Hydrodynamics and flow in ultrarelativistic heavy-ion collisions

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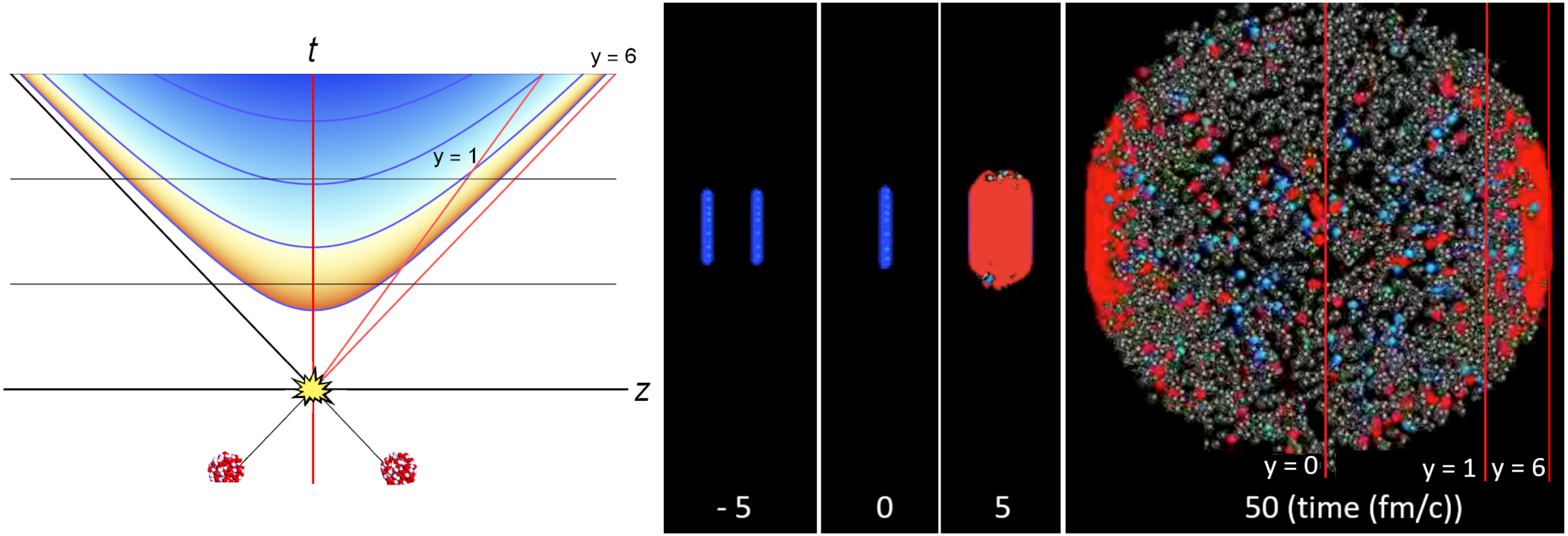
RHIC Beam Energy Scan: Theory and Experiment
Seminar series III, Oct.12 , 2021

Sketch of a Pb+Pb collision at LHC



Busza et al. 1802.04801

- The collision creates strongly-coupled quark-gluon matter, governed by strong interactions, which expands into the vacuum. ~30000 particles produced at the end.
- The best theoretical description is a macroscopic one: a small lump of fluid.

Ideal and viscous hydrodynamics

Fluid dynamics is a macroscopic description. One does not follow the particles individually, but only the evolution in space and time of the energy and momentum.

Energy-momentum tensor $T_{\mu\nu}$ = ideal fluid+viscous corrections.

$$T_{\mu\nu} = \varepsilon\, u_\mu u_\nu + p[\varepsilon]\Delta_{\mu\nu} - \eta[\varepsilon]\,\sigma_{\mu\nu} - \zeta[\varepsilon]\Delta_{\mu\nu}\nabla_\mu u^\mu + \mathcal{O}(\partial^2)$$

Ideal fluid.
equation of state of QCD

Shear viscosity

Bulk viscosity

Hydrodynamics applied to heavy ions

- One chooses an initial condition, inspired by what one knows about the early stages of the collision, before a fluid is formed, usually allowing for event-by-event fluctuations.
- The expansion into the vacuum is modeled by the equations of hydrodynamics, which involve an equation of state and transport coefficients (viscosities).
- The fluid freezes out into individual hadrons (hadronization) which may undergo further interactions, and eventually decay into stable hadrons.
- The output of the calculation is the momentum distribution of identified hadrons in every event, which can be compared with experimental data: particle spectra, and correlations (typically anisotropic flow).

Approaches to hydro/data comparison

- There are many parameters entering the hydrodynamic calculations, and many experimental observables which can be compared with calculations. This has motivated the development of global Bayesian analyses.

  Novak Novak Pratt Vredevoogd Coleman-Smith 1303.5769
  Bernhard Moreland Bass Liu Heinz 1605.03954
  Nijs van der Schee Gürsoy Snellings 2010.15134
  JETSCAPE Collaboration 2011.01430

- In this talk, I describe simpler approaches, by selecting specific bulk observables, showing that they depend on specific parameters, and studying this dependence in unprecedented detail.

Outline

1. Measuring the equation of state of QCD using $<p_t>$ and $dN/d\eta$ of charged hadrons.

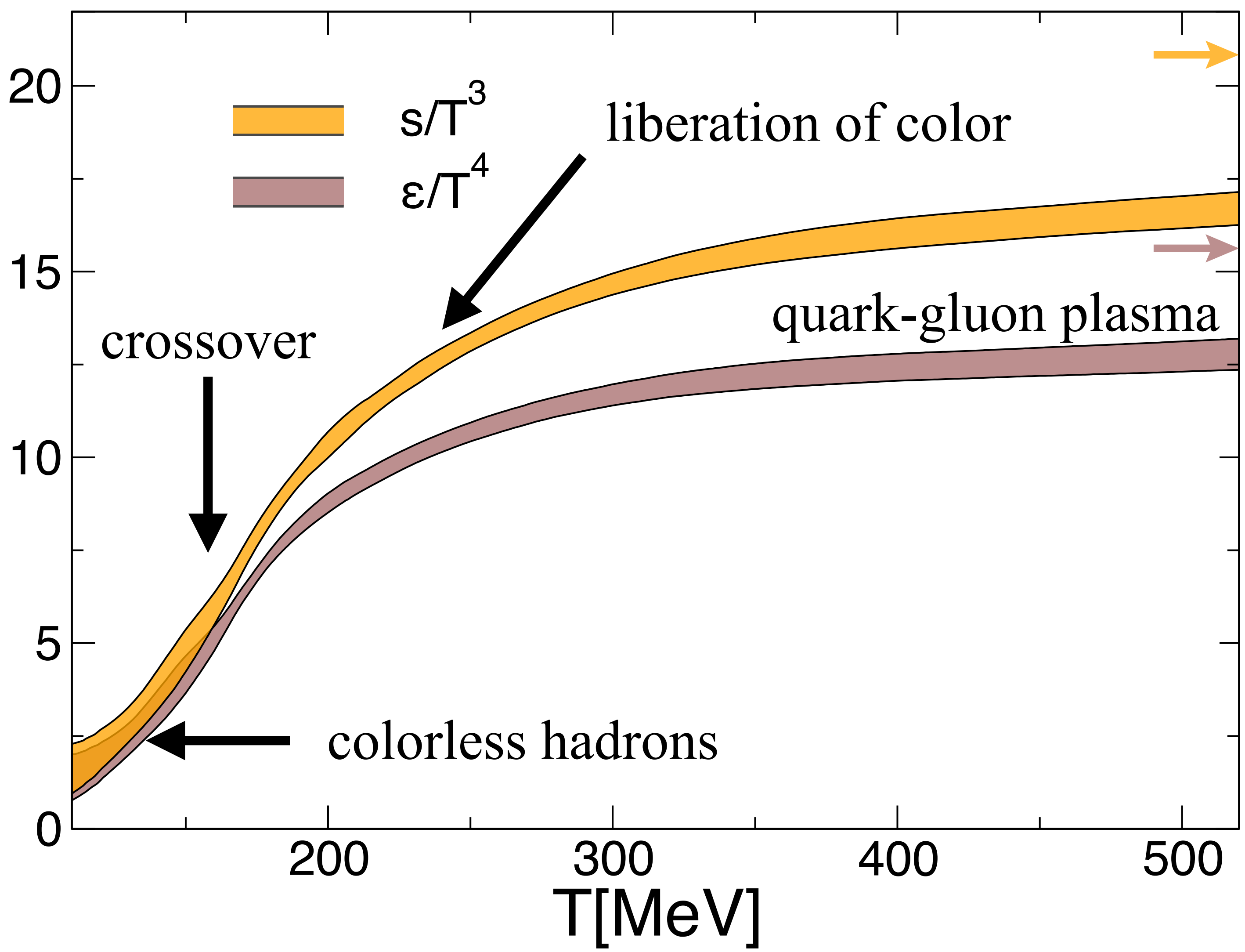
2. What we can learn about temperature-dependent shear and bulk viscosities using v_2 and v_3 of charged hadrons.

3. Event-to-event initial state fluctuations: the success of hydrodynamics in describing anisotropic flow fluctuations in proton-nucleus and nucleus-nucleus collisions.

1. Measuring the equation of state of QCD using $<p_t>$ and $dN/d\eta$ of charged hadrons.

1908.09728, with Fernando Gardim, Giuliano Giacalone, Matt Luzum,
Nature Physics 16 (2020) 6, 615-619

The equation of state of strong-interaction matter



Now accurately calculated from first principles using lattice QCD

Can we confirm some of these results with heavy-ion data ?

Borsanyi et al,1309.5258

What was already known

State-of-the-art hydrodynamic simulations of nucleus-nucleus collisions all use an equation of state taken from (or inspired by) these lattice QCD results. They do a good job in reproducing experimental data.

Experienced hydro practitioners have known for decades that if one runs hydro with a very different equation of state, the calculated p_t spectra differ from the measured ones.

Our contribution: find a simple and robust correspondence between equation of state and data, which allows for quantitative comparison, including realistic error estimates.

An old idea by Léon Van Hove (1982)

ELSEVIER

Physics Letters B
Volume 118, Issues 1–3, 2 December 1982, Pages 138-140

Multiplicity dependence of p_t spectrum as a possible signal for a phase transition in hadronic collisions

L. Van Hove

Abstract

It is argued that the flattening of the transverse momentum (pt) spectrum for increasing multiplicity n, observed at the CERN proton-antiproton collider for charged particles in the central rapidity region, may serve as a probe for the equation of state of hot hadronic matter. We discuss the possibility that this pt versus n correlation could provide a signal for the deconfinement transition of hadronic matter.

An old idea by Léon Van Hove (1982)

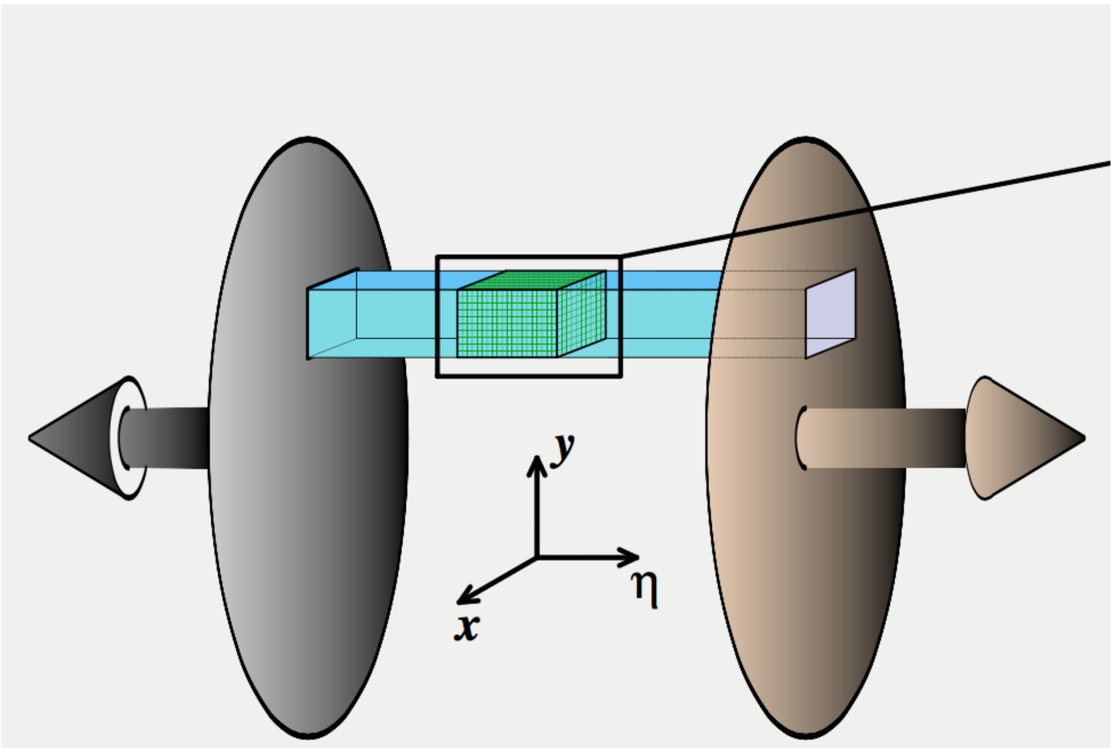
The mean transverse momentum of (almost massless) particles seen in detector, $\langle p_t \rangle$, is a fraction of the energy per particle: proportional to temperature T.

The multiplicity N_{ch} is proportional to the entropy density s if the volume is fixed.

Vary $\sqrt{s}$ of central Pb+Pb collisions: volume is ~fixed:
$\langle p_t \rangle$ vs. N_{ch} gives T versus s = equation of state.

A thermodynamic argument, which we have rephrased in the framework of the hydrodynamical description.

Longitudinal cooling



(illustration by François Gelis)

In heavy-ion collisions, we observe a slice of fluid near mid-rapidity.
Its energy decreases according to $dE=-PdV$ as the system expands (Bjorken, 1983).
Its entropy increases slightly due to viscosity.

Since hadrons are emitted at the end of the evolution, we expect that their $\langle p_t \rangle$ is determined by the entropy and energy of the fluid at freeze-out.

Effective temperature, effective volume

We define the effective temperature, $T_{\rm eff}$, and the effective volume, $V_{\rm eff}$, of the quark-gluon plasma, as those of a uniform fluid at rest which would have the same energy E and entropy S as the fluid at freeze-out. *(extensive quantities E, S, $V_{\rm eff}$ are meant per unit rapidity)*

$$E = \int_{\rm f.o.} T^{0\mu} d\sigma_\mu = \epsilon(T_{\rm eff}) V_{\rm eff} ,$$
$$S = \int_{\rm f.o.} s u^\mu d\sigma_\mu = s(T_{\rm eff}) V_{\rm eff} ,$$

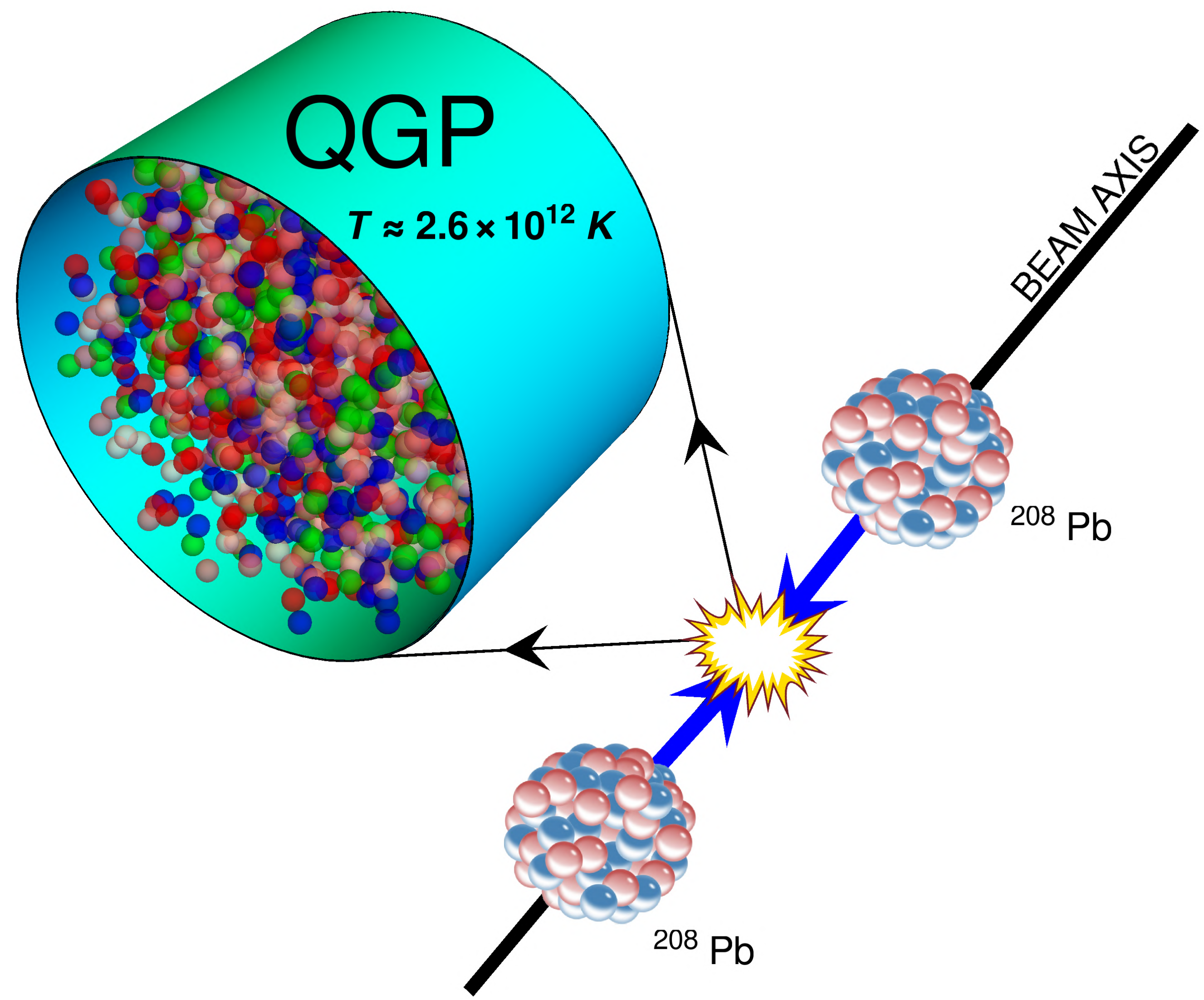
These equations are solved to calculate $T_{\rm eff}$ and $V_{\rm eff}$
= simple quantities, yet non-trivial ones. Hydro practitioners: calculate them!
$T_{\rm eff}$ is smaller than the initial temperature due to longitudinal cooling
larger than the freeze-out temperature due to transverse flow.

I show how $T_{\rm eff}$ and $s(T_{\rm eff})$ relate to experimental observables.

Effective temperature, effective volume

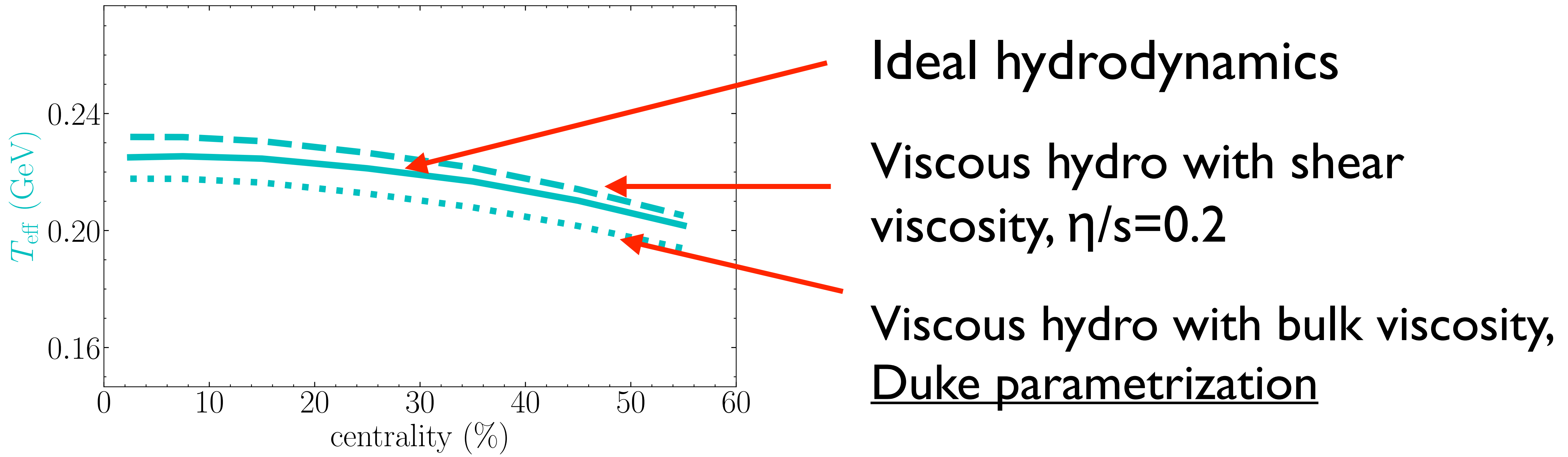
Put the total energy and entropy contained in one rapidity unit of the fluid, just before it transforms into hadrons, into a uniform cylinder.

Effective temperature and volume are those of this cylinder.



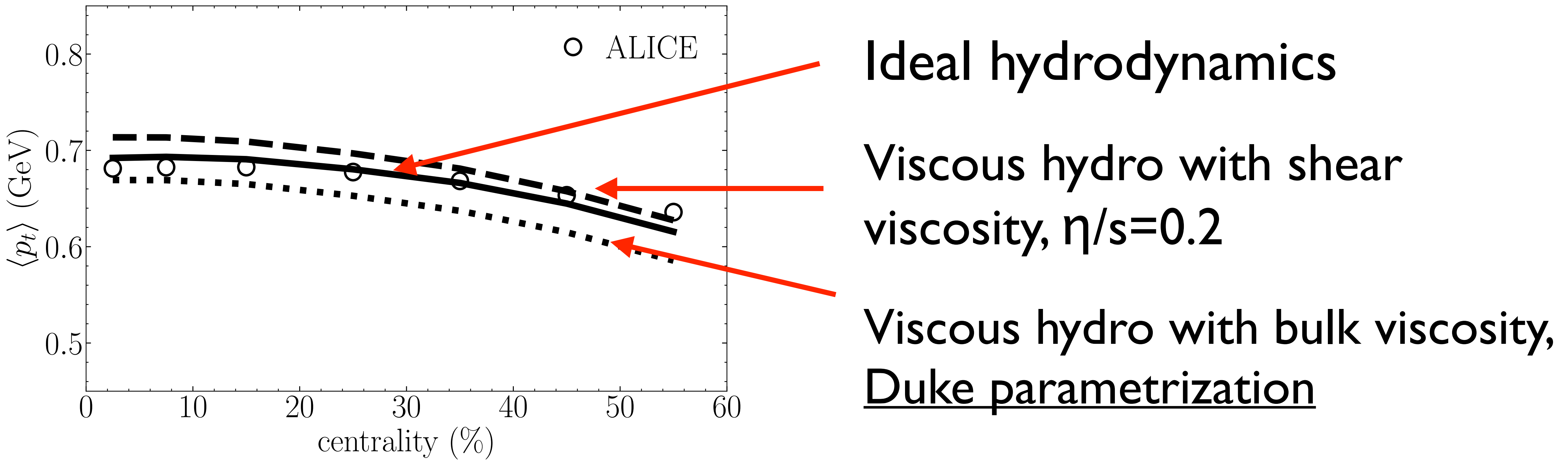
Value of T_{eff} in hydro simulations of Pb+Pb @ 5.02 TeV

Hydro code = MUSIC.
Smooth initial density profile, normalization tuned to reproduce the charged multiplicity measured by ALICE.



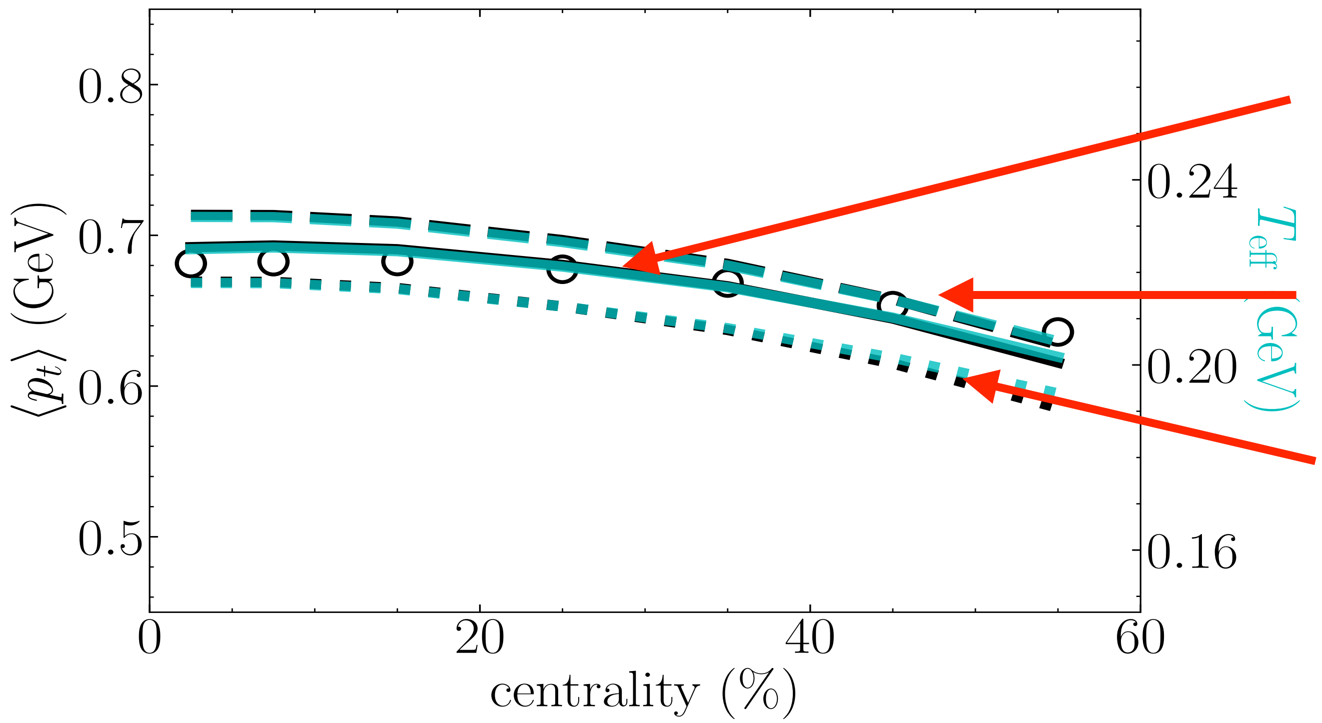
Value of <p_t> in hydro simulations of Pb+Pb @ 5.02 TeV

We compute the average transverse momentum of particles at the end of the fluid expansion (and after resonance decays)



Reviving Van Hove's idea

$\langle p_t \rangle = 3.07\, T_{eff}$ for all centralities, irrespective of bulk and shear viscosity!



Ideal hydrodynamics

Viscous hydro with shear viscosity, η/s=0.2

Viscous hydro with bulk viscosity, Duke parametrization

Value of T_{eff} from experiment

Extraction from data is straightforward.
ALICE measures $\langle p_t \rangle$ = 681 MeV
in Pb+Pb @ 5.02 TeV in 0-5% centrality bin.

This implies $T_{eff} = \langle p_t \rangle / 3.07$ = 222 ± 9 MeV,

where the error is estimated by varying the freeze-out temperature.

Note that $\langle p_t \rangle$, hence T_{eff}, depends very little on the collision centrality.
It depends also very little on the system size (e.g. Xe-Xe)

Next step: entropy density at T_{eff}

Entropy density = S/V_{eff}

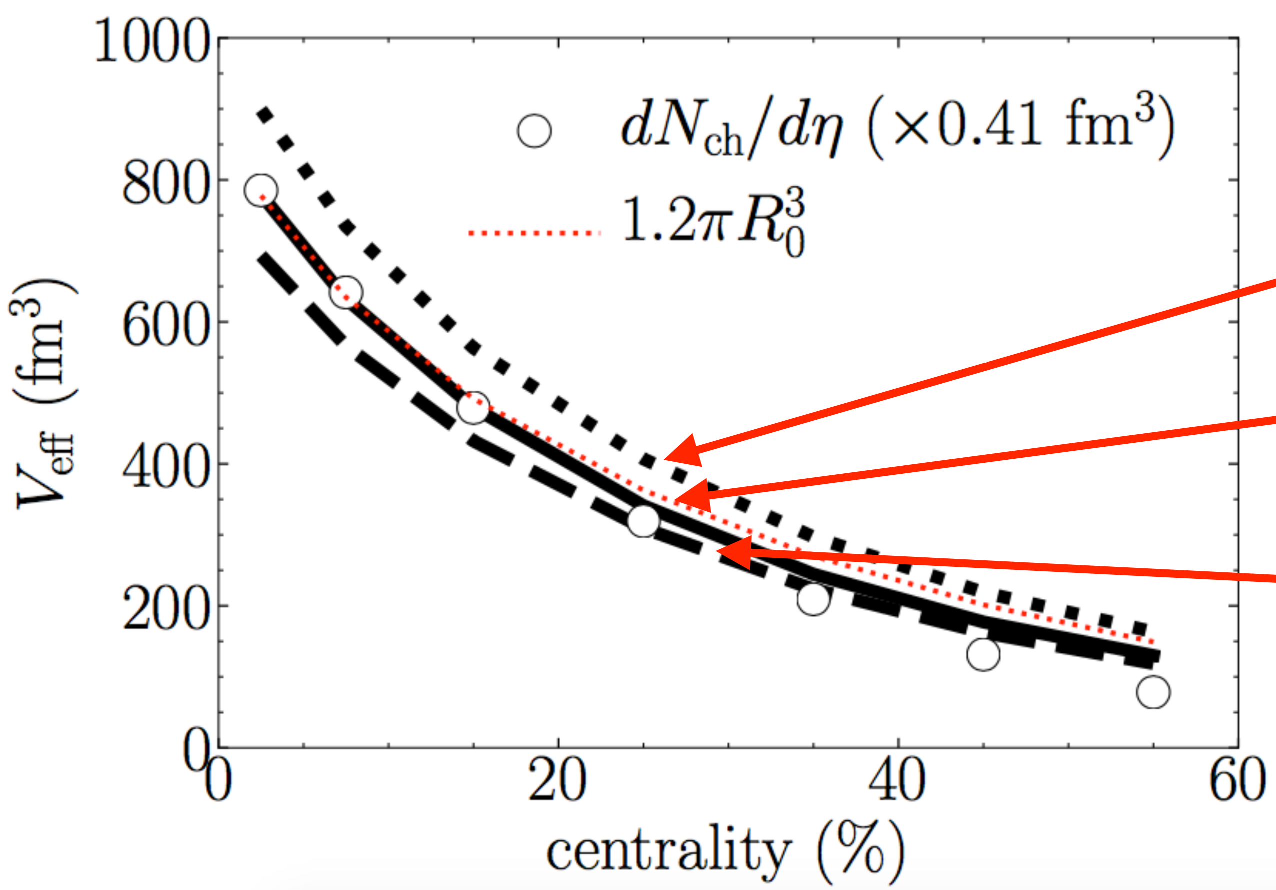
S =entropy *at freeze-out*, by definition of V_{eff} and T_{eff}
$S/N_{ch} = 6.7 \pm 0.8$ after resonance decays,

Hanus Mazeliauskas Reygers, 1908.02792

N_{ch} is measured, therefore, S is known

Effective volume V_{eff} *cannot be extracted from data.*
Comes from a hydrodynamic *calculation.*

Estimating the effective volume



Viscous hydro with bulk viscosity, Duke parametrization

Ideal hydrodynamics

Viscous hydro with shear viscosity, η/s=0.2

V_{eff} varies with centrality like R_0^3, where R_0 = initial transverse radius.

~ 6 fm for central Pb+Pb collisions

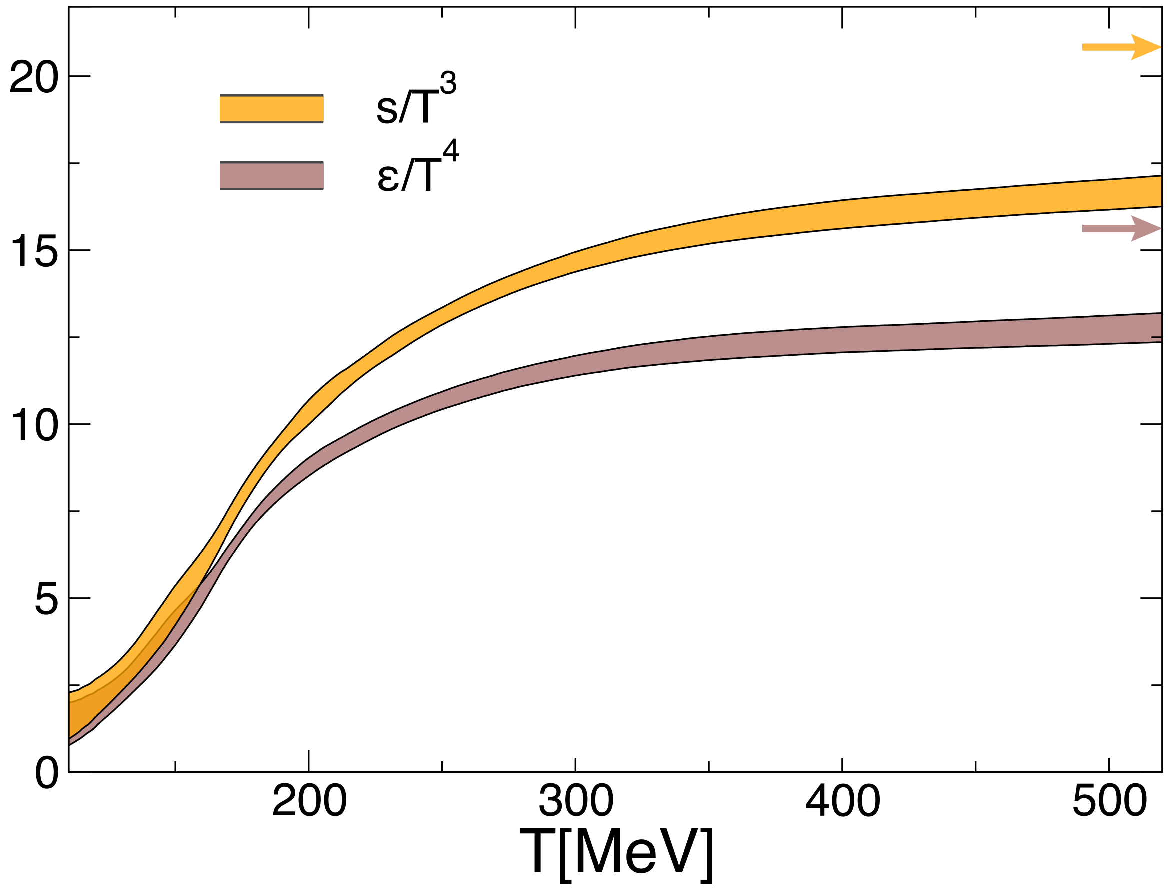
Entropy density at T_{eff}

We obtain $S/V_{eff} = s(T_{eff}) = 20 \pm 5\ \mathrm{fm}^{-3}$.

error on V_{eff} : 40% from initial size R_0, which depends on the model of initial conditions

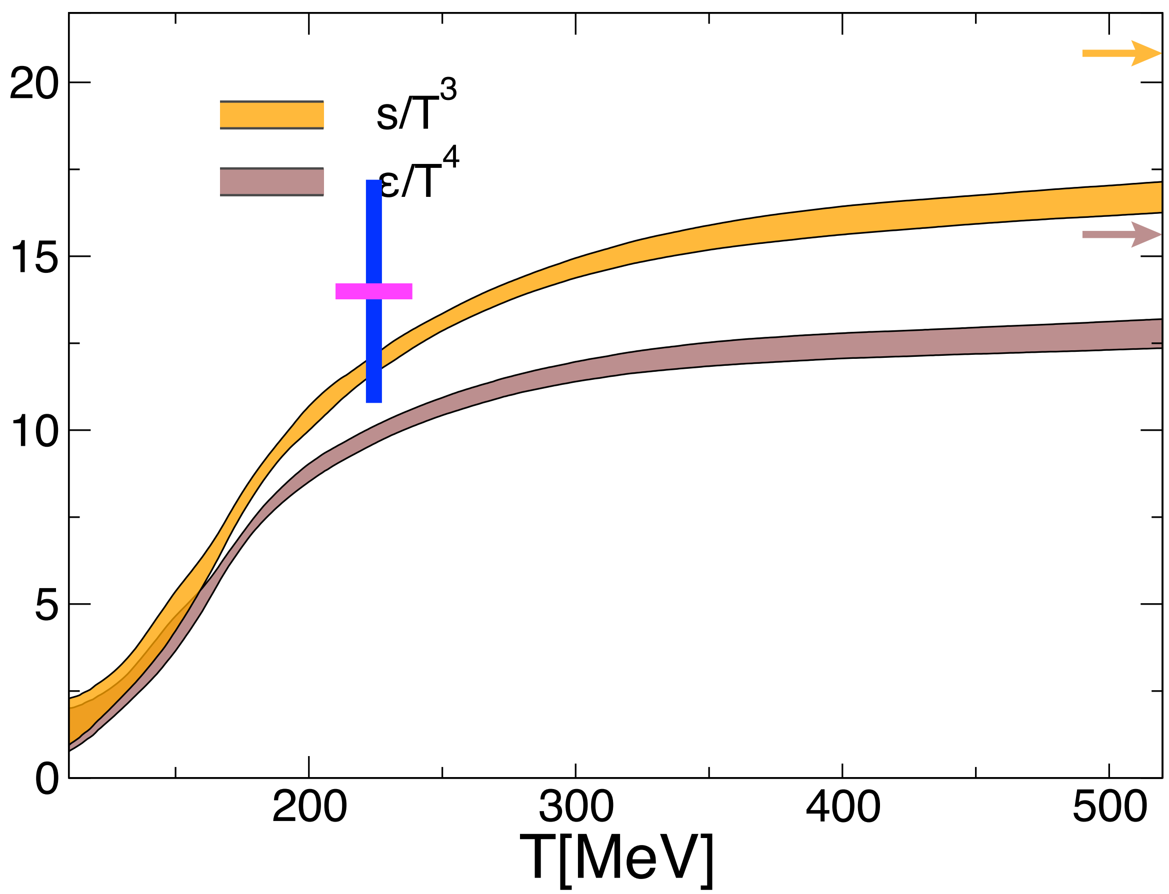
60% from transport coefficients, which modify V_{eff} / R_0^3

Comparison with lattice QCD



Entropy density

Comparison with lattice QCD



T_{eff}=222 ± 9 MeV

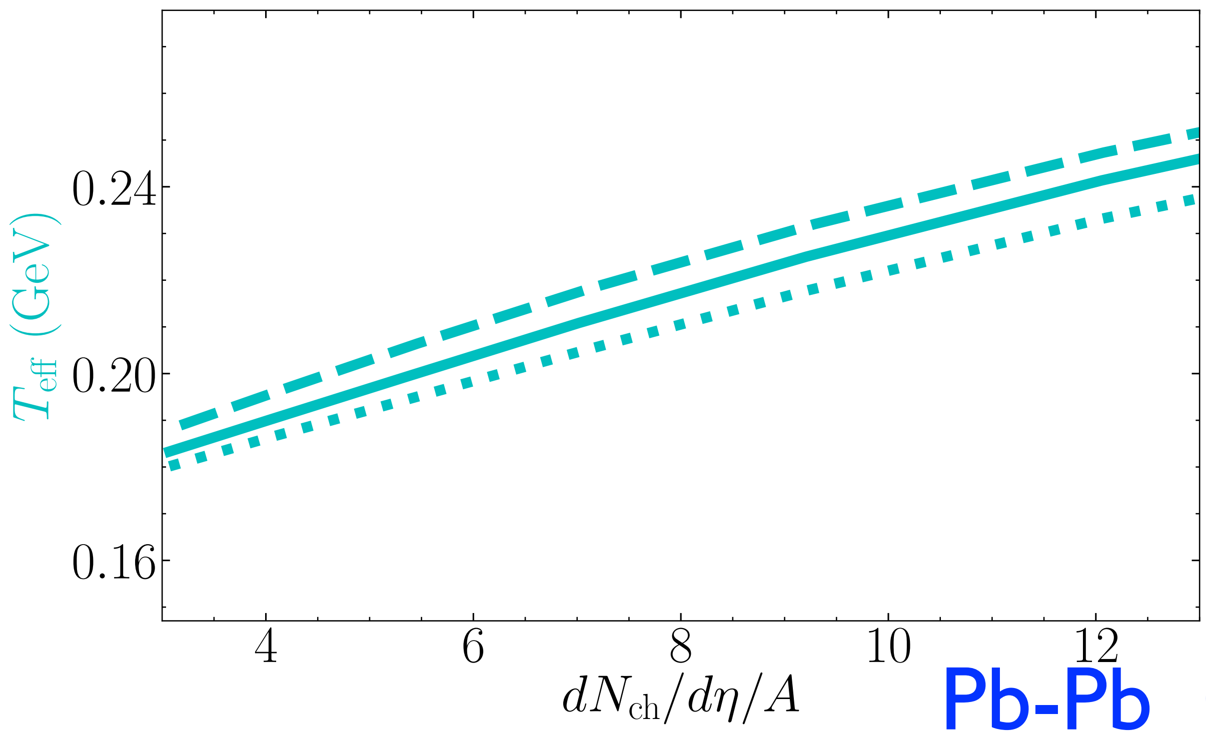
$s(T_{eff})/T_{eff}^3$ = 14 ± 3.5

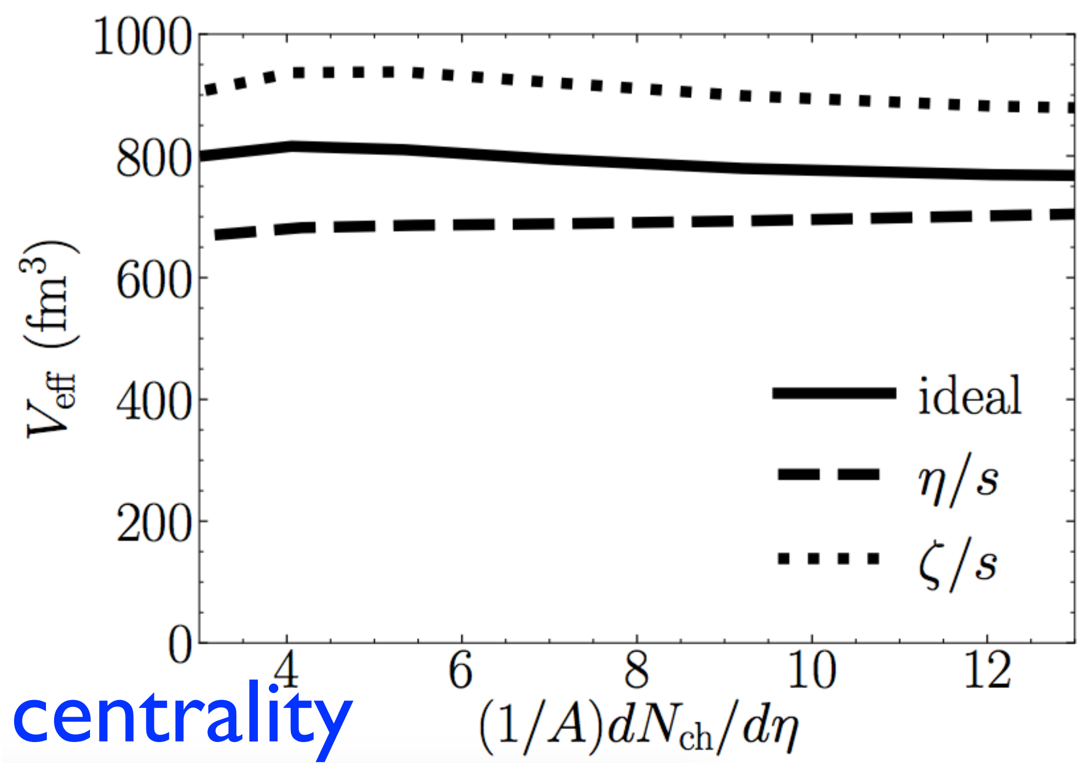
compatible with lattice.

Confirms large number of degrees of freedom, implying that color is liberated: ***deconfinement observed!***

Varying the collision energy

(results are plotted as a function of number of produced particles, which itself depends on collision energy)



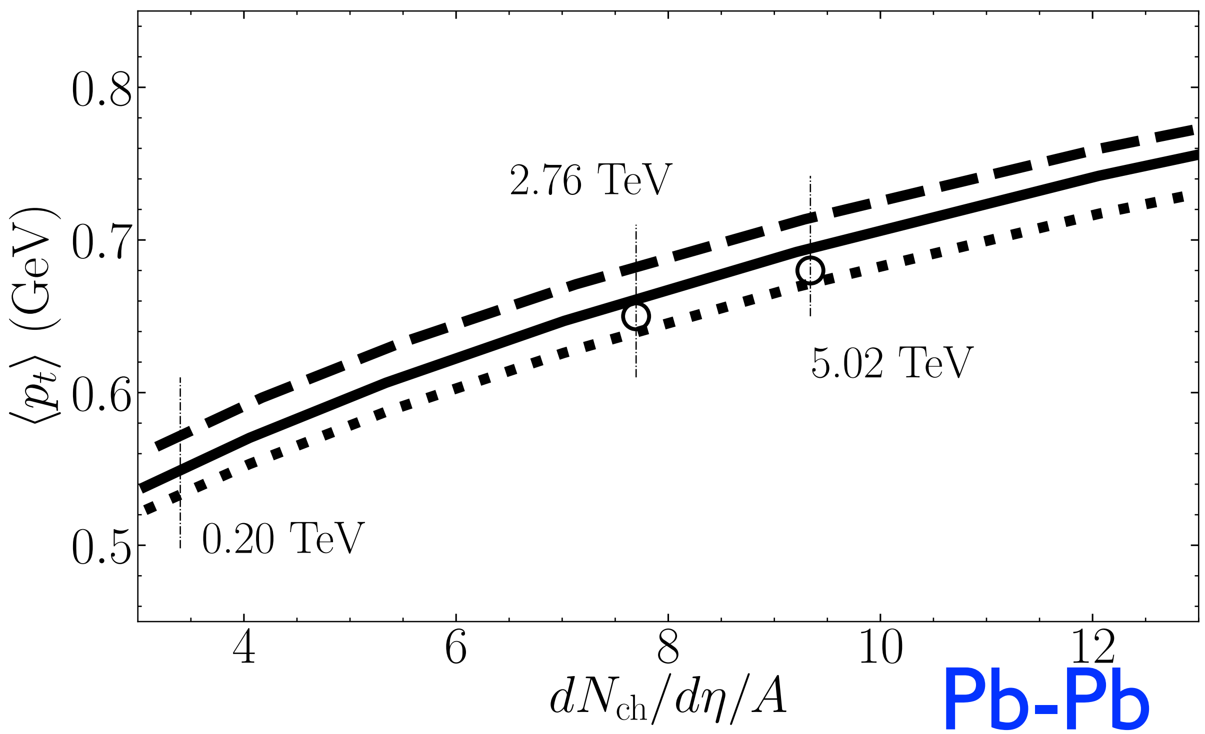


Pb-Pb 0-5% centrality

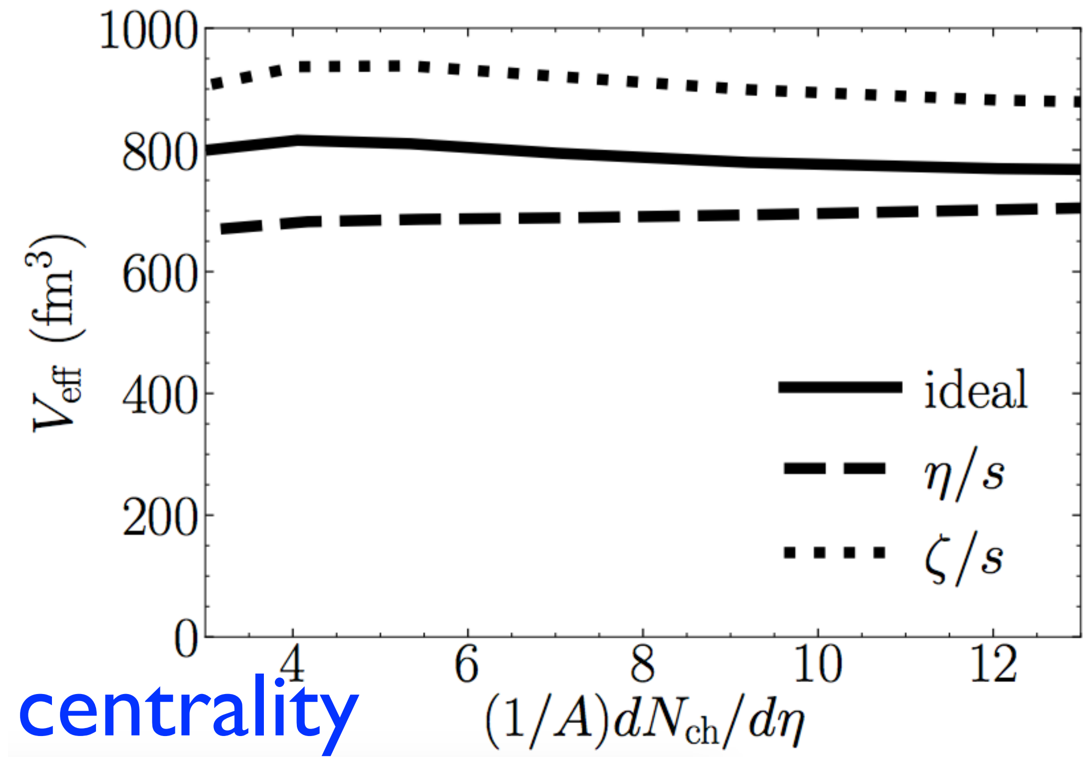
As energy increases, T_{eff} increases, V_{eff} remains constant. Increasing the collision energy amounts to heating the system at constant volume.

Varying the collision energy

(results are plotted as a function of number of produced particles, which itself depends on collision energy)



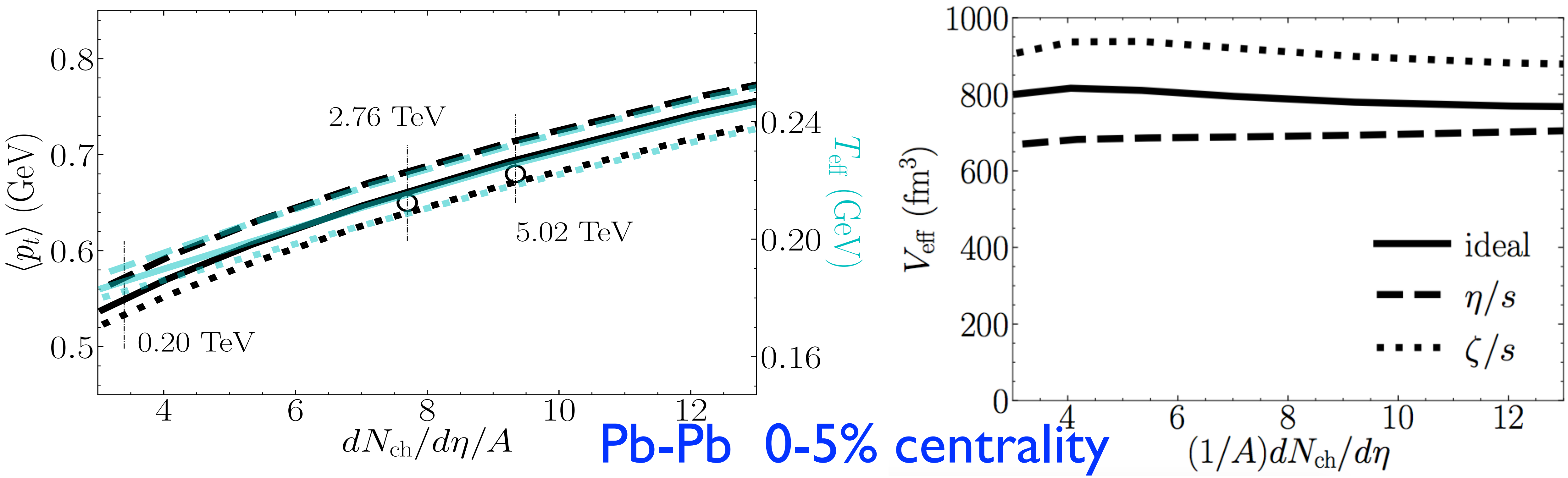
Pb-Pb 0-5% centrality



The variation of $<p_t>$ still closely follows that of T_{eff}

Varying the collision energy

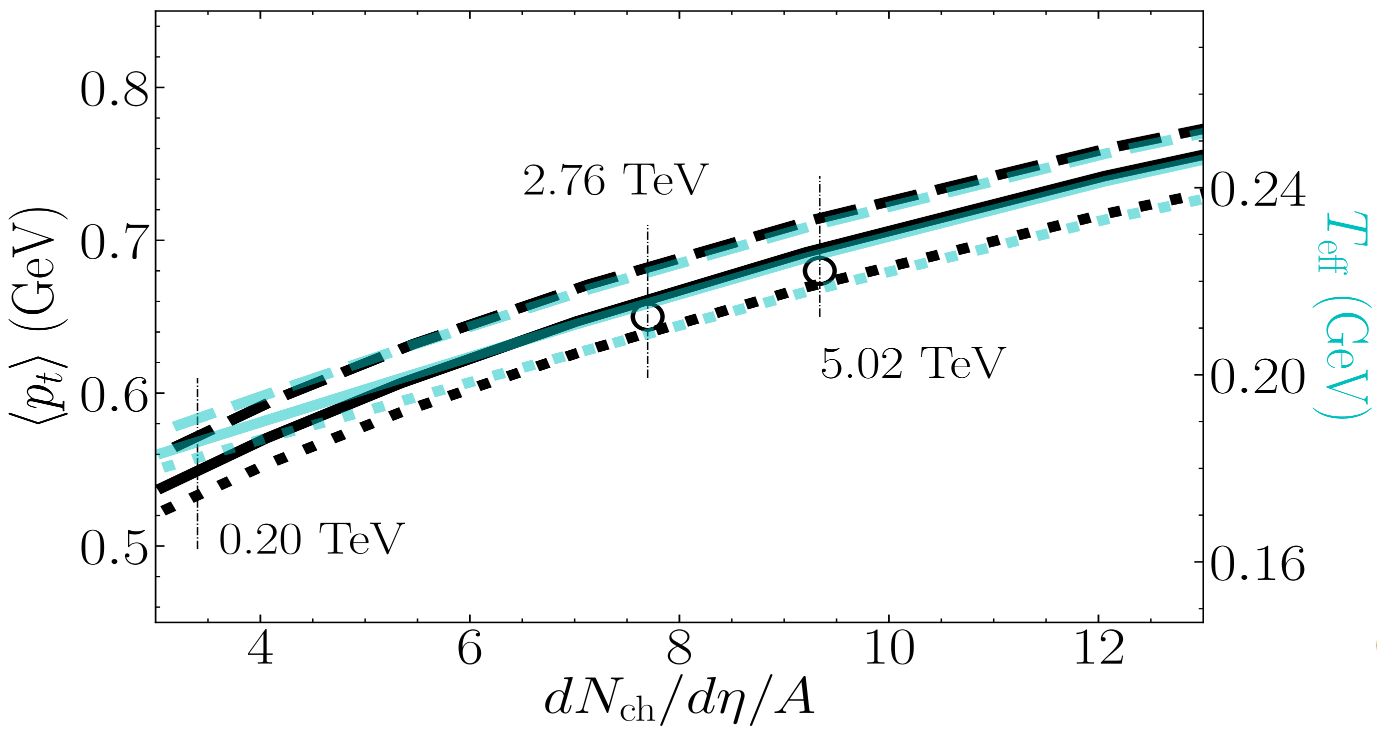
(results are plotted as a function of number of produced particles, which itself depends on collision energy)



Pb-Pb 0-5% centrality

Deviations from <p_t>=3.07 T_{eff} are negligible at LHC energy and beyond

Speed of sound c_s in the QGP



The math:

$T_{\rm eff}$ proportional to $\langle p_t \rangle$

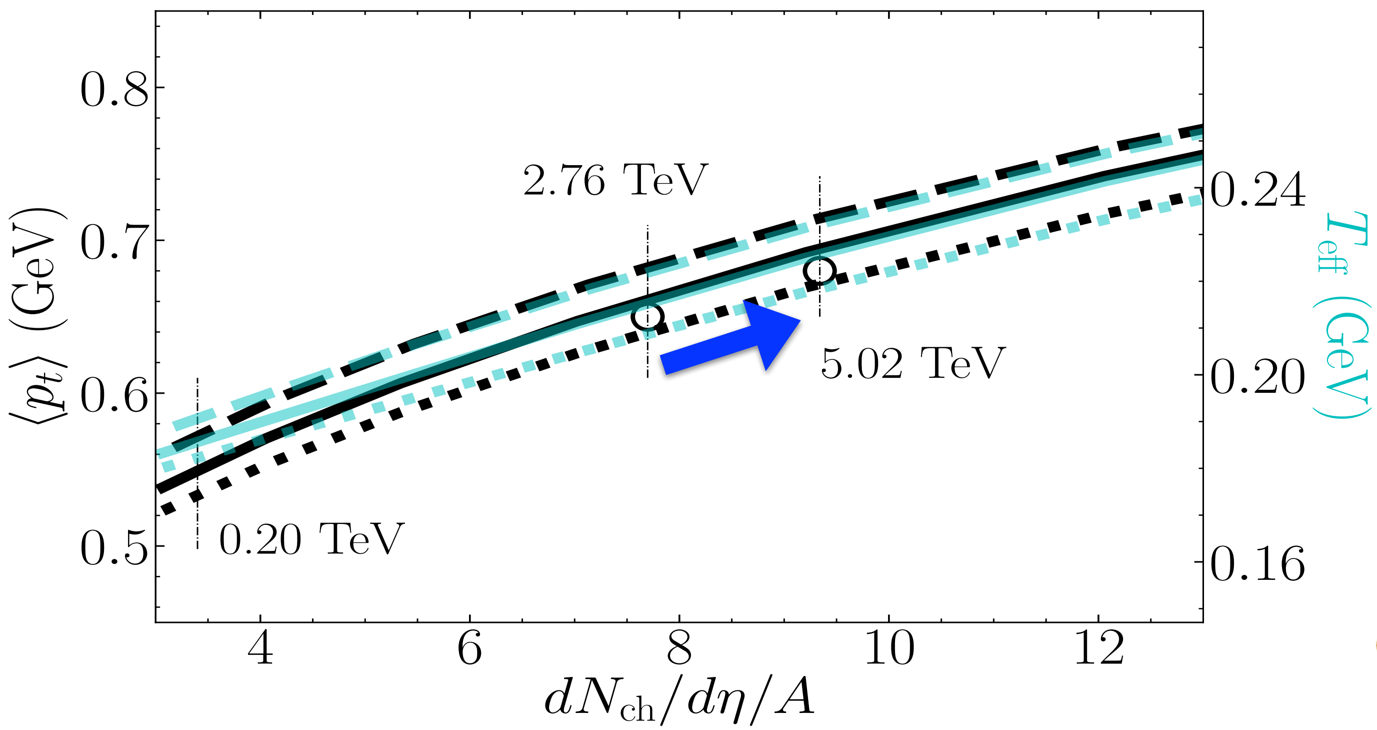
$s(T_{\rm eff})$ proportional to $dN_{ch}/d\eta$

$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \left.\frac{sdT}{Tds}\right|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}$$

The physics:

Increasing the collision energy amounts to putting more energy into a fixed volume. Gives direct access to the compressibility=speed of sound.

Speed of sound c_s in the QGP



T_{eff} proportional to $<p_t>$

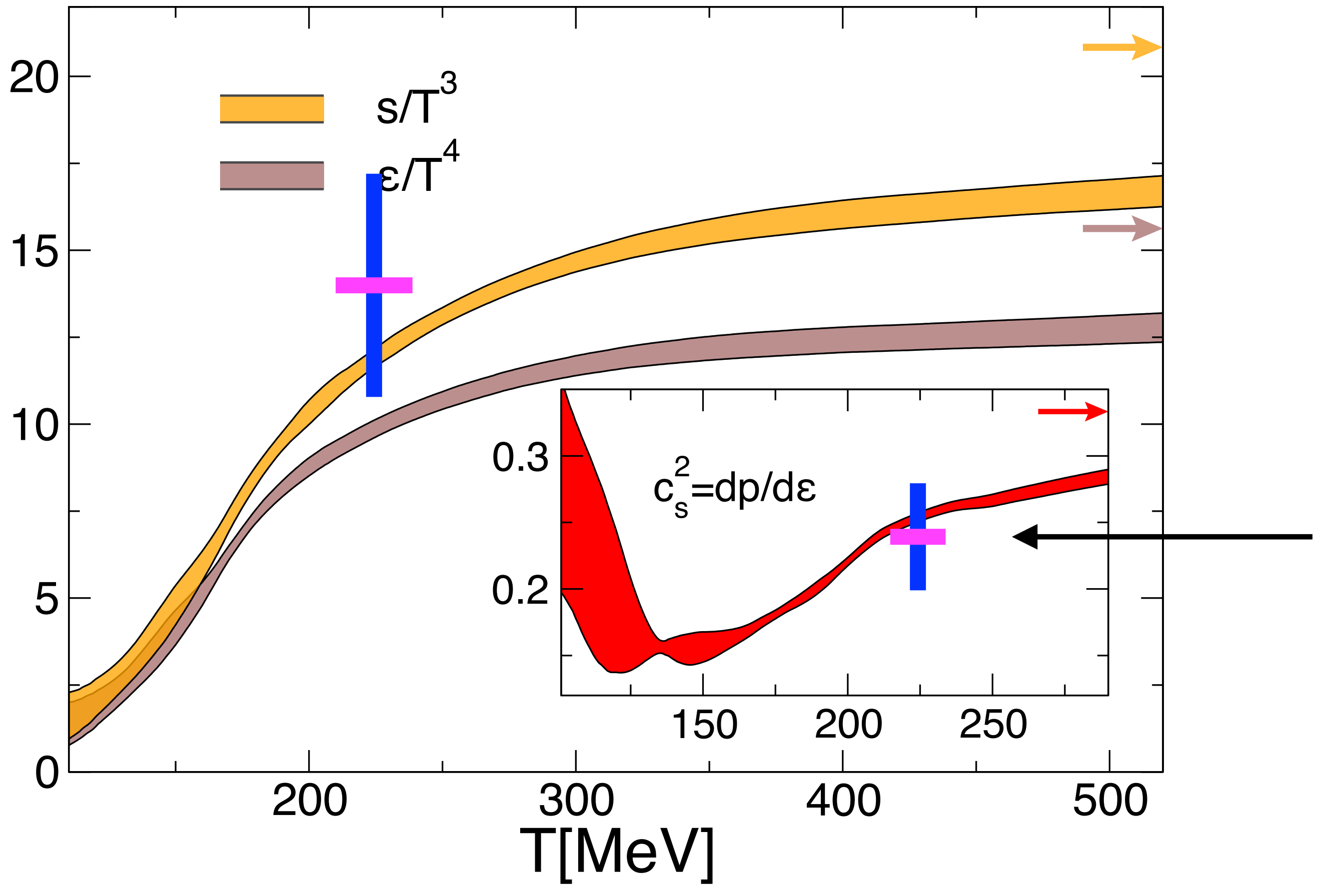
$s(T_{eff})$ proportional to $dN_{ch}/d\eta$

$$c_s^2(T_{\rm eff}) \equiv \frac{dP}{d\varepsilon} = \left.\frac{sdT}{Tds}\right|_{T_{\rm eff}} = \frac{d\ln\langle p_t\rangle}{d\ln(dN_{\rm ch}/d\eta)}$$

we obtain $c_s^2(T_{eff}) = 0.24 \pm 0.04$

(error from variation of V_{eff})

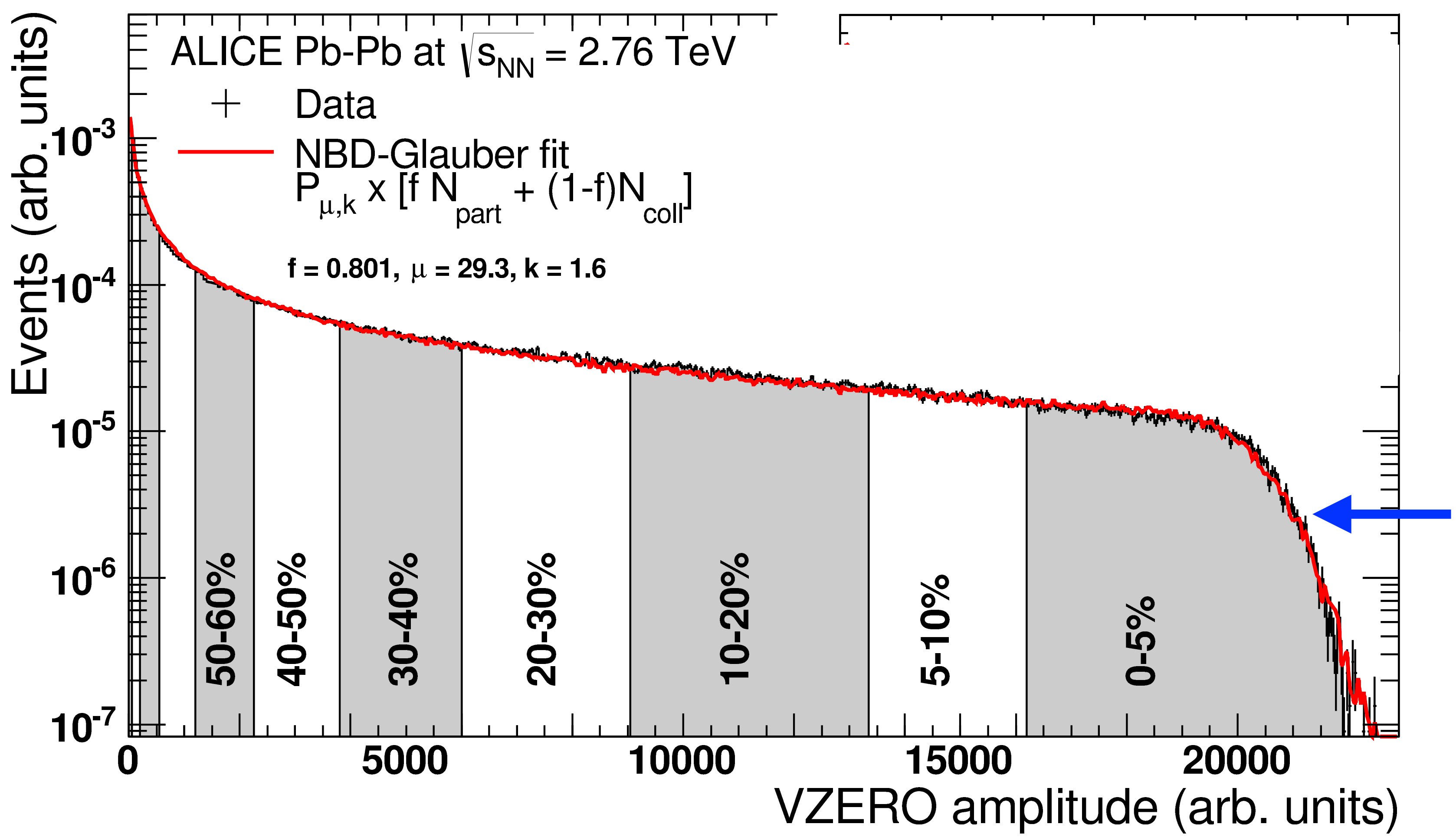
Comparison with lattice QCD



compatible with lattice

Predictions for ultracentral collisions

An alternative method for measuring the speed of sound.
Fix the collision energy, but increase the multiplicity by selecting *ultracentral* collisions



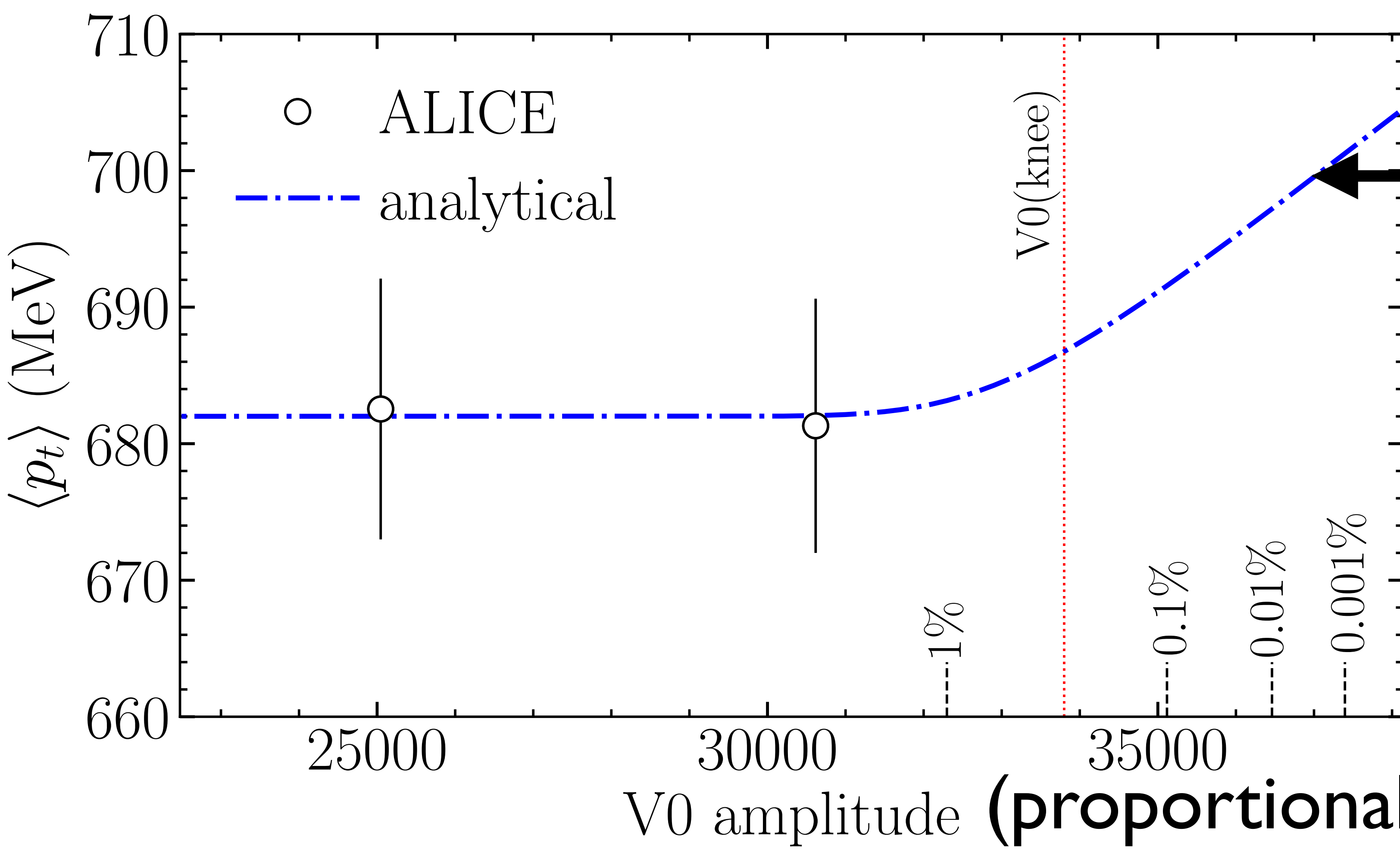
ALICE arXiv:1301.4361

VZERO amplitude =quantity used by ALICE to determine the centrality

Ultracentral collisions: beyond the *knee*, impact parameter is close to 0 but multiplicity keeps increasing

Predictions for ultracentral collisions

We predict an increase of <p_t> in ultracentral collisions.
No hydro, no free parameter. We take c_s^2 from lattice EOS.



c_s^2=d log<p_t>/d logV0

(proportional to multiplicity)

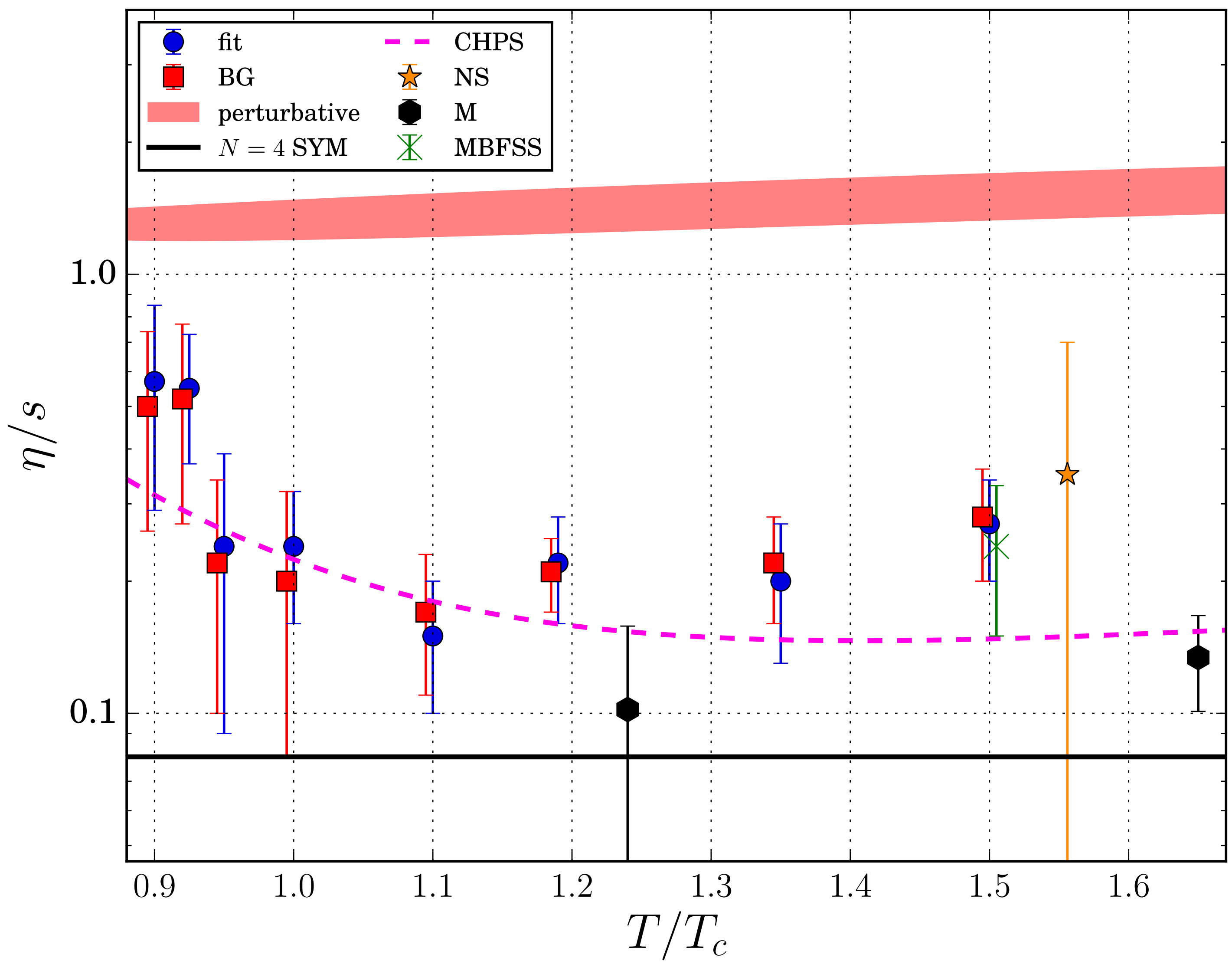
Gardim Giacalone JYO 1909.11609

2. What we can learn about temperature-dependent shear and bulk viscosities using v_2 and v_3 of charged hadrons.

2010.11919, with Fernando Gardim.

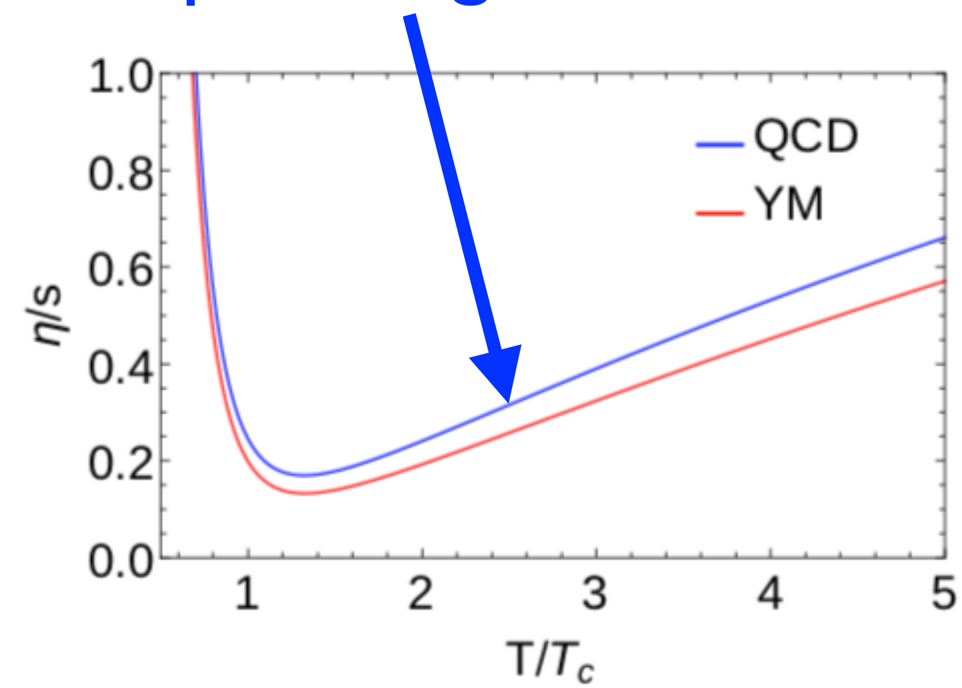
Shear viscosity of QCD

gluons only



Astrakhantsev et al. 1701.02266

quarks+gluons

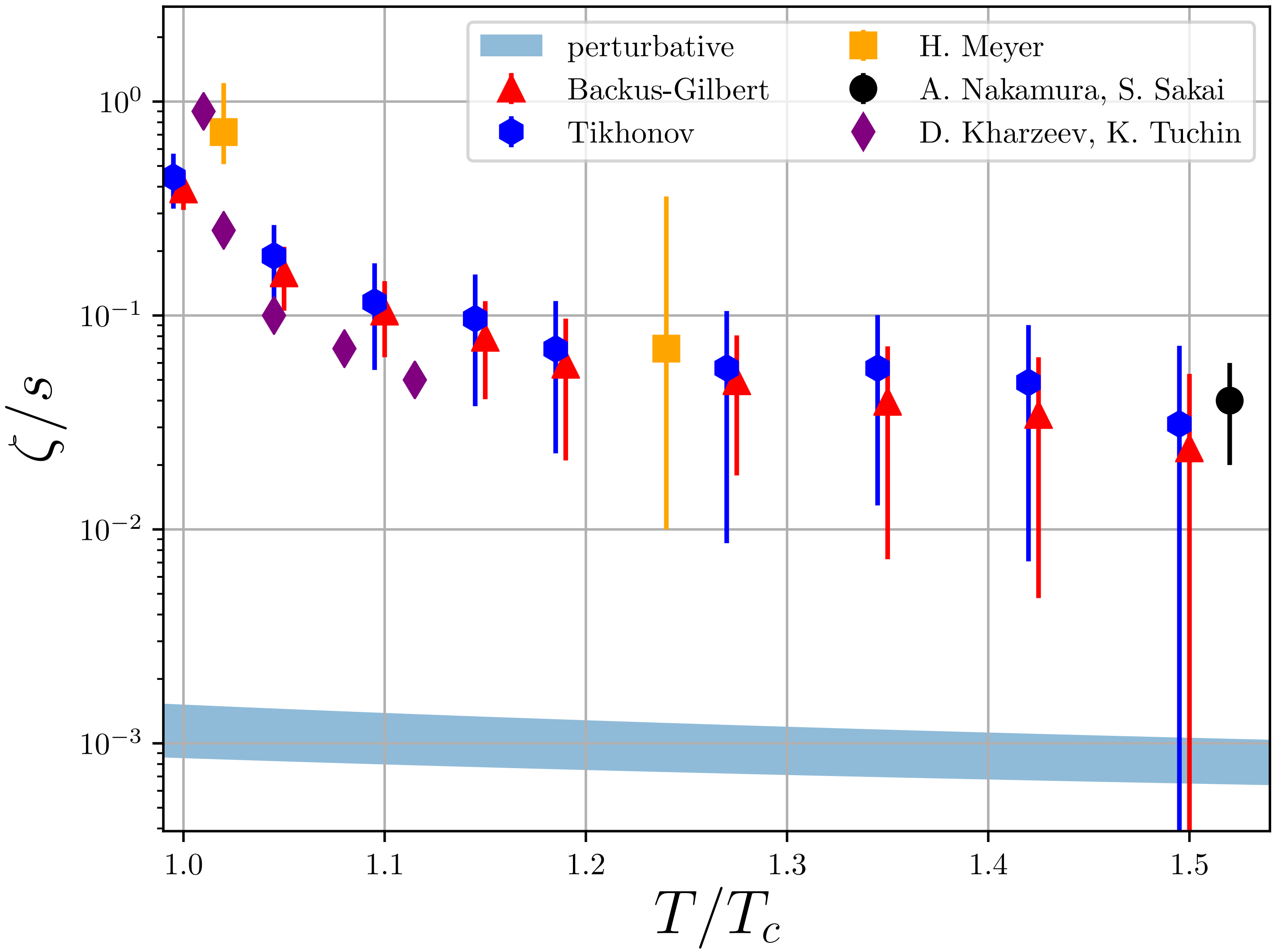


Christiansen et al. 1411.7986

Uncertainties are large.

Bulk viscosity of QCD

gluons only



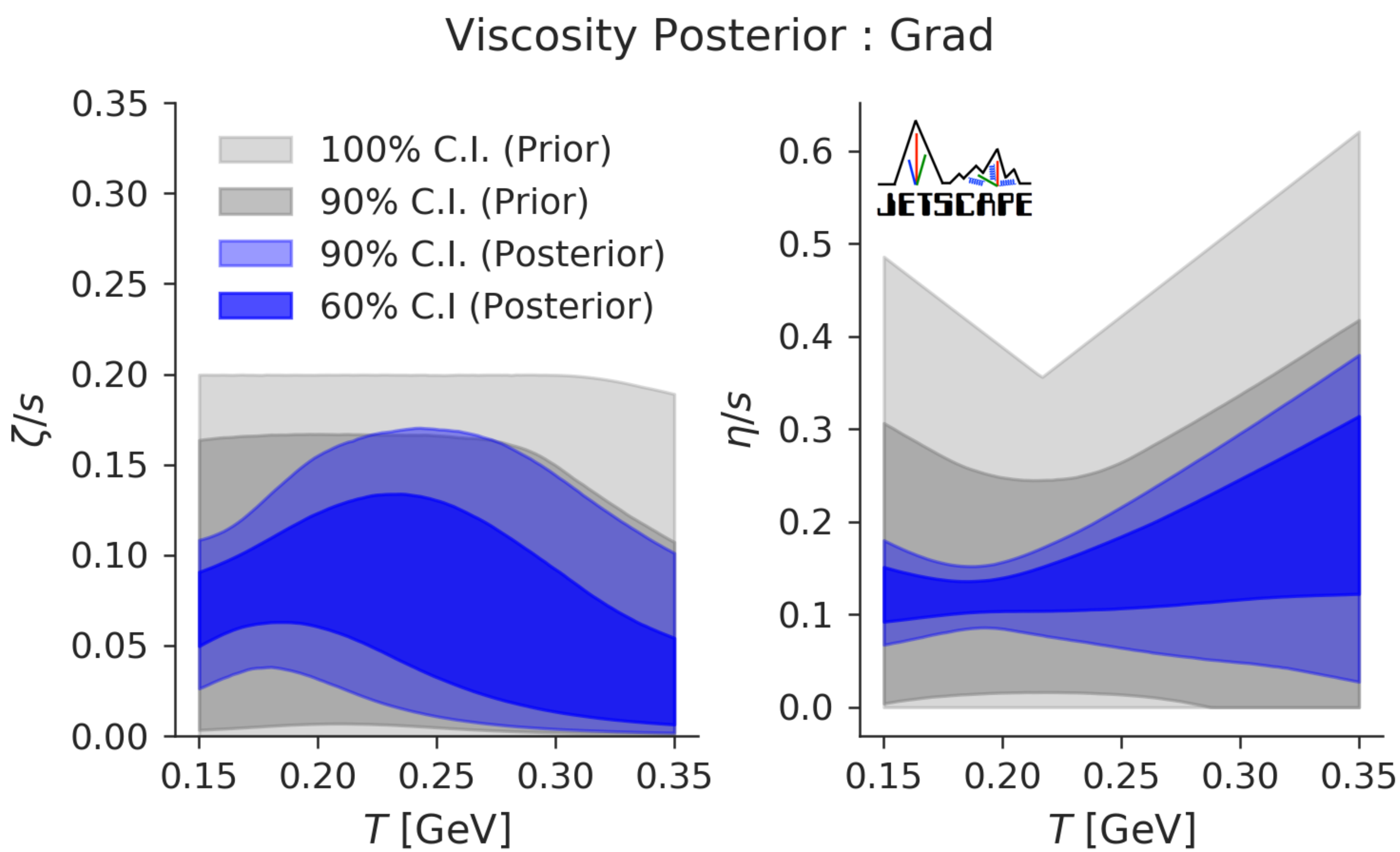
Astrakhantsev et al. [1804.02382]

Typically smaller than shear except around T_c.

Uncertainties are large. Not known with quarks+gluons

Viscosity from LHC data

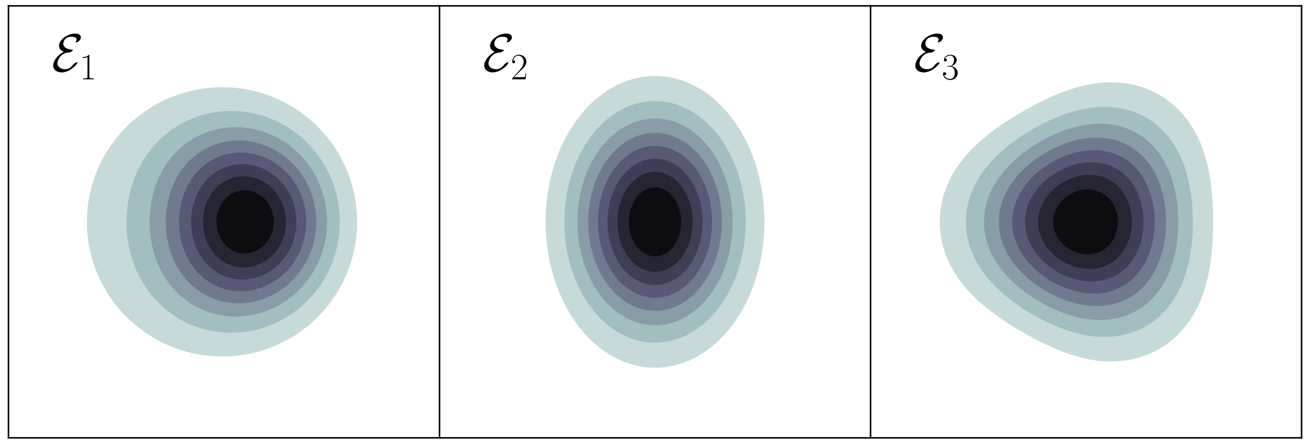
Global analysis. Model-to-data comparison with Bayesian inference



Everett et al. 2011.01430

Reasonable constraints in the range 150<T<200 MeV
Uncertainties on η/s and ς/s are similar in absolute value.

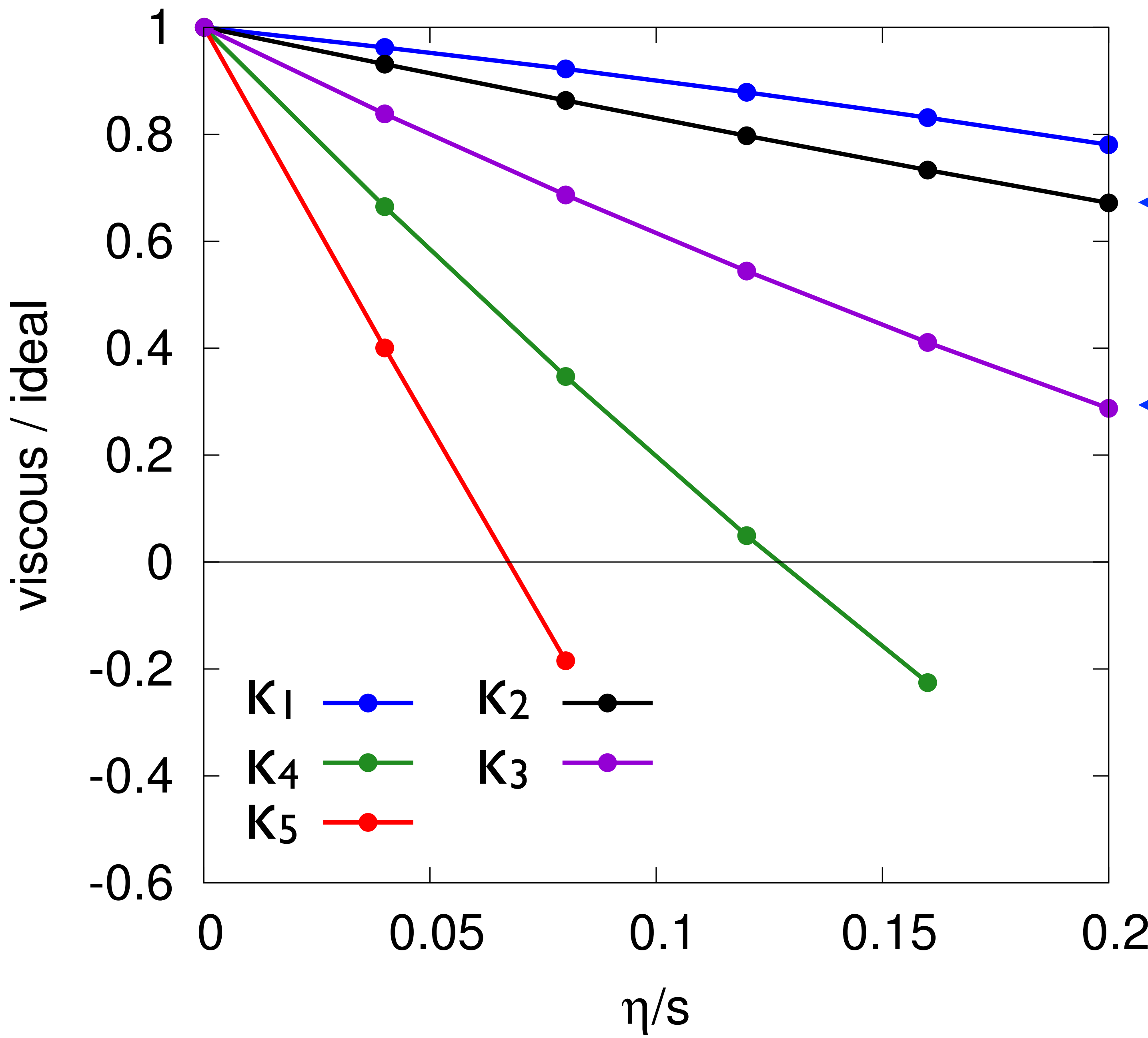
Elliptic flow v_2, triangular flow v_3



Giacalone 2101.00168

- Azimuthal anisotropy in the initial density profile, characterized by the Fourier coefficients ε_n, is converted into azimuthal anisotropy in momentum state, $v_n = \langle\cos(n\varphi)\rangle$, through pressure gradients.
- In hydrodynamics, $v_n = \kappa_n \varepsilon_n$, where κ_n is the hydrodynamic response coefficient.
- The sensitivity of observables to viscosity lies mostly in κ_n.
- I focus on the largest two harmonics v_2 and v_3.

Dependence of v_n on η/s



Teaney & Yan 1206.1905

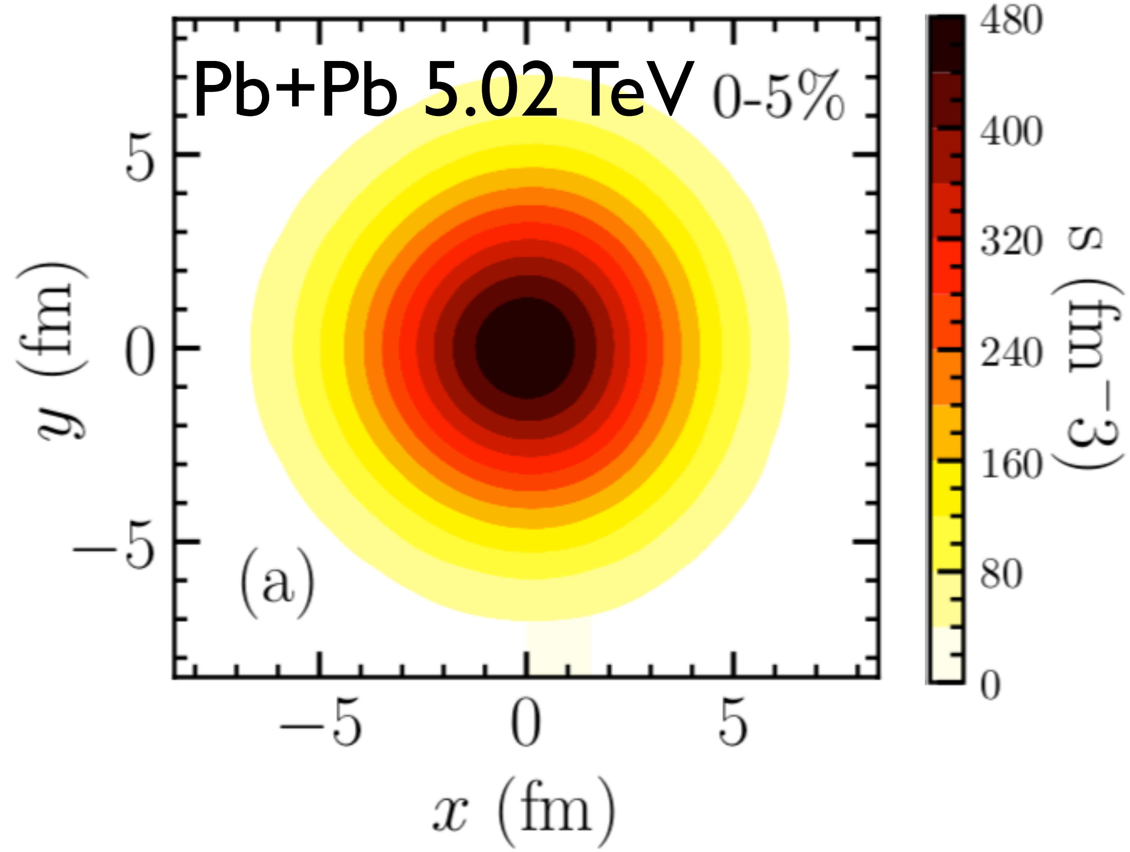
Damping of v_n, linear in η/s for small η/s

← v_2

← v_3

We generalize this to a temperature-dependent $\eta/s(T)$, and we study bulk viscosity in the same way.

Method (1/3)



We evolve this initial density profile, which has ε_2=0.085 and ε_3=0.075, through ideal and viscous hydrodynamics (boost invariant, MUSIC code)

We evaluate v_2 and v_3 of charged hadrons at freeze-out after resonance decays

We compute the relative variation Δ_n of v_n due to viscosity, so that the dependence on ε_2 and ε_3 cancels.

$$\Delta_n \equiv \ln(v_n(\text{viscous})/v_n(\text{ideal})) \approx v_n(\text{viscous})/v_n(\text{ideal})-1$$

Method (2/3)

To leading order in viscosity, one expects by linearity

$v_n(\text{viscous})/v_n(\text{ideal})-1 = \int [w_n^{(\eta)}(T)(\eta/s)(T)+w_n^{(\varsigma)}(T)(\varsigma/s)(T)]\, dT,$

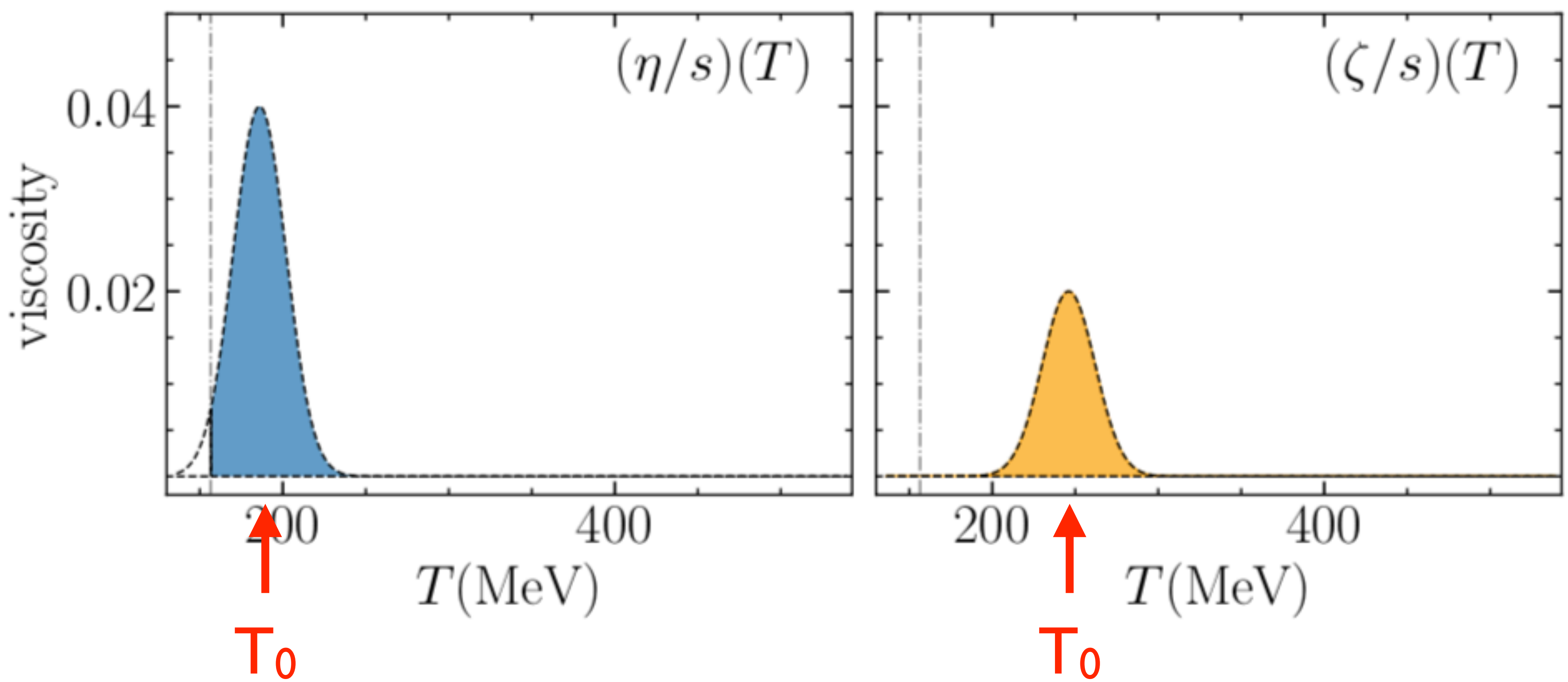
where $w_2^{(\eta)}(T)$, $w_3^{(\eta)}(T)$, $w_2^{(\varsigma)}(T)$,$w_3^{(\varsigma)}(T)$ are four functions. Once these functions are known, we know the dependence of v_n on viscosity for an arbitrary temperature dependence of bulk and shear viscosities, provided that they are small enough.

Since the integral over T starts at the freeze-out temperature T_f, $w_n^{(\eta,\varsigma)}(T)$ is the sum of:

- a discrete term proportional to $\delta(T-T_f)$, representing the contribution of freeze-out to the viscous correction.
- a smooth function of T for $T>T_f$.

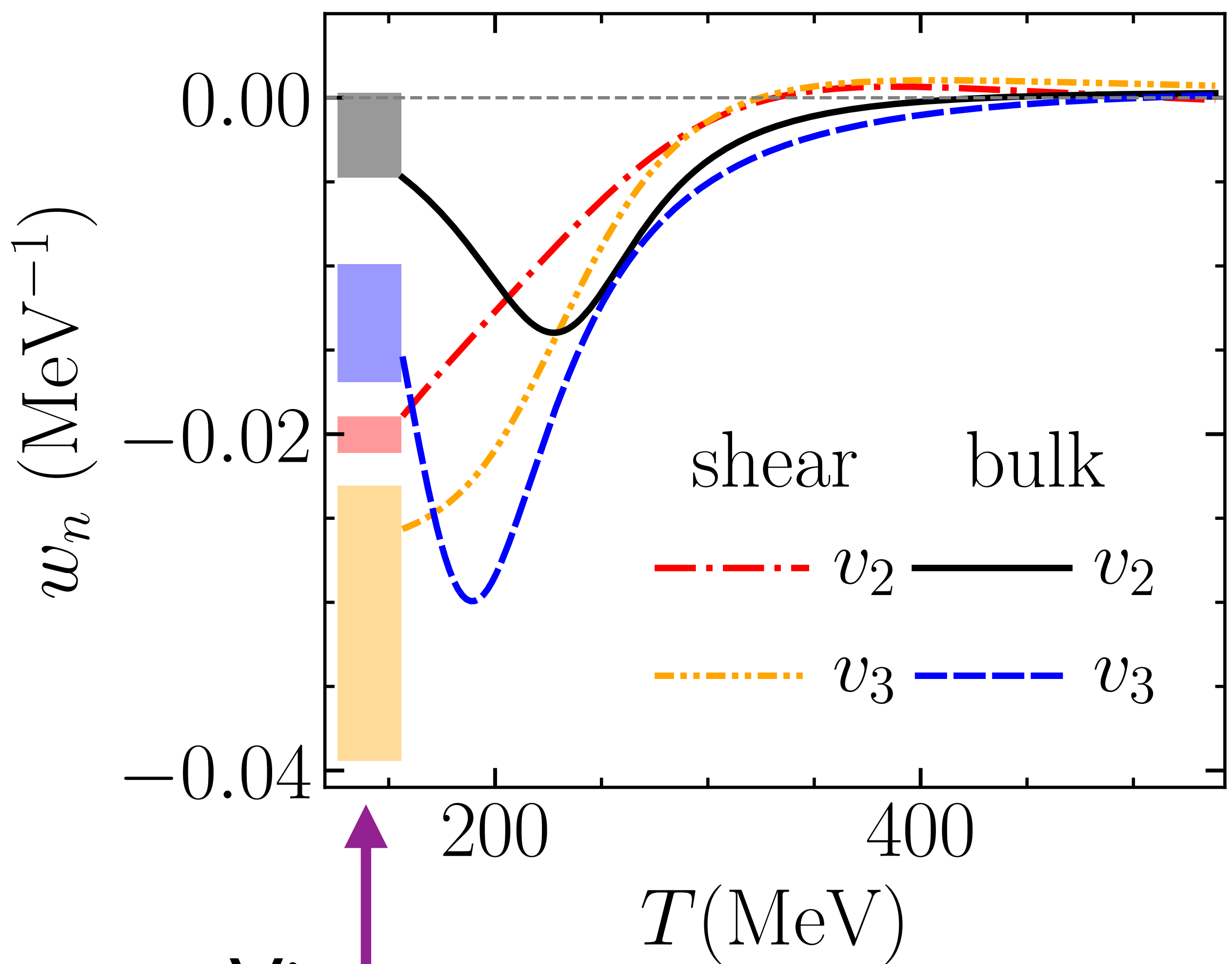
Method (3/3)

Idea: in order to determine $w_n^{(\eta)}(T)$, we switch on shear viscosity only in a narrow temperature interval around a temperature T_0. Same for bulk. Thus we isolate the effect of viscosity around T_0.



We then vary T_0 and repeat the calculation.

Result



Viscous correction at T_f

- Viscous suppression is a factor ~2 larger for v_3 than v_2.
- Similar magnitude for shear and bulk.
- The freeze-out contribution is less than 20% of the integral. Good news since this part is not robust (depends on details of hadronic interactions).
- Weights are large only below 250 MeV. Explains why Bayesian inference only constrains viscosity in this range.

Effective viscosities

Even if η/s depends on temperature T, we can define an effective shear viscosity as the weighted average:

$(\eta/s)_{n,\mathrm{eff}} \equiv \int w_n^{(\eta)}(T)\,(\eta/s)(T)\,dT \,/\, \int w_n^{(\eta)}(T)dT$

and same for bulk. In practice almost identical for n=2 and n=3.

The variation of v_n due to viscosity is proportional to the sum of effective shear and bulk viscosities.

$v_n(\mathrm{viscous})/v_n(\mathrm{ideal})-1 \sim (\eta/s)_{n,\mathrm{eff}} + (\zeta/s)_{n,\mathrm{eff}}$.

This implies that v_n data can only constrain the sum of effective shear and bulk viscosities, not the whole temperature dependence.
Lattice QCD for pure glue gives $(\zeta/s)_{n,\mathrm{eff}} << (\eta/s)_{n,\mathrm{eff}}$

Centrality and system-size (in)dependence

- We have seen that $\langle p_t \rangle$ and the effective temperature T_{eff} depend little on centrality and system size at a given collision energy.

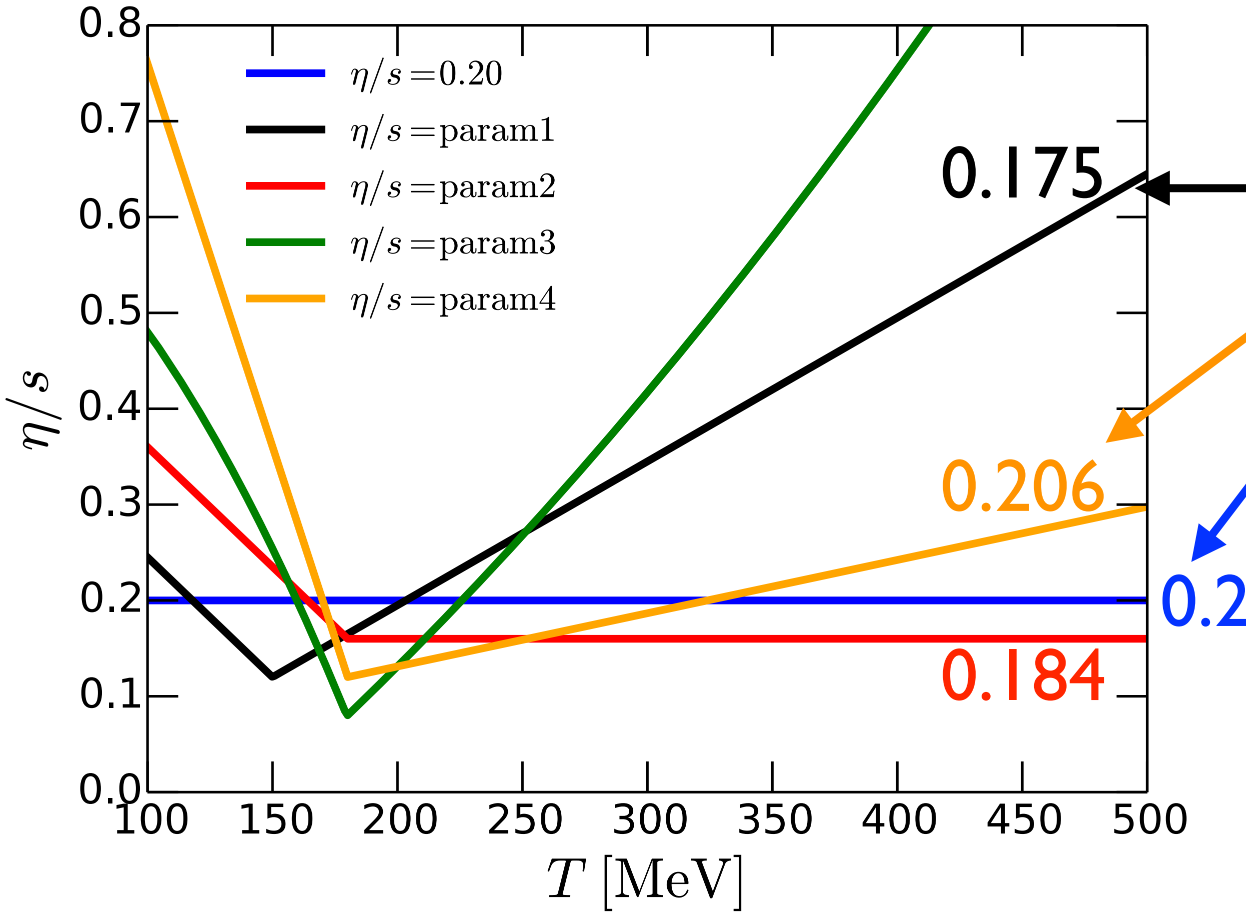
- In the same way, effective viscosities are independent of centrality and system size, to a very good approximation. We have checked this explicitly by repeating some of the calculations in the 20-30% centrality range.

- Viscous damping *does depend* on centrality and system size. We have checked that v_n(viscous)/v_n(ideal)-1 varies with the transverse size R_0 *precisely* like $1/R_0$, as expected by dimensional analysis (*Reynolds number scaling*).

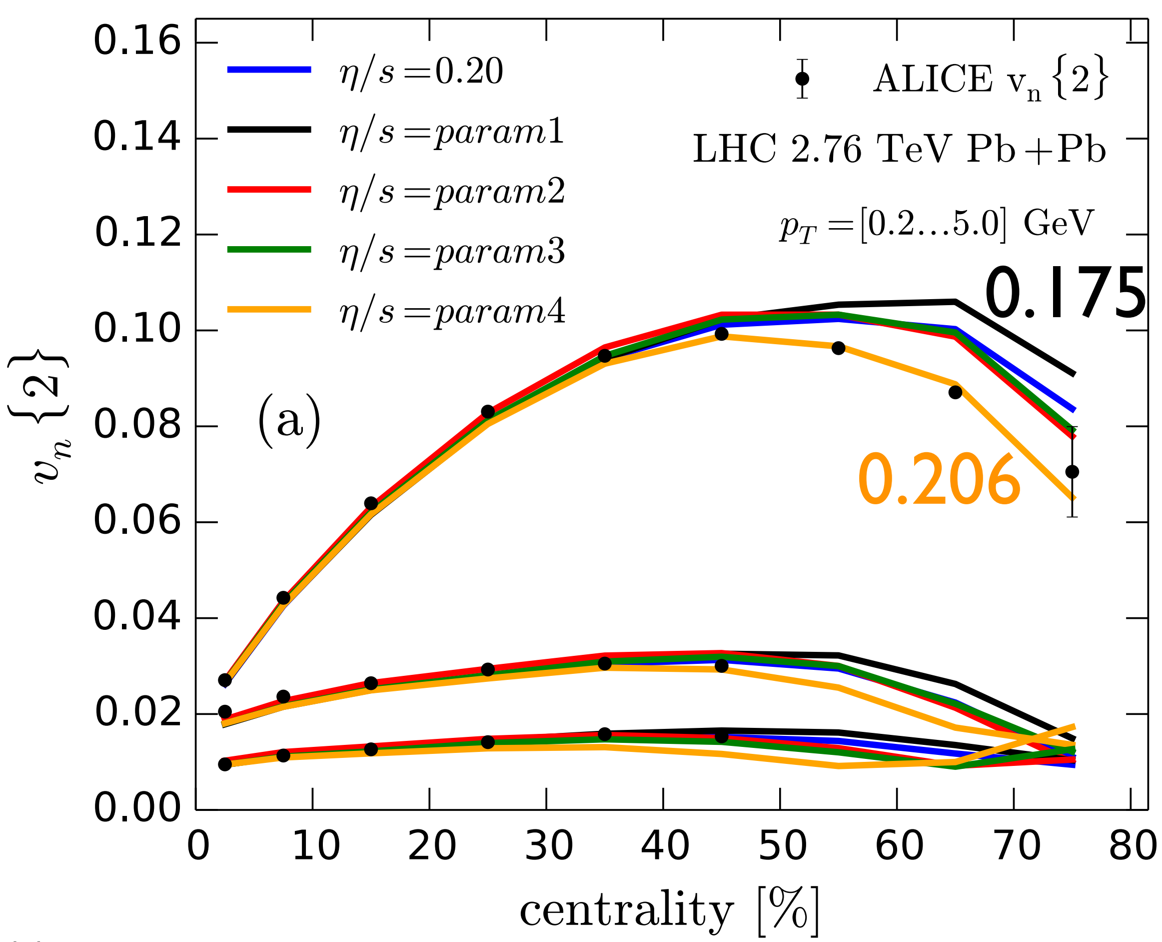
- A useful order of magnitude estimate, which works within ~30%: $\ln[v_n(\text{viscous})/v_n(\text{ideal})] = -[(\eta/s)_{n,eff} + (\varsigma/s)_{n,eff}]\, n^2/(T_{eff} R_0)$

Effectiveness of effective viscosity

Niemi et al. 1505.02677



Values of $(\eta/s)_{2,\text{eff}}$



All profiles have similar $(\eta/s)_{2,\text{eff}}$.
Explains why they all give similar v_2.
The small differences for peripheral collisions are also explained by the slight differences in $(\eta/s)_{2,\text{eff}}$

3. Event-to-event initial state fluctuations: the success of hydrodynamics in describing anisotropic flow fluctuations in proton-nucleus and nucleus-nucleus collisions.

Old theory predictions: 1312.6555, with Li Yan, 1702.01730, with Giuliano Giacalone and Jaki Noronha-Hostler, meet recent LHC data.

Event-by-event fluctuations

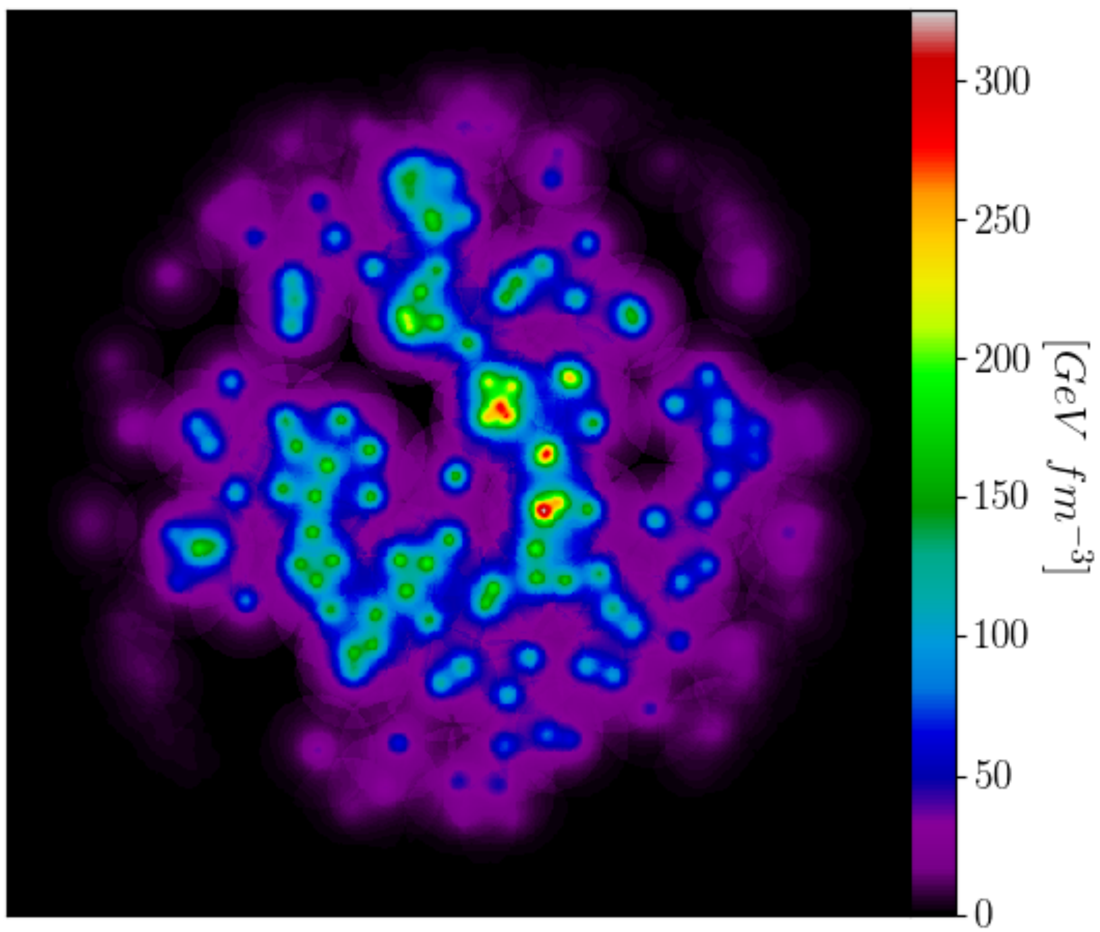


Illustration by G. Giacalone

The initial density profile fluctuates event to event.

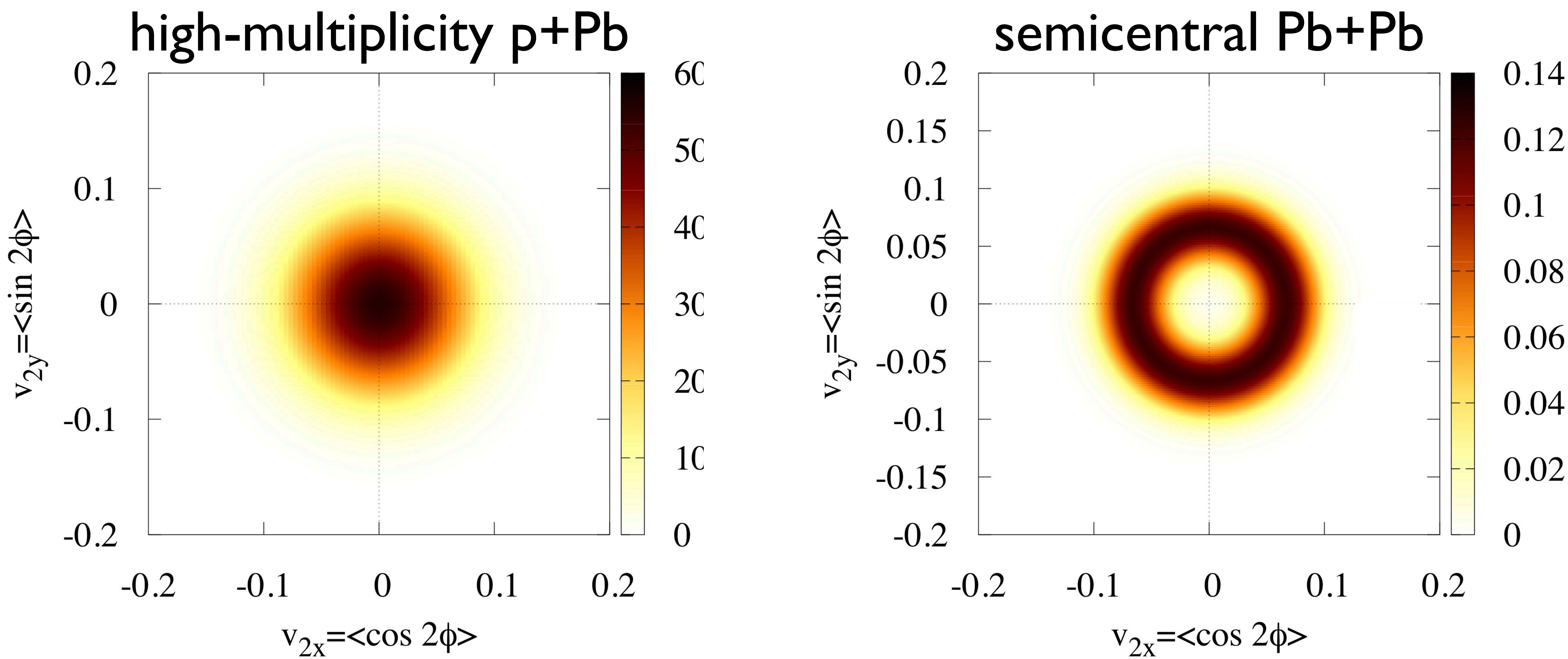
Elliptic flow in a *hydro event* is obtained by evaluating

v_{2x} = <cos 2φ>
v_{2y} = <sin 2φ>

= a 2-d vector whose magnitude and orientation fluctuates.

$v_2=\sqrt{v_{2x}^2+v_{2y}^2}$

Typical distributions of (v_{2x},v_{2y})



v_2 from fluctuations only

v_2 from reaction plane eccentricity: magnitude fluctuates less.

Distributions characterized by cumulants: $v_2\{2\}$, $v_2\{4\}$, $v_2\{6\}$, $v_2\{8\}$.

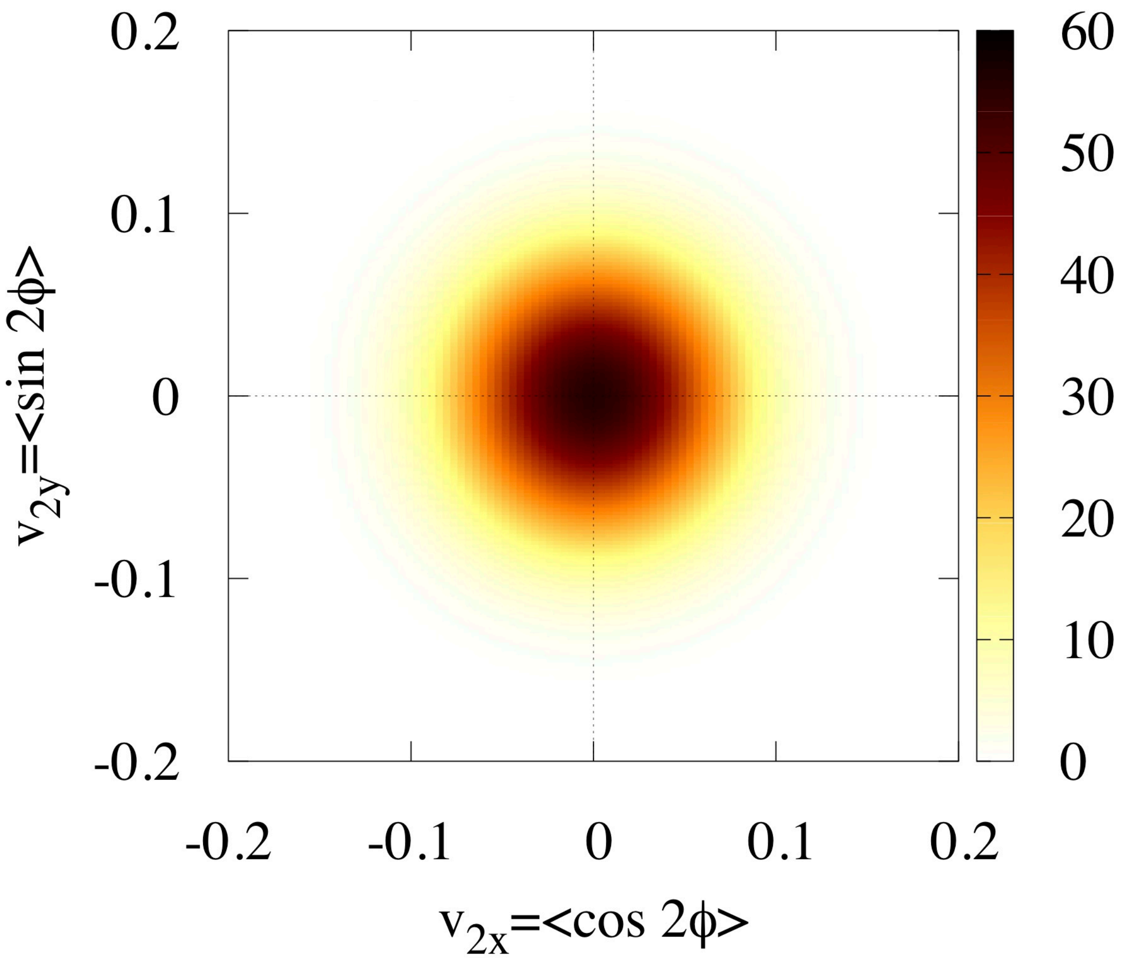
Crash course on cumulants

1. Take the Fourier transform of this distribution,
   $F(k_x,k_y) = < \exp(i\, k_x\, v_{2x} + i\, k_y\, v_{2y}) >$.
   By azimuthal symmetry, it only depends on $k^2=k_x^2+k_y^2$.

2. Expand $\log F(k_x,k_y)$ in powers of k^2.
   Term proportional to k^2 defines $v_2\{2\}$
   Term proportional to k^4 defines $v_2\{4\}$ etc.

3. Normalization is such that if all events have the same v_2, then $v_2\{n\}=v_2$ for all n.

p+Pb collisions



v_{2x},v_{2y} only from fluctuations

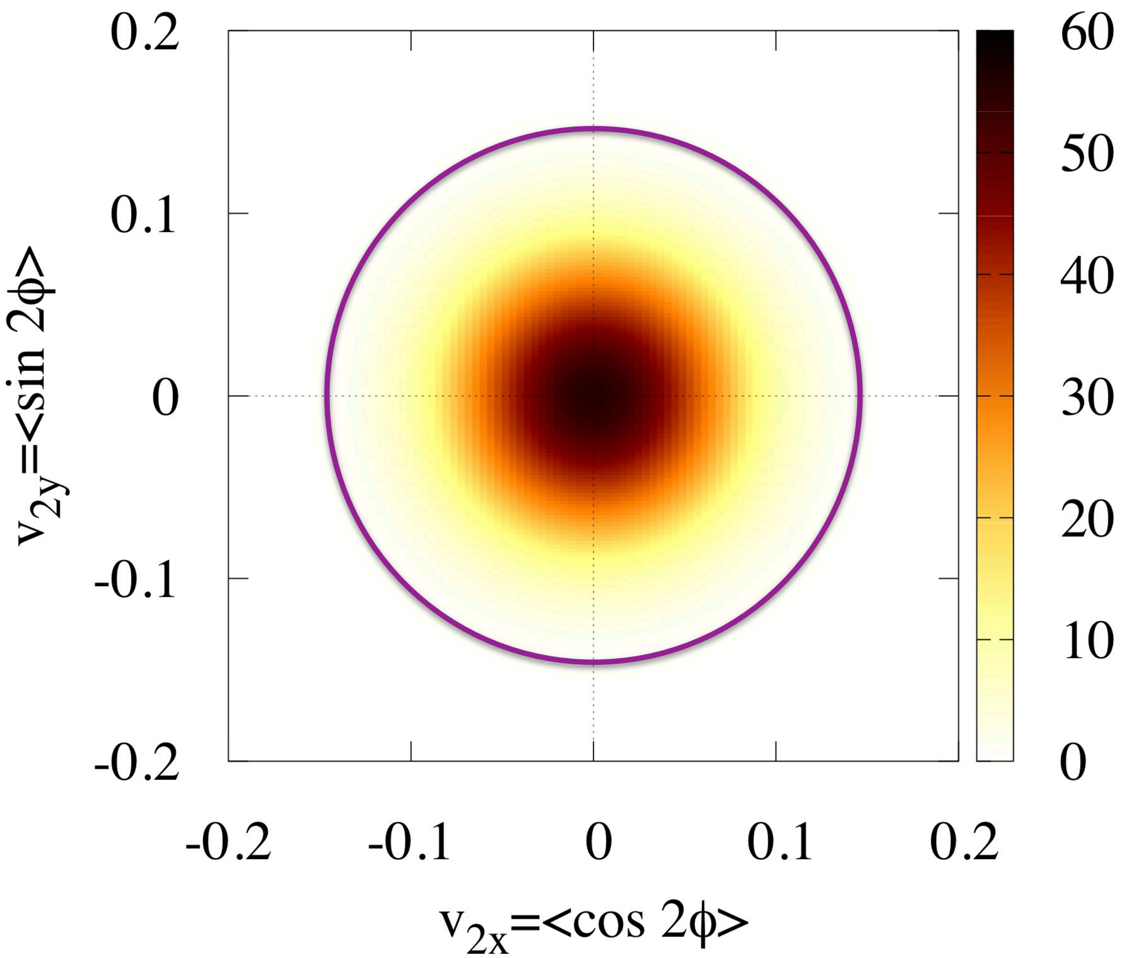
If the distribution of (v_{2x},v_{2y}) is a 2-d Gaussian,

then its Fourier transform F(k_x,k_y) is also a 2-d Gaussian, i.e. log F(k_x,k_y) proportional to k^2,

which implies
$v_2\{4\}=v_2\{6\}=v_2\{8\}=0$

The observation of non-zero $v_2\{4\}$ implies non-Gaussian fluctuations

The origin of non-Gaussianity



In hydrodynamics, $v_2=\kappa_2\varepsilon_2$, where $\varepsilon_2<1$ by construction.
Therefore $v_2<\kappa_2$.
No such bound for a Gaussian.

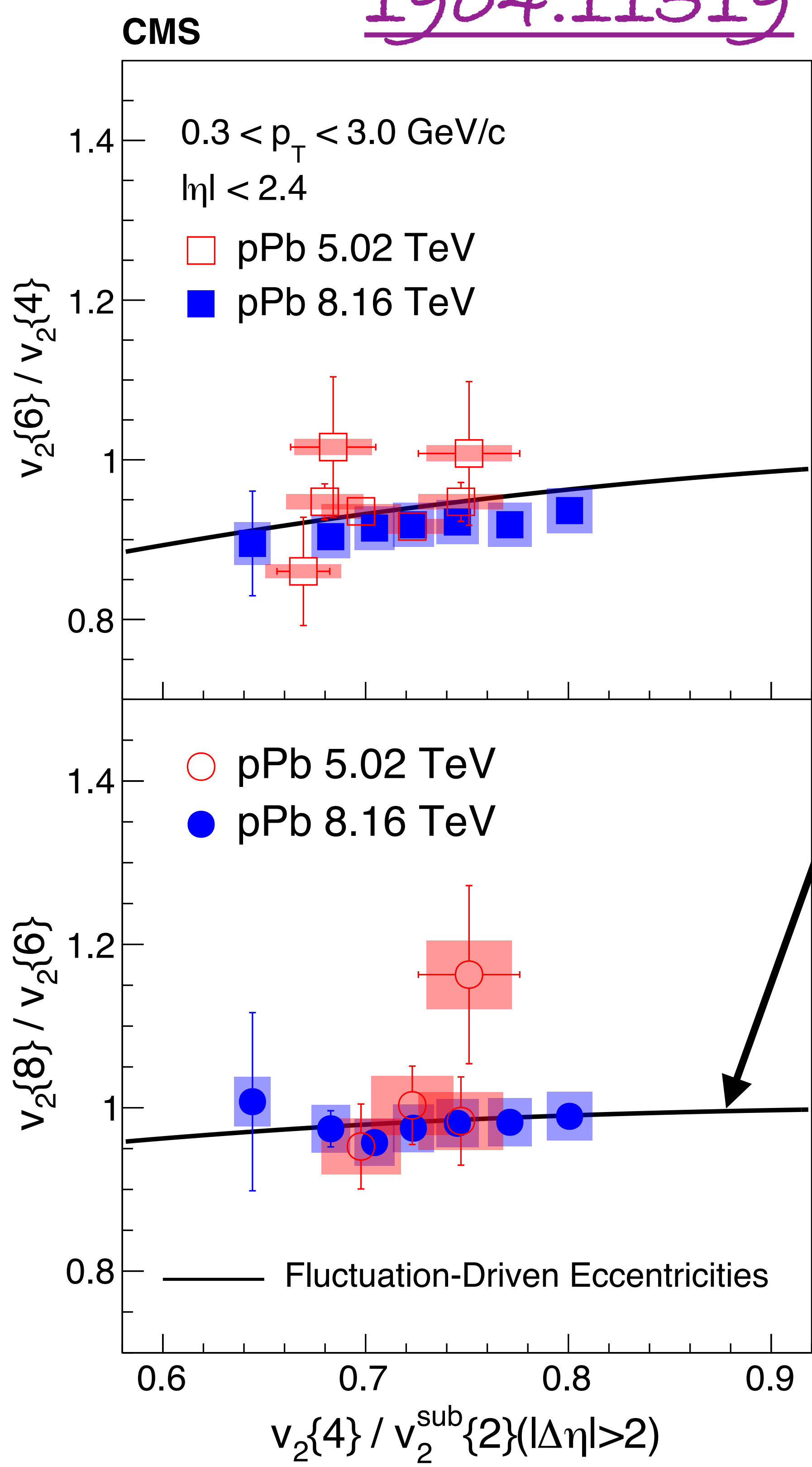
The generic shape of the distribution is instead of the form $(1-\varepsilon_2{}^2)^\alpha=(1-v_2{}^2/\kappa_2{}^2)^\alpha$.

Ratios of cumulants, such as
$v_2\{4\}/v_2\{2\}$,
$v_2\{6\}/v_2\{4\}$
$v_2\{8\}/v_2\{6\}$,
are simple analytic functions of α.

Li Yan, JYO, 1312.6555

Experimental results in p+Pb

1904.11519



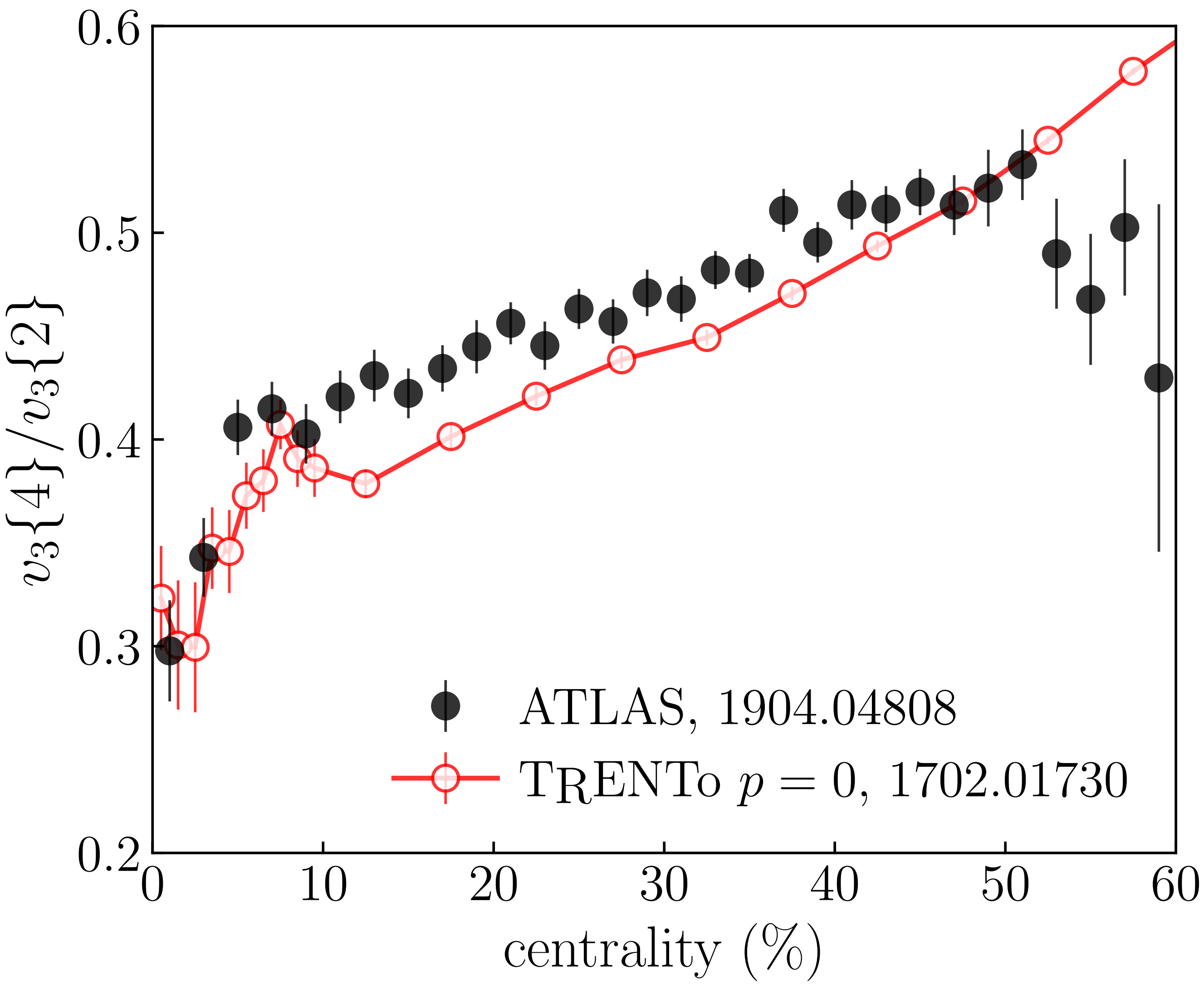
This simple prediction of hydrodynamics accurately matches the latest CMS data

Most solid evidence that a small fluid is formed in p+Pb.

My guess: the very slight disagreement is due to the fact that $v_2\{4\}$, $v_2\{6\}$, $v_2\{8\}$, are analyzed without a rapidity gap, unlike $v_2\{2\}$.

$v_3\{4\}/v_3\{2\}$ in Pb+Pb collisions

Like v_2 in p+Pb, v_3 in Pb+Pb is only from fluctuations: non-zero $v_3\{4\}$ = non-Gaussian fluctuations of v_3

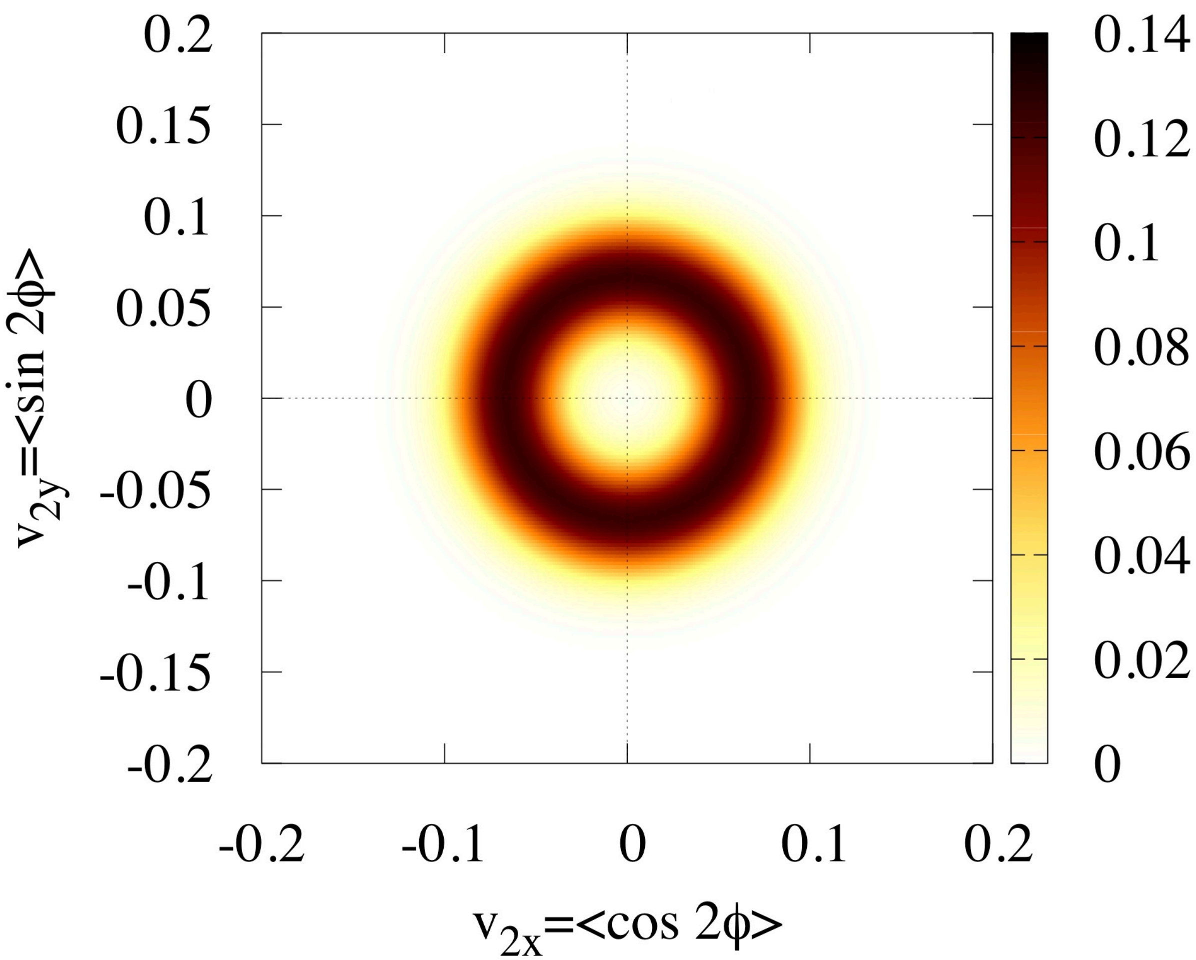


Hydrodynamics predicts $v_3\{4\}/v_3\{2\}=\varepsilon_3\{4\}/\varepsilon_3\{2\}$ and initial-state models quantitatively predicted the experimental result.

see also Carzon et al. 2007.00780

Figure by Giuliano Giacalone

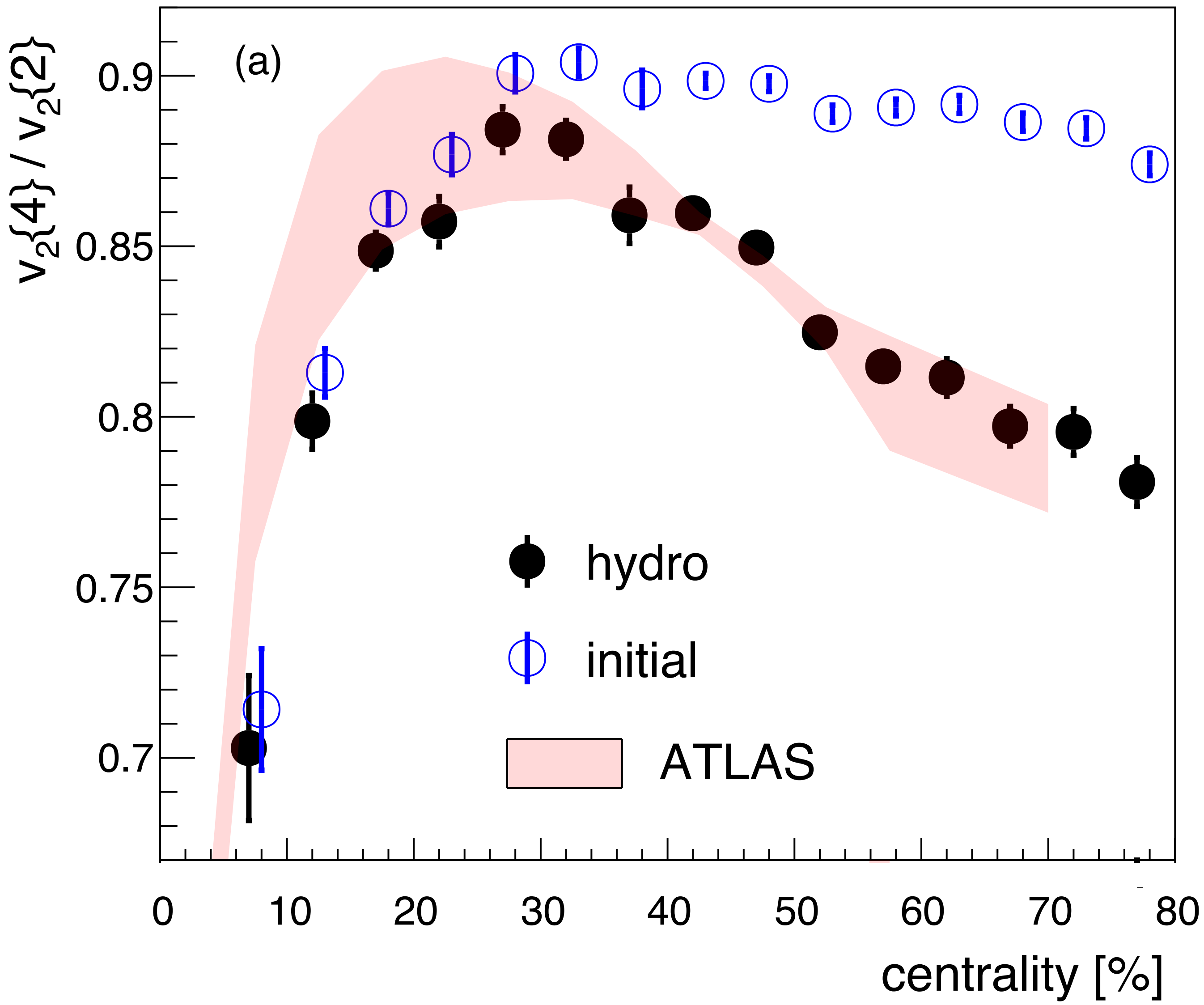
v_2 fluctuations in Pb+Pb collisions



A different situation:
In semi-central to peripheral collisions, large v_2 from the reaction plane eccentricity, small fluctuations on top of it.

One expects
$v_2\{2\} \approx v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$

$v_2\{4\}/v_2\{2\}$ in Pb+Pb collisions



$\varepsilon_2\{4\}/\varepsilon_2\{2\}$

differs somewhat from

$v_2\{4\}/v_2\{2\}$ in e-by-e hydro.

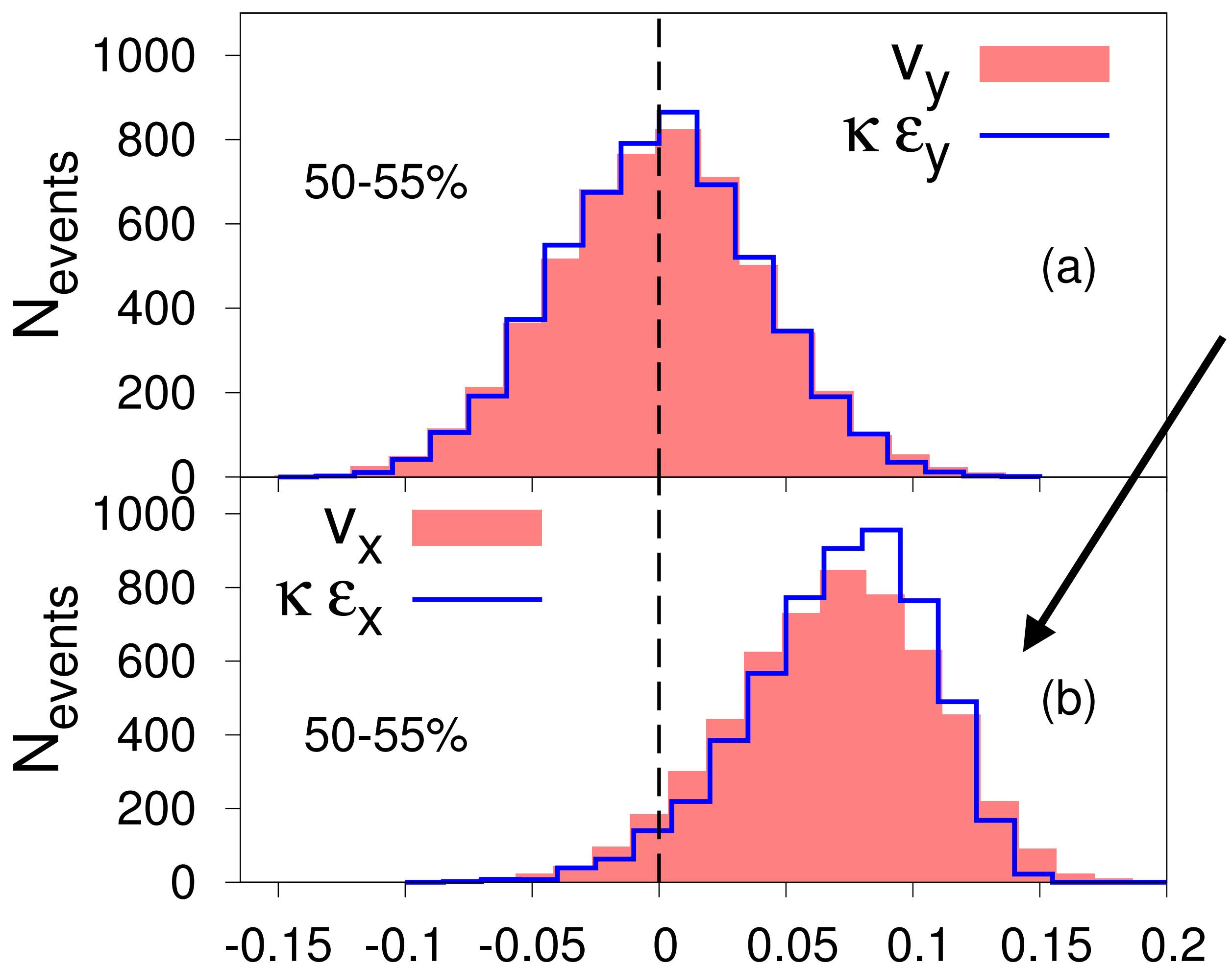
Giacalone Noronha-Hostler JYO

1702.01730

Comparison with data suggests that we see effects of hydro beyond the linear response to initial eccentricity.

Non-Gaussian v_2 fluctuations in Pb+Pb

here, x≡reaction plane.



Hydrodynamic calculations predict that the probability distribution of elliptic flow in the reaction plane, v_{2x}=<cos(2φ)>, is asymmetric, and has negative skew.

The skewness γ_1 and kurtosis γ_2 of the distribution of v_{2x} can be inferred from the small splittings between $v_2\{4\}$, $v_2\{6\}$, and $v_2\{8\}$:

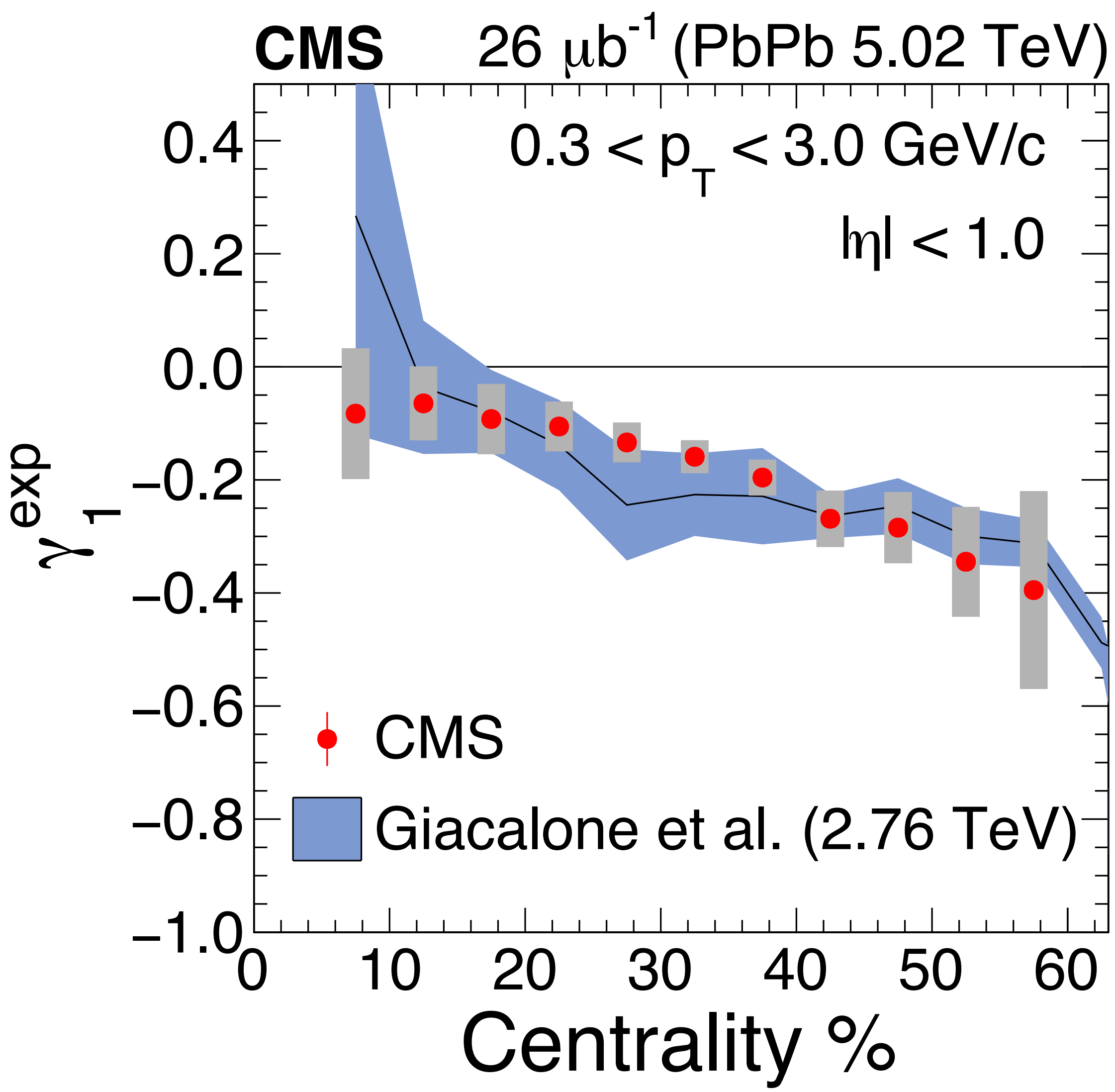
$$\gamma_1 \simeq \gamma_1^{\rm expt} \equiv -2^{3/2}\frac{v_2\{4\}^3 - v_2\{6\}^3}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}},$$

$$\gamma_2 \simeq \gamma_2^{\rm expt} \equiv -\frac{3}{2}\frac{v_2\{4\}^4 - 12 v_2\{6\}^4 + 11 v_2\{8\}^4}{(v_2\{2\}^2 - v_2\{4\}^2)^2}.$$

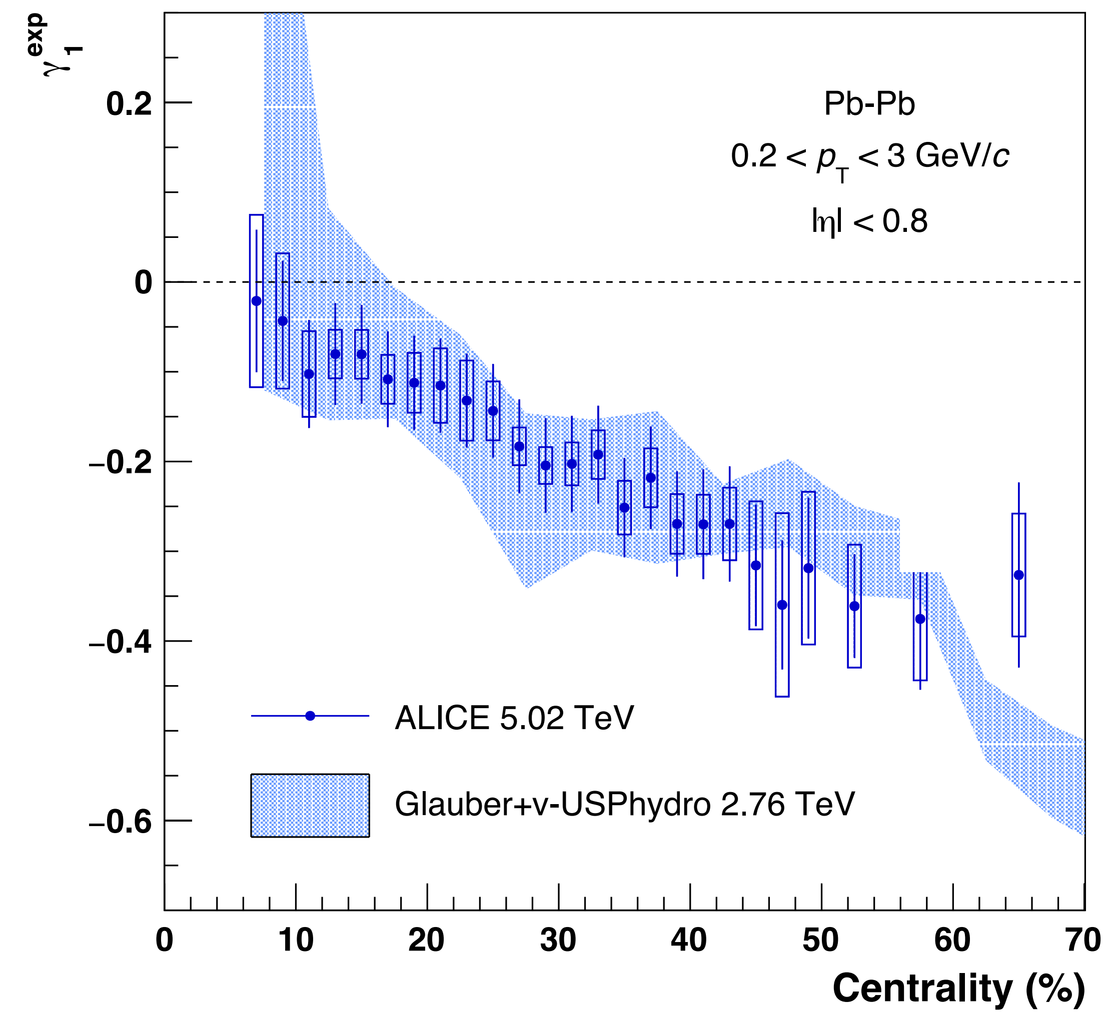
1608.01823

1811.00837

Skewness of v_2 fluctuations



CMS 1711.05594



ALICE 1804.02944

Measured skewness in good agreement with hydrodynamic predictions.
Kurtosis (more difficult) will hopefully be measured soon.

The problem of hadronization

- In this talk, I have only covered bulk observables, averaged over all charged particles: $\langle p_t \rangle$, v_2, v_3.
- The reason is that they have limited sensitivity to how the fluid is converted into hadrons (freeze-out), as we have seen explictly for the effective viscosities.
- At freeze-out, the fluid falls out of equilibrium, therefore, the momentum distributions deviate from a thermal distribution. How it deviates (the δf correction at freeze-out) depends on the details of hadronic interactions.
- We have really *no idea* how δf depends on the particle momentum, and this is crucial for all the differential observables: p_t spectra, $v_n(p_t)$, and identified particle analyses in general.

Dusling Schäfer 1109.5181

Molnar 2012.15574

Conclusions

- Success of hydro in describing anisotropic flow fluctuations in p+Pb and Pb+Pb, which is fully non trivial and does not rely on model details.
- Quantitative information about the equation of state can be obtained from data by varying the collision energy.
- At each energy, one can at best extract one effective viscosity, which is a weighted average of the temperature-dependent viscosities. Shear viscosity is likely to dominate.

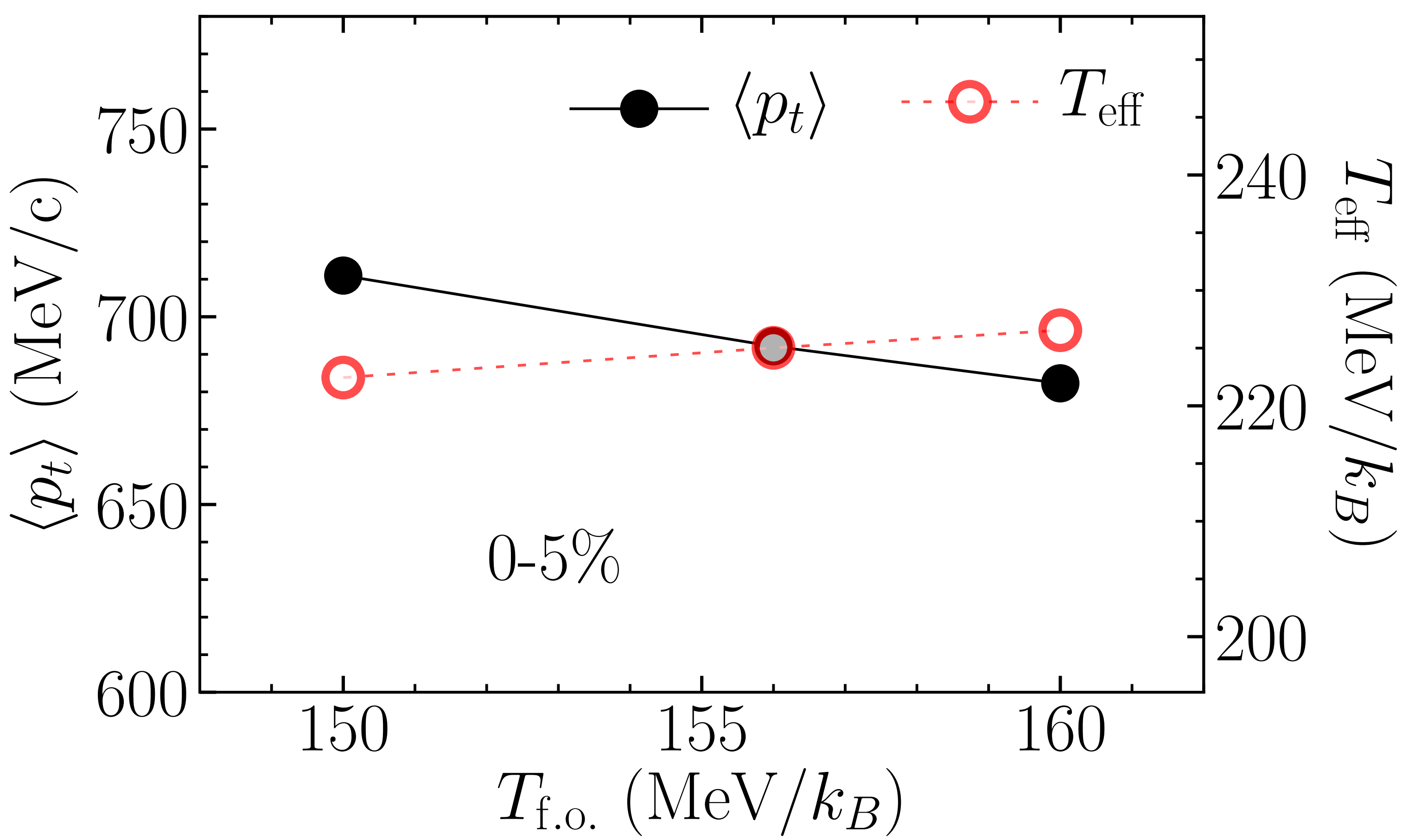
Not covered in this talk:

- More theory work needed on other harmonics v_1, v_4, v_5, v_6…
- $<p_t>$ fluctuations are typically overestimated in hydro. Largely an open question, and much activity lately.

Supplementary material

Varying the freeze-out temperature

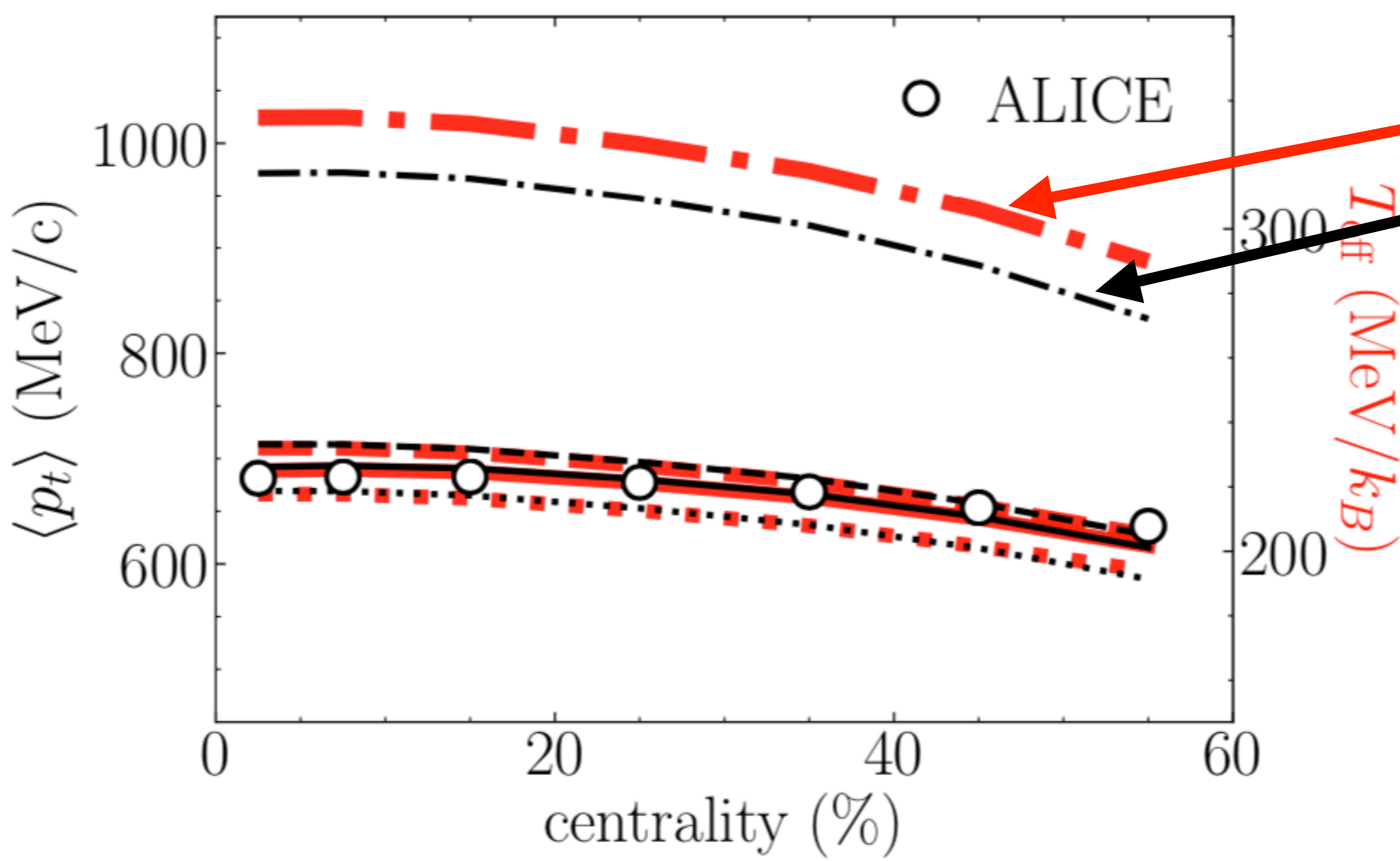
Freeze-out temperature $T_{f.o.}$= temperature at which one converts the fluid to particles = some arbitrariness here



T_{eff} is remarkably independent of the freeze-out temperature,

Changing the equation of state

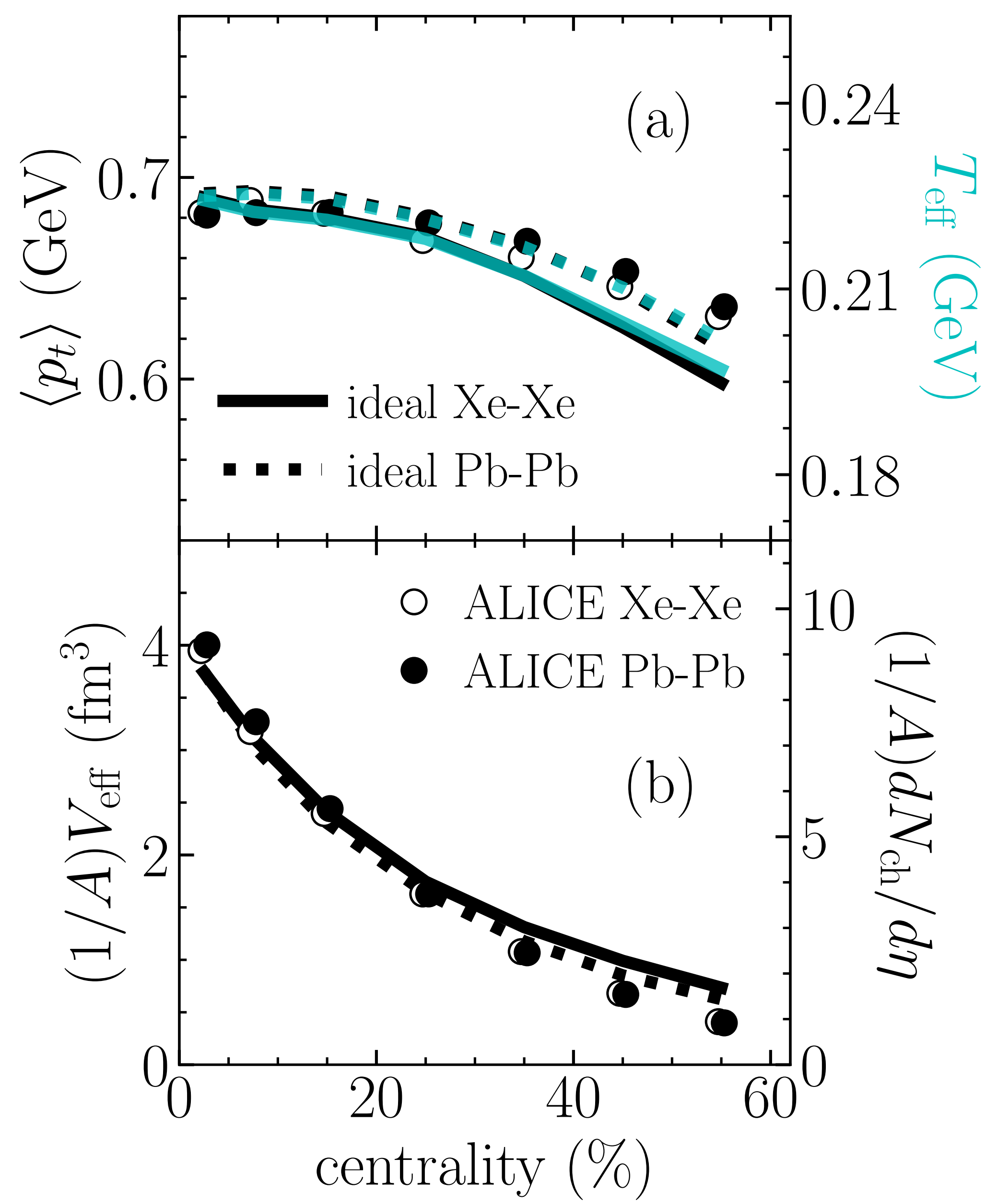
We test the robustness of the correspondence between $<p_t>$ and T_{eff} by running ideal hydro with a stiff equation of state ε=3P+const.



Both T_{eff} and $<p_t>$ increase.

$<p_t>$ /T_{eff} is slightly smaller: 2.90 instead of 3.07

System-size (in)dependence

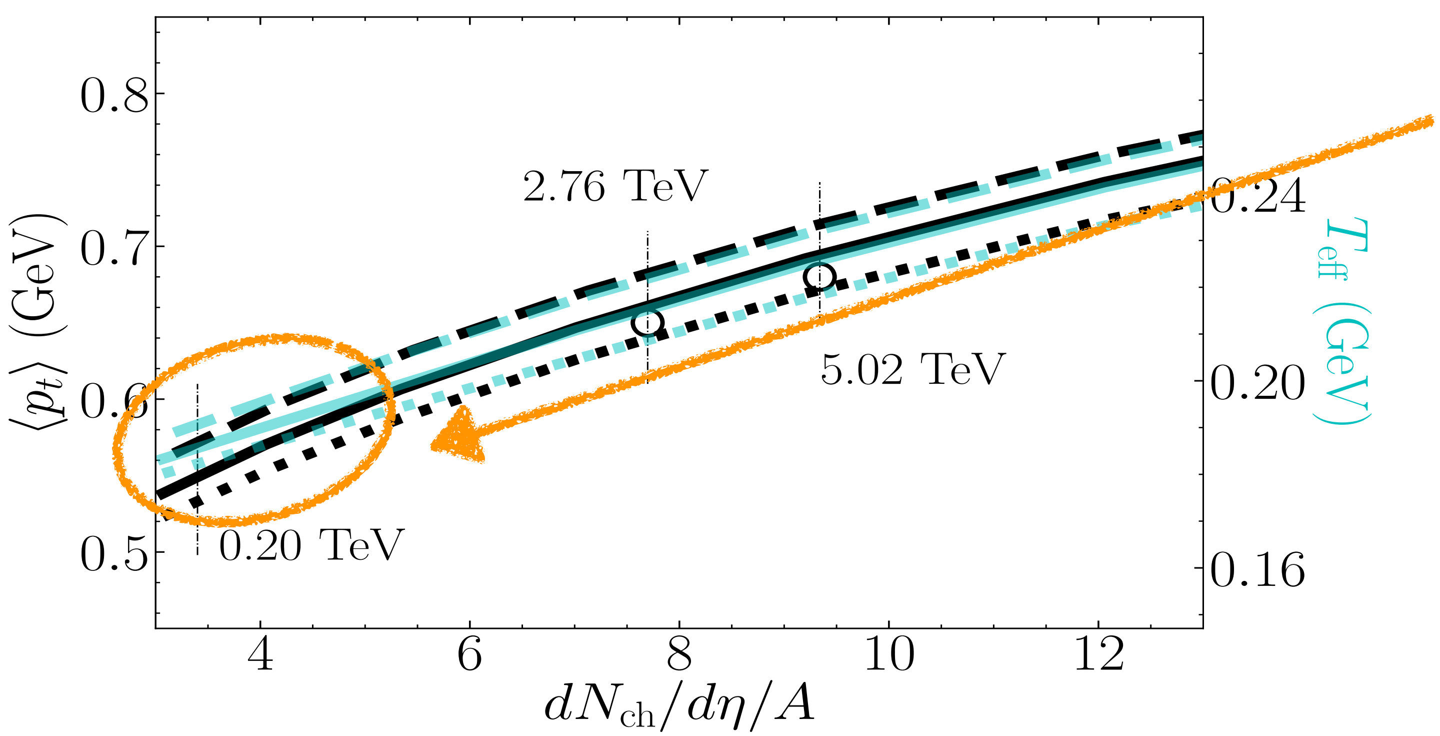


In hydro, $\langle p_t \rangle / T_{eff}$ is identical in Pb+Pb and Xe+Xe collisions.

In experiment, $\langle p_t \rangle$ is essentially the same in both systems, therefore T_{eff} is also the same.

V_{eff} and the multiplicity are both proportional to A at a given centrality percentile.

The equation of state at RHIC

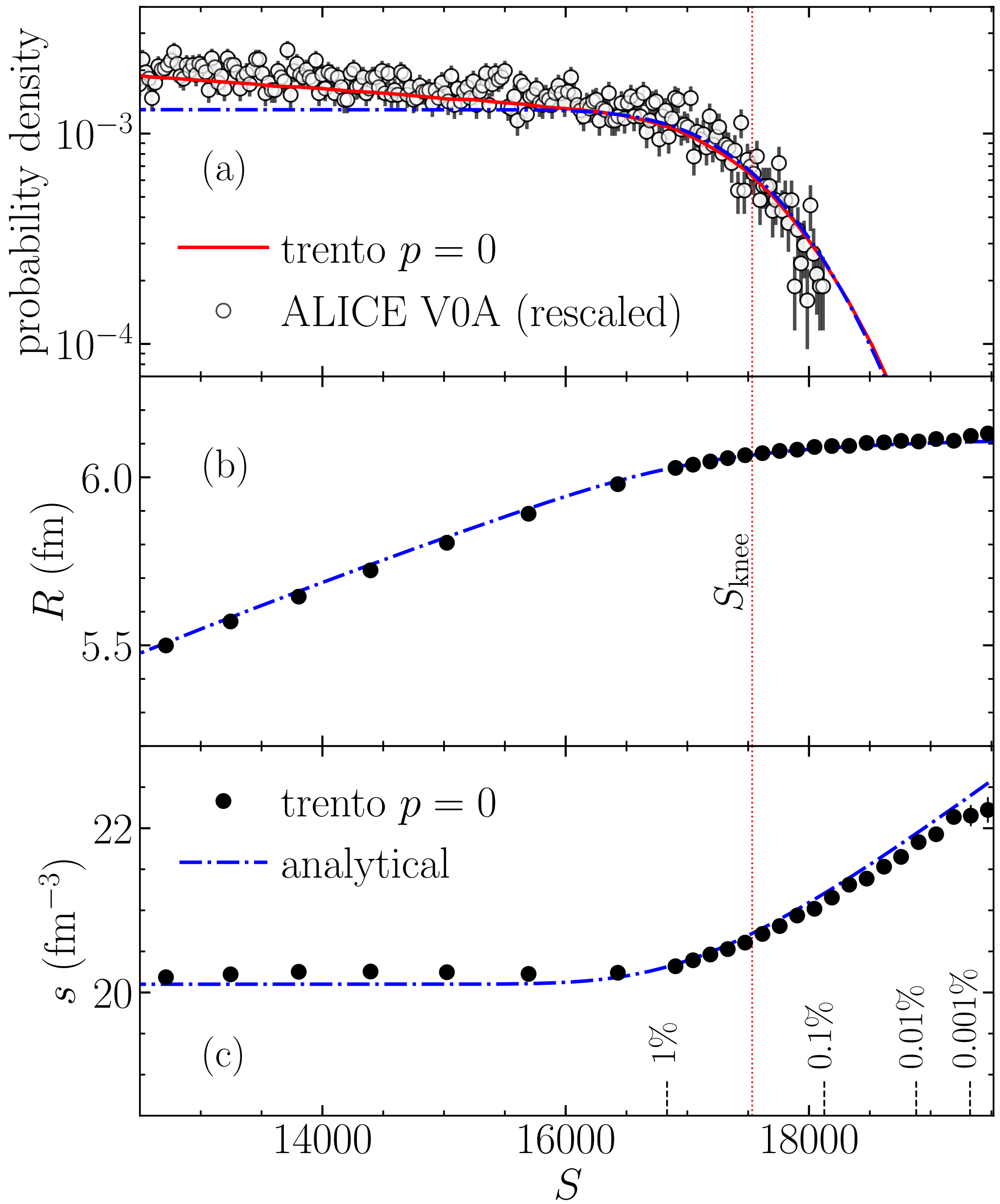


At RHIC energies, $<p_t>$ is slightly steeper than T.

Around the transition region, $<p_t>$ follows the energy over entropy ratio ε/s, rather than T.

In a baryonless plasma, $\frac{3}{4}T < \varepsilon/s < T$ so ε/s and T are almost proportional.
Hadron to QGP transition: T is almost constant, but ε/s keeps increasing.
This is probably what we see here.

Predictions for ultracentral collisions



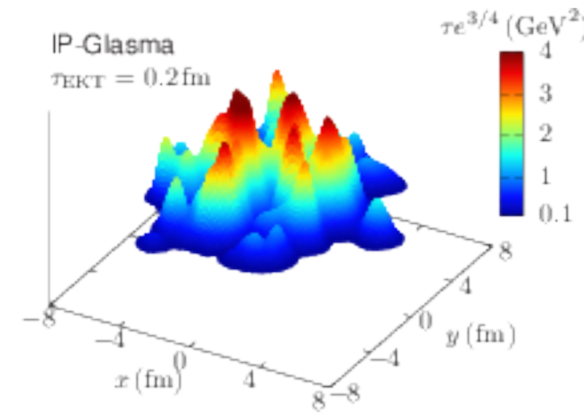
Entropy per unit rapidity

Zoom of the V0 distribution in ultracentral collisions

In a model of initial state (Trento model) tuned to reproduce the V0 distribution,the transverse radius saturates beyond the knee

The entropy density increases beyond the knee: hence T_{eff} increases

Event-by-event fluctuations

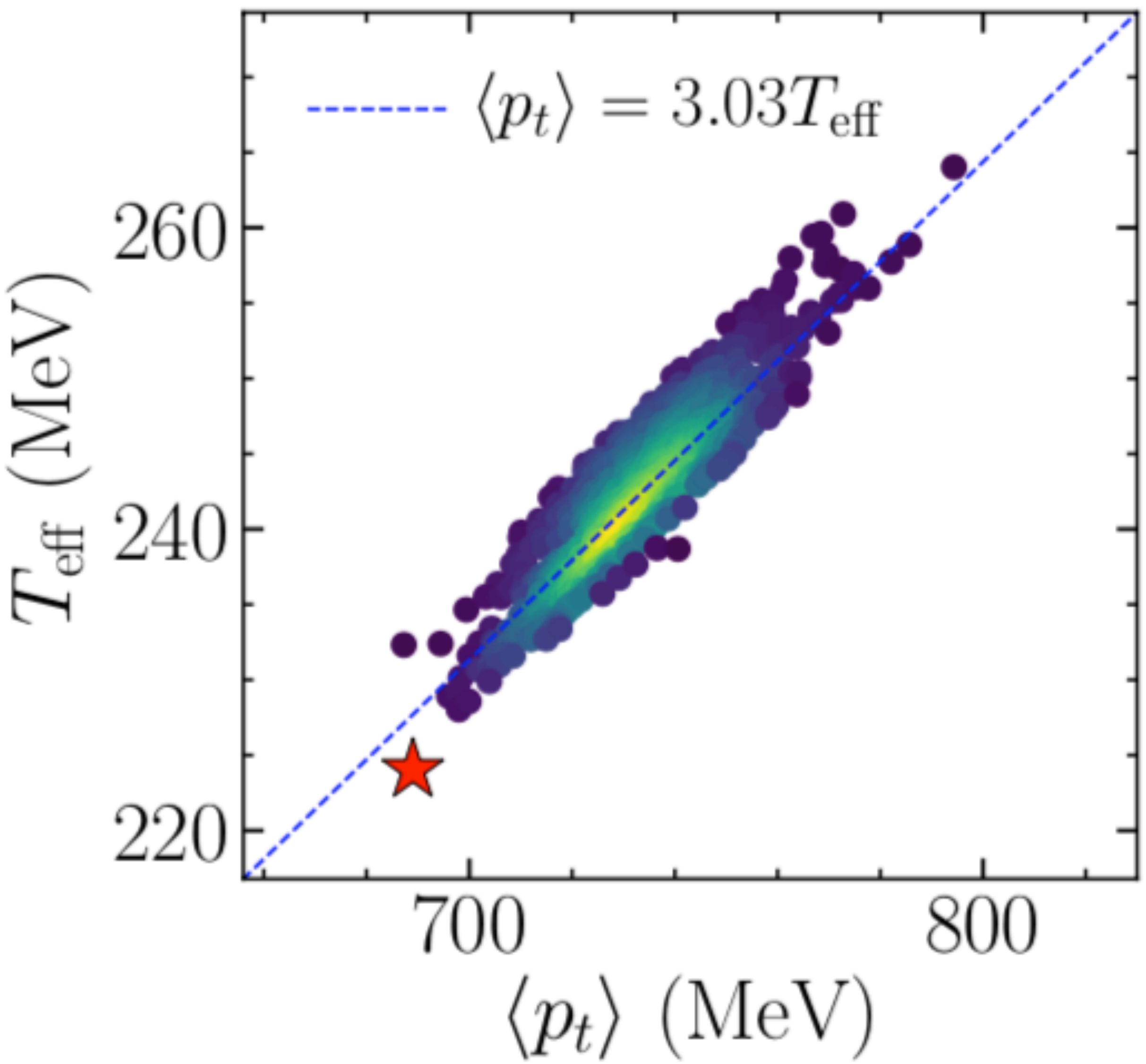


- Our calculations so far were done with a smooth initial density profile, depending only on impact parameter
- If the initial density fluctuates event to event:
  - Is the ratio between $\langle p_t \rangle$ and T_{eff} modified?
  - Do the event-by-event fluctuations of $\langle p_t \rangle$ follow those of T_{eff} ?

Event-by-event fluctuations

We use the trento model of initial conditions.
We fix both the impact parameter and the total entropy, then run ideal hydrodynamics

Moreland Bernhard Bass 1412.4708



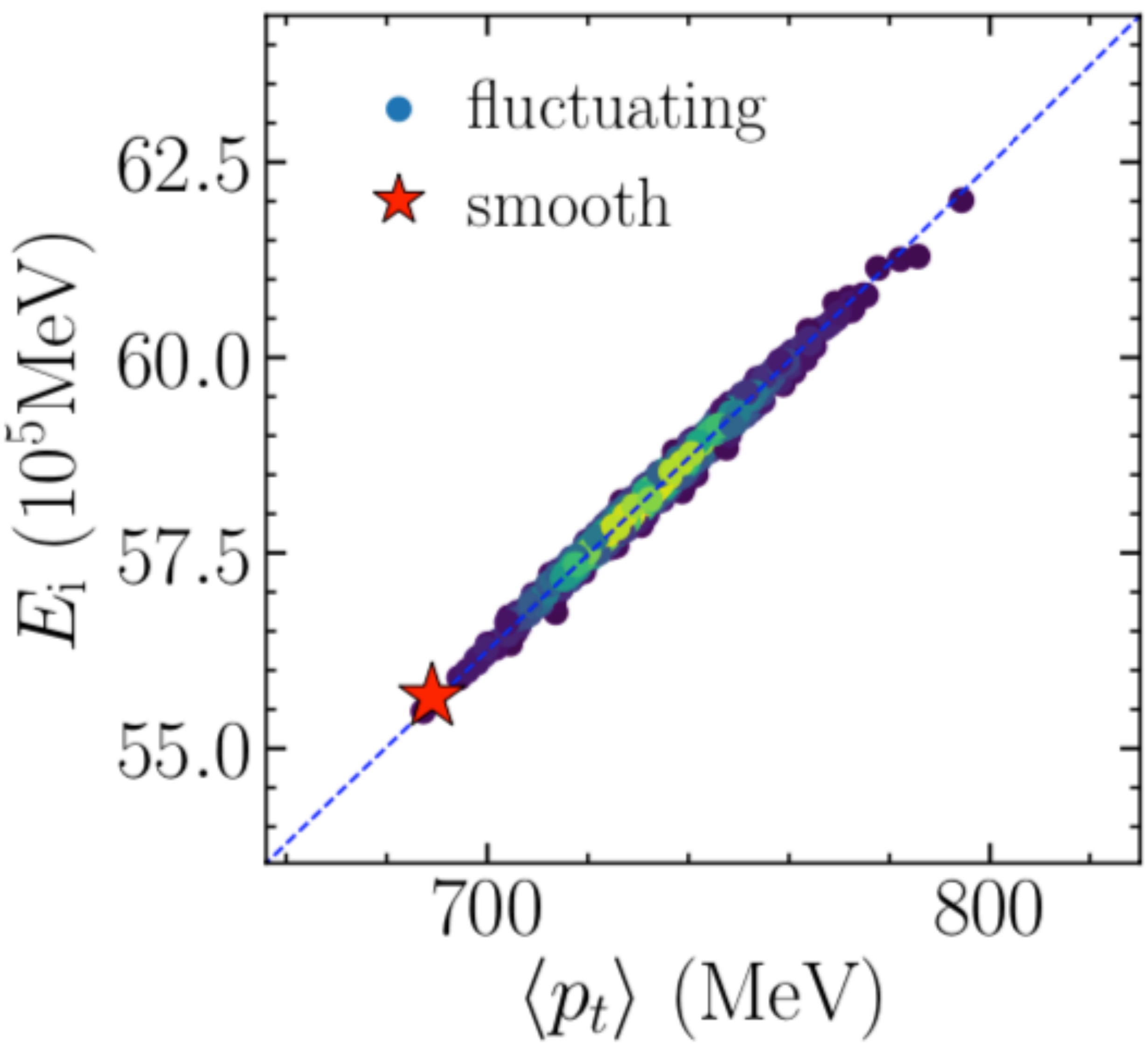
Each point corresponds to 1 event.

The ratio between $<p_t>$ and T_{eff} is almost unchanged (3.03 vs 3.07)

Event-by-event fluctuations of $<p_t>$ are well correlated with those of T_{eff}

Gardim et al. 2002.07008

Event-by-event fluctuations



Surprise: we find a much tighter correlation between $<p_t>$ and the initial energy of the fluid E_i.

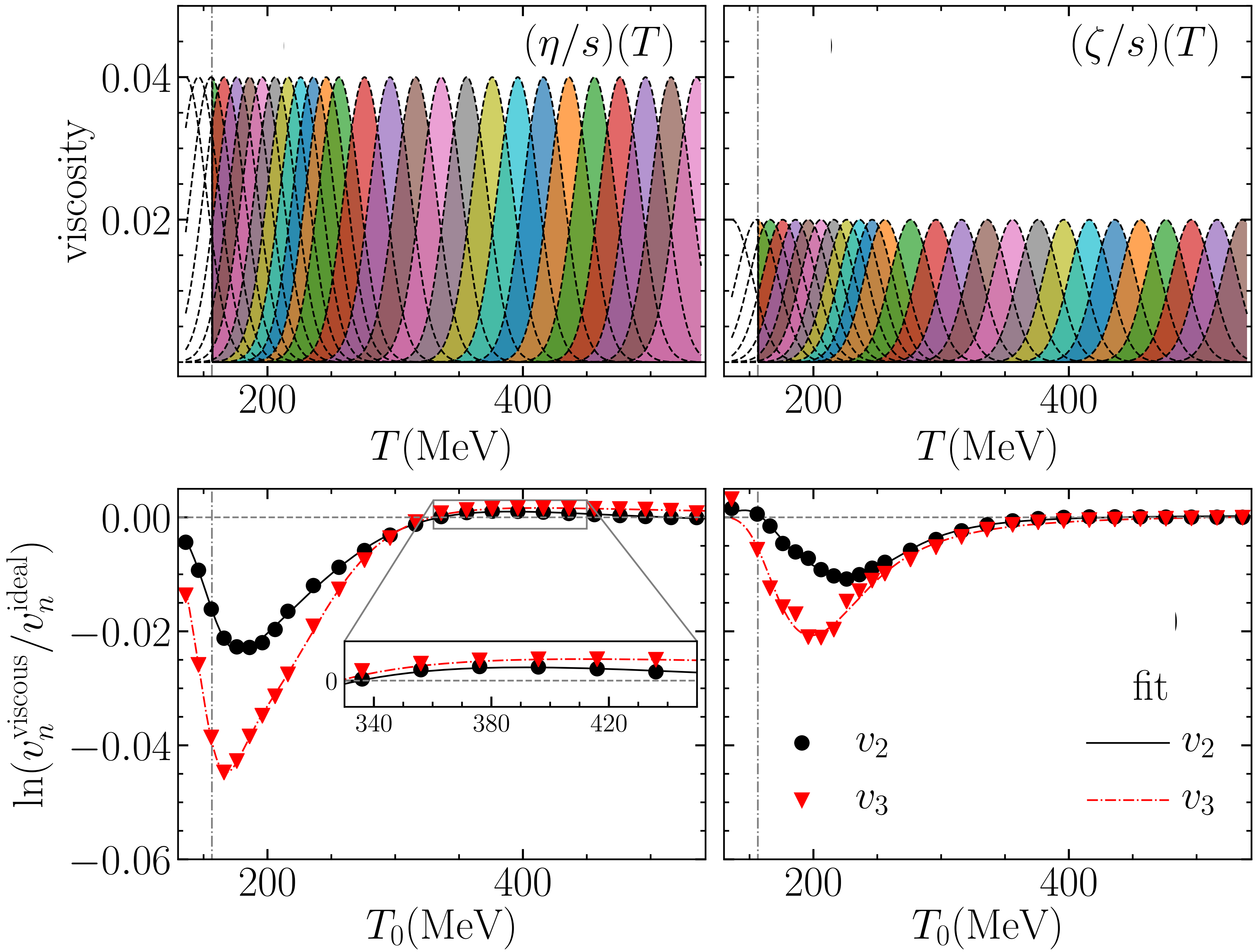
At fixed total entropy, the fluctuations in E_i are determined by the initial temperature (locally: dE=TdS)

Therefore, event-to-event $<p_t>$ fluctuations may reveal information about the early thermodynamics

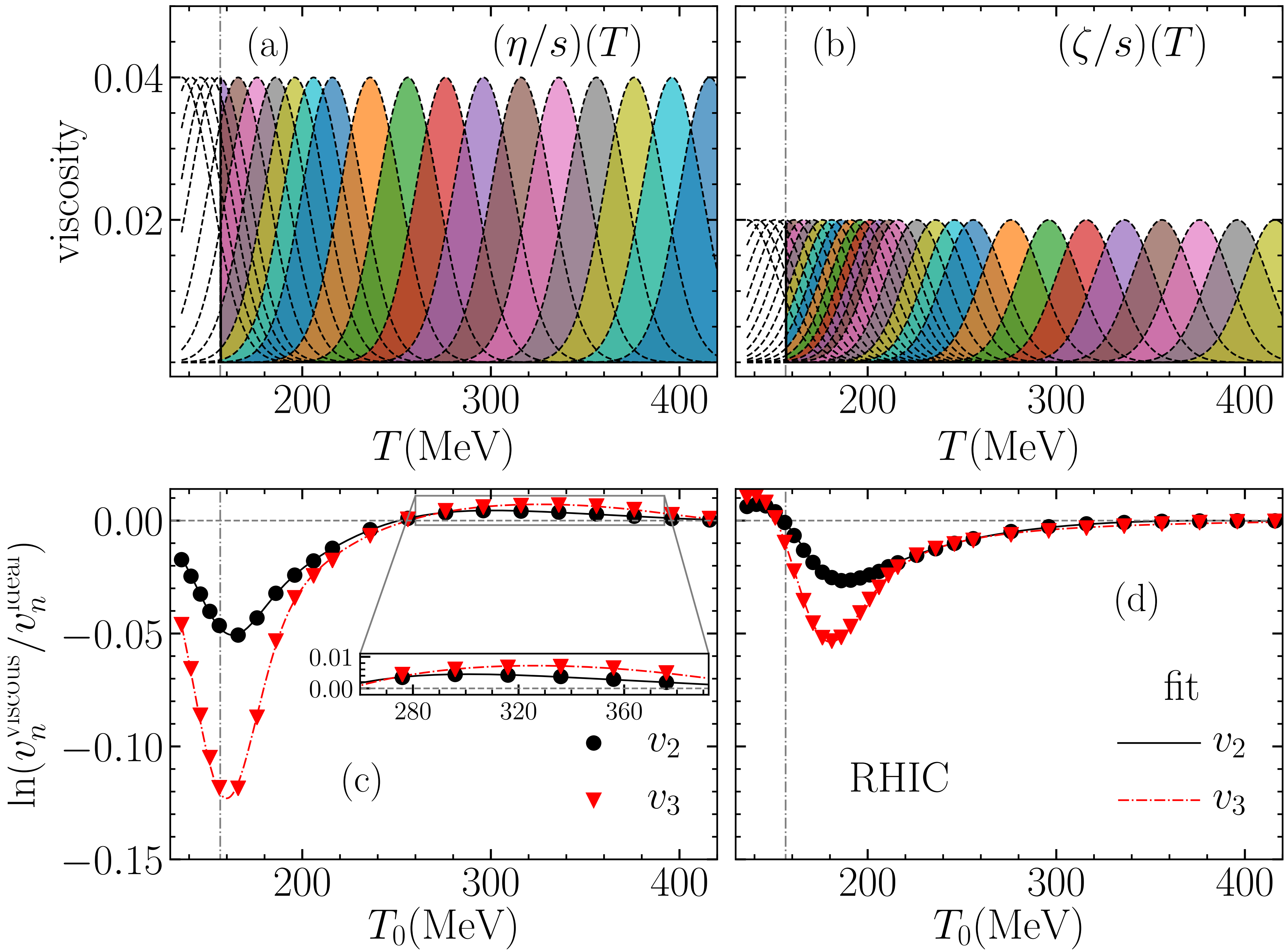
Giacalone et al. 2004.09799

Gardim et al. 2002.07008

Detailed results for narrow temperature-dependent viscosities (LHC)

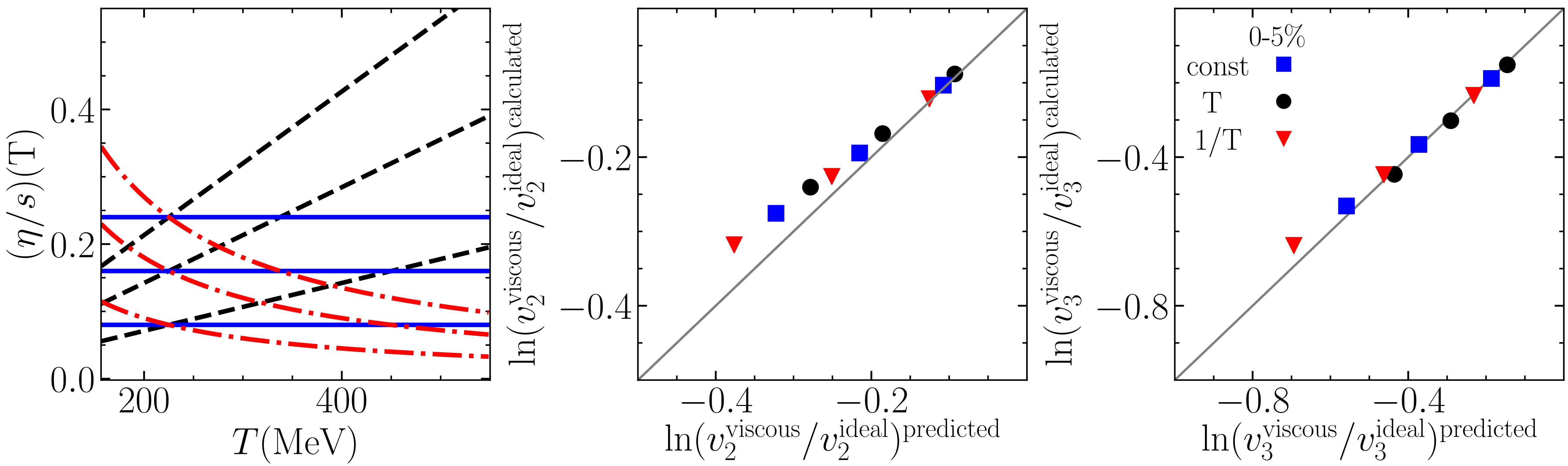


Detailed results for narrow temperature-dependent viscosities (RHIC)



Tests of linearity (1/2)

We check the validity of
$\ln[v_n(\text{viscous})/v_n(\text{ideal})] = \int w_n^{(\eta)}(T)(\eta/s)(T)\, dT$
for various $(\eta/s)(T)$ profiles

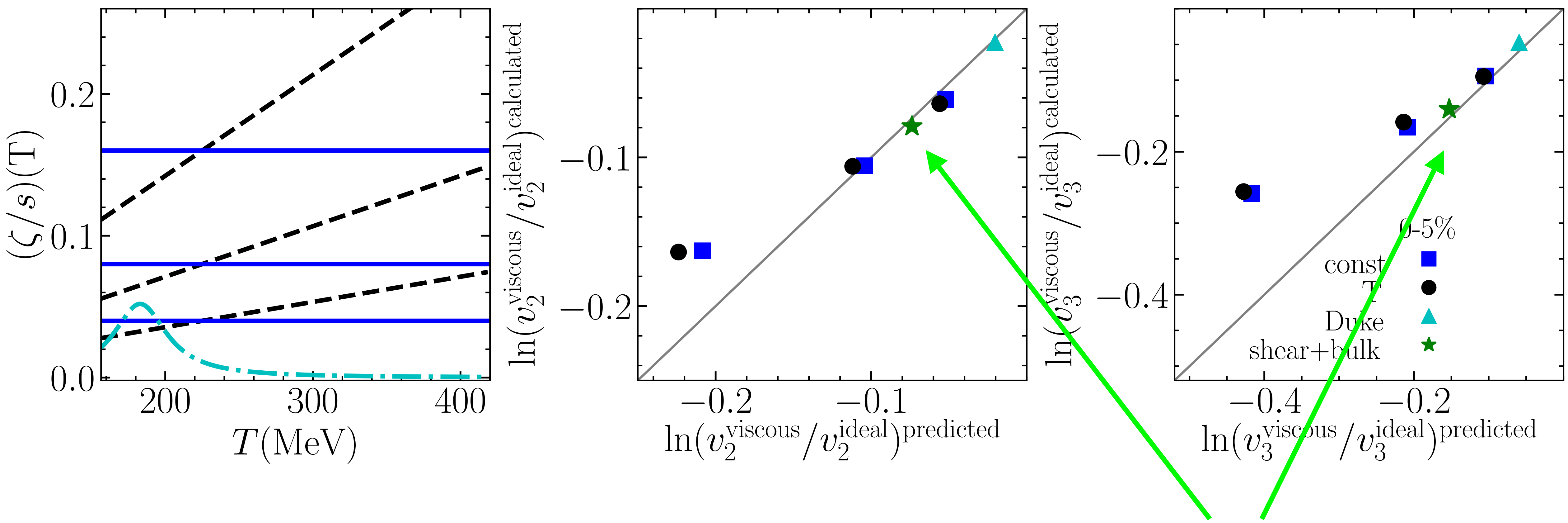


Even in the nonlinear regime where viscous suppression is large, the effective viscosity remains an excellent predictor.

Tests of linearity (2/2)

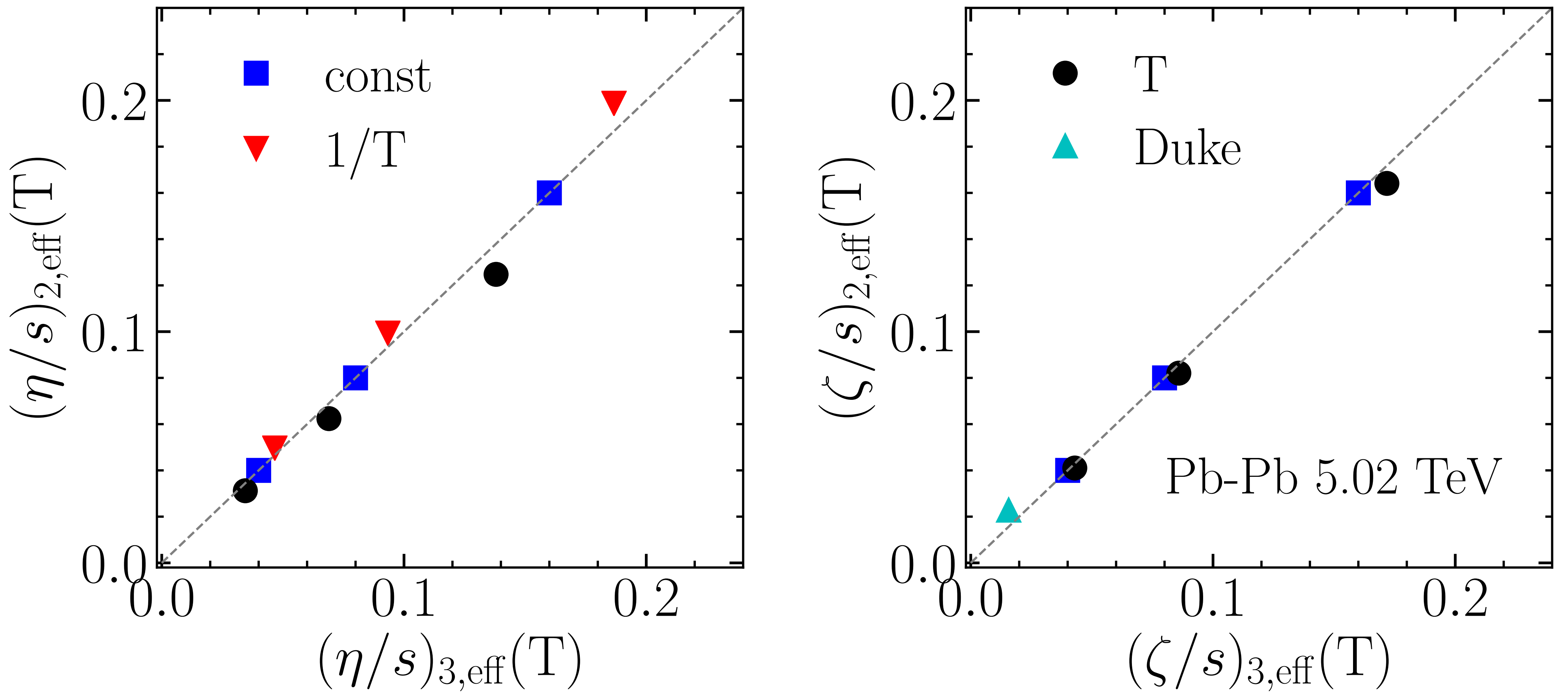
Same with bulk and shear+bulk

$\ln[v_n(\text{viscous})/v_n(\text{ideal})] = \int w_n^{(\eta)}(T)(\eta/s)(T) + w_n^{(\varsigma)}(T)(\varsigma/s)(T)]\, dT,$

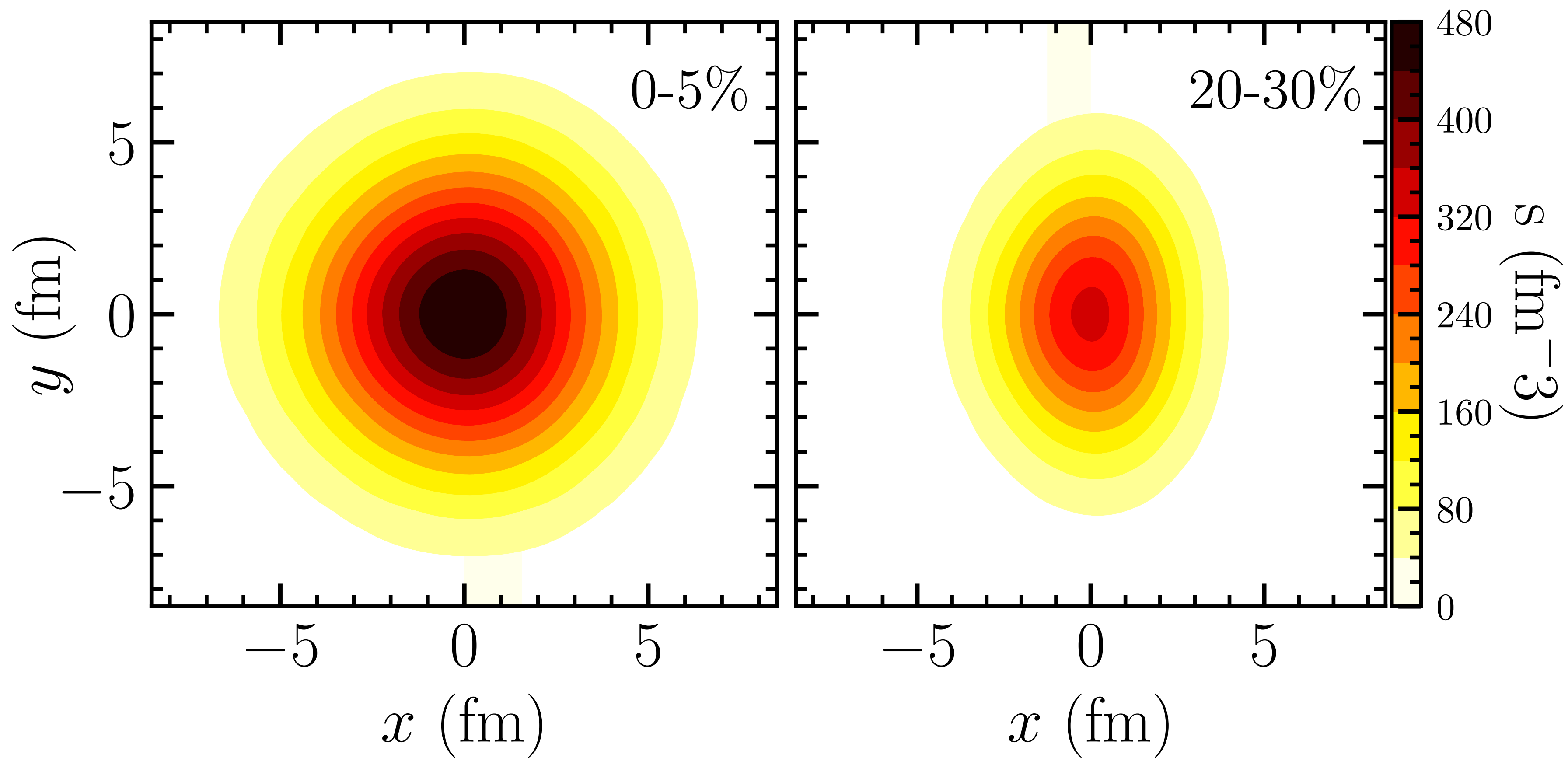


We confirm that the relative variation of v_n is the sum of the contributions of shear and bulk.

Comparison between effective viscosities for v_2 and v_3



Centrality dependence (1/3)

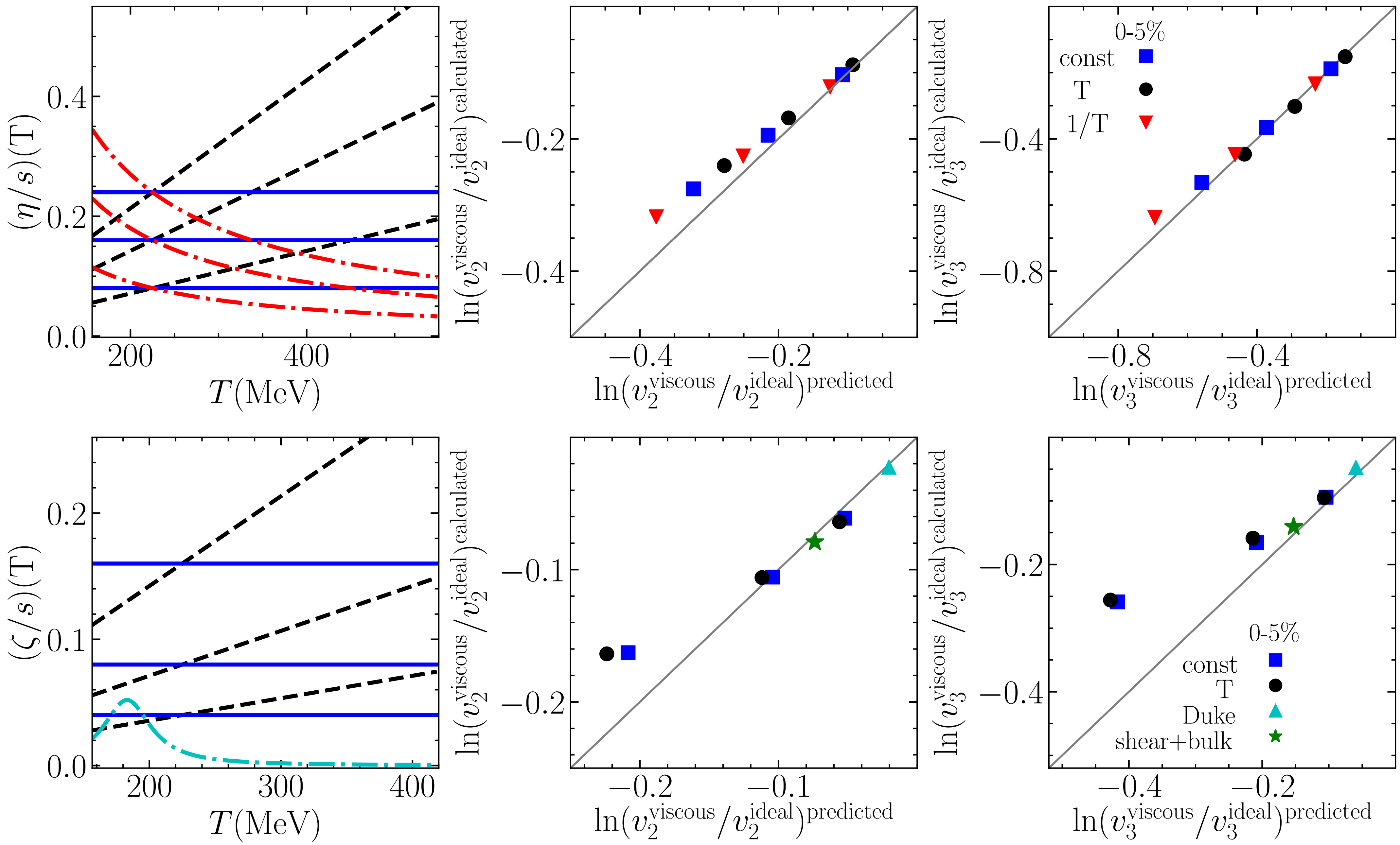


<p_t> is almost identical for these 2 centrality windows. Implies that they probe the same temperature interval.

The only change comes from the transverse size R.
Viscosity is the first gradient correction to ideal hydro.
v_n(viscous)/v_n(ideal)-1 should be proportional to 1/R.
(Reynolds number scaling)

Centrality dependence (2/3)

Using our results for the 0-5 % centrality window
Prediction for 20-30% based on 1/R scaling: global factor 1.32



Centrality dependence (2/3)

Using our results for the 0-5 % centrality window

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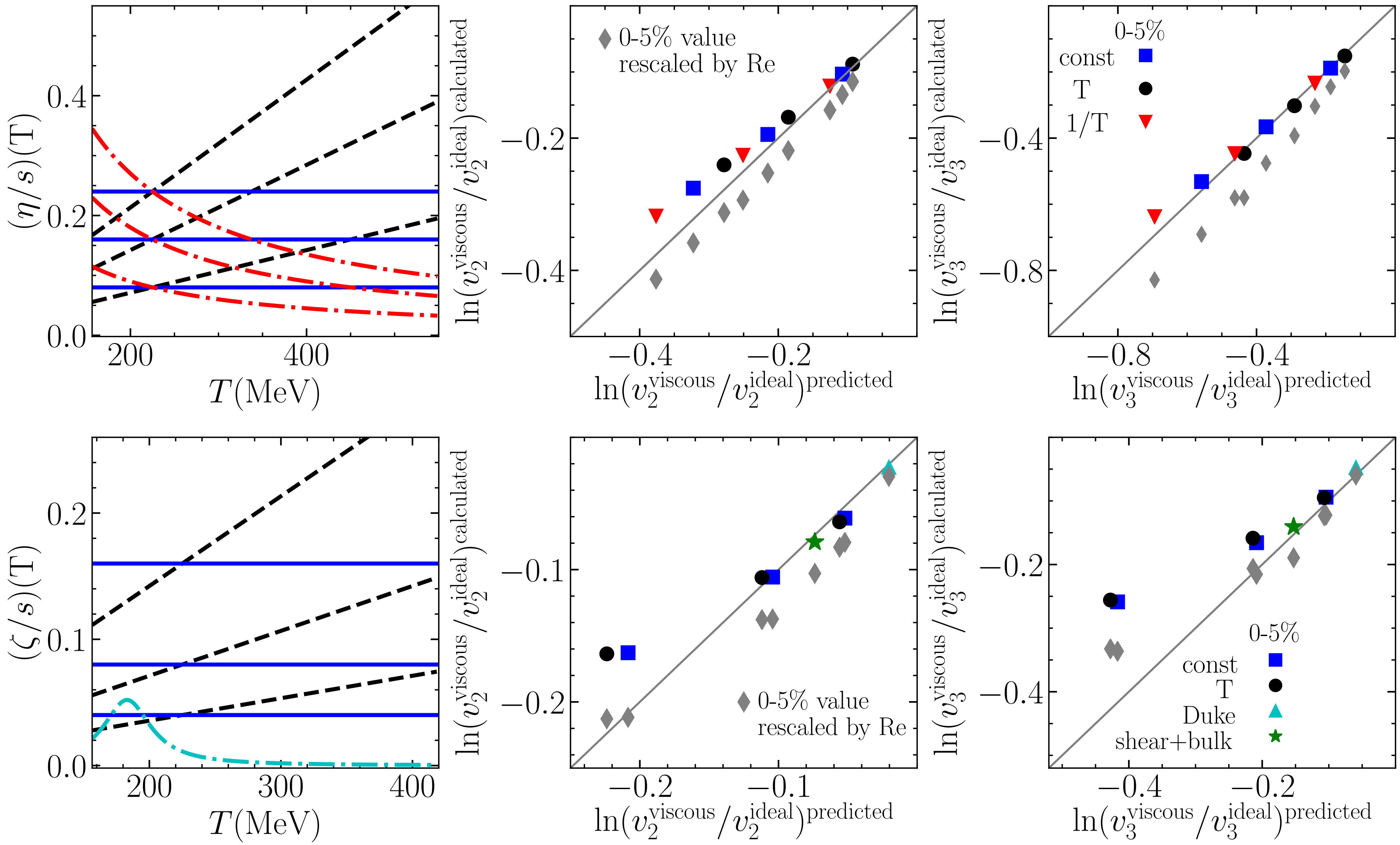
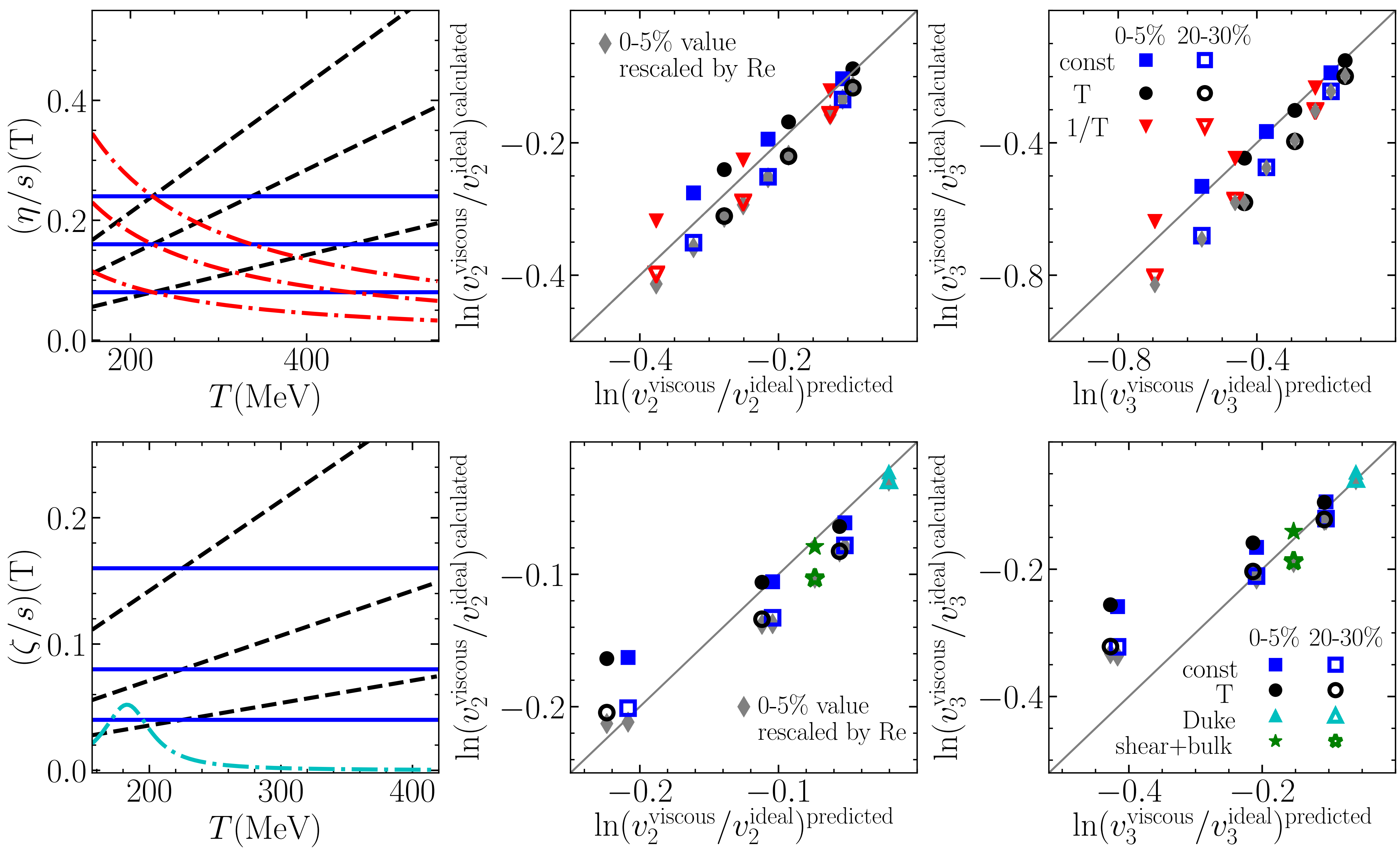
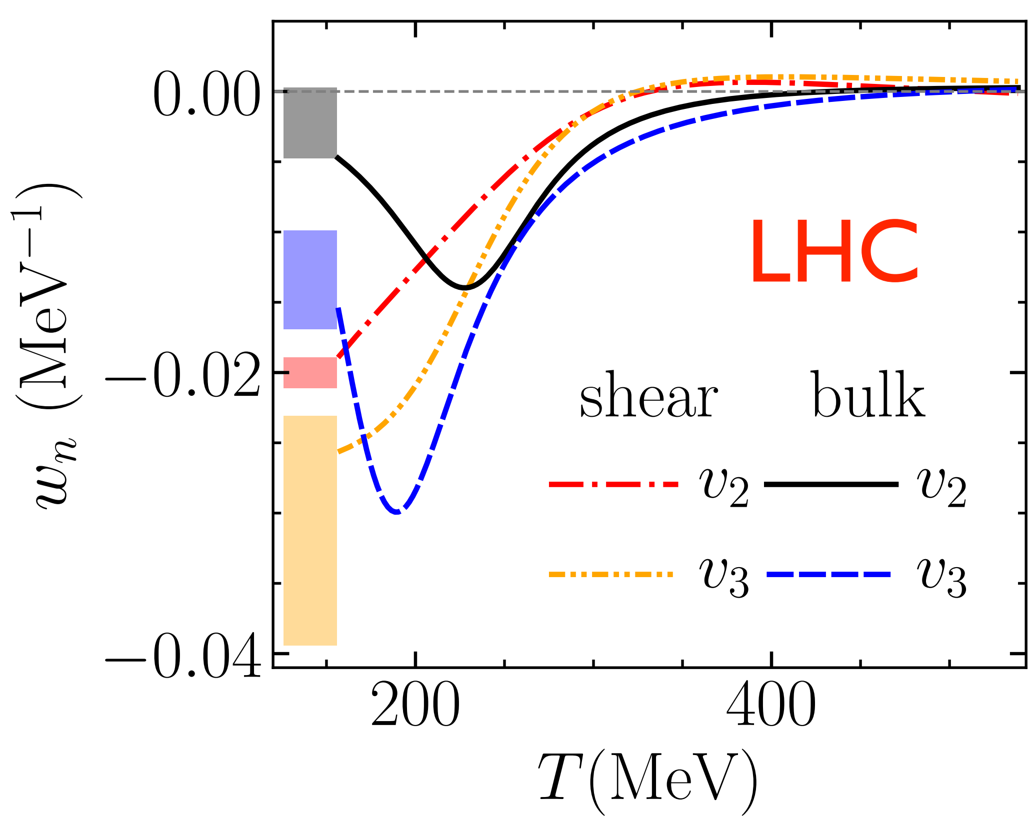


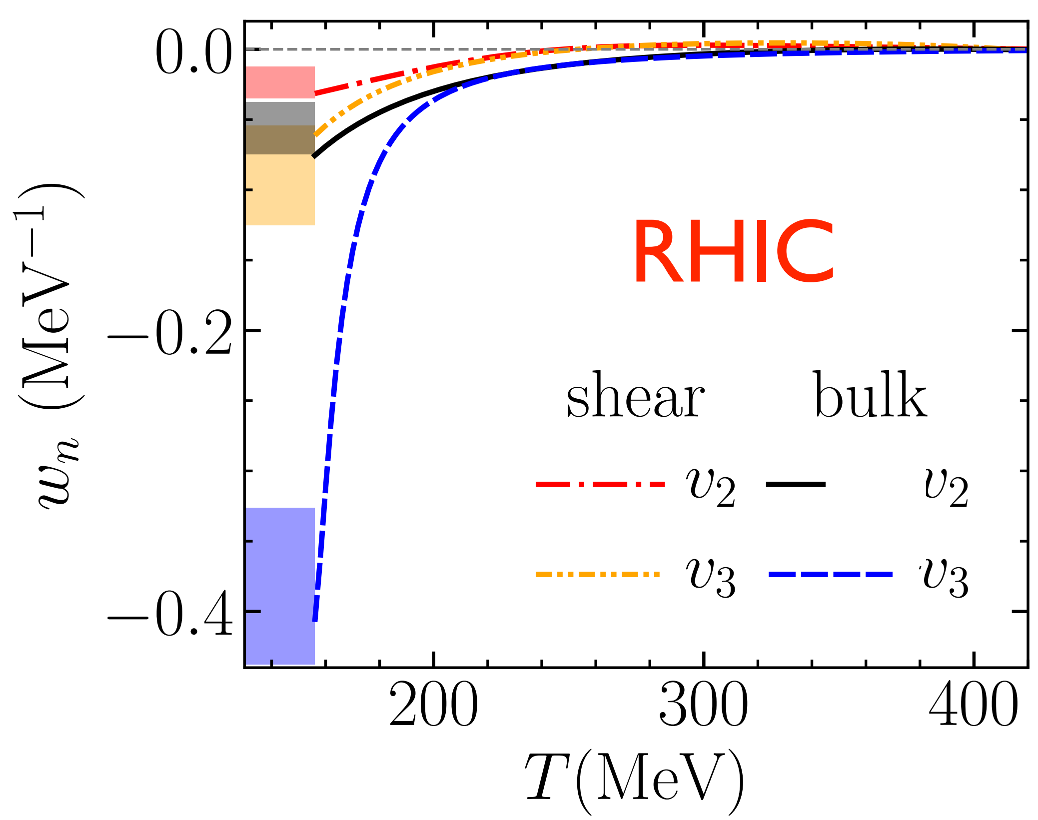
Centrality dependence (3/3)

Numerical calculations confirm the expected scaling.
Implies that effective viscosities are independent of centrality.



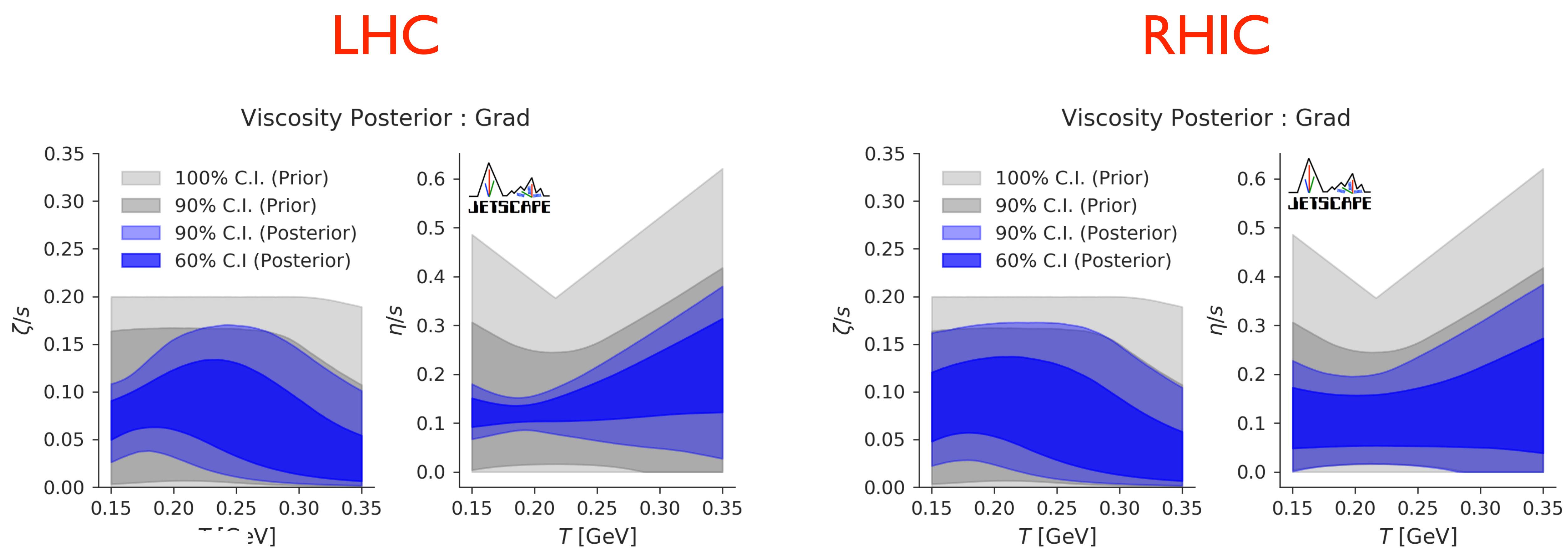
Effective viscosities at RHIC





The main difference is that freeze-out is no longer a small contribution to the viscous damping.
And it is the non-robust part. This implies that it will be harder to constrain the viscosity from RHIC data.

LHC versus RHIC



Everett et al. 2011.01430

The global study based on Bayesian inference reaches a similar conclusion: allowed band using RHIC data is broader.