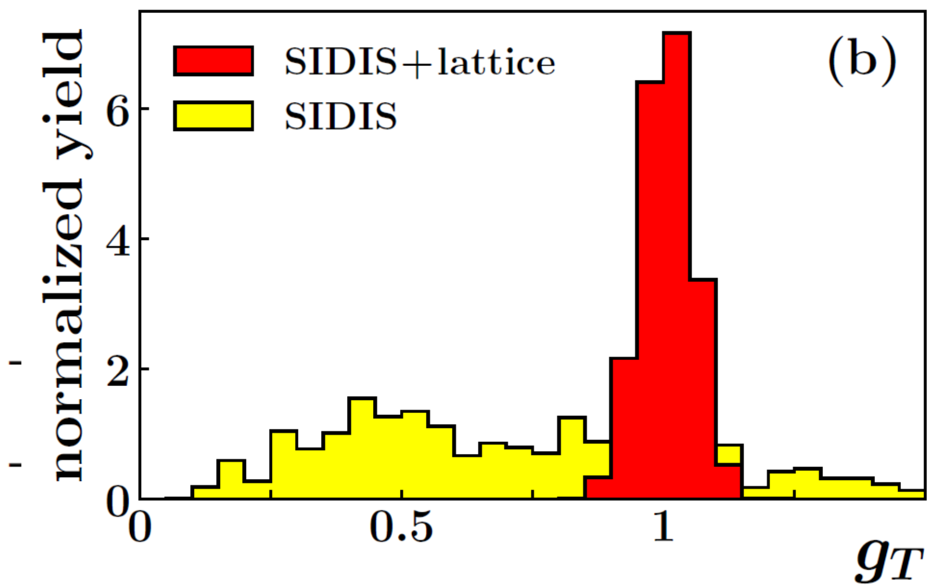
S. Mondal et al (PNDME), 2005.13779

From Moments to PDFs

§ Improved **transversity** distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

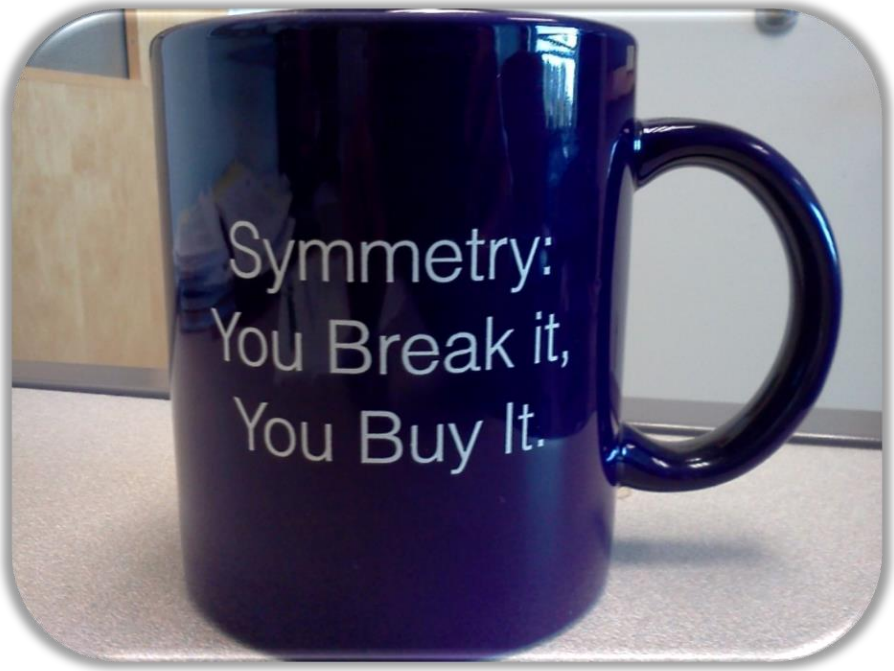




Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea



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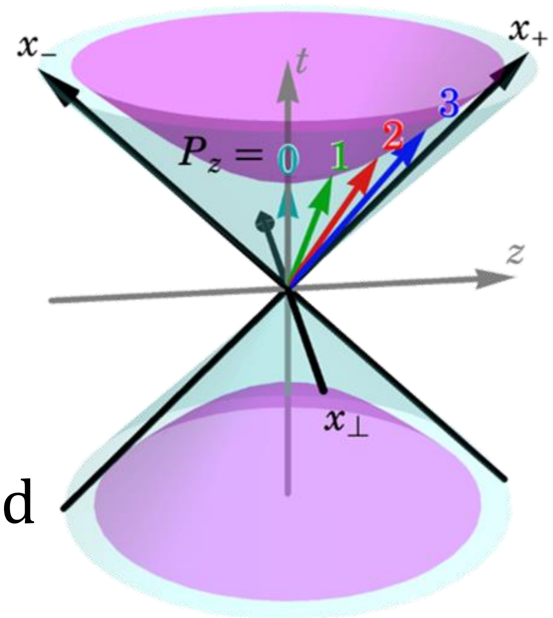


- **New Strategy:** Xiangdong Ji, PRL 111, 039103 (2013);
  - Adopt lightcone description for PDFs
  - Calculate finite-boost quark distribution
    - In $P_z \to \infty$ limit, parton distribution recovered
    - For finite P_z, corrections are applied through effective theory
  - Feasible with today's resources!

Bjorken-x Dependent of Nucleon/Pion/Kaon PDFs

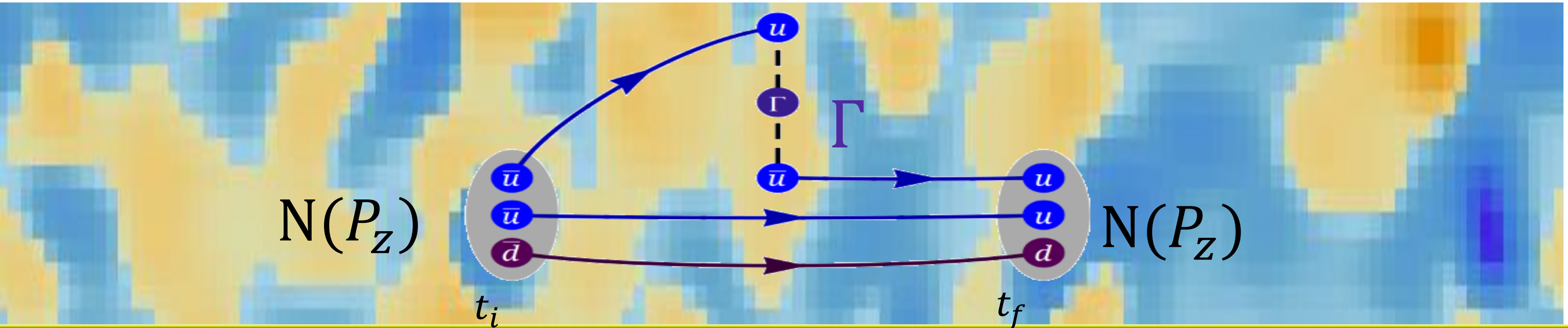
Due to time constraints, I will quickly show a number of recent results

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (**LaMET**)
  (X. Ji, 2013; See 2004.03543 for review)



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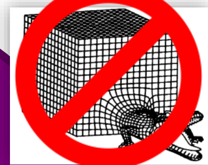
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!



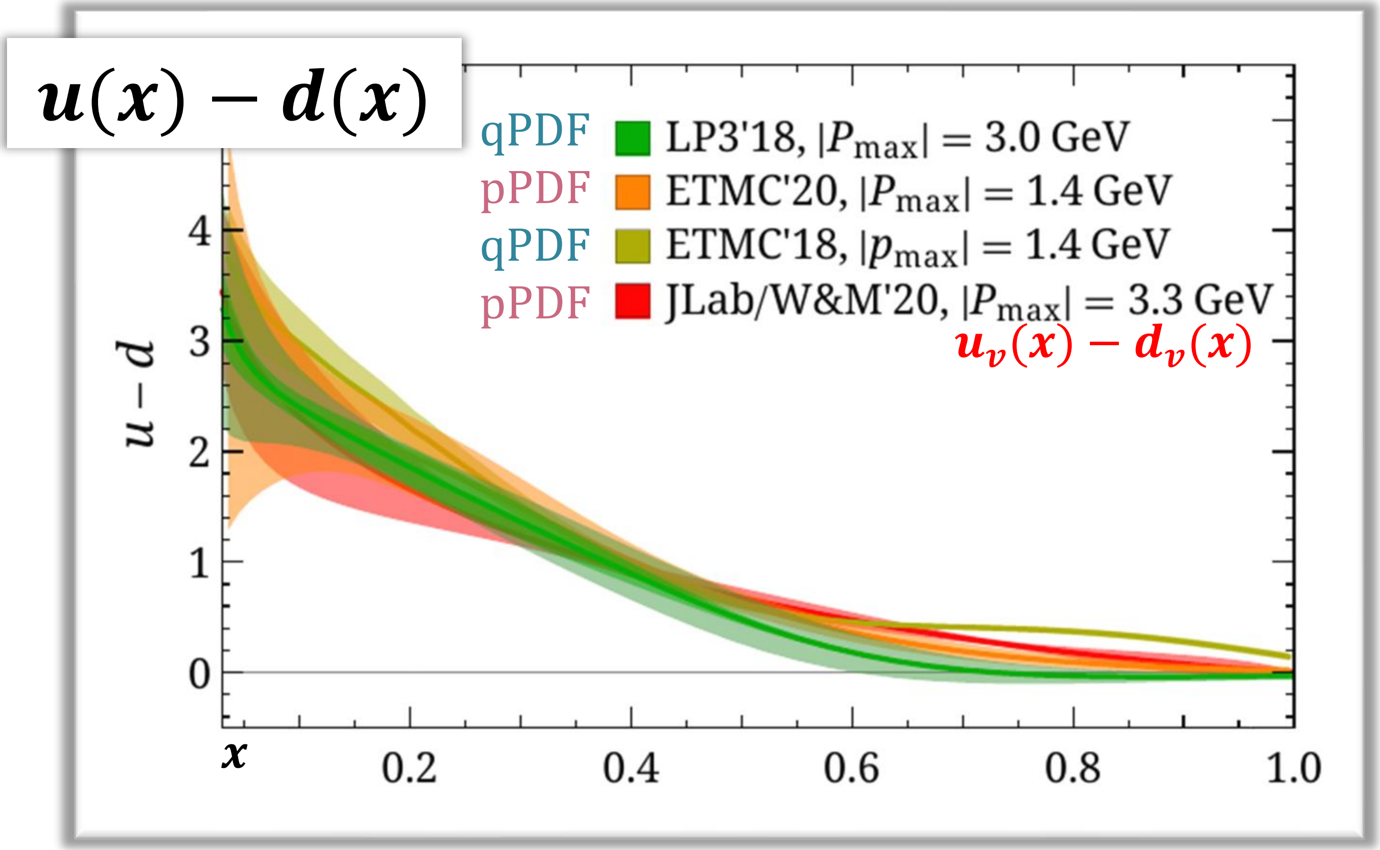
Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

- **Kernel is a complicated object**
  - Mostly calculated up to one-loop level; two-loops results available recently
- **Inverse problem to extract the wanted distribution**
  - Slightly different approaches from each group; systematics vary
- **Larger momentum the better**
  - Smaller systematics: $O\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$
  - Needed in the lattice calculations in all methods to reach small-*x* region
    - Current projects focus on mid- to large-*x*

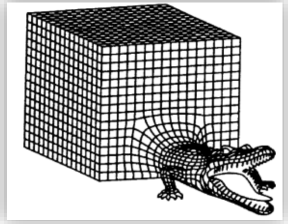
Physical Pion Mass Results

§ Summary of **physical pion mass** results

- Recent study increase boost momenta $P_z > 3$ GeV



2006.08636, PDFLattice2019 report



Finite volume,
Discretization,
…

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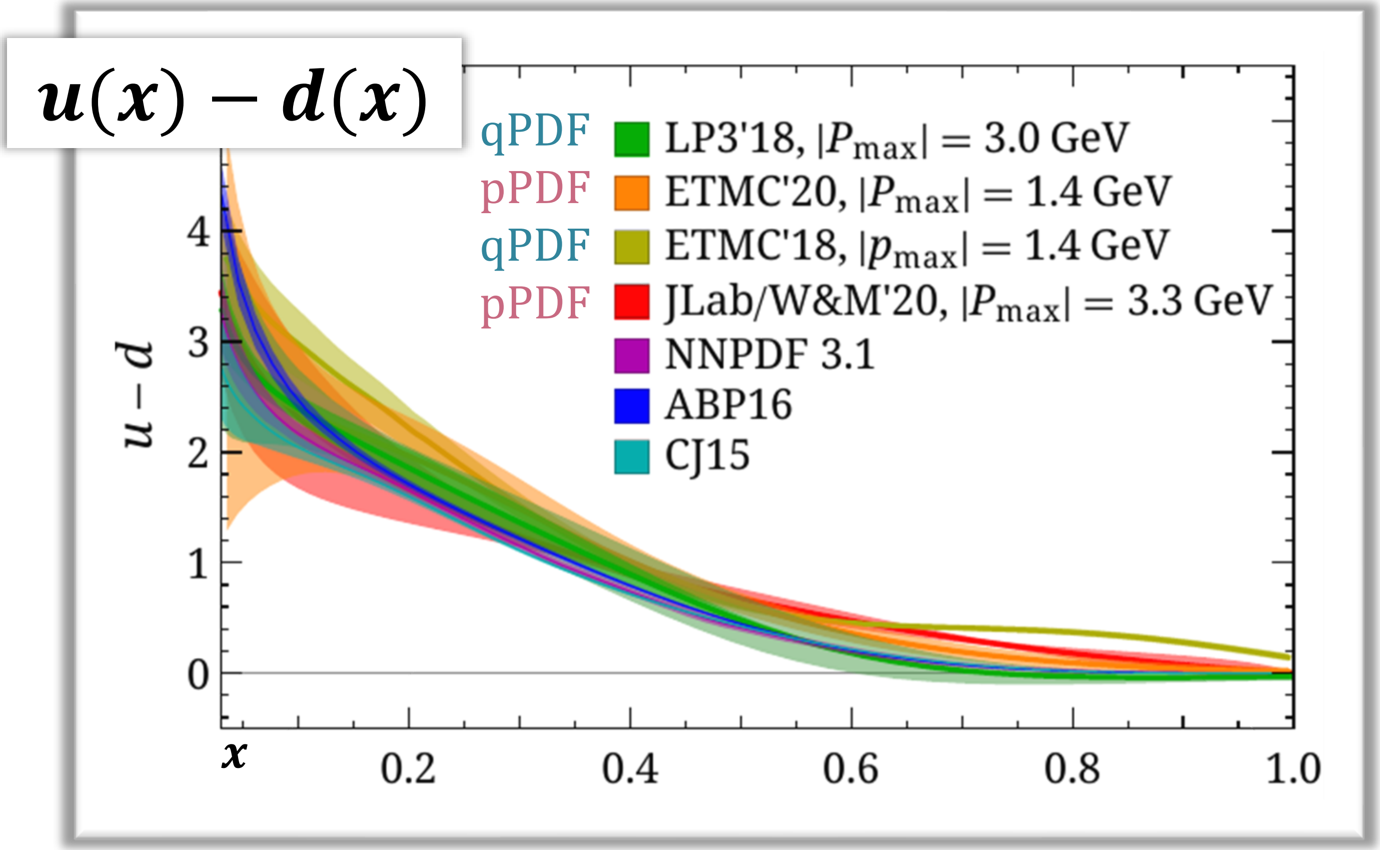

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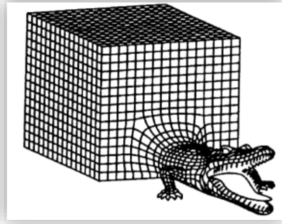

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Discretization,
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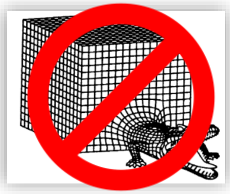
First Continuum PDF

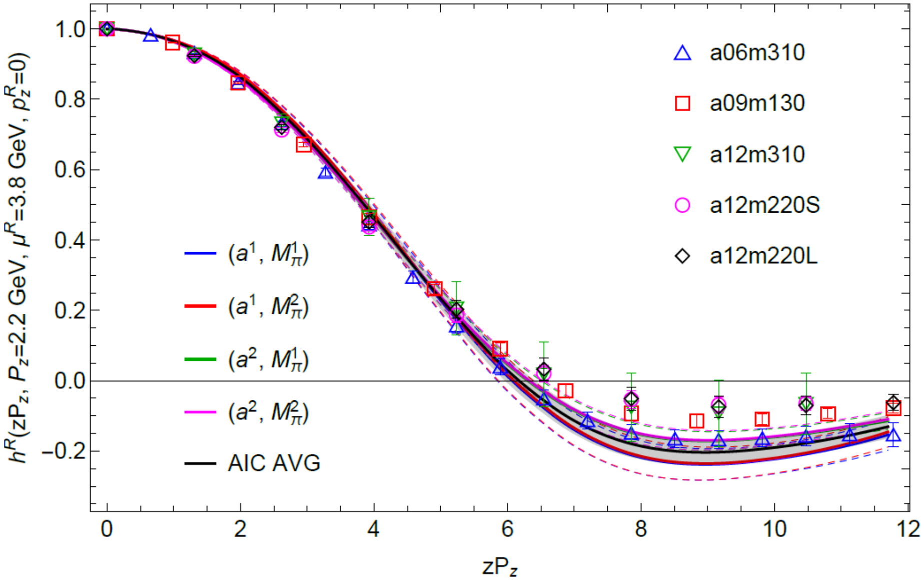
§ Nucleon isovector PDFs using quasi-PDFs in the continuum limit

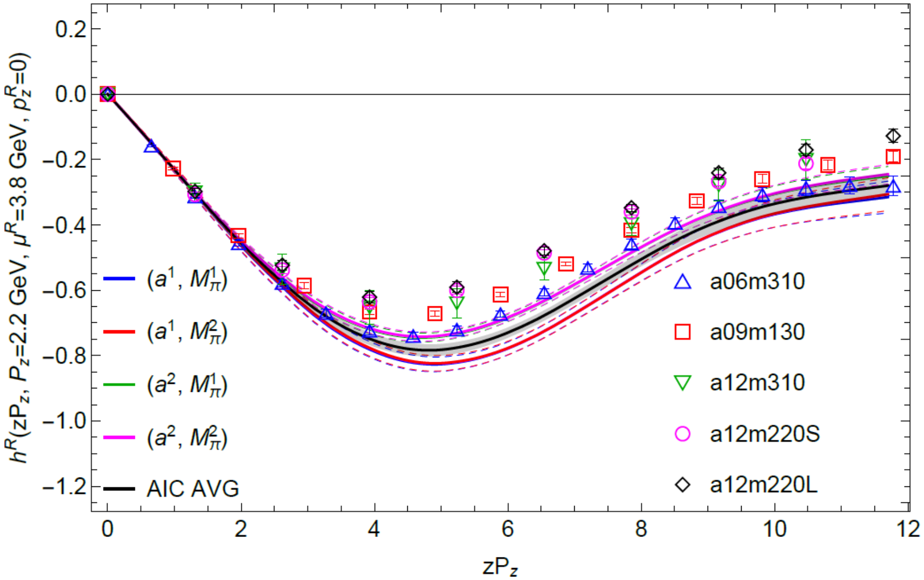
- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.09, 0.12\}$ fm,
  $M_\pi \in \{\mathbf{135}, 220, 310\}$-MeV pion,
  $M_\pi L \in \{3.3, 5.5\}$, $P_z \approx 2.2, 2.6$ GeV



- Naïve extrapolation to physical-continuum limit

2011.14971, HL et al (MSULat)





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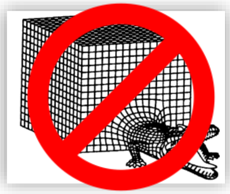

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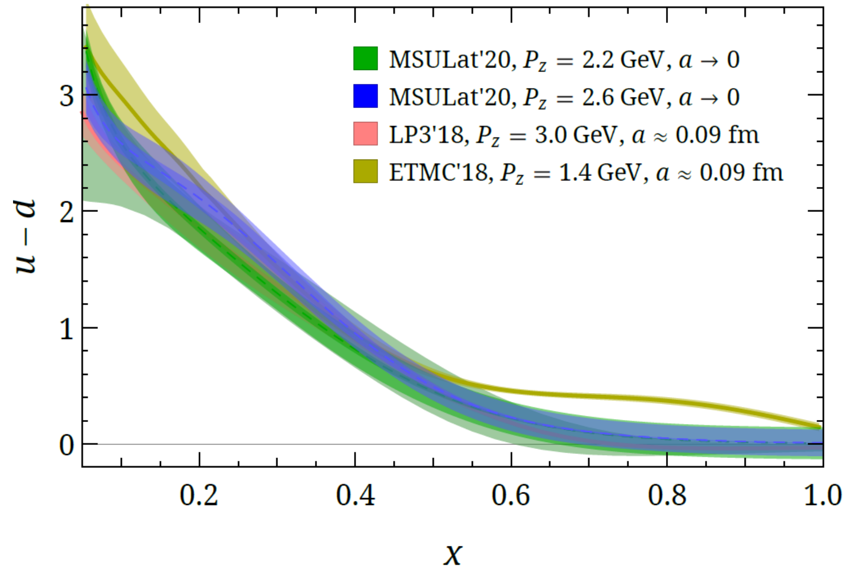
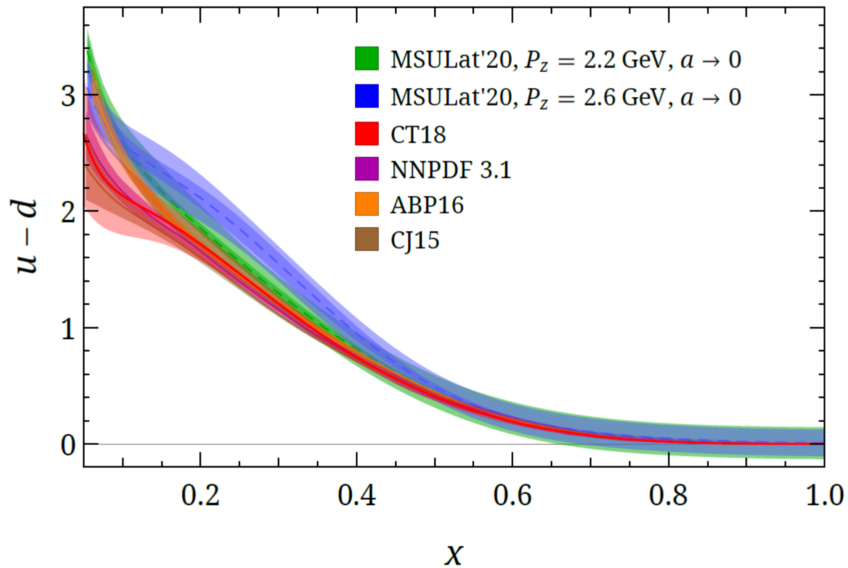
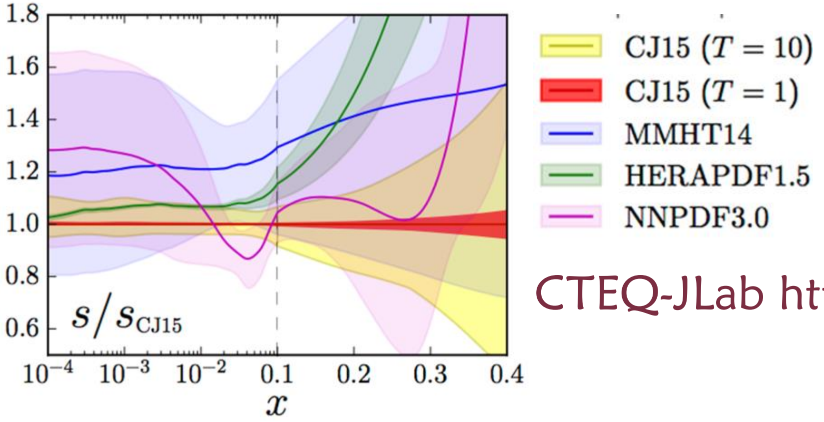




Also see ETMC's continuum-limit work at 370 MeV with $P_z \approx 1.8$ GeV, 2011.00964

First Lattice Strange PDF

§ Large uncertainties in global PDFs



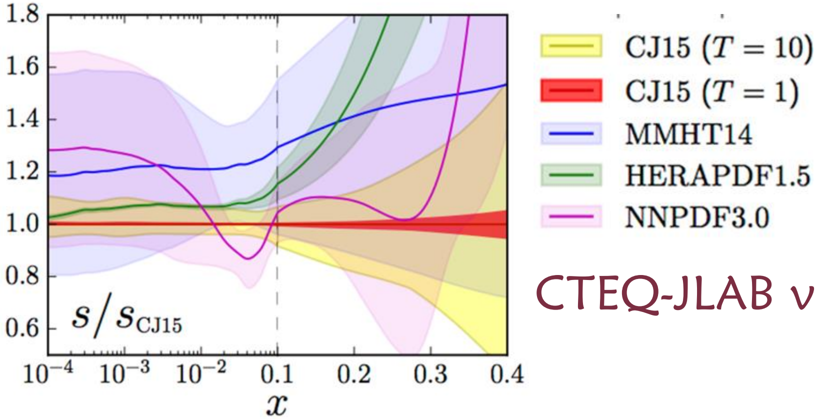
- Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

CTEQ-JLab https://www.jlab.org/theory/cj

First Lattice Strange PDF

§ Strange PDFs have large uncertainties in global PDFs



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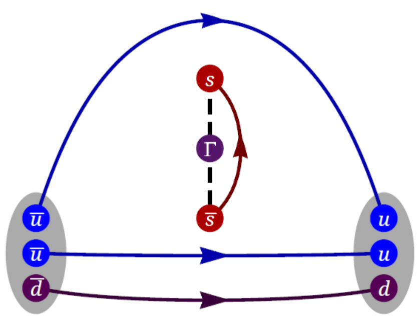
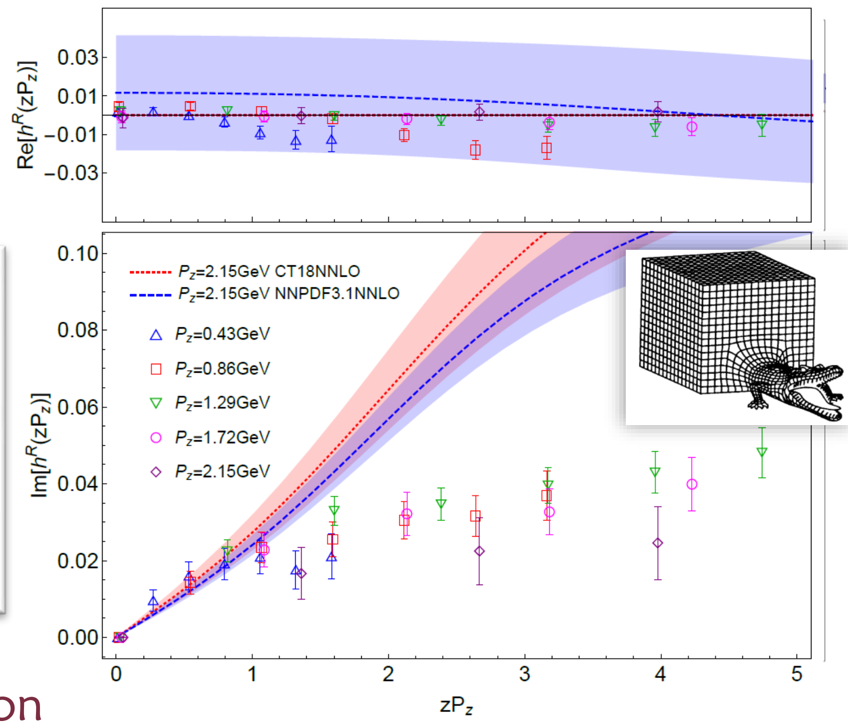
CTEQ-JLAB vhttps://www.jlab.org/theory/cj/

§ First lattice results by MSULat



$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$

$$\mathrm{Im}[h(z)] \propto \int dx\,(s(x) + \bar{s}(x))\sin(xzP_z)$$

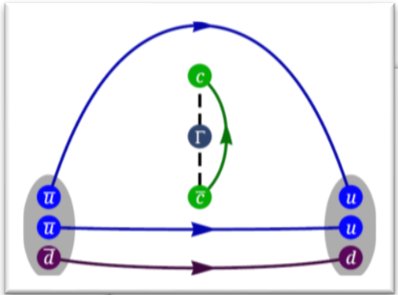


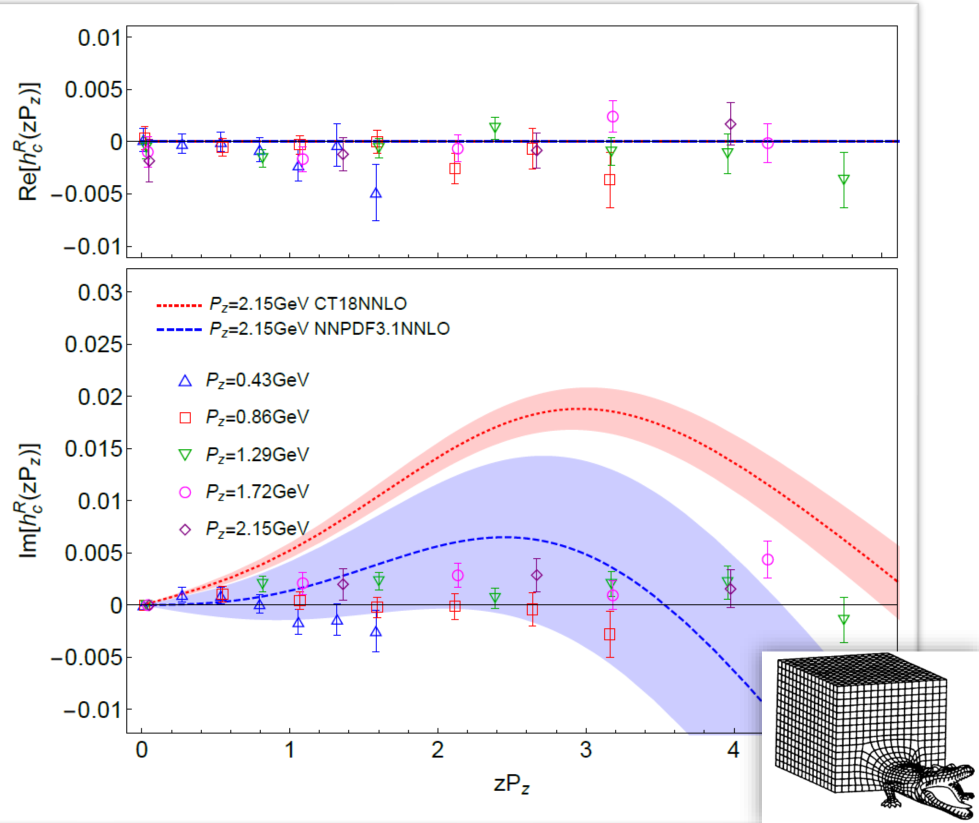
2005.12015, Zhang, Lin, Yoon

First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method

2005.12015, R. Zhang et al (MSULat)





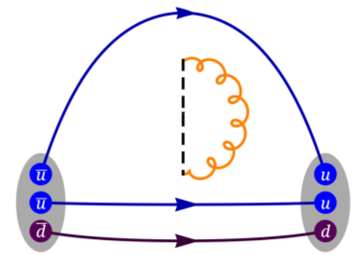
- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF

First Lattice Nucleon Gluon PDF

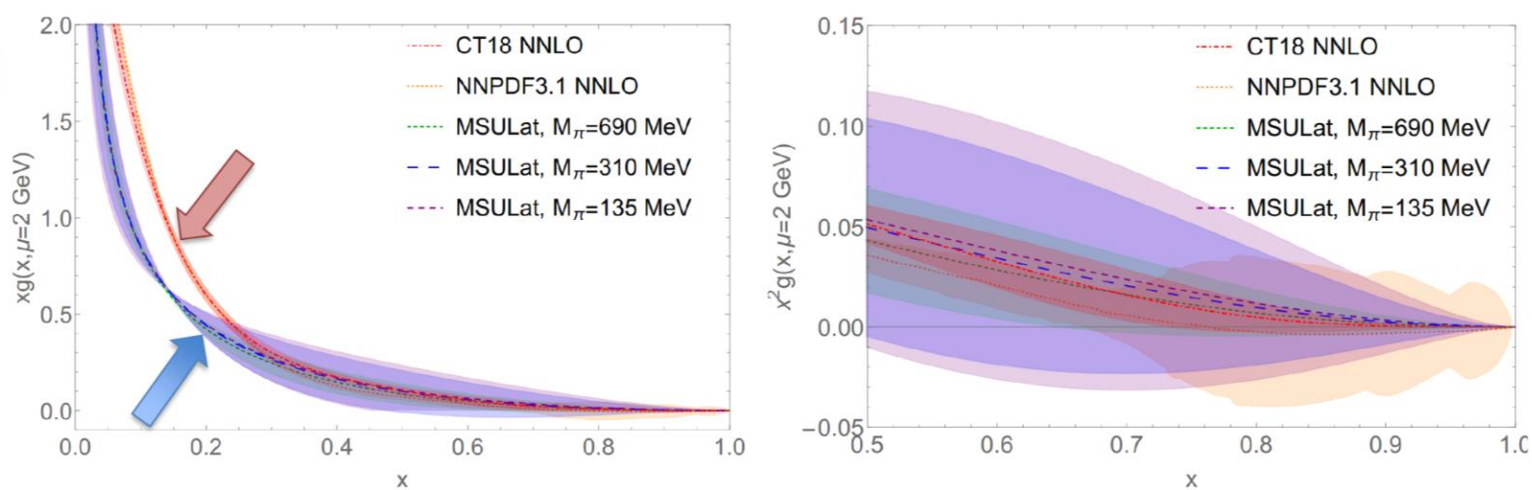
§ Gluon PDF using pseudo-PDF

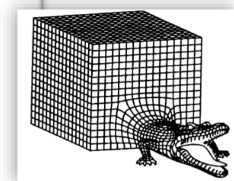
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

Z. Fan. et al (MSULat), 2007.16113



The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\text{MS}}$ scheme.





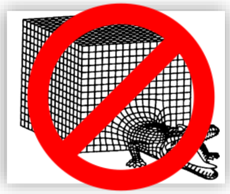
Zhouyou Fan (MSU)

Slide by Zhouyou Fan@2020 APS DNP Meeting

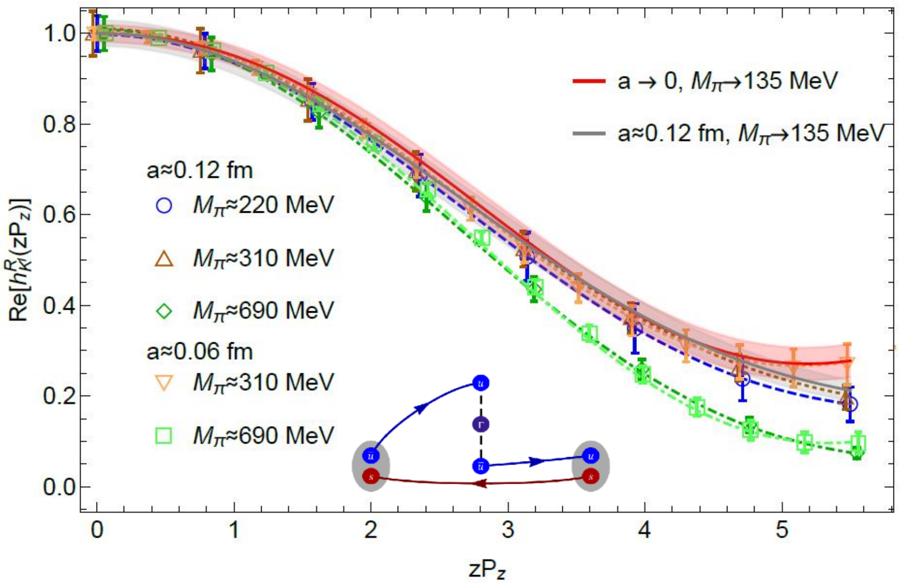
Pion and Kaon PDFs

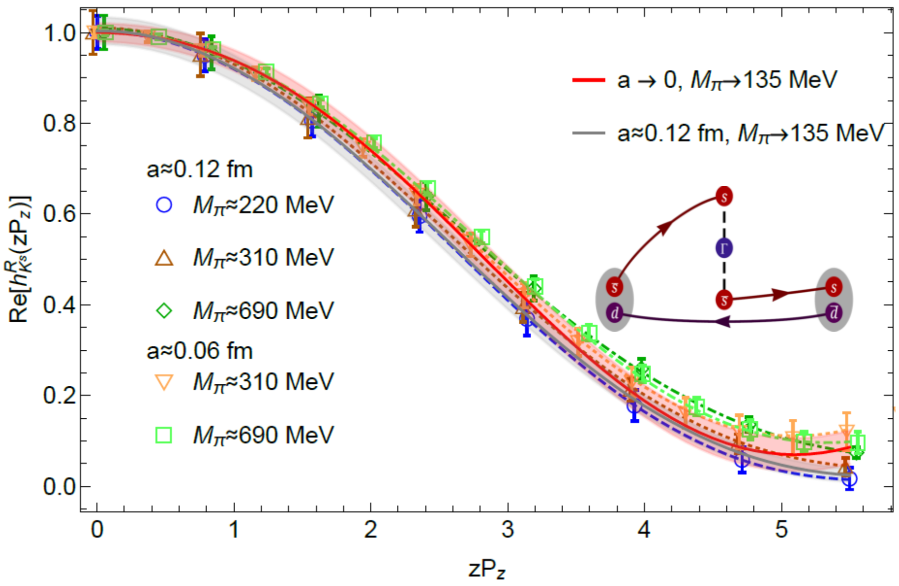
§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)

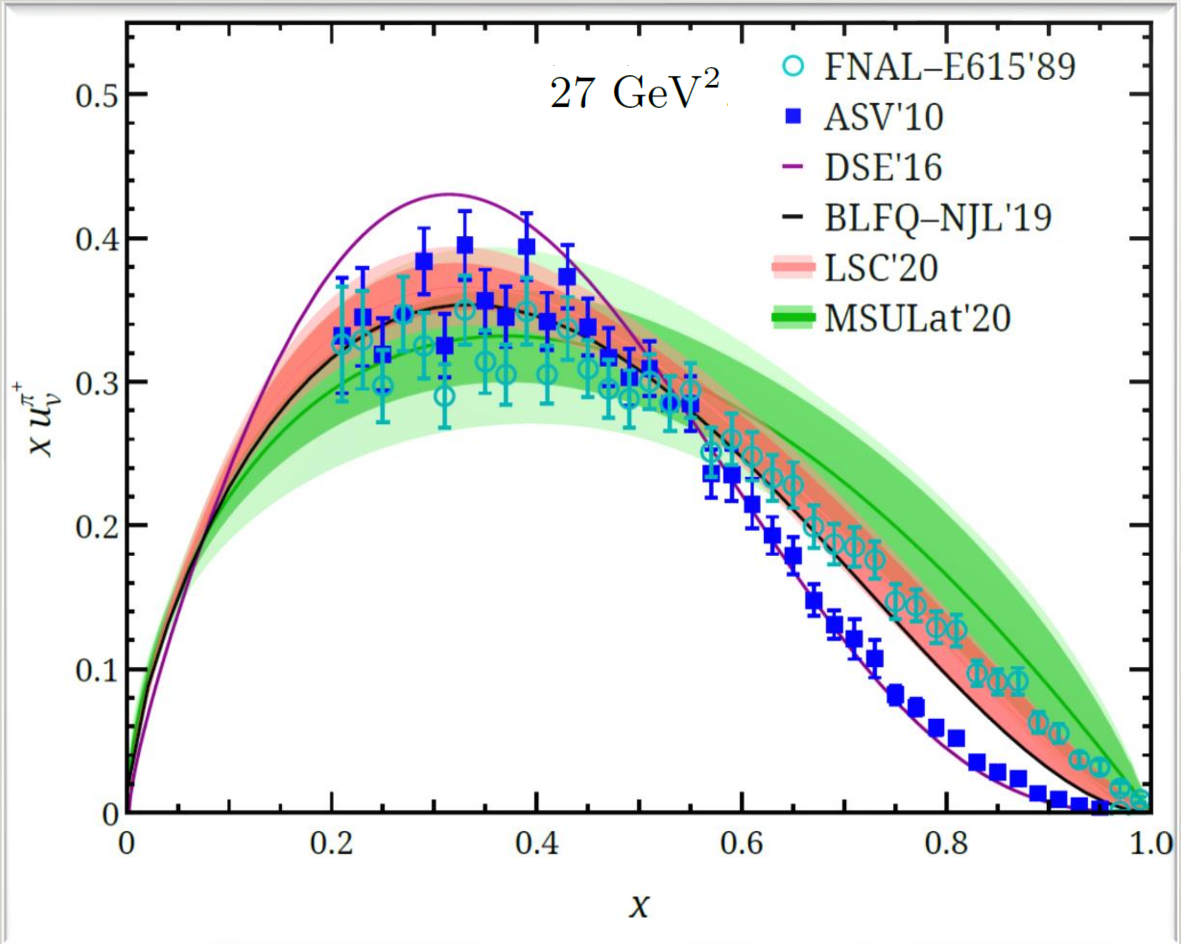


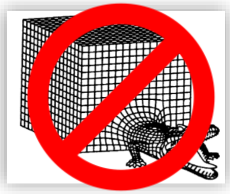


Pion Valence Quark PDFs

§ **Pion**/Kaon PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)





2003.14128 HL et al (MSULat)

See S. Mukherjee's talk for BNL group's work at 310 MeV with a ≈ 0.04 and 0.06 fm, 2007.06590

Kaon Valence-Quark PDFs

§ Pion/**Kaon** PDFs using quasi-PDF in the continuum limit

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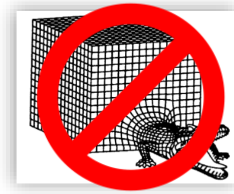
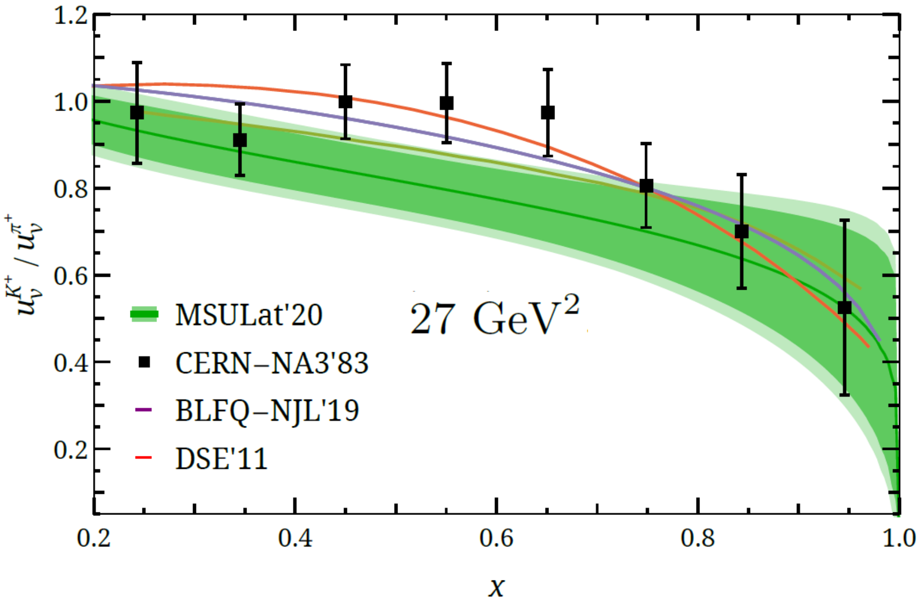
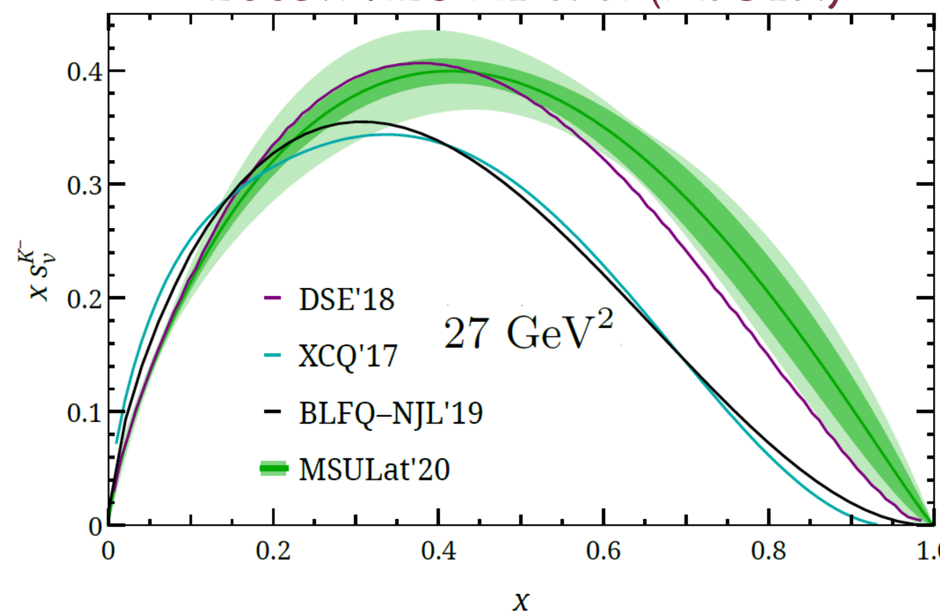


2003.14128 HL et al (MSULat)





Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
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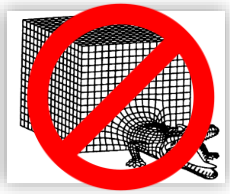
  $P_z \approx \{1.3, 1.7\}$ GeV



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$ 2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle \left(u_v^{K^+}\right)$	$\langle x^n \rangle \left(s_v^{K^-}\right)$
1	$0.192(8)_{\mathrm{stat}}(6)_{\mathrm{syst}}$	$0.261(8)_{\mathrm{stat}}(8)_{\mathrm{syst}}$
2	$0.080(7)_{\mathrm{stat}}(6)_{\mathrm{syst}}$	$0.120(7)_{\mathrm{stat}}(9)_{\mathrm{syst}}$
3	$0.041(6)_{\mathrm{stat}}(4)_{\mathrm{syst}}$	$0.069(6)_{\mathrm{stat}}(8)_{\mathrm{syst}}$

Kaon Valence-Quark PDFs

- Pion/**kaon** PDFs using quasi-PDF in the continuum limit
  - Lattice details: clover/2+1+1 HISQ (MSULat)
    $a \approx \{0.06, 0.12\}$ fm,
    $M_\pi \in \{220, 310, 690\}$-MeV pion
    $P_z \approx \{1.3, 1.7\}$ GeV

- First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$
- Later ETMC **260**-MeV results on $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle \left(u_v^{K^+}\right)$	$\langle x^n \rangle \left(s_v^{K^-}\right)$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
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3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

2010.0349, 2104.02247

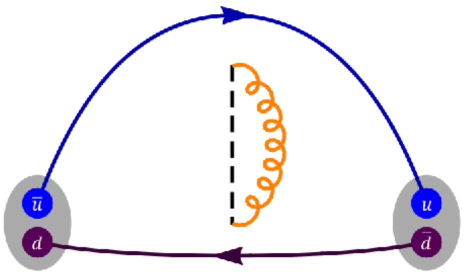
n	$\langle x^n \rangle \left(u_v^{K^+}\right)$	$\langle x^n \rangle \left(s_v^{K^-}\right)$
1	$0.246(2)_{\text{stat}}(2)_{\text{syst}}$	$0.317(2)_{\text{stat}}(1)_{\text{syst}}$
2	$0.093(5)_{\text{stat}}(3)_{\text{syst}}$	$0.134(5)_{\text{stat}}(2)_{\text{syst}}$
3	$0.035(6)_{\text{stat}}(3)_{\text{syst}}$	$0.075(5)_{\text{stat}}(1)_{\text{syst}}$

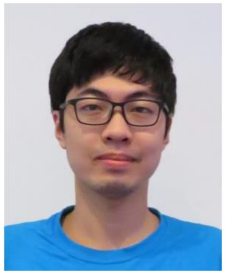
First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

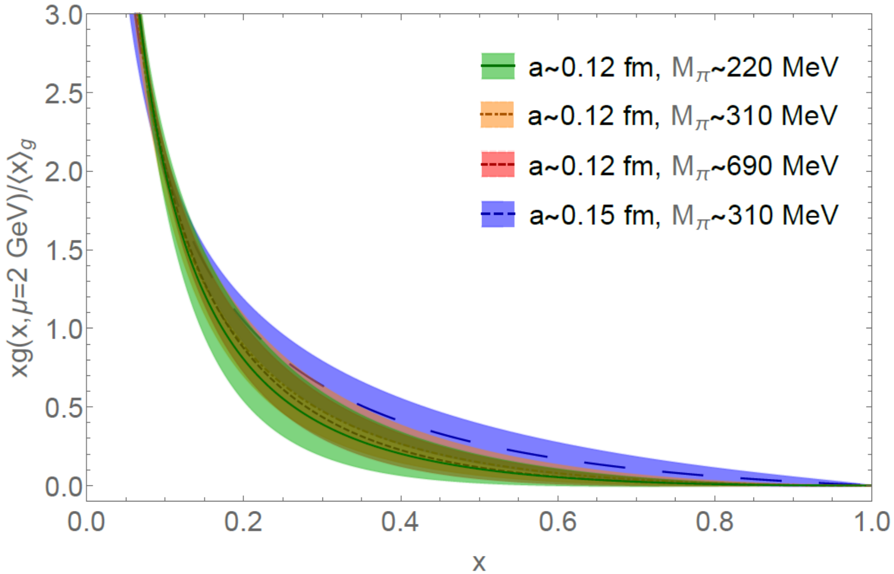
- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.12, 0.15\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_{z,\max} \approx 2.3$ GeV

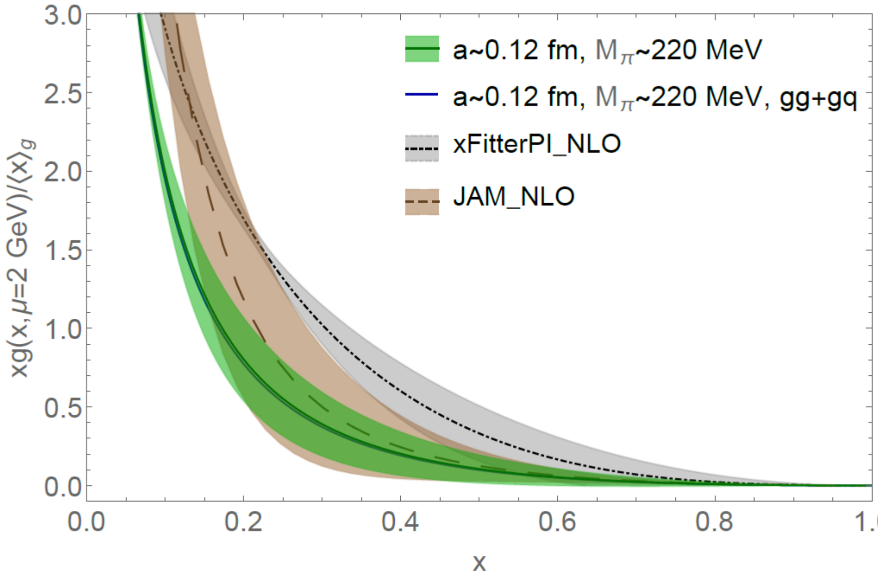
  2104.06372, Fan, HL(MSULat)





Zhouyou Fan (MSU)

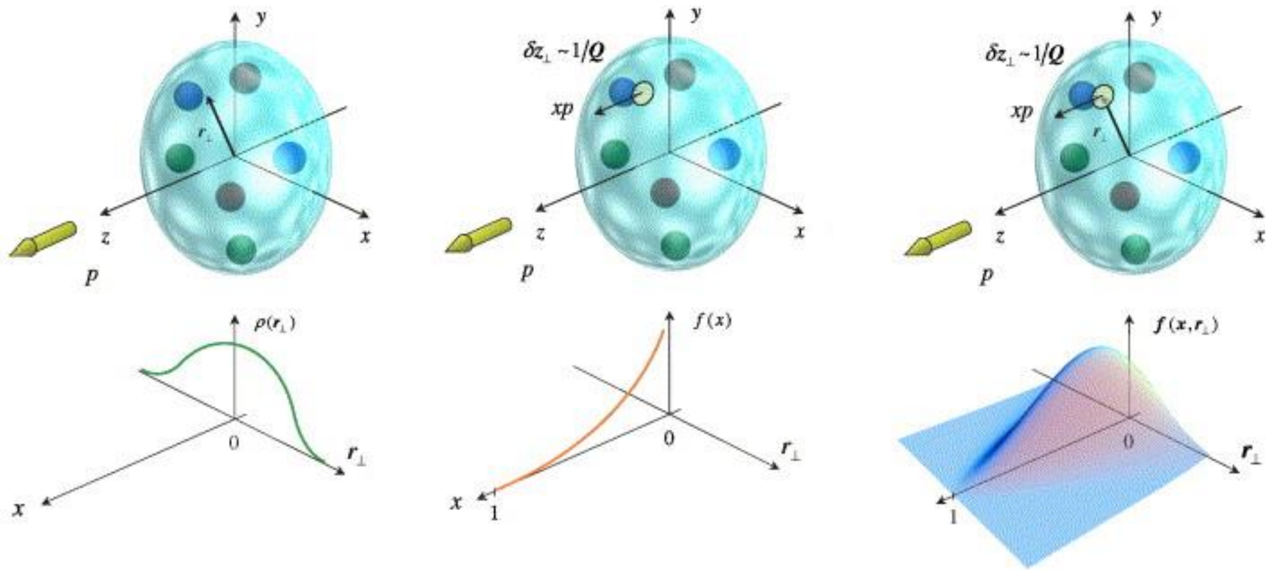




Bjorken-x Dependent GPDs

Due to time constraints, I only have time to show selected recent results

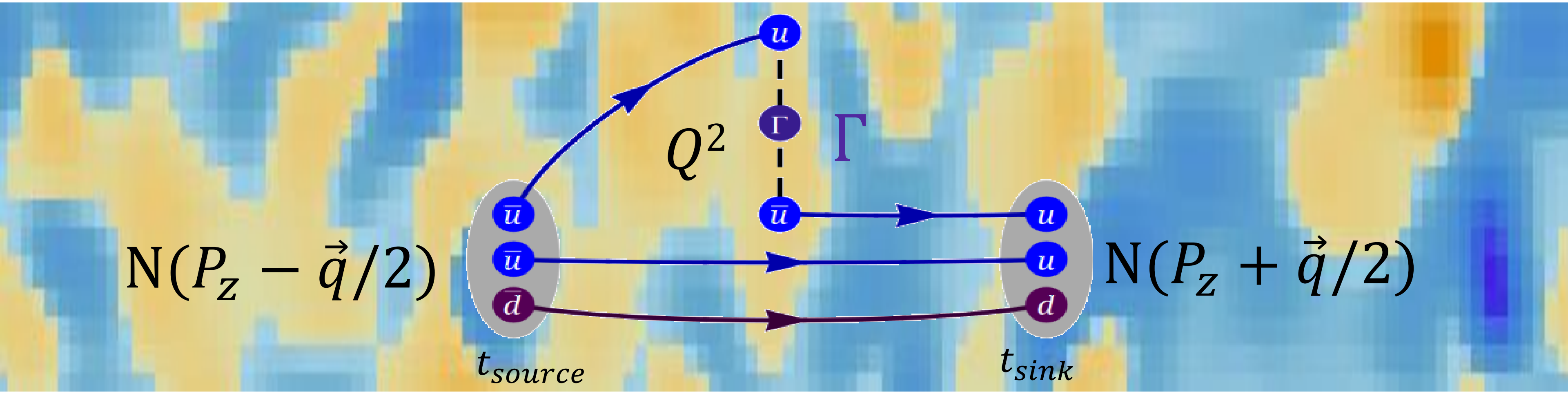
Bjorken-x Dependent GPDs



Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)



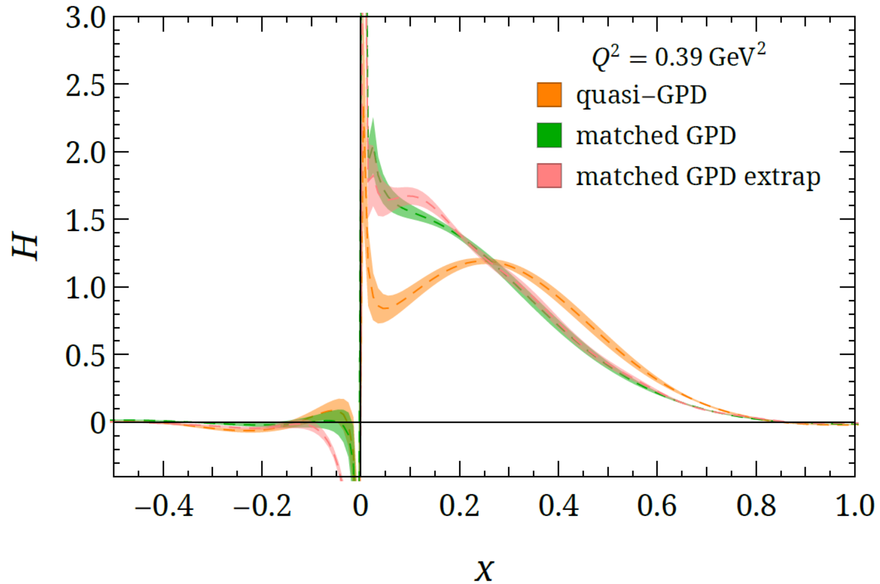
$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_z)$$
$$= \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(H(x, \tilde{\xi}, t, \bar{P}_z) \gamma^0 + E(x, \tilde{\xi}, t, \bar{P}_z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

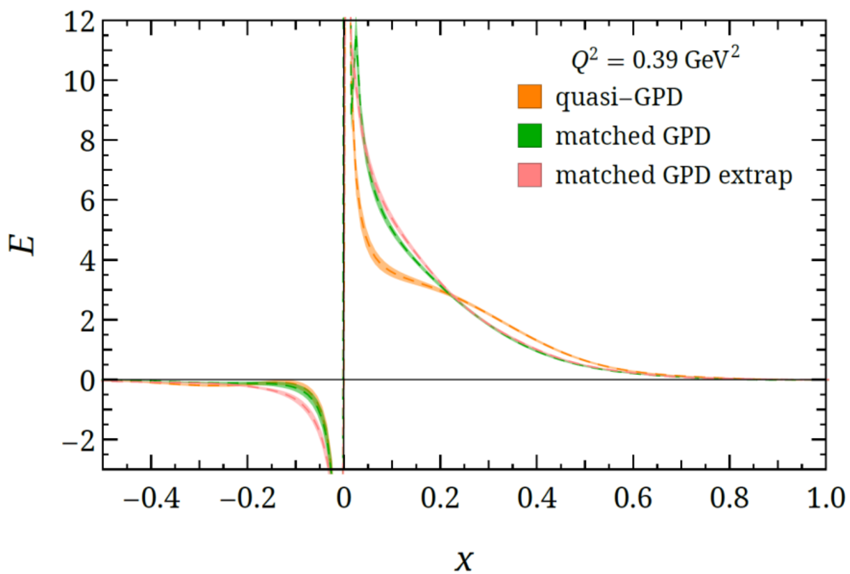
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results





Extrap: $c_1(-izP_z)^{-d_1} + c_2 e^{izP_z}(izP_z)^{-d_2}$

X. Ji et al, 2008.03886

HL (MSULat), 2008.12474, to appear on PRL

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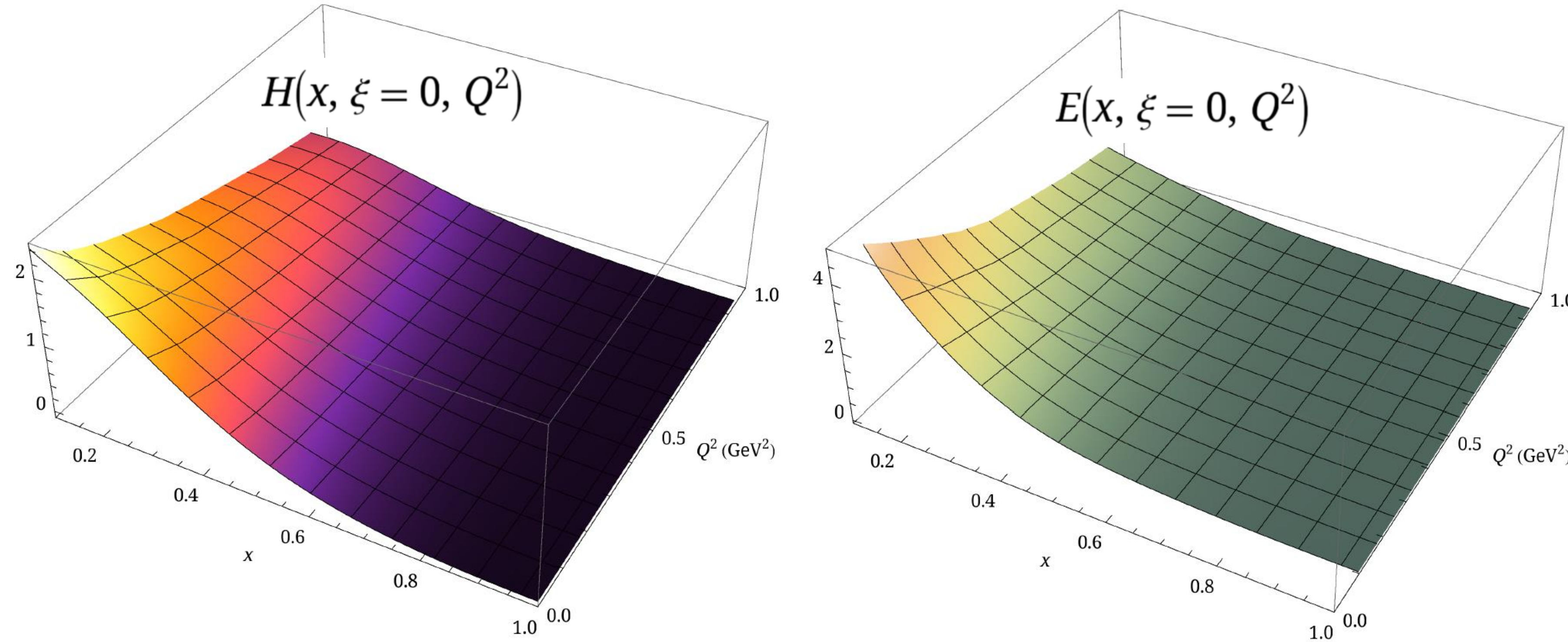

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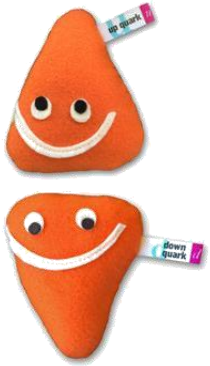
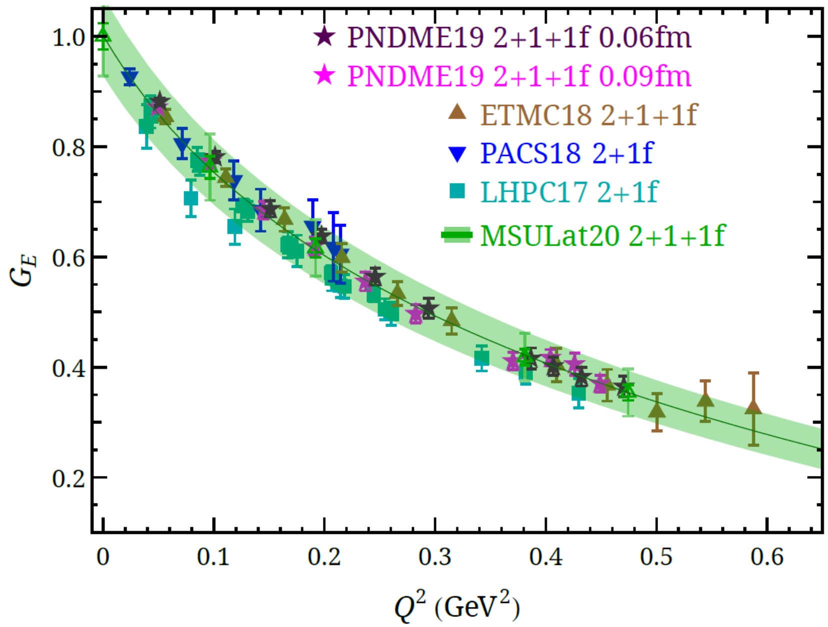
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} H^q(x,\xi,t) = \sum_{i=0,\mathrm{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \mathrm{even}}$$



$n = 1$



HL (MSULat), 2008.12474, to appear on PRL

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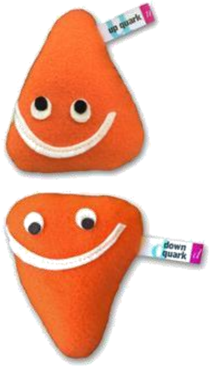
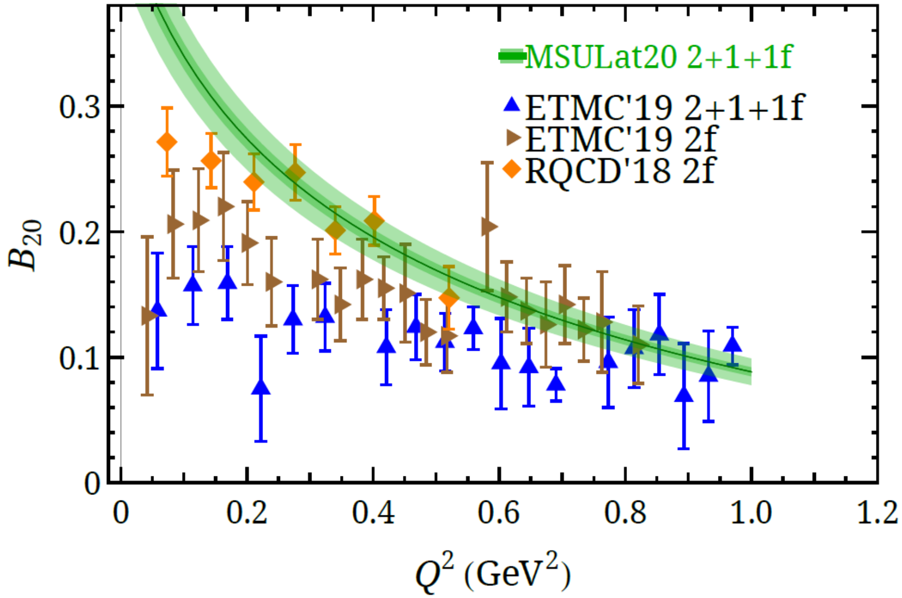
$$\int_{-1}^{+1} dx\, x^{n-1} E^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t)\Big|_{n\ \text{even}}$$



$n = 2$



HL (MSULat), 2008.12474, to appear on PRL

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

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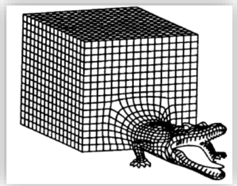
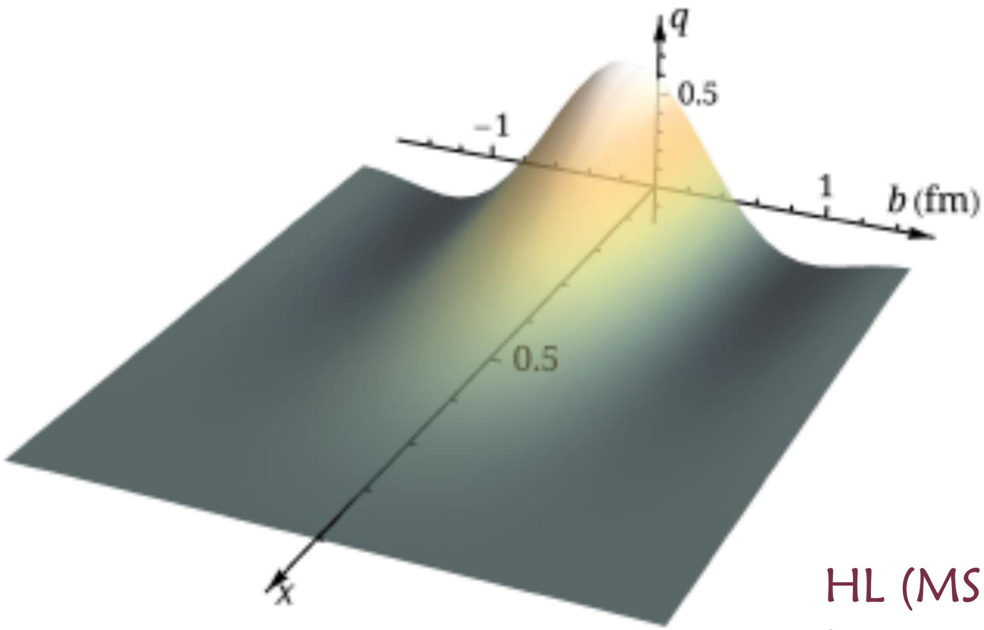


finite-volume, discretization, …

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^2\right) e^{i\vec{q}\cdot\vec{b}}$$

M. Burkardt, hep-ph/0207047



HL (MSULat), 2008.12474, to appear on PRL

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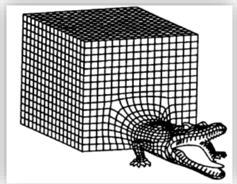

finite-volume, discretization, …

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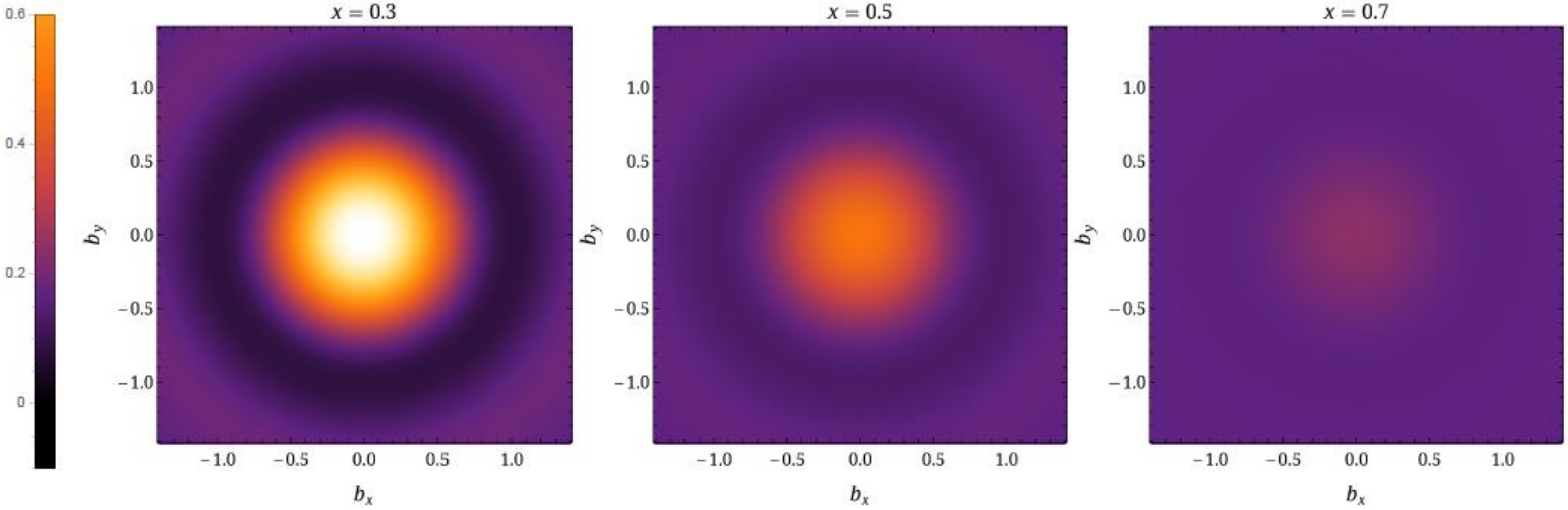

HL (MSULat), 2008.12474, to appear on PRL

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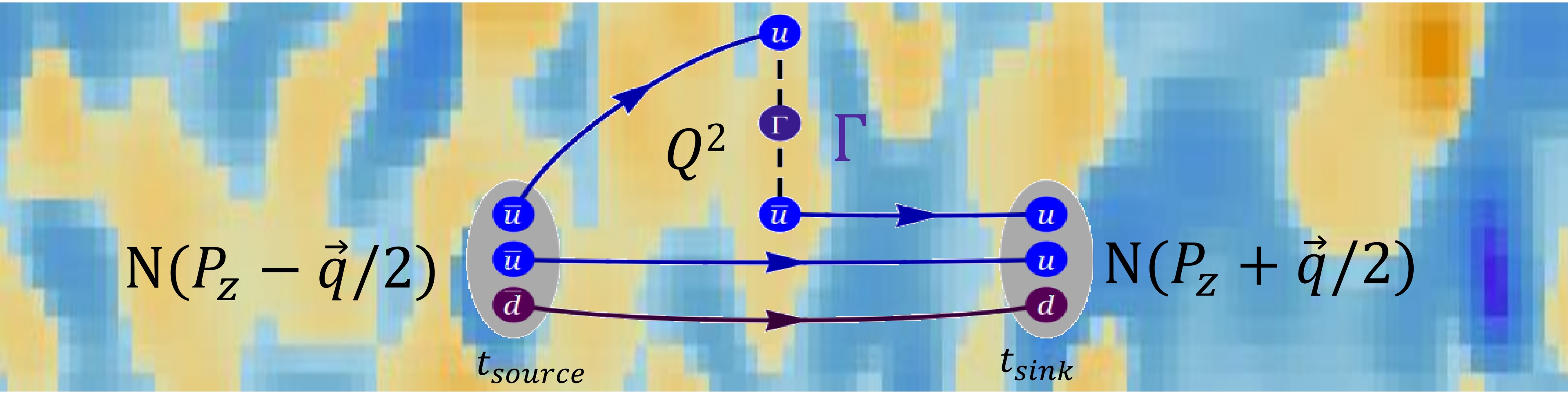


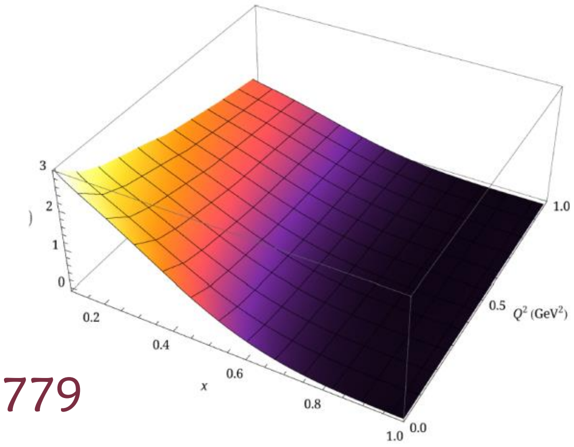
$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_Z)$$
$$= \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_Z}\langle P'|\tilde{O}_{\gamma_5\gamma_z}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_Z)\gamma_5\gamma_z + \tilde{E}(x,\tilde{\xi},t,\bar{P}_Z)\frac{i\gamma_5\Delta_z}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

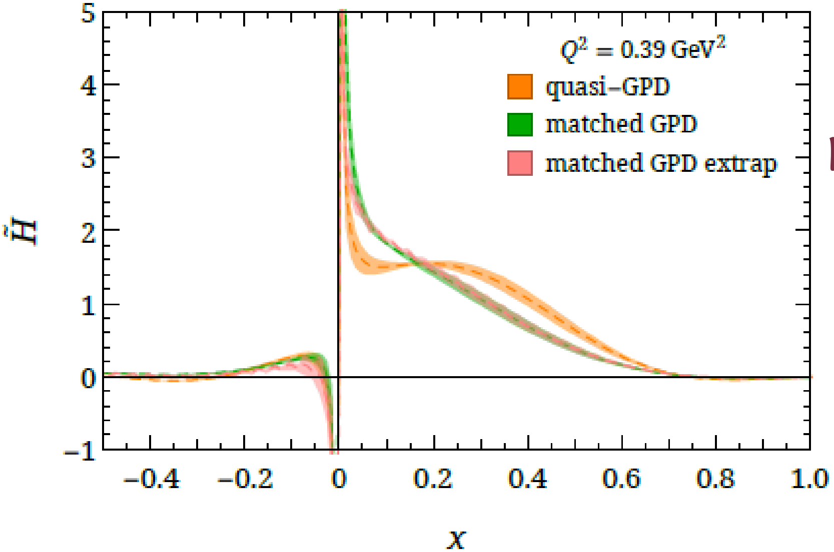
Nucleon Polarized GPDs

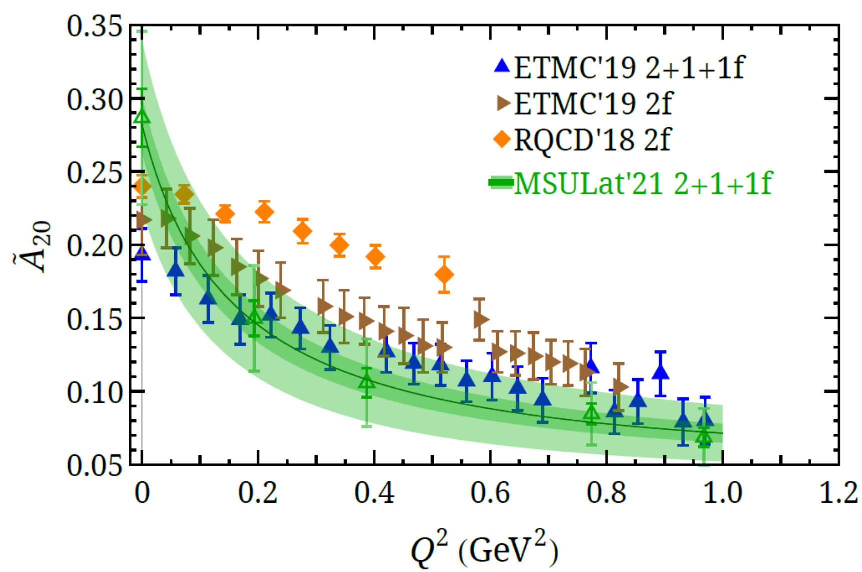
§ Helicity GPD ($\widetilde{H}$) using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results



HL (MSULat), 2110.06779





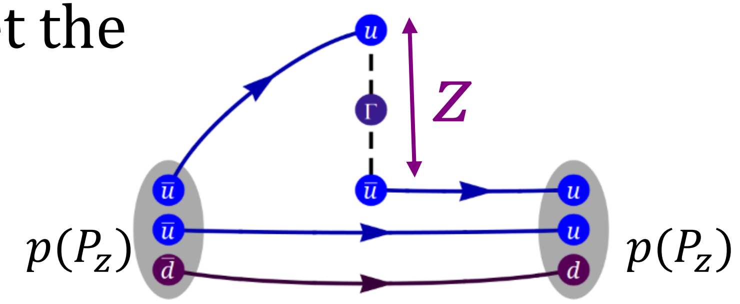
"Extrap": $c_1(-izP_z)^{-d_1} + c_2 e^{izP_z}(izP_z)^{-d_2}$

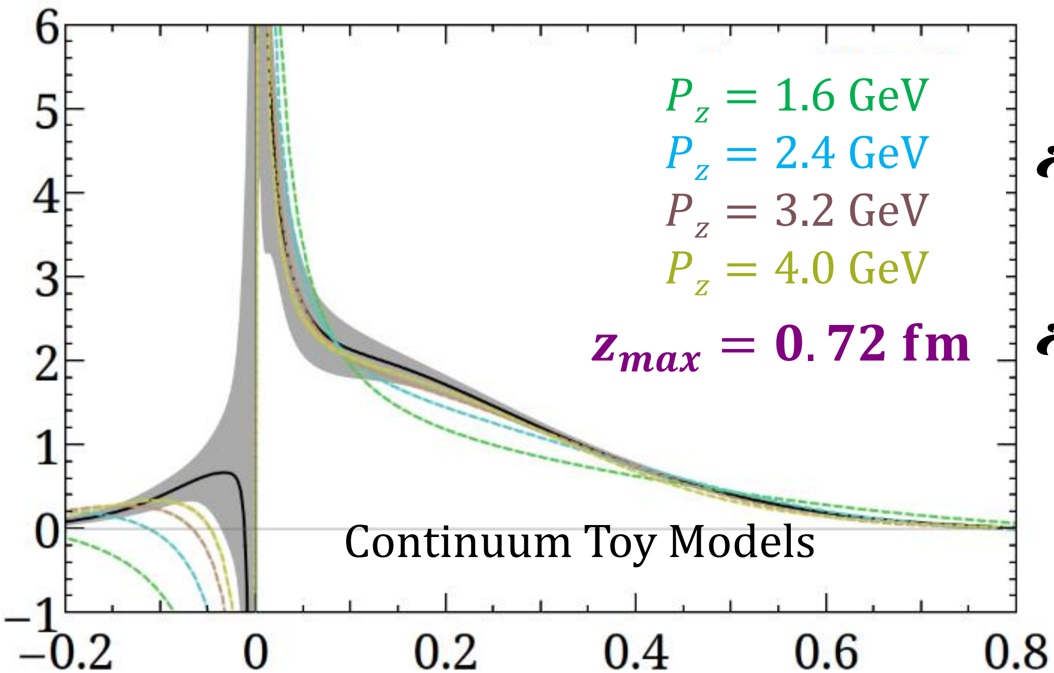
X. Ji et al, 2008.03886

Challenge Ahead

§ Reach small-x regions and predict on antiquark distribution

- One needs large momentum just to get the sign of the antiquark correct!
- With small zP_z, we miss over the majority of the x range





HL et al, 1708.05301

- Not just a quasi-PDF problem
- Going to large P_z is unavoidable for all x-dependent methods

Challenge Ahead

§ Reach small-x regions and predict on antiquark distribution

(P_z)

6
5
4
3
2
1
0
−1
−0.2

Hard problems require diverse ways of thinking!

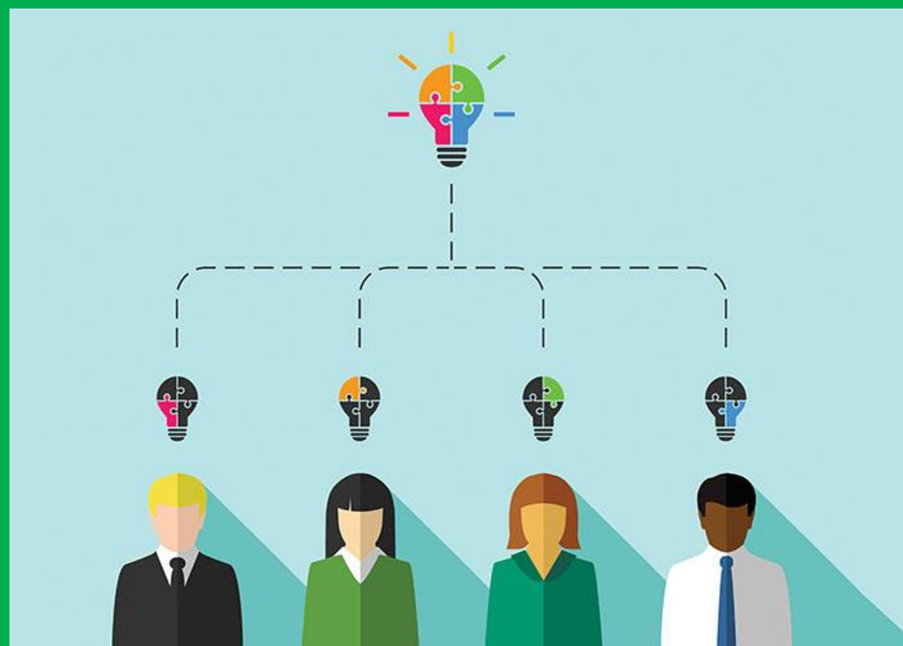


Image credit: Physics World

A more diverse workfoce is needed!

HL et al, 1708.05301

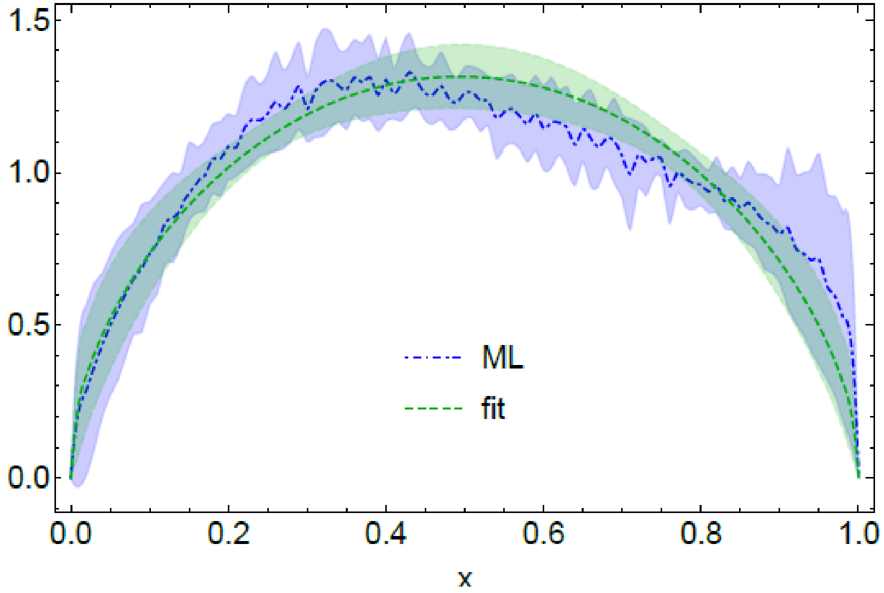
Machine Learning Application

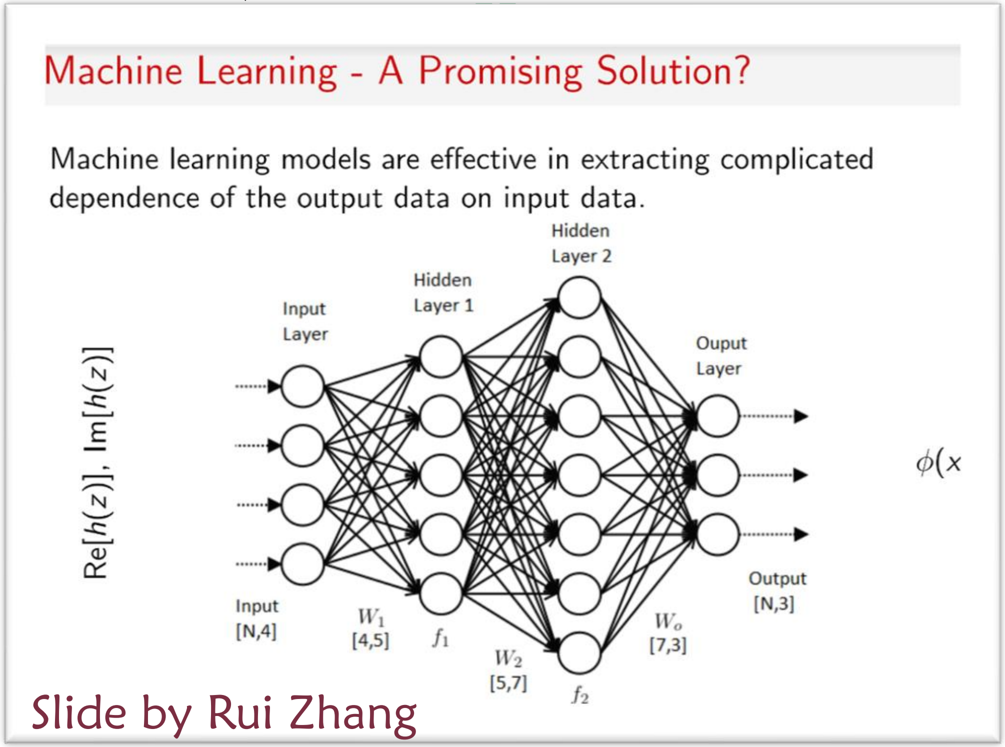
§ Extract the DA distribution from the physical-continuum matrix elements

R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

Pion Distribution Amplitude





Slide by Rui Zhang

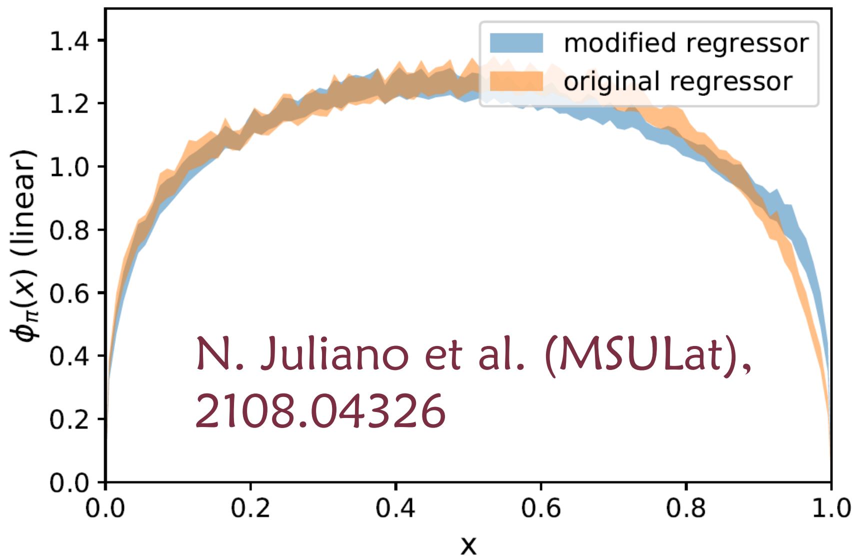
Machine Learning Application

- Extract the DA distribution from the physical-continuum matrix elements

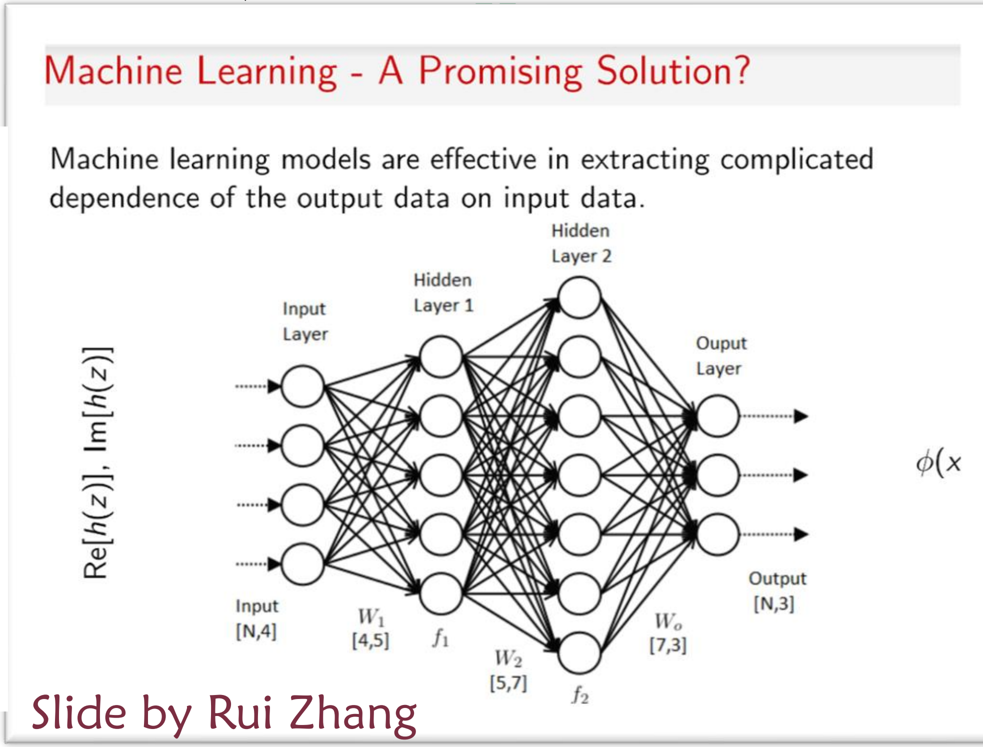
R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

Pion Distribution Amplitude



N. Juliano et al. (MSULat), 2108.04326



Slide by Rui Zhang

Summary

§ Exciting era using LQCD to study PDFs

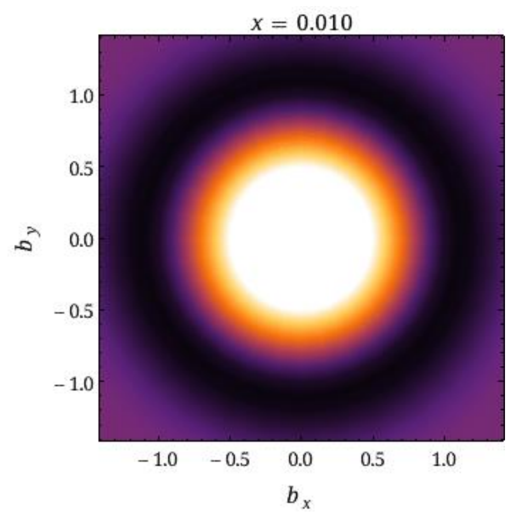
- Well-studied systematics → precision structures
- More nucleon matrix elements with physical pion masses

§ Overcoming longstanding limitations of moment method

- Bjorken-*x* dependence of parton distributions are widely studied with LaMET and its variants
- More study of systematics planned for the near future
- Start to address neglected disconnected contributions obtaining flavor-dependent quantities

§ Stay tuned for more updates from LQCD

Titan @ORNL, IC@LANL



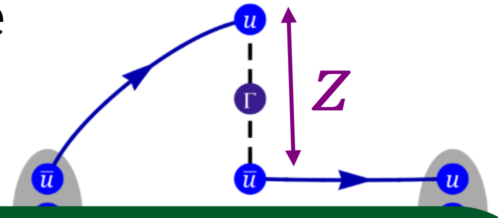
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

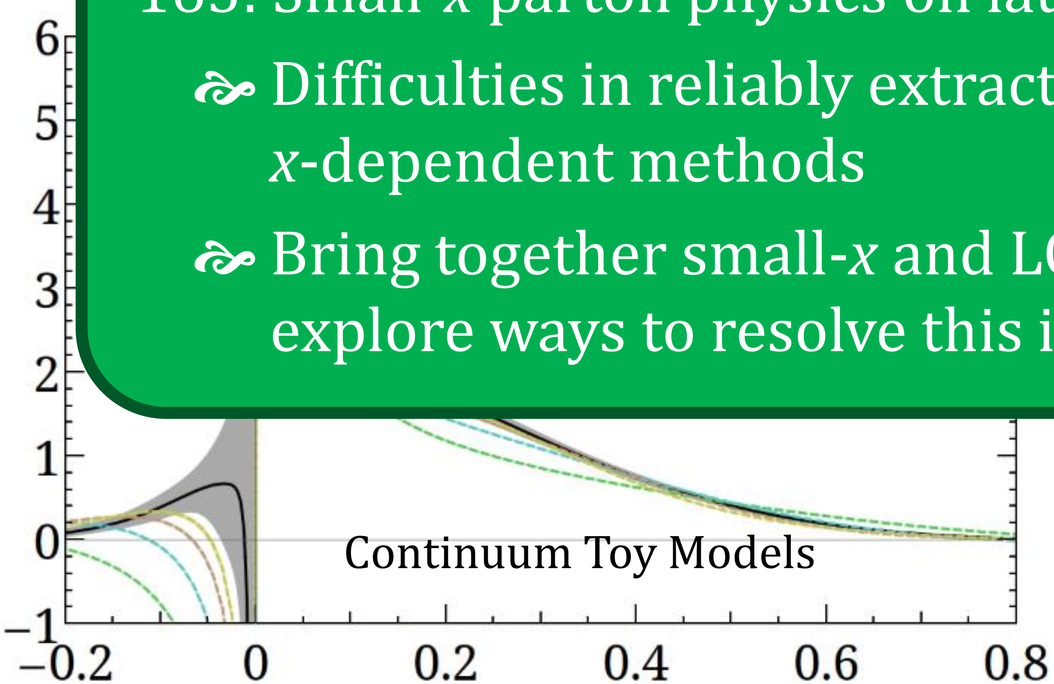
The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award

Challenge Ahead

§ Reach small-x regions and predict on antiquark distribution

- One needs large momentum just to get the sign of the antiquark correct!
- With small zP_z, we miss over





HL et al, 1708.05301

unavoidable for all x-dependent methods

165: Small-x parton physics on lattice (EF6) by Ji et al.

- Difficulties in reliably extracting small-x in current x-dependent methods
- Bring together small-x and LQCD communities to explore ways to resolve this in the near future

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary

- § Exciting era using LQCD to study PDFs
  - Well-studied systematics → precision structures
  - More nucleon matrix elements with physical pion masses
- § Overcoming longstanding limitations of moment method
  - Bjorken-x dependence of parton distributions are widely studied with LaMET and its variants
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  - Start to address neglected disconnected contributions
    obtaining flavor-dependent quantities
- § Stay tuned for more updates from LQCD

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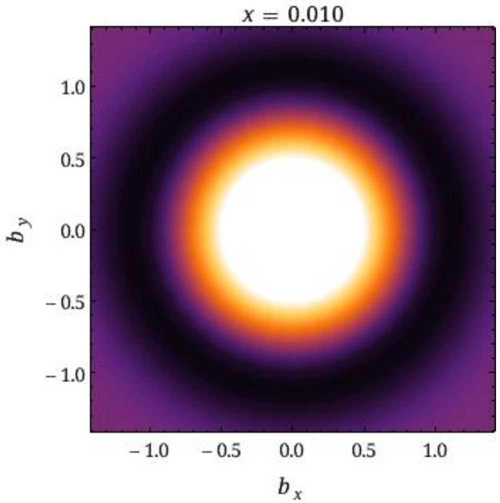

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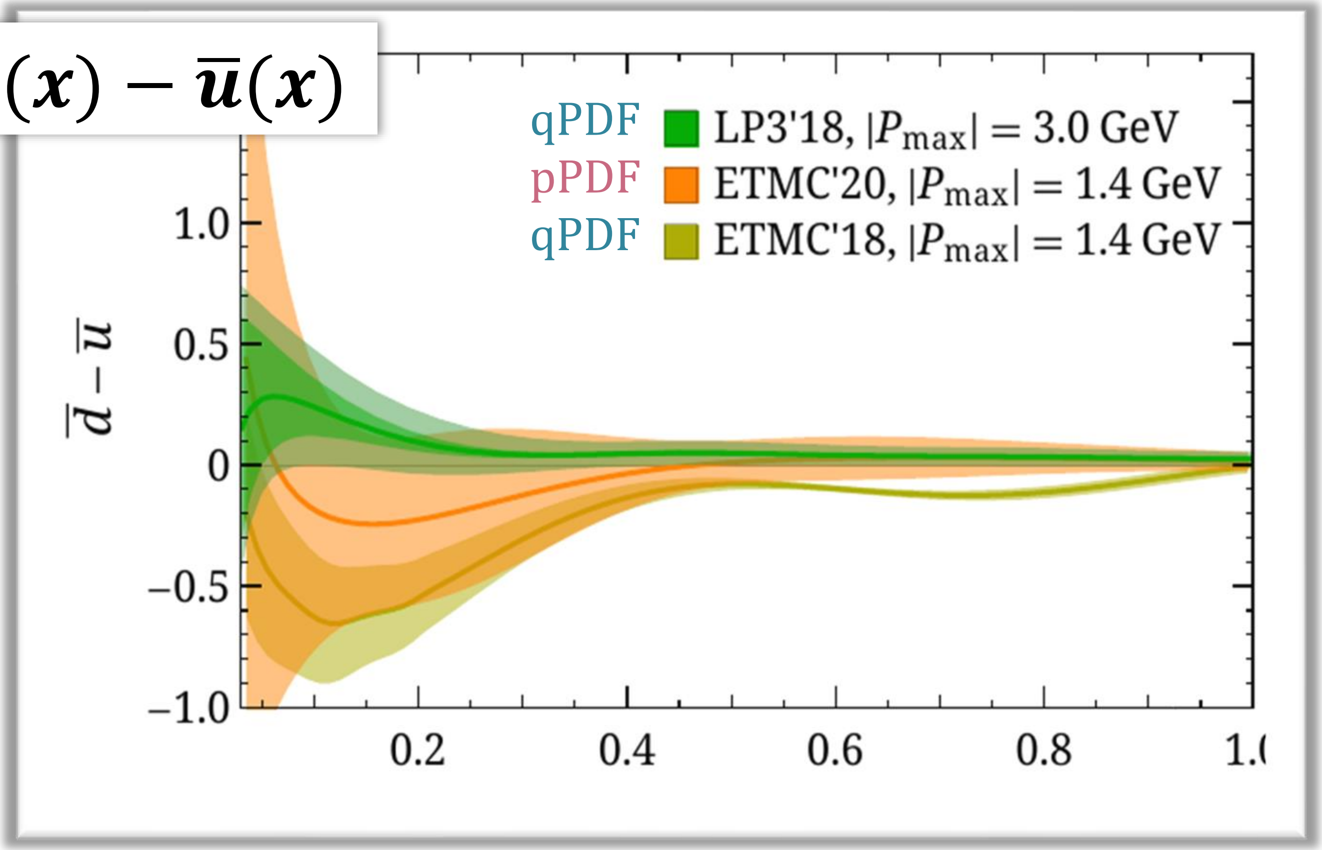
Backup Slides

Physical Pion Mass Results

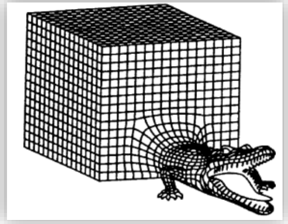
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



Finite volume,
Discretization,
…

Physical Pion Mass Results

§ Summary of physical pion mass results

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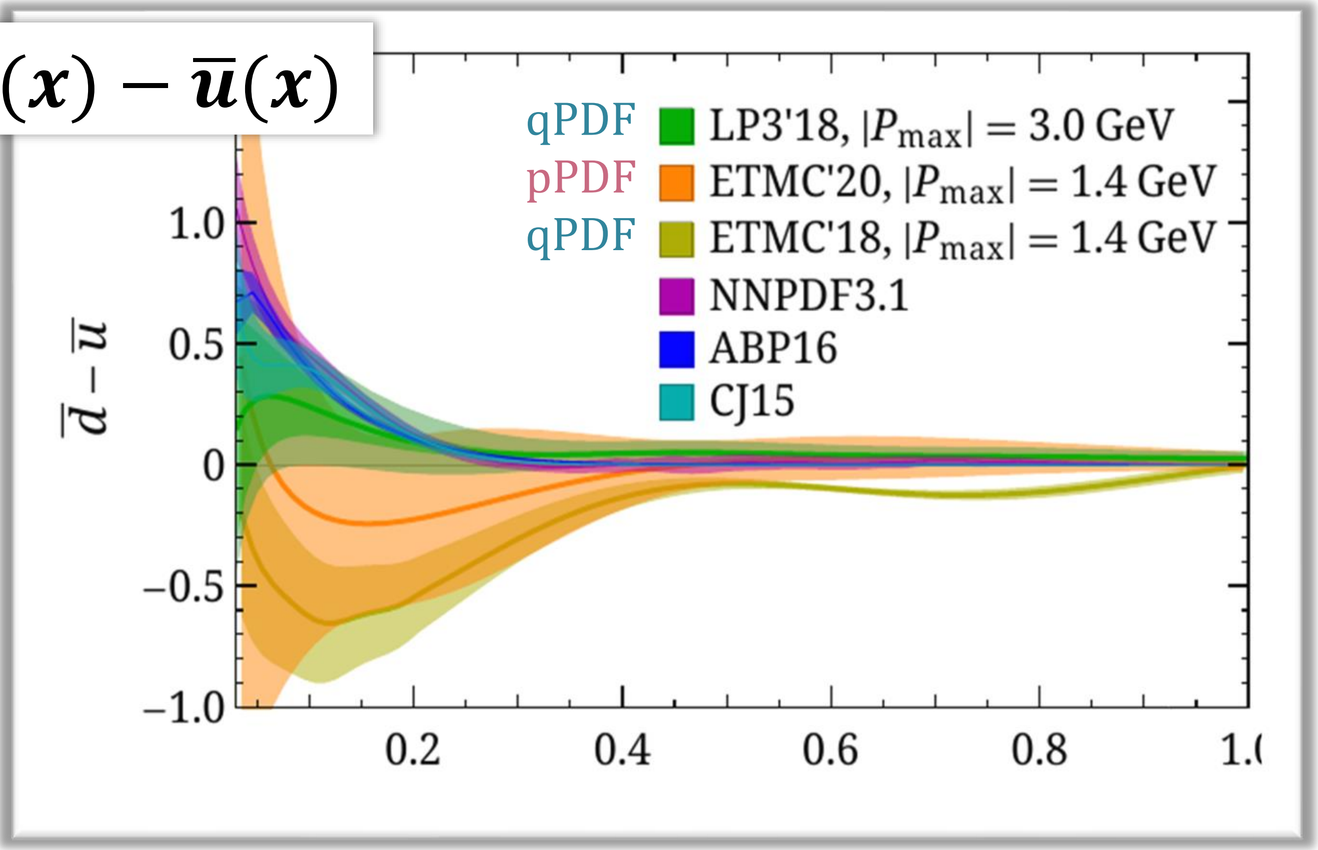
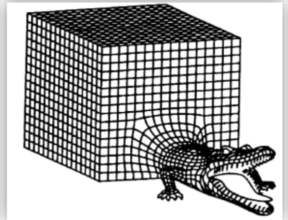




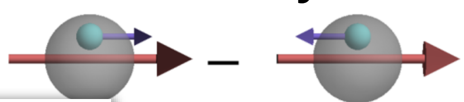
Finite volume,
Discretization,
…

2006.08636, PDFLattice2019 report

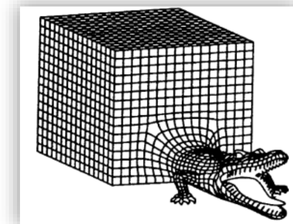
Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

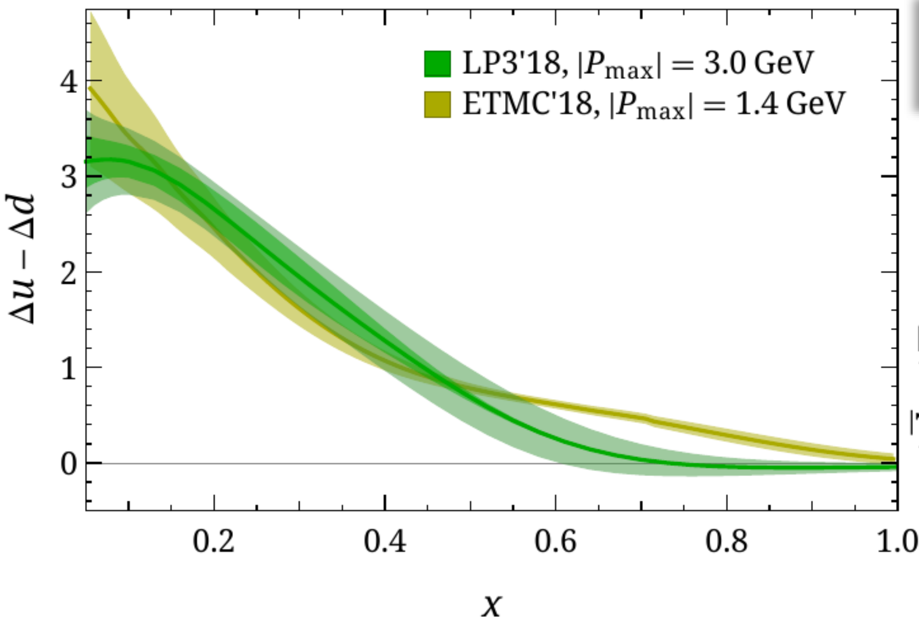


Helicity
long. polarized

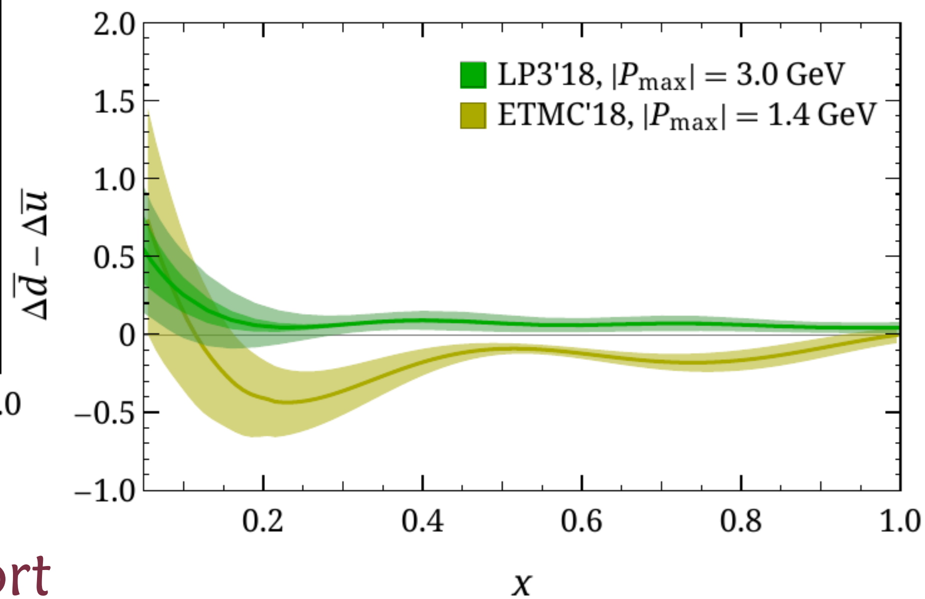


Finite volume,
Discretization,
…

$\Delta u(x) - \Delta d(x)$



$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$



2006.08636, PDFLattice2019 report

Polarized PDFs

§ Summary of physical pion mass results

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Helicity
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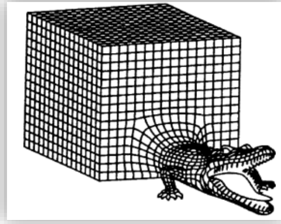

Finite volume,
Discretization,
…

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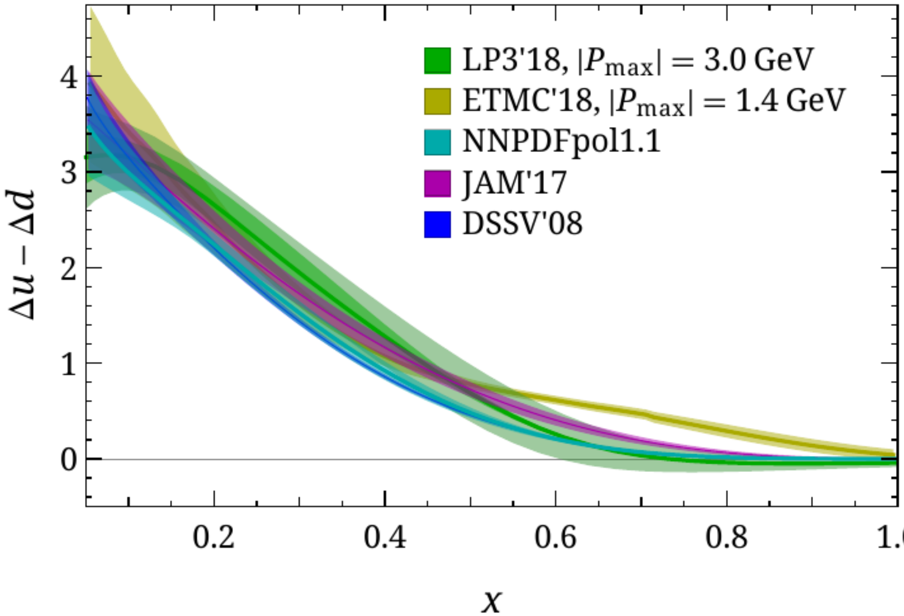
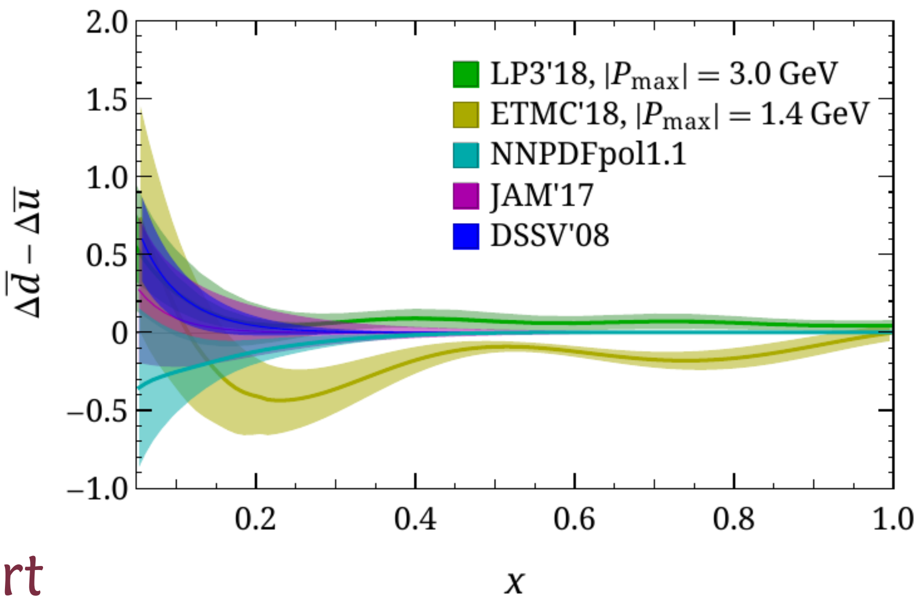


$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$



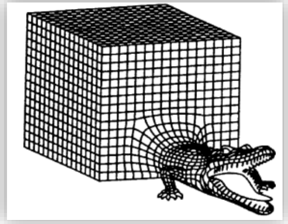
2006.08636, PDFLattice2019 report

§ Summary of physical pion mass results

- Quasi-PDF method only

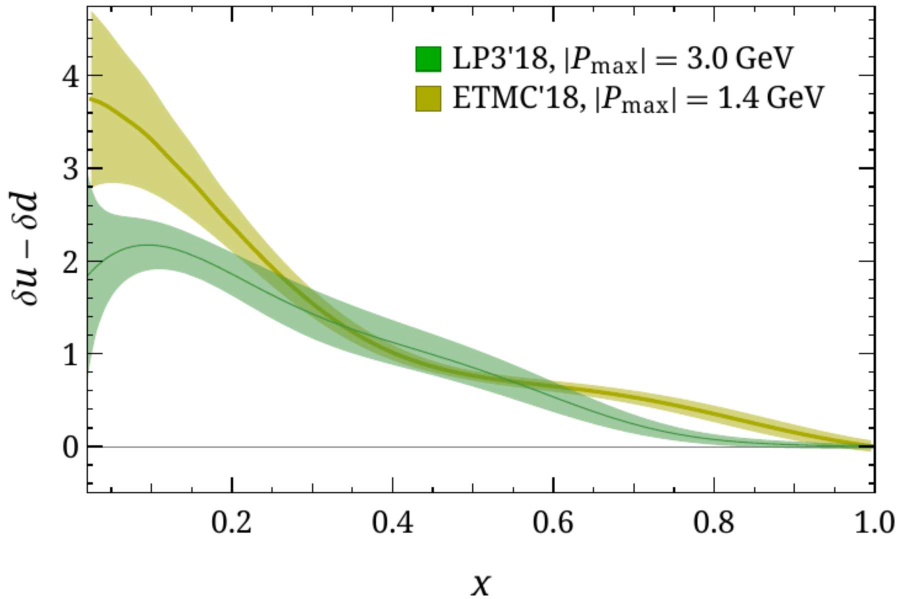
$\boldsymbol{\delta u(x) - \delta d(x)}$

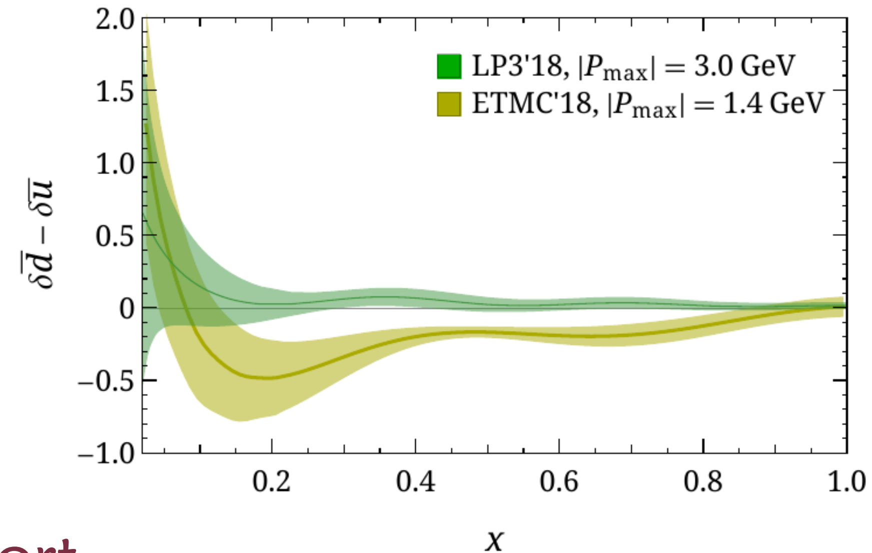
Transversity



Finite volume,
Discretization,
...

$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$





2006.08636, PDFLattice2019 report

Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

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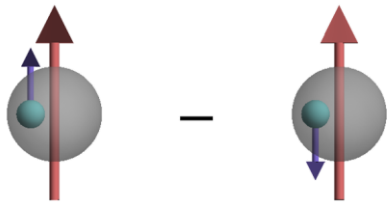

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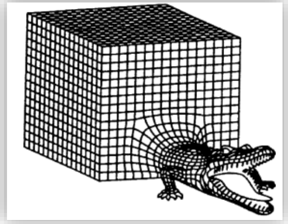

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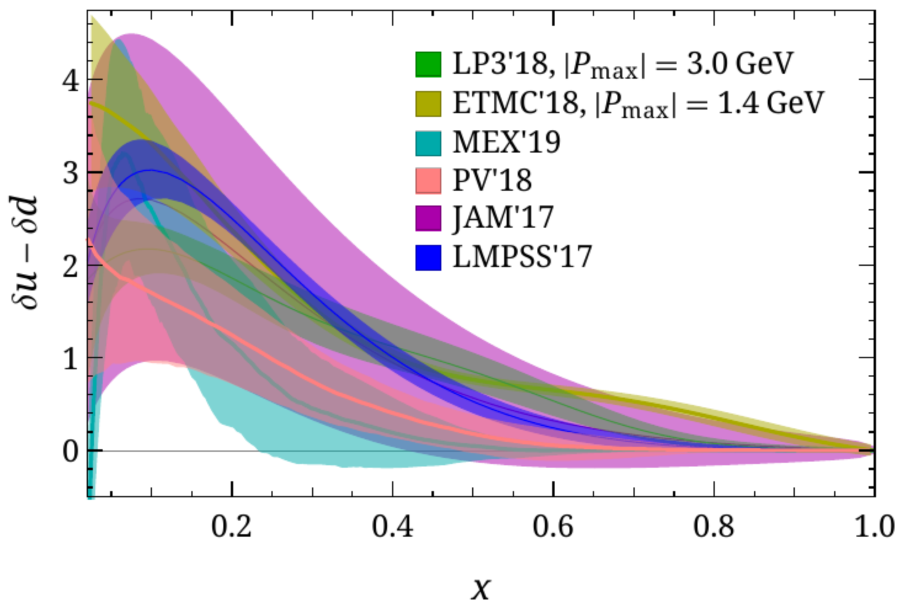
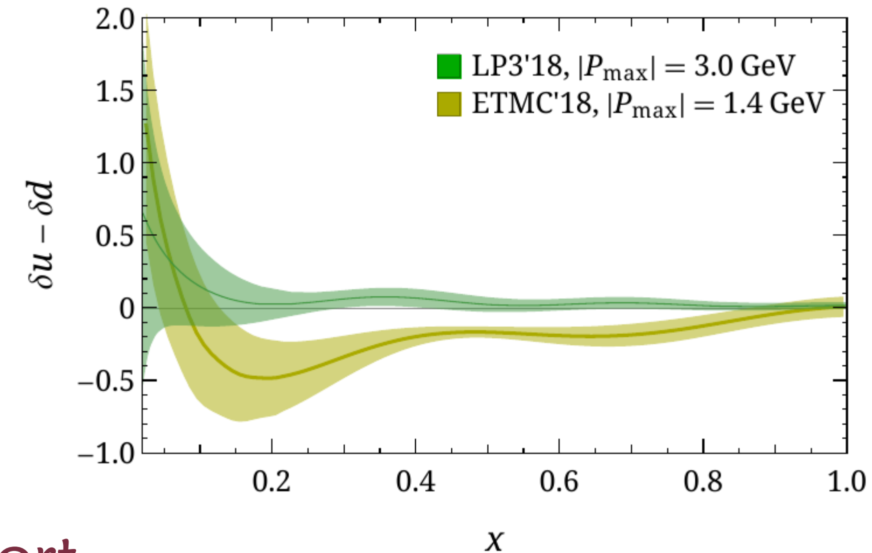
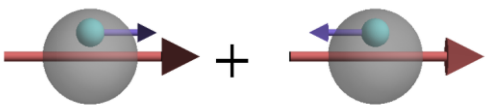




2006.08636, PDFLattice2019 report

Moments of PDFs

§ First moments are most commonly done



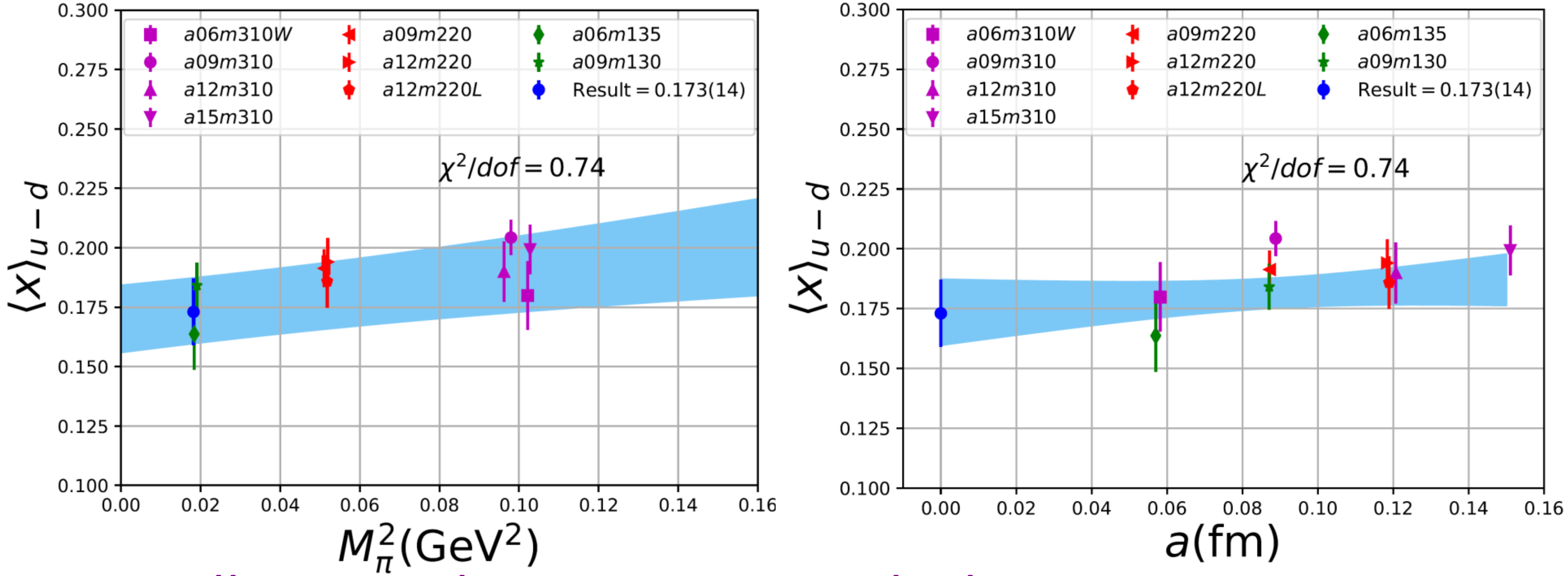
$+$

§ State-of-the art example

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

❧ Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779



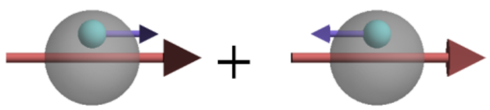
§ Usually more than one LQCD calculation

❧ Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

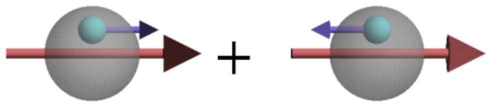
Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★		0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★		0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.051(26)(5)	
$\langle x\rangle_{g}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★	**	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★		0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■		0.47(4)(11)	

** No quenching effects are seen.

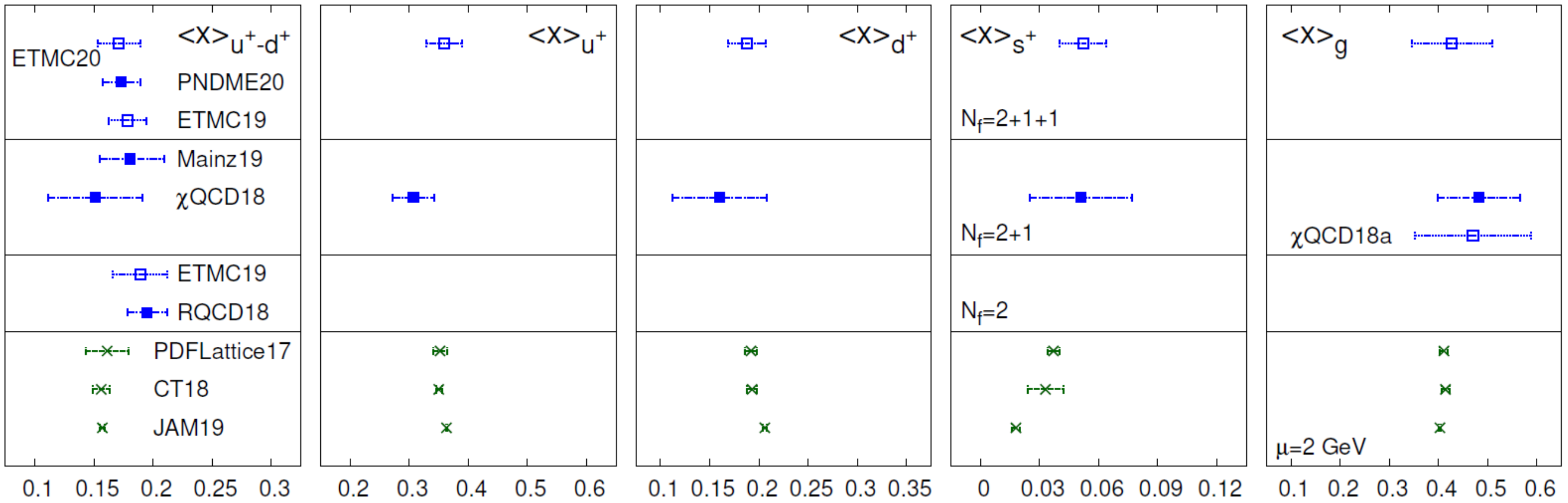
Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636



Moments of PDFs

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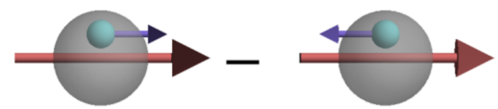
$$\langle x^{n-1} \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



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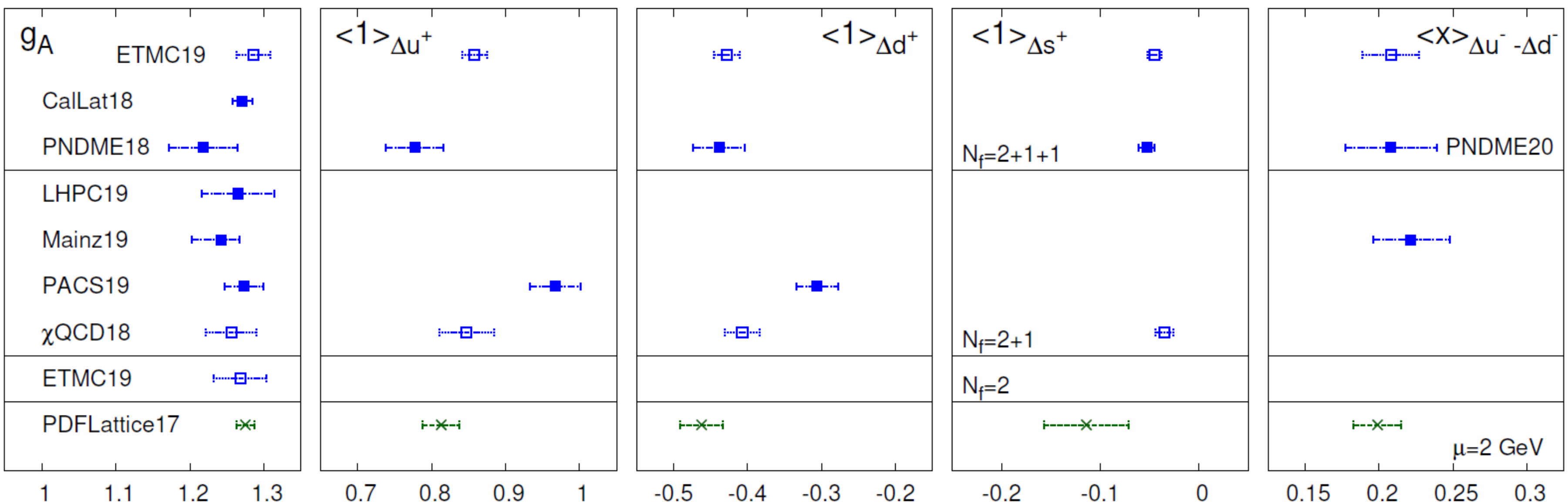
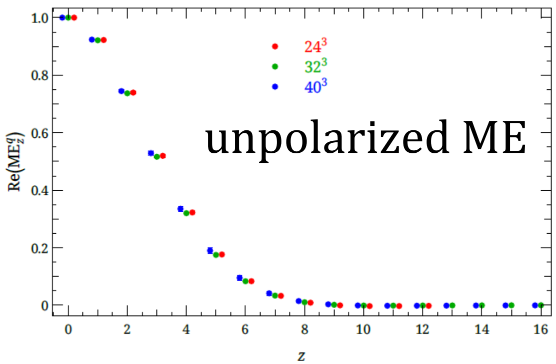


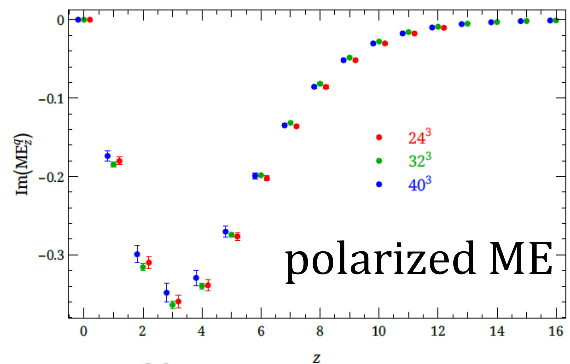
Systematics Study

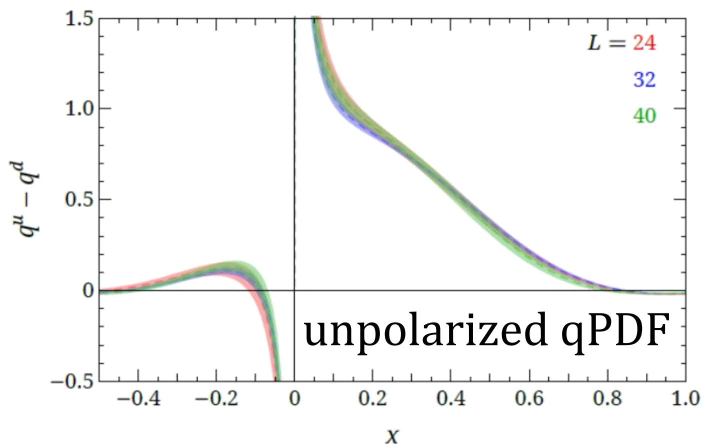
§ First finite-volume study in quasi-PDFs

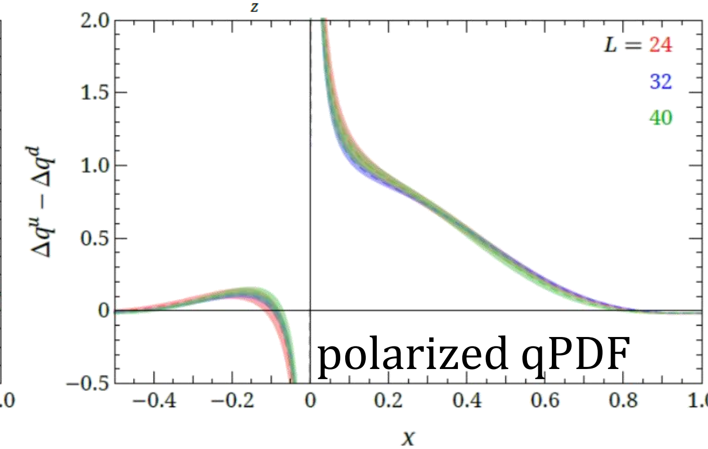
- Clover on 2+1+1 HISQ, $M_\pi \approx \mathbf{220}$ MeV, $a \approx 0.12$ fm
- $M_\pi L \approx 3.3, 4.4, 5.5$, $P_z \approx 1.3$ GeV

HL, R, Zhang, Phys.Rev.D 100 (2019) 7, 074502







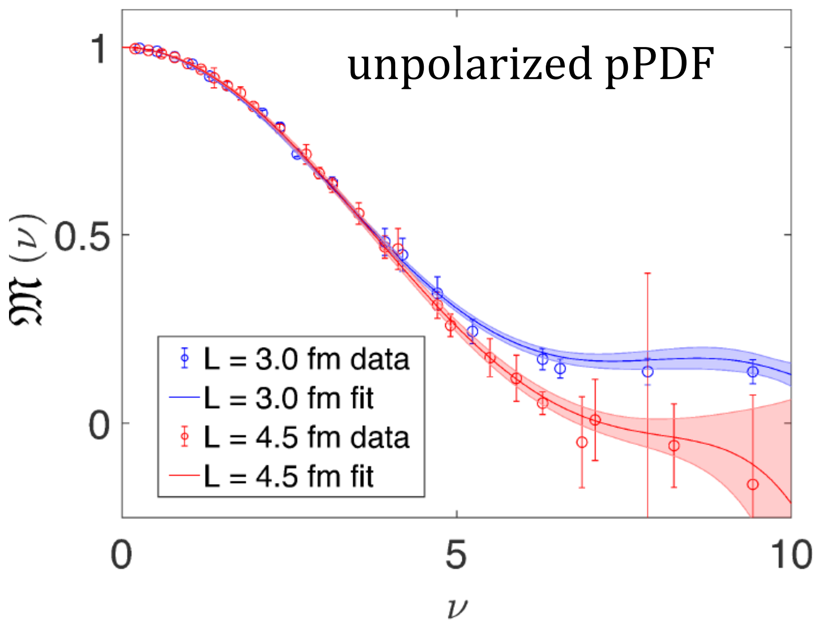


Systematics Study

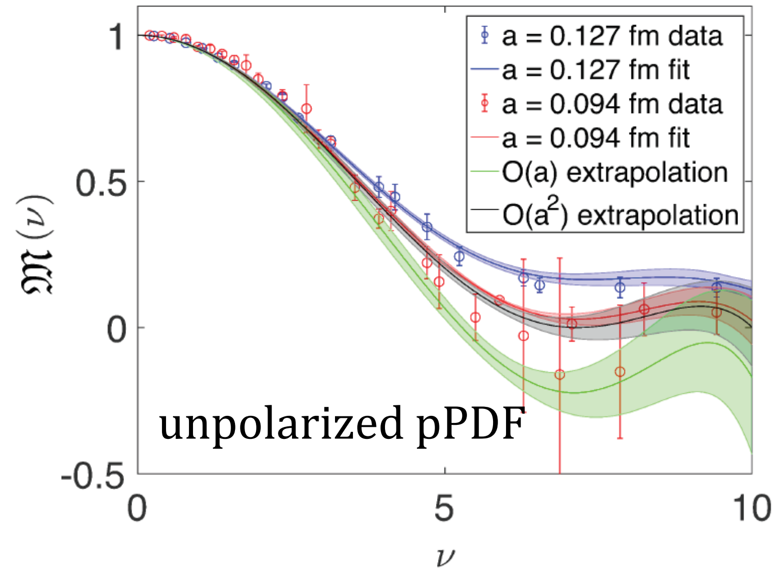
§ Finite-volume study in unpolarized pseudo-PDFs

- 2+1f clover, $M_\pi \approx \mathbf{415}$ MeV, $a \approx 0.127$ fm
- Two volumes used: $L \approx 3, 4.5$ fm

B. Joo et al (Jlab/W&M) 1908.09771



§ Also see strong lattice-spacing dependence



§ Lattice artifacts are sensitive to the simulated QCD vacuum

- Each group will have to check their own systematics carefully