Hidden-heavy and doubly-heavy molecular tetraquarks

in a quasipotential Bethe-Salpeter equation approach

Jun He

Nanjing Normal University

Collaborators: Zuo-Ming Ding, Han-Yu Jiang, Dan Song

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Outline

- Introduction
- Hidden(doubly)-heavy tetraquark without strangeness

  from $D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}$ and $D^{(*)}D^{(*)}/B^{(*)}B^{(*)}$ interactions
- Hidden(doubly)-heavy tetraquark with strangeness

  from $D_{(s)}^{(*)}\bar{D}_s^{(*)}/B_{(s)}^{(*)}\bar{B}_s^{(*)}$ and $D_{(s)}^{(*)}D_s^{(*)}/B_{(s)}^{(*)}B_s^{(*)}$ interactions
- Summary

Hadronic molecular state

F.K.Guo 第7届XYZ粒子研讨会

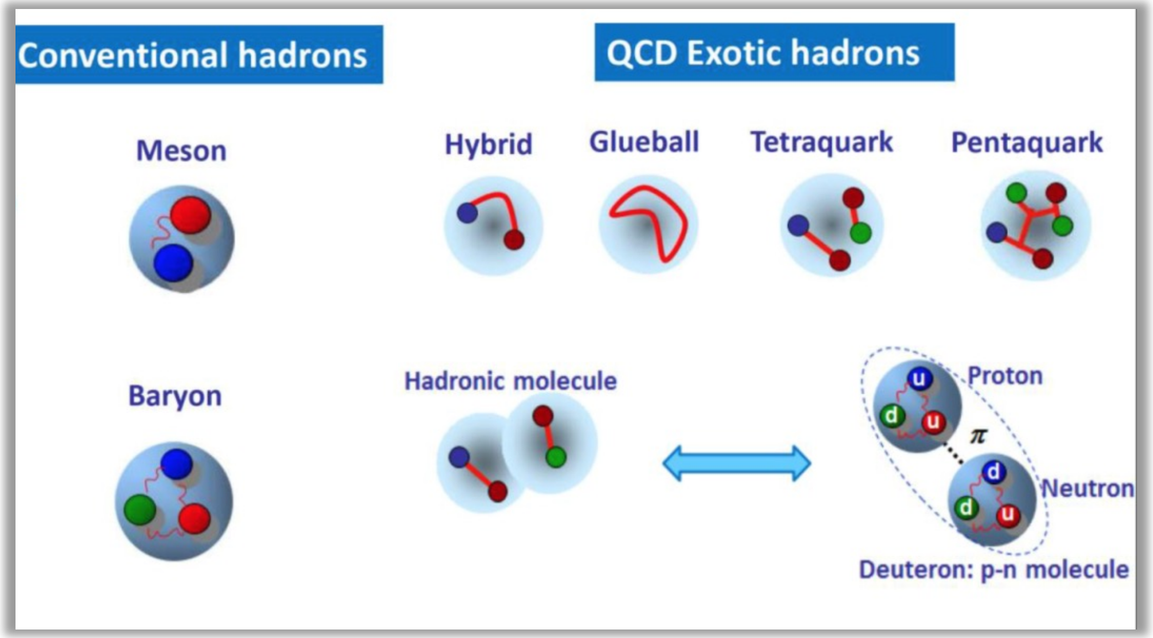
Exotic hadrons beyond conventional quark model

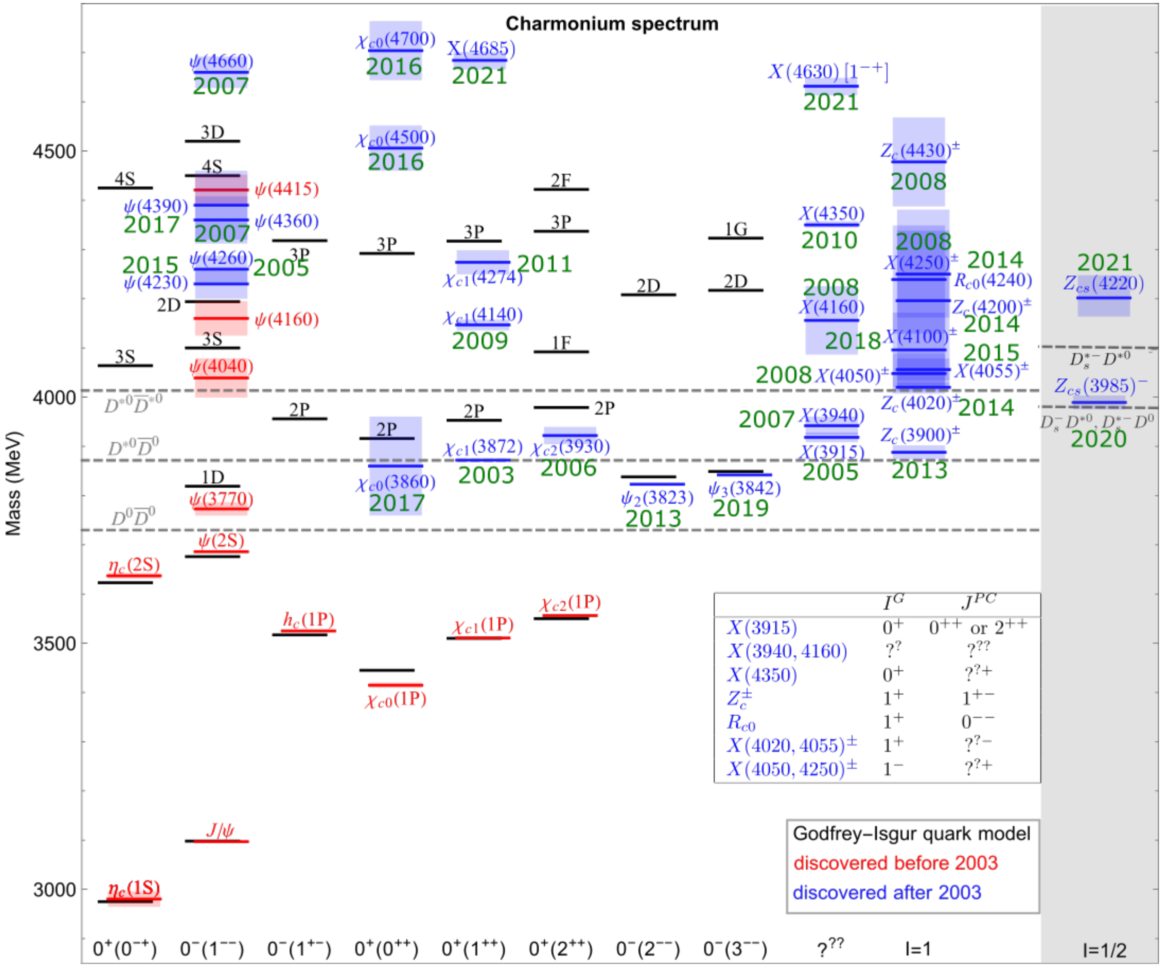
- States with exotic quantum numbers

  Meson：0^{--}, 0^{+-}, 1^{-+}, 2^{+-},…… ;

  Charged charmonium-like and bottomonium-like states

- States with mass different from predictions in conventional QM

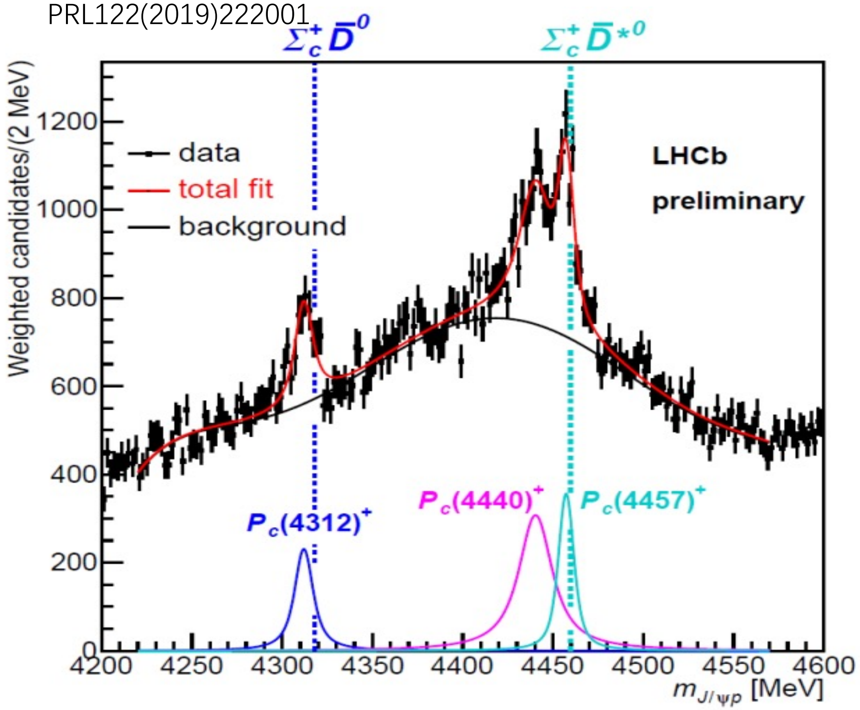




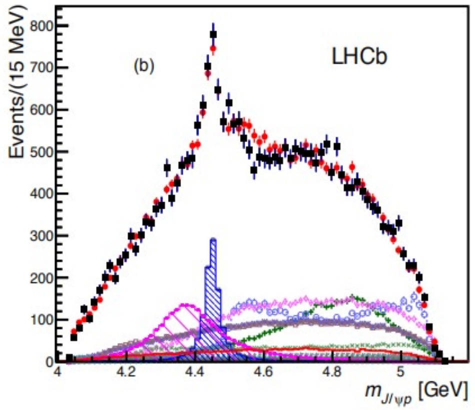
Many exotic hadrons are close to the thresholds

A typical spectrum supporting molecular states: LHCb Pentaquarks

PRL122(2019)222001

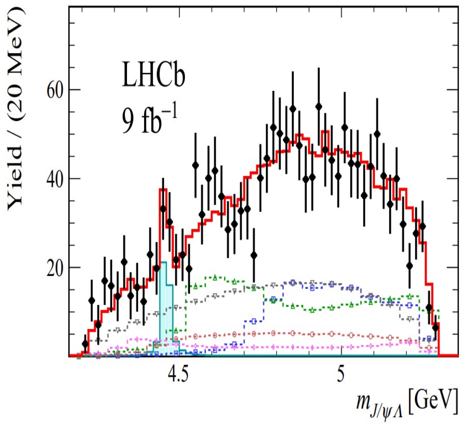


PRL115(2015)072001

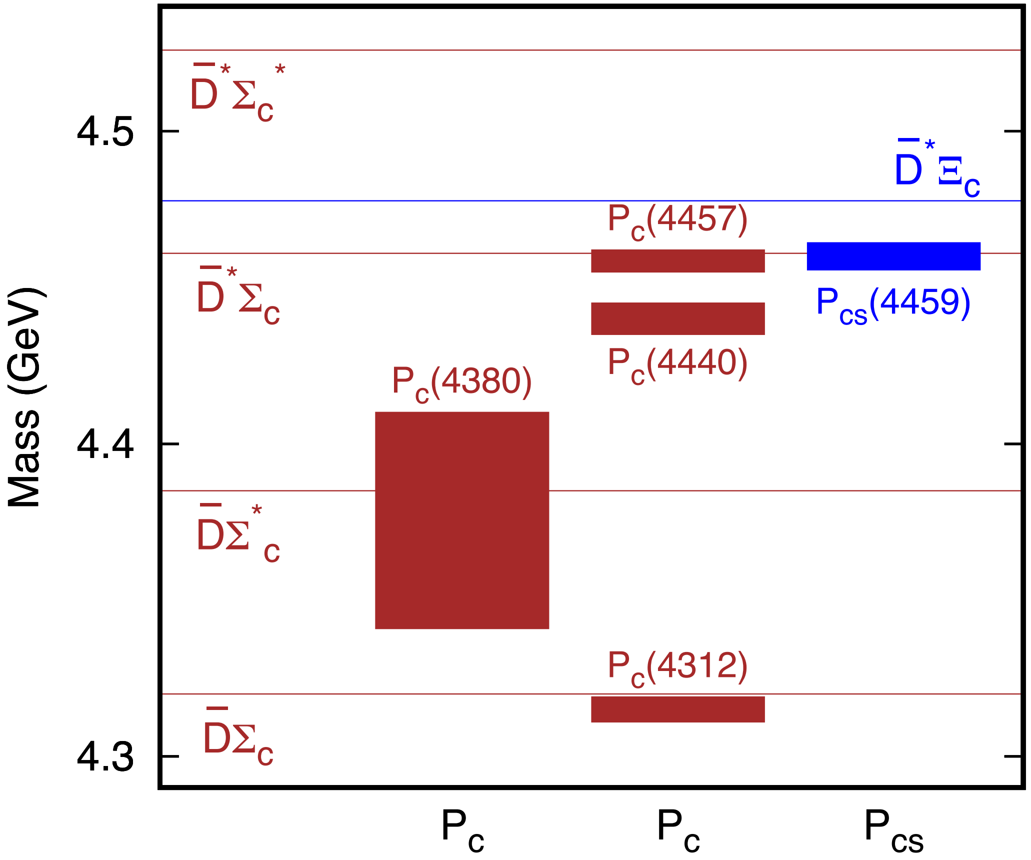


$P_c(4380)$

Sci.Bull. 66 (2021) 1278



$P_{cs}(4459)$



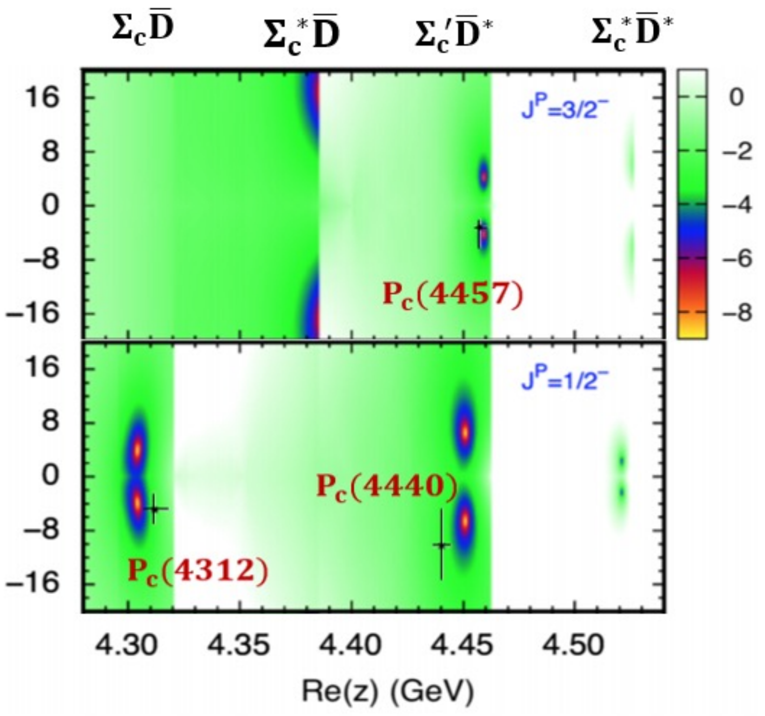
- J.J.Wu, R.Molina, E.Oset and B.S.Zhou, Phys.Rev.Lett.105,232001;
- J.J.Wu, T.-S.H. Lee, and B.S.Zou, Phys.Rev.C85,044002
- Z.C.Yang, Z.F.Sun, J.He, X.Loiu and S.L.Zhu, Chin.Phys.C36,6
- W.L.Wang, F.Huang, Z.Y.Zhang and B.S.Zou, Phys.Rev.D84,015203
- M. Z. Liu, et al., Phys. Rev. Lett. 122, 242001 (2019)
- H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D 100, 051501 (2019)
- J. He Eur. Phys. J.C 79, no.5, 393 (2019)
- ……

P_c and P_{cs} as molecular states

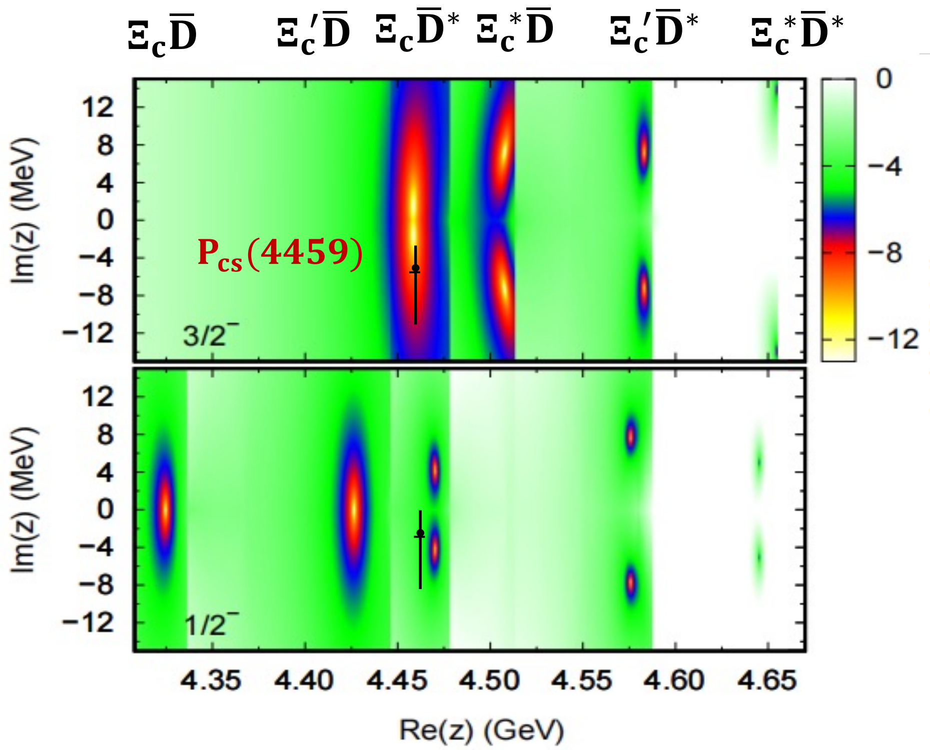
P_c

P_{cs}

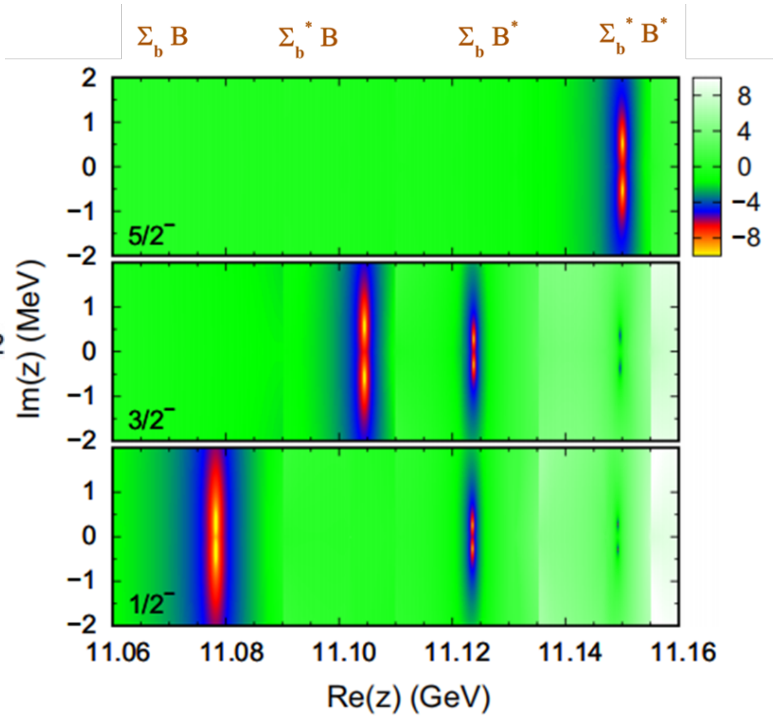
P_b



JH, Chen, EPJC79 (2019)887



Zhu, Song, JH, PRD103(2021)074007



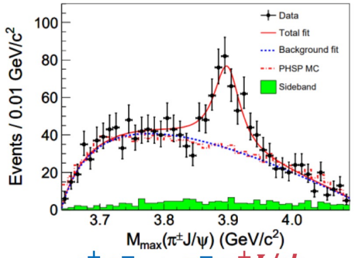
Zhu, Kong, Liu, JH,EPJC80(2020)1016

Possible molecular tetraquarks

Z_c/Z_{cs}

$Z_c(3900)^+$

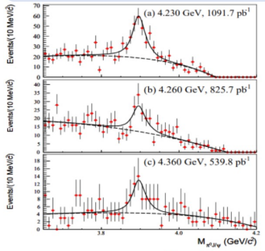
PRL 110, 252001 (2013)



$e^+e^- \to \pi^-\pi^+ J/\psi$

$Z_c(3900)^0$

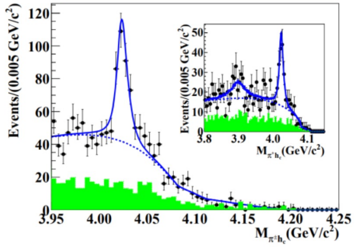
PRL 115, 112003 (2015)



$e^+e^- \to \pi^0\pi^0 J/\psi$

$Z_c(4020)^+$

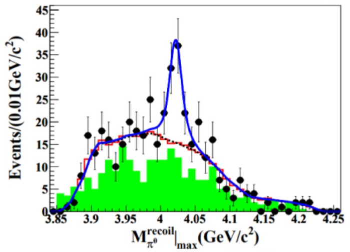
PRL 111, 242001 (2013)



$e^+e^- \to \pi^-\pi^+ h_c$

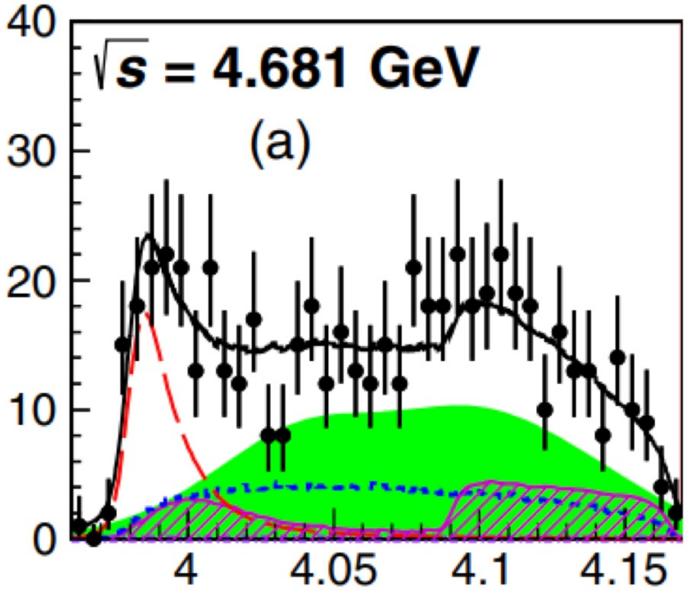
$Z_c(4020)^0$

PRL 113, 212002 (2014)



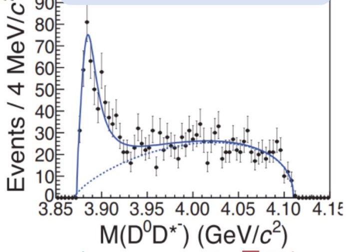
$e^+e^- \to \pi^0\pi^0 h_c$

$Z_{cs}(3985)$



$Z_c(3885)^+$

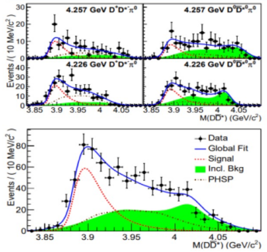
ST: PRL 112, 022001 (2014)
DT: PRD 92, 092006 (2014)



$e^+e^- \to \pi^-(D\bar{D}^*)^+$

$Z_c(3885)^0$

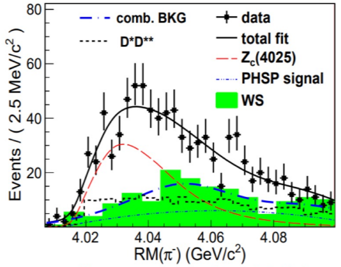
PRL 112, 022001 (2014)



$e^+e^- \to \pi^0(D^*\bar{D})^+$

$Z_c(4025)^+$

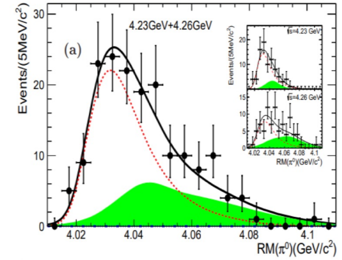
PRL 112, 132001 (2014)



$e^+e^- \to \pi^-(D^*\bar{D}^*)^+$

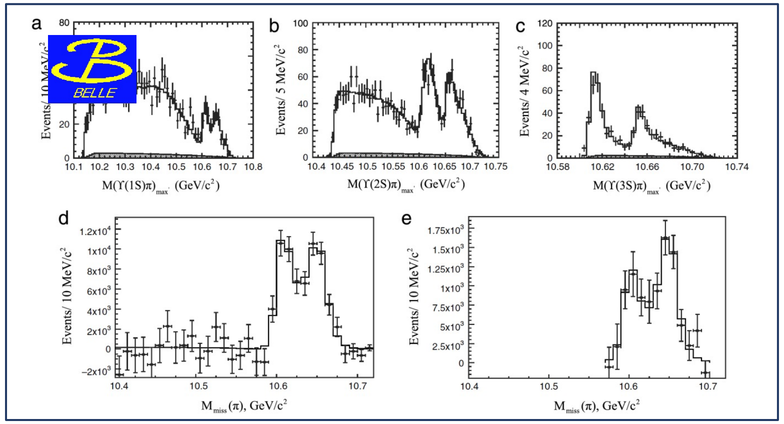
$Z_c(4025)^0$

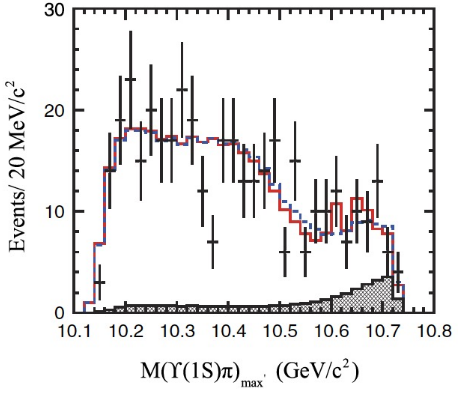
PRL 115, 182002 (2015)

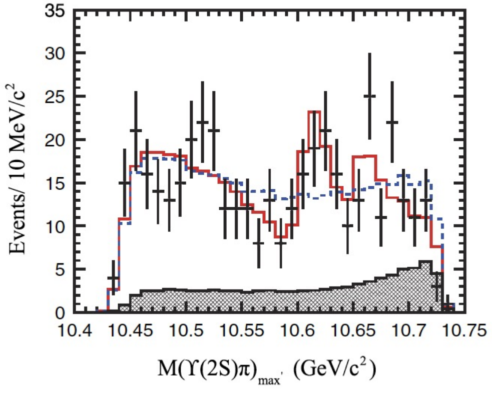


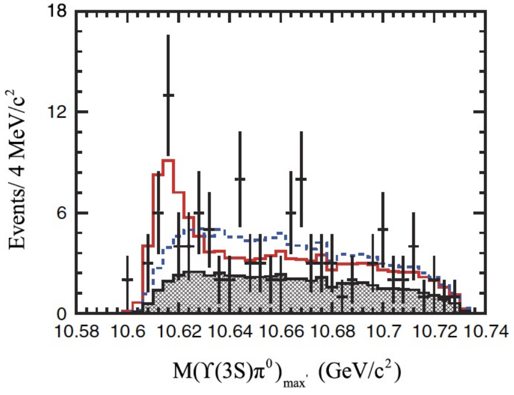
$e^+e^- \to \pi^0(D^*\bar{D}^*)^0$

Z_b

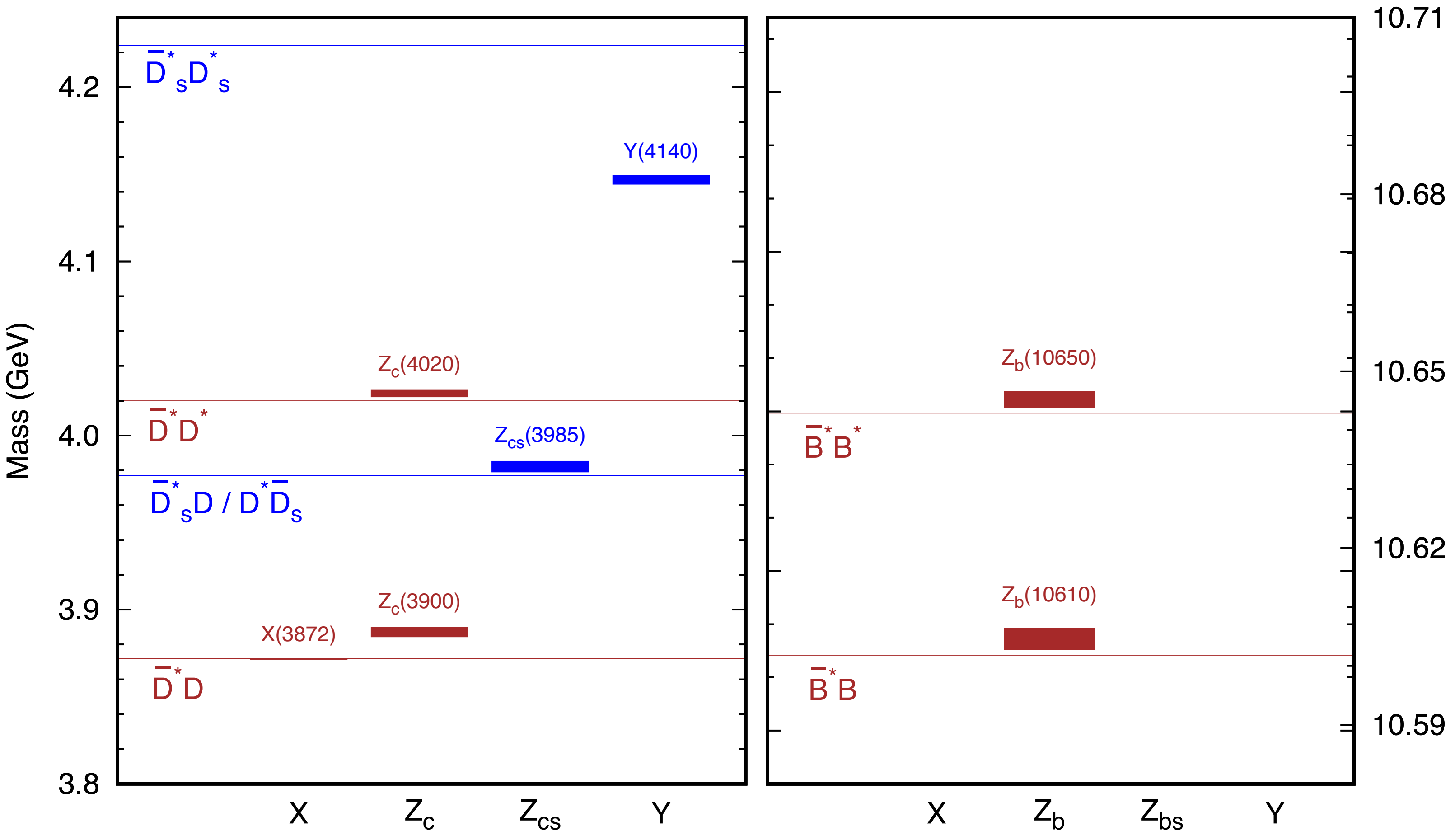








Z_c and Z_b states and relevant thresholds



Theoretical work in molecular state picture

Tornqvist, Phys.Lett.B 590 (2004) 209-215

Liu, et al. Eur.Phys.J.C 56 (2008) 63-73

Wang, Huang, Eur.Phys.J.C 74 (2014) 5, 2891

Aceti, et al., Phys.Rev.D 90 (2014) 1, 016003

Zhang, Phys.Rev.D 87 (2013) 11, 116004

Sun, et al., Phys.Rev.D 84 (2011) 054002

Chen etl al. Phys.Rev.D 92 (2015) 5, 054002

Aceti, et al., Eur.Phys.J.A 50 (2014) 103

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Our previous works about the molecular states from $B\bar{B}^*, D\bar{D}^*$, and $K\bar{K}^*$ Interaction

Sun, **JH**, Liu, Luo, Zhu, Phys. Rev. D84 (2011) 054002
JH, Phys.Rev. D92 (2015) 034004, Phys.Rev. D90 (2014) 076008
Lu, **JH**, Euro. Phys. J. A52(2016) 359

The experimentally observed states can be reproduced.
The results in different sectors are similar.

For doubly-heavy systems BB^*, DD^*, the theoretical frame are almost the same.

Theoretically, the doubly heavy state can be obtained by replacing one constituent particle by its antiparticle.

	$B\bar{B}^*(b\bar{b}q\bar{q})$		$D\bar{D}^*(c\bar{c}q\bar{q})$		$K\bar{K}^*(s\bar{s}q\bar{q})$	
$I^G(J^{PC})$	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.
$0^-(0^{--})$						
$0^+(0^{-+})$						
$0^-(1^{--})$						
$0^+(1^{-+})$						
$0^-(1^{+-})$	✓	?	✓	?	✓	$h_1(1380)$
$0^+(1^{++})$	✓	?	✓	$X(3872)$	✓	$f_1(1285)$
$1^+(0^-)$						
$1^-(0^-)$						
$1^+(1^-)$						
$1^-(1^-)$						
$1^+(1^+)$	✓	$Z_b(10610)$	✓	$Z_c(3900)$	✓	$b_1(1235)$
$1^-(1^+)$						

BB^*? DD^*?

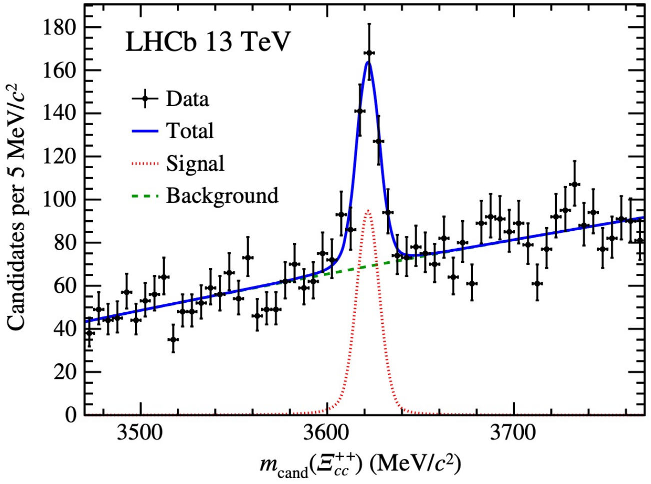
Existence of the doubly heavy states predicted in the same model can be expected.

Observations of Ξ_{cc}^{++} and T_{cc}^{+}

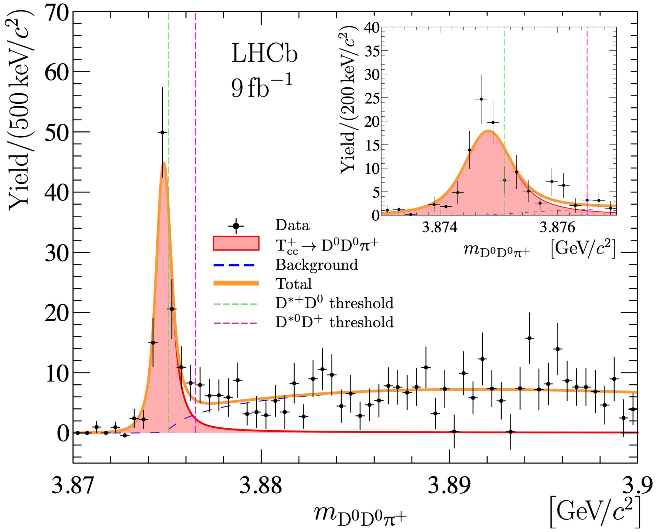
- Because a charm quark pair can be created at the same time, a hidden-charm molecular state can be formed with other light quarks easily.
- However, two charm quarks in doubly-charm molecular states are from two charm quark pairs.
- Compared with the hidden-charm states, the doubly-charm states should be difficult to be produced in experiment.

- Recently the doubly baryons Ξ_{cc}^{++} have been observed at LHCb.
- It suggest that the experimental observation of the doubly charm molecular state becomes possible!
- Very recently, the Tcc+ was observed. It is soon explained as a DD* molecular state.

PRL119 (2017)112001



arXiv:2109.01038



Theoretical works

$B^{(*)}B^{(*)}$ interaction was studied by solving the Schödinger equation

Barnes, N. Black, D.J. Dean, E.S. Swanson, Phys. Rev. C 60, 045202 (1999).

Coupled-channel analysis of the possible $D^{(*)}D^{(*)}, B^{(*)}B^{(*)}, D^{(*)}B^{(*)}$ molecular states.

$T_{cc}(3875)$

N. Li, Z.F. Sun, X. Liu, S.L. Zhu, Phys. Rev. D 88(11), 114008 (2013)

DD and BB interactions within heavy meson chiral effective field theory

B. Wang, Z.W. Liu, X. Liu, Phys. Rev. D **99**(3), 036007 (2019)

DD^* potentials in chiral perturbation theory

H. Xu, B. Wang, Z.W. Liu, X. Liu, Phys. Rev. D **99**(1), 014027 (2019)

Possible molecular states in $B^{(*)}\ B^{(*)}$ scatterings

M.T. Yu, Z.Y. Zhou, D.Y. Chen, Z. Xiao. Phys. Rev. D **101**(7), 074027 (2020).

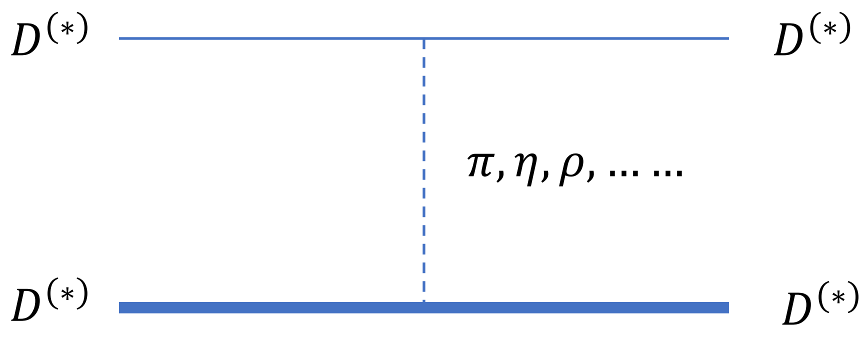
A survey of heavy-heavy hadronic molecules

Dong Guo, Zou, arXiv: 2108.02673

Hidden(doubly)-heavy tetraquark without strangeness from $D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}$ and $D^{(*)}D^{(*)}/B^{(*)}B^{(*)}$ interactions

Ding, Jiang, **JH**, Eur.Phys.J.C 80 (2020) 12, 1179

One boson exchange model



Wave function

$$|X^0_{D\bar{D}^*}\rangle_{I=0} = \frac{1}{2}\Big[\big(|D^{*+}D^-\rangle + |D^{*0}\bar{D}^0\rangle\big) + c\big(|D^+D^{*-}\rangle + |D^0\bar{D}^{*0}\rangle\big)\Big],$$
$$|X^0_{D\bar{D}^*}\rangle_{I=1} = \frac{1}{2}\Big[\big(|D^{*+}D^-\rangle - |D^{*0}\bar{D}^0\rangle\big) + c\big(|D^+D^{*-}\rangle - |D^0\bar{D}^{*0}\rangle\big)\Big],$$
$$|X^+_{D\bar{D}^*}\rangle_{I=1} = \frac{1}{\sqrt{2}}\big(|D^{*+}\bar{D}^0\rangle + c|D^+\bar{D}^{*0}\rangle\big),$$
$$|X^-_{D\bar{D}^*}\rangle_{I=1} = \frac{1}{\sqrt{2}}\big(|D^{*-}\bar{D}^0\rangle + c|D^-\bar{D}^{*0}\rangle\big), \qquad (1)$$

$$|X^0_{D\bar{D}}\rangle_{I=0} = \frac{1}{\sqrt{2}}\Big(|D^+\bar{D}^-\rangle + |D^0\bar{D}^0\rangle\Big),$$
$$|X^0_{D\bar{D}}\rangle_{I=1} = \frac{1}{\sqrt{2}}\Big(|D^+\bar{D}^-\rangle - |D^0\bar{D}^0\rangle\Big),$$
$$|X^+_{D\bar{D}}\rangle_{I=1} = |D^+\bar{D}^0\rangle,$$
$$|X^-_{D\bar{D}}\rangle_{I=1} = |D^-\bar{D}^0\rangle.$$

$$|X^+_{D^{(*)}D^{(*)}}\rangle_{I=0} = \frac{1}{\sqrt{2}}\Big(|D^{(*)+}D^{(*)0}\rangle - |D^{(*)0}D^{(*)+}\rangle\Big),$$
$$|X^+_{D^{(*)}D^{(*)}}\rangle_{I=1} = \frac{1}{\sqrt{2}}\Big(|D^{(*)+}D^{(*)0}\rangle + |D^{(*)0}D^{(*)+}\rangle\Big),$$
$$|X^0_{D^{(*)}D^{(*)}}\rangle_{I=1} = |D^{(*)0}D^{(*)0}\rangle,$$
$$|X^{++}_{D^{(*)}D^{(*)}}\rangle_{I=1} = |D^{(*)+}\bar{D}^{(*)+}\rangle.$$

Lagrangian

with chiral and heavy quark symmetries

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{P}} = -i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\lambda\dagger}\partial^\nu\mathbb{P}_{ba} + i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\lambda}\partial^\nu\mathbb{P}_{ab},$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{P}} = -\frac{2g}{f_\pi}\left(\mathcal{P}_b\mathcal{P}_{a\lambda}^{*\dagger} + \mathcal{P}_{b\lambda}^*\mathcal{P}_a^\dagger\right)\partial^\lambda\mathbb{P}_{ba} + \frac{2g}{f_\pi}\left(\widetilde{\mathcal{P}}_{a\lambda}^{*\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_{b\lambda}^*\right)\partial^\lambda\mathbb{P}_{ab}.$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V\mathcal{P}_b\mathcal{P}_a^\dagger v\cdot\mathbb{V}_{ba} + \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab},$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = -2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}\left(\mathcal{P}_b\mathcal{P}_a^{*\mu\dagger} + \mathcal{P}_b^{*\mu}\mathcal{P}_a^\dagger\right)(\partial^\alpha\mathbb{V}^\beta)_{ba} - 2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}\left(\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b^{*\mu}\right)(\partial^\alpha\mathbb{V}^\beta)_{ab},$$

$$\begin{aligned}\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}} = {} & \sqrt{2}\beta g_V\mathcal{P}_b^*\cdot\mathcal{P}_a^{*\dagger}v\cdot\mathbb{V}_{ba} \\ & - i2\sqrt{2}\lambda g_V\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\nu\dagger}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ba} \\ & - \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^{*\dagger}\widetilde{\mathcal{P}}_b^* v\cdot\mathbb{V}_{ab} \\ & - i2\sqrt{2}\lambda g_V\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\nu}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ab}.\end{aligned}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V\mathcal{P}_b\mathcal{P}_a^\dagger v\cdot\mathbb{V}_{ba} + \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab},$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = -2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}\left(\mathcal{P}_b\mathcal{P}_a^{*\mu\dagger} + \mathcal{P}_b^{*\mu}\mathcal{P}_a^\dagger\right)(\partial^\alpha\mathbb{V}^\beta)_{ba} - 2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}\left(\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b^{*\mu}\right)(\partial^\alpha\mathbb{V}^\beta)_{ab},$$

$$\begin{aligned}\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}} = {} & \sqrt{2}\beta g_V\mathcal{P}_b^*\cdot\mathcal{P}_a^{*\dagger}v\cdot\mathbb{V}_{ba} \\ & - i2\sqrt{2}\lambda g_V\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\nu\dagger}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ba} \\ & - \sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^{*\dagger}\widetilde{\mathcal{P}}_b^* v\cdot\mathbb{V}_{ab} \\ & - i2\sqrt{2}\lambda g_V\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\nu}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ab}.\end{aligned}$$

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = -2g_s\mathcal{P}_b\mathcal{P}_b^\dagger\sigma - 2g_s\widetilde{\mathcal{P}}_b\widetilde{\mathcal{P}}_b^\dagger\sigma,$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\sigma} = 2g_s\mathcal{P}_b^*\cdot\mathcal{P}_b^{*\dagger}\sigma + 2g_s\widetilde{\mathcal{P}}_b^*\cdot\widetilde{\mathcal{P}}_b^{*\dagger}\sigma,$$

$$\begin{aligned}\mathcal{L}_{D^*_{(s)}\bar{D}^*_{(s)}J/\psi} = {} & -ig_{D^*_{(s)}D^*_{(s)}\psi}\big[\psi\cdot\bar{D}^*\overleftrightarrow{\partial}\cdot D^* \\ & -\psi^\mu\bar{D}^*\cdot\overleftrightarrow{\partial}^\mu D^* + \psi^\mu\bar{D}^*\cdot\overleftrightarrow{\partial}D^{*\mu})\big],\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{D^*_{(s)}\bar{D}_{(s)}J/\psi} = {} & g_{D^*_{(s)}D_{(s)}\psi}\ \epsilon_{\beta\mu\alpha\tau}\partial^\beta\psi^\mu(\bar{D}\overleftrightarrow{\partial}^\tau D^{*\alpha} \\ & +\bar{D}^{*\alpha}\overleftrightarrow{\partial}^\tau D),\end{aligned}$$

$$\mathcal{L}_{D_{(s)}\bar{D}_{(s)}J/\psi} = ig_{D_{(s)}D_{(s)}\psi}\psi\cdot\bar{D}\overleftrightarrow{\partial}D,$$

β	g	g_V	λ	g_s			
0.9	0.59	5.9	0.56	0.76			
β_S	ℓ_S	g_1	λ_S	β_B	ℓ_B	g_4	λ_I
-1.74	6.2	-0.94	-3.31	$-\beta_S/2$	$-\ell_S/2$	$3g_1/(2\sqrt{2})$	$-\lambda_S/\sqrt{8}$

$$\frac{g_{D^*D^*\psi}}{m_{D^*}} = \frac{g_{D_{(s)}D_{(s)}\psi}}{m_D} = g_{D^*_{(s)}D_{(s)}\psi} = 2g_2\sqrt{m_\psi},$$

Chen et al. PRD47(1993)1030; Yan et al.PRD46(1992)1148; Wise, PRD45(1992)2188; Casalbuoni et al. Phys. Rept. 281(1997)145

Potential

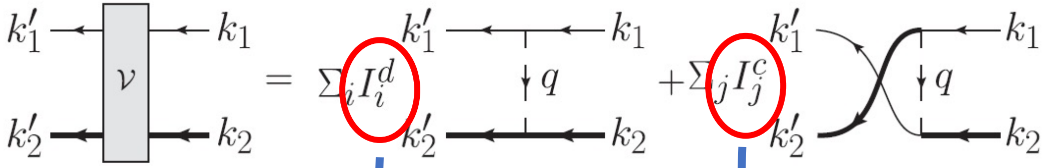


Table 1 The isospin factors I_i^d and I_i^c for direct and cross diagrams and different exchange mesons.

	I_i^d						I_i^c					
	π	η	ρ	ω	σ	J/ψ	π	η	ρ	ω	σ	J/ψ
$[\mathcal{P}\bar{\mathcal{P}}]^T$	--	--	$-\frac{1}{2}$	$\frac{1}{2}$	1	1	--	--	--	--	--	--
$[\mathcal{P}\bar{\mathcal{P}}]^S$	--	--	$\frac{3}{2}$	$\frac{1}{2}$	1	1	--	--	--	--	--	--
$[\mathcal{P}\bar{\mathcal{P}}^*]^T$	--	--	$-\frac{1}{2}$	$\frac{1}{2}$	1	1	$-\frac{c}{2}$	$\frac{c}{6}$	$-\frac{c}{2}$	$\frac{c}{2}$	--	c
$[\mathcal{P}\bar{\mathcal{P}}^*]^S$	--	--	$\frac{3}{2}$	$\frac{1}{2}$	1	1	$\frac{3}{2}c$	$\frac{c}{6}$	$\frac{3}{2}c$	$\frac{c}{2}$	--	c
$[\mathcal{P}^*\bar{\mathcal{P}}^*]^T$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	1	--	--	--	--	--	--
$[\mathcal{P}^*\bar{\mathcal{P}}^*]^S$	$\frac{3}{2}$	$\frac{1}{6}$	$\frac{3}{2}$	$\frac{1}{2}$	1	1	--	--	--	--	--	--
$[\mathcal{P}\mathcal{P}]^T$	--	--	$\frac{1}{2}$	$\frac{1}{2}$	1	1	--	--	$\frac{1}{2}$	$\frac{1}{2}$	1	1
$[\mathcal{P}\mathcal{P}]^S$	--	--	$-\frac{3}{2}$	$\frac{1}{2}$	1	1	--	--	$\frac{3}{2}$	$-\frac{1}{2}$	1	1
$[\mathcal{P}\mathcal{P}^*]^T$	--	--	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	--	1
$[\mathcal{P}\mathcal{P}^*]^S$	--	--	$-\frac{3}{2}$	$\frac{1}{2}$	1	1	$\frac{3}{2}$	$-\frac{1}{6}$	$\frac{3}{2}$	$-\frac{1}{2}$	--	1
$[\mathcal{P}^*\mathcal{P}^*]^T$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
$[\mathcal{P}^*\mathcal{P}^*]^S$	$-\frac{3}{2}$	$\frac{1}{6}$	$-\frac{3}{2}$	$\frac{1}{2}$	1	1	$\frac{3}{2}$	$-\frac{1}{6}$	$\frac{3}{2}$	$-\frac{1}{2}$	1	1

Potential

$$\mathcal{V}_{\mathbb{P},\sigma} = I_i^{(d,c)} \Gamma_1 \Gamma_2 P_{\mathbb{P},\sigma} f(q^2), \quad \mathcal{V}_{\mathbb{V}} = I_i^{(d,c)} \Gamma_{1\mu} \Gamma_{2\nu} P_{\mathbb{V}}^{\mu\nu} f(q^2),$$

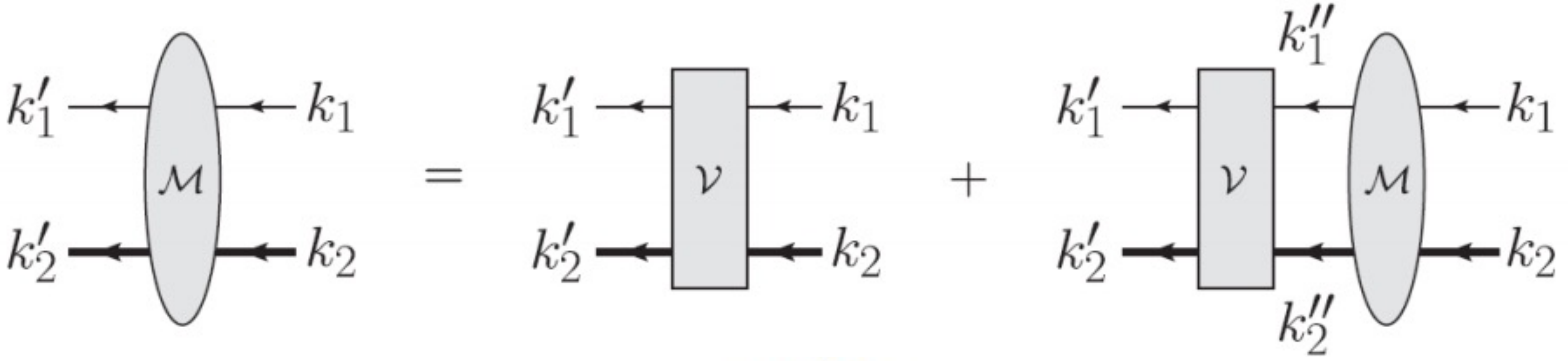
Propagator

$$P_{\mathbb{P},\sigma} = \frac{i}{q^2 - m_{\mathbb{P},\sigma}^2}, \qquad P_{\mathbb{V}}^{\mu\nu} = i\frac{-g^{\mu\nu} + q^\mu q^\nu / m_{\mathbb{V}}^2}{q^2 - m_{\mathbb{V}}^2},$$

Form factor

$$f(q^2) = \frac{\Lambda_e^2}{q^2 - \Lambda_e^2}$$

Quasipotential Bethe-Salpeter Equation



$$\mathcal{M}(k_1'k_2', k_1k_2; P) \quad = \quad \mathcal{V}(k_1'k_2', k_1k_2; P) + \int \frac{d^4k_2''}{(2\pi)^4}\mathcal{V}(k_1'k_2', k_1''k_2''; P)G(k_1''k_2''; P)\mathcal{M}(k_1''k_2'', k_1k_2; P),$$

$$G_0(k'') = \frac{\delta^+(k_h''^2 - m_h^2)}{k''^2_l - m_l^2} \quad , \quad E_h = \sqrt{m^2 + \vec{k}_h''^2}$$

The 4-dimensional BSE → Quasipotential approximation → The 3-dimensional equation

$$i\mathcal{M}(k', k) \quad = \quad i\mathcal{V}(k', k) + \int \frac{dk''}{(2\pi)^3} i\mathcal{V}(k', k'')G_0(k'')i\mathcal{M}(k'', k),$$

Partial wave decomposition

$$iM(k',k) = iV(k',k) + \int \frac{dk''}{(2\pi)^3} iV(k',k'')G_0(k'')iM(k'',k),$$

The 3-dimensional equation

Partial wave decomposition

$$\mathcal{V}_{\lambda'\lambda}(\boldsymbol{k}',\boldsymbol{k}) = \sum_{J\lambda_R} \frac{2J+1}{4\pi} D^{J*}_{\lambda_R,\lambda'}(\phi',\theta',-\phi')\mathcal{V}^J_{\lambda'\lambda,\lambda_R}(\mathrm{k}',\mathrm{k})D^J_{\lambda_R,\lambda}(\phi,\theta,-\phi)$$

$$\to \mathcal{V}^J_{\lambda'\lambda,\lambda_R}(\mathrm{k}',\mathrm{k}) = \frac{2J+1}{4\pi}\int d\Omega' d\Omega D^{J*}_{\lambda_R,\lambda'}(\phi',\theta',-\phi')\mathcal{V}_{\lambda'\lambda}(\boldsymbol{k}',\boldsymbol{k})D^J_{\lambda_R,\lambda}(\phi,\theta,-\phi)$$

$$\to \mathcal{V}^J_{\lambda'\lambda}(\mathrm{k}',\mathrm{k}) = 2\pi \int d\cos\theta_{k,k'} d^J_{\lambda,\lambda'}(\theta_{k',k})\mathcal{V}_{\lambda'\lambda}(\boldsymbol{k}',\boldsymbol{k})$$

$$i\mathcal{M}_{\lambda',\lambda}(\mathrm{k}',\mathrm{k}) = i\mathcal{V}_{\lambda'\lambda}(\mathrm{k}',\mathrm{k}) + \int \frac{\mathrm{k}''^2 d\mathrm{k}''}{(2\pi)^3} i\mathcal{V}_{\lambda'\lambda''}(\mathrm{k}',\mathrm{k}'')G_0(\boldsymbol{k}'')i\mathcal{M}_{\lambda''\lambda}(\mathrm{k}'',\mathrm{k})$$

The 1-dimensional equation

Transformation to matrix euqation

Transformed as a matrix equation by Gauss discretization

$$iM_{ik} = iV_{ik} + \sum_{j=0}^{N} iV_{ij}G_j iM_{jk} \Rightarrow M = V + VGM$$

$$G_j = \begin{cases} -\frac{i\bar{q}}{32\pi^2 W} + \sum_j \left[\frac{w(q_j)}{(2\pi)^3}\frac{\bar{q}^2}{2W(q_j^2-\bar{q}^2)}\right] & \text{for } j=0, \text{ if } Re(W) > m_1 + m_2, \\ \frac{w(q_j)}{(2\pi)^3}\frac{q_j^2}{2E(q_j)[(W-E(q_j))^2-\omega^2(q_j)]} & \text{for } j \neq 0 \end{cases}$$

$$\text{with } \bar{q} = \frac{1}{2W}\sqrt{[W^2-(m_1+m_2)^2][W^2-(m_1-m_2)^2]}.$$

Search for the pole

$$M = (1-VG)^{-1}V \quad \Rightarrow \quad |1-VG| = 0$$

Results for observed states

	Full(V)		Full(B)		No $J/\psi(\Upsilon)$		$J/\psi(\Upsilon)$	
$[I^G(J^P)]$	Λ	W	Λ	W	Λ	W	Λ	W
$D\bar{D}^*[0^+(1^+)]$	--	--	0.5	2.3	0.5	1.5	--	--
$X(3872)$	--	--	0.7	13.4	0.7	7.8	--	--
	0.2	*2.6*	0.8	23.3	0.8	15.3	--	--
$D\bar{D}^*[1^+(1^+)]$	*1.4*	*22.8*	1.9	2.4	--	--	2.5	1.1
$Z_c(3900)$	*1.5*	*8.3*	2.0	7.1	--	--	2.6	6.6
	1.6	*2.3*	2.1	17.1	--	--	2.7	17.3
$D^*\bar{D}^*[1^+(1^+)]$	*1.4*	*23.4*	2.3	2.8	--	--	3.6	1.7
$Z_c(4020)$	*1.6*	*5.3*	2.6	10.2	--	--	3.8	6.8
	1.8	*0.5*	2.8	20.0	--	--	4.0	18.4
$B\bar{B}^*[0^+(1^+)]$	--	--	0.3	1.5	0.3	0.8	--	--
	--	--	0.5	11.9	0.5	10.6	--	--
	0.1	*4.4*	0.6	21.4	1.5	20.2	--	--
$B\bar{B}^*[1^+(1^+)]$	*0.9*	*19.5*	1.6	2.4	2.2	0.8	3.6	1.2
$Z_b(10610)$	*1.0*	*4.4*	1.9	10.9	2.6	8.4	3.9	9.8
	1.1	*1.2*	2.1	22.7	3.0	25.4	4.1	21.5
$B^*\bar{B}^*[1^+(1^+)]$	*0.8*	*16.3*	1.5	0.8	2.2	1.2	4.1	0.8
$Z_b(10650)$	*0.9*	*7.9*	2.0	13.5	2.7	8.3	4.6	9.4
	1.0	*1.4*	2.2	21.1	3.1	20.4	5.0	23.2

Λ for $D\bar{D}^*$ and $D^*\bar{D}^*$ states with $1^+(1^+)$ are about 2 GeV, larger than those to produce $0^+(1^+)$ state.

→The $Z_c(3900)$ as a virtual state.
Guo, Hidalgo-Duque, Nieves,Valderrama, Phys.Rev.D 88 (2013) 054007

If we assume that the $Z_c(3900)$ and $Z_c(4020)$ are virtual states , the gap of the cutoff will reduce.

We consider the heavy meson exchange.

Without the J/ψ exchange, the two hidden–charm states with $1^+(1^+)$ can not be produced while without the Υ exchange the two hidden–bottom states are still found with a little larger cutoff.

Only with the $J/\psi(\Upsilon)$ exchange, the bound state can be found in both cases while the hidden–bottom states requires larger cutoffs.

The J/ψ exchange in the charm sector is more important than the Υ exchange in the bottom sector.

Results for all channels

	$D\bar{D}(G=+)$		$D\bar{D}(G=-)$		DD	
$I(J^P)$	Λ	W	Λ	W	Λ	W
$0(0^+)$	0.6	2.1			−−	−−
	0.8	10.7			−−	−−
	0.9	18.0			−−	−−
$1(0^+)$			3.5	0.4	−−	−−
			3.9	11.0	−−	−−
			4.1	22.2	−−	−−
	$D\bar{D}^*(G=+)$		$D\bar{D}^*(G=-)$		DD^*	
$0(1^+)$	0.5	2.3	0.6	1.5	0.8	1.2
	0.7	13.4	0.8	12.0	1.0	12.7
	0.8	23.3	0.9	22.7	1.1	27.9
$1(1^+)$	1.9	2.4	−−	−−	−−	−−
	2.0	7.1	−−	−−	−−	−−
	2.1	17.1	−−	−−	−−	−−
	$D^*\bar{D}^*(G=+)$		$D^*\bar{D}^*(G=-)$		D^*D^*	
$0(0^+)$	0.8	0.8			−−	−−
	1.4	14.0			−−	−−
	2.1	22.4			−−	−−
$0(1^+)$			0.6	1.0	0.9	1.0
			1.0	13.0	1.1	14.6
			1.2	23.5	1.2	33.7
$0(2^+)$	0.5	2.4			0.7	0.3
	0.7	14.0			1.0	15.2
	0.8	25.6			1.1	27.5
$1(0^+)$			1.8	1.9	−−	−−
			2.1	11.5	−−	−−
			2.3	23.1	−−	−−
$1(1^+)$	2.3	2.8			−−	−−
	2.6	10.2			−−	−−
	2.8	20.0			−−	−−
$1(2^+)$			4.5	1.6	1.75	1.0
			4.9	10.4	1.80	5.9
			5.0	18.2	1.90	41.4

T_{cc}

	$B\bar{B}(G=+)$		$B\bar{B}(G=-)$		$\bar{B}\bar{B}$	
$I(J^P)$	Λ	W	Λ	W	Λ	W
$0(0^+)$	0.3	1.1			−−	−−
	0.5	9.2			−−	−−
	0.6	17.2			−−	−−
$1(0^+)$			3.5	1.7	−−	−−
			4.0	11.1	−−	−−
			4.3	22.9	−−	−−
	$B\bar{B}^*(G=+)$		$B\bar{B}^*(G=-)$		$\bar{B}\bar{B}^*$	
$0(1^+)$	0.3	1.5	1.2	2.2	0.4	1.0
	0.5	11.9	1.4	12.6	0.5	9.3
	0.6	21.4	1.5	22.0	0.7	18.2
$1(1^+)$	1.6	2.4	−−	−−	−−	−−
	1.9	10.9	−−	−−	−−	−−
	2.1	22.7	−−	−−	−−	−−
	$B^*\bar{B}^*(G=+)$		$B^*\bar{B}^*(G=-)$		$\bar{B}^*\bar{B}^*$	
$0(0^+)$	0.4	1.3			−−	−−
	0.8	9.5			−−	−−
	1.2	20.9			−−	−−
$0(1^+)$			0.3	0.7	0.9	2.2
			0.6	10.3	1.1	11.6
			0.8	23.0	1.2	19.7
$0(2^+)$	0.3	1.7			0.4	1.5
	0.5	13.6			0.6	11.7
	0.6	25.4			0.7	22.1
$1(0^+)$			1.1	1.5	−−	−−
			1.5	12.3	−−	−−
			1.7	24.4	−−	−−
$1(1^+)$	1.5	0.8			−−	−−
	2.0	13.5			−−	−−
	2.2	21.1			−−	−−
$1(2^+)$			3.5	0.4	1.2	0.7
			4.4	13.8	1.4	5.5
			4.7	21.3	1.6	19.1

Hidden(doubly)-heavy tetraquark with strangeness from $D_{(s)}^{(*)}\bar{D}_{s}^{(*)}/B_{(s)}^{(*)}\bar{B}_{s}^{(*)}$ and $D_{(s)}^{(*)}D_{s}^{(*)}/B_{(s)}^{(*)}B_{s}^{(*)}$ interactions

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Wave function

$$|X^{-}_{D^*\bar{D}_s+D\bar{D}_s^*}\rangle = \frac{1}{\sqrt{2}}(|D^{*0}D_s^-\rangle + \eta|D^0D_s^{*-}\rangle),$$

$$|X^{0}_{D^*\bar{D}_s+D\bar{D}_s^*}\rangle = \frac{1}{\sqrt{2}}(|D^{*+}D_s^-\rangle + \eta|D^+D_s^{*-}\rangle).$$

where $\eta = \pm$ corresponds to the Z_{cs} state with $G_{U/V} = \pm$

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$$|X^{-}_{D^*\bar{D}_s^*}\rangle = |D^{*0}D_s^{*-}\rangle, \quad |X^{0}_{D_s\bar{D}_s}\rangle = |D_s^-D_s^+\rangle,$$

$$|X^{0}_{D^*\bar{D}_s^*}\rangle = |D^{*+}D_s^{*-}\rangle.$$

$$|X^{0}_{D_s\bar{D}_s^*}\rangle = \frac{1}{\sqrt{2}}(|D_s^-D_s^{*+}\rangle - C|D_s^{*-}D_s^+\rangle).$$

$$|X^{+}_{D^*D_s+DD_s^*}\rangle = \frac{1}{\sqrt{2}}(|D^{*0}D_s^+\rangle + \eta|D^0D_s^{*+}\rangle),$$

$$|X^{++}_{D^*D_s+DD_s^*}\rangle = \frac{1}{\sqrt{2}}(|D^{*+}D_s^+\rangle + \eta|D^+D_s^{*+}\rangle).$$

$$|X^{+}_{D^*D_s^*}\rangle = |D^{*0}D_s^{*+}\rangle, \quad |X^{+}_{DD_s}\rangle = |D^0D_s^+\rangle,$$

$$|X^{++}_{D^*D_s^*}\rangle = |D^{*+}D_s^{*+}\rangle, \quad |X^{++}_{DD_s}\rangle = |D^+D_s^+\rangle.$$

$$|X^{++}_{D_s^{(*)}D_s^{(*)}}\rangle = |D_s^{(*)+}D_s^{(*)+}\rangle.$$

Single channel results

$Z_{cs}(3895)$

The calculation supports the existence of a state with $G_{U/V} = +$ at a cutoff of about 3 GeV, which is relevant to the state $Z_{cs}(3895)$ observed at BESIII recently.

	$D\bar{D}_s$		$D_s\bar{D}_s$		DD_s		D_sD_s	
J^P	Λ	W	Λ	W	Λ	W	Λ	W
0^+	4.4	1.1	1.6	0.3	⋯	⋯	⋯	⋯
	4.7	8.8	2.1	8.4	⋯	⋯	⋯	⋯
	4.9	18.4	2.4	19.5	⋯	⋯	⋯	⋯
	$B\bar{B}_s$		$B_s\bar{B}_s$		BB_s		B_sB_s	
J^P	Λ	W	Λ	W	Λ	W	Λ	W
0^+	⋯	⋯	1.0	1.4	⋯	⋯	⋯	⋯
	⋯	⋯	1.5	9.8	⋯	⋯	⋯	⋯
	⋯	⋯	1.8	18.7	⋯	⋯	⋯	⋯

	$D^*\bar{D}_s + D\bar{D}_s^*$				$D^*D_s + DD_s^*$			
	$G_{U/V} = +$		$G_{U/V} = -$		$G_{U/V} = +$		$G_{U/V} = -$	
J^P	Λ	W	Λ	W	Λ	W	Λ	W
1^+	3.1	1.4	⋯	⋯	⋯	⋯	2.0	0.4
	3.3	10.8	⋯	⋯	⋯	⋯	2.3	10.6
	3.4	19.1	⋯	⋯	⋯	⋯	2.4	17.7
	$D_s\bar{D}_s^*$				$D_sD_s^*$			
	$C = +$		$C = -$		$C = -$			
J^P	Λ	W	Λ	W	Λ	W		
1^+	1.4	0.8	1.8	1.0	3.40	0.1		
	1.7	9.7	2.3	7.8	3.42	10.5		
	1.8	15.8	2.6	16.8	3.43	18.1		
	$B^*\bar{B}_s + B\bar{B}_s^*$				$B^*B_s + BB_s^*$			
	$G_{U/V} = +$		$G_{U/V} = -$		$G_{U/V} = +$		$G_{U/V} = -$	
J^P	Λ	W	Λ	W	Λ	W		
1^+	⋯	⋯	⋯	⋯	⋯	⋯	0.8	0.9
	⋯	⋯	⋯	⋯	⋯	⋯	1.2	10.0
	⋯	⋯	⋯	⋯	⋯	⋯	1.4	19.6
	$B_s\bar{B}_s^*$				$B_sB_s^*$			
	$C = +$		$C = -$		$C = -$			
J^P	Λ	W	Λ	W	Λ	W		
1^+	0.9	0.9	0.8	0.2	2.84	1.5		
	1.3	8.1	1.4	6.3	2.90	12.8		
	1.6	20.1	1.8	15.0	2.95	23.5		

	$D^*\bar{D}_s^*$		$D_s^*\bar{D}_s^*$		$D^*D_s^*$		$D_s^*D_s^*$	
J^P	Λ	W	Λ	W	Λ	W	Λ	W
0^+	4.2	1.3	1.3	0.9	⋯	⋯	⋯	⋯
	4.5	7.8	1.6	7.8	⋯	⋯	⋯	⋯
	4.7	15.4	1.8	16.6	⋯	⋯	⋯	⋯
J^P	Λ	W	Λ	W	Λ	W	Λ	W
1^+	4.2	1.7	1.4	0.6	2.95	0.5	3.00	0.4
	4.5	8.4	1.8	7.7	3.03	8.2	3.20	11.0
	4.8	22.6	2.1	18.5	3.07	23.2	3.25	18.8
J^P	Λ	W	Λ	W	Λ	W	Λ	W
2^+	4.1	0.8	2.0	1.1	3.21	0.4	3.00	0.9
	4.5	9.2	2.8	8.4	3.23	10.2	3.10	11.2
	4.7	18.1	3.2	16.1	3.24	25.7	3.15	21.6
	$B^*\bar{B}_s^*$		$B_s^*\bar{B}_s^*$		$B^*B_s^*$		$B_s^*B_s^*$	
J^P	Λ	W	Λ	W	Λ	W	Λ	W
0^+	⋯	⋯	0.8	0.7	3.9	1.4	⋯	⋯
	⋯	⋯	1.2	8.2	4.2	8.7	⋯	⋯
	⋯	⋯	1.5	20.5	4.4	20.3	⋯	⋯
J^P	Λ	W	Λ	W	Λ	W	Λ	W
1^+	⋯	⋯	0.8	0.4	1.2	0.7	1.8	0.7
	⋯	⋯	1.2	5.7	1.7	8.8	2.3	6.8
	⋯	⋯	1.6	18.3	2.0	19.8	2.6	14.3
J^P	Λ	W	Λ	W	Λ	W	Λ	W
2^+	⋯	⋯	1.0	0.7	2.0	0.6	2.1	0.2
	⋯	⋯	1.8	8.1	2.7	8.7	2.5	11.5
	⋯	⋯	2.2	13.8	2.9	15.3	2.6	19.2

$Z_{cs} \longleftrightarrow Z_c$

- $D^*\bar{D}_s + D\bar{D}_s^*$ bound state with $G_{U/V} = +$ can be related to the $Z_{cs}(3895)$ state observed at BESIII.
- it is suggested as the $U/V-$spin partner of the $Z_c(3900)$

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- The J/ψ exchange is important to reproduce the $Z_c(3900)$.
- Only J/ψ exchange involves in Z_{cs} state.
- Hence, the J/ψ exchange is essential to reproduce both $Z_{cs}(3895)$ and $Z_c(3900)$.

Table 4 The binding energies of some bound states with selected value of cutoff Λ. The result for the $D^{(*)}\bar{D}_s^{(*)}$ systems are listed in the $2-4^{th}$ columns. The results for the $D^{(*)}\bar{D}^{(*)}$ only with J/ψ exchange are listed in $5-7^{th}$ columns. The cutoff Λ and binding energy E_B are in the units of GeV and MeV.

J^{PG}	Z_{cs}			Z_c only with J/ψ		
	system	Λ	E_B	system	Λ	E_B
0^+	$D\bar{D}_s$	4.6	5.4	$D\bar{D}$	4.6	0.3
1^{++}	$D^*\bar{D}_s + D\bar{D}_s^*$	3.2	5.0	$D\bar{D}^*$	3.2	0.8
0^+	$D^*\bar{D}_s^*$	4.4	4.8	$D^*\bar{D}^*$	4.4	0.6
1^+	$D^*\bar{D}_s^*$	4.4	5.5	$D^*\bar{D}^*$	4.4	0.9
2^+	$D^*\bar{D}_s^*$	4.4	6.1	$D^*\bar{D}^*$	4.4	1.1

Generally speaking, there is no significant difference between two cases as expected, which means it is reasonable to consider Z_{cs} state as strange partner of Z_c state in the $SU(3)_F$ symmetry.

Moreover, we can expect that the binding of the hidden charmed strange system Z_{cs} is loosely than the hidden charmed system Z_c because more light exchanges are allowed for the latter state.

The role played by the σ exhange

Table 6 The binding energies of some bound states with some selected values of cutoff Λ. The results with σ exchange are listed $3-4^{rd}$ columns. The result without σ exchange are listed $5-6^{th}$ columns. The cutoff Λ and binding energy E_B are in the units of GeV and MeV.

system	$J^{PC/G}$	with σ		without σ	
		Λ	E_B	Λ	E_B
$D\bar{D}_s$	0^+	4.4	6.7	4.4	1.1
$D^*\bar{D}_s+D\bar{D}_s^*$	1^{++}	3.1	5.9	3.1	1.4
$D^*\bar{D}_s^*$	0^+	4.2	6.1	4.2	1.3
$D^*\bar{D}_s^*$	1^+	4.2	6.8	4.2	1.7
$D^*\bar{D}_s^*$	2^+	4.1	5.1	4.1	0.8
DD_s	0^+	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D^*D_s+DD_s^*$	1^{++}	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D^*D_s^*$	0^+	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D^*D_s^*$	1^+	2.95	1.2	2.95	0.5
$D^*D_s^*$	2^+	3.21	1.5	3.21	0.4
$D_s\bar{D}_s$	0^+	1.6	1.1	1.6	0.3
$D_s\bar{D}_s^*$	1^{++}	1.4	1.9	1.4	0.8
$D_s^*\bar{D}_s^*$	0^+	1.3	1.9	1.3	0.9
$D_s^*\bar{D}_s^*$	1^+	1.4	1.6	1.4	0.6
$D_s^*\bar{D}_s^*$	2^+	2.0	2.6	2.0	1.1
D_sD_s	0^+	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_sD_s^*$	1^+	3.41	6.1	3.41	1.2
$D_s^*D_s^*$	0^+	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$D_s^*D_s^*$	1^+	3.0	0.4	3.0	0.4
$D_s^*D_s^*$	2^+	3.0	1.3	3.0	0.9

In our calculation we adopt the widely used assumption that no $s\bar{s}$ component in σ meson.

It can not be exchanged between the mesons considered in the current work.

In Ref [arXiv: 2102.13058] where the J/ψ exchange was not included, the σ exchange is proposed to play the most important factor to form a molecular state to interpret the $Z_{cs}(3895)$.

Here, the Lagrangians for the vertices $PP\sigma$ and $P^*P^*\sigma$ are also applied to the heavy–strange meson.

One can find that there is no significant difference between cases with and without σ exchange.

It suggests that if we include the J/ψ exchange, the contribution from the σ exchange may be smeared in our theoretical frame.

Coupled-channel results

0^+			1^+			
Λ	$M_{th}-z$		Λ	$M_{th}-z$		
	$D\bar{D}_s$	$D^*\bar{D}_s^*$		$D^*\bar{D}_s$	$D\bar{D}_s^*$	$D^*\bar{D}_s^*$
4.2	1.9	$1.7+4.2i$	3.7	0.9	$\cdots$	$\cdots$
4.3	6.0	$3.8+7.0i$	3.9	10.0	$1.0+0.87i$	$\cdots$
4.4	13.4	$5.8+11.1i$	4.0	19.2	$2.8+0.00i$	$0.2+0.04i$
			4.1	35.3	$4.1+0.00i$	$0.7+0.13i$

$Z_{cs}(3895)$

0^+			1^+		
Λ	$M_{th}-z$		Λ	$M_{th}-z$	
	$D_s\bar{D}_s$	$D_s^*\bar{D}_s^*$		$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$
1.6	0.3	$7.8+0.11i$	1.4	0.8	$0.6+0.00i$
1.7	1.0	$11.7+0.21i$	1.5	2.5	$1.7+0.01i$
1.8	2.2	$16.6+0.37i$	1.6	5.4	$3.2+0.01i$

0^+			1^+			
Λ	$M_{th}-z$		Λ	$M_{th}-z$		
	$B\bar{B}_s$	$B^*\bar{B}_s^*$		$B^*\bar{B}_s$	$B\bar{B}_s^*$	$B^*\bar{B}_s^*$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

0^+			1^+		
Λ	$M_{th}-z$		Λ	$M_{th}-z$	
	$B_s\bar{B}_s$	$B_s^*\bar{B}_s^*$		$B_s^*\bar{B}_s$	$B_s^*\bar{B}_s^*$
1.0	0.5	$3.4+0.00i$	1.0	1.8	$2.2+0.01i$
1.1	2.5	$5.4+0.00i$	1.1	3.3	$3.7+0.02i$
1.2	3.8	$8.2+0.00i$	1.2	5.3	$5.7+0.02i$

0^+			1^+			
Λ	$M_{th}-z$		Λ	$M_{th}-z$		
	DD_s	$D^*D_s^*$		D^*D_s	DD_s^*	$D^*D_s^*$
$\cdots$	$\cdots$	$\cdots$	2.20	1.6	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	2.33	16.7	$0.6+1.7i$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	2.40	32.5	$5.2+0.0i$	$1.5+3.9i$
$\cdots$	$\cdots$	$\cdots$	2.50	$\cdots$	20.4+0.0i	$4.4+5.4i$

0^+			1^+		
Λ	$M_{th}-z$		Λ	$M_{th}-z$	
	D_sD_s	$D_s^*D_s^*$		$D_s^*D_s$	$D_s^*D_s^*$
$\cdots$	$\cdots$	$\cdots$	2.30	$\cdots$	$0.2+1.8i$
$\cdots$	$\cdots$	$\cdots$	2.70	$\cdots$	$35.0+6.9i$
$\cdots$	$\cdots$	$\cdots$	3.31	0.1	$\cdots$
$\cdots$	$\cdots$	$\cdots$	3.40	21.9	$\cdots$

0^+			1^+			
Λ	$M_{th}-z$		Λ	$M_{th}-z$		
	BB_s	$B^*B_s^*$		B^*B_s	BB_s^*	$B^*B_s^*$
$\cdots$	$\cdots$	$\cdots$	1.2	$\cdots$	$4.9+0.0i$	$1.7+2.2i$
$\cdots$	$\cdots$	$\cdots$	1.3	$\cdots$	$7.7+0.0i$	$4.3+3.0i$
$\cdots$	$\cdots$	$\cdots$	1.4	0.1	$11.7+0.0i$	$5.2+4.2i$

0^+			1^+		
Λ	$M_{th}-z$		Λ	$M_{th}-z$	
	B_sB_s	$B_s^*B_s^*$		$B_s^*B_s$	$B_s^*B_s^*$
$\cdots$	$\cdots$	$\cdots$	1.2	$\cdots$	$0.2+0.66i$
$\cdots$	$\cdots$	$\cdots$	2.1	$\cdots$	$36.7+7.7i$
$\cdots$	$\cdots$	$\cdots$	2.6	4.8	$\cdots$
$\cdots$	$\cdots$	$\cdots$	2.8	21.0	$\cdots$

Here we consider the $D^*\bar{D}_s$ and $D\bar{D}_s^*$ as sperated channel and coupled each other to discuss the effect of violation of symmetry. Generally speaking, the results with such treatment are consistent with single-channel calculation with $G_{U/V}$ parity.

Summary

The Hidden(doubly)-heavy molecular state from $D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}/B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}$ and $D_{(s)}^{(*)}D_{(s)}^{(*)}/B_{(s)}^{(*)}B_{(s)}^{(*)}$ interactions are studied systematically.

- $X(3872), Z_c(3900)$ and $Z_c(4020)$ can be well reproduced as hidden-charm molecular states. The latter two may be virtual states.

- The $T_{cc}(3875)$ can be interpretated as a doubly-charm DD^* state.

- The $Z_{cs}(3895)$ can be interpretated as a $D^*\bar{D}_s + D\bar{D}_s^*$ state with $G_{U/V} = +$ as strange partner of $Z_c(3900)$ state in the $SU(3)_F$ symmetry

- Other molecular states are predicted

THANK YOU!