Fix the phase angles in proton electromagnetic form factors from charge asymmetry distribution in process $e^+e^- \to p\bar{p}\gamma$

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Outline

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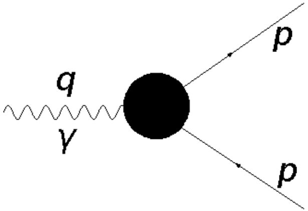
2. *The main process*($e^- + e^+ \to p + \overline{p} + \gamma$)
3. *Result*
4. *Details in calculation*
5. *Summary*

Motivation

- Proton, being one of the fundamental building blocks of the world, its internal structure and dynamics are still confusing the human.
- Electromagnetic form factors (FFs), $G_E(q^2)$, $G_M(q^2)$ are a bridge connecting the experiment and theory.

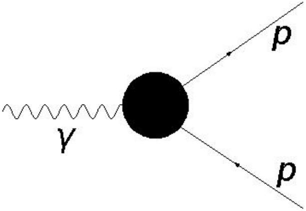


$$ie[\gamma_\mu F_1(q^2) + iF_2(q^2)\sigma_{\mu\nu}q^\nu]$$

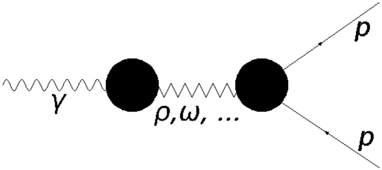
region	obtain	value
spacelike($q^2 < 0$)	$ep \to ep$	real
timelike($q^2 > 0$)	$e^+e^- \to p\bar{p}$	complex

Motivation

- In fact, the appearance of these phase angles is not strange:



(a) Complicated strong interactions in proton



(b) Example: vector meson dominance

$$\begin{cases} F_1 = e^{i\theta_2} \dfrac{4M_p^2|G_E| - s|G_M|e^{i\theta_1}}{4M_p^2 - s}, \\ F_2 = e^{i\theta_2} \dfrac{2M_p(|G_M|e^{i\theta_1} - |G_E|)}{4M_p^2 - s}. \end{cases} \quad (1)$$

θ_1: the relative phase angle θ_2: the additional phase angle

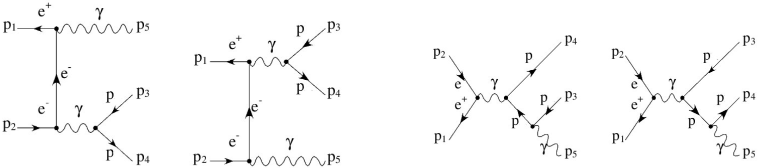
Motivation

How can we find these phase angles?

- for the annihilation process $e^+e^- \to p\bar{p}$
  (I) No polarization proton: the FFs also only appear in the form of moduli;
  (II) Polarizated proton: the relative phase angle can be decided.[hep-ph/0311355]
  (III) Hard to achieve
- $e^- + e^+ \to p + \overline{p} + \gamma$

The main process $(e^- + e^+ \to p + \overline{p} + \gamma)$

Figure: The Feynman diagrams of $e^- + e^+ \to p + \overline{p} + \gamma$



(a) Only contain on-shell protons

(b) Contain one off-shell proton as propagator

$$d\sigma = d\sigma_a + d\sigma_b + d\sigma_{ab} \tag{2}$$

$d\sigma_a$: no relation with phase angles

$d\sigma_b$: small

$d\sigma_{ab}$(the interference term):

- $\sigma_{ab} = 0$
- the $p\bar{p}$ is produced with charge asymmetry: $C = -1$(ISR), $C = +1$(FSR).
- Charge aysmmetry $\to$ Final states charge parity transformation:

$$d\sigma_{ab} = -d\sigma_{ab}^{c(FS)} \tag{3}$$

The interference term

The charge asymmetry is a kind of forward-backward asymmetry and the cancellation is happened in the phase space.

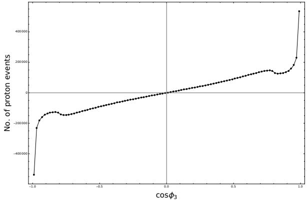


Figure: The distribution of $cos\phi_3$

$$cos\phi_3 = \frac{(\boldsymbol{p}_e \times \boldsymbol{p}_\gamma) \cdot (\boldsymbol{p}_p \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e \times \boldsymbol{p}_\gamma||\boldsymbol{p}_p \times \boldsymbol{p}_\gamma|}, \quad (cos\phi_3)^c = cos\phi_3 \qquad (4)$$

Do the final states charge parity transformation:

$$(cos\phi_3)^{c(FS)} = \frac{(\boldsymbol{p}_e \times \boldsymbol{p}_\gamma) \cdot ((-\boldsymbol{p}_p - \boldsymbol{p}_\gamma) \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e \times \boldsymbol{p}_\gamma||(-\boldsymbol{p}_p - \boldsymbol{p}_\gamma) \times \boldsymbol{p}_\gamma|} = -cos\phi_3. \qquad (5)$$

The interference term

Final states charge parity transformation:

$$(cos\phi_3)^{c(FS)} = -cos\phi_3$$

Charge aysmmetry:

$$d\sigma_{ab} = -d\sigma_{ab}^{c(FS)}$$

Hence,

$$d\sigma_{ab}(cos\phi_3) = -d\sigma_{ab}(-cos\phi_3) \tag{6}$$

- the C parity asymmetry or forward-backward asymmetry $\neq$ C parity violation.
- to test C parity conservation:

$$cos\beta = \frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\gamma}{|\boldsymbol{p}_e||\boldsymbol{p}_\gamma|}. \tag{7}$$

.

For the process $e^- + e^+ \rightarrow p + \overline{p}$:

$$cos\theta_p = \frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_p}{|\boldsymbol{p}_e||\boldsymbol{p}_p|} \tag{8}$$

$$(cos\theta_p)^c = cos\theta_p \quad , \quad (cos\theta_p)^{c(FS)} = -cos\theta_p \tag{9}$$

Hence,
the C parity asymmetry or forward-backward asymmetry $\neq$ C parity violation too.

Defining a value to reveal this asymmetry:

$$\sigma'_{ab} \equiv \sigma_{ab}(cos\phi_3 > 0) - \sigma_{ab}(cos\phi_3 < 0) \tag{10}$$

Meanwhile,

$$\sigma_a(cos\phi_3 > 0) - \sigma_a(cos\phi_3 < 0) = 0, \tag{11}$$

$$\sigma_b(cos\phi_3 > 0) - \sigma_b(cos\phi_3 < 0) = 0. \tag{12}$$

Hence one can conclude

$$\sigma' \equiv \sigma(cos\phi_3 > 0) - \sigma(cos\phi_3 < 0) = \sigma'_{ab}. \tag{13}$$

Now, one can define a ratio

$$R \equiv \frac{\sigma'}{\sigma} = \frac{\sigma(cos\phi_3 > 0) - \sigma(cos\phi_3 < 0)}{\sigma(cos\phi_3 > 0) + \sigma(cos\phi_3 < 0)}, \tag{14}$$

which represents the asymmetry.

Results(The internal propagator is proton)

We first fix $\theta_2 = 0$ in the following computation.

$$R = 0.0134 * sin(\theta_1 - 0.5\pi) + 0.1788. \tag{15}$$

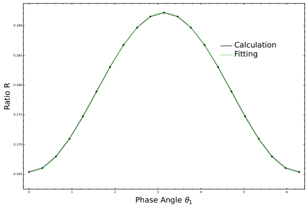


Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.1$ GeV and $(\delta k)^2 = 29$ GeV^2

The range of the ratio R is from 0.165 to 0.192, where the maximum asymmetry point is $\theta_1 = \pi$, $E_5 > 0.1$ GeV and $(\delta k)^2 = 29$ GeV^2. At this point, the cross section is $\sigma = 0.945$ pb and there can be 3018 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

Then we change and fix $\theta_2 = \pi/2$ in the following computation.

$$R = -0.0300 * sin(\theta_1). \tag{16}$$

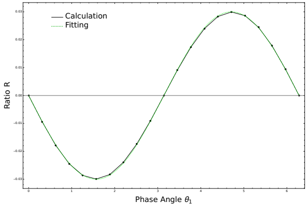


Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.3$ GeV and $(\delta k)^2 = 33$ GeV^2

The range of the ratio R is from -0.0300 to 0.0300, where the maximum asymmetry point is $\theta_1 = 0.5\pi$ or $\theta_1 = 1.5\pi$, $E_5 > 0.3$ GeV, $(\delta k)^2 = 33$ GeV^2.

At this point, the cross section is $\sigma = 0.572$ pb and there can be 1824 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

The conclusion(The internal propagator is proton)

As is shown above:

- $\theta_2 = 0$: the range of R is from 0.165 to 0.192 with the change of θ_1
- $\theta_2 = \pi/2$: the range is from -0.0300 to 0.0300 with the change of θ_1

The conclusion:

- The additional phase angle θ_2 has more influence than the relative phase angle θ_1 for the process $e^- + e^+ \to p + \overline{p} + \gamma$.
- The additional phase θ_2 can not be ignored and can be even more important in some process.

Details in calculation(The internal propagator is proton)

The group (b) contains one off-shell proton(k_4) as propagator.

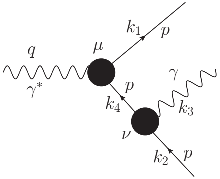


Figure: part of group (b)

We consider the influence of k_4 by leading to a Compressibility Factor (CF), which can be placed in the corresponding fermion line.

$$CF(k_4^2) = exp(-\frac{(k_4^2 - M_p^2)}{(\delta k)^2}), \quad (17)$$

where δk is a constant to be determined by experiment.

Details in calculation(The internal propagator is proton)

The internal propagator is proton:
The factors can influence the ratio R:

- CF or $(\delta k)^2$
- the minimum limit of photon energy
- the relative phase θ_1 and the additional phase θ_2

Here we mainly consider that group(a) and group(b) have same phase angles.
The FFs are from the newly datum collected with the BESIII detector operating at the BESIII collider [hep-ex/1905.09001].

Results(The internal propagator is N(1710))

- For group (b), the propagator corresponding k_4 can also be some excited states N*, such as N(1440), N(1710), N(1880) ...
- These states have Breit-Winger full width Γ, and the propagator:

$$\frac{1}{\not{p} - (m - i\Gamma)}. \tag{18}$$

- Meanwhile, the contribution of these diagrams shall also be small, so there shall exist a factor K in front of the amplitude of each diagram.
- K is a small constant and also needs to be determined by experiment.

Results(The internal propagator is N(1710))

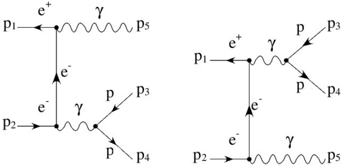
The internal propagator is N(1710):
The factors can influence the ratio R:

- K
- the minimum limit of photon energy
- the relative phase θ_1 and the additional phase θ_2

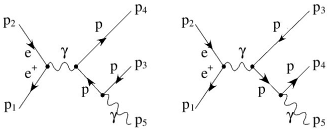
Here we mainly consider that group (a) and group (b) have different θ_1 and same θ_2.

Results(The internal propagator is N(1710))

Case 1:
Here we mainly consider the θ_1 of group (b) is fixed to 0 and the θ_1 of group (a) changes.



(a) Only contain on-shell protons



(b) Contain one off-shell proton as propagator

Results(The internal propagator is N(1710))

We first fix $\theta_2 = 0$ in the following computation.

$$R = 0.00875 * sin(\theta_1 - 0.15\pi) - 0.02295. \tag{19}$$

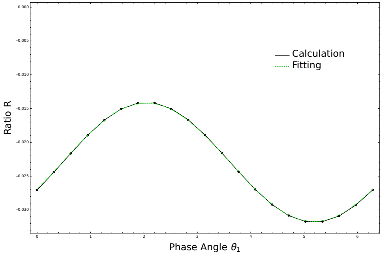


Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.2$ GeV and K=0.15

The range of the ratio R is from -0.0142 to -0.0317 and the maximum asymmetry point $\theta_1 = 1.6\pi$, $E_5 > 0.2$ GeV and K=0.15.
At this point, the cross section is $\sigma = 0.696$ pb and there can be 2223 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

Results(The internal propagator is N(1710))

Then we change and fix $\theta_2 = \pi/2$ in the following computation.

$$R = 0.0089 * sin(\theta_1 - 0.65\pi) - 0.0292. \quad (20)$$

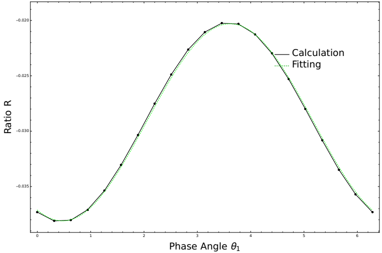


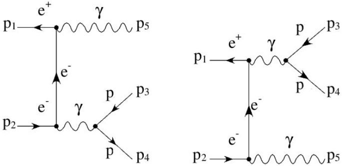
Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.25$ GeV and K=0.14

The range of the ratio R is from -0.0203 to -0.0381 and the maximum asymmetry point $\theta_1 = 0.1\pi$, $E_5 >$ 0.25 GeV and K=0.14.
At this point, the cross section is $\sigma = 0.623$ pb and there can be 1990 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

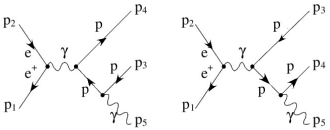
Results(The internal propagator is N(1710))

Case 2:

Here we mainly consider the θ_1 of group (a) is fixed to 0 and the θ_1 of group (b) changes.



(a) Only contain on-shell protons



(b) Contain one off-shell proton as propagator

Results(The internal propagator is N(1710))

We first fix $\theta_2 = 0$ in the following computation.

$$R = 0.0135 * sin(\theta_1 + 0.5\pi) - 0.0204. \tag{21}$$

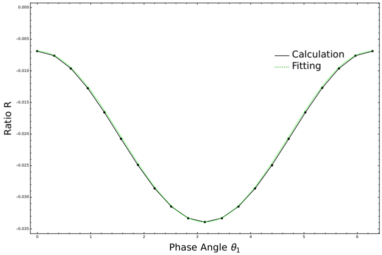


Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.5$ GeV and K=0.09

The range of the ratio R is from -0.0069 to -0.0339 and the maximum asymmetry point $\theta_1 = \pi$, $E_5 >$ 0.5 GeV and K=0.09.
At this point, the cross section is $\sigma = 0.464$ pb and there can be 1482 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

Results(The internal propagator is N(1710))

Then we change and fix $\theta_2 = \pi/2$ in the following computation.

$$R = 0.02785 * sin(\theta_1 - 0.5\pi) - 0.00945. \quad (22)$$

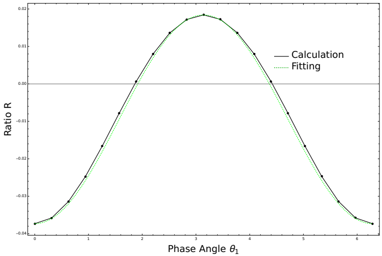


Figure: The relation between phase angle θ_1 and ratio R, with $E_5 > 0.25$ GeV and K=0.14

The range of the ratio R is from -0.0373 to 0.0184 and the maximum asymmetry point $\theta_1 = 0$, $E_5 >$ 0.25 GeV and K=0.14.
At this point, the cross section is $\sigma = 0.623$ pb and there can be 1990 events in the experiment ($\sqrt{s} = 4.180$ GeV, L=3194.5 pb^{-1}).

The conclusion(The internal propagator is N(1710))

- As a preliminary study, when the internal propagator is N*, the value of the additional phase angle θ_2 influence not much as the condition that the internal propagator is proton.
- The contribution to the asymmetry is also small with a perspective of the maximum possible degree. The results are consistent with our expectations that the internal excited states propagator shall contribution less than internal proton propagator.
- We need to pay more attention to the internal proton propagator condition.
- The contribution of σ_b is limited by $\sigma_b/\sigma_a < 0.1$.
- if $\sigma_b/\sigma_a < 0.01$, the ratio is about $\sqrt{10}$ times smaller.

Summary

- Two phase angles, the relative phase angle and the additional phase angle, in proton electromagnetic form factors are studied in the process $e^+e^- \to p\bar{p}\gamma$.
- The results show that different value of the phase angles can give very different asymmetry distribution which could be obtained from experimental measurement in electron-positron collider.
- The results also show that the additional phase may have more influence to this process than the relative phase, which means the additional phase can not be ignored and can be even more important in some process.
- There is a chance that the asymmetry can reach nearly 20 percent degrees and the events can reach several thousands at the BESIII experiment.

The End
Thank you very much

Appendix(The interaction of photon and proton)

The photon proton interaction can be interpreted as an electromagnetic current, which can be written in the usual form as:

$$J_\mu(k_1, k_2) = \overline{u}(k_1) ie[\gamma_\mu F_1(q^2) + iF_2(q^2)\sigma_{\mu\nu} q^\nu] u(k_2), \tag{23}$$

where k_1 and k_2 are the momentum of the protons or anti-protons, demand

$$k_1^2 = M_p^2, \quad k_2^2 = M_p^2. \tag{24}$$

q is the momentum of the photon, $F_1(q^2)$ and $F_2(q^2)$ are the usual Dirac and Pauli form factors.
None of form factor $F_1(q^2)$ or $F_2(q^2)$ corresponds to the Sachs electric form-factor $G_E(q^2)$ or magnetic form-factor $G_M(q^2)$ alone:

$$\begin{cases} |G_E(q^2)| = |F_1(q^2) + \frac{s}{2M_p} F_2(q^2)| \\ |G_M(q^2)| = |F_1(q^2) + 2M_p F_2(q^2)|. \end{cases} \tag{25}$$

$$\begin{cases} |G_E(q^2)| = |F_1(q^2) + \frac{s}{2M_p} F_2(q^2)| \\ |G_M(q^2)| = |F_1(q^2) + 2M_p F_2(q^2)|. \end{cases} \tag{26}$$

By introducing two phase angles θ_1 and θ_2:

$$\begin{cases} |G_E| e^{i\theta_2} = F_1 + \frac{s}{2M_p} F_2 \\ |G_M| e^{i(\theta_1+\theta_2)} = F_1 + 2M_p F_2. \end{cases} \tag{27}$$

Hence, one can obtain

$$\begin{cases} F_1 = \frac{4M_p^2 |G_E| e^{i\theta_2} - s|G_M| e^{i(\theta_1+\theta_2)}}{4M_p^2 - s} = e^{i\theta_2} \frac{4M_p^2 |G_E| - s|G_M| e^{i\theta_1}}{4M_p^2 - s} \\ F_2 = \frac{2M_p(|G_M| e^{i(\theta_1+\theta_2)} - |G_E| e^{i\theta_2})}{4M_p^2 - s} = e^{i\theta_2} \frac{2M_p(|G_M| e^{i\theta_1} - |G_E|)}{4M_p^2 - s}. \end{cases} \tag{28}$$

Appendix(The interaction of photon and proton)

Considering the Feynman diagram only contains the fermion line representing the electromagnetic current and summing over the spinors:

$$
\begin{aligned}
|M|^2_{\mu_1\mu_2} &= \sum_{spins} J_{\mu_1} J^*_{\mu_2} \\
&= \sum_{s_1,s_2} \overline{u}(n_1,k_1,s_1)(ie[\gamma_{\mu_1} F_1(q^2) + iF_2(q^2)\sigma_{\mu_1\nu_1} q^{\nu_1}])u(n_2,k_2,s_2) \\
&\qquad (\overline{u}(n_1,k_1,s_1)(ie[\gamma_{\mu_2} F_1(q^2) + iF_2(q^2)\sigma_{\mu_2\nu_2} q^{\nu_2}])u(n_2,k_2,s_2))^* \\
&= f(q_{\mu_1} q_{\mu_2}, q'_{\mu_1} q'_{\mu_2}, g_{\mu_1\mu_2}, s, |G_M|^2, |G_E|^2).
\end{aligned}
\tag{29}
$$

One can get a conclusion that in diagrams level, if the summation of all diagrams can be factored into two parts, in which one part is the fermion line representing the electromagnetic current, and the other part has no relation to this fermion line, the electromagnetic FFs can only appear with the form of moduli in the cross section.

Appendix(The comparison of ϕ_3 and φ_3)

For comparison and readability, we can transform $cos\varphi_3$ and rewrite $cos\phi_3$ here:

$$cos\varphi_3 = \frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_p}{|\boldsymbol{p}_e||\boldsymbol{p}_p|}, \tag{30}$$

$$cos\phi_3 = \frac{(\boldsymbol{p}_e \times \boldsymbol{p}_\gamma) \cdot (\boldsymbol{p}_p \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e \times \boldsymbol{p}_\gamma||\boldsymbol{p}_p \times \boldsymbol{p}_\gamma|}. \tag{31}$$

The initial process is

$$e^-(p_e) + e^+(p_{e^+}) \to p(p_p) + \overline{p}(p_{\overline{p}}) + \gamma(p_\gamma). \tag{32}$$

Now,

$$cos\varphi_3 = \frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_p}{|\boldsymbol{p}_e||\boldsymbol{p}_p|}, \tag{33}$$

$$cos\phi_3 = \frac{(\boldsymbol{p}_e \times \boldsymbol{p}_\gamma) \cdot (\boldsymbol{p}_p \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e \times \boldsymbol{p}_\gamma||\boldsymbol{p}_p \times \boldsymbol{p}_\gamma|}. \tag{34}$$

Appendix(The comparison of ϕ_3 and φ_3)

Do the final states charge parity transformation, the corresponding process is

$$e^-(p_e) + e^+(p_{e^+}) \to \overline{p}(p_p) + p(p_{\overline{p}}) + \gamma(p_\gamma). \tag{35}$$

Now,

$$(cos\varphi_3)^{c(FS)} = \frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_{\overline{p}}}{|\boldsymbol{p}_e||\boldsymbol{p}_{\overline{p}}|} = \frac{\boldsymbol{p}_e \cdot (-\boldsymbol{p}_p - \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e||\boldsymbol{p}_p + \boldsymbol{p}_\gamma|} \neq cos\varphi_3, \tag{36}$$

$$(cos\phi_3)^{c(FS)} = \frac{(\boldsymbol{p}_e \times \boldsymbol{p}_\gamma) \cdot (\boldsymbol{p}_{\overline{p}} \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e \times \boldsymbol{p}_\gamma||\boldsymbol{p}_{\overline{p}} \times \boldsymbol{p}_\gamma|} = -cos\phi_3. \tag{37}$$

where the derivation is happened in the e^-e^+ c.m.s. Obviously, $cos\phi_3$ is odd in final states charge parity transformation, while $cos\varphi_3$ is not odd or even. Hence $cos\phi_3$ is better to reveal the forward-backward asymmetry, to reveal the charge asymmetry, and to reveal the interference term.

Appendix(The comparison of ϕ_3 and φ_3)

Do the complete charge parity transformation, the corresponding process is

$$e^+(p_e) + e^-(p_{e^+}) \to \overline{p}(p_p) + p(p_{\overline{p}}) + \gamma(p_\gamma). \quad (38)$$

Now,

$$(cos\varphi_3)^c = \frac{\boldsymbol{p}_{e^+} \cdot \boldsymbol{p}_{\overline{p}}}{|\boldsymbol{p}_{e^+}||\boldsymbol{p}_{\overline{p}}|} = \frac{(-\boldsymbol{p}_e) \cdot (-\boldsymbol{p}_p - \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_e||\boldsymbol{p}_p + \boldsymbol{p}_\gamma|} \neq cos\varphi_3, \quad (39)$$

$$(cos\phi_3)^c = \frac{(\boldsymbol{p}_{e^+} \times \boldsymbol{p}_\gamma) \cdot (\boldsymbol{p}_{\overline{p}} \times \boldsymbol{p}_\gamma)}{|\boldsymbol{p}_{e^+} \times \boldsymbol{p}_\gamma||\boldsymbol{p}_{\overline{p}} \times \boldsymbol{p}_\gamma|} = cos\phi_3. \quad (40)$$

Obviously, $cos\phi_3$ is even in complete charge parity transformation, while $cos\varphi_3$ is not odd or even. Hence $cos\phi_3$ or $cos\varphi_3$ can not test the charge parity conservation.