Recent investigations for the beam-beam instability

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Overview

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Head on collision

Nowadays, high-luminosity e+ e colliding rings are being considered seriously. Small bunch spacing is useful because collisions occur more frequently. This causes the problem of parasitic collisions: Bunches may interact with each other not only at the interaction point (IP) but also at points around the IP.

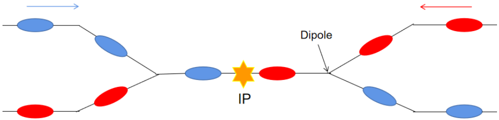


图: Head-on collision versus collision with crossing angle

Collison with a crossing angle

These can be avoided by collision with a crossing angle. It has been noticed that increasing Piwinski angle($\theta_P = \frac{\sigma_z}{\sigma_x}\theta_c$) may help to increase luminosity[1]. However, only with a large Piwinski angle, there exist very strong transverse nonlinear resonances coming from beam-beam interaction.

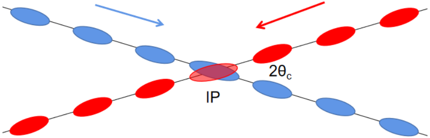


图: Collision with a crossing angle

[1] **Ruggiero**, **F. and Zimmermann**, **F.PhysRevSTAB.5.061001**.

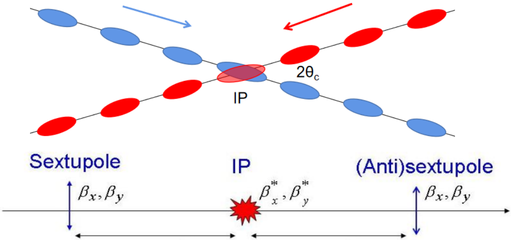


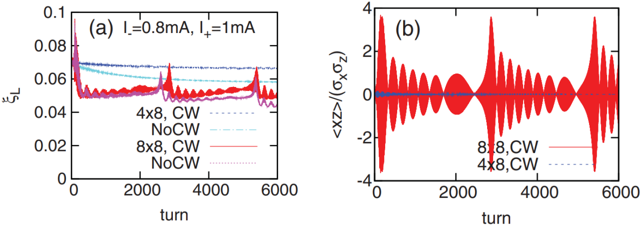
图: Collision with a crossing angle

The crab-waist scheme is proposed to suppress the resonances by placing sextupole before and after interaction point (IP) with proper phase advances, which also help increase the beam-beam parameter. The crab waist collision scheme has been successfully tested to achieve a high luminosity[2]. Several future colliders would adopt this collision scheme.

[2] **M. Zobov et al., Phys. Rev. Lett. 104, 174801 (2010).**.

Coherent head-tail instability (X-Z instability)

With a large Piwinski angle, the horizontal beam-beam parameter is normally very low ($\xi_x < 0.01$). Usually, it is believed that the horizontal oscillation of colliding bunches would be very stable. However, during the study of FCC-ee in 2016, the simulations[3,4] show that there exists a coherent head-tail instability (X-Z instability) in collision with large Piwinski angle. The instability is observed when the strong-strong model is used.



Strong-strong simulations[5] shows a correlated coherent head-tail instability (x-z instability) in case of large crossing angle. The crab waist is not a primary reason for the instability, but it has an effect on the luminosity.

[3] K.Ohmi, Beambeam and electron cloud effects in CEPC/FCC-ee.

[4] K.Ohmi, Int. J. Mod. Phys. A, 31, 1644014 (2016).

[5] K.Ohmi et al., Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle, PRL (2017).

Effects of longitudinal impedance

The "cross-wake force" induced by beam-beam interaction has been introduced and a mode-coupling theory based on the localized wake force has been developed to explain the instability[6]. However, simulations[7] have further revealed that the longitudinal impedance strongly impacts the beam stability in which the horizontal stable tune areas are squeezed seriously. The above mentioned mode-coupling theory could not include the effects of longitudinal impedance.

Therefore, we try to develop a new transverse mode-coupling analysis method to study the beam-beam instability with and without longitudinal impedance.

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[6] **K.Ohmi et al.**, **Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle**, **PRL (2017)**.

[7] **Y.Zhang et al.**, **PhysRevAccelBeams.23.104402**.

Overview

Coherent instability with a large Piwinski angle

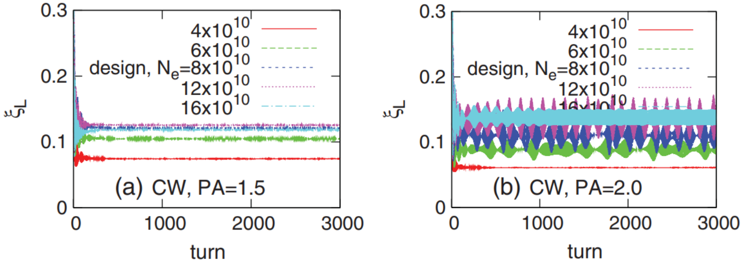


图: The evolutions of the beam-beam parameter of FCC-ee-H for PA is 1.5 and 2

Simulations[8] show that the coherent X-Z instability occurs in large Piwinski angle (PA$=\frac{\sigma_z}{\sigma_x}\theta_c$). In conventional e^+e^- storage ring colliders, $PA < 1$. For CEPC-Z CDR, $PA \approx 23$.

[8] **K.Ohmi et al.**, **Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle**, **PRL (2017)**.

Lorentz transformation

The Lorentz transformation[9] transforms the collision with an angle to a head-on collision between bunches tilted horizontally.

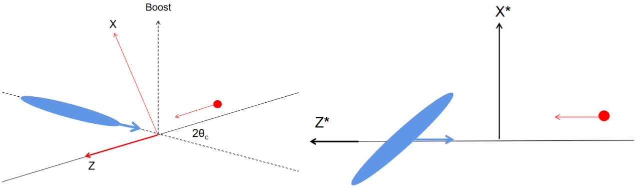


图: Original frame versus Lorentz boosted frame

Thanks to the Lorentz boosted frame, we could analyze beam-beam interaction just like in a head-on frame. it is relatively easy

[9] **K.Hirata**, **Analysis of Beam-Beam Interactions with a Large Crossing Angle**, **PRL**, **(1995)**.

Cross-wake force induced by beam-beam interaction

The cross-wake force is defined as follows[10]:

$$\begin{aligned}\Delta p_x^{(-)} &\equiv \delta p_x^{(-)}\left(x_- - x_+ - \Delta x\right) - \delta p_x^{(-)}\left(x_- - x_+\right) \\ &= -W^{(-)}\left(z - z'\right)\rho^{(+)}\Delta x_+ \delta z\end{aligned} \tag{1}$$

where

$$W_x^{(-)}\left(z - z'\right) = -\frac{N^{(+)} r_e}{\gamma^{(-)}} \frac{\partial F_x}{\partial x}\bigg|_{x=(z-z')\theta_c} \tag{2}$$

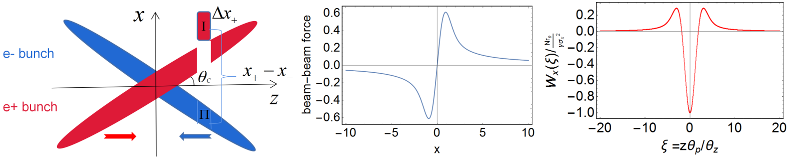


图: Illustration of the evaluation of cross wake force (left). Beam-beam force on a single test particle produced by a round beam (middle). Cross wake force $W_x(z)$ (right).

The total beam-beam force can be expressed as a form of ordinary transverse wake force:

$$\Delta p_x^{(\pm)}(z) = -\int_{-\infty}^{\infty} W_x^{(\pm)}\left(z - z'\right)\rho_x^{(\mp)}\left(z'\right) dz' \tag{3}$$

where $\rho_x(z) = \rho(z)\cdot x(z)$

[10] **K.Ohmi et al., Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle, PRL (2017).**

σ mode vs. π mode

Assuming the bunch dipole moment distribution to be σ mode $\rho_x^{(+)}(z) = \rho_x^{(-)}(z)$ and π mode $\rho_x^{(+)}(z) = -\rho_x^{(-)}(z)$, the beam-beam momentum kick can be expressed by usual formula of a normal wake force

$$\Delta p_x(z) = \mp \int_{-\infty}^{\infty} W_x^{(\pm)}\left(z - z'\right) \rho_x\left(z'\right) dz' \tag{4}$$

Choosing the sign of the wake force, the behavior of the σ/π mode can be studied as exhibiting transverse single-bunch instability.

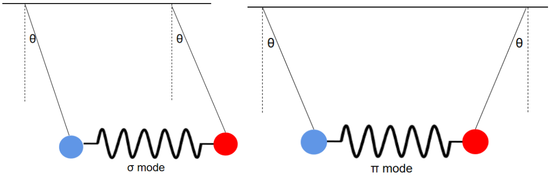


图: Two coupled oscillators with normal modes

Eigenvalues problem

$$\Delta p_x(z) = \mp \int_{-\infty}^{\infty} W_x^{(\pm)}\left(z - z'\right) \rho \cdot x\left(z'\right) dz'$$

Note that $\Delta p_x(z)$ is linear depedent on betatron oscillation x. We are able to rewrite the beam-beam kick in a matrix formalism. The stability of the colliding ring is determined by the eigenvaluec $\lambda's$ of the one-turn transfer matrix M_{total}.

Parameters		
Circumference	$L(km)$	50
Synchrotron tune	ν_s	0.014
Beta function at IP	$\beta_x^*(m)$	0.15
Beam energy	$E(GeV)$	45.5
Horizontal emittance	$\epsilon_x(nm)$	0.18
Half crossing angle	$\theta_c(mrad)$	16.5
rms bunch length	$\sigma_z(mm)$	7.35
Energy spread	$\sigma_\delta(10^{-3})$	1.055
Bunch population	$N_b(10^{10})$	8

The CEPC-Z parameters[11] are used in the following presentation.

[11] **CEPC Study Group**, **CEPC conceptual design report: Volume 1Accelerator**, **arXiv:1809.00285.**.

Analysis without impedance

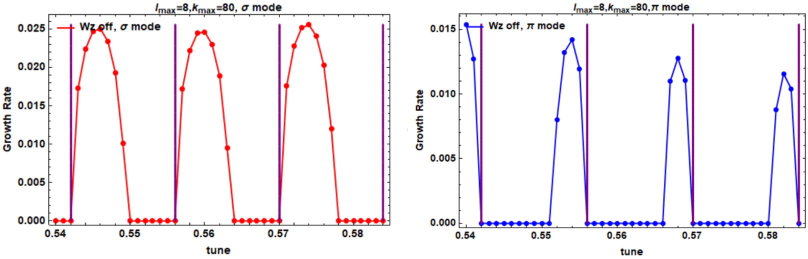


图: Growth rate versus horizontal tune without impedance. The growth rate is defined as the largest $log|\lambda|$. The red is σ mode, and the blue is π mode.

There are stable horizontal tunes for σ/π mode when we do not consider the longidudinal impedance. The stale tune areas are separated by $\nu_s = 0.014$, which is the nominal synchrotron tune.

Overview

Wakefield and longitudinal impedance

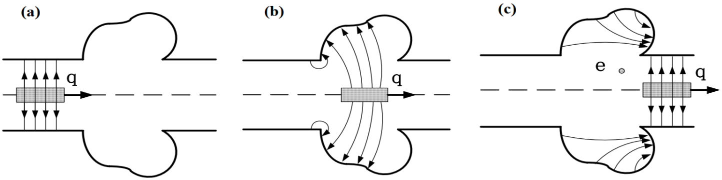


图: Wake field generation diagram: (a) before the beam passes through the vacuum chamber discontinuity, (b) the beam is passing through the discontinuity, and (c) after the beam passes through the discontinuity.

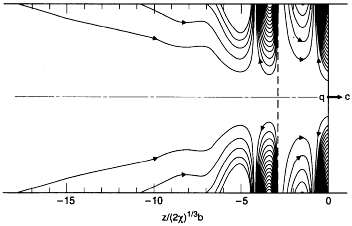
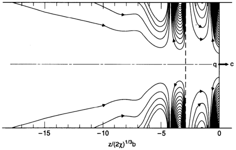


图: Wake electric field lines in a resistive wall pipe generated by a point charge q. The field pattern shows oscillatory behavior.

Wakefield and longitudinal impedance



The descriptions of the wake force in terms of wake functions $W'_m(z)$ in the time domain,

$$\int_{-L/2}^{L/2} dsF_{\|} = -eI_m W'_m(z) r^m \cos m\theta, \tag{5}$$

and in terms of impedances $Z_m^{\|}(\omega)$ in the frequency domain,

$$Z_m^{\|}(\omega) = \int_{-\infty}^{\infty} \frac{dz}{c} e^{-i\omega z/c} W'_m(z). \tag{6}$$

Collective beam instability occurs only when the beam does not move at the speed of light and the beam pipe is not a smooth and complete conductor.

Simulation procedures

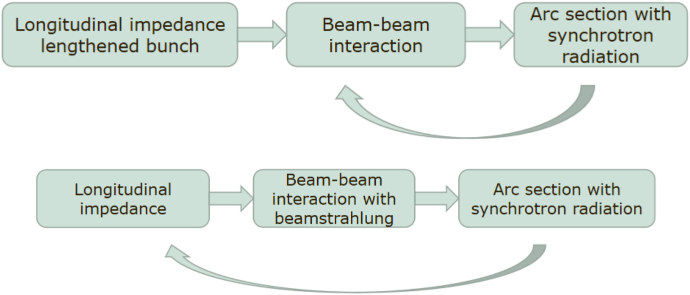
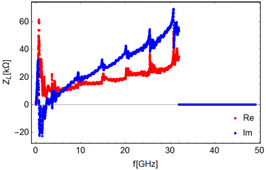


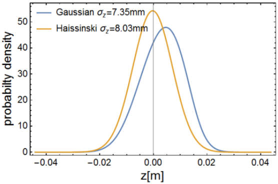
图: Simulation proedures: conventional v.s. new

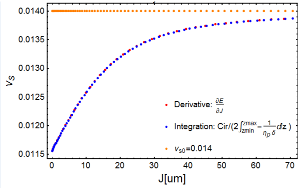
In conventional e^+e^- storage ring colliders, we usually use natural bunch length or impedance lengthened bunch length in beam-beam simulation instead of considering impedance directly. In the future colliders, the beam-beam interaction becomes essentially three dimensional. With extreme parameters, such as very intense beams, low emittances, small beta functions, large Piwinski angle, and so on, several new effects become significant in the collider performance, for example beamstrahlung, coherent X-Z instability, so that the longitudinal beam dynamics should be also treated in a self-consistent manner.

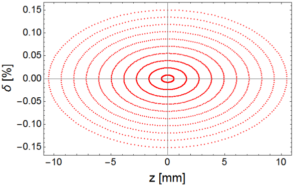
Effects of longitudinal impedance

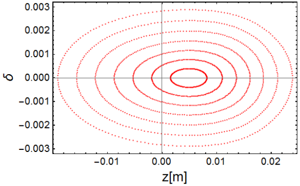
The longitudinal impedance lengthens the bunch, deform the longitudinal distribution, produce incoherent synchrotron tune shift and distort the longitudinal trajectory.



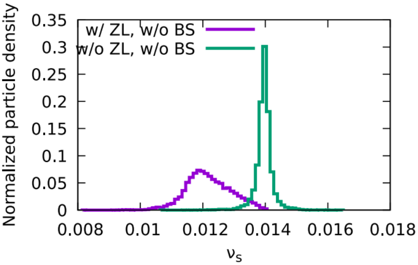








Simulation results



The stability of horizontal motion is sensitive to longitudinal dynamics. The figure[12] shows the collision stability versus horizontal tune with and without longitudinal impedance. There exist some apparent changes when the impedance is considered: (1) the shift of stable tune area; (2) the squeeze of stable tune area; (3) decrease of growth rate. It seems that the width of stable working point is too narrow for the considered configurations. It is necessary to re-optimize the machine parameters and to reduce further the impedance budget.

[12] **Y.Zhang et al.**, **Self-consistent simulations of beam-beam interaction in future e+e- circular colliders including beamstrahlung and longitudinal coupling impedance**, **PRAB (2020)**.

Mode coupling analytical method

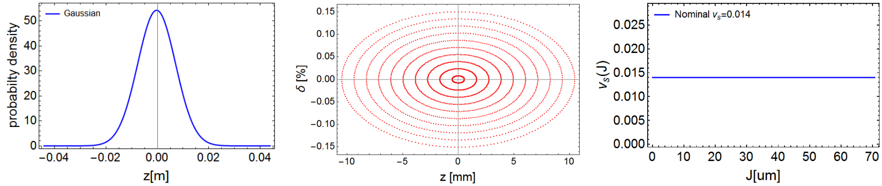


图: Longitudinal beam dynmics without longitudinal impedance

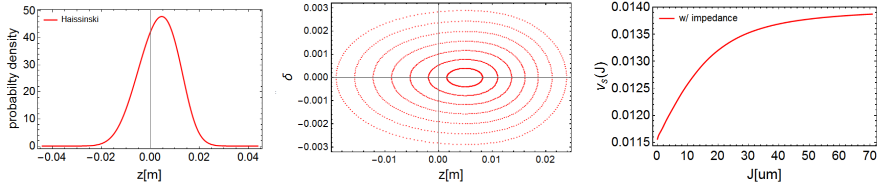


图: Longitudinal beam dynmics with longitudinal impedance

In conventional anlytical method, the longtidudinal beam parameters used in analysis are the ones which are unperturbed. It is natural to use the parameters perturbed by the longitudinal impedance.

Cross-check without impedance

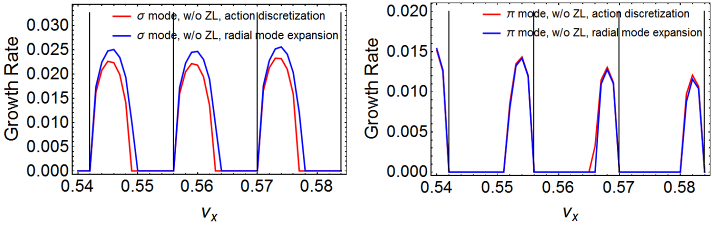


图: Growth rate as a function of tune ν_x without impedance.

Above plots are obtained by our action discretization method, and below plots are obtained by the conventional radial mode expansion method. The agreement between the two methods is satisfying.

Analysis with impedance

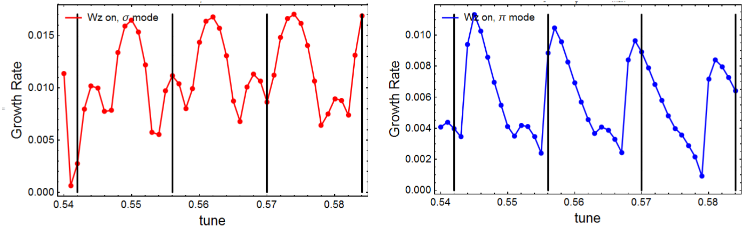


图: Growth rate as a function of tune ν_x considering the longitudinal wake. Red is for σ mode, and blue is for π mode.

Comparing to the results without impedance, the gap $\Delta\nu$ between two neighboring peaks is reduced from 0.01. The original stable areas turn unstable for both σ mode and π mode when we consider the influence of impedance

Overview

Summary

1. Beam beam collision with a large Piwinski angle is strongly dependent on the longitudinal beam dynamics.
2. Simulation has found a new instability induced by the coupling of beam-beam interaction and longitudinal impedance.
3. A new analytical mehtod has been developed to explain this instability.
4. This mehtod can be used to study transverse instability induced by general wakefields.

The parameters used in the presentation are CDR parameters. New parameters has been updated.

This work is submitted to PRAB.