Recent progress on light pseudoscalar and axial vector meson spectra

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Outline

1. The long-standing $\eta(1405)/\eta(1475)$ puzzle and pseudoscalar glueball search
2. The presence of the "triangle singularity"
3. Reconsile the the dynamical model calculations with LQCD simulations
4. Further evidences for the TS mechanism
5. Brief summary

1. The $\eta(1405)/\eta(1475)$ puzzle and the pseudoscalar glueball search

Hadrons beyond the conventional QM and…

Exotics of Type-I:

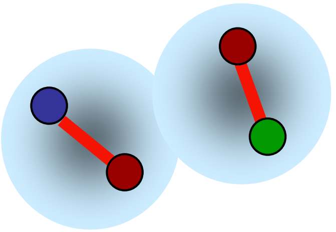
J^{PC} are not allowed by $Q\bar{Q}$ configurations, e.g. $0^{--}, 1^{-+}$ …

- **Direct observation**

Exotics of Type-II:

J^{PC} are the same as $Q\bar{Q}$ configurations

- **Outnumbering of conventional QM states?**
- **Peculiar properties?**

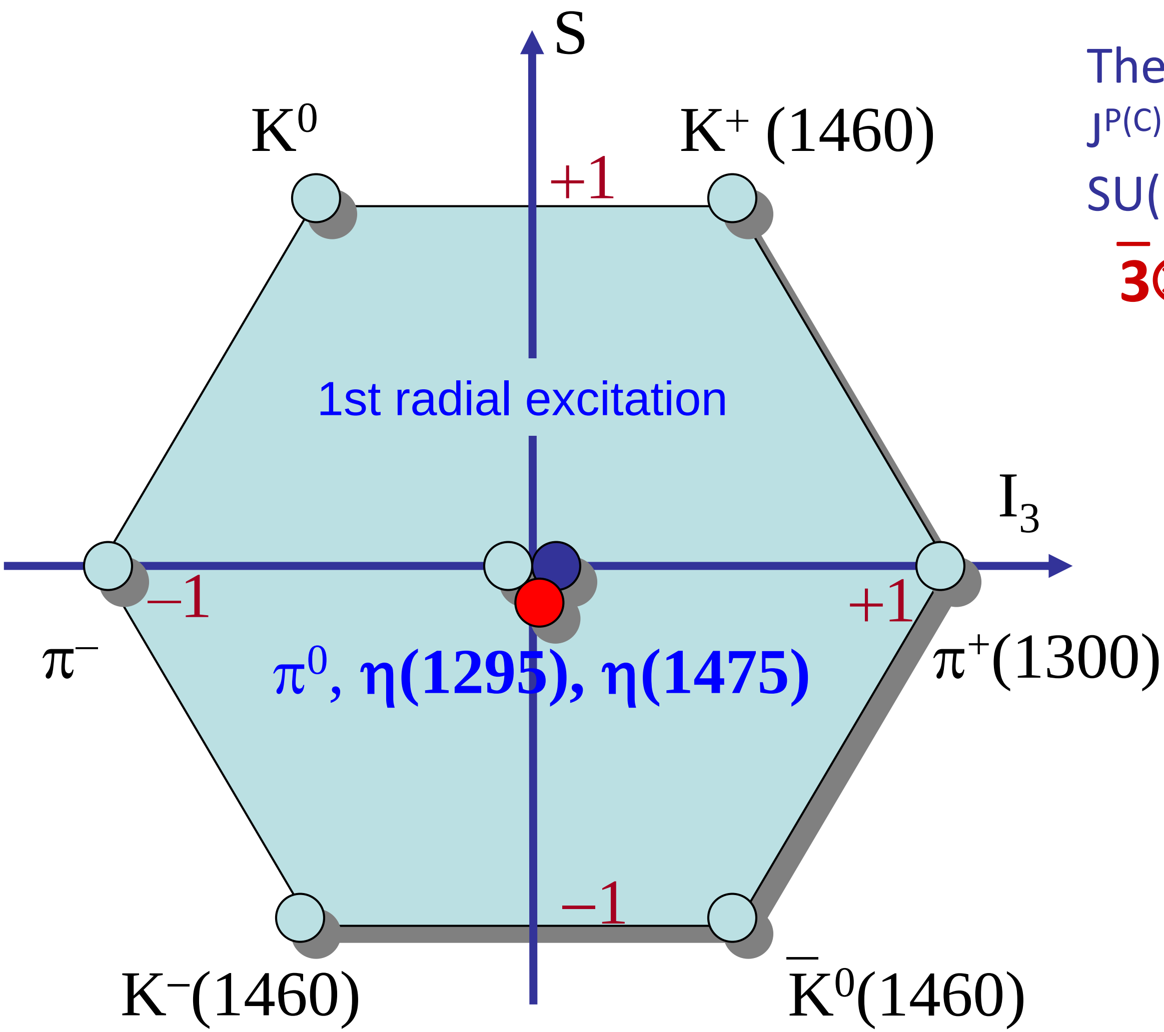


"Exotics" of Type-III:

Leading kinematic singularity can cause measurable effects, e.g. the triangle singularity.

- **What's the impact?**
- **How to distinguish a genuine state from kinematic effects?**

Exotics of Type II:
The abundance of 0^{-+} (I=0) states implies an exotic candidate



The first radial excitation of $J^{P(C)}=0^{-(+)}$ states also make a $\bar{q}q$ SU(3) flavor nonet:

$$\bar{3}\otimes 3 = 1 \oplus 8$$

Three η states have been listed by Particle Data Group around 1.2 ~ 1.5 GeV: **η(1295)**, **η(1405)**, and **η(1475)**

The arising of the E-ι puzzle:

E meson was first observed in 1965 in $p\,\bar{p} \rightarrow (K\,\bar{K}\pi)\,\pi^{+}\pi^{-}$.

Observation of ι(1440) at Mark II (left, 1980) and Crystal Ball (right, 1982)

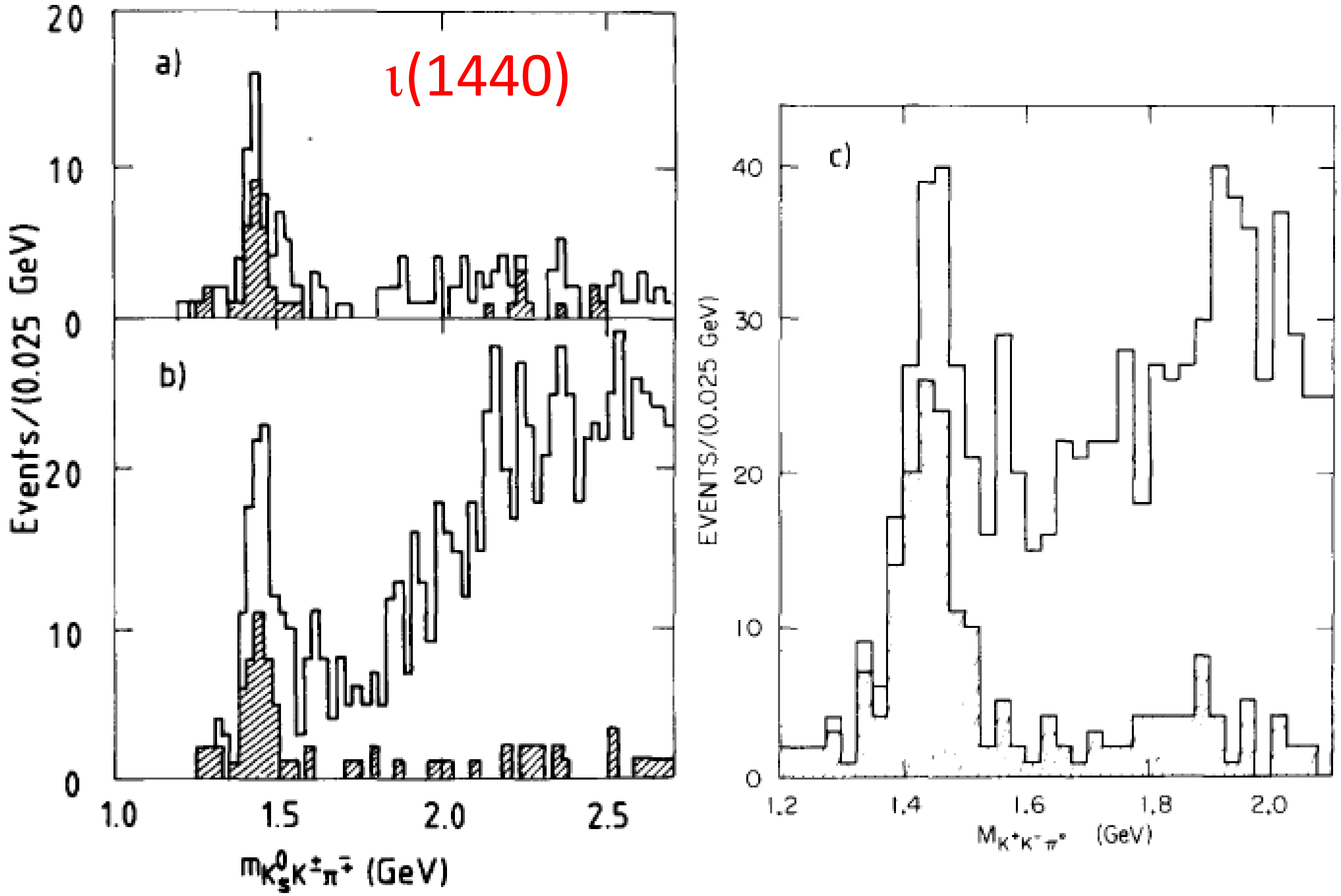
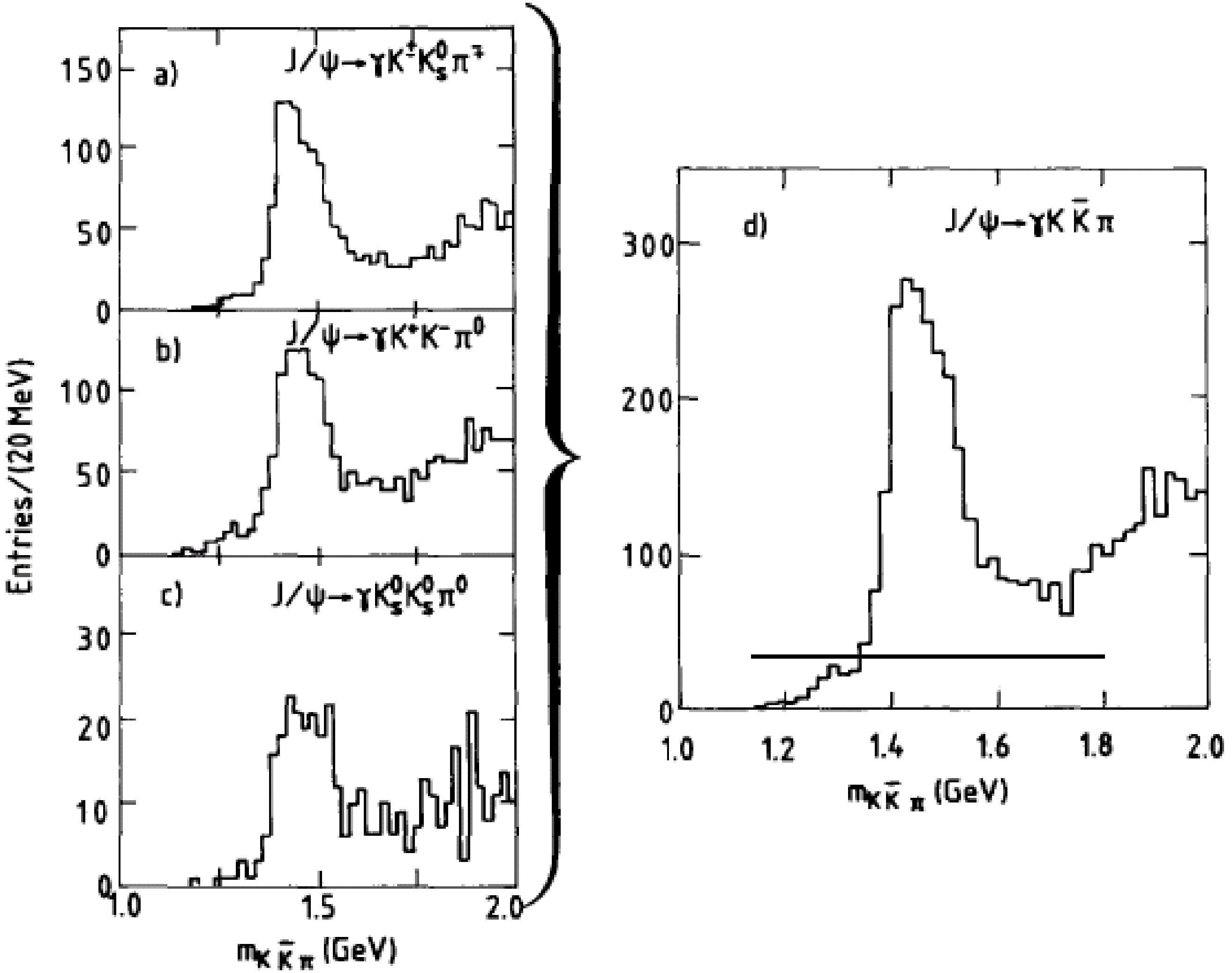
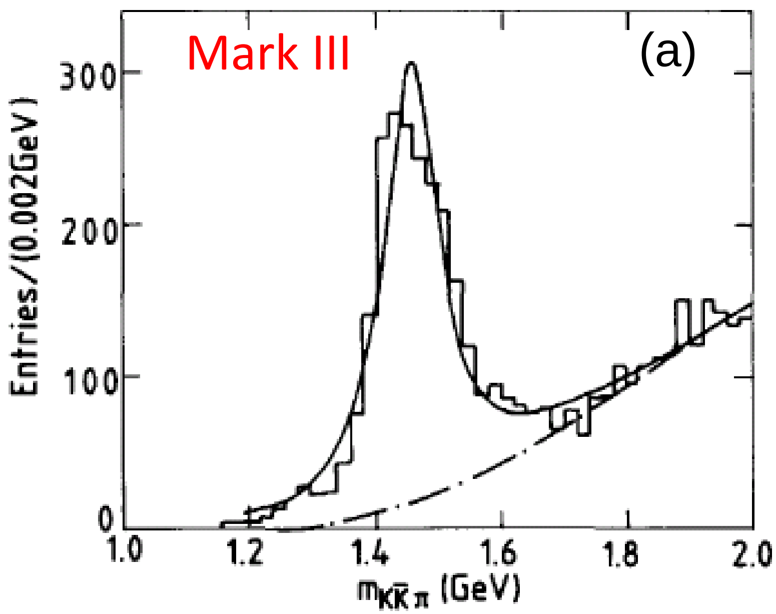


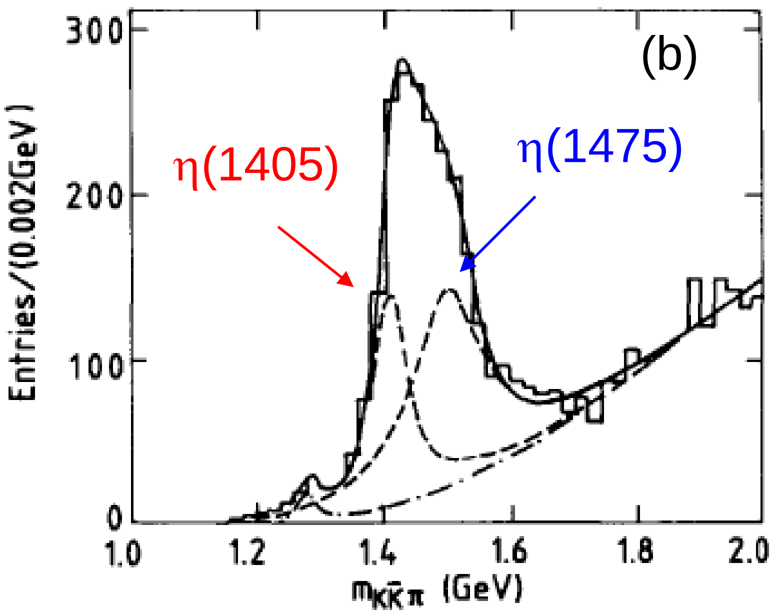
Fig. 69. Observation of the η(1440) by Mark II and Crystal Ball. (a) Mark II, radiative photon detection required, (b) Mark II, photon detection not required. The events in the shaded region have $m_{K\bar{K}} < 1.05$ GeV ("delta cut"). (c) Crystal Ball, events in the shaded region have $m_{K\bar{K}} < 1.125$ GeV.

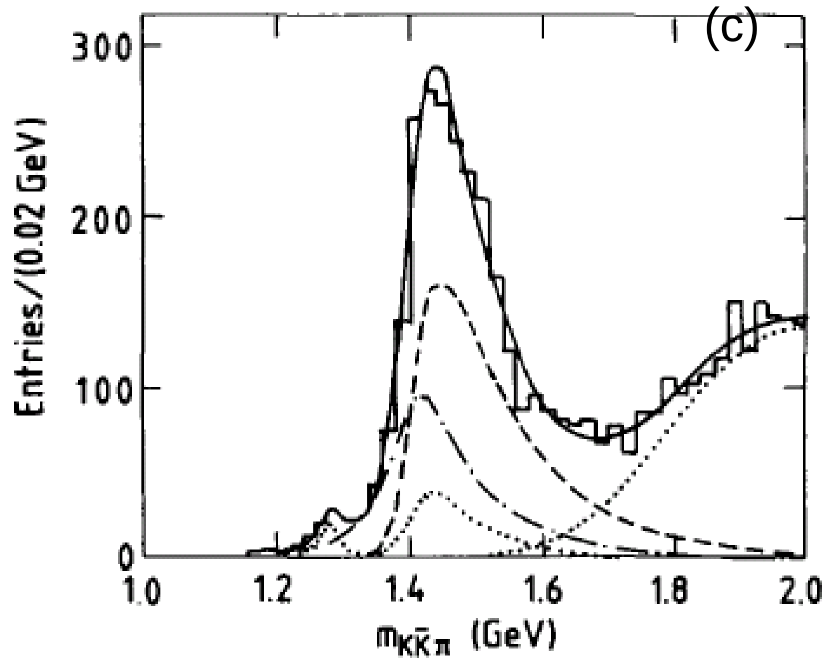
Confirmation of η(1440) at Mark III in 1987



Distorted lineshape?







(a) A single Breit-Wigner fit

(b) Two interfering B-W fit

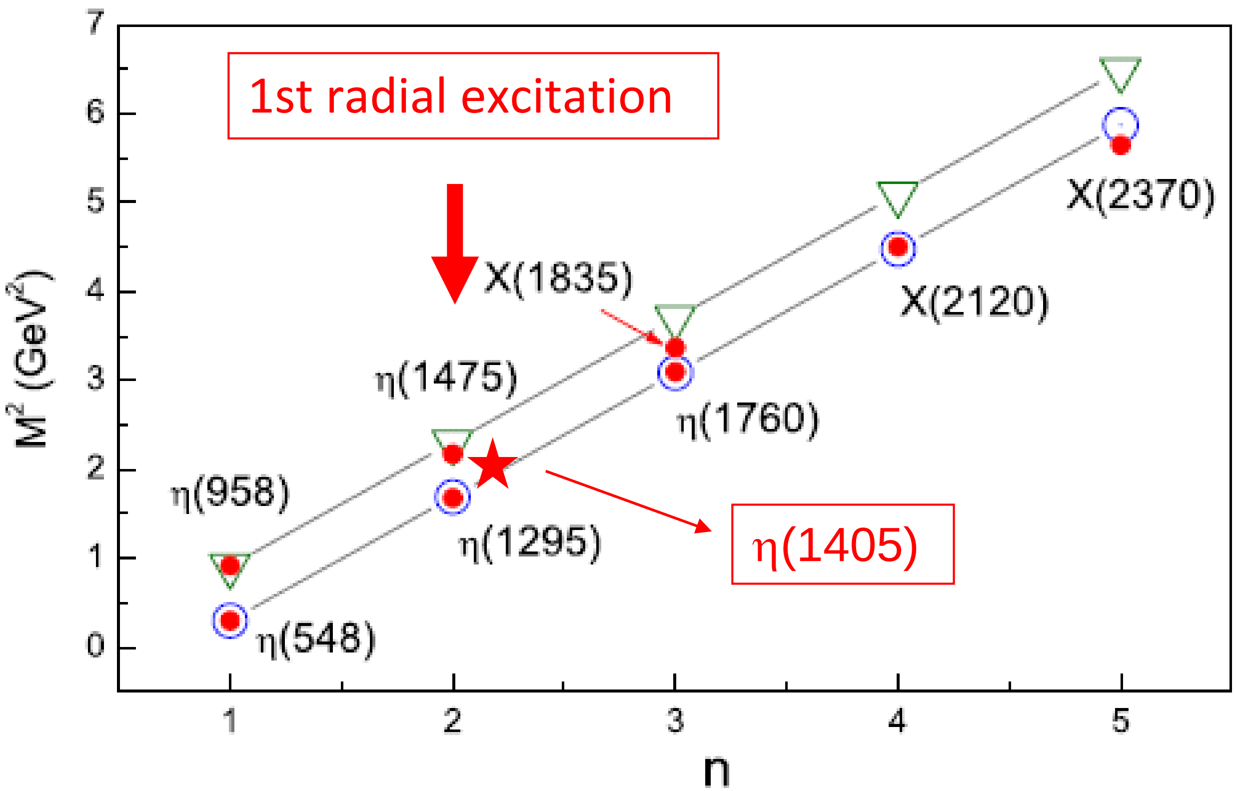
(c) Coupled channel B-W fit

$$M = 1416 \pm 8^{+7}_{-5};\ \Gamma = 91^{+67}_{-31}{}^{+15}_{-38}\ \mathrm{MeV}/c^2$$

$$M = 1490^{+14+3}_{-8-6};\ \Gamma = 54^{+37+13}_{-21-24}\ \mathrm{MeV}/c^2$$

Also "confirmed" by Obelix collaboration

- Regge trajectory for the η/η′ mass spectrum



$$M^2 = M_0^2 + (n-1)\mu^2 \ (\mu^2 = 1.39\ \text{GeV})$$

- How to understand the presence of η(1405) ?

J.S. Yu, Z.F. Sun, X. Liu, and Q. Z., PRD83, 114007 (2011)

The abundance of 0^{-+} (I=0) states implies a glueball candidate?

Positive:
- Flux tube model favors $M_G \cong 1.4$ GeV [1]
- A dynamical model based on $U_A(1)$ anomaly gives a similar mass [2].

Caveat:
- LQCD favors $M_G \cong 2.4 - 2.6$ GeV [3,4,5]

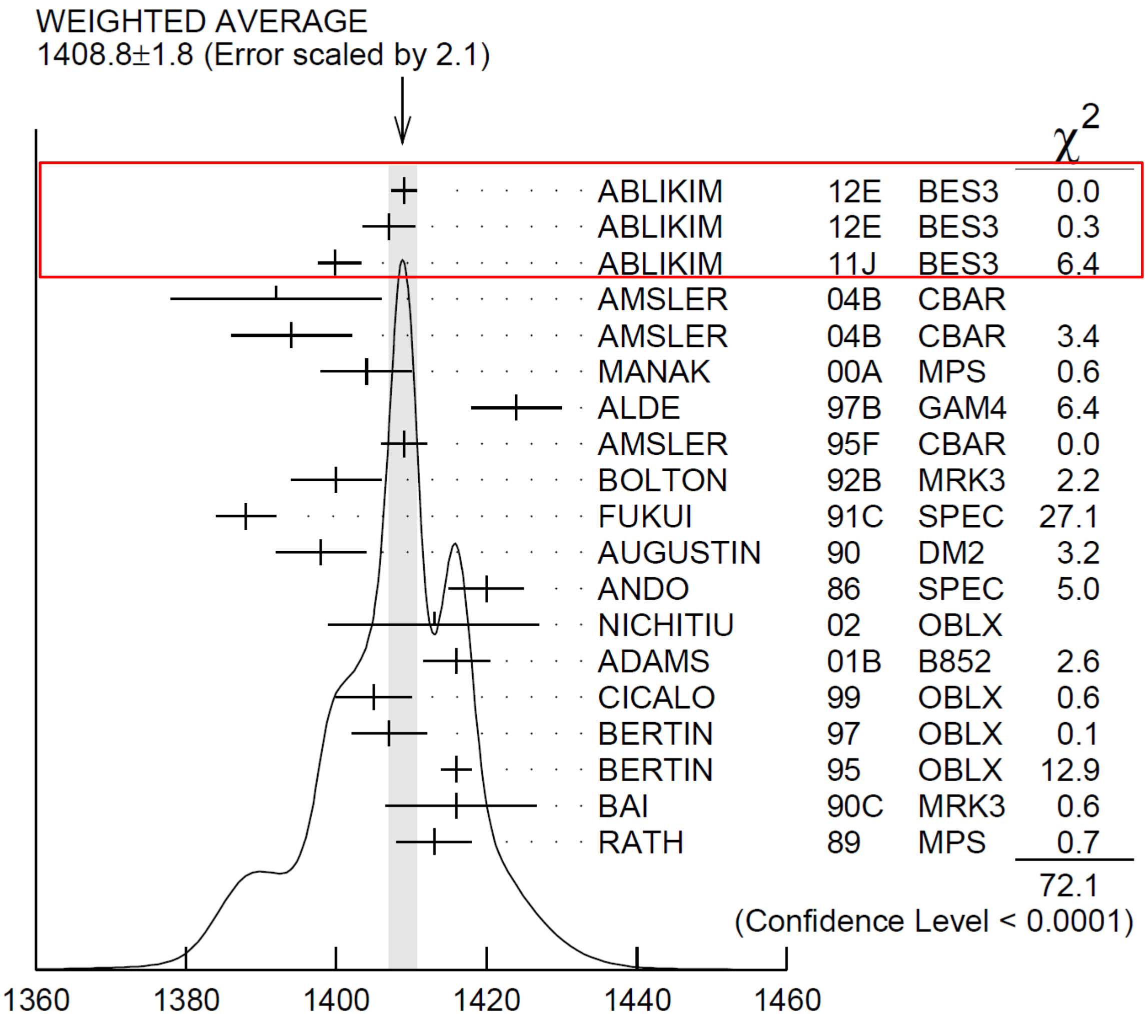
◆ **What can we learn from modern high-precision data? E.g. BESIII, Belle, LHCb…**

◆ **How to understand the HUGE difference between the dynamical calculations and LQCD results?**

[1] Faddeev, Niemi, and Wiedner, PRD70, 114033 (2004)
[2] H. Y. Cheng, H. n. Li, and K. F. Liu, Phys. Rev. D 79, 014024 (2009)
[3] Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)
[4] Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)
[5] W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

$\eta(1405)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$



$\eta(1405)$ mass (MeV)

$\eta(1405)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$\eta\pi\pi$	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\eta(\pi\pi)_{S\text{-wave}}$	seen
Γ_5	$f_0(980)\eta$	seen
Γ_6	4π	seen
Γ_7	$\rho\rho$	<58 %
Γ_8	$\gamma\gamma$	
Γ_9	$\rho^0\gamma$	seen
Γ_{10}	$\phi\gamma$	
Γ_{11}	$K^*(892)K$	seen

$\eta(1405)$

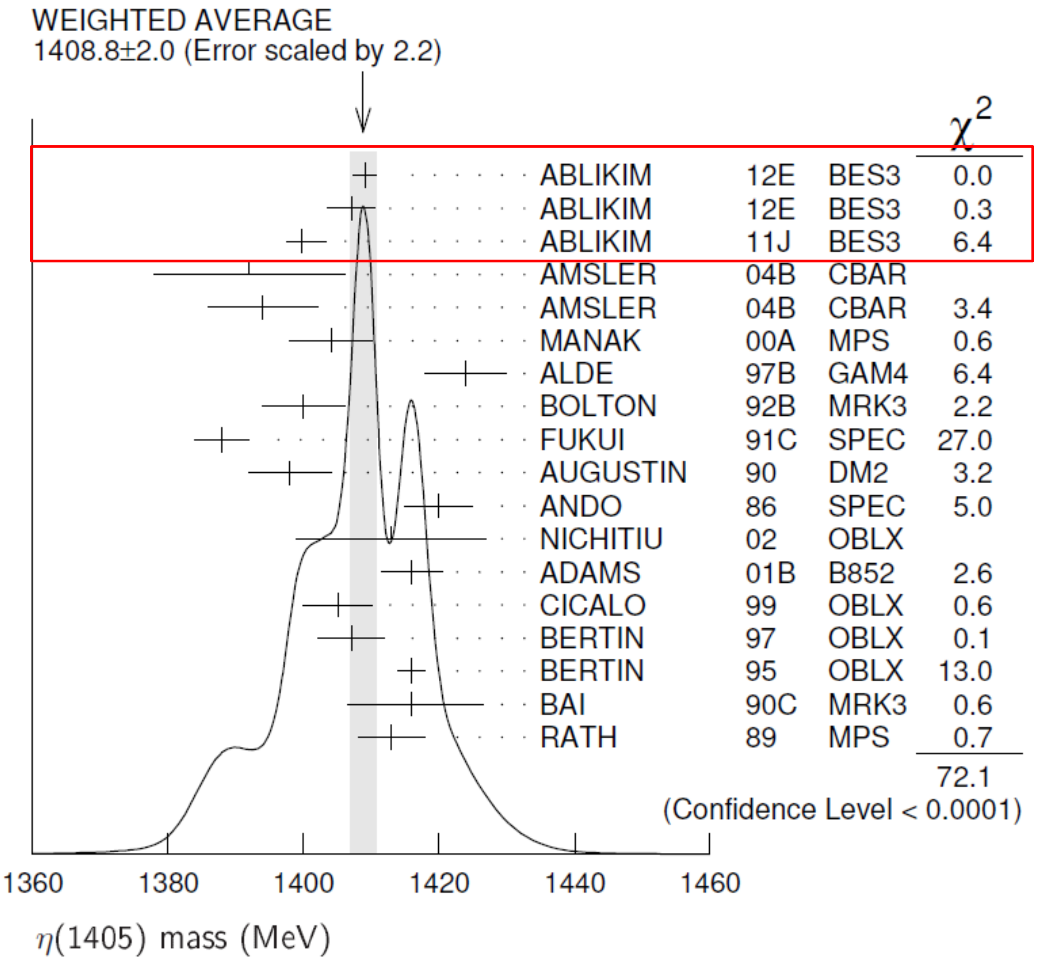
$I^G(J^{PC}) = 0^+(0^{-+})$

P.A. Zyla et al. (Particle Data Group),
Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

$\eta(1405)$ MASS

VALUE (MeV)	*DOCUMENT ID*
1408.8±2.0 OUR AVERAGE	Includes data from the 2 datablocks that follow this one.

Error includes scale factor of 2.2. See the ideogram below.



	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$\eta\pi\pi$	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\eta(\pi\pi)_{S\text{-wave}}$	seen
Γ_5	$f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$	not seen
Γ_6	$f_0(980)\eta$	seen
Γ_7	4π	seen
Γ_8	$\rho\rho$	<58 %
Γ_9	$\gamma\gamma$	
Γ_{10}	$\rho^0\gamma$	seen
Γ_{11}	$\phi\gamma$	
Γ_{12}	$K^*(892)K$	seen

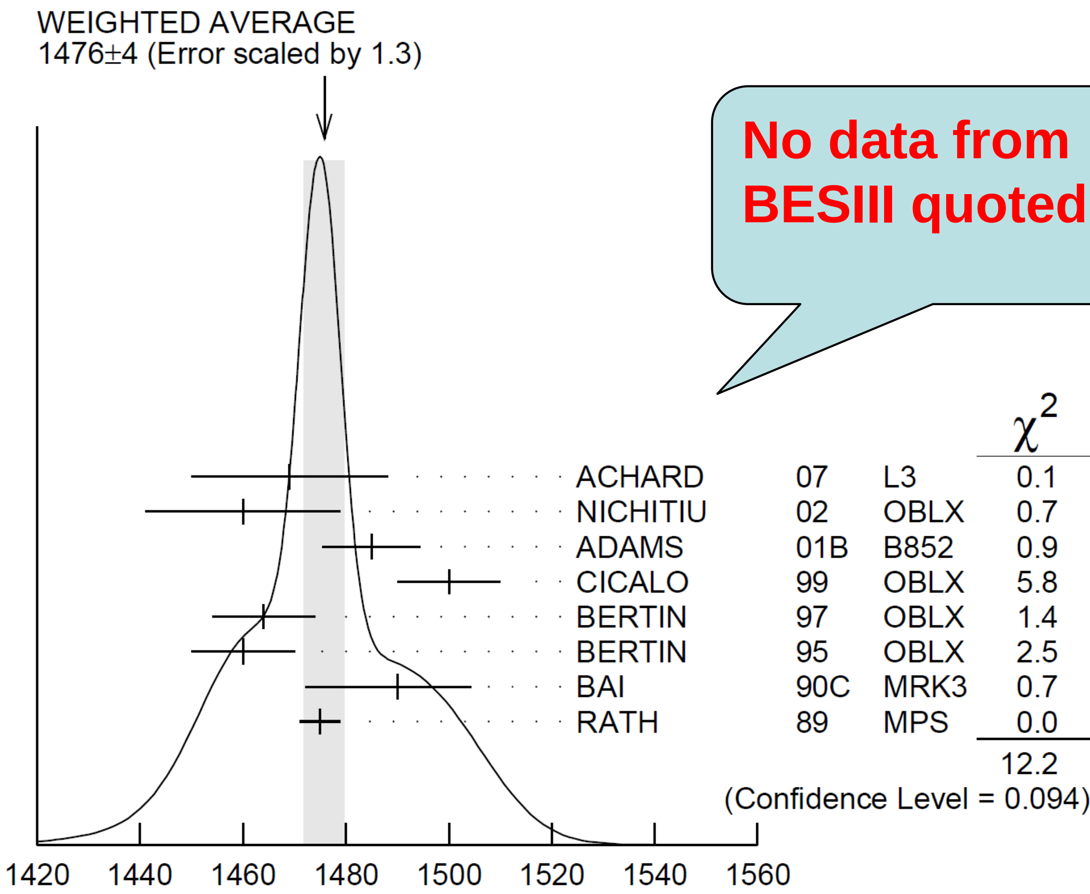
Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

$\eta(1475)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$

Apparent inconsistency between the analyses for η(1405) and η(1475)

No data from BESIII quoted!!!

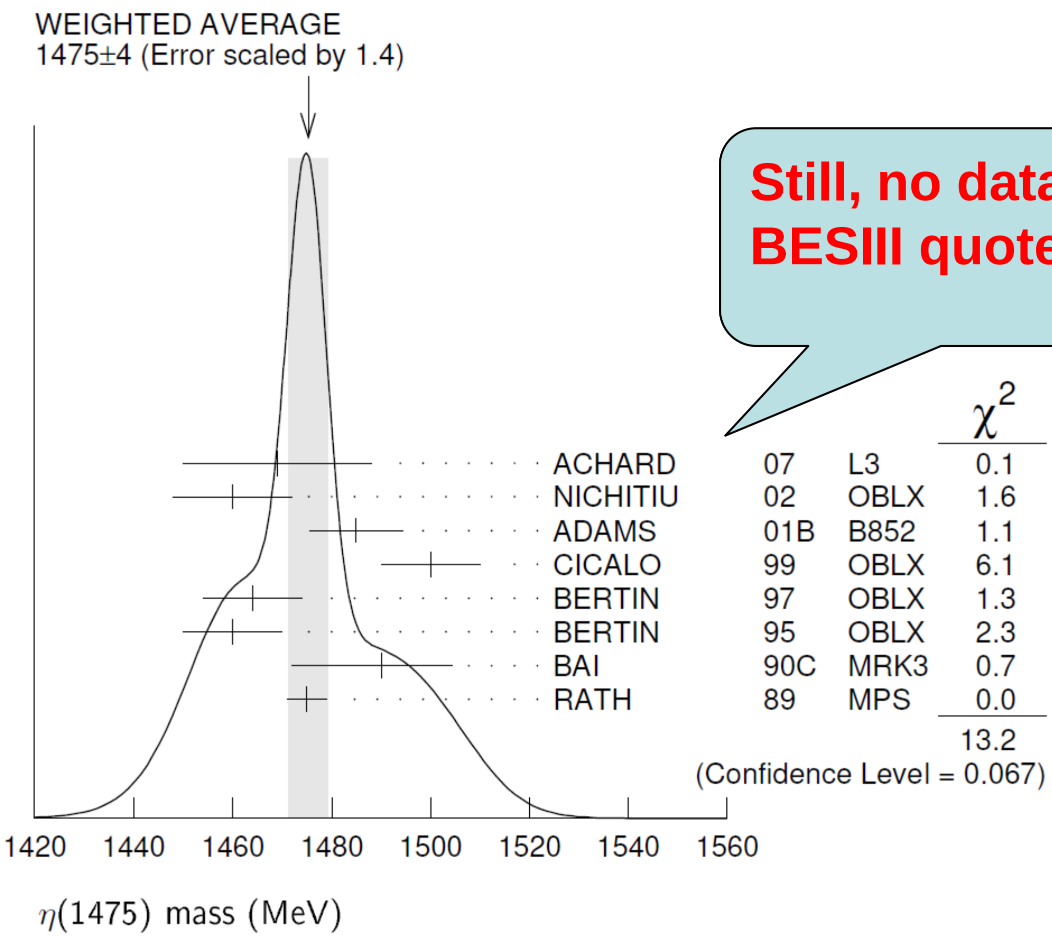


	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	dominant
Γ_2	$K\overline{K}^*(892)+$ c.c.	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$K^0_S K^0_S \eta$	possibly seen

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

$\eta(1475)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$



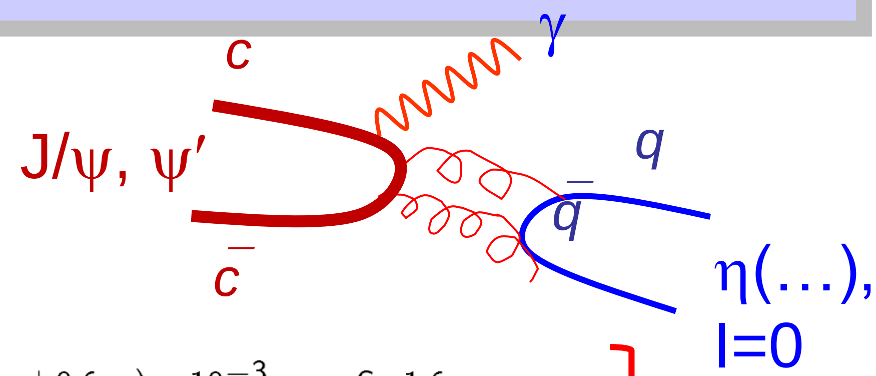
Still, no data from BESIII quoted!!!

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$K\overline{K}^*(892)+$ c.c.	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$K^0_S K^0_S \eta$	possibly seen
Γ_6	$\gamma\phi(1020)$	possibly seen

BESIII

Only a single state is observed in the J/ψ and ψ′ decays at BESIII

PDG 2016



$J/\psi(1S)$

$I^G(J^{PC}) = 0^-(1^{--})$

Γ_{151}	$\gamma\eta(1405/1475) \to \gamma K\overline{K}\pi$	[d]	(2.8	±0.6	) $\times 10^{-3}$	S=1.6
Γ_{152}	$\gamma\eta(1405/1475) \to \gamma\gamma\rho^0$		(7.8	±2.0	) $\times 10^{-5}$	S=1.8
Γ_{153}	$\gamma\eta(1405/1475) \to \gamma\eta\pi^+\pi^-$		(3.0	±0.5	) $\times 10^{-4}$	
Γ_{154}	$\gamma\eta(1405/1475) \to \gamma\gamma\phi$		< 8.2		$\times 10^{-5}$	CL=95%
Γ_{165}	$\gamma\eta(1405/1475) \to \gamma\rho^0\rho^0$		(1.7	±0.4	) $\times 10^{-3}$	S=1.3
Γ_{87}	$\phi\eta(1405) \to \phi\eta\pi^+\pi^-$		(2.0	±1.0	) $\times 10^{-5}$	

$\psi(2S)$

Γ_{94}	$\omega X(1440) \to \omega K^0_S K^-\pi^+ +$ c.c.	(1.6	±0.4	) $\times 10^{-5}$	
Γ_{95}	$\omega X(1440) \to \omega K^+K^-\pi^0$	(1.09	±0.26	) $\times 10^{-5}$	
Γ_{155}	$\gamma\eta(1405)$				
Γ_{156}	$\gamma\eta(1405) \to \gamma K\overline{K}\pi$	< 9		$\times 10^{-5}$	CL=90%
Γ_{157}	$\gamma\eta(1405) \to \eta\pi^+\pi^-$	(3.6	±2.5	) $\times 10^{-5}$	
Γ_{158}	$\gamma\eta(1475)$				
Γ_{159}	$\gamma\eta(1475) \to K\overline{K}\pi$	< 1.4		$\times 10^{-4}$	CL=90%
Γ_{160}	$\gamma\eta(1475) \to \eta\pi^+\pi^-$	< 8.8		$\times 10^{-5}$	CL=90%

BES-III

BES-II

$J/\psi(1S)$

$I^G(J^{PC}) = 0^-(1^{--})$

Γ_{121}	$\phi\eta(1405) \to \phi\eta\pi^+\pi^-$		$(2.0 \pm 1.0) \times 10^{-5}$		
Γ_{216}	$\gamma\eta(1405/1475) \to \gamma K \bar{K} \pi$	[d]	$(2.8 \pm 0.6) \times 10^{-3}$	S=1.6	
Γ_{217}	$\gamma\eta(1405/1475) \to \gamma\gamma\rho^0$		$(7.8 \pm 2.0) \times 10^{-5}$	S=1.8	
Γ_{218}	$\gamma\eta(1405/1475) \to \gamma\eta\pi^+\pi^-$		$(3.0 \pm 0.5) \times 10^{-4}$		
Γ_{219}	$\gamma\eta(1405/1475) \to \gamma\gamma\phi$		$< 8.2 \times 10^{-5}$	CL=95%	
Γ_{220}	$\gamma\eta(1405) \to \gamma\gamma\gamma$		$< 2.63 \times 10^{-6}$	CL=90%	
Γ_{221}	$\gamma\eta(1475) \to \gamma\gamma\gamma$		$< 1.86 \times 10^{-6}$	CL=90%	
Γ_{232}	$\gamma\eta(1405/1475) \to \gamma\rho^0\rho^0$		$(1.7 \pm 0.4) \times 10^{-3}$	S=1.3	

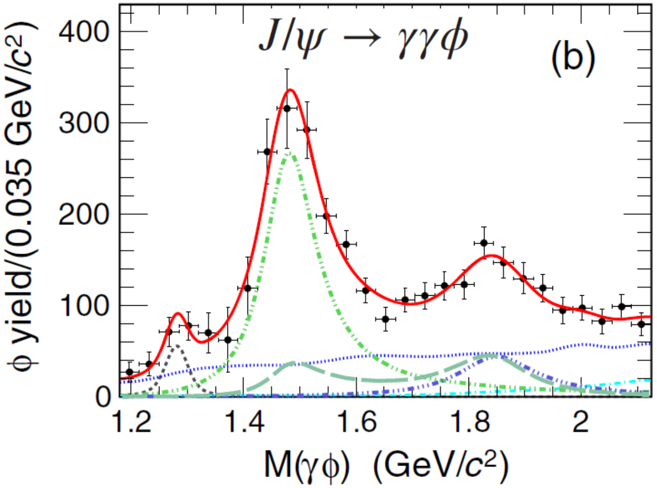
$\psi(2S)$

Γ_{107}	$\omega X(1440) \to \omega K^0_S K^-\pi^+ +$ c.c.	$(1.6 \pm 0.4) \times 10^{-5}$	
Γ_{108}	$\omega X(1440) \to \omega K^+ K^- \pi^0$	$(1.09 \pm 0.26) \times 10^{-5}$	
Γ_{135}	$\phi\eta(1405) \to \phi\pi^+\pi^-\eta$	$(8.5 \pm 1.7) \times 10^{-6}$	
Γ_{174}	$\gamma\eta(1405)$		
Γ_{175}	$\gamma\eta(1405) \to \gamma K\bar{K}\pi$	$< 9 \times 10^{-5}$	CL=90%
Γ_{176}	$\gamma\eta(1405) \to \eta\pi^+\pi^-$	$(3.6 \pm 2.5) \times 10^{-5}$	
Γ_{177}	$\gamma\eta(1405) \to \gamma f_0(980)\pi^0 \to \gamma\pi^+\pi^-\pi^0$	$< 5.0 \times 10^{-7}$	CL=90%
Γ_{178}	$\gamma\eta(1475)$		
Γ_{179}	$\gamma\eta(1475) \to K\bar{K}\pi$	$< 1.4 \times 10^{-4}$	CL=90%
Γ_{180}	$\gamma\eta(1475) \to \eta\pi^+\pi^-$	$< 8.8 \times 10^{-5}$	CL=90%

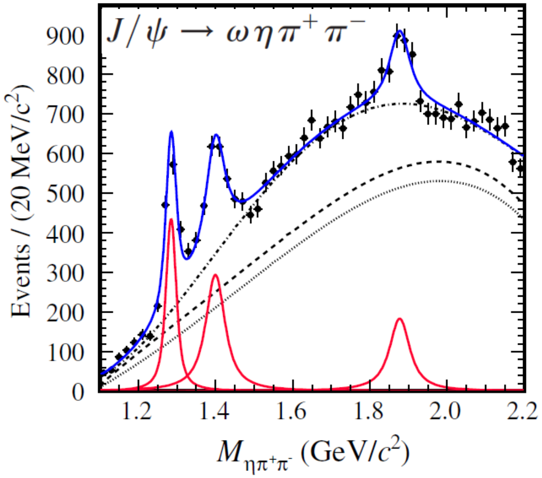
BESIII, PRD96, 112008(2017)

Invariant mass spectra measured at BES-III

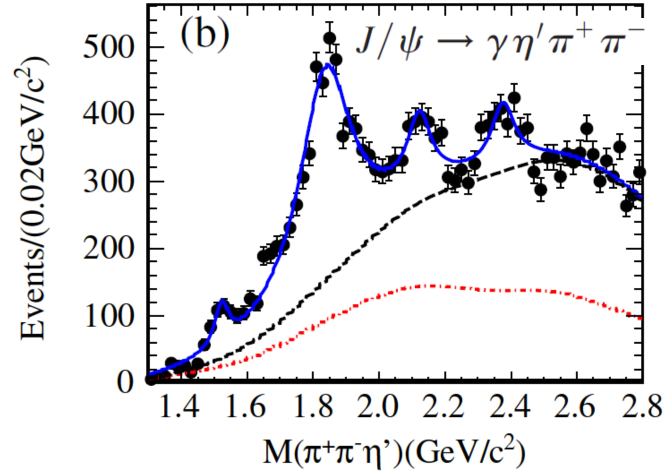
PRD97, 051101(R)(2018)



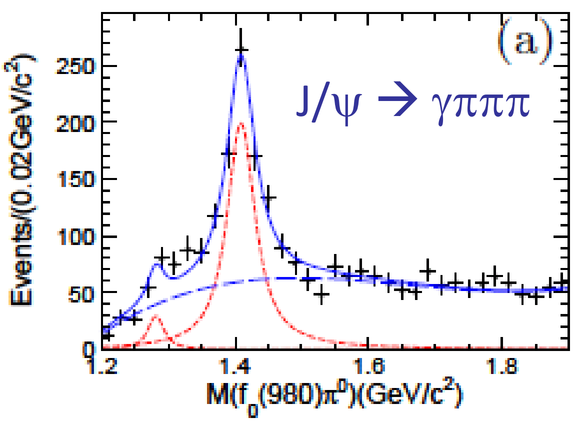
PRL 107, 182001 (2011)

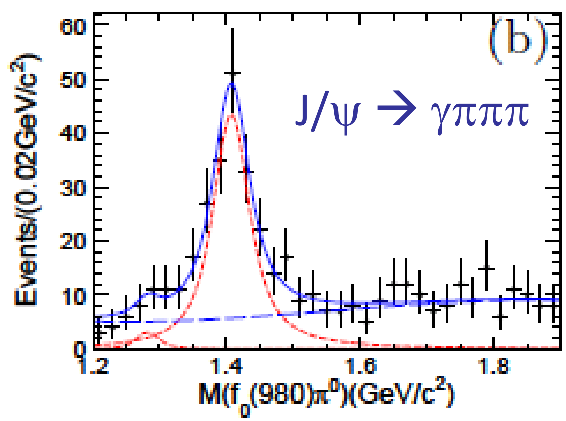


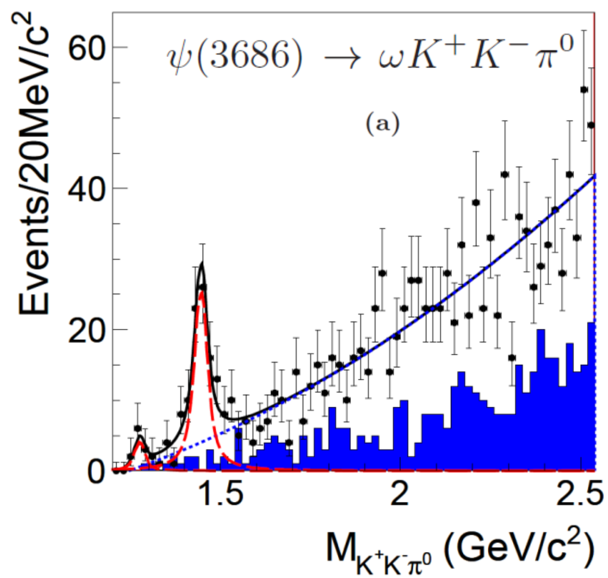
PRL 106, 072002 (2011)



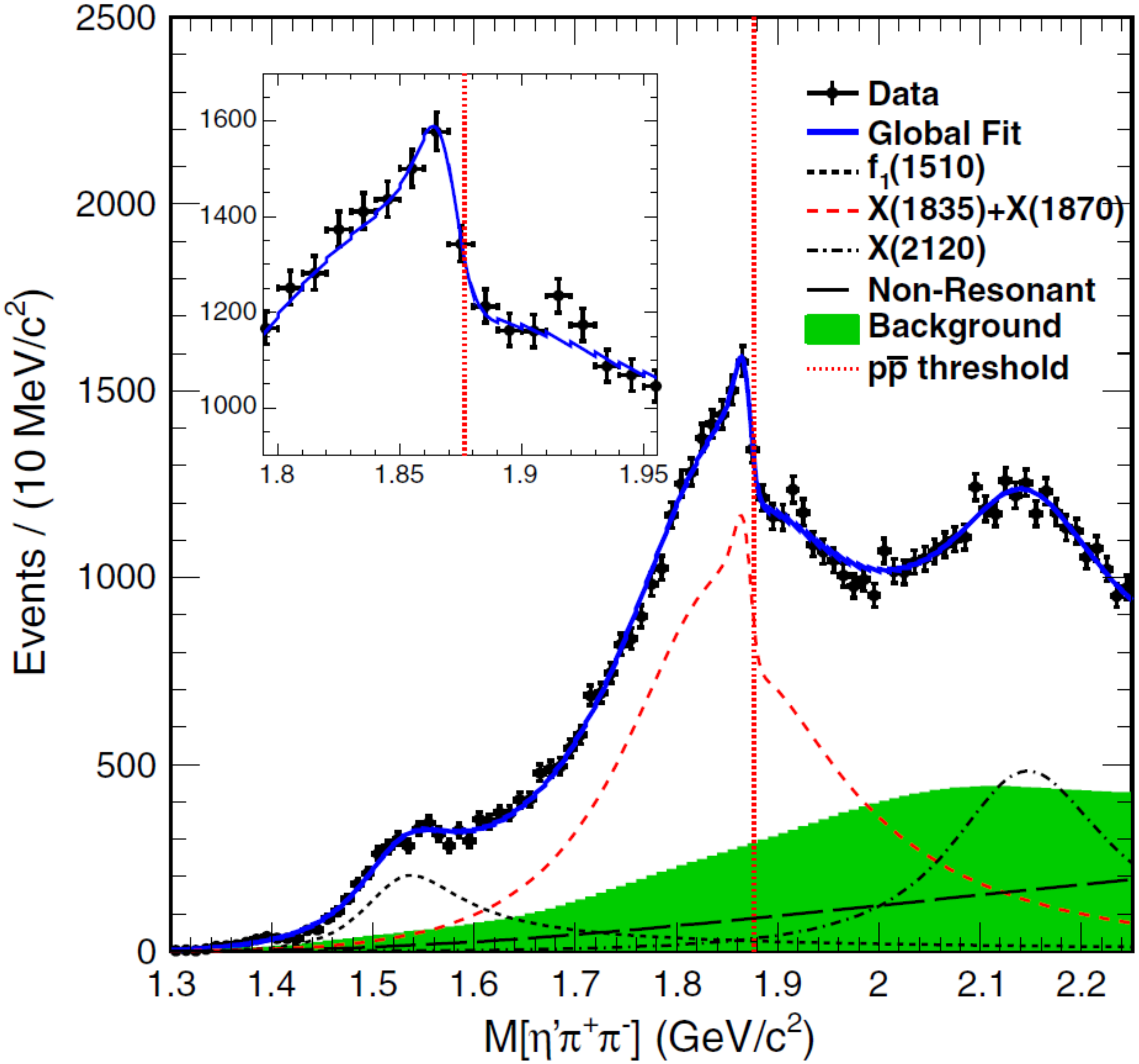
PRL108, 182001 (2012)

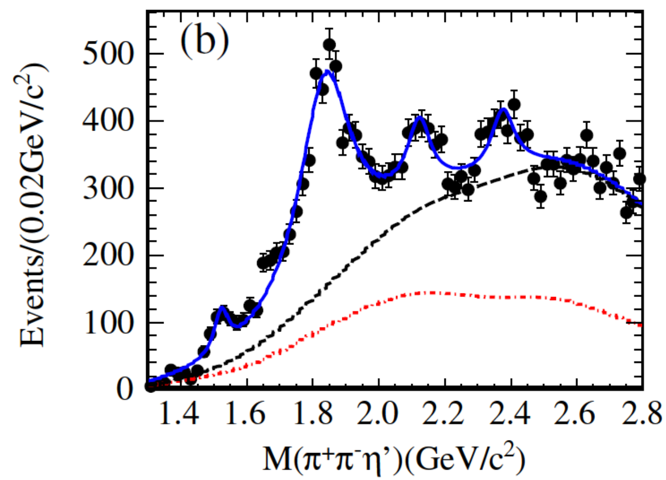






Observation of an Anomalous Line Shape of the $\eta'\pi^+\pi^-$ Mass Spectrum near the $p\bar{p}$ Mass Threshold in $J/\psi \to \gamma\eta'\pi^+\pi^-$





The $\eta(1405)$ and $\eta(1475)$ paradox:

- No experimental evidence for $\eta(1405)$ and $\eta(1475)$ to be present in the same decay channel!
- One state/two states?
- Where to look for the pseudoscalar glueball?

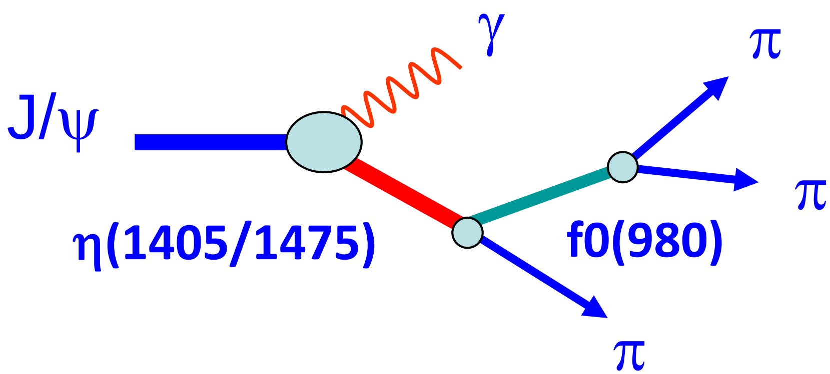
[W. Qin, Q.Z., and X.H. Zhong, PRD 97, 096002 (2018) , J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012); X.G. Wu, J.-J. Wu, Q. Z., and B.-S. Zou, PRD87, 014023 (2013), M.-C. Du and Q.Z., PRD100, 036005 (2019)]

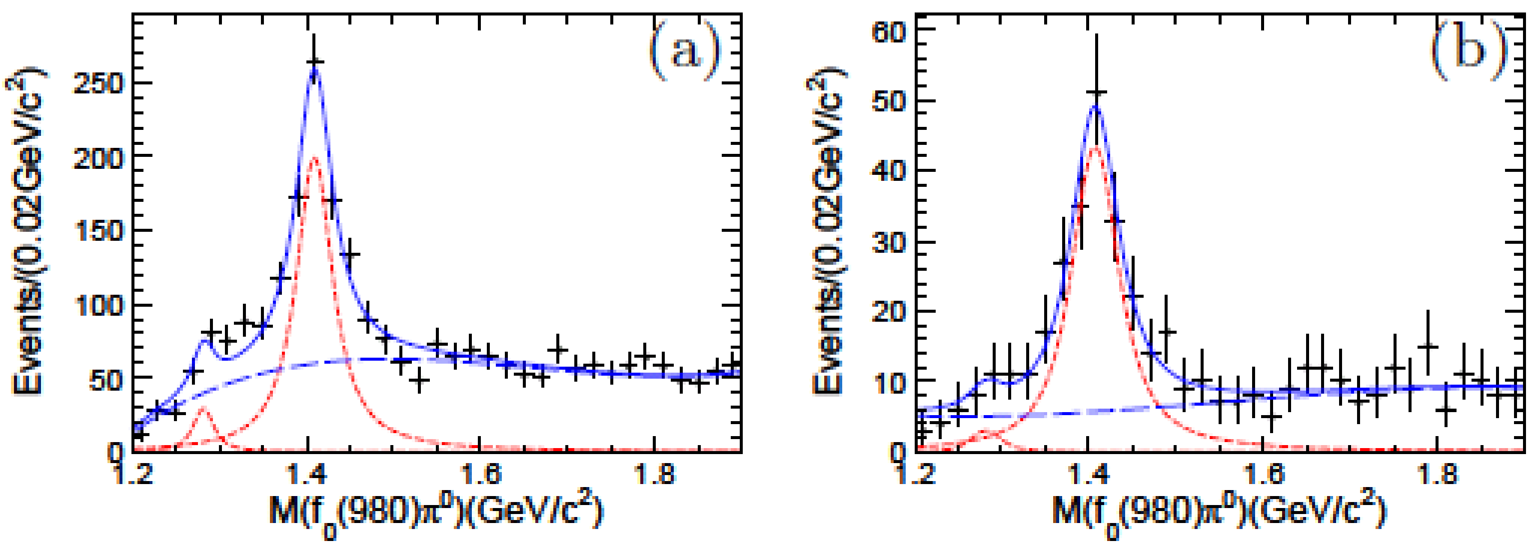
2. The presence of the “triangle singularity”

PRL **108,** 182001 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

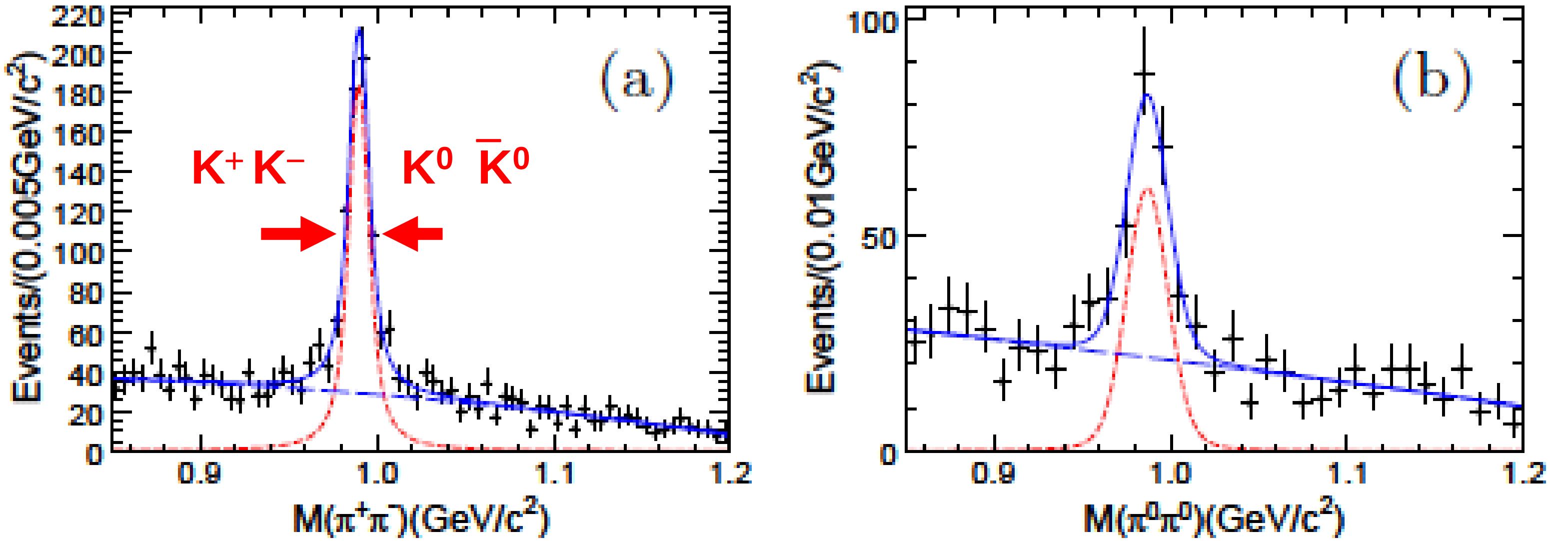
First Observation of $\eta(1405)$ Decays into $f_0(980)\pi^0$

Isospin-violating decay of $J/\psi \rightarrow \gamma\, \eta(1405) \rightarrow \gamma\pi\pi\pi$





BES-III Collaboration, Phys. Rev. Lett. 108, 182001 (2012)



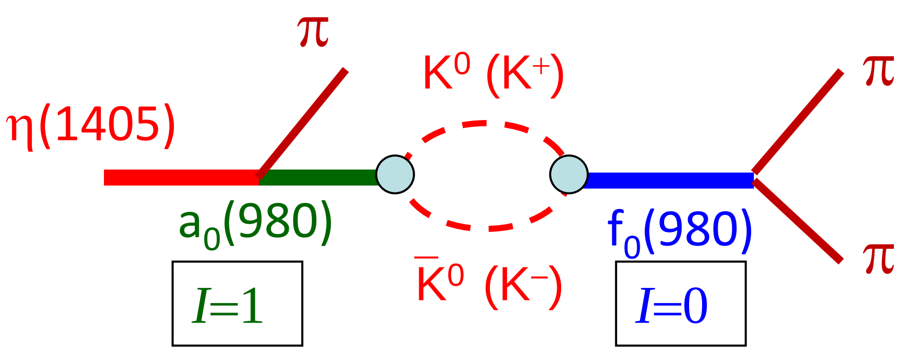
- $f_0(980)$ is extremely narrow: $\Gamma \cong 10$ MeV !

PDG: $\Gamma \cong 40\sim100$ MeV.

- Anomalously large isospin violation!

$$\frac{Br(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\eta(1405) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

"$a_0(980)$-$f_0(980)$ mixing" gives only ~1% isospin violation effects !

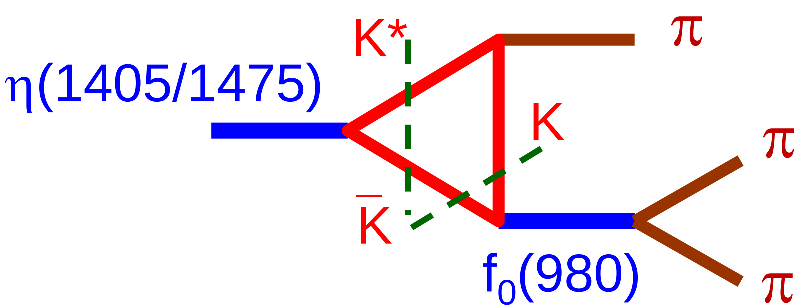


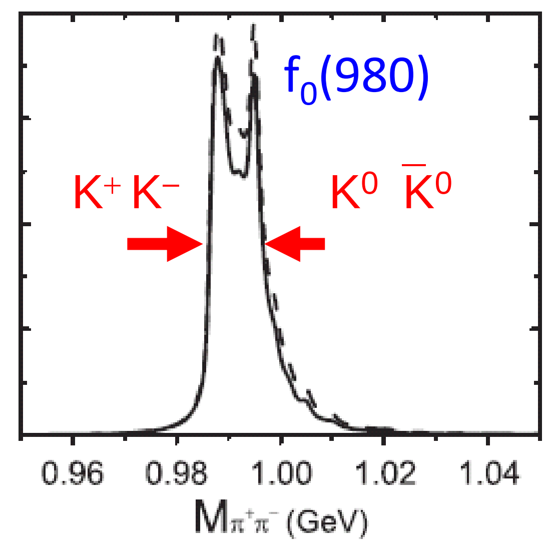
$$g(a_0K^+K^-)\, g(f_0K^+K^-) = -g(a_0K^0\bar{K}^0)\, g(f_0K^0\bar{K}^0)$$

$$M(K^0)-M(K^\pm) = m_d-m_u$$

"Triangle singularity"

Internal $\bar{K}K^*(K)$ approach the on-shell condition simultaneously!





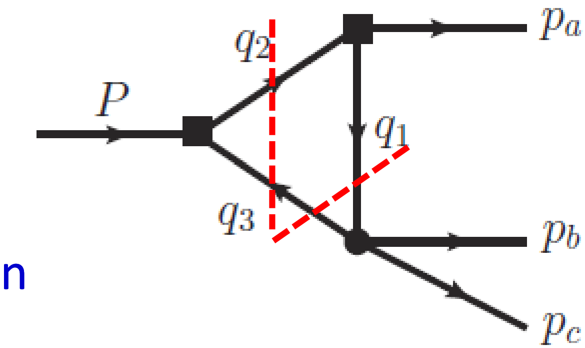
Manifestation of Landau singularity!

J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012);
X.G. Wu, J.-J. Wu, Q. Z., and B.-S. Zou, PRD87, 014023 (2013)

“Exotics” of Type-III:
Peak structures caused by kinematic effects, in particular, by triangle singularity.

$$\Gamma_3(s_1,s_2,s_3) = \frac{1}{i(2\pi)^4}\int \frac{d^4q_1}{(q_1^2-m_1^2+i\epsilon)(q_2^2-m_2^2+i\epsilon)(q_3^2-m_3^2+i\epsilon)}$$
$$= \frac{-1}{16\pi^2}\int_0^1\int_0^1\int_0^1 da_1\, da_2\, da_3\, \frac{\delta(1-a_1-a_2-a_3)}{D-i\epsilon},$$

$$D \equiv \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2}\left[m_i^2+m_j^2-(q_i-q_j)^2\right]$$

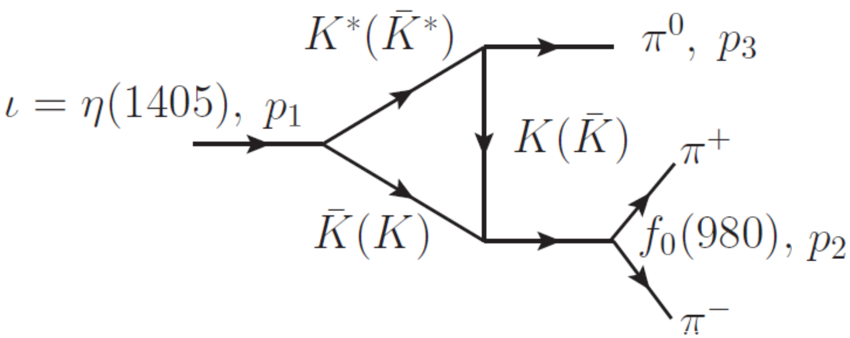


The TS occurs when all the three internal particles can approach their on-shell condition simultaneously:

$\partial D/\partial a_j = 0$ for all j=1,2,3. $\Rightarrow$ $\det[Y_{ij}] = 0$

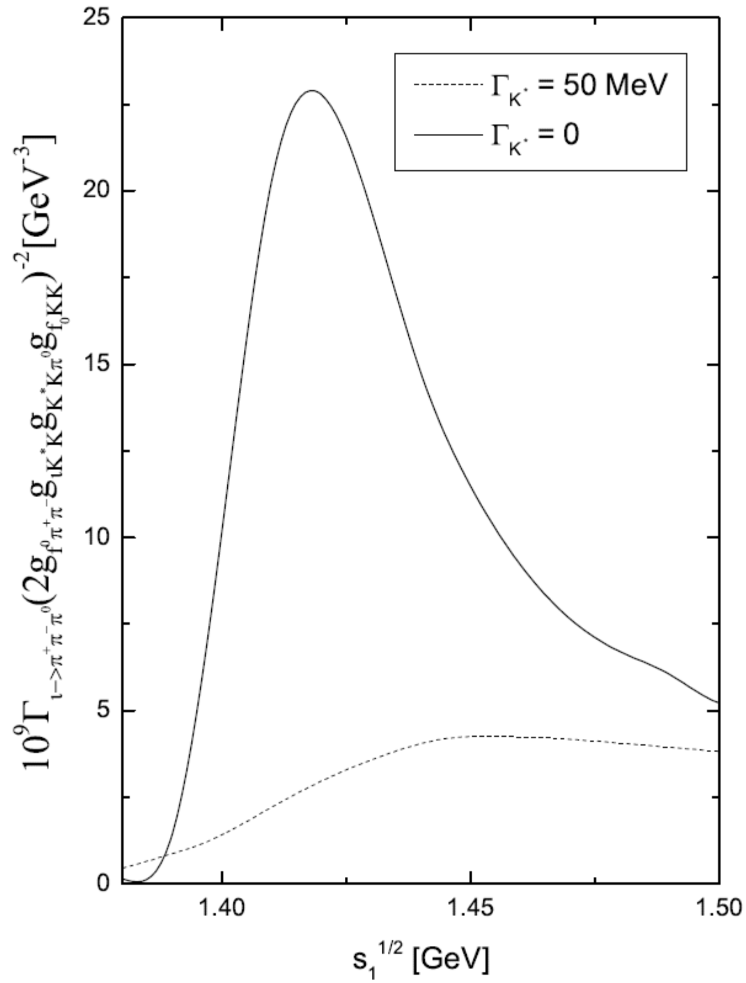
L. D. Landau, Nucl. Phys. 13, 181 (1959);
J.J. Wu, X.-H. Liu, Q. Zhao, B.-S. Zou, Phys. Rev. Lett. 108, 081003 (2012);
Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013); Phys. Lett. B 725, 106 (2013)
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016);
F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv:1705.00141[hep-ph], Rev. Mod. Phys. 90, 015004 (2018); F.K. Guo, X.-H. Liu, S. Sakai, arXiv:1912.07030[hep-ph]

Understanding the width effects from the intermediate K* in η(1405/1475)→ 3π, K ππ, ηππ



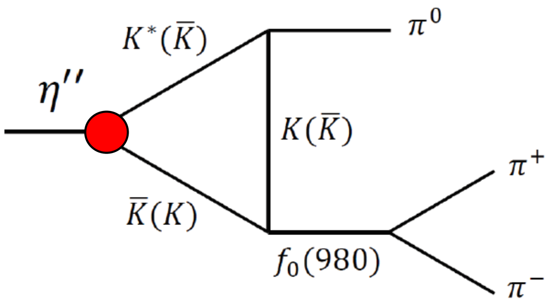
$a_0(980) - f_0(980)$ mixing is required to be enhanced.

However, experimental data do not support large b.r. for $\eta(1405)/\eta(1475) \to a_0(980)\pi$!



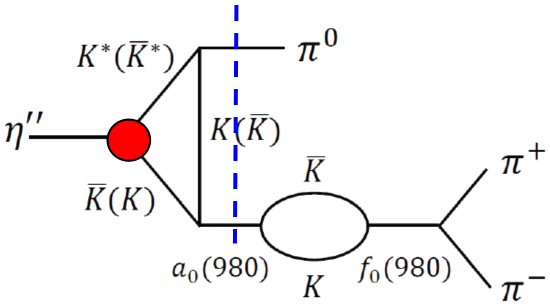
N.N. Achasov et al., PRD92, 036003 (2015)

Updated study of $\eta(1405/1475) \rightarrow 3\pi$, $K\bar{\pi}\pi$, $\eta\pi\pi$ with width effects



(a)

- **Direct isospin breaking via the TS mechanism**

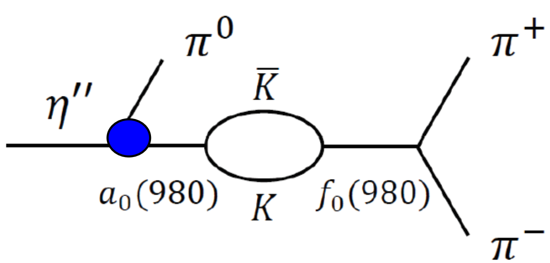


(b)

- **a0-f0 mixing enhanced by the TS mechanism**

-- Unitarized treatment for a0 and f0;

-- To separate (b) and (c) allows a self-contained evaluation of the TS and a0-f0 mixing contributions.



(c)

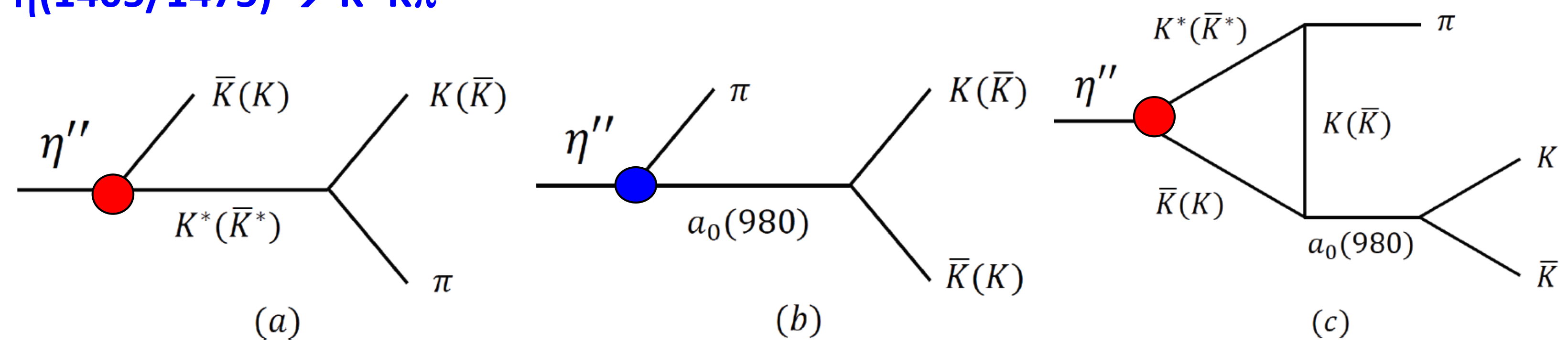
- **a0-f0 mixing at tree level**

M.C. Du and Q.Z., PRD100, 036005 (2019).

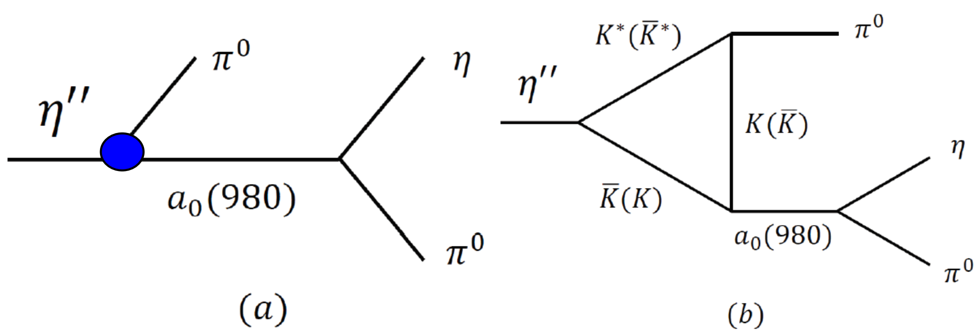
See also N.N. Achasov et al., PRD92, 036003 (2015)

Updated study of η(1405/1475)→ 3π, K ¯ππ, ηππ with width effects

η(1405/1475) → K $\bar{K}\pi$



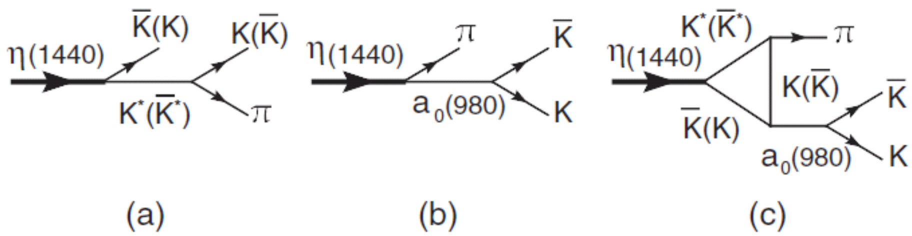
η(1405/1475) → ηππ



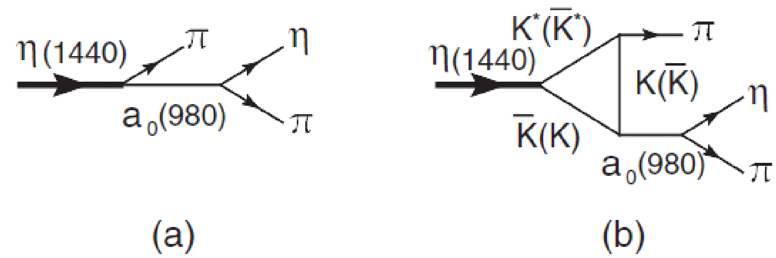
M.C. Du and Q.Z., PRD100, 036005 (2019).

Interferences from the TS mechanism

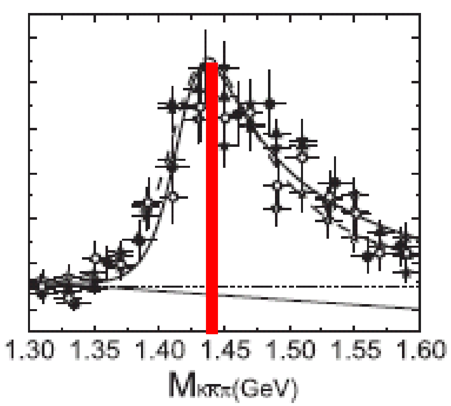
$\eta(1405/1475) \rightarrow K\bar{K}\pi$

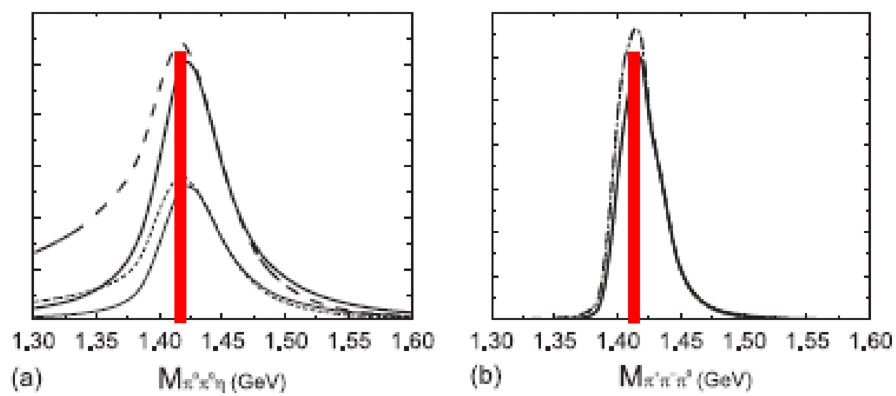


$\eta(1405/1475) \rightarrow \eta\pi\pi$



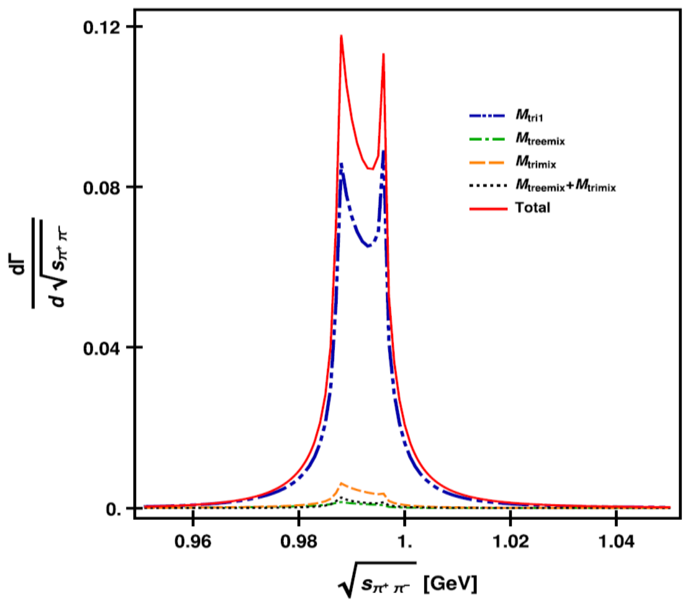
- The "Triangle Singularity" mechanism can shift the peak positions exclusive channels.
- Different lineshapes in difference channels, i.e. $K\bar{K}\pi$, $\eta\pi\pi$, and 3π.
- No obvious need for two independent states, $\eta(1405)$ and $\eta(1475)$!



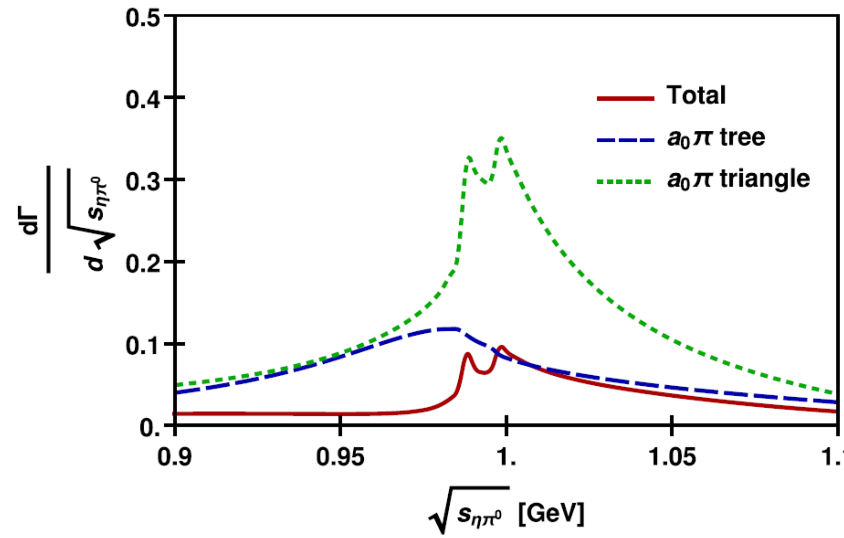


Still the dominance of the TS is present in η(1405/1475) → 3π with the width effects

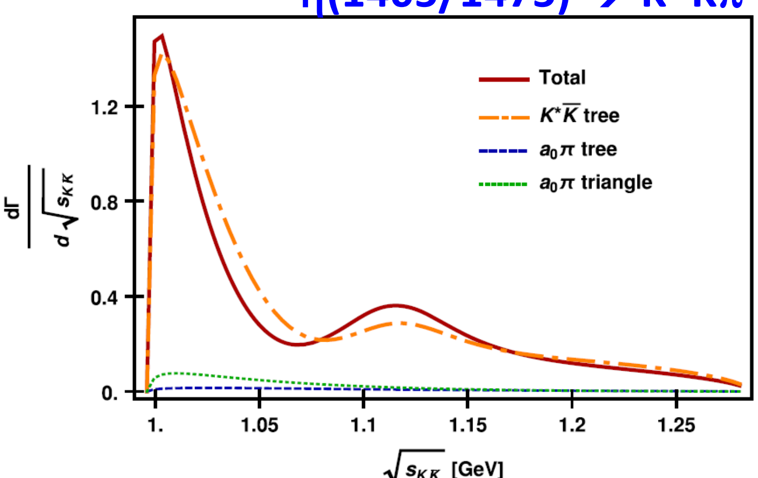
η(1405/1475) → 3π

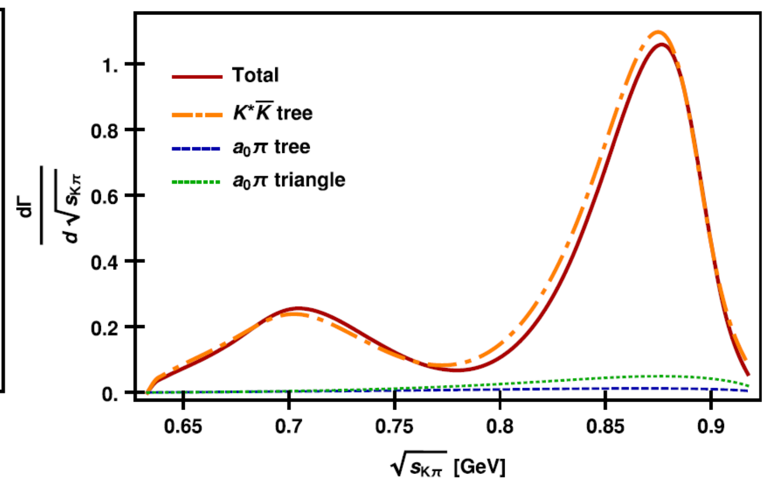


η(1405/1475) → ηππ



η(1405/1475) → K $\bar{K}$π

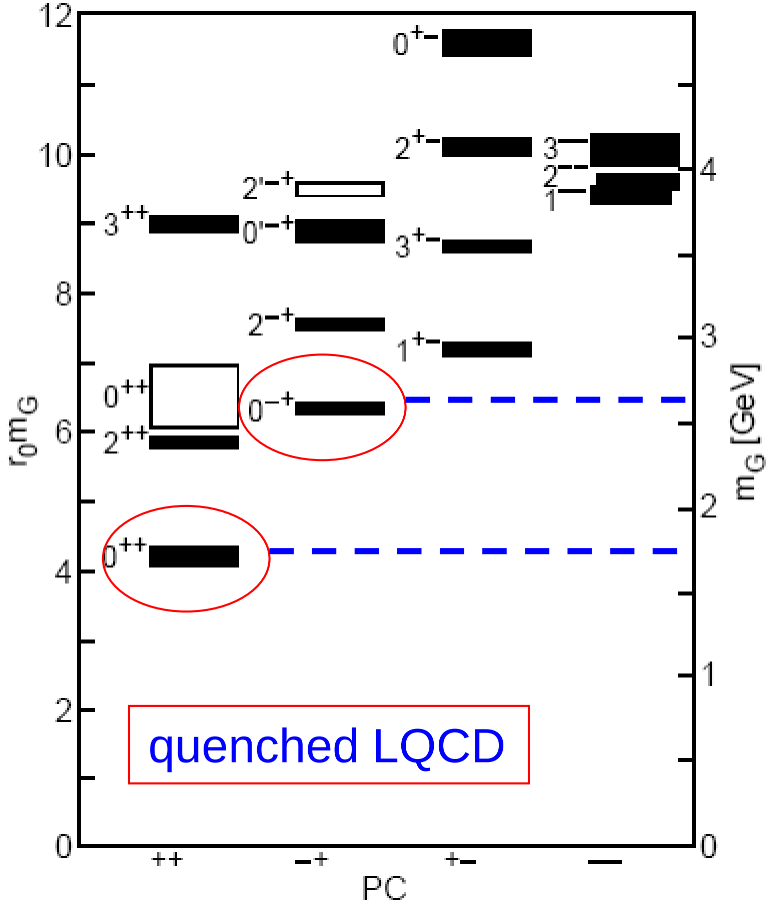


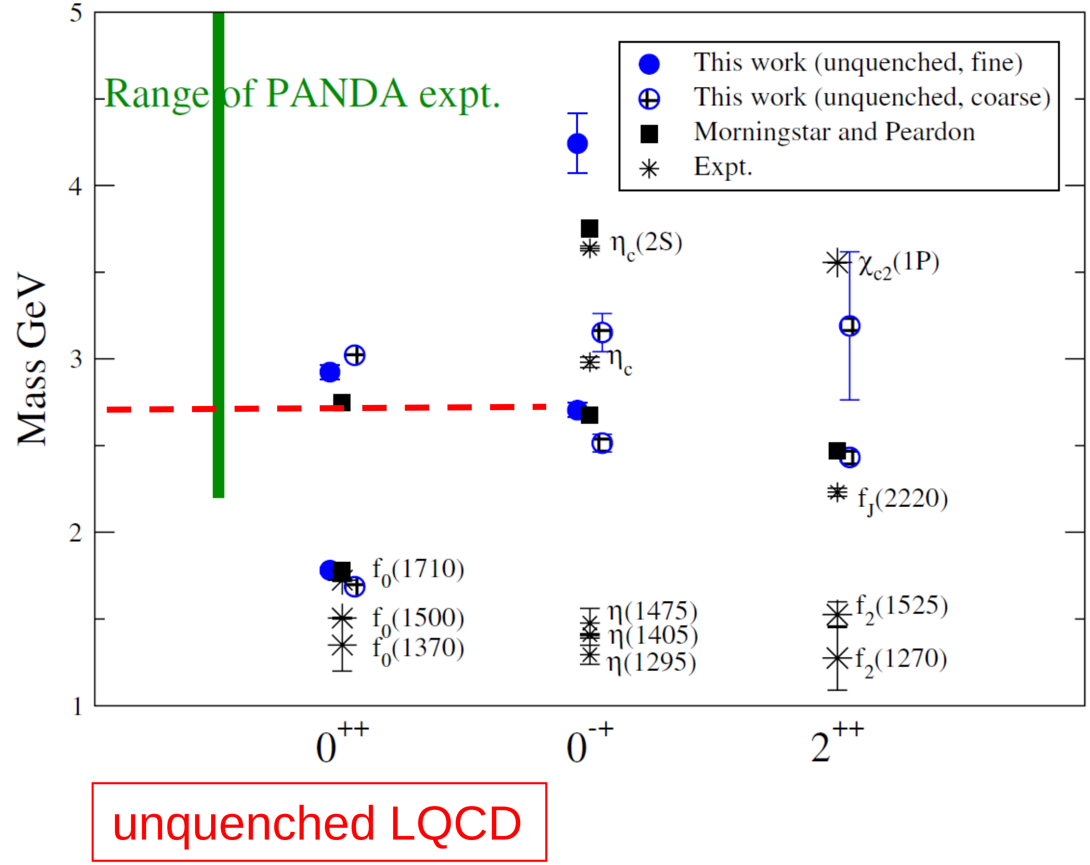


3. Reconsile the the dynamical model calculations with LQCD simulations

A brief status review: Qin, QZ, and Zhong, PRD 97, 096002 (2018)

Stable PG masses from LQCD simulations

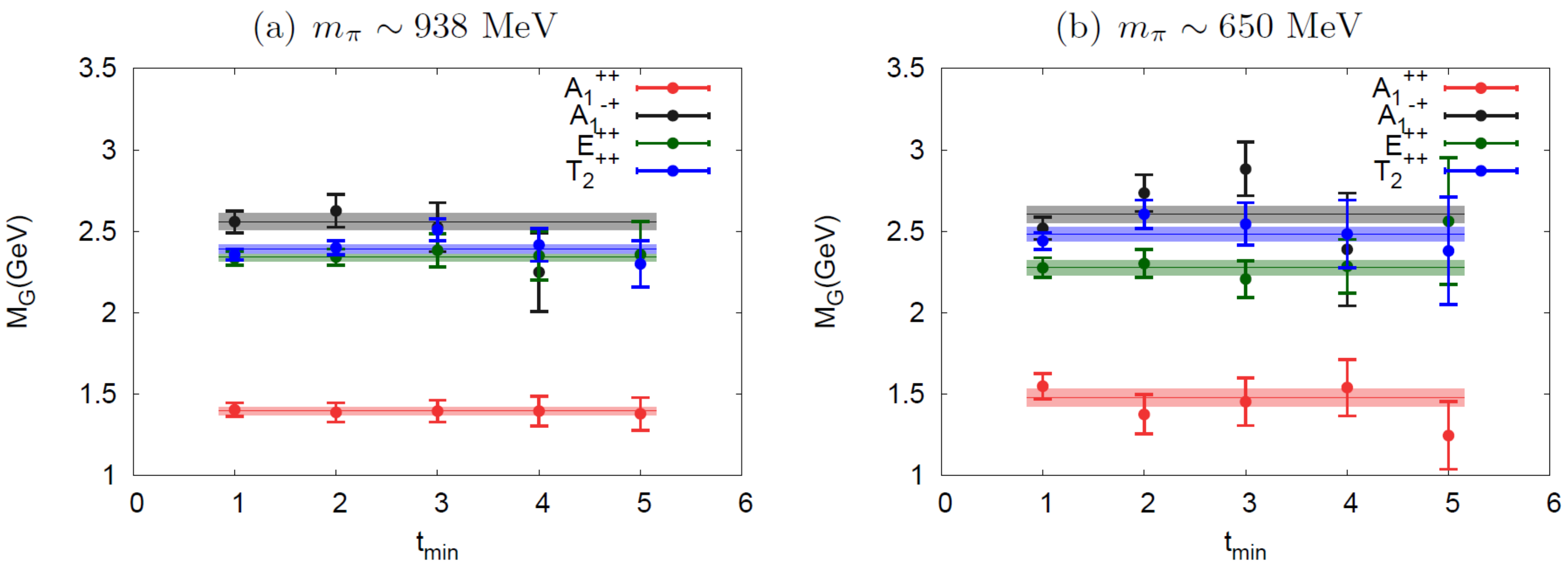




Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)

Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)

N_f = 2 LQCD study on anisotropic lattices



	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$ [13]	360	1795(60)	2620(50)	—
quenched [8]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [9]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

Can mixing bring down the PG mass in a dynamical calc.?

- $\eta(1295)$ and $\eta(1475)$ are the 1st radial excitation between the flavor singlet and octet with I=0.

$$\begin{cases} \eta(1295) & = & \cos\alpha n\bar{n} - \sin\alpha s\bar{s} \\ \eta(1440) & = & \sin\alpha n\bar{n} + \cos\alpha s\bar{s} \end{cases}$$

- $\eta(1405)$ is a pseudoscalar glueball candidate which favors to mix with the ground states $\eta(547)$ and $\eta'(958)$.
- **Caution:** Lattice QCD gives the pseudoscalar glueball mass of ~2.4 GeV.

$$\begin{pmatrix} \eta \\ \eta' \\ \eta'' \end{pmatrix} = U \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}$$

- G. Li, Q. Zhao, C.H. Chang, JPG35, 055002 (2008); hep-ph/0701020
- C. Thomas, JHEP 0710:026, 2007
- R. Escribano, EPJC65, 467 (2010)
- H.Y. Cheng, H.n. Li and K.F. Liu, PRD79, 014024 (2009)
- … …

- One can even include η_c ($\bar{c}c$) in the mixing scheme.

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \\ |\eta_c\rangle \end{pmatrix} = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q)\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix},$$

$M_G \cong 2.4$ GeV
$M\eta_c = 2.98$ GeV

$$U_{34}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_{14}(\phi_G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G & 0 \\ 0 & -\sin\phi_G & \cos\phi_G & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_{12}(\phi_Q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi_Q & \sin\phi_Q \\ 0 & 0 & -\sin\phi_Q & \cos\phi_Q \end{pmatrix}. \quad \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix} = U_{34}(\theta_i)\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

Constraints on the η and η′, but not strongly on a glueball candidate!

Y.-D. Tsai, H.-n. Li and Q.Z., PRD85, 034002 (2011)
Re-investigated in Qin, QZ, and Zhong, PRD 97, 096002 (2018)

Assuming that the decay constants in the flavor basis follow the same mixing pattern of the particle states, we have

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_G^q & f_G^s & f_G^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = U \begin{pmatrix} f_q & 0 & 0 \\ 0 & f_s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_c \end{pmatrix}$$

T. Feldmann, P. Kroll, and B. Stech, PRD 58, 114006 (1998); PLB 449, 339 (1999)

where

$$\begin{aligned} U(\theta, \phi_G, \phi_Q) &= U_{34}(\theta) U_{14}(\phi_G) U_{12}(\phi_Q) U_{34}(\theta_i), \\ &= \begin{pmatrix} c\theta c\theta_i - s\theta c\phi_G s\theta_i & -c\theta s\theta_i - s\theta c\phi_G c\theta_i & -s\theta s\phi_G c\phi_Q & -s\theta s\phi_G s\phi_Q \\ s\theta c\theta_i + c\theta c\phi_G s\theta_i & -s\theta s\theta_i + c\theta c\phi_G c\theta_i & c\theta s\phi_G c\phi_Q & c\theta s\phi_G s\phi_Q \\ -s\phi_G s\theta_i & -s\phi_G c\theta_i & c\phi_G c\phi_Q & c\phi_G s\phi_Q \\ 0 & 0 & -s\phi_Q & c\phi_Q \end{pmatrix} \end{aligned}$$

The axial vector anomaly is given by the $U_A(1)$ Ward identity:

$$\partial^\mu J^j_{\mu 5} = \partial^\mu (\bar{j}\gamma_\mu \gamma_5 j) = 2m_j(\bar{j} i\gamma_5 j) + \frac{\alpha_s}{4\pi} G\tilde{G}$$

The axial vector anomaly can then relate the pseudoscalar meson masses to the flavor singlet pseudoscalar densities and the topological charge density:

$$\langle 0|\partial^\mu J^j_{\mu 5}|P\rangle = M_P^2 f_P^j$$

where $$M_P^2 \equiv \begin{pmatrix} M_\eta^2 & 0 & 0 & 0 \\ 0 & M_{\eta'}^2 & 0 & 0 \\ 0 & 0 & M_G^2 & 0 \\ 0 & 0 & 0 & M_{\eta_c}^2 \end{pmatrix}$$

And $$\mathcal{M}_{qsgc} = U^\dagger M_P^2 U$$ **--- (A)**

Meanwhile, the axial vector anomaly gives:

$$\tilde{\mathcal{M}}_{qsgc} = \begin{pmatrix} m_{qq}^2 + \sqrt{2}G_q/f_q & m_{sq}^2 + G_q/f_s & m_{cq}^2 + G_q/f_c \\ m_{qs}^2 + \sqrt{2}G_s/f_q & m_{ss}^2 + G_s/f_s & m_{cs}^2 + G_s/f_c \\ m_{qg}^2 + \sqrt{2}G_g/f_q & m_{sg}^2 + G_g/f_s & m_{cg}^2 + G_g/f_c \\ m_{qc}^2 + \sqrt{2}G_c/f_q & m_{sc}^2 + G_c/f_s & m_{cc}^2 + G_c/f_c \end{pmatrix}$$ **--- (B)**

The equivalence of Eqs. (A) and (B) gives:

$$U^\dagger \begin{pmatrix} M_\eta^2 & 0 & 0 & 0 \\ 0 & M_{\eta'}^2 & 0 & 0 \\ 0 & 0 & M_G^2 & 0 \\ 0 & 0 & 0 & M_{\eta_c}^2 \end{pmatrix} U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} m_{qq}^2 + \sqrt{2}G_q/f_q & m_{sq}^2 + G_q/f_s & m_{cq}^2 + G_q/f_c \\ m_{qs}^2 + \sqrt{2}G_s/f_q & m_{ss}^2 + G_s/f_s & m_{cs}^2 + G_s/f_c \\ m_{qg}^2 + \sqrt{2}G_g/f_q & m_{sg}^2 + G_g/f_s & m_{cg}^2 + G_g/f_c \\ m_{qc}^2 + \sqrt{2}G_c/f_q & m_{sc}^2 + G_c/f_s & m_{cc}^2 + G_c/f_c \end{pmatrix}$$

with

$$\left\{ \begin{aligned} m_{qq,qs,qg,qc}^2 &\equiv \frac{\sqrt{2}}{f_q} \langle 0 | m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d | \eta_q, \eta_s, g, \eta_Q \rangle \\ m_{sq,ss,sg,sc}^2 &\equiv \frac{2}{f_s} \langle 0 | m_s \bar{s} i\gamma_5 s | \eta_q, \eta_s, g, \eta_Q \rangle, \\ m_{cq,cs,cg,cc}^2 &\equiv \frac{2}{f_c} \langle 0 | m_c \bar{c} i\gamma_5 c | \eta_q, \eta_s, g, \eta_Q \rangle, \\ G_{q,s,g,c} &\equiv \frac{\alpha_s}{4\pi} \langle 0 | G\tilde{G} | \eta_q, \eta_s, g, \eta_Q \rangle. \end{aligned} \right.$$

This allows a relation for the physical glueball mass and the topological charge density in association with the other constrained parameters:

$$\begin{aligned}\tilde{\mathcal{M}}^{31}_{qsgc} &= m^2_{qg} + \sqrt{2} G_g / f_q \\ &= -M^2_\eta (c\theta c\theta_i - s\theta c\phi_G s\theta_i) s\theta s\phi_G c\phi_Q + M^2_{\eta'} (s\theta c\theta_i + c\theta c\phi_G s\theta_i) c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G s\theta_i c\phi_Q,\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{M}}^{32}_{qsgc} &= m^2_{sg} + G_g / f_s \\ &= M^2_\eta (c\theta s\theta_i + s\theta c\phi_G c\theta_i) s\theta s\phi_G c\phi_Q + M^2_{\eta'} (-s\theta s\theta_i + c\theta c\phi_G c\theta_i) c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G c\theta_i c\phi_Q.\end{aligned}$$

$$\Rightarrow \quad \hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}^{31}_{qsgc}}{\tilde{\mathcal{M}}^{32}_{qsgc}} = \frac{m^2_{qg} + \sqrt{2} G_g / f_q}{m^2_{sg} + G_g / f_s} = \hat{R}_{41/42}$$

$$\Rightarrow \quad \begin{aligned} M^2_G &= -\frac{1}{\cos\phi_G \sin\theta_i \cos\phi_Q} \left\{ \frac{\sqrt{2} G_g / f_q}{\sin\phi_G} - [-M^2_\eta (\cos\theta \cos\theta_i - \sin\theta \cos\phi_G \sin\theta_i) \sin\theta \cos\phi_Q \right. \\ &\quad \left. + M^2_{\eta'} (\sin\theta \cos\theta_i + \cos\theta \cos\phi_G \sin\theta_i) \cos\theta \cos\phi_Q] \right\}. \\ &\approx -\frac{1}{\sin\theta_i} \left\{ \frac{\sqrt{2} G_g / f_q}{\sin\phi_G} - M^2_{\eta'} \sin\theta_i - (M^2_{\eta'} - M^2_\eta) \sin\theta \cos(\theta + \theta_i) \right\} \end{aligned}$$

With the LQCD results for the topological charge density, we can fit the parameters:

TABLE I. The numerical values of all the parameters with $G_g = -0.054$ GeV^3 and $\phi_G = 12°$ fixed. The two quantities, m_{qc}^{2*} and m_{sc}^{2*} involve more complicated issues and are sensitive to m_{cc}^2 and ϕ_G. Further detailed discussions can be found in the context.

f_s/f_q	M_G (GeV)	m_{qq}^2 (GeV)^2	m_{ss}^2	m_{sg}^2	m_{cg}^2	m_{qc}^{2*}	m_{sc}^{2*}	m_{cq}^2	m_{cs}^2	G_q (GeV)^3	G_s	G_c
1.2	2.1	0.055	0.45	−0.041	−0.81	0.87	0.50	−0.24	−0.15	0.060	0.035	−0.092
1.3	2.1	0.0012	0.47	−0.067	−0.81	0.87	0.46	−0.25	−0.15	0.065	0.035	−0.092

where we have applied the condition: $\boxed{m_{qs,sq}^2 \ll m_{qg}^2 \ll m_{qq}^2}$

Note: $m_{qg}^2 \ll m_{sg}^2$

$$\hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}_{qsgc}^{31}}{\tilde{\mathcal{M}}_{qsgc}^{32}} = \frac{m_{qg}^2 + \sqrt{2}G_g/f_q}{m_{sg}^2 + G_g/f_s}$$

If $m_{qg}^2 \sim m_{sg}^2 \ll G_g/f_q \sim G_g/f_s$ ⇨ $\hat{R}_{31/32} \simeq \sqrt{2}f_s/f_q$

⇨ $M_G \sim 1.4$ GeV !

Inappropriate approx. made in

H.-Y. Cheng, H.-n. Li and K.-F. Liu, PRD79, 014024 (2009)

Y.-D. Tsai, H.-n. Li and Q.Z., PRD85, 034002 (2011)

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f_s/f_q	M_G (GeV)	m_{qq}^2 (GeV)^2	m_{ss}^2	m_{sg}^2	m_{cg}^2	m_{qc}^{2*}	m_{sc}^{2*}	m_{cq}^2	m_{cs}^2	G_q (GeV)^3	G_s	G_c
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However, the approximation does not hold for $\hat{R}_{41/42}$.

$$\hat{R}_{41/42} = \frac{m_{qc}^2 + \sqrt{2}G_c/f_q}{m_{sc}^2 + G_c/f_s} \neq \sqrt{2}f_s/f_q$$

Inappropriate approx. made in

H.-Y. Cheng, H.-n. Li and K.-F. Liu, PRD79, 014024 (2009)

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f_s/f_q	M_G (GeV)	m^2_{qq} (GeV)^2	m^2_{ss}	m^2_{sg}	m^2_{cg}	m^{2*}_{qc}	m^{2*}_{sc}	m^2_{cq}	m^2_{cs}	G_q (GeV)^3	G_s	G_c
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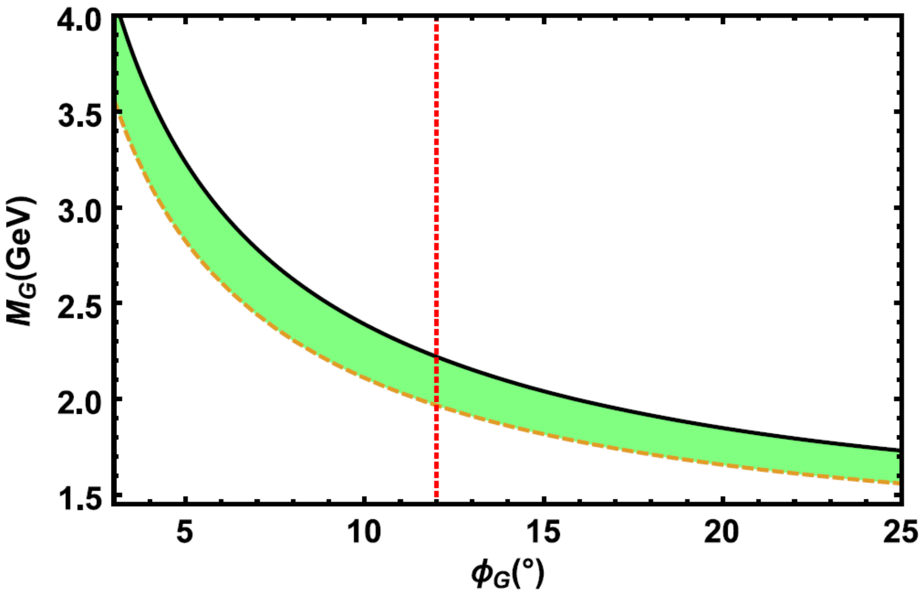
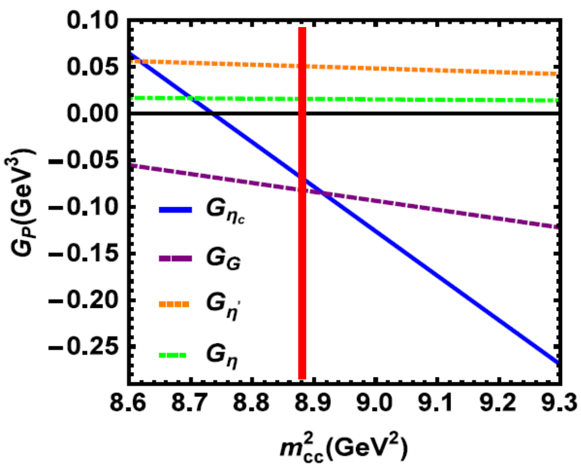


$$\hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}^{31}_{qsgc}}{\tilde{\mathcal{M}}^{32}_{qsgc}} = \frac{m^2_{qg} + \sqrt{2}G_g/f_q}{m^2_{sg} + G_g/f_s}$$

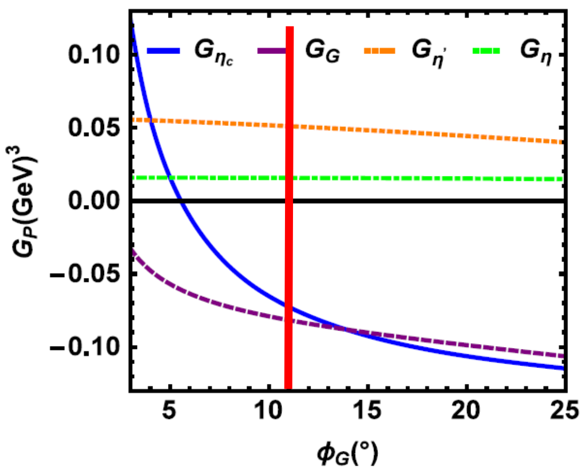
FIG. 1. The physical glueball mass M_G varies with $\phi_G \in$ (3–25)°, with $\theta = -11°$, $\phi_Q = 11.6°$, and $f_q = 131$ MeV.

The dependence of G_P on m_{cc}^2, ϕ_G, and ϕ_Q

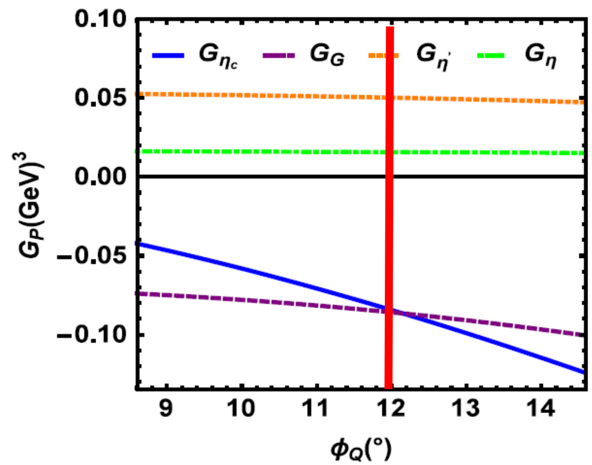
$\phi_G = 12°$ and $\phi_Q = 11.6°$



$m_{cc}^2 = M_{\eta_c}^2$, and $\phi_Q = 11.6°$



$\phi_G = 12°$ and $m_{cc}^2 = M_{\eta_c}^2$



The topological susceptibility can be extracted for the pseudoscalar mesons:

$$\left\{\begin{aligned} \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle &= 0.016\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle &= 0.051\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle &= -0.084\ \mathrm{GeV}^3, \\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_c\rangle &= -0.079\ \mathrm{GeV}^3, \end{aligned}\right.$$

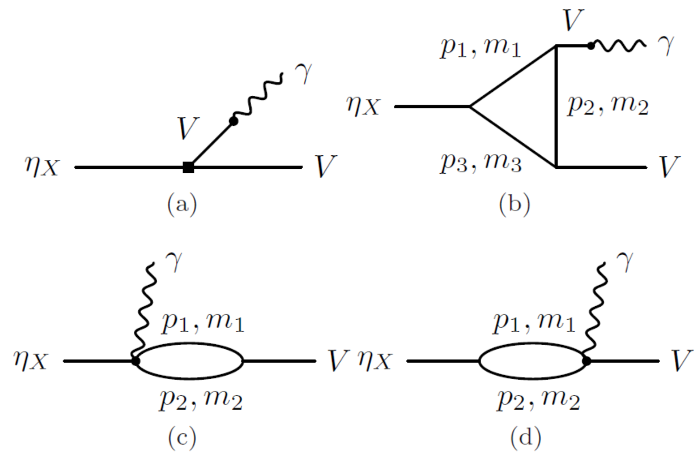
LQCD results:

$$\left\{\begin{aligned} &\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle \approx 0.021\ \mathrm{GeV}^3 \\ &\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle \approx 0.035\ \mathrm{GeV}^3 \\ &\boxed{G_g = -(0.054 \pm 0.008)\ \mathrm{GeV}^3} \end{aligned}\right.$$

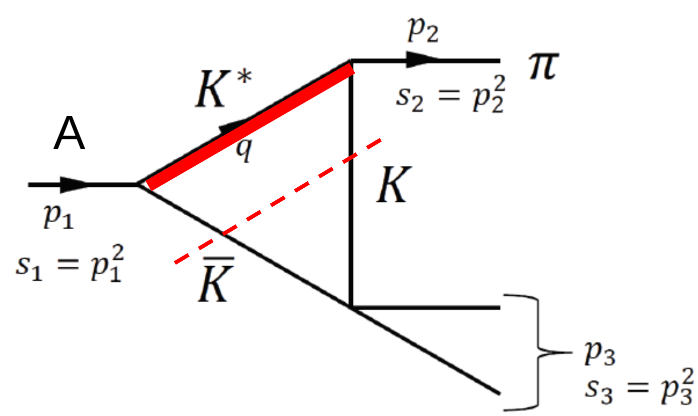
- **Low mass pseudoscalar glueball is unlikely to be favored!**
- **Similar conclusion from V. Vento et al.**

4. Further evidence for the TS mechanism in pseudoscalar meson radiative decays

- **Radiative decays of $\eta(1295)$ and $\eta(1405/1475)$ into γV**
  [Y. Cheng and Q. Zhao, arXiv:2106.12483v1 [hep-ph]]



- **Further evidence for the TS mechanism in axial vector meson decays**
  [M.-C. Du and Q. Zhao, PRD 100, 036005 (2019)]



5. Brief summary

I) We have to alter our view of the pseudoscalar spectrum dramatically even for the 1st radial excitations!

- **The $\eta(1405)$ puzzle is originated from the triangle singularity mechanism.**
- **The dynamical calculations of the PG mass are consistent with the LQCD expectations if an inappropriate approx. is corrected.**
- **The evidence of the TS mechanism also exits in the light axial vector meson spectra.**

(See a brief status review: Qin, Zhao, and Zhong, PRD 97, 096002 (2018) and Du and Zhao, PRD100, 036005 (2019))

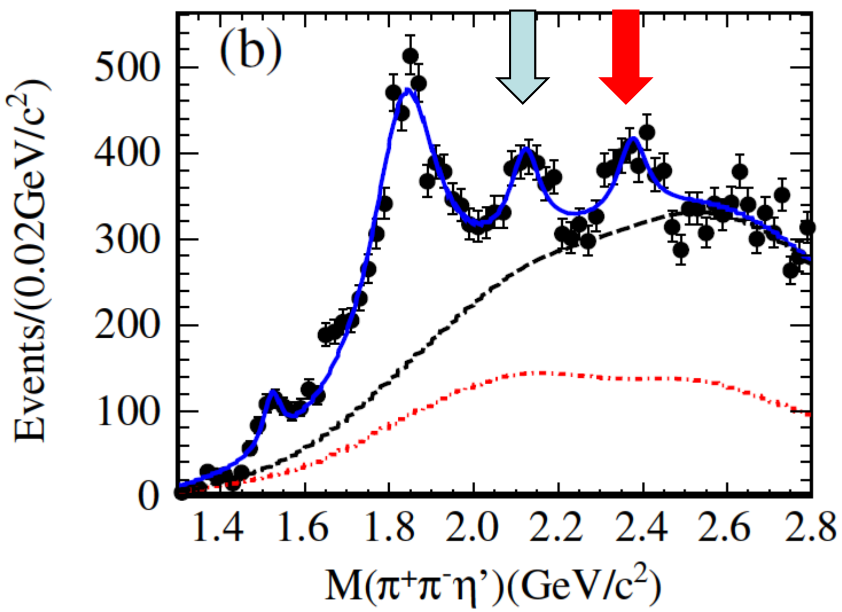
5. Brief summary

II) Where to look for the pseudoscalar glueball candidate?

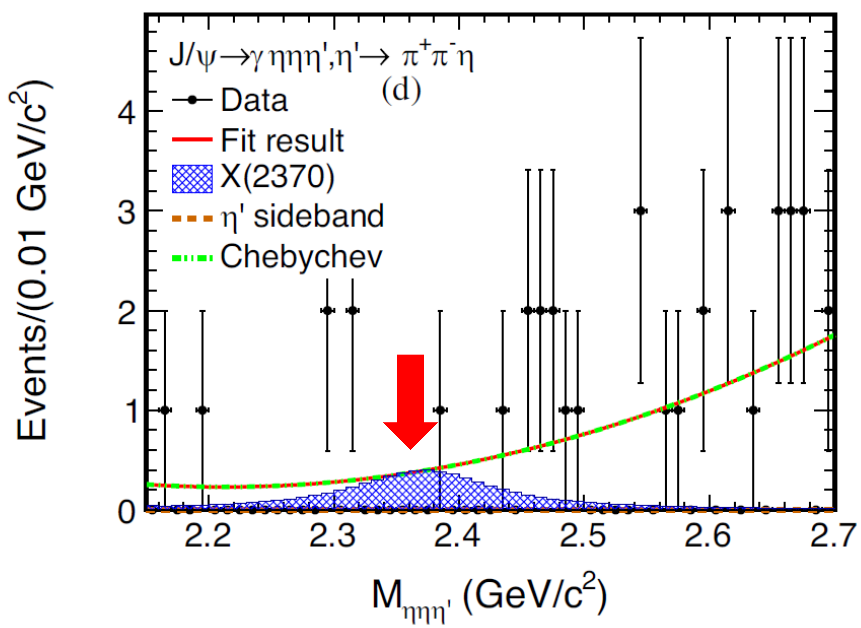
Isoscalar pseudoscalars with higher masses above 2 GeV, e.g.

X(2120), X(2370) …

BESIII, PRL 106, 072002 (2011)



BESIII, PRD103, 012009 (2021)



[1] W.I. Eshraim et al., PRD 87, 054036 (2013)

[2] H.-N. Li, arXiv:2109.04956v1 [hep-ph]

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Thanks for your attention!