Light tetraquark states with the exotic quantum number $J^{PC} = 3^{-+}$

Niu Su (苏妞)

Southeast University

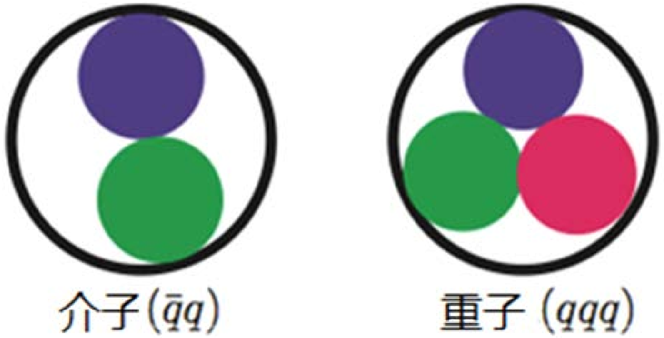
Collaborators: Rui-Rui Dong, Hua-Xing Chen, Wei Chen, Er-Liang Cui

BESIII轻强子物理研讨会

Contents

Introduction

- **Traditional quark model**

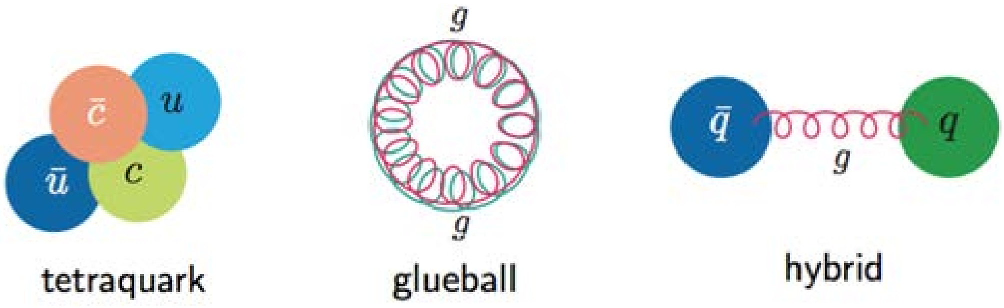


- **BESIII Collaboration are carefully examining the physics happening in the energy region around 2.0GeV.**

BESIII, Phys. Rev. Lett. 124, 112001 (2020)
BESIII, Phys. Rev. Lett. 117, 042002 (2016)
BESIII, Phys. Rev. Lett. 115, 091803(2015)
BESIII, Phys. Rev. Lett. 91, 022001 (2003)
……

Introduction

- **Exotic hadron: tetraquark, hybrid state, glueball, etc.**



……

- **Exotic spin-parity quantum numbers**

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+} \quad ……$$

Introduction

- **The hybrid states of** $J^{PC}=1^{-+}$ Prog. Part. Nucl. Phys. 82,21 (2015).

- **The light tetraquark of** $J^{PC}=1^{-+}$ $J^{PC}=0^{--}$ $J^{PC}=0^{+-}$

  Phys. Rev. D 78,054017 (2008). Phys. Rev. D 78,117502 (2008).
  Phys. Rev. D 79,114034 (2009). Phys. Rev. D 95,076017 (2017).
  New Phys. Sae Mulli 70, 836 (2020). Chin. Phys. C 37, 033104 (2013).

- **We will investigate the** $sq\overline{sq}$ **tetraquark state with the exotic quantum number** $J^{PC}=3^{-+}$ **using the method of QCD sum rules.**

Introduction

- **There has not been any theoretical study on the $sq\bar{s}\bar{q}$ tetraquark state with the exotic quantum number $J^{PC}=3^{-+}$.**

- **Wei Zhu, Yan-Rui Liu, Tao Yao used the one-boson-exchange model to study $D^{*}\bar{D}_{2}^{*}$ the molecular state of $J^{PC}=3^{-+}$.**

  Chin. Phys. C 39,023101(2015)

- **There was a Lattice QCD study on the $J^{PC}=3^{-+}$ glueball .**

  High Energy Phys. Nucl. Phys. 8, 573 (1984)

Contents

QCD sum rules

- In QCD sum rule analyses, we consider **two-point correlation functions**:

$$\Pi_{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}(q^2) \equiv i\int d^4x e^{iqx}\langle 0|\mathbf{T}[\eta_{\alpha_1\alpha_2\alpha_3}(x)\eta^\dagger_{\beta_1\beta_2\beta_3}(0)]|0\rangle = (-1)^J \mathcal{S}'[\tilde{g}_{\alpha_1\beta_1}\tilde{g}_{\alpha_2\beta_2}\tilde{g}_{\alpha_3\beta_3}]\Pi(q^2),$$

where η is the current which can couple to **hadronic states**.

- At the **hadron level**: described by the **dispersion relation**.

$$\Pi(q^2) = \int_{4m_s^2}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds,$$

- At the **quark-gluon level**: evaluated via **operator product expansion(OPE).**

QCD sum rules

SVZ sum rule (Shifman et al 1979)

Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{OPE}(q^2) \xrightarrow[s\,=\,-q^2]{\text{dispersion relation}} \rho_{OPE}(s) = a_n s^n + a_{n-1} s^{n-1}$$

Quark-Hadron Duality

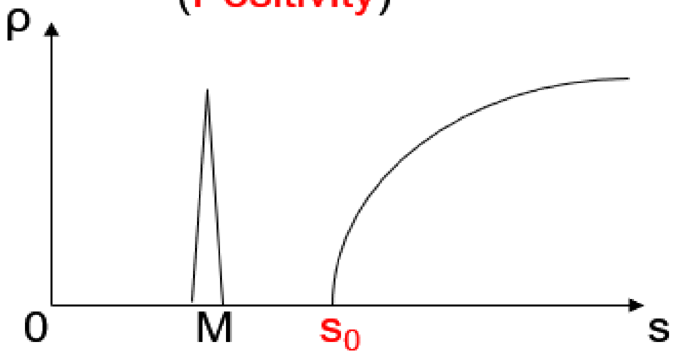
Hadron Level

$$\Pi_{phys}(q^2) = f_P^2 \frac{\not{q} + M}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \cdots$$

(Positivity)

(for baryon case)

(Sufficient amount of Pole contribution)



QCD sum rules

- Perform the **Borel transform** to correlation function at both hadron and quark-gluon levels.

$$\Pi(s_0, M_B^2) \equiv f_X^2 e^{-M_X^2/M_B^2} = \int_{4m_s^2}^{s_0} e^{-s/M_B^2} \rho(s) ds.$$

- **Two parameters**： M_B、s_0

- **Criteria**：

  1. Positivity of spectral density
  2. Convergence of OPE
  3. Sufficient amount of pole contribution
  4. The dependence of mass on parameters M_B、s_0

Contents

- **Introduction**
- **Method of the QCD sum rules**
- **Numerical analyses**
- **Decay behavior**
- **More studies on light tetraquark states**

Numerical analyses

- **Interpolating currents**

Diquark-antidiquark currents

$$\eta = [q_a^T C\Gamma_1 \overleftrightarrow{D}_\alpha s_b](\bar{q}_c \Gamma_2 C \bar{s}_d^T),$$

$$\eta' = (q_a^T C\Gamma_1 s_b)[\bar{q}_c \Gamma_2 C \overleftrightarrow{D}_\alpha \bar{s}_d^T],$$

$$\eta'' = [(q_a^T C\Gamma_3 s_b)\overleftrightarrow{D}_\alpha(\bar{q}_c \Gamma_4 C \bar{s}_d^T)],$$

$$\eta^1_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times \mathcal{S}\Big\{[q_a^T C\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\gamma_{\alpha_2} C\bar{s}_d^T) + (q_a^T C\gamma_{\alpha_1} s_b)[\bar{q}_c\gamma_{\alpha_2} C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

$$\eta^2_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times \mathcal{S}\Big\{[q_a^T C\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\gamma_{\alpha_2} C\bar{s}_d^T) + (q_a^T C\gamma_{\alpha_1} s_b)[\bar{q}_c\gamma_{\alpha_2} C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

$$\eta^3_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times \mathcal{S}\Big\{[q_a^T C\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\gamma_{\alpha_2}\gamma_5 C\bar{s}_d^T) + (q_a^T C\gamma_{\alpha_1}\gamma_5 s_b)[\bar{q}_c\gamma_{\alpha_2}\gamma_5 C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

$$\eta^4_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times \mathcal{S}\Big\{[q_a^T C\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\gamma_{\alpha_2}\gamma_5 C\bar{s}_d^T) + (q_a^T C\gamma_{\alpha_1}\gamma_5 s_b)[\bar{q}_c\gamma_{\alpha_2}\gamma_5 C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

$$\eta^5_{\alpha_1\alpha_2\alpha_3} = \epsilon^{abe}\epsilon^{cde} \times g^{\mu\nu}\mathcal{S}\Big\{[q_a^T C\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\sigma_{\alpha_2\nu} C\bar{s}_d^T) + (q_a^T C\sigma_{\alpha_1\mu} s_b)[\bar{q}_c\sigma_{\alpha_2\nu} C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

$$\eta^6_{\alpha_1\alpha_2\alpha_3} = (\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) \times g^{\mu\nu}\mathcal{S}\Big\{[q_a^T C\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3} s_b](\bar{q}_c\sigma_{\alpha_2\nu} C\bar{s}_d^T) + (q_a^T C\sigma_{\alpha_1\mu} s_b)[\bar{q}_c\sigma_{\alpha_2\nu} C\overleftrightarrow{D}_{\alpha_3}\bar{s}_d^T]\Big\},$$

Numerical analyses

- **Interpolating currents**

Color-singlet-color-singlet currents

$$\xi^1_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{(\bar{q}_a\gamma_{\alpha_1}q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\gamma_{\alpha_2}s_b)\Big\}\,,$$

$$\xi^2_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{(\bar{q}_a\gamma_{\alpha_1}\gamma_5q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\gamma_{\alpha_2}\gamma_5s_b)\Big\}\,,$$

$$\xi^3_{\alpha_1\alpha_2\alpha_3} = g^{\mu\nu}\mathcal{S}\Big\{(\bar{q}_a\sigma_{\alpha_1\mu}q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\sigma_{\alpha_2\nu}s_b)\Big\}\,,$$

$$\xi = [\bar{q}q]\overleftrightarrow{D}[\bar{s}s]$$

$$\xi^4_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{[\bar{q}_a\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3}s_a](\bar{s}_b\gamma_{\alpha_2}q_b) - (\bar{q}_a\gamma_{\alpha_1}s_a)[\bar{s}_b\gamma_{\alpha_2}\overleftrightarrow{D}_{\alpha_3}q_b]\Big\}\,,$$

$$\xi^5_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{[\bar{q}_a\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3}s_a](\bar{s}_b\gamma_{\alpha_2}\gamma_5q_b) - (\bar{q}_a\gamma_{\alpha_1}\gamma_5s_a)[\bar{s}_b\gamma_{\alpha_2}\gamma_5\overleftrightarrow{D}_{\alpha_3}q_b]\Big\}\,,$$

$$\xi^6_{\alpha_1\alpha_2\alpha_3} = g^{\mu\nu}\mathcal{S}\Big\{[\bar{q}_a\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3}s_a](\bar{s}_b\sigma_{\alpha_2\nu}q_b) - (\bar{q}_a\sigma_{\alpha_1\mu}s_a)[\bar{s}_b\sigma_{\alpha_2\nu}\overleftrightarrow{D}_{\alpha_3}q_b]\Big\}\,.$$

$$\xi' = [\bar{q}\overleftrightarrow{D}s][\bar{s}q] - [\bar{q}s][\bar{s}\overleftrightarrow{D}q]$$

Numerical analyses

- **Interpolating currents**

Fierz transformation

$$\begin{pmatrix} \eta^1_{\alpha_1\alpha_2\alpha_3} \\ \eta^2_{\alpha_1\alpha_2\alpha_3} \\ \eta^3_{\alpha_1\alpha_2\alpha_3} \\ \eta^4_{\alpha_1\alpha_2\alpha_3} \\ \eta^5_{\alpha_1\alpha_2\alpha_3} \\ \eta^6_{\alpha_1\alpha_2\alpha_3} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 & -1 & 0 \end{pmatrix} \begin{pmatrix} \xi^1_{\alpha_1\alpha_2\alpha_3} \\ \xi^2_{\alpha_1\alpha_2\alpha_3} \\ \xi^3_{\alpha_1\alpha_2\alpha_3} \\ \xi^4_{\alpha_1\alpha_2\alpha_3} \\ \xi^5_{\alpha_1\alpha_2\alpha_3} \\ \xi^6_{\alpha_1\alpha_2\alpha_3} \end{pmatrix}$$

Numerical analyses

- **The OPE of current** $\eta^{1}_{\alpha_1\alpha_2\alpha_3}$:

$$
\begin{aligned}
\Pi_{11} \;=\; & \int_{4m_s^2}^{s_0}\Bigg[\frac{s^5}{691200\pi^6}-\frac{m_s^2 s^4}{14336\pi^6}+\Big(-\frac{179\langle g_s^2GG\rangle}{5806080\pi^6}+\frac{m_s^4}{2016\pi^6}-\frac{m_s\langle\bar{q}q\rangle}{720\pi^4}+\frac{m_s\langle\bar{s}s\rangle}{1512\pi^4}\Big)s^3 \\
&+\Big(\frac{37\langle g_s^2GG\rangle m_s^2}{122880\pi^6}-\frac{91m_s\langle g_s\bar{q}\sigma Gq\rangle}{30720\pi^4}+\frac{m_s^3\langle\bar{q}q\rangle}{80\pi^4}-\frac{m_s^3\langle\bar{s}s\rangle}{240\pi^4}+\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{60\pi^2}\Big)s^2 \\
&+\Big(-\frac{\langle g_s^2GG\rangle m_s^4}{18432\pi^6}+\frac{3m_s^3\langle g_s\bar{q}\sigma Gq\rangle}{256\pi^4}+\frac{5\langle g_s^2GG\rangle m_s\langle\bar{q}q\rangle}{3456\pi^4}-\frac{7\langle g_s^2GG\rangle m_s\langle\bar{s}s\rangle}{8640\pi^4}+\frac{5\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{288\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{12\pi^2} \\
&+\frac{5\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{288\pi^2}\Big)s+\Big(\frac{\langle g_s^2GG\rangle m_s\langle g_s\bar{q}\sigma Gq\rangle}{4608\pi^4}-\frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{12\pi^2}+\frac{\langle g_s^2GG\rangle m_s^3\langle\bar{s}s\rangle}{13824\pi^4}-\frac{3m_s^2\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{128\pi^2} \\
&-\frac{\langle g_s^2GG\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{324\pi^2}+\frac{17\langle g_s\bar{q}\sigma Gq\rangle\langle g_s\bar{s}\sigma Gs\rangle}{3456\pi^2}-\frac{m_s^2\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{576\pi^2}\Big)\Bigg]e^{-s/M^2}ds \\
&+\Big(-\frac{m_s^2\langle g_s\bar{q}\sigma Gq\rangle^2}{24\pi^2}+\frac{2m_s\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{9}\Big)\,,
\end{aligned}
$$

Numerical analyses

- **Positivity of spectral density**

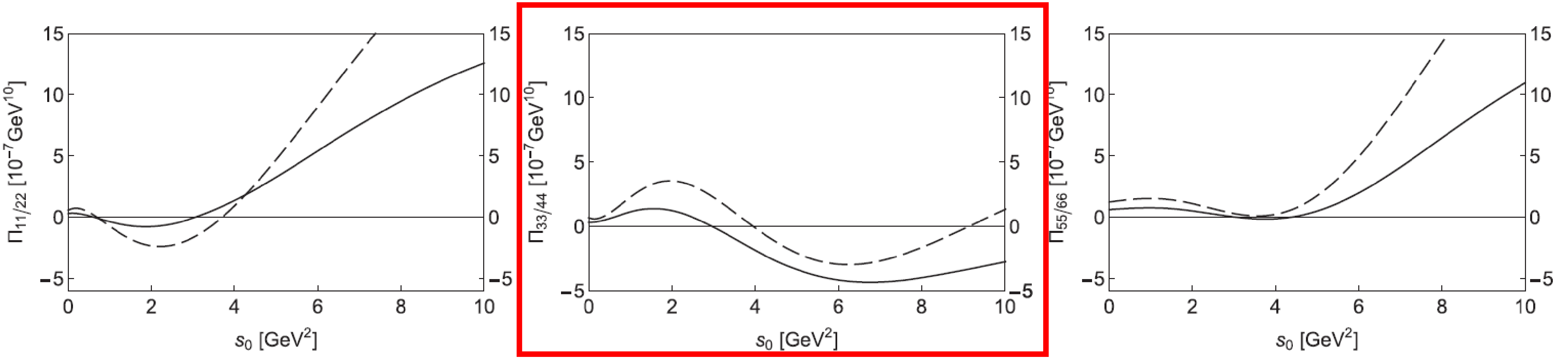


FIG. 1. The two-point correlation functions, $\Pi_{11}(s_0, M_B^2)$ (left-solid), $\Pi_{22}(s_0, M_B^2)$ (left-dashed), $\Pi_{33}(s_0, M_B^2)$ (middle-solid), $\Pi_{44}(s_0, M_B^2)$ (middle-dashed), $\Pi_{55}(s_0, M_B^2)$ (right-solid), and $\Pi_{66}(s_0, M_B^2)$ (right-dashed), as functions of the threshold value s_0. These curves are obtained by setting $M_B^2 = 1.4$ GeV^2.

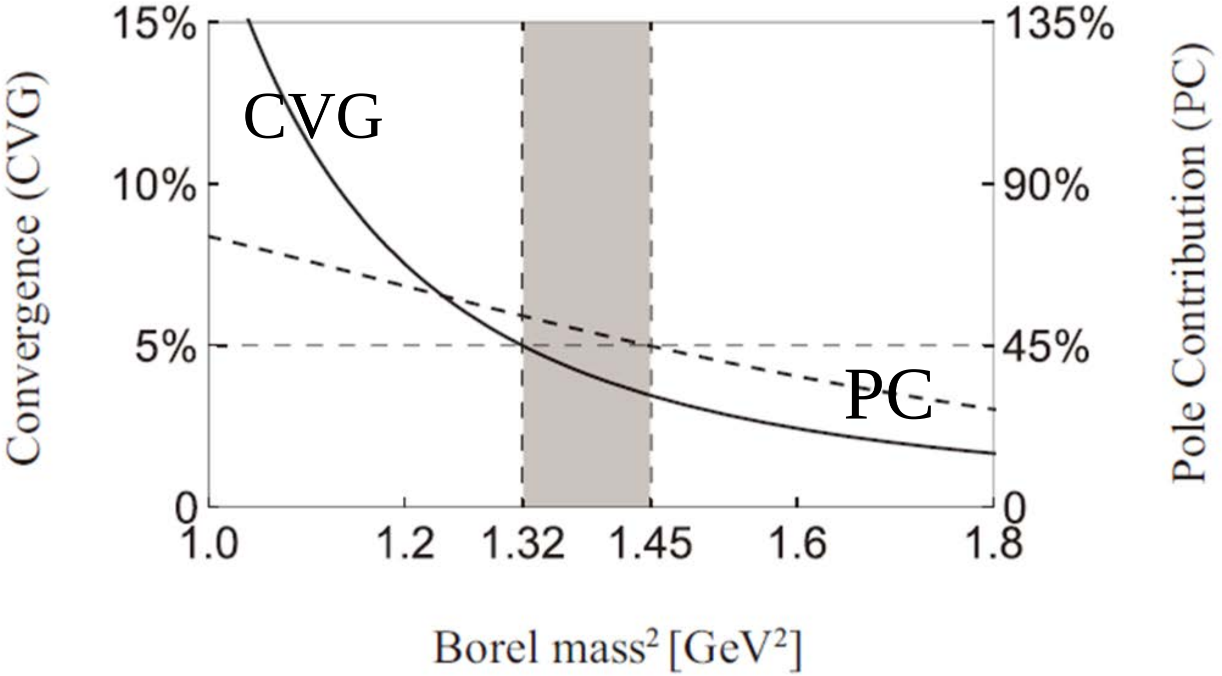
Numerical analyses

- **Convergence of OPE**

$$\mathrm{CVG} \equiv \left| \frac{\Pi_{11}^{D=10}(s_0, M_B^2)}{\Pi_{11}(s_0, M_B^2)} \right| \leq 5\% \,.$$

- **Sufficient amout of pole contribution**

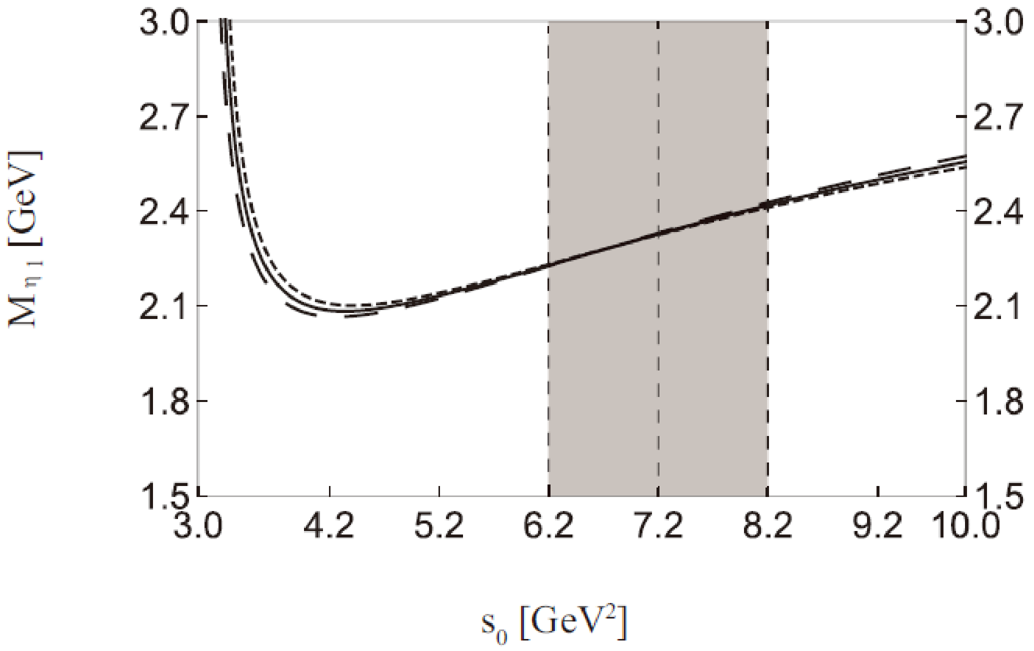
$$\mathrm{PC} \equiv \left| \frac{\Pi_{11}(s_0, M_B^2)}{\Pi_{11}(\infty, M_B^2)} \right| \geq 45\% \,.$$

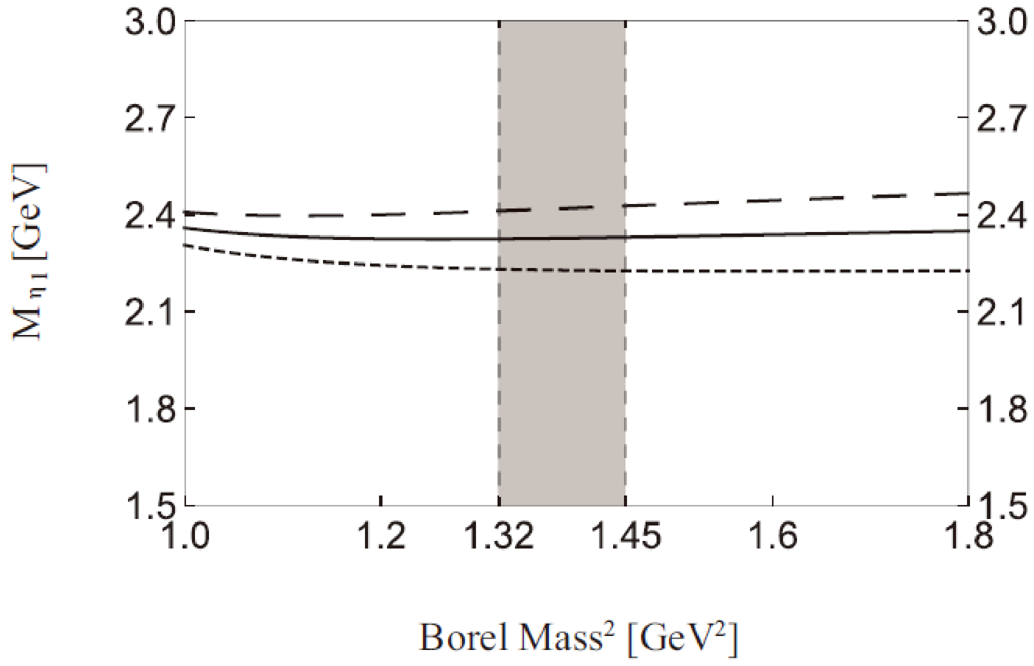


$$1.32\,\mathrm{GeV}^2 < M_B^2 < 1.45\,\mathrm{GeV}^2$$

Numerical analyses

- **The dependence of mass on parameters** $M_{\eta_1} = 2.33^{+0.19}_{-0.16}$ GeV





Numerical analyses

- **Mass extracted from currents**

TABLE I: Masses extracted from the currents $\eta^{1,2,5,6}_{\alpha_1\alpha_2\alpha_3}$.

Currents	M_B^2 [GeV^2]	s_0 [GeV^2]	Pole [%]	Mass [GeV]
$\eta^1_{\alpha_1\alpha_2\alpha_3}$	1.32-1.45	7.2 ± 1.0	44.9-53.3	$2.33^{+0.19}_{-0.16}$
$\eta^2_{\alpha_1\alpha_2\alpha_3}$	1.33-1.48	7.6 ± 1.0	45.1-54.1	$2.45^{+0.27}_{-0.18}$
$\eta^5_{\alpha_1\alpha_2\alpha_3}$	1.46-1.60	9.6 ± 1.0	45.1-53.4	$2.72^{+0.11}_{-0.12}$
$\eta^6_{\alpha_1\alpha_2\alpha_3}$	1.45-1.58	9.4 ± 1.0	45.2-53.1	$2.67^{+0.11}_{-0.12}$

The mass of $sq\bar{s}\bar{q}$ tetraquark state with the exotic quantum number $J^{PC} = 3^{-+}$ is 2.33 GeV.

Contents

- **Introduction**
- **Method of the QCD sum rules**
- **Numerical analyses**
- **Decay behavior**
- **More studies on light tetraquark states**

Decay behavior

$$\xi^1_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{(\bar{q}_a\gamma_{\alpha_1}q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\gamma_{\alpha_2}s_b)\Big\}\,,$$

$$\xi^2_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{(\bar{q}_a\gamma_{\alpha_1}\gamma_5 q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\gamma_{\alpha_2}\gamma_5 s_b)\Big\}\,,$$

$$\xi^3_{\alpha_1\alpha_2\alpha_3} = g^{\mu\nu}\mathcal{S}\Big\{(\bar{q}_a\sigma_{\alpha_1\mu}q_a)\overleftrightarrow{D}_{\alpha_3}(\bar{s}_b\sigma_{\alpha_2\nu}s_b)\Big\}\,,$$

$$\xi^4_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{\big[\bar{q}_a\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_3}s_a\big](\bar{s}_b\gamma_{\alpha_2}q_b) - (\bar{q}_a\gamma_{\alpha_1}s_a)\big[\bar{s}_b\gamma_{\alpha_2}\overleftrightarrow{D}_{\alpha_3}q_b\big]\Big\}\,,$$

$$\xi^5_{\alpha_1\alpha_2\alpha_3} = \mathcal{S}\Big\{\big[\bar{q}_a\gamma_{\alpha_1}\gamma_5\overleftrightarrow{D}_{\alpha_3}s_a\big](\bar{s}_b\gamma_{\alpha_2}\gamma_5 q_b) - (\bar{q}_a\gamma_{\alpha_1}\gamma_5 s_a)\big[\bar{s}_b\gamma_{\alpha_2}\gamma_5\overleftrightarrow{D}_{\alpha_3}q_b\big]\Big\}\,,$$

$$\xi^6_{\alpha_1\alpha_2\alpha_3} = g^{\mu\nu}\mathcal{S}\Big\{\big[\bar{q}_a\sigma_{\alpha_1\mu}\overleftrightarrow{D}_{\alpha_3}s_a\big](\bar{s}_b\sigma_{\alpha_2\nu}q_b) - (\bar{q}_a\sigma_{\alpha_1\mu}s_a)\big[\bar{s}_b\sigma_{\alpha_2\nu}\overleftrightarrow{D}_{\alpha_3}q_b\big]\Big\}\,.$$

$$\xi = [\bar{q}q]\overleftrightarrow{D}[\bar{s}s]$$

$$\xi' \quad [\bar{q}\overleftrightarrow{D}s][\bar{s}q] - [\bar{q}s][\bar{s}\overleftrightarrow{D}q]$$

- **It well decay into the P-wave** $\rho\phi\,/\,\omega\phi$ **channel but not into the** $\rho f_2(1525)\,/\,\omega f_2(1525)\,/\,\phi f_2(1525)$ **channels.**
- **It well decay into** $K^*(892)\bar{K}^*(1430)$ **channel but not into the P-wave** $K^*(892)\bar{K}^*(892)$ **channel.**

- **We use the method of QCD sum rules to study light tetraquark states $sq\bar{s}\bar{q}$ with the exotic quantum number $J^{PC}=3^{-+}$.**

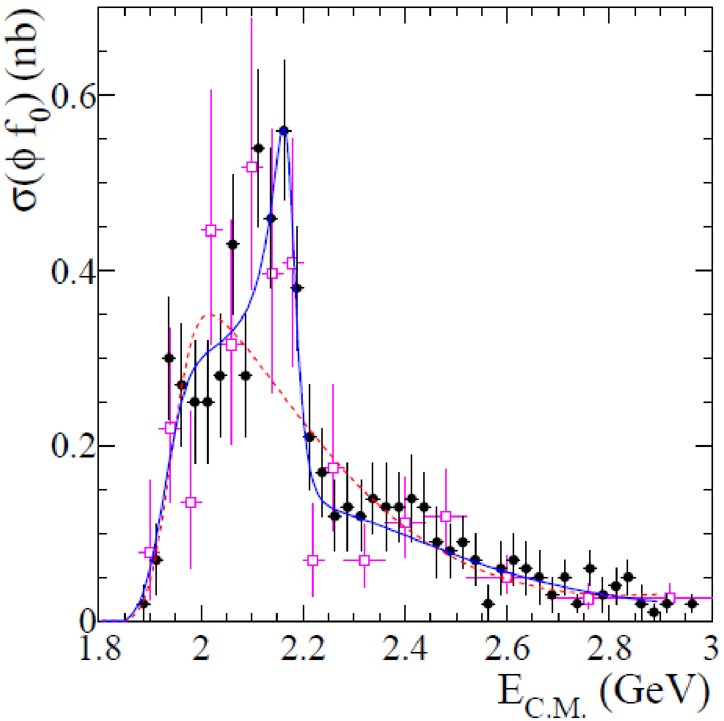
- **We calculate the mass of this state, and predict its decay behavior.**

- **We propose to investigate the P-wave $\rho\phi/\omega\phi$ channel in future BESIII, if there existed a narrower resonance of $J^{PC}=3^{-+}$, it would be more likely to be a compact $sq\bar{s}\bar{q}$ tetraquark state.**

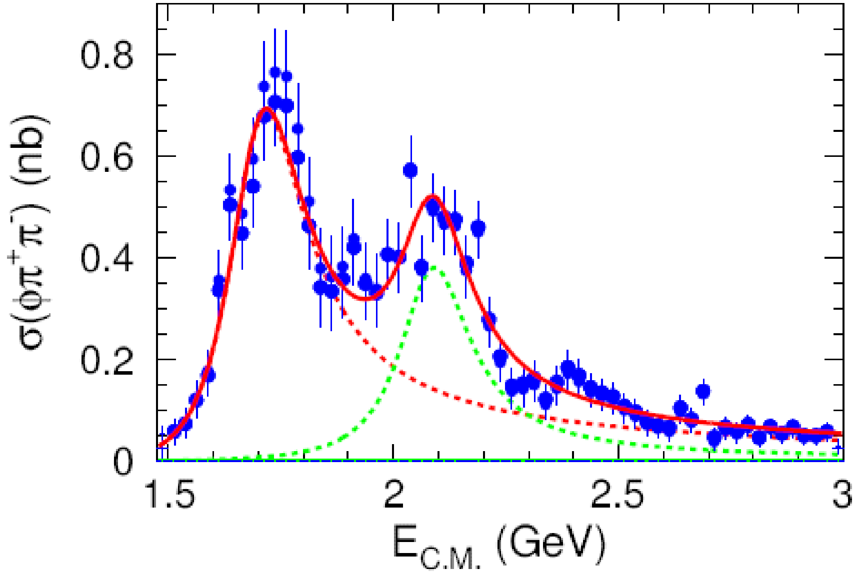
Contents

- **Introduction**
- **Method of the QCD sum rules**
- **Numerical analyses**
- **Decay behavior**
- **More studies on light tetraquark states**

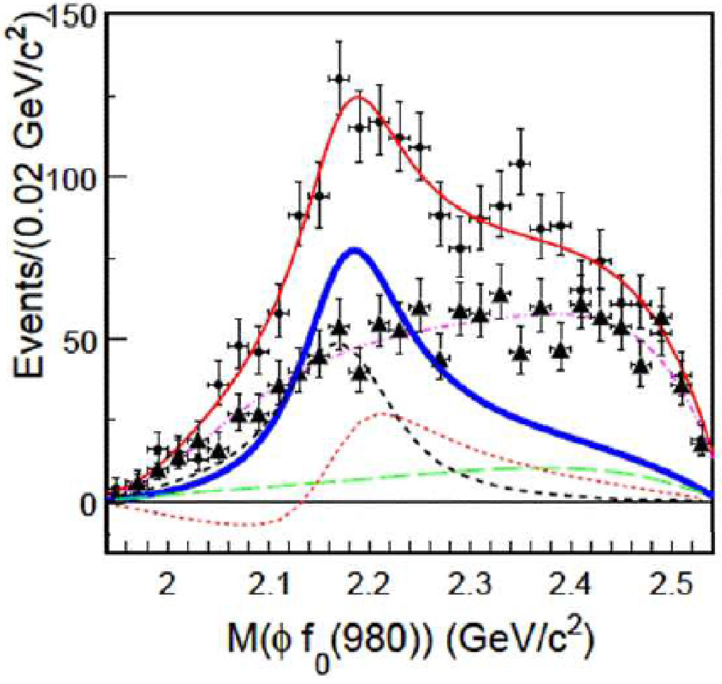
More studies on light tetraquark states



(a)The $e^+e^- \to \phi f_0(980)$ cross section. Taken from BaBar [11].



(c)The $e^+e^- \to \phi\pi^+\pi^-$ cross section with two incoherent Briet-Wigner functions, the $\phi(1680)$ and the $Y(2175)$. Taken from Belle [16].



(f)The $\phi f_0(980)$ invariant mass spectrum. Taken from BESIII [17].

More studies on light tetraquark states

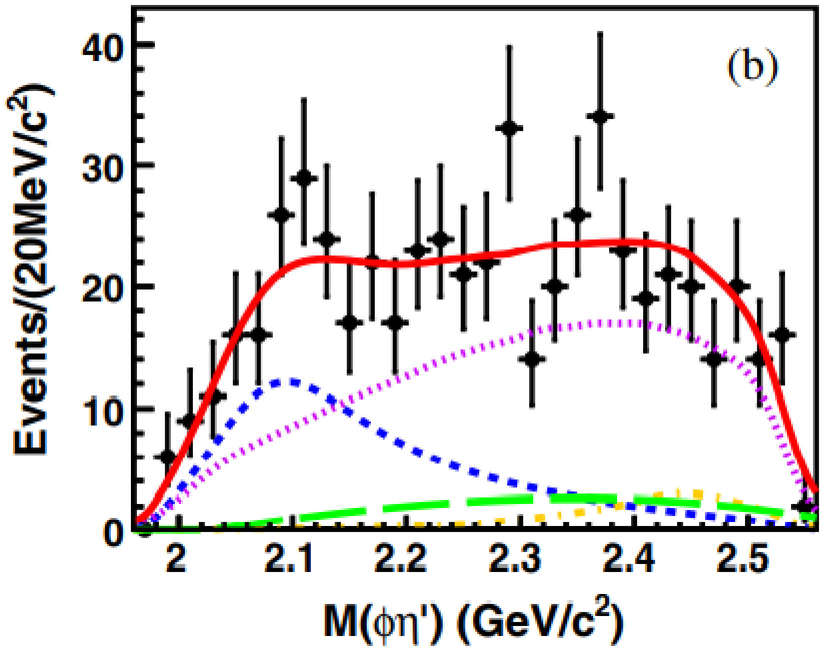
- **There are two $ss\bar{s}\bar{s}$ interpolating currents of $J^{PC}=1^{--}$:**

$$\eta_{1\mu}=(s_a^T C\gamma_5 s_b)(\bar{s}_a\gamma_\mu\gamma_5 C\bar{s}_b^T)-(s_a^T C\gamma_\mu\gamma_5 s_b)(\bar{s}_a\gamma_5 C\bar{s}_b^T)$$
$$\eta_{2\mu}=(s_a^T C\gamma^\nu s_b)(\bar{s}_a\sigma_{\mu\nu} C\bar{s}_b^T)-(s_a^T C\sigma_{\mu\nu} s_b)(\bar{s}_a\gamma^\nu C\bar{s}_b^T)$$

- **After mixing, the results are more reliable(reasonable), the mass is extracted to be**

$$J_{1\mu} = \cos\theta\ \eta_{1\mu} + \sin\theta\ i\ \eta_{2\mu}\,, \qquad M_{Y_1} = 2.41 \pm 0.25 \text{ GeV}\,,$$
$$J_{2\mu} = \sin\theta\ \eta_{1\mu} + \cos\theta\ i\ \eta_{2\mu}\,. \qquad M_{Y_2} = 2.34 \pm 0.17 \text{ GeV}\,.$$

More studies on light tetraquark states



$J/\psi \to \eta Y \to \eta\phi\eta'$
BESIII, PRD99 (2019) 112008

Possibility A: $J^P = 1^-$

$$M_{1^-} = 2002.1 \pm 27.5 \pm 15.0 \text{ MeV},$$
$$\Gamma_{1^-} = 129 \pm 17 \pm 7 \text{ MeV}.$$

Possibility B: $J^P = 1^+$

$$M_{1^+} = 2062.8 \pm 13.1 \pm 4.2 \text{ MeV},$$
$$\Gamma_{1^+} = 177 \pm 36 \pm 20 \text{ MeV}.$$

More studies on light tetraquark states

- **There are two $ss\bar{s}\bar{s}$ interpolating currents of $J^{PC}=1^{+-}$:**

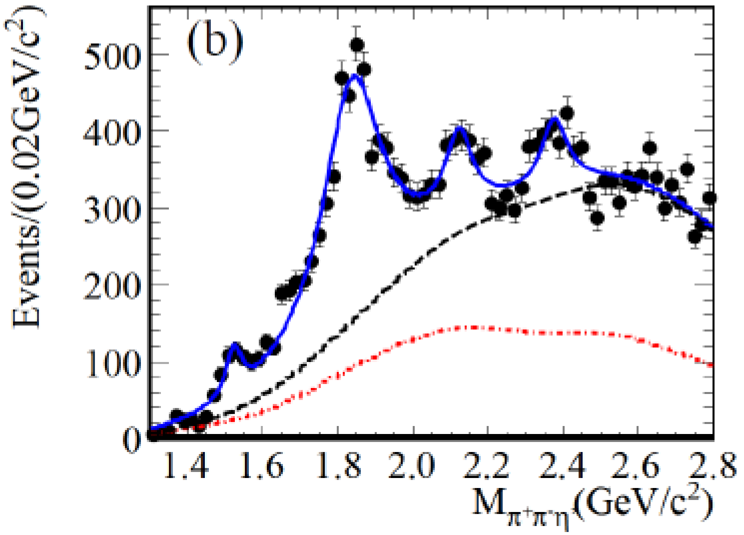
$$\eta_{1\mu}=(s_a^T C s_b)(\bar{s}_a\gamma_\mu\gamma_5 C\bar{s}_b^T)-(s_a^T C\gamma_\mu\gamma_5 s_b)(\bar{s}_a C\bar{s}_b^T)$$

$$\eta_{2\mu}=(s_a^T C\gamma^\nu s_b)(\bar{s}_a\sigma_{\mu\nu}\gamma_5 C\bar{s}_b^T)-(s_a^T C\sigma_{\mu\nu}\gamma_5 s_b)(\bar{s}_a\gamma^\nu C\bar{s}_b^T)$$

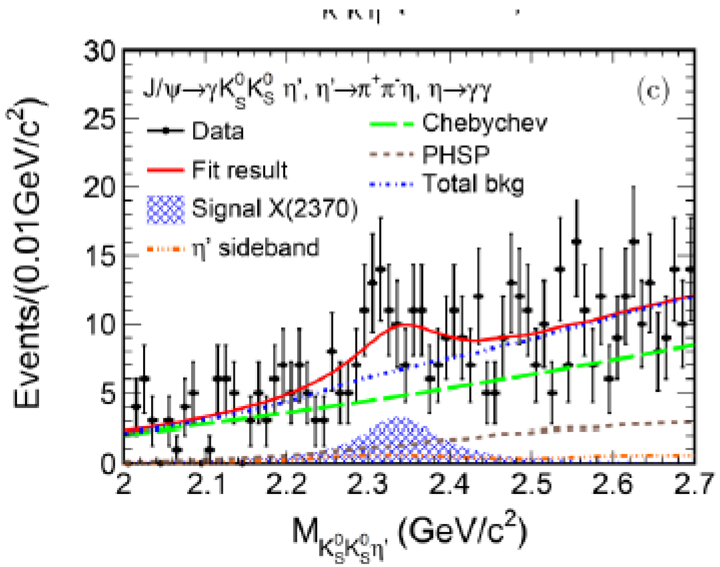
- **Only one of them, $\eta_{2\mu}$ leads to reliable(reasonable) sum rules:**

$$M_{\eta_2}=2.00^{+0.10}_{-0.09}\ \text{GeV}$$

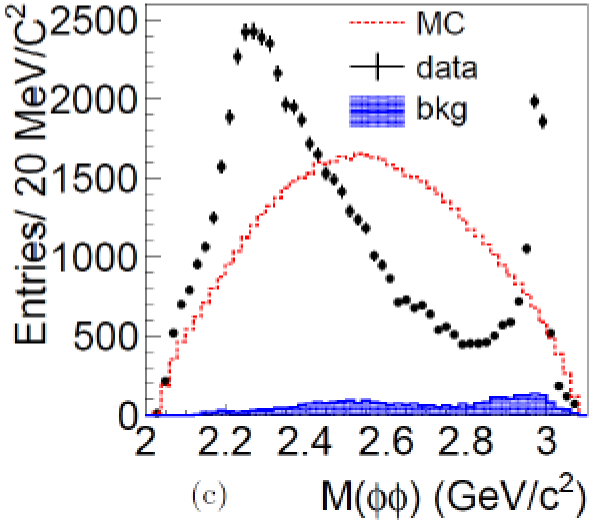
More studies on light tetraquark states



$J/\psi \to \gamma(X(2370) \to \pi^+\pi^-\eta')$
BESIII, PRL 106 (2011) 072002



$J/\psi \to \gamma(X(2370) \to K\bar{K}\eta')$
BESIII, EPJC80 (2020) 8, 746



$J/\psi \to \gamma(X(2500) \to \phi\phi)$
BESIII, PRD93, 112011 (2016)

More studies on light tetraquark states

- **There are two $ss\bar{s}\bar{s}$ interpolating currents of $J^{PC}=0^{-+}$:**

$$\eta_1 = (s_a^T C s_b)(\bar{s}_a \gamma_5 C \bar{s}_b^T) + (s_a^T C \gamma_5 s_b)(\bar{s}_a C \bar{s}_b^T)\,,$$
$$\eta_2 = (s_a^T C \sigma_{\mu\nu} s_b)(\bar{s}_a \sigma^{\mu\nu} \gamma_5 C \bar{s}_b^T)\,.$$

- **After mixing, one leades to reliable(reasonable) sum rules:**

$$M_{J_2} = 2.51^{+0.15}_{-0.12}\ \text{GeV}$$

While the other leads to:

$$M_{J_1} > 3.0\,\text{GeV}$$

Advantages of rich strangeness states:

- **Experimentally, their widths are possibly not too broad, so they are capable of being observed.**

- **Theoretically, their internal structures are simpler due to the Pauli principle restriction, on their potential number is limited.**

Thank You for your attention !