Electromagnetic form factors of Λ hyperon in the vector meson dominance model

Zhong-Yi Li (李中义)
arXiv:2107.10499

Institute of Modern Physics, Chinese Academy of Sciences
In collaboration with An-Xin Dai and Ju-Jun Xie

October , 2021

Outline

Electromagnetic form factors

The electromagnetic current of baryon with spin-1/2 in terms of the Dirac form factors $F_1(Q^2)$ and Pauli form factors $F_2(Q^2)$ can be written as

$$J^\mu = \gamma^u F_1(Q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2m_B} F_2(Q^2),$$

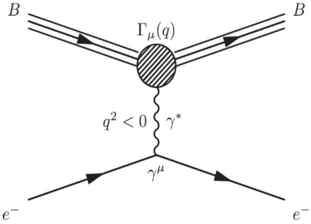


Figure: The reaction $e^- B \to e^- B$

The observed Sachs form factors G_E and G_M can be obtained by the relations

$$C_E\left(Q^2\right) = F_1\left(Q^2\right) - \tau F_2\left(Q^2\right), \quad G_M\left(Q^2\right) = F_1\left(Q^2\right) + F_2\left(Q^2\right)$$

Electromagnetic form factors

We obtain the differential cross section, in terms of the electromagnetic FFs G_E and G_M, i.e. the so-called Rosenbluth formula

$$\frac{d\sigma}{d\Omega_e} = \frac{\alpha^2}{Q^2}\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 \left[2\tau G_M^2 + \frac{\cot^2(\theta_e/2)}{1+\tau}\left(G_E^2 + \tau G_M^2\right)\right]$$

$$= \frac{d\sigma_M}{d\Omega_e}\left[2\tau G_M^2 \tan^2(\theta_e/2) + \frac{G_E^2 + \tau G_M^2}{1+\tau}\right]$$

M. Rosenbluth, Phys.Rev. 79, 615–619(1950).

Electromagnetic form factors

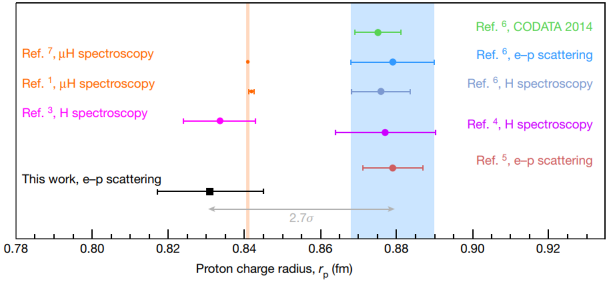
Measurement Method 1: Electron - proton elastic scattering

$$r_p \equiv \left(-6 \frac{dG_E\left(Q^2\right)}{dQ^2} \bigg|_{Q^2=0} \right)^{1/2}$$

Measurement Method 2: Lamb-Shift transition energy spectrum of hydrogen atom

$$\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_p^2 \delta_{l0}$$

Xiong, W. and others, Nature 575, 147 (2019)



Time like form factors

By adopting the one-photon approximation, one gets the following differential cross section for the spin-1/2 $B\bar{B}$-pair production,

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2\beta}{4q^2} C\left(q^2\right)\left\{\left|G_M\left(q^2\right)\right|^2\left(1+\cos^2\theta\right) + \left|G_E\left(q^2\right)\right|^2 \frac{1}{\tau}\sin^2\theta\right\}$$

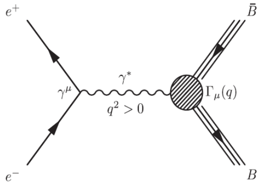


Figure: The reaction $e^+e^- \to B\bar{B}$

$$\beta = \sqrt{1-4M_B^2/q^2} = |p|/E,$$

$$C(y) = \begin{cases} \frac{y}{(1-e^{-y})} & \text{for charged particle} \\ 1 & \text{for neutral particle} \end{cases}$$

$$y = 2\frac{M_B}{q}\frac{\alpha\pi}{\beta}$$

$$\sigma = \frac{4\pi\alpha^2\beta}{3q^2} C\left(q^2\right)\left[\left|G_M\left(q^2\right)\right|^2 + \frac{\left|G_E\left(q^2\right)\right|^2}{2\tau}\right]$$

Effective form factors

Such an expression suggests to define an effective form factor in the TL region by dividing the actual total cross section, by the point-like one. Then, one can write

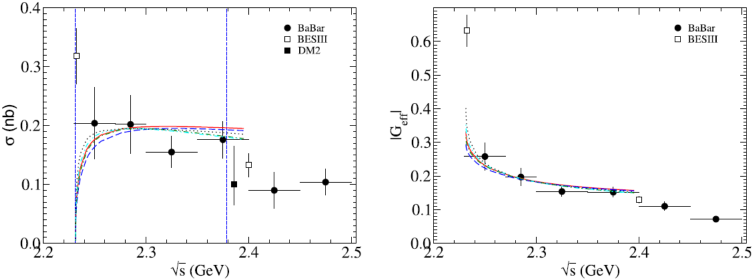
$$\sigma_{e^+e^-\to B\bar{B}} = \frac{4\pi\alpha^2\beta}{3q^2}\left[1+\frac{1}{2\tau}\right]\left|G_{eff}\left(q^2\right)\right|^2 = \sigma_{point}\left(q^2\right)\left|G_{eff}\left(q^2\right)\right|^2$$

$$\sigma_{point}\left(q^2\right) = \frac{4\pi\alpha^2\beta}{3q^2}\left(q^2\right)\left[1+\frac{1}{2\tau}\right]$$

$$\left|G_{eff}\left(q^2\right)\right| = \sqrt{\frac{\sigma_{e^+e^-\to B\bar{B}}}{\sigma_{point}\left(q^2\right)}} = \sqrt{\frac{2\tau\left|G_M\left(q^2\right)\right|^2+\left|G_E\left(q^2\right)\right|^2}{1+2\tau}}$$

$e^+e^- \to \Lambda\bar{\Lambda}$

The BESIII collaboration measured the $e^+e^- \to \Lambda\bar{\Lambda}$ reaction with much improved precision. The Born cross-section at the center of mass energy $\sqrt{s}$=2.2324 GeV is determined to be $305 \pm 45^{+66}_{-36}$ pb.



J. Haidenbauer and U. G. Meißner, Phys. Lett. B 761, 456-461(2016).

Figure: Total cross section (left) and effective form factor |Geff|(right) for $e^+e^- \to \Lambda\bar{\Lambda}$.

DM2(D. Bisello, et al., DM2 Collaboration, Z. Phys. C 48, 23(1990).)
BarBar(B. Aubert, et al., BaBar Collaboration, Phys. Rev. D 76, 092006(2007).)
BESIII(C. Morales Morales, BESIII Collaboration, AIP Conf. Proc. 1735, 050006(2016).)

$e^+e^- \to \Lambda\bar{\Lambda}$

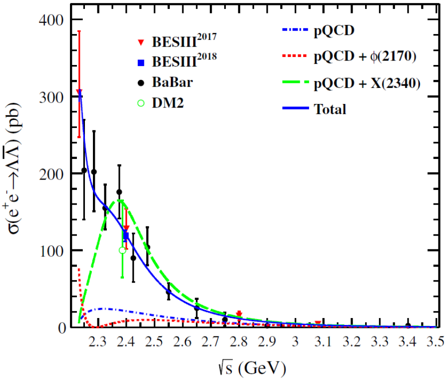


Figure: Fit to the data with pQCD.

X. Cao, J. P. Dai and Y. P. Xie, Phys. Rev. D 98, 094006 (2018).

$e^+e^- \to \Lambda\bar{\Lambda}$

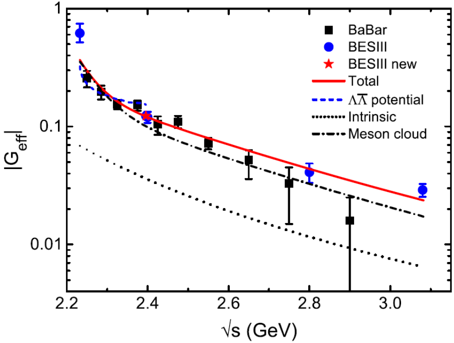


Figure: Fit to the data with VMD.

Y. Yang, D. Y. Chen and Z. Lu, Phys. Rev. D 100, 073007 (2019)

The electromagnetic (EM) interaction of hadrons, at low energy, is unexpectedly well described by the vector meson dominance (VMD) model. In VMD model the interaction is mediated by vector mesons that couple directly to the baryon.

Figure: Feynman diagram of process $e^+e^- \to \gamma \to B\bar{B}$ and $e^+e^- \to \gamma \to V \to B\bar{B}$

Parameterization

In the VMD model, the virtual photon couples to Λ hyperon through vector mesons, thus the Dirac and Pauli form factors are parametrized as following

$$
\begin{aligned}
F_1(Q^2) &= g(Q^2)[-\beta_\omega - \beta_\phi + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} + \beta_x \frac{m_x^2}{m_x^2 + Q^2}] \\
F_2(Q^2) &= g(Q^2)[(\mu_\Lambda - \alpha_\phi) \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2 + Q^2} + \alpha_x \frac{m_x^2}{m_x^2 + Q^2}] \\
g(Q^2) &= 1/(1 + \gamma Q^2)^2
\end{aligned}
$$

In addition to these contributions from ground ω and ϕ meson, we introduce also a new narrow vector meson with mass around the mass threshold of $\Lambda\bar{\Lambda}$.

In the time like region we consider also the width of vector mesons to introduce the complex structure of the electromagnetic form factors of Λ hyperon. For this purpose, we need to replace

$$
\begin{aligned}
\mathrm{g}(\mathrm{Q}^2) &\rightarrow \frac{1}{(1-\gamma \mathrm{q}^2)^2}, \\
\frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2+\mathrm{Q}^2} &\rightarrow \frac{\mathrm{m}_\omega^2}{\mathrm{m}_\omega^2-\mathrm{q}^2-\mathrm{i}\mathrm{m}_\omega\Gamma_\omega}, \\
\frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2+\mathrm{Q}^2} &\rightarrow \frac{\mathrm{m}_\phi^2}{\mathrm{m}_\phi^2-\mathrm{q}^2-\mathrm{i}\mathrm{m}_\phi\Gamma_\phi}, \\
\frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2+\mathrm{Q}^2} &\rightarrow \frac{\mathrm{m}_\mathrm{x}^2}{\mathrm{m}_\mathrm{x}^2-\mathrm{q}^2-\mathrm{i}\mathrm{m}_\mathrm{x}\Gamma_\mathrm{x}},
\end{aligned}
$$

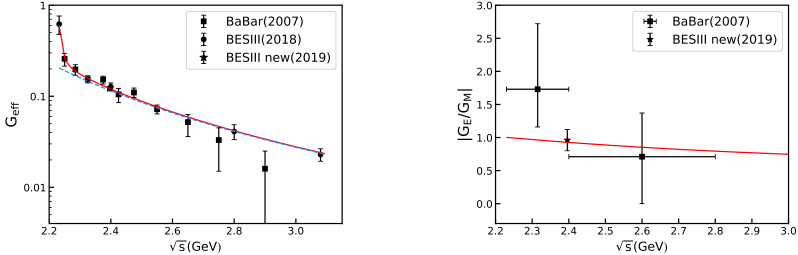


Figure: Fit to Geff of Λ(left).Fit to $|G_E/G_M|$ of Λ(right). The red solid line represents the contribution ω, ϕ and X(2231). The blue dashed line is the result that we remove the contribution of X(2231).

Numerical results: Cross section

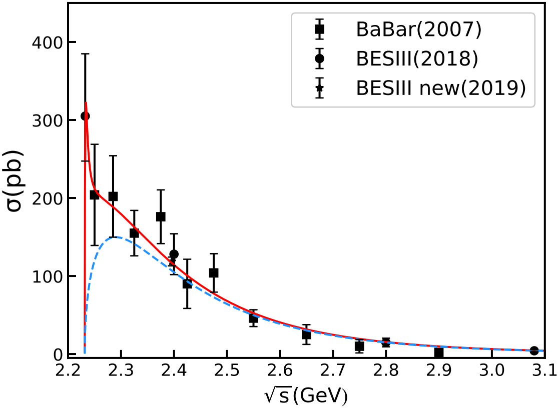


Figure: Cross section of the reaction $e^+e^- \to \Lambda\bar{\Lambda}$. The red solid line represents the contribution ω, ϕ and X(2231). The blue dashed line is the result that we remove the contribution of X(2231).

Parameters

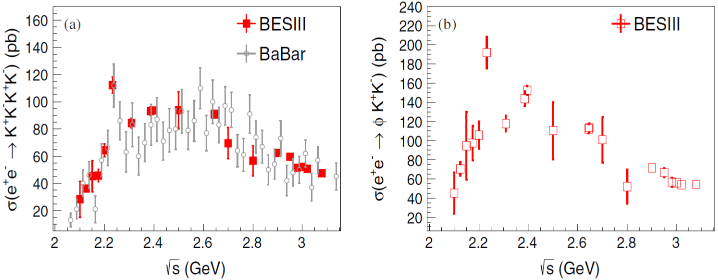
Table: Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.426	β_ω	-1.132
β_ϕ	1.350	α_ϕ	-0.401
β_x	0.0015	$\mathrm{m_x}$ (GeV)	2.2309
Γ_x(GeV)	0.0047		

By some statistical means, we obtain the errors of parameters $\mathrm{m_x}$ and Γ_x, which are: $\mathrm{m_x} = 2230.9^{+3.4}_{-3.5}$MeV, and $\Gamma_\mathrm{x} = 4.7^{+2.2}_{-4.7}$MeV.

(1)According to Chin. Phys., C43(11):113105, 2019, there are two strangeonium meson resonances $\phi(3^3S_1)$ and $\phi(2^3D_1)$, both with masses around 2.2 GeV and $J^P = 1^-$, as a possible source of the observed threshold enhancement.

(2)

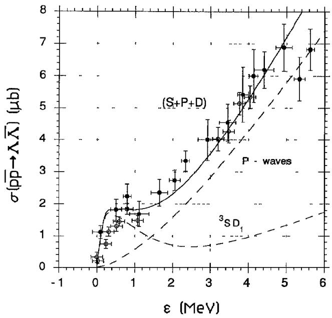


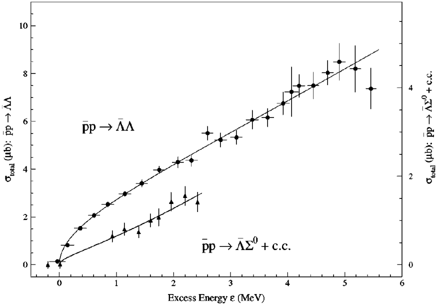
Ablikim, M. and others. Phys. Rev. D 100, 032009(2019).

Figure: (a) Comparison of the measured Born cross section of $e^+e^- \to K^+K^-K^+K^-$ to that of previous measurements. (b)Born cross section of $e^+e^- \to \phi K^+K^-$ obtainedin this work.

Some related studies

P.D. Barnes, et al., Phys. Rev. C 62, 055203(2000).





J. Carbonell, K.V. Protasov, O.D. Dalkarov, Phys. Lett.B 306,407(1993).

Figure: Experimental reaction cross section compared to the CCM calculations(left). Final total cross section results for the $p\bar{p} \to \Lambda\bar{\Lambda}$reaction and the $p\bar{p} \to \bar{\Lambda}\Sigma^0 +$ c.c. reactions(right).

Some related studies

Through the analysis of experimental data, their results show the possibility to explain the nearthreshold structure by the existence of a narrow $\Lambda\bar{\Lambda}$ subthreshold state of quasinuclear nature. This resonance, produced in the $^3SD_1$ partial wave, has quantum numbers $J^{PC} = 1^{--}$. The reasons for the smallness of its width are the D-wave dominance of its wavefunction (96 %).ϵ =- 2.0 MeV and $\Gamma = 1.8$ MeV

Some experimental possibilities to find this resonance are proposed:

1. $p\bar{p} \to \Lambda\bar{\Lambda}$
2. $p\bar{p} \to K\bar{K} + n\pi (n = 2, 3, 4)$
3. $e^+e^- \to \Lambda\bar{\Lambda}$

J. Carbonell, K.V. Protasov, O.D. Dalkarov, Phys. Lett.B 306,407(1993).

We propose a possible $\Lambda\bar{\Lambda}$ quasi-bound state X(2231) in the $e^+e^- \to \Lambda\bar{\Lambda}$, $m_x = 2230.9^{+3.4}_{-3.5}$MeV, and $\Gamma_x = 4.7^{+2.2}_{-4.7}$MeV.

It could be also the one shown in $e^+e^- \to K^+K^-K^+K^-(\phi K^+K^-)$ reaction.

Some physicist suggested that these phenomena may be caused by dynamics, this requires experimental and theoretical research, too.

Thanks