TMD Phenomenology

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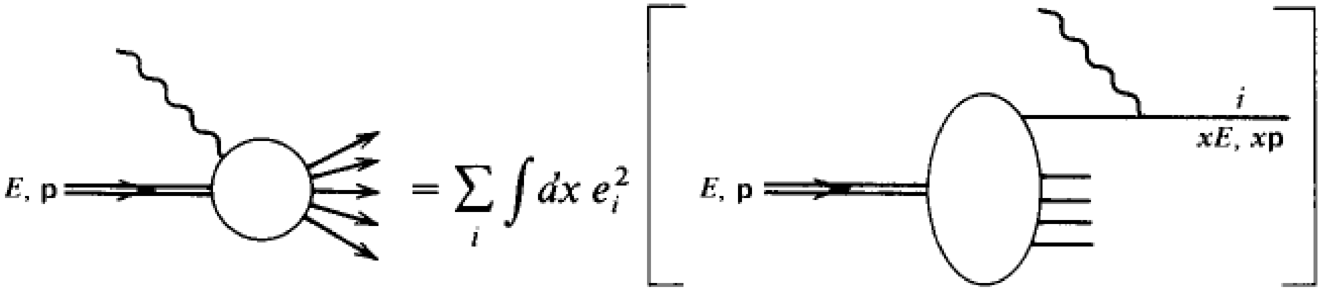
outline

- Overview on status of hadron structure in terms of transverse momentum distributions (TMDs)

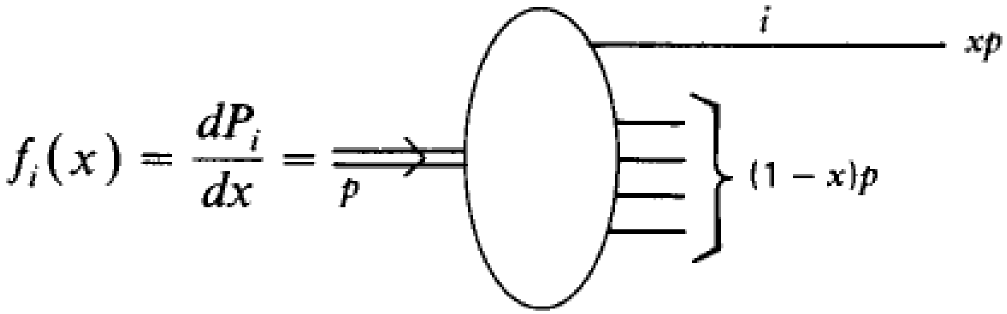
- Phenomenology —— Physical processes and observables within TMD evolution: selected results

- Summary

parton model

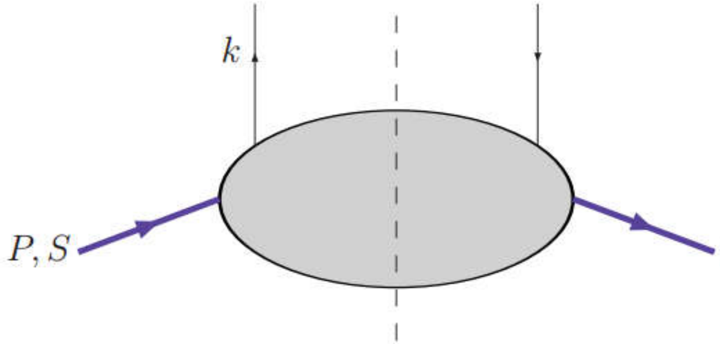


Parton distribution functions (PDF)



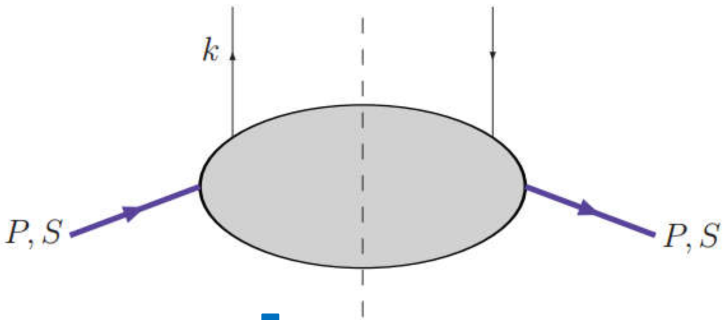
one dimension

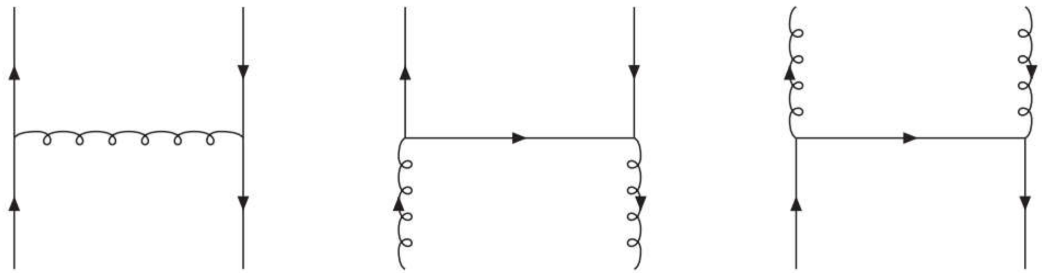
Cut diagram of Forward amplitude



$$f(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle$$

Including spin of Nucleon and quark





Order α_s evolution

$$
\begin{aligned}
\Phi_{ij}(k;P,S) &= \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X) \langle PS|\overline{\Psi}_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle \\
&= \int \mathrm{d}^4\xi\, e^{ik\cdot\xi} \langle PS|\overline{\Psi}_j(0)\Psi_i(\xi)|PS\rangle
\end{aligned}
$$

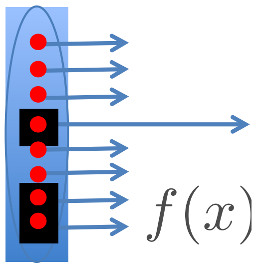
$$
\Phi(x,S) = \frac{1}{2}\left[\underbrace{f_1(x)}_{q} \not{n}_+ + S_L \underbrace{g_{1L}(x)}_{\Delta q} \gamma^5 \not{n}_+ + \underbrace{h_{1T}}_{\Delta_T q} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu \right]
$$

Collinear PDFs:

- Universality: once measured, can be applied to other processes
- Evolution : governed by the DGLAP/BFKL equations

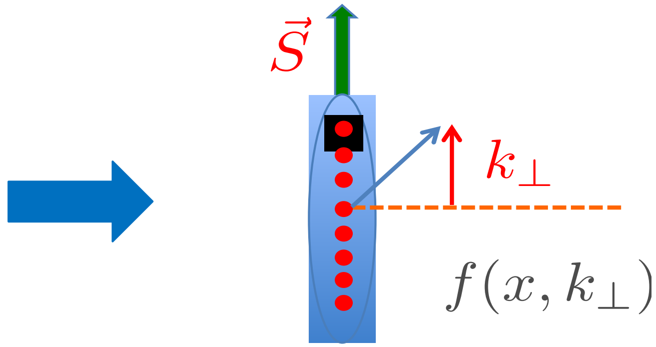
$$
F(x, Q_i) \longrightarrow F(x, Q_f)
$$

Parton's transverse motion



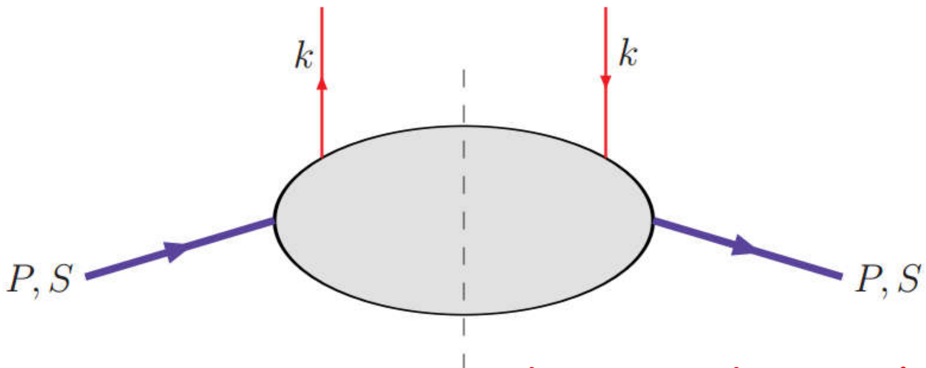
$p_a \approx xP_A$

Longitudinal motion only



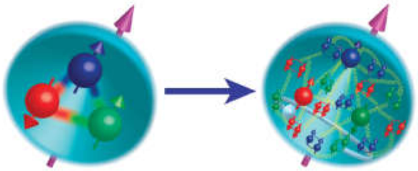
Longitudinal + transverse motion

$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) = \frac{1}{2}\Big[& f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \Big(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \Big) \gamma^5 \not{n}_+ \\
& + h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \Big(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \Big) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
& + h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]
\end{aligned}
$$

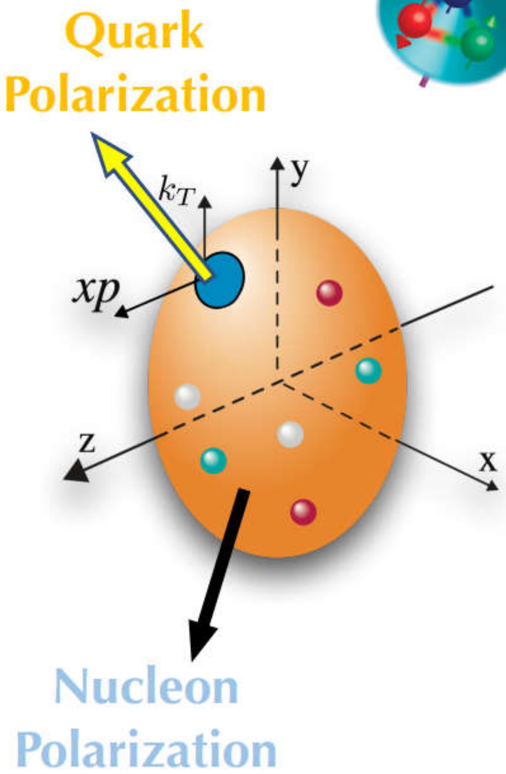


Transverse momentum dependent (TMD) PDFs

Probability interpretation of TMDPDFs



Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons



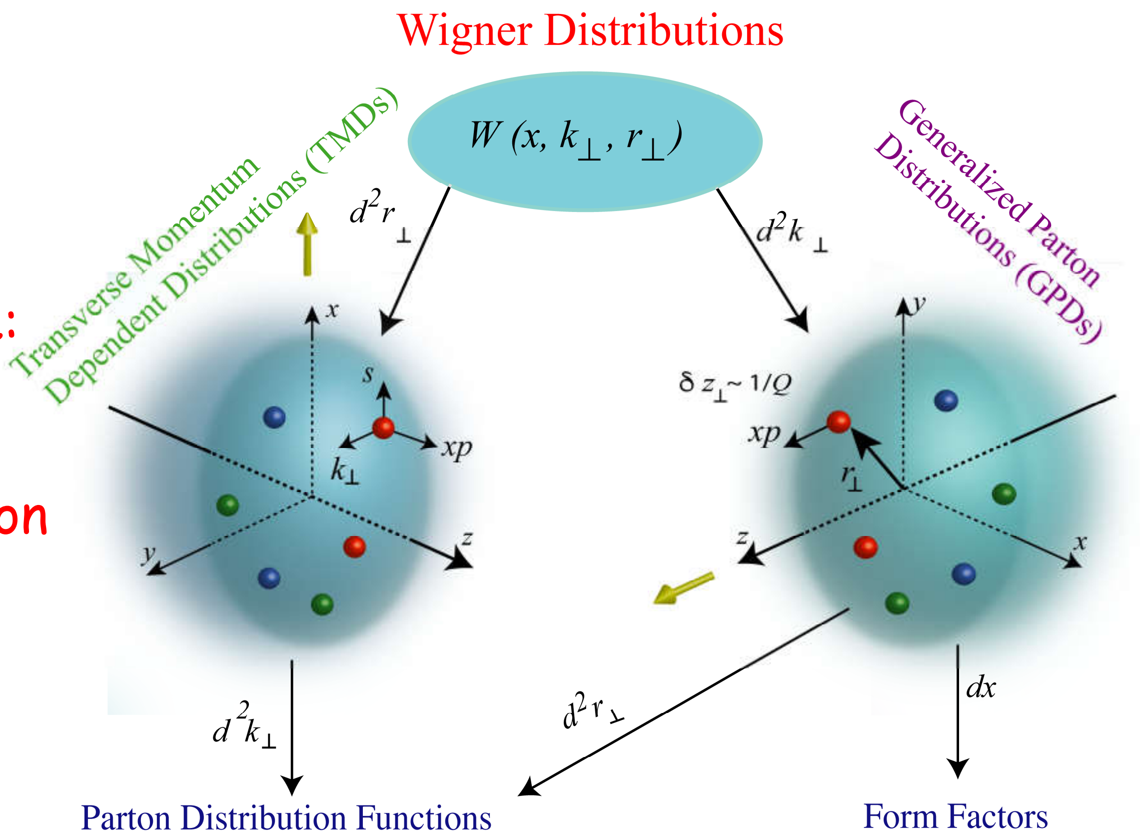
Nucleon Polarization	Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	$f_1(x,k_T^2)$		$h_1^\perp(x,k_T^2)$ - **Boer-Mulders**
L		$g_1(x,k_T^2)$ - **Helicity**	$h_{1L}^\perp(x,k_T^2)$ -
T	$f_{1T}^\perp(x,k_T^2)$ - **Sivers**	$g_{1T}(x,k_T^2)$ -	$h_1(x,k_T^2)$ - **Transversity** $h_{1T}^\perp(x,k_T^2)$ - **Pretzelosity**

analogous table exists for gluon TMDs

TMDPDFs provide new structure of nucleon – 3D structure: both longitudinal + transverse momentum dependent structure (confined motion in a nucleon)

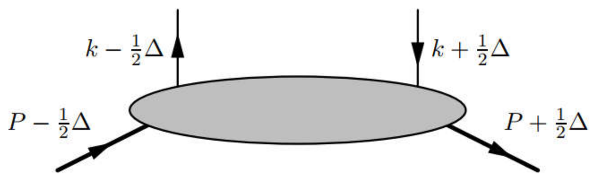
in a more generalized picture

semi-inclusive:
SIDIS
Drell-Yan
e^+e^- annihilation



exclusive:
DVCS
HEMP

Wigner distribution: F.T. of GTMD

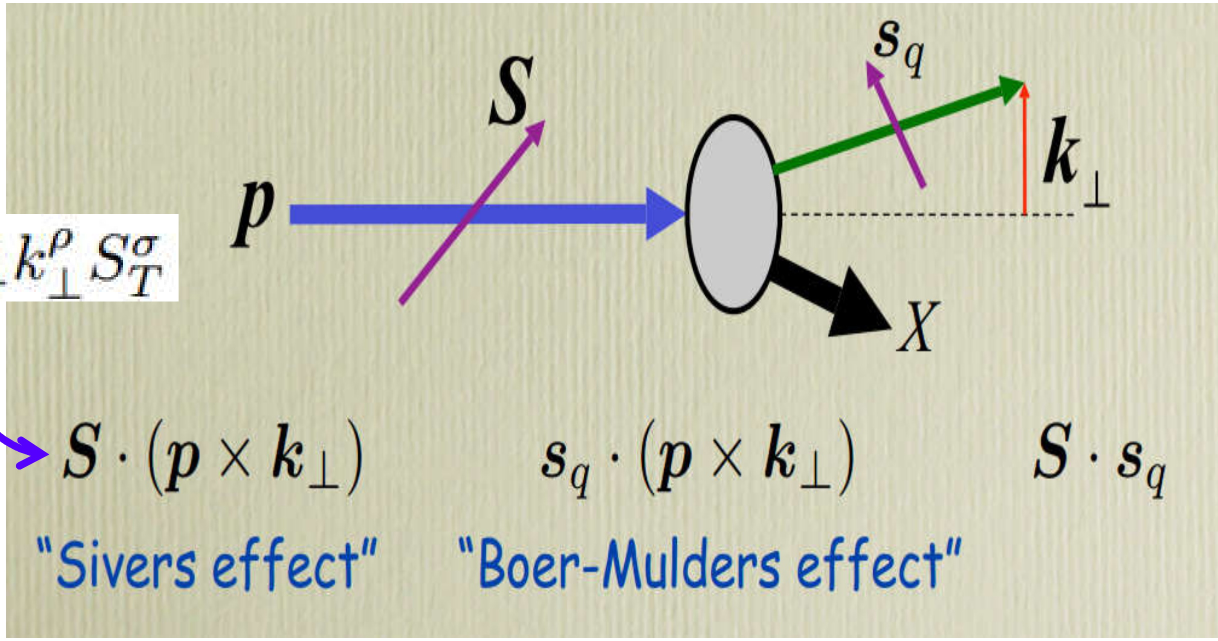


$$\frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p', \lambda' | \bar{\psi}\left(-\frac{1}{2}z\right) \Gamma\, \mathcal{W}\left(-\frac{1}{2}z, \frac{1}{2}z \,|\, n\right) \psi\left(\frac{1}{2}z\right) |p, \lambda\rangle$$

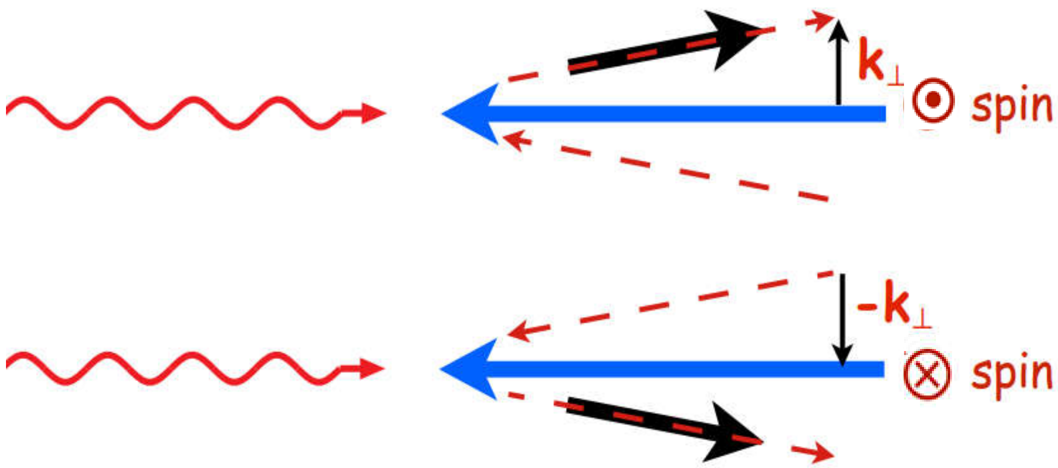
Spin-orbit correlation —— T-odd TMDs

- Sivers function: correlation between the transverse spin of the nucleon and parton transverse momentum
- Boer-Mulders function: correlation between the transverse spin of the quark and quark transverse momentum

$\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu}n_{+}^{\nu}k_{\perp}^{\rho}S_{T}^{\sigma}$



T-odd TMDs—— Physical picture



$f_{1T}^{\perp q}$ or $h_1^{\perp q}$ >0 : the quark prefers to move upwards if the proton is moving to the left and the proton/quark spin is pointing towards the observer.

$$f_{q/p^\uparrow}(x,k_T) - f_{q/p^\uparrow}(x,-k_T) = \Delta^N f_{q/p^\uparrow}(x,k_T^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_T)\cdot\boldsymbol{S}}{|\boldsymbol{k}_T|}$$

$$\Delta^N f_{q/p^\uparrow}(x,k_T^2) = -\frac{2|\boldsymbol{k}_T|}{M}\,f_{1T}^{\perp q}(x,k_T^2)$$

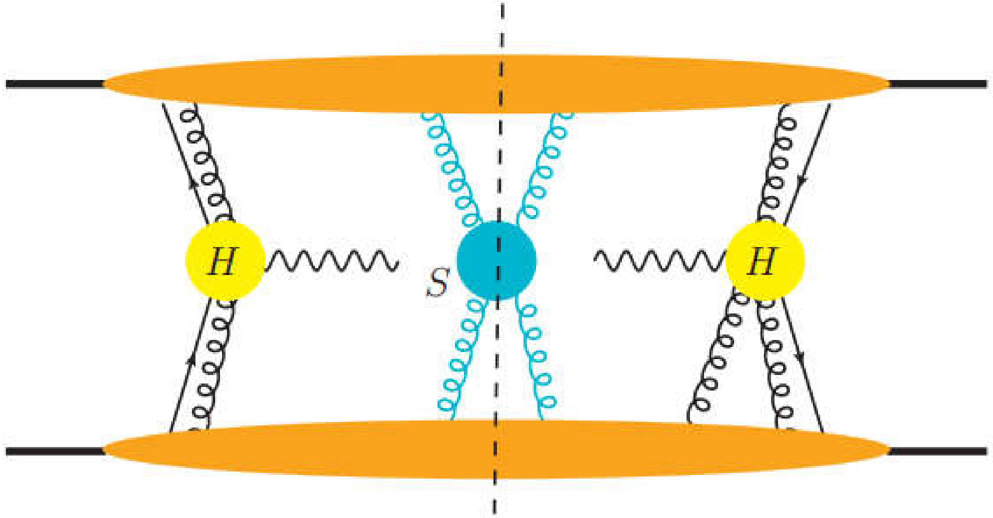
Give rise to SSA or Azimuthal asymmetry

$$f_{q^\uparrow/p}(x,k_T) - f_{q^\uparrow/p}(x,-k_T) = \Delta^N f_{q^\uparrow/p}(x,k_T^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_T)\cdot\boldsymbol{S}_q}{|\boldsymbol{k}_T|}$$

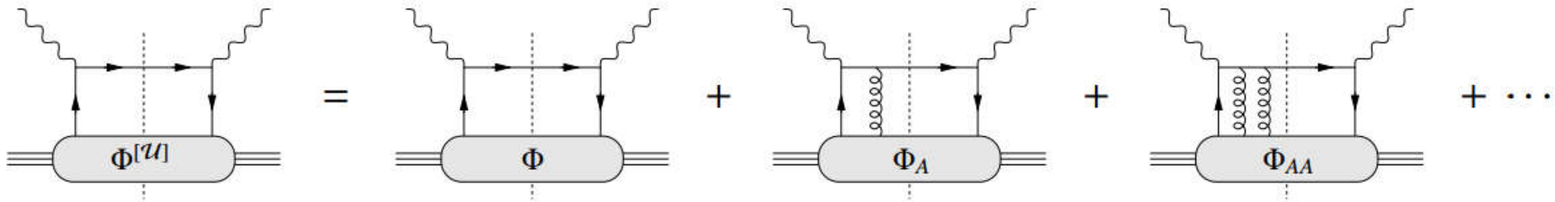
$$\Delta^N f_{q^\uparrow/p}(x,k_T^2) = -\frac{|\boldsymbol{k}_T|}{M}\,h_1^{\perp q}(x,k_T^2)$$

analogous pictures for the Collins and Sivers-type FF

Role of gauge field of QCD---gluon

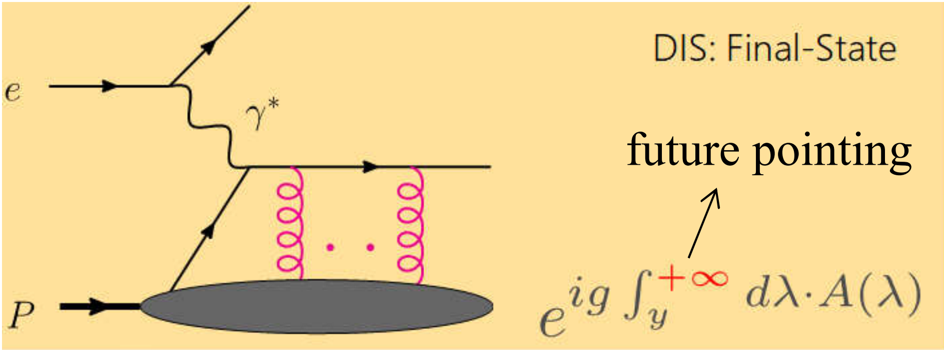


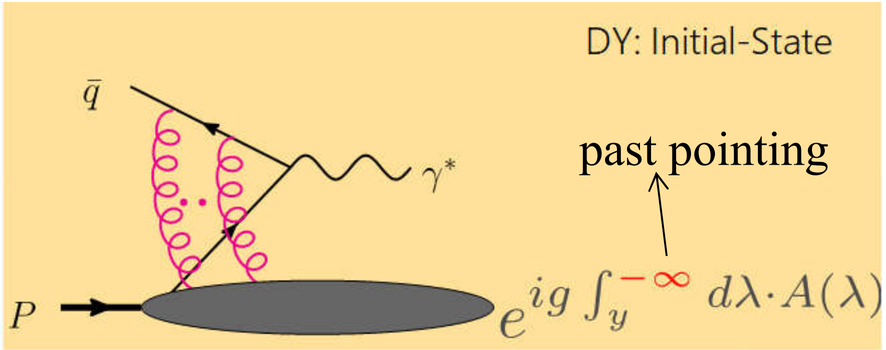
➢ gluon rescattering between the hard part (H) and the target spectator: form the Wilson lines (or gauge-link) to ensure the gauge invariance of TMDs



$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\mathcal{U}^{n-}_{(0,+\infty)}\mathcal{U}^{n-}_{(+\infty,\xi)}\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

Non-Universality of TMD distributions





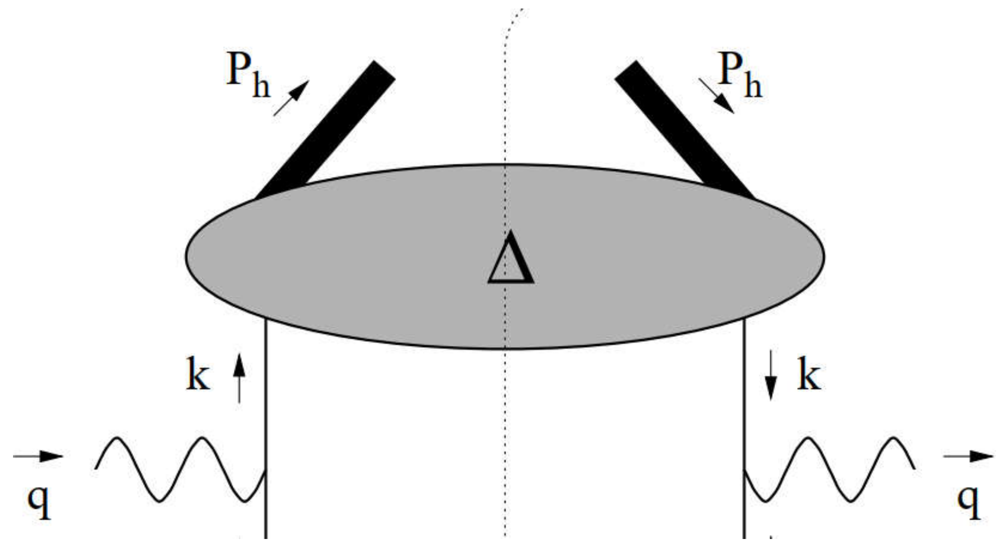
$$\frac{1}{[l^+ - i\epsilon]} \to 2\pi i\, \delta[l^+]$$ SIDIS

$$\frac{1}{[l^+ + i\epsilon]} \to -2\pi i\, \delta[l^+]$$ Drell-Yan

Sivers function $$f_{1T}^{\perp \mathrm{DIS}}(x, k_\perp) = -f_{1T}^{\perp \mathrm{DY}}(x, k_\perp)$$

- open issues regarding TMDs
  - sign change of T-odd TMDs between SIDIS and DY
  - Universality: TMD *might not be universal* when probed through different hard scattering processes

TMD fragmentation functions



$$\Delta_{ij}(z,k_T) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\xi)}\,\psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=0}$$

spin-0:

$$\Delta(z,k_T) = \frac{1}{2}\left\{D_1\,\not{n}_- + iH_1^\perp\frac{[\not{k}_T,\not{n}_-]}{2M_h}\right\}$$
$$+\frac{M_h}{2P_h^-}\left\{E + D^\perp\frac{\not{k}_T}{M_h} + iH\frac{[\not{n}_-,\not{n}_+]}{2} + G^\perp\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\right\},$$

TMD fragmentation--probability interpretation

spin-1/2

polarization: quark	hadron	pictorially	TMD FFs (8)
U	U		$D_1(z,k_{F\perp})$
	T		$D_{1T}^{\perp}(z,k_{F\perp})$
L	L		$G_{1L}(z,k_{F\perp})$
	T		$G_{1T}^{\perp}(x,k_{\perp})$
T	U		$H_1^{\perp}(z,k_{F\perp})$
	$T(//)$		$H_{1T}(z,k_{F\perp})$
	$T(\perp)$		$H_{1T}^{\perp}(z,k_{F\perp})$
	L		$H_{1L}^{\perp}(z,k_{F\perp})$

unpolarized FF

Sivers-type FF
(Λ transverse polarization)

Collins Function

Model calculations and extractions for TMDs

- model calculations for the nucleon:
  - spectator model: Gamberg, Goldstein 02; Bacchetta, Schaefer, Yang 03; Gamberg, Goldstein, Schlegel 07; Bacchetta, Conti, Radici 08.
  - constituent (light-cone) quark model: Courtoy, Scopetta, Vento 09; Pasquini and Yuan 10.
  - bag model: Yuan 03; Courtoy, Scopetta, Vento 09
  - baryon-meson fluctuation model for the sea quarks: ZL, Ma, Schmidt 07.

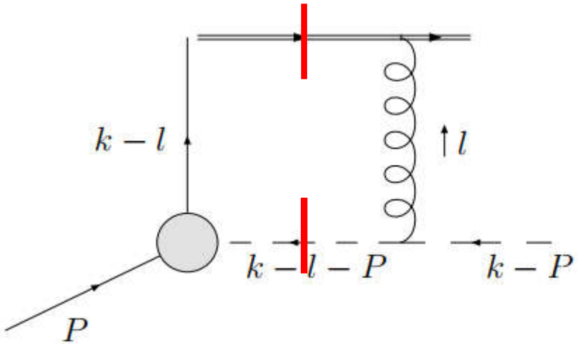
  chiral quark soliton model: Wakamatus 09, Lorce, Pasquini 11

- model calculations for the pion: ZL, Ma 04; Meissner, Metz, Schlegel, Goeke 08; Gamberg, Schlegel 10. Bastami et.al. 20

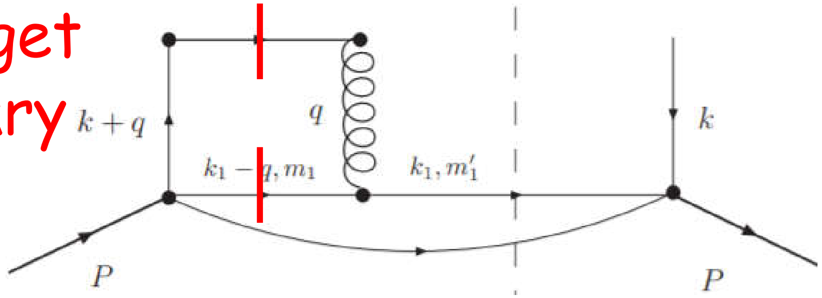
Models calculations provide important insights on the size and the sign of TMDs before the era of global fit

modeling T-odd TMD: Sivers function

T-odd distribution require the presence of the gauge-link

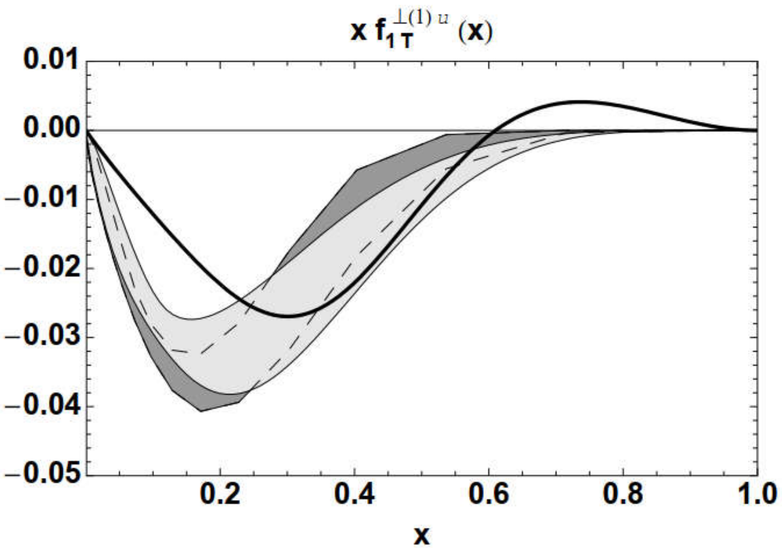


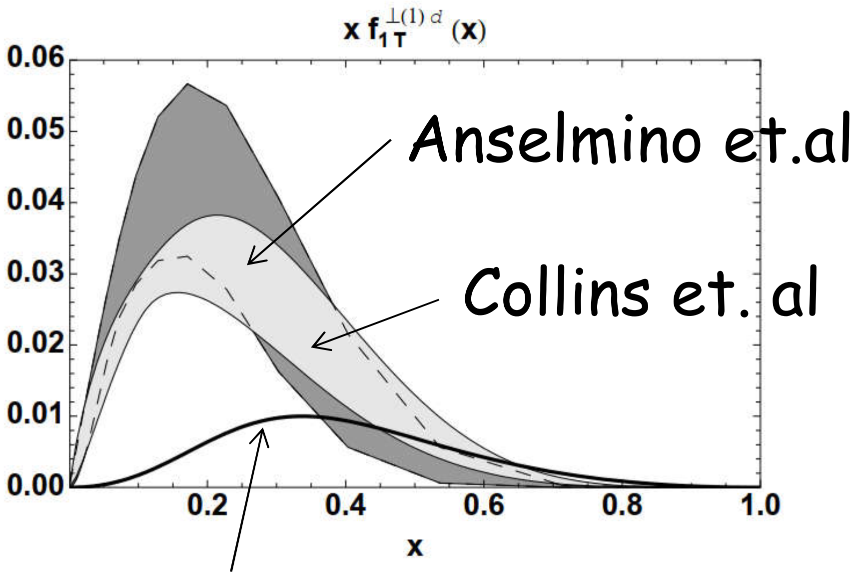
cut to get imaginary part



diquark model

constituent/lignt-cone quark model





Bacchetta et. al

outline

- Overview on status of hadron structure in terms of transverse momentum distributions(TMDs)
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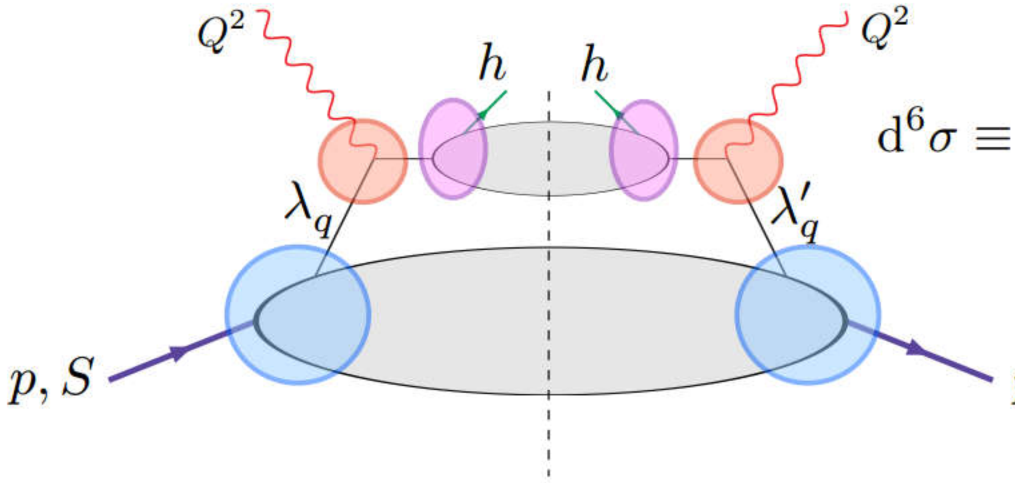
Physical process: SIDIS



$$\mathrm{d}^6\sigma \equiv \frac{\mathrm{d}^6\sigma^{\ell p^\uparrow \to \ell h X}}{\mathrm{d}x_B\, \mathrm{d}Q^2\, \mathrm{d}z_h\, \mathrm{d}^2\boldsymbol{P}_T\, \mathrm{d}\phi_S}$$

$$\boldsymbol{P}_T = \boldsymbol{p}_\perp + z\boldsymbol{k}_\perp$$

TMD factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

Two scales: $P_T \ll Q^2$

TMD-PDFs hard scattering TMD-FFs

$$\mathrm{d}\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes \mathrm{d}\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

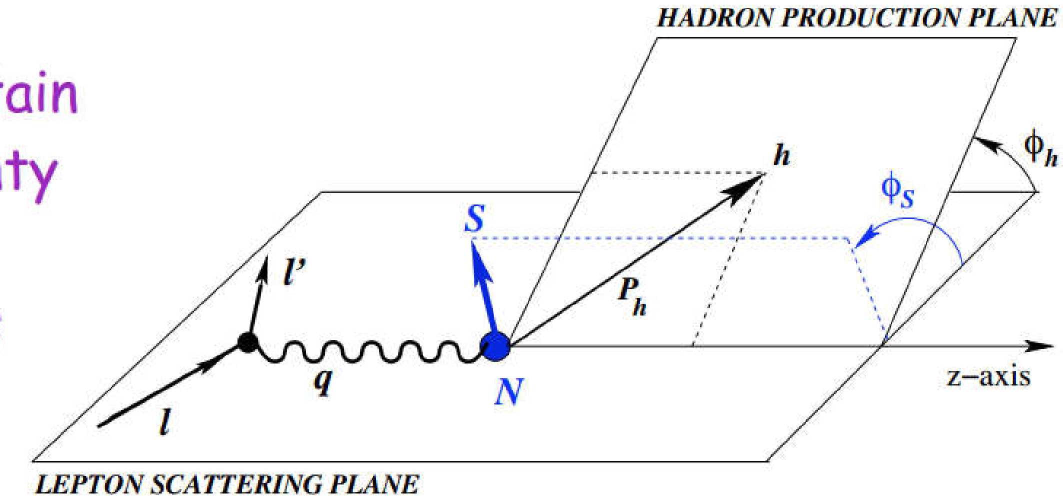
(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz...)

HERMES, COMPASS, JLab12, EIC, EicC...

Observables in SIDIS:

$$
\begin{aligned}
\frac{d\sigma}{d\phi} = {} & F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\, F_{UU}^{\cos\phi} + \lambda\,\frac{1}{Q}\sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
& + S_T \Big\{ \underbrace{\sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)}}_{\text{Sivers}} + \underbrace{\sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)}}_{\text{Collins}} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \frac{1}{Q}\left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
& + \lambda \left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Big\}
\end{aligned}
$$

the $F^{(\cdots)}_{S_B\,S_T}$ contain the TMDs; plenty of Spin Asymmetries



Mulders, Tangerman, Diehl, Bacchetta, Kotzinian ...

single transverse-spin asymmetries in SIDIS:

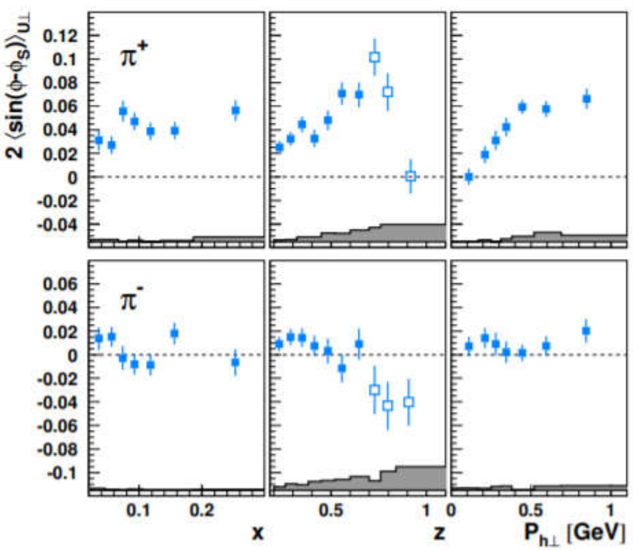
$$A_{UT} = \frac{d\sigma(S_T) - d\sigma(-S_T)}{d\sigma(S_T) + d\sigma(-S_T)}$$

Sivers asymmetry

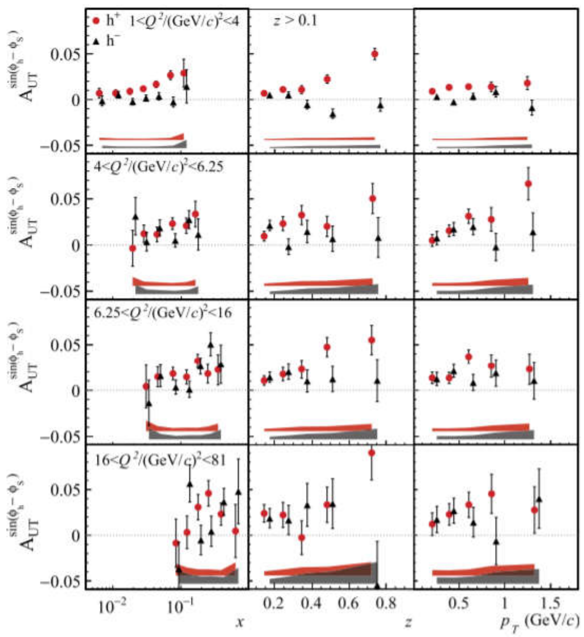
$$A_{UT}^{\sin(\phi_h - \phi_S)} = \frac{F_{UT}^{\sin(\phi_h - \phi_S)}}{F_{UU}}, \qquad F_{UT,T}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

Collins asymmetry

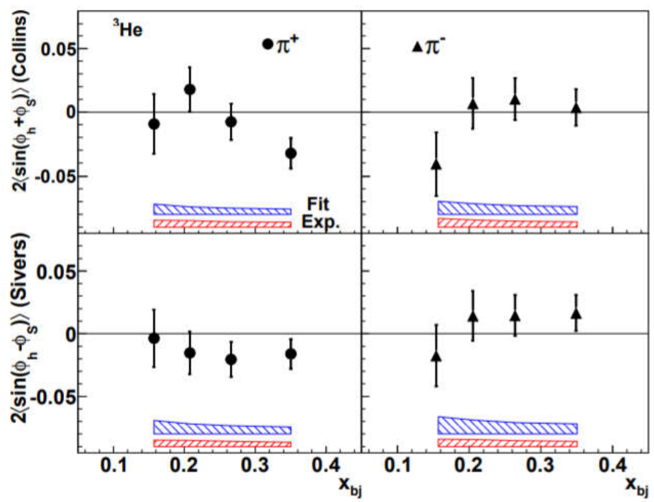
$$A_{UT}^{\sin(\phi_h + \phi_S)} = \frac{F_{UT}^{\sin(\phi_h + \phi_S)}}{F_{UU}}, \qquad F_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$



HERMES 20



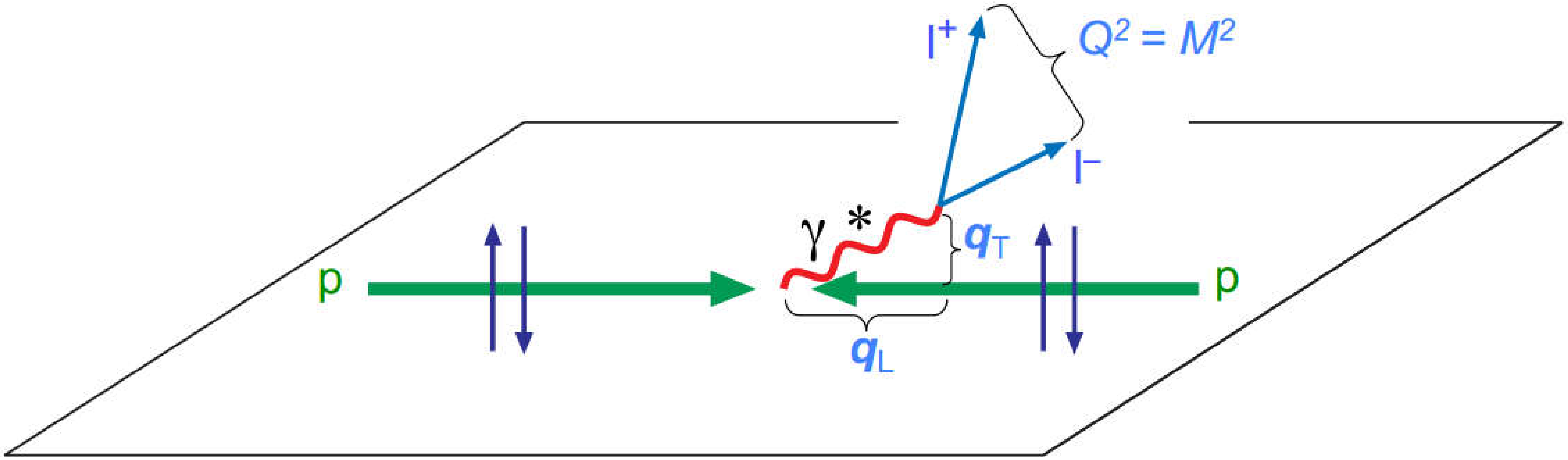
COMPASS 17



JLab 11

Physical process: Drell-Yan

COMPASS, RHIC, Fermilab, NICA, AFTER...



factorization holds, two scales, M^2, and $q_T \ll M$

$$\mathrm{d}\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, \mathrm{d}\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs, no fragmentation process

Observables in Drell-Yan

Case of one polarized nucleon only

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha^2}{\Phi\,q^2}\Big\{(1+\cos^2\theta)\,F_U^1 + (1-\cos^2\theta)\,F_U^2 + \sin 2\theta\cos\phi\,F_U^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_U^{\cos 2\phi}$$

B-M ⊗ B-M

$$+\; S_L\left(\sin 2\theta\sin\phi\,F_L^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_L^{\sin 2\phi}\right)$$

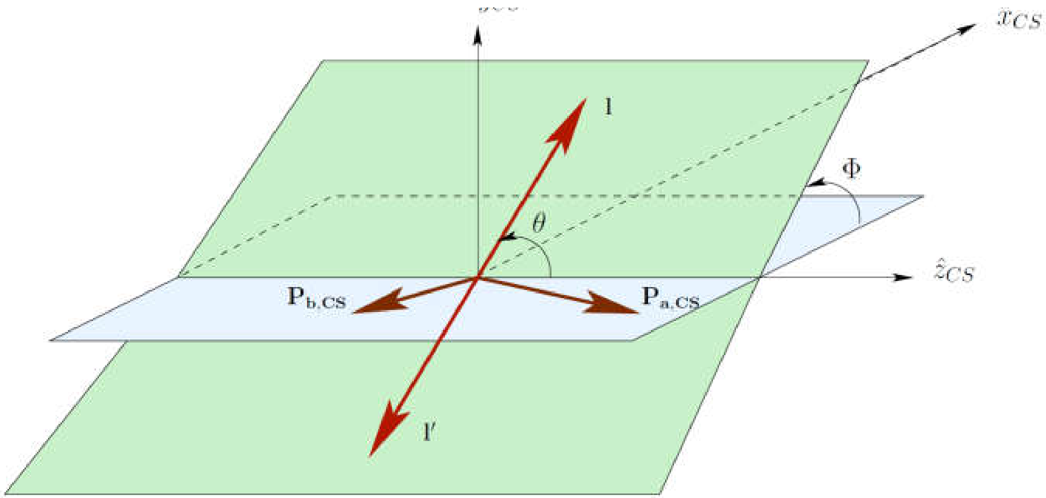
$$+\; S_T\Big[\Big(F_T^{\sin\phi_S} + cos^2\theta\,\tilde{F}_T^{\sin\phi_S}\Big)\sin\phi_S + \sin 2\theta\Big(\sin(\phi+\phi_S)\,F_T^{\sin(\phi+\phi_S)}$$

$$+\; \sin(\phi-\phi_S)\,F_T^{\sin(\phi-\phi_S)}\Big)$$

Sivers

$$+\; \sin^2\theta\Big(\sin(2\phi+\phi_S)\,F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\,F_T^{\sin(2\phi-\phi_S)}\Big)\Big]\Big\}$$

$$A\,(P_1) + B\,(P_2) \to l^+\,(\ell) + l^-\,(\ell') + X\,,$$



Collins-Soper frame

Spin/azimuthal asymmetries

- General form of the cross section (Target: unpolarized/transversely polarized)

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha^2}{Fq^2}\hat{\sigma_U}\Big\{\Big(1+\cos^2(\theta)+\sin^2(\theta)A_{UU}^{\cos(2\phi)}\cos(2\phi)\Big)$$
$$+S_T\Big[(1+\cos^2(\theta))A_{UT}^{\sin(\phi_S)}\sin(\phi_S)$$
$$+\sin^2(\theta)\Big(A_{UT}^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S)+A_{UT}^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Big\}$$

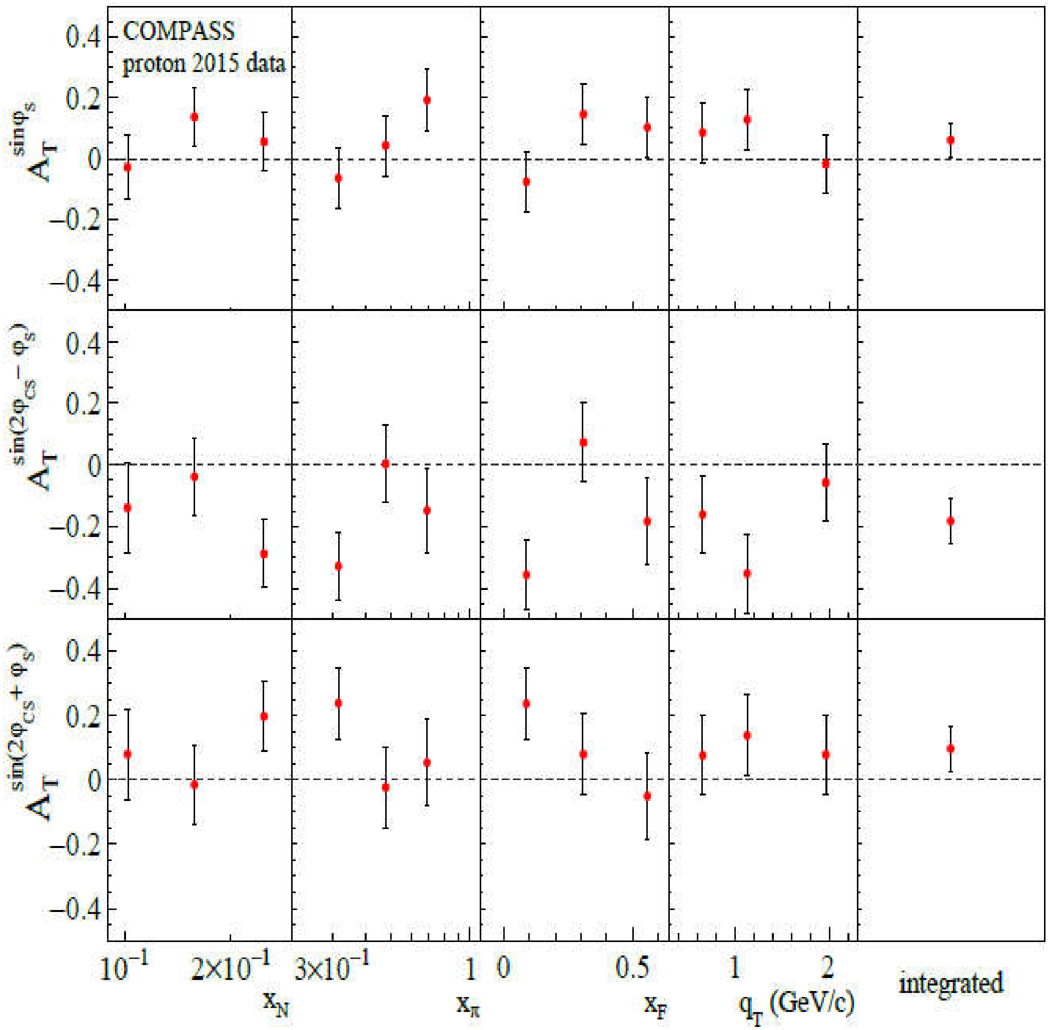
Leading-twist

- Asymmetries

					Beam	Target
$A_{UU}^{\cos(2\phi)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1,p}^{\perp q}$		Boer-Mulders	Boer-Mulders
$A_{UT}^{\sin(\phi_S)}$	$\propto f_{1,\pi}^{q}$	$\otimes$	$f_{1T,p}^{\perp q}$		$f_{1,\pi}^{q}$	Sivers
$A_{UT}^{\sin(2\phi-\phi_S)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1,p}^{q}$		Boer-Mulders	Transversity
$A_{UT}^{\sin(2\phi+\phi_S)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1T,p}^{\perp q}$		Boer-Mulders	Pretzelosity

SSAs measured at COMPASS Drell-Yan

- Experimental measurements (transversely polarized target)



Phys. Rev. Lett. 119, 112002 (2017)

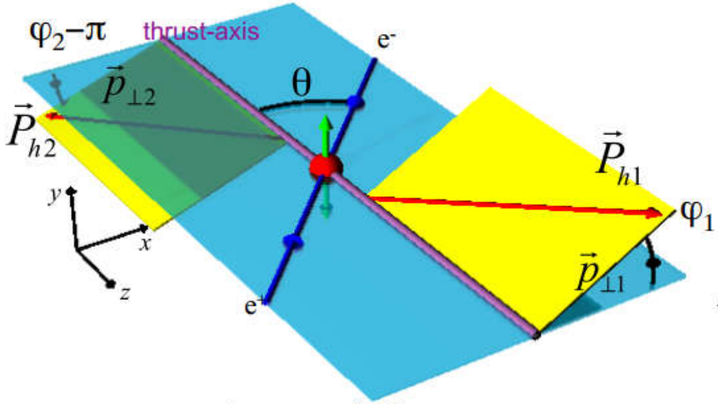
$$\propto f_{1,\pi}^{q} \quad \otimes \quad f_{1T,p}^{\perp q}$$

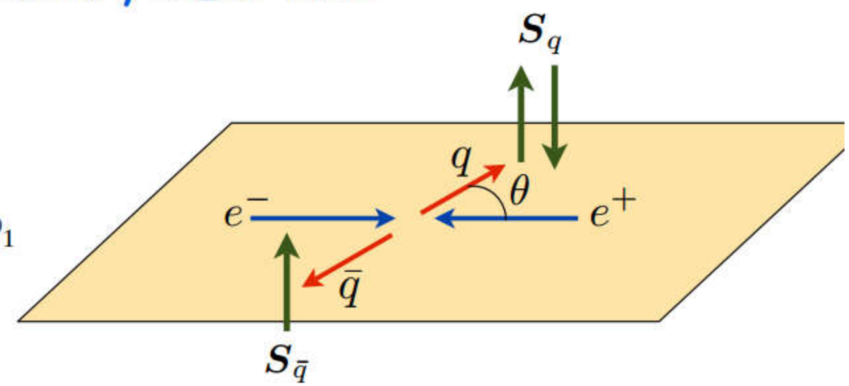
$$\propto h_{1,\pi}^{\perp q} \quad \otimes \quad h_{1,p}^{q}$$

$$\propto h_{1,\pi}^{\perp q} \quad \otimes \quad h_{1T,p}^{\perp q}$$

Physical process: e⁺e⁻ annihilation

Belle, BaBar, BES-III





$$\frac{\mathrm{d}\sigma^{e^+e^-\to q^\uparrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2 \cos^2\theta \qquad \frac{\mathrm{d}\sigma^{e^+e^-\to q^\downarrow \bar{q}^\uparrow}}{\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2$$

$$A_{12}(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{4}\frac{\sin^2\theta}{1+\cos^2\theta}\cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

TMD FFs

	Quark Polarization		
	U	L	T
Pion	D_1		$H_1^\perp$ Collins

TMD factorization/evolution

- General form of the differential cross section (CSS formalism)

Collins, Soper and Sterman, NPB 250 (1985) 199

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{\boldsymbol{q}}_\perp \cdot \vec{\boldsymbol{b}}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q, q_\perp),$$

Studied by Collins et. al., PRD94,034014, 16'

- $\widetilde{W}_{UU}(Q;b)$ dominates in the region $q_\perp \ll Q$, all-order resummation
- $Y_{UU}(Q, q_\perp)$ provides corrections at $q_\perp \sim Q$

Unpolarized Drell-Yan process

- The structure function $\widetilde{W}_{UU}$ can be written as (in b space)

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \tilde{f}^{\rm sub}_{1\,\bar{q}/\pi}(x_\pi, b; \mu, \zeta_F) \tilde{f}^{\rm sub}_{1\,q/p}(x_p, b; \mu, \zeta_F),$$

- $\tilde{f}^{\rm sub}_{1\,q/H}$ is the subtracted distribution function in the b-space and universal.

- $H_{UU}(Q;\mu)$ is the factor associated with hard scattering and scheme-dependent.

- scale dependence of TMDs is governed by the TMD evolution

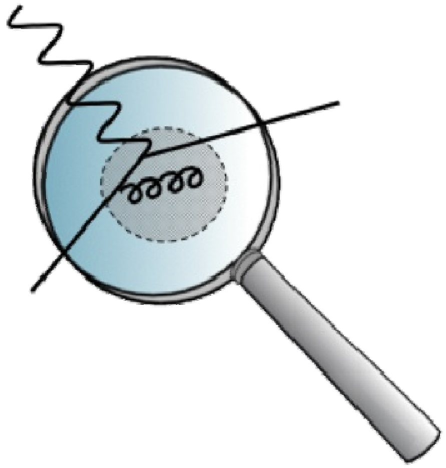
  The way to regularize light-cone singularity in TMD definition and subtract soft gluon contribution defines the scheme for the TMD factorization

TMD evolution

Collinear PDFs

$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative



TMDs

$$F(x, k_\perp; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i) \;\downarrow\; R^{\rm coll}(x, Q_i, Q_f) \;\downarrow\; F(x, Q_f)$$

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

$$F(x, k_\perp, Q_i) \;\downarrow\; R^{\rm TMD}(x, k_\perp, Q_i, Q_f) \;\downarrow\; F(x, k_\perp, Q_f)$$

TMD evolution

- TMD evolution for the ζ_F -dependence (energy evolution)

$$\frac{\partial \ln \tilde{f}^{\text{sub}}(x,b;\mu,\zeta_F)}{\partial \sqrt{\zeta_F}} = \tilde{K}(b;\mu)$$

Collins, Soper 81'
Idilbi, Ji, Ma, Yuan 04'

$$\tilde{K}(b;\mu) = -\frac{\alpha_s C_F}{\pi}[\ln(\mu^2 b^2) - \ln 4 + 2\gamma_E] + \mathcal{O}(\alpha_s^2).$$

- Evolution for the μ-dependence

$$\frac{d\,\tilde{K}}{d\,\ln\mu} = -\gamma_K(\alpha_s(\mu)),$$

$$\gamma_K = 2\frac{\alpha_s C_F}{\pi} + \mathcal{O}(\alpha_s^2),$$

$$\frac{d\,\ln \tilde{f}^{\text{sub}}(x,b;\mu,\zeta_F)}{d\,\ln\mu} = \gamma_F(\alpha_s(\mu);\frac{\zeta_F^2}{\mu^2}),$$

$$\gamma_F = \alpha_s\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\left(\frac{\zeta_F}{\mu^2}\right)\right) + \mathcal{O}(\alpha_s^2).$$

- General structure of the solution

$$f(x,b,Q) = \mathcal{F} \times e^{-S} \times f(x,b,\mu_b)$$

TMD evolution

formalism of TMD evolution

Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation

Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q)=\frac{1}{(2\pi)^2}\int d^2be^{ik_\perp\cdot b}F(x,b;Q)=\frac{1}{2\pi}\int_0^\infty db\,bJ_0(k_\perp b)F(x,b;Q)$$

$$F(x,b;Q)\approx \underbrace{C\otimes F(x,c/b^*)}_{\text{longitudinal/collinear part}}\times \underbrace{\exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}}_{\text{transverse part (perturbative)}}\times \underbrace{\exp\left(-S_{\text{non-pert}}(b,Q)\right)}_{\text{transverse part Non-perturbative: fitted from data}}$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Pavia group 17… mostly for the proton case

nonperturbative part

$S_{\text{non-pert}}(b, Q)$ parameterized as $g_{j/P}(x, b) + g_K(b) \ln \frac{Q}{Q_0}$

$g_{j/P}(x, b)$ parametrize the nonperturbative large b behavior that is intrinsic to the proton target (intrinsic transverse momentum)

$g_K(b)$ parametrize the nonperturbative large b behavior of the evolution kernel $\tilde{K}(b; \mu)$
should be universal: independent of hadron types, partons, poliarizations

several parameterizations of $S_{\text{non-pert}}(b,Q)$

- Brock-Landry-Nadolsky-Yuan (BLNY) parametrization 02

$$(g_1 + g_2 \ln(Q/2Q_0) + g_1 g_3 \ln(100\, x_1 x_2)))b^2$$

- Sun-Isaacson-Yuan-Yuan (SIYY) parametrization 14

$$g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda\right)$$

- Echevarria-Idilbi-Kang-Vitev (EIKV parametrization) 14

$$S_{\rm NP}^{\rm pdf}(b,Q) = b^2\left(g_1^{\rm pdf} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)$$

- Bacchetta,-Delcarro-Pisano-Radici-Signori/Pavia group (BDPRS parametrization) 17

$$S_{\rm NP}^{\rm pdf}(b,Q) = \frac{1}{4}g_1 b^2 - \ln\left(1 - \frac{\lambda g_1^2}{1+\lambda g_1}\frac{b^2}{4}\right) + \frac{g_2}{2} b^2 \ln\frac{Q}{Q_0}$$

$g_K(b)$ almost same ($g_2 b^2$) in different parametrizations

$g_{j/P}(x,b)$ quite different

TMD fit of unpolarized data

in most cases the TMD evolution is implemented

	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✘	✘	✔	✔	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	>100 (?)
Pavia 2013 arXiv:1309.3507	LO	W	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	LO	W	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✘	✘	✔	✔	457

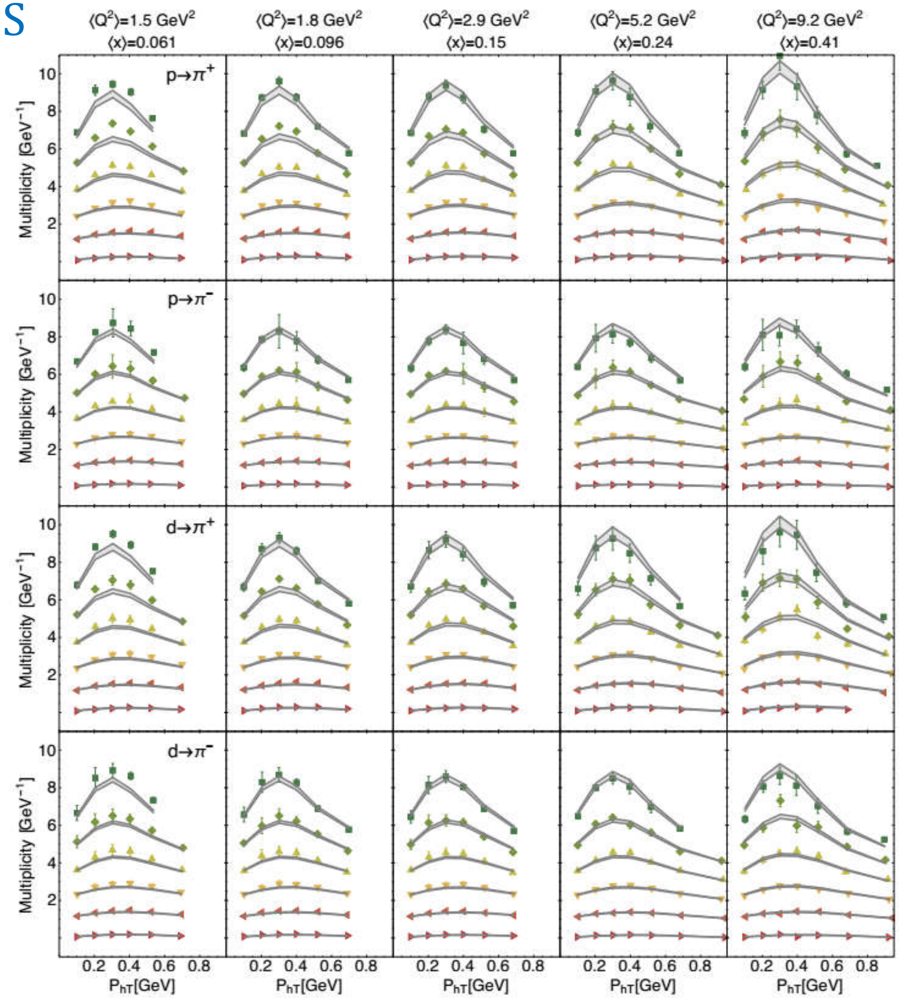
hard factors

perturbative part of Sudakov form factor

TMD effects in unpolarized SIDIS and pp Drell-Yan

P_T distribution in lepton-Nucleon SIDIS

HERMES
data



Pavia 2017
JHEP 1706, 081

bands are the 68% C.L. uncertainty from the 200 replicas of the fit function

- $\langle z\rangle$=0.24 (offset=5)
- $\langle z\rangle$=0.28 (offset=4)
- $\langle z\rangle$=0.34 (offset=3)
- $\langle z\rangle$=0.43 (offset=2)
- $\langle z\rangle$=0.54 (offset=1)
- $\langle z\rangle$=0.70 (offset=0)

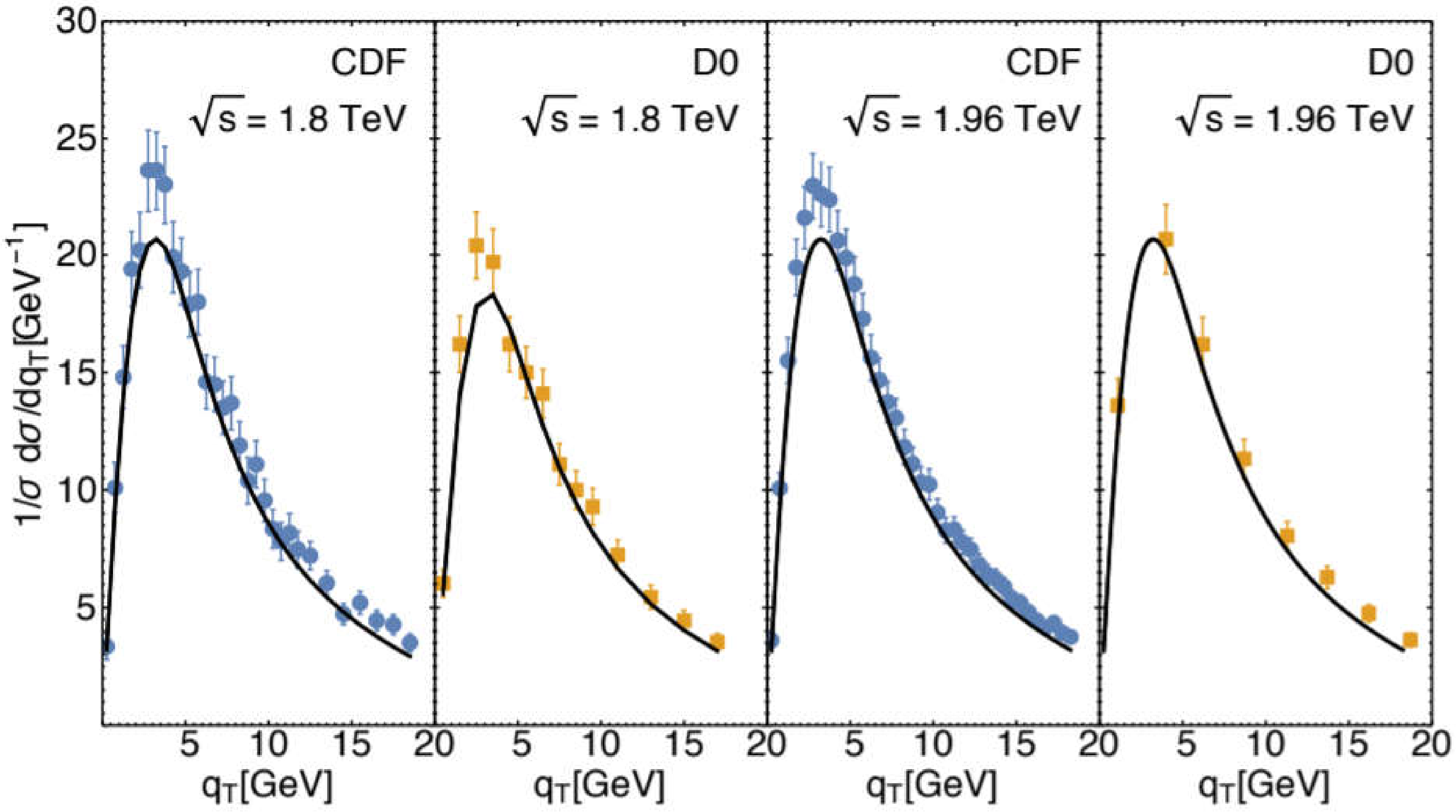
$$\tilde{f}_1^a(x, b^2; Q^2) = f_1^a(x; \mu^2) e^{-S(\mu^2, Q^2)} e^{\frac{1}{2} g_K(b) \ln(Q^2/Q_0^2)} \tilde{f}_{1\mathrm{NP}}^a(x, b^2),$$

$$g_K = -g_2 b^2/2.$$

Nonperturbative form factor

Q_T dependence in Z production

Bacchetta et al, JHEP 1706, 081



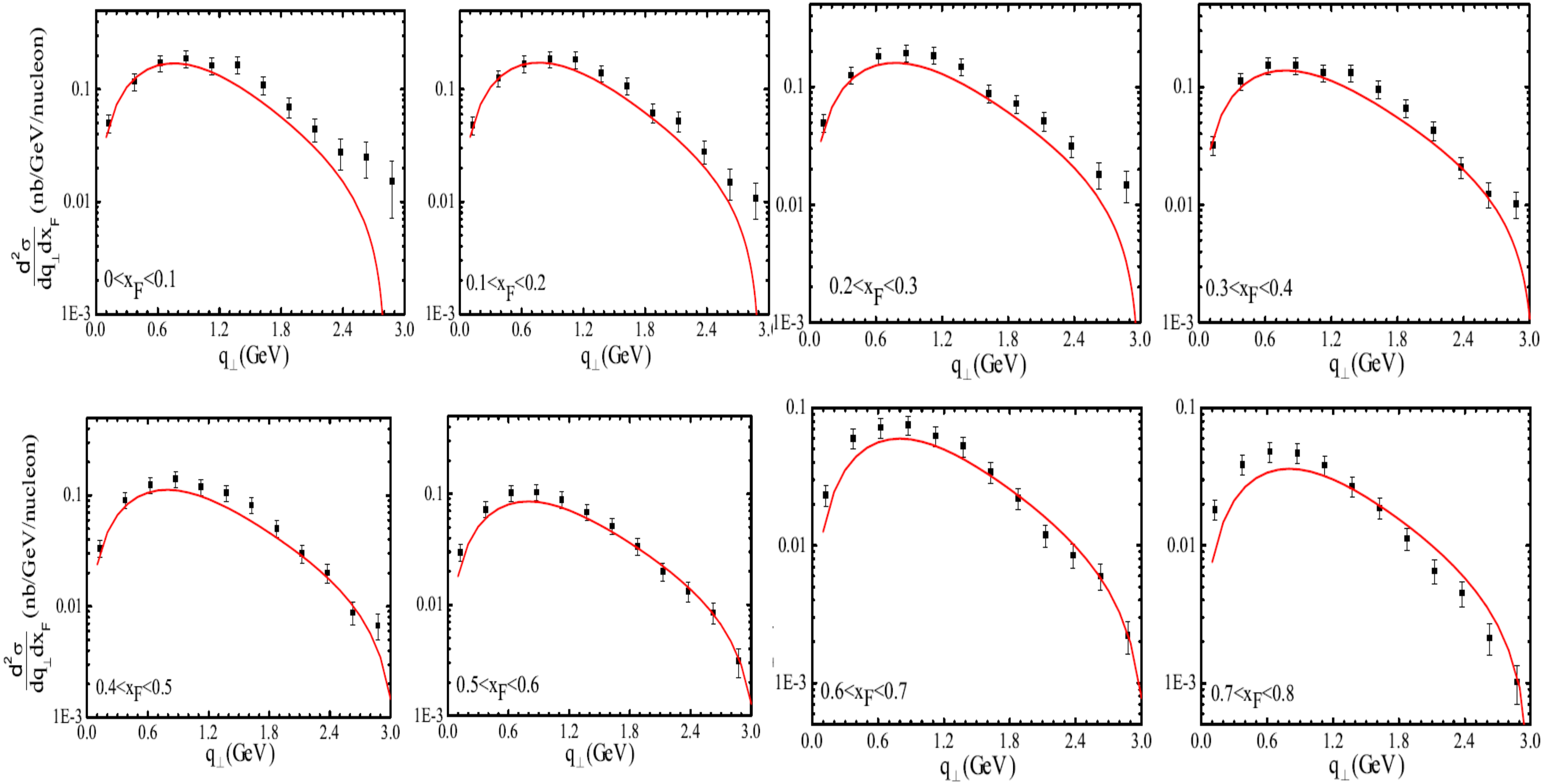
Due to the higher $Q = M_Z$, the range explored in q_T is much larger compared to all the other observables

Unpolarized process: pi-Nucleon Drell-Yan

- Fit $S_{\mathrm{NP}}^{f_1^{q/\pi}}(Q,b)$ to E615 q_T data, use SIYY parameterization Wang, ZL, Schmidt, 17

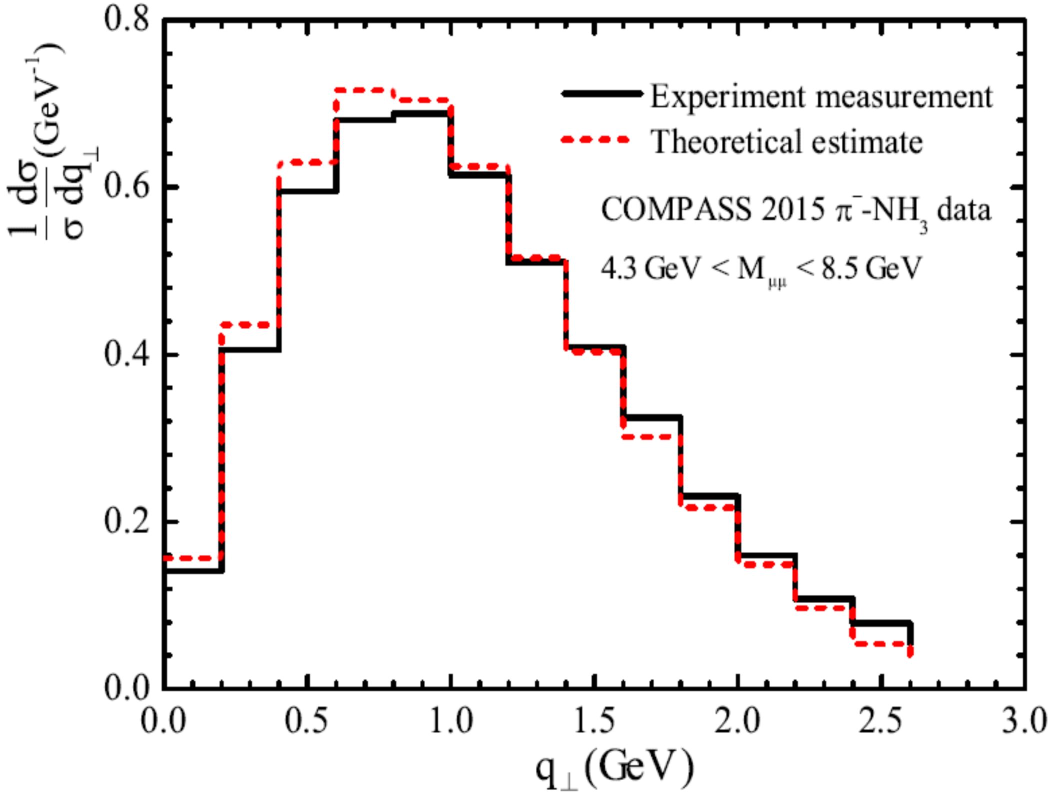
$$g_1^\pi = 0.082 \pm 0.022, \quad g_2^\pi = 0.394 \pm 0.103, \quad g_1^\pi < g_1^p, \quad g_2^\pi \approx g_2^p$$

$q_T < 3$ **GeV**



prediction of q_T distribution at COMPASS DY

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int_0^\infty \frac{dbb}{2\pi} J_0(q_\perp b) \times \widetilde{W}_{UU}(Q;b)$$

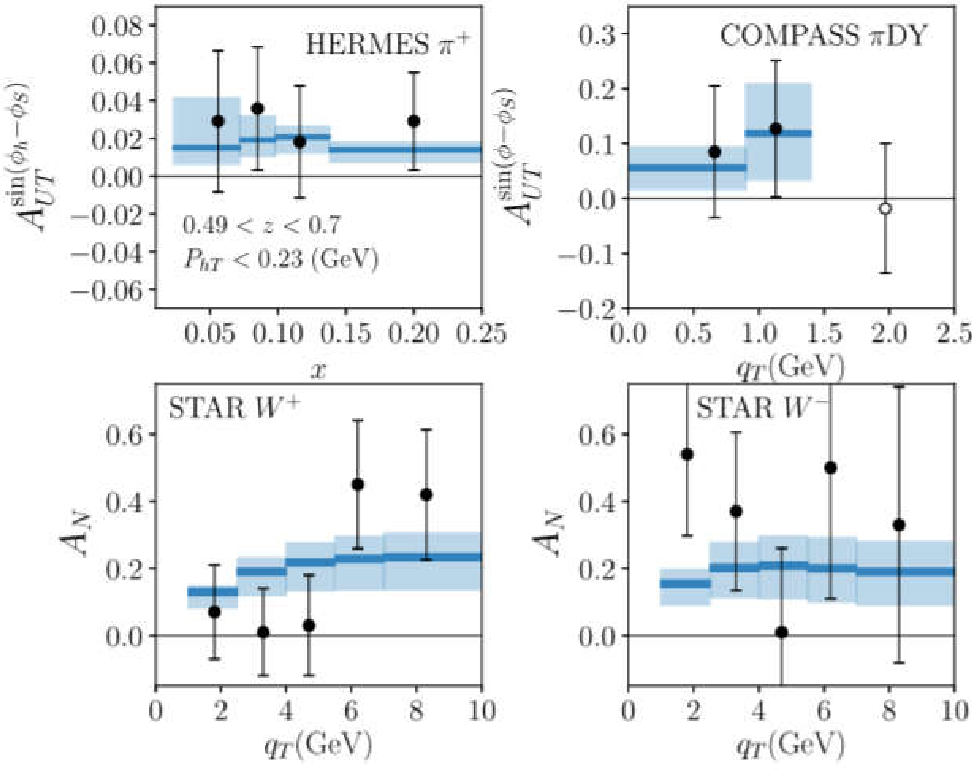


Wang, ZL, Schmidt, 17

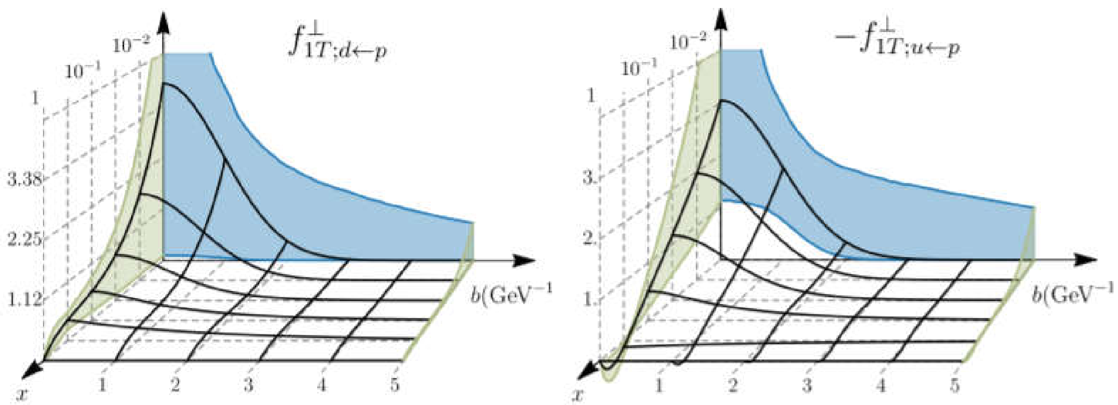
extraction of polarized TMDs

	Type of TMDs	SIDIS	$e^+ e^-$	Drell-Yan	TMD evolution
Torino 2013	transversity & Collins FF	✔	✔	✘	✘
KPSY 2015	transversity & Collins FF	✔	✔	✘	✔
Torino 2013	Sivers	✔	✘	✘	✘
BPV 2021	Sivers	✔	✘	✔	✔
ZLMS 2009	Boer-Mulders	✘	✘	✔	✘
BMP 2010	Boer-Mulders	✔	✘	✔	✘
Cagliari 2020	$D_{1T}^{\perp}(z,k_{F\perp})$	✘	✔	✘	✘
CKT 2020	$D_{1T}^{\perp}(z,k_{F\perp})$	✘	✔	✘	✘
CLPSW 2020	$D_{1T}^{\perp}(z,k_{F\perp})$	✘	✔	✘	✘

extraction of sivers function with TMD evolution



Dataset name	Ref.	Reaction	# Points	Av.Uncertainty
Compass08	[36]	$d^\uparrow + \gamma^* \to \pi^+$	1 / 9	1.2%
		$d^\uparrow + \gamma^* \to \pi^-$	1 / 9	1.1%
		$d^\uparrow + \gamma^* \to K^+$	1 / 9	3.4%
		$d^\uparrow + \gamma^* \to K^-$	1 / 9	5.1%
Compass16	[39]	$p^\uparrow + \gamma^* \to h^+$	5 / 40	1.6%
		$p^\uparrow + \gamma^* \to h^-$	5 / 40	2.0%
Hermes	[35]	$p^\uparrow + \gamma^* \to \pi^+$	11 / 64	2.6%
		$p^\uparrow + \gamma^* \to \pi^-$	11 / 64	3.1%
		$p^\uparrow + \gamma^* \to K^+$	12 / 64	6.1%
		$p^\uparrow + \gamma^* \to K^-$	12 / 64	10.8%
JLab	[41, 42]	${}^3He^\uparrow + \gamma^* \to \pi^+$	1 / 4	13.9%
		${}^3He^\uparrow + \gamma^* \to \pi^-$	1 / 4	8.0%
		${}^3He^\uparrow + \gamma^* \to K^+$	1 / 4	7.0%
		${}^3He^\uparrow + \gamma^* \to K^-$	0 / 4	—
SIDIS total			63	
CompassDY	[40]	$\pi^- + d^\uparrow \to \gamma^*$	2 / 3	12.2%
Star.W+	[43]	$p^\uparrow + p \to W^+$	5 / 5	16.1%
Star.W-		$p^\uparrow + p \to W^-$	5 / 5	32.2%
Star.Z		$p^\uparrow + p \to \gamma^*/Z$	1 / 1	33.%
DY total			13	
Total			76	



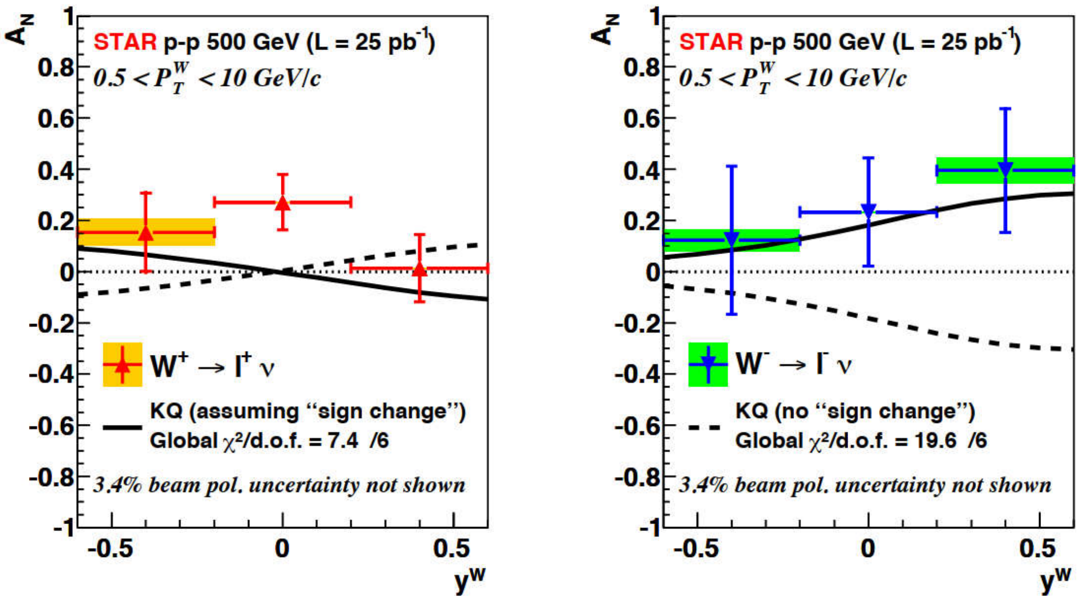
Bury,Prokudin,Vladimirov 21

$[f_{1T}^{q\perp}]_{\rm SIDIS} = -[f_{1T}^{q\perp}]_{\rm DY}$ used

N^{3}LL accuracy

test the sign change of the Sivers function

STAR Collaboration, PRL 116 (2016) 132301



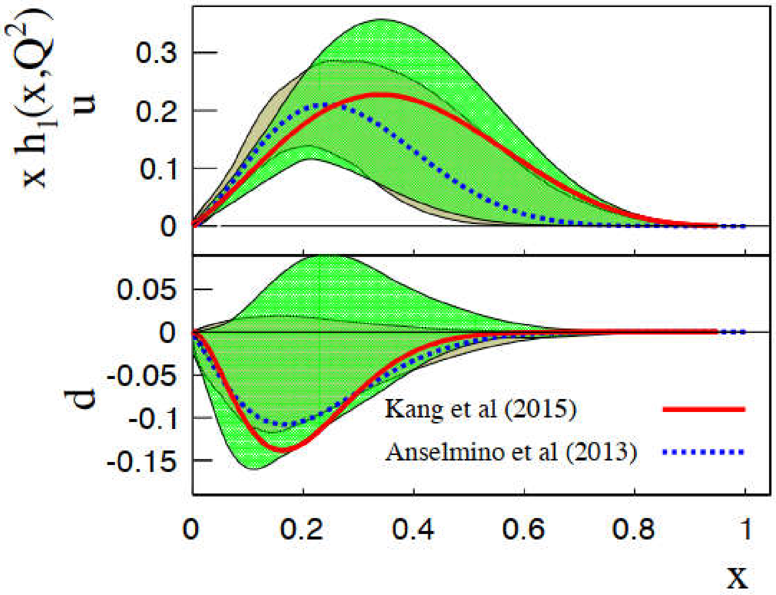
global analysis with and without sign change

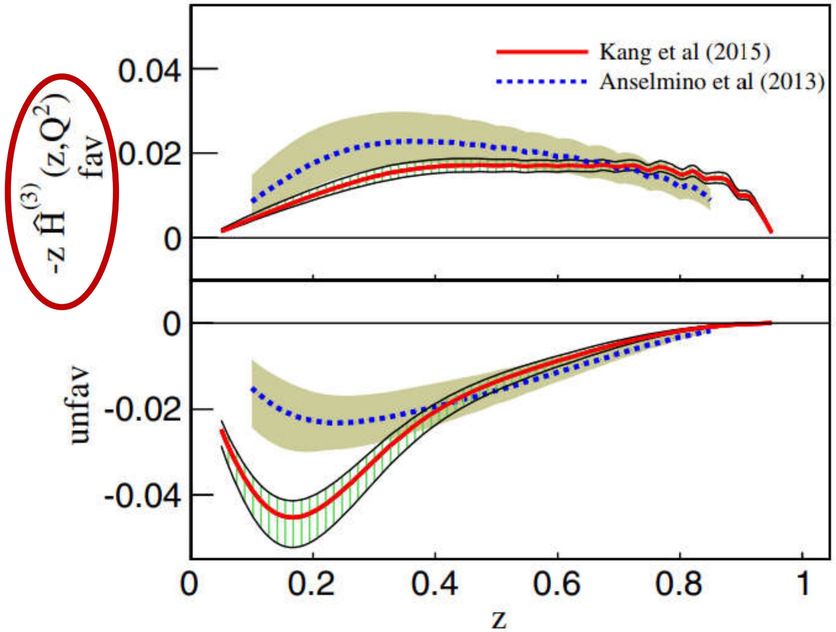
Bury,Prokudin,Vladimirov 21

	$f_{1T[DY]}^{\perp} = -f_{1T[SIDIS]}^{\perp}$	$f_{1T[DY]}^{\perp} = +f_{1T[SIDIS]}^{\perp}$
χ^2/N_{pt}	$0.88^{+0.16}_{+0.06}$	$1.00^{+0.22}_{+0.08}$
p-value (CF)	0.74	0.44
p-value 68%CI	[0.60, 0.34]	[0.28, 0.08]
p-value 68%CI (SIDIS)	[0.67, 0.42]	[0.53, 0.11]
p-value 68%CI (DY)	[0.56, 0.17]	[0.68, 0.02]

No conclusion on the sign-change yet

Global fit of tranversity & Collins FF





from SIDIS + e^+e^- data:
TMD evolution (—) and
PDF evolution (····)

$$\hat{H}^{(3)}_{h/j}(z_h) = \int d^2 p_\perp \frac{|p_\perp^2|}{M_h} H^\perp_{1h/j}(z_h, p_\perp).$$

$$\hat{H}^{(3)}_{h/j}(z_h) = -2zM_h H^{\perp(1)}_{1h/j}(z_h)|_{\text{Trento}}.$$

different extractions in good agreetment

summary

- Study on hadronic structure, particular the 3D structure in momentum and position space, are extremely active in the past few years

- Future measurement on unpolarized and polarized SIDIS and Drell-Yan at existing and planned facilities combined with phenomenological analysis can provide more precise information on the parton structure inside hadrons

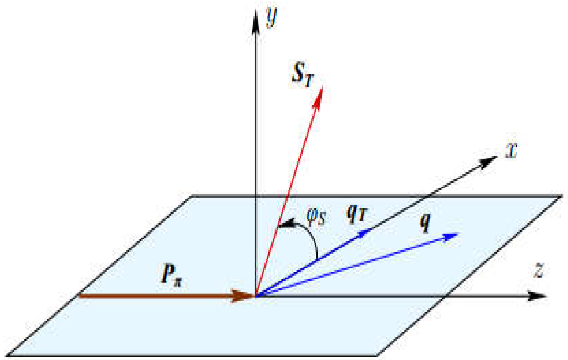
Thank you

谢谢大家！

Backup slides

Transversely polarized process

- The transverse single spin asymmetry can be defined as



$$A_{UT} = \frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \Big/ \frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp},$$

Spin-dependent

Spin-independent(Unpolarized)

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_\perp).$$

$$\frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \epsilon_\perp^{\alpha\beta} S_\perp^\alpha \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UT}^\beta(Q;b) + Y_{UT}^\beta(Q,q_\perp).$$

Sivers Asymmetry

Transversely polarized process

- Spin-dependent structure function

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = H_{UT}(Q;\mu)\sum_{q,\bar{q}} e_q^2 \tilde{f}_{1\bar{q}/\pi}(x_\pi, b;\mu,\zeta_F)\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x_p, b;\mu,\zeta_F).$$

$$H_{UT}(Q;\mu) = H_{UU}(Q;\mu)$$

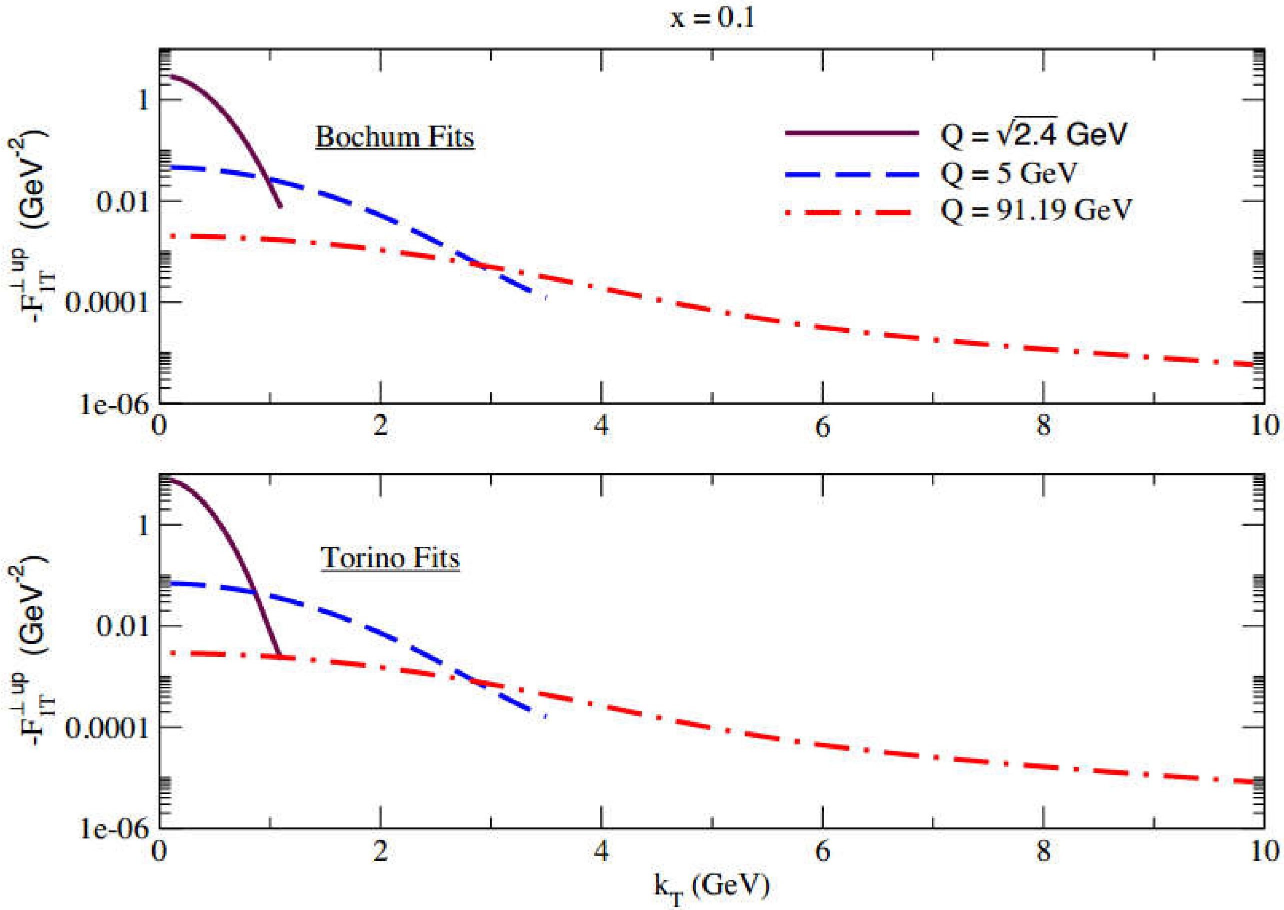
$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b;\mu,\zeta_F) = \int d^2\boldsymbol{k}_\perp e^{-i\vec{k}_\perp\cdot\vec{b}}\frac{k_\perp^\alpha}{M_p} f_{1T,q/p}^{\perp(\mathrm{DY})}(x,\boldsymbol{k}_\perp;\mu),$$

Sivers function in b-space

TMDs follows the same evolution equation in the perturbative region. The evolution for $\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}$ can be written in a similar form.

Echevarria, Idilbi, Kang, and Vitev, 14' $f(x,b,Q) = \mathcal{F}\times e^{-S}\times f(x,b,\mu_b)$

TMD evolution of up quark Sivers function

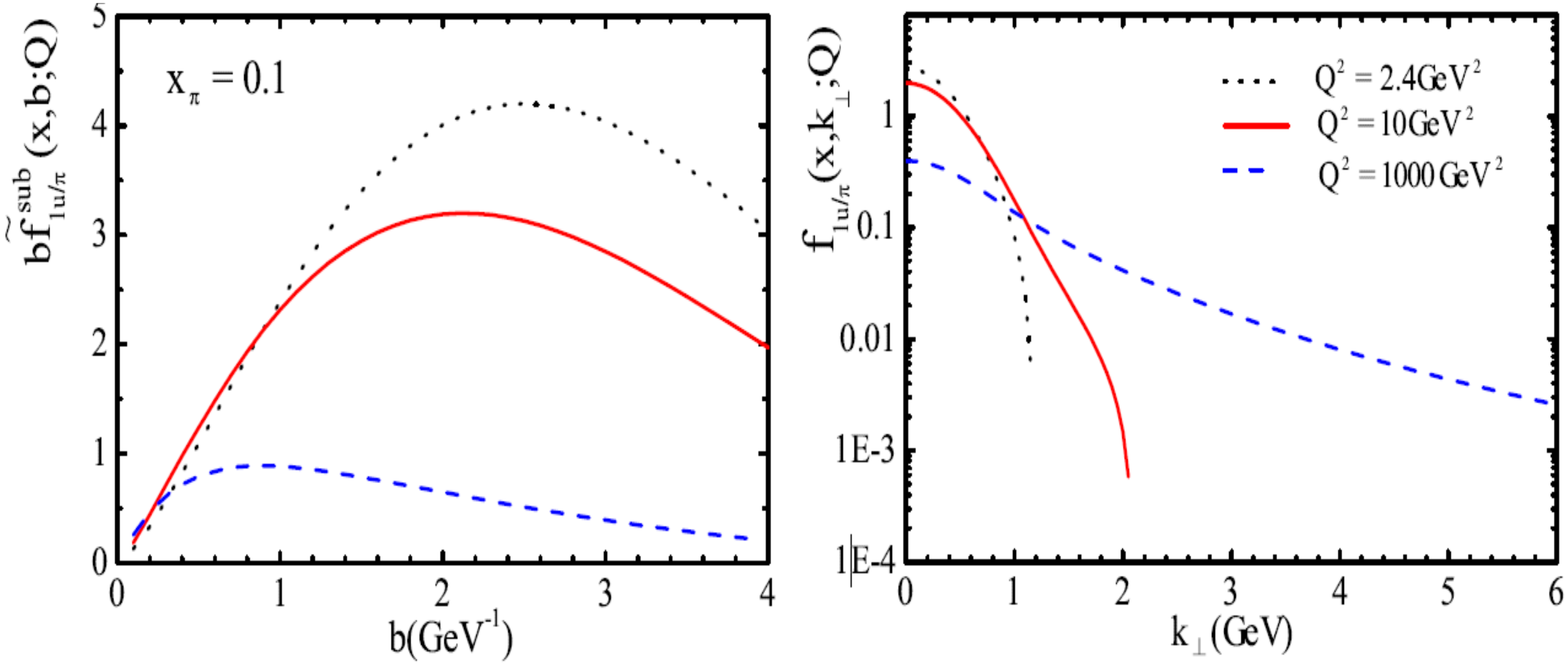


Aybat, Collins, Qiu, Rogers, Phys. Rev. D85 (2012) 034043

$$F(x,b;Q) \approx C \otimes F(x,c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

Evolution of pion TMD

$$\tilde{f}_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*) - S_{\rm NP}^{f_1^{q/\pi}}(Q,b)}\,\mathcal{F}(\alpha_s(Q)) \sum_i C_{q\leftarrow i}^{f_1} \otimes f_1^{i/\pi}(x,\mu_b)$$

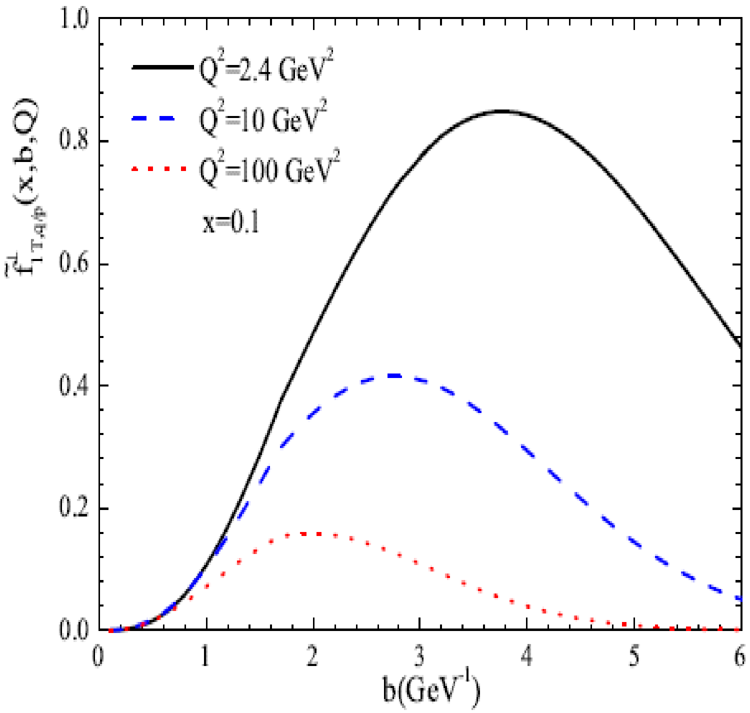


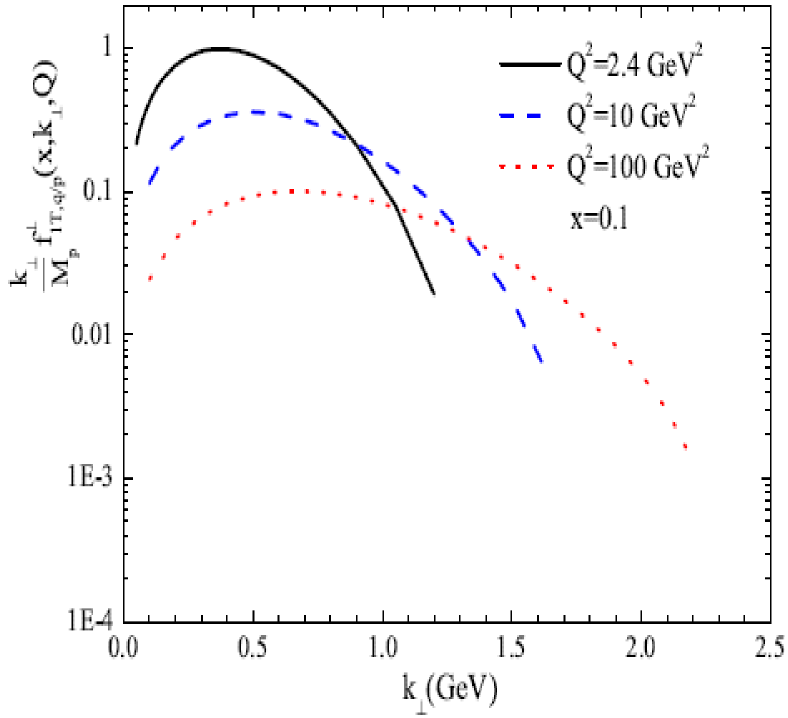
- the peak of the b-dependent distribution function moves towards the higher b region when decreasing energy scales
- At high energy scale the distribution has a tail falling off slowly at large k_T

Wang, Lu, Schmidt, JHEP 1708 (2017) 137

Transversely polarized process

- TMD evolution of the Sivers function





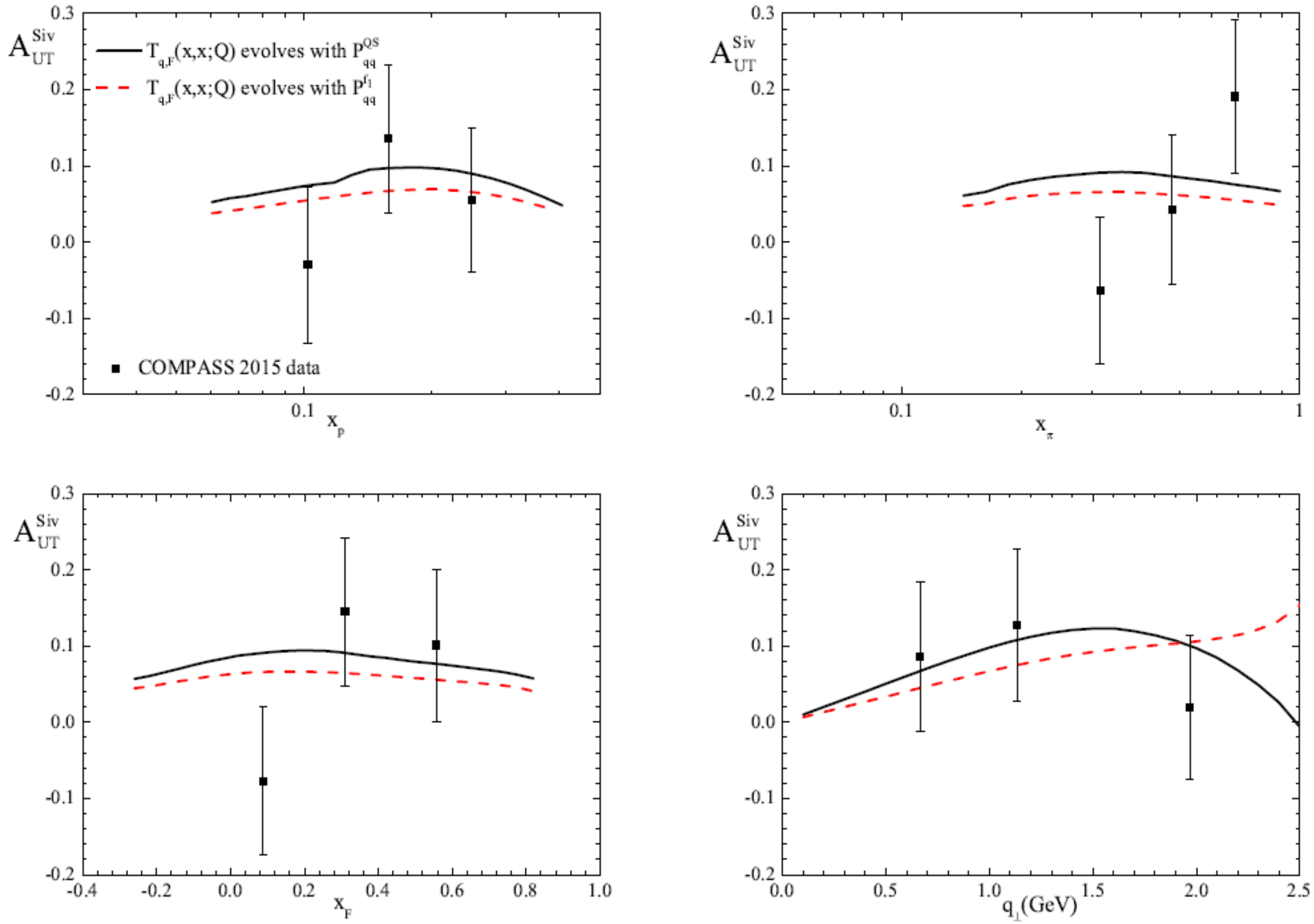
$$P_{qq}^{\mathrm{QS}} \approx P_{qq}^{f_1} - \frac{N_c}{2}\frac{1+z^2}{1-z} - N_c\delta(1-z),$$

Wang, ZL, PRD 97, 054005 (2018)

◆Sivers asymmetry in πN Drell-Yan

$$A_{UT} = \frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \Big/ \frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp},$$

Wang, ZL, PRD 97, 054005 (2018)



twist-3 distributions

- quark-quark correlator at twist-3 (Bacchetta et.al, 06):

$$\Phi(x, p_T) = \ldots + \frac{M}{2P^+}\left\{ e - i\, e_s\, \gamma_5 - e_T^\perp \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} \right.$$
$$+ f^\perp \frac{\not{p}_T}{M} - f_T'\, \epsilon_T^{\rho\sigma} \gamma_\rho S_{T\sigma} - f_s^\perp \frac{\epsilon_T^{\rho\sigma} \gamma_\rho p_{T\sigma}}{M}$$
$$+ g_T'\, \gamma_5\, \not{S}_T + g_s^\perp \gamma_5 \frac{\not{p}_T}{M} - g^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma} \gamma_\rho p_{T\sigma}}{M}$$
$$\left. + h_s \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} + h_T^\perp \frac{[\not{S}_T, \not{p}_T]\gamma_5}{2M} + i\, h \frac{[\not{n}_+, \not{n}_-]}{2} \right\}$$

T-even

$q \setminus N$	U	L	T
U	$f^\perp$		
L		$g_L^\perp$	g_T, $g_T^\perp$
T	e	h_L	h_T, $h_T^\perp$

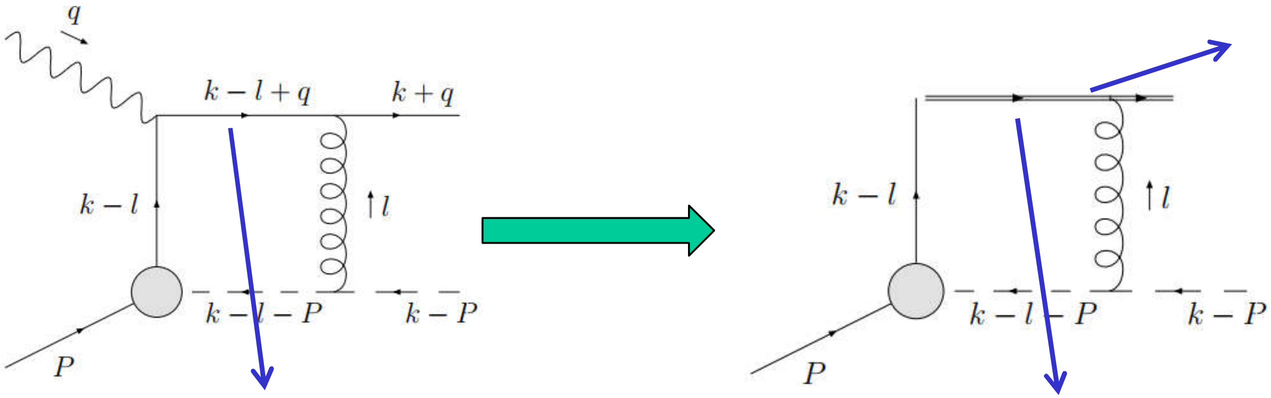
T-odd

$q \setminus N$	U	L	T
U		$f_L^\perp$	f_T, $f_T^\perp$
L	$g^\perp$		
T	h	e_L	e_T, $e_T^\perp$

no parton probability intepretation

Wilson lines -- details

LO:

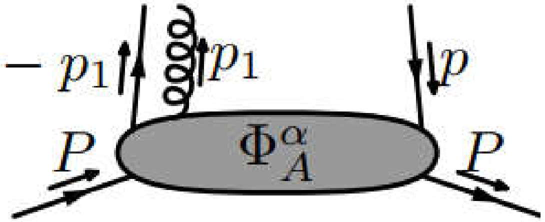


eikonal line

final state interaction (in SIDIS)

$$\frac{\mathrm{i}(\not k+\not q-\not l+m)}{(k+q-l)^2-m^2+\mathrm{i}\epsilon}\approx \mathrm{i}\,\frac{(k+q)^-\gamma^+}{-2l^+(k+q)^-+\mathrm{i}\epsilon}=\frac{\mathrm{i}}{2}\,\frac{\gamma^+}{-l^++\mathrm{i}\epsilon}$$

eikonal propagator



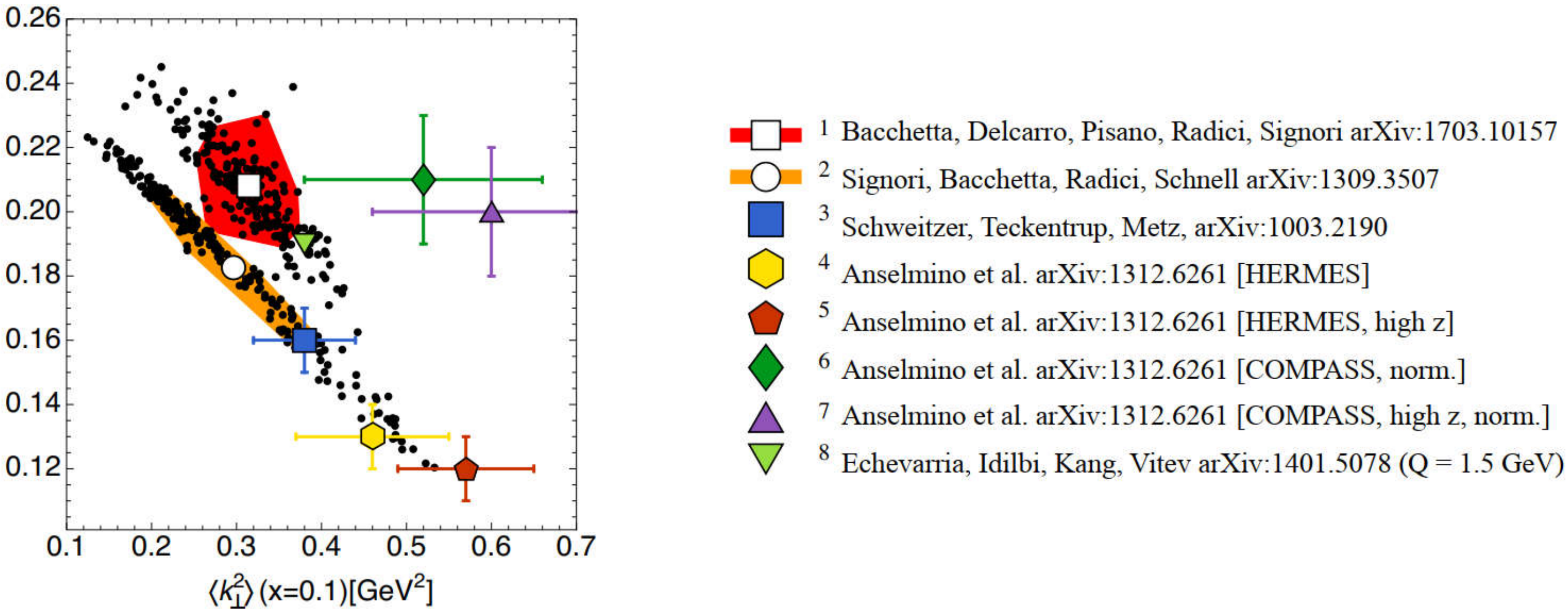
$$\Phi^{a\rho}_{A\,ij}(p,p_1)=\int\frac{d^4\xi}{(2\pi)^4}\,\frac{d^4\eta}{(2\pi)^4}\;e^{i(p-p_1)\cdot\xi}\,e^{ip_1\cdot\eta}\,\langle P,S\,|\,\overline{\psi}_j(0)\,A^{a\rho}(\eta)\,\psi_i(\xi)\,|P,S\,\rangle$$

$$\Phi^{[U^n_{[\infty;\xi]}]}_{(g)}(p)=\frac{1}{P{\cdot}n}\int d^4p_1\,\frac{g\,t^a}{x_1-i\epsilon}\,n_\lambda\Phi^{a\lambda}_A(p,p_1)\qquad \frac{1}{2\pi i}\int dx_1\frac{e^{x_1P^+(\eta^--\xi^-)}}{x_1-i\epsilon}=\theta(\eta^--\xi^-)$$

$$=\int\frac{d^4\xi}{(2\pi)^4}\,e^{ip\cdot\xi}\,\langle P,S\,|\,\overline{\psi}(0)\left[-ig\int_{+\infty}^{0}d\lambda\;n{\cdot}A^a(\xi{+}\lambda n)\,t^a\right]\psi(\xi)\,|P,S\,\rangle$$

gauge invariant

correlation in mean transvers momentum



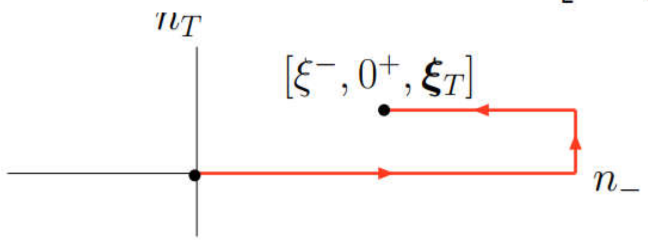
Transverse momentum
in PDFs

Non-Universality of TMD distributions

$$\mathcal{U}^{n_-}_{(0,+\infty)} = \mathcal{U}^{n_-}(0^-,\infty^-;\mathbf{0}_T)\,\mathcal{U}^T(\mathbf{0}_T,\boldsymbol{\infty}_T;\infty^-),$$

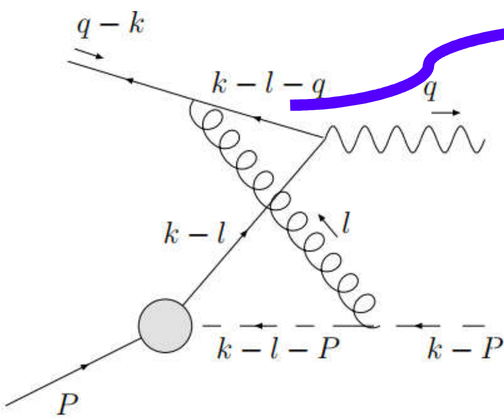
$$\mathcal{U}^{n_-}(a^-,b^-;\boldsymbol{c}_T) = \mathcal{P}\exp\left[-ig\int_{a^-}^{b^-} d\zeta^- A^+(\zeta^-,0^+,\boldsymbol{c}_T)\right],$$

$$\mathcal{U}^T(\boldsymbol{a}_T,\boldsymbol{b}_T;c^-) = \mathcal{P}\exp\left[-ig\int_{\boldsymbol{a}_T}^{\boldsymbol{b}_T} d\boldsymbol{\zeta}_T\cdot \boldsymbol{A}_T(c^-,0^+,\boldsymbol{\zeta}_T)\right].$$

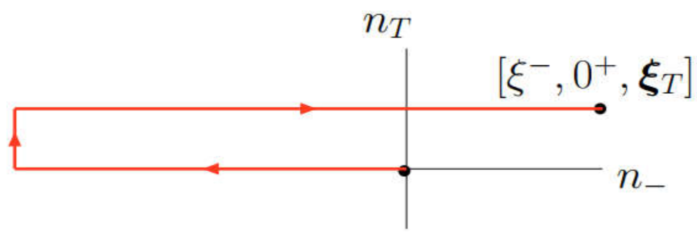


gauge-link in SIDIS (future-pointing)

in Drell-Yan process ($h_1 h_2 \to l^+ l^- X$)



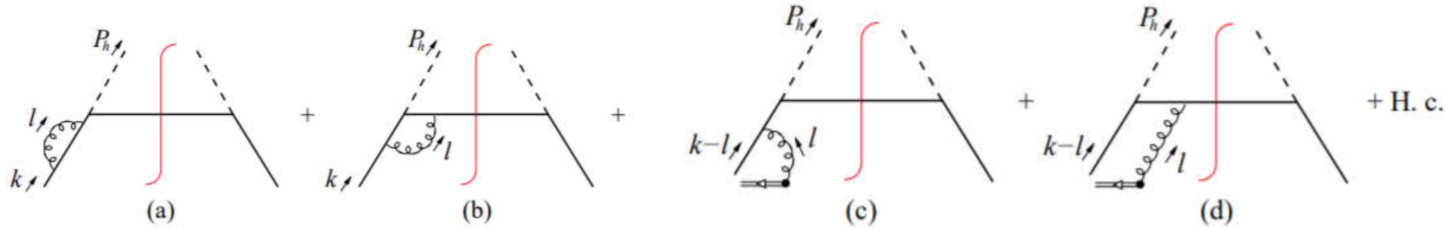
$$\approx \mathrm{i}\,\frac{-(q-k)^-\gamma^+}{2l^+(q-k)^- + \mathrm{i}\epsilon} = \frac{\mathrm{i}}{2}\,\frac{\gamma^+}{-l^+ - \mathrm{i}\epsilon}$$



gauge-link in Drell-Yan (past-pointing)

T-odd fragmentation functions

- four diagrams contributes to the imaginary part (unlike the distribution functions)

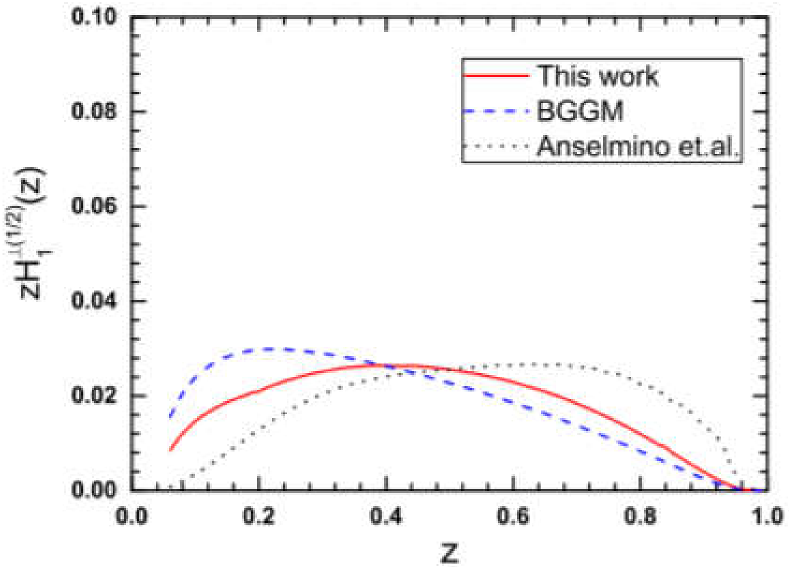


Collins(-related) function: chiral-odd

$$\frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp(z, k_T^2) = \frac{1}{2}\,\mathrm{Tr}[\Delta(z, k_T) i\sigma^{i-}\gamma_5].$$

half k_T-moment

$$H_1^{\perp(1/2)}(z) = z^2 \int d^2\boldsymbol{k}_T \frac{|\boldsymbol{k}_T|}{2m_h} H_1^\perp(z, k_T^2)$$



Unpolarized process—solution

- **Solution in *b* space** Collins, Soper 81' Collins, Soper, Sterman 85' Collins, 11' Collins, Rogers 15' Ji,Ma,Yuan 04'

$$\tilde{f}_1^{u/p}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*)-S_{\rm NP}^{f_1^{q/p}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/p}(x,\mu_b)$$

CSS prescription $b_* = b/\sqrt{1+b^2/b_{\rm max}^2}$ $\quad b_* \approx b$ at low b $\quad b_* \approx b_{\rm max}$ at large b $\quad \mu_b = c_0/b_*$

- The way to regularize light-cone singularity in TMD definition and subtract soft gluon contribution defines the scheme for the TMD factorization

- $\mathcal{F}(\alpha_s(Q))$, $H_{UU}(Q;\mu)$: scheme-dependent coefficients/factors

Prokudin, Sun, Yuan 15'

  - Ji-Ma-Yuan (JMY) scheme: Ji,Ma,Yuan, PRD71, 034005; PLB 597,299
  - Collins-11(JCC) scheme: J. C. Collins, Foundations of perterbative QCD
  - Lattice (LAT) scheme: Ji,Ma,Yuan, PRD91, 074009

Sudakov form factor of the pion

- Propose a similar non-perturbative Sudakov form factor $S_{\rm NP}^{f_1^{q/\pi}}(Q,b)$ for pion TMD

$$S_{\rm NP}^{f_1^{q/\pi}} = g_1^\pi b^2 + g_2^\pi \ln\frac{b}{b_*}\ln\frac{Q}{Q_0}.$$

Wang, Lu, Schmidt, JHEP 1708, 137

$$g_1^\pi(b) = g_1^\pi b^2$$

contains information on the nonperturbative transverse motion of partons inside pion

- The unpolarized TMD distribution for the pion

$$f_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*)-S_{\rm NP}^{f_1^{q/\pi}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/\pi}(x,\mu_b)$$

$$f_{1q/\pi}(x,k_\perp;Q) = \int_0^\infty \frac{dbb}{2\pi}J_0(k_\perp b)\tilde{f}_{1q/\pi}^{\rm sub}(x,b;Q).$$

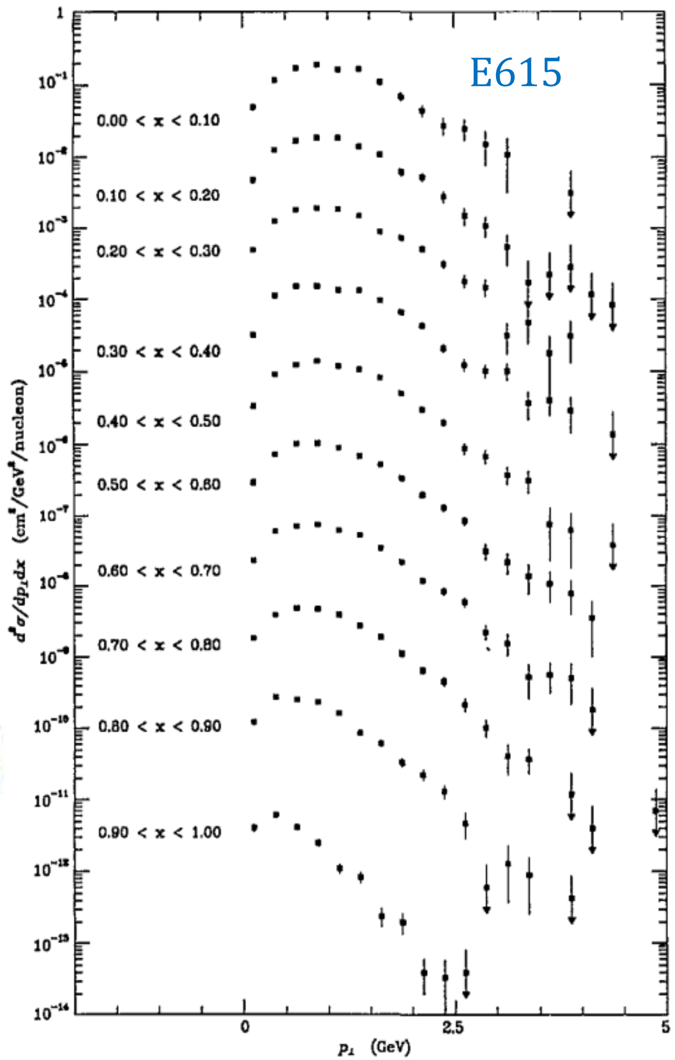
Experimental measurements

- Pion-N Drell-Yan: early measurements (Unpolarized)

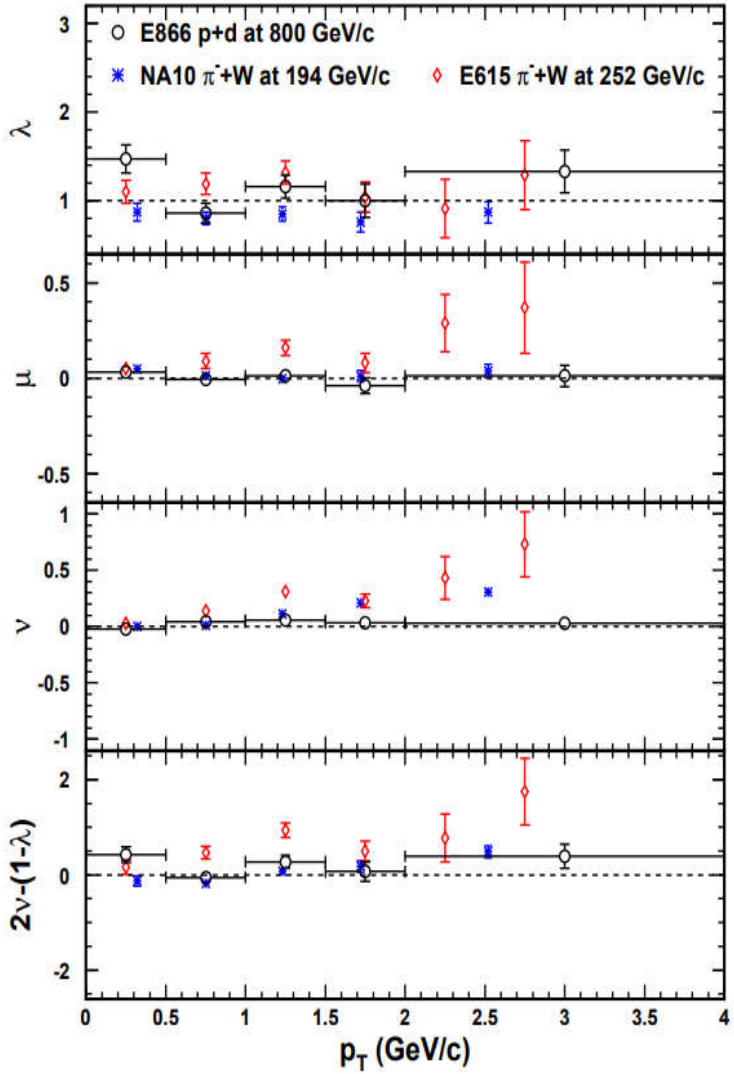
  - E615
  - NA10

$$\frac{dN}{d\Omega} \equiv \frac{d\sigma}{d^4q\, d\Omega} \Big/ \frac{d\sigma}{d^4q}$$

$$= \frac{3}{4\pi} \frac{1}{\lambda+3} \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi\right)$$



q_T distribution of dilepton

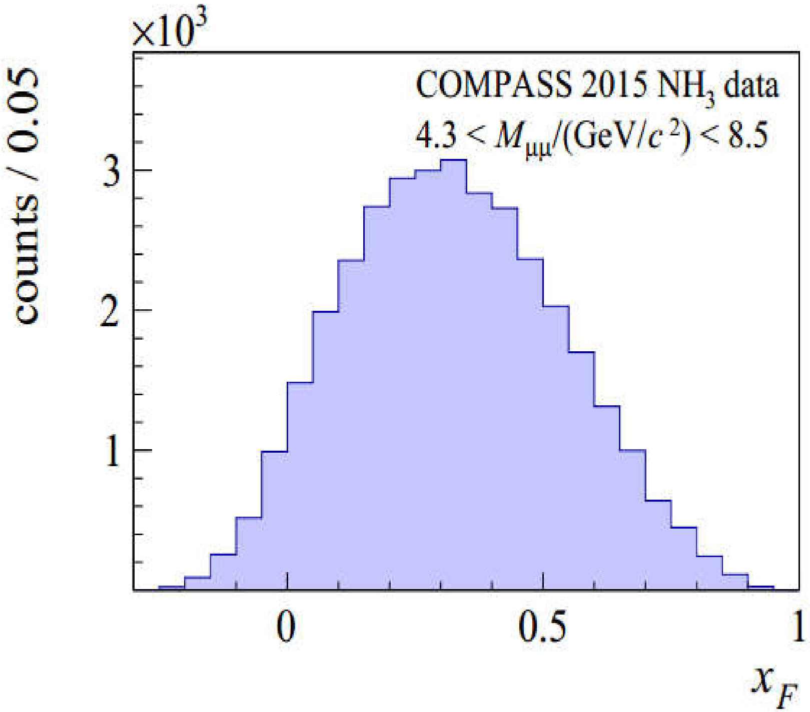


Angular coefficients

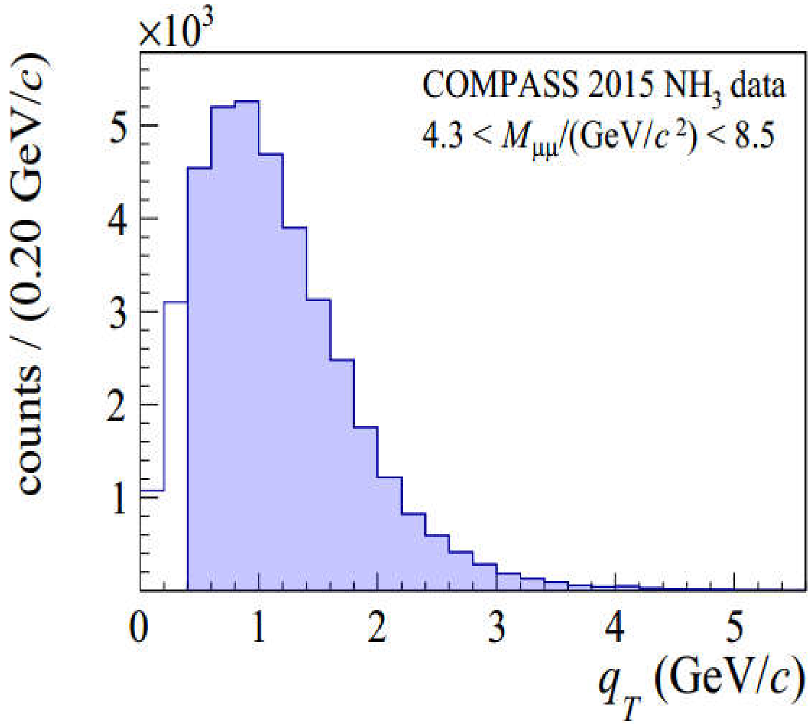
Recent COMPASS measurements

- Pion-N Drell-Yan: Experimental measurements (Unpolarized)

PRL119, 112002 (2017)

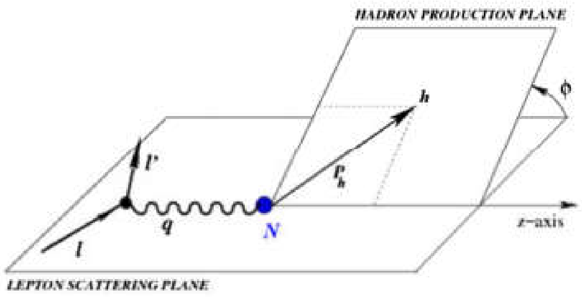


x_F distribution of dilepton events



q_T distribution

beam-spin asymmetry (twist-3)



SIDIS cross section for an unpolarized target:

→ Contains model independent structure functions

$$\frac{d\sigma}{dx_B\, dQ^2\, dz\, d\phi_h\, dp_{h\perp}^2} = K(x,y,Q^2)\Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\sin\phi_h \underline{F_{LU}^{\sin\phi_h}} \Big\}$$

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\mathcal{C}\left(-\frac{\hat{\mathbf{h}}\cdot\mathbf{k_T}}{M_h}\left(xeH_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p_T}}{M}\left(xg^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z}\right)\right)$$

twist-3 pdf

Collins FF

unpolarized dist.

twist-3 FF

twist-3 t-odd dist. funcion

Boer-Mulders

twist-3 FF

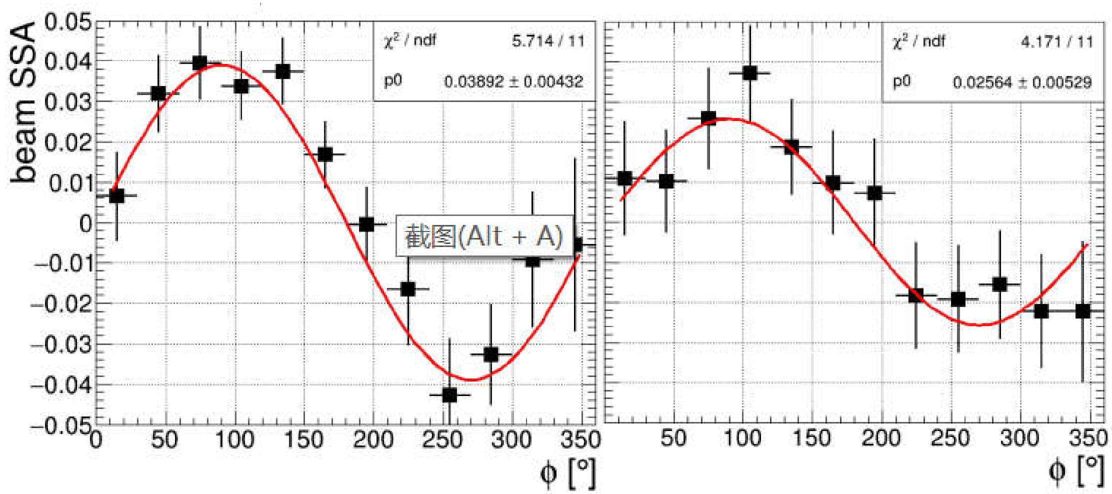
CLAS12 measurement

$$d\sigma = d\sigma_0(1 + A_{UU}^{\cos\phi}\cos\phi + A_{UU}^{\cos 2\phi}\cos 2\phi + \lambda_e A_{LU}^{\sin\phi}\sin\phi)$$

$$BSA = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} = \frac{A_{LU}^{\sin\phi}\sin\phi}{1 + A_{UU}^{\cos\phi}\cos\phi + A_{UU}^{\cos(2\phi)}\cos(2\phi)}$$

$$A_{LU}^{\sin\phi} = \sqrt{2\varepsilon(1-\varepsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU}}$$

$$BSA_i = \frac{1}{P_e}\cdot\frac{N_i^+ - N_i^-}{N_i^+ + N_i^-}$$



data compared to models

Mao,ZL 14

Mao,ZL 14

Bastami et al, 20