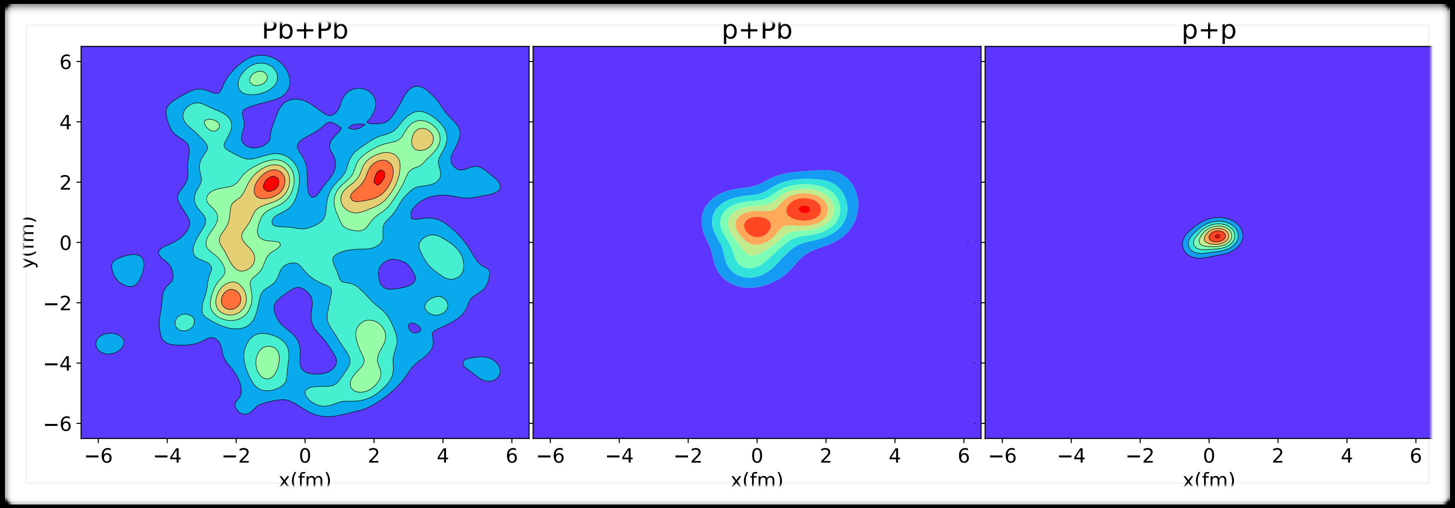
On-line seminar series III on “RHIC Beam Energy Scan: Theory and Experiment” 2021

Collective flow in Large and Small systems



You Zhou

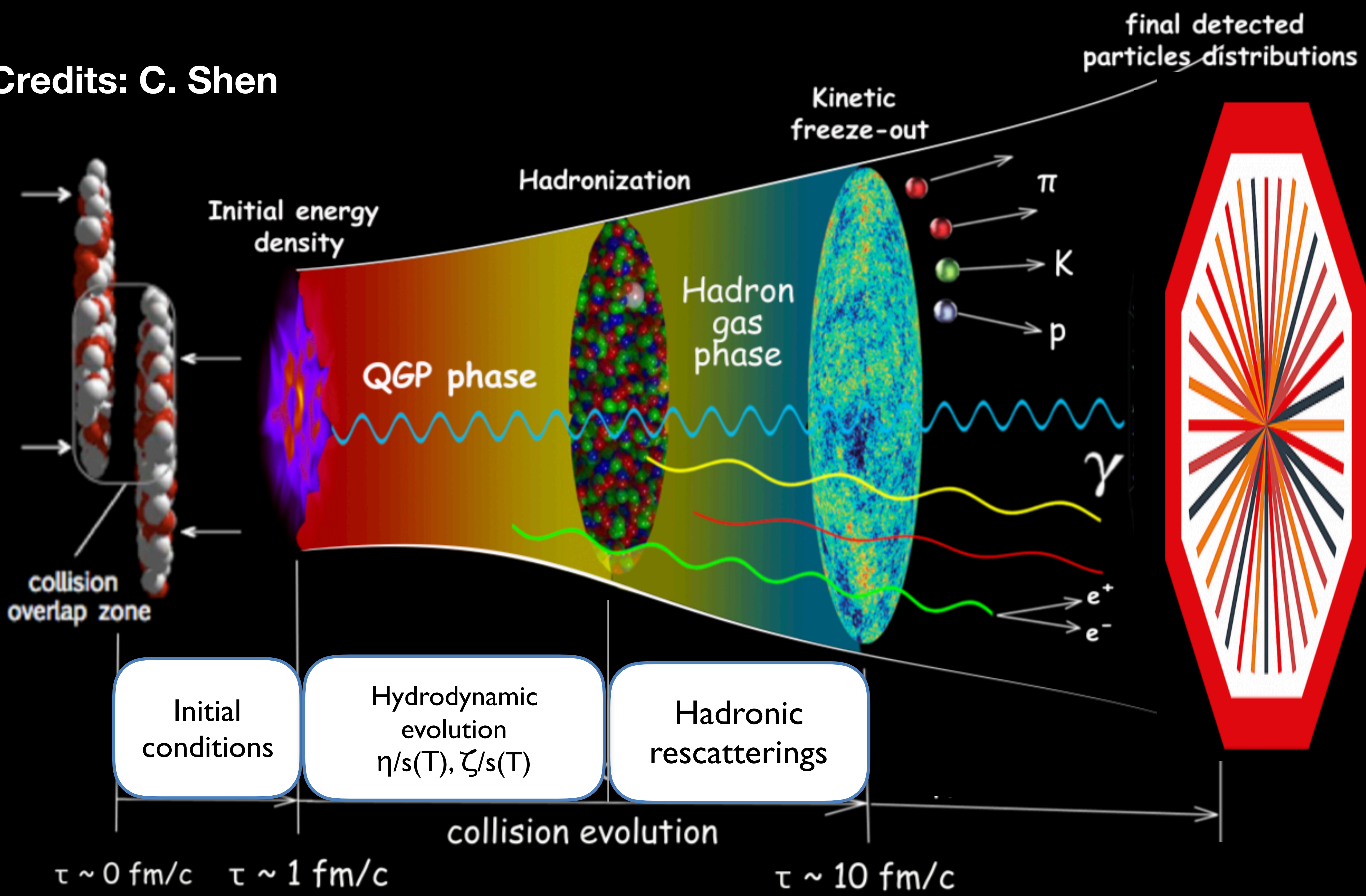
Niels Bohr Institute

UNIVERSITY OF COPENHAGEN

THE VELUX FOUNDATIONS
VILLUM FONDEN VELUX FONDEN

Evolution in the *Little Bang*

Credits: C. Shen



Current status of initial state models

Credits: G. Giacalone

THERE ARE CURRENTLY THREE CATEGORIES OF MODELS.

- **"sharp" models: IP-GLASMA and TRENTo 2016** **(v-USPhydro)**
  [Schenke, Shen, Tribedy **2005.14682**]
  [Bass, Bernhard, Moreland **1605.03954**]

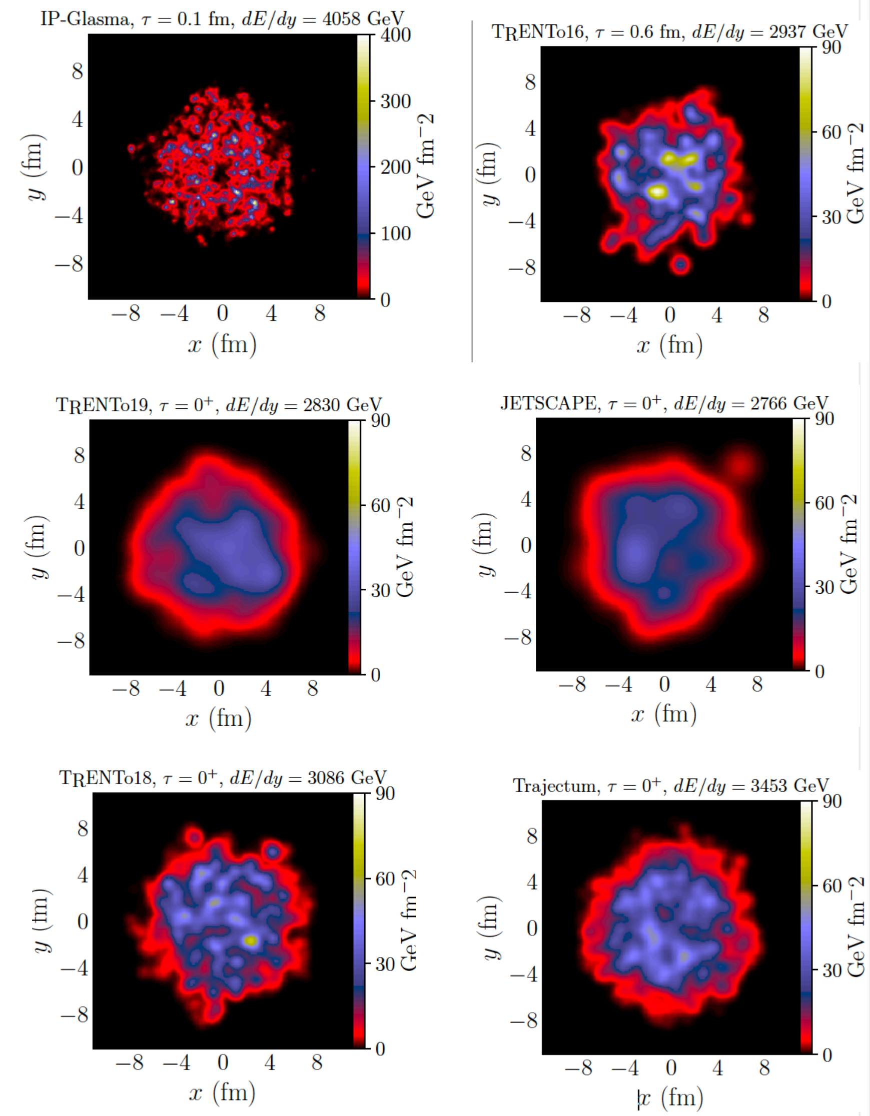
  Nucleons have a width of ~0.5fm (trento), 3 sub-nucleons with size ~0.3fm (IP-Glasma). Trento is used for the entropy density at the beginning of hydro. <u>IP-Glasma is the only model which incorporates a realistic pre-equilibrium evolution with longitudinal cooling.</u>

- **"fat" models: TRENTo 2019 and JETSCAPE**
  [Bass, Bernhard, Moreland **Nature Phys. 15 (2019)**]
  [JETSCAPE Collaboration **2011.01430**, **2010.03928**]

  The Trento parametrization is now used for the energy density at tau=0+. There is no substructure. The nucleon width is now ~1fm. Very smooth profiles.

- **"lumpy fat" models: TRENTo 2018 and Trajectum**
  [Bass, Bernhard, Moreland **1808.02106**]
  [Nijs, van der Schee, Gürsoy, Snellings **2010.15130**, **2010.15134**]

  The Trento parametrization is the energy density at tau=0+. Substructure is included: 4-6 constituents with width ~0.5fm. Profiles with some 'old school' lumpiness.



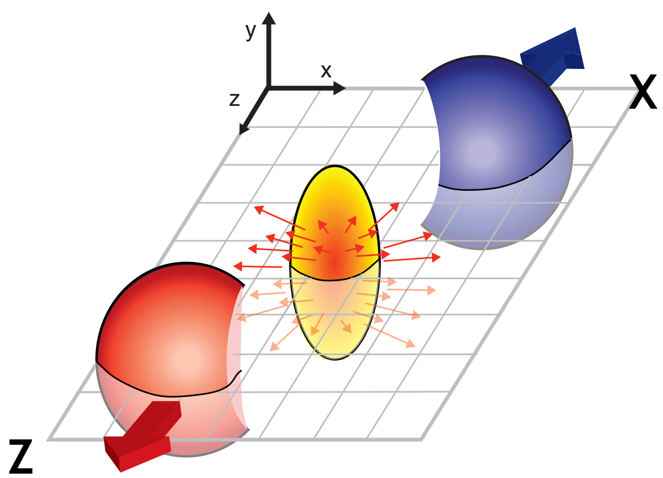
How can we access the initial conditions in EXP ?

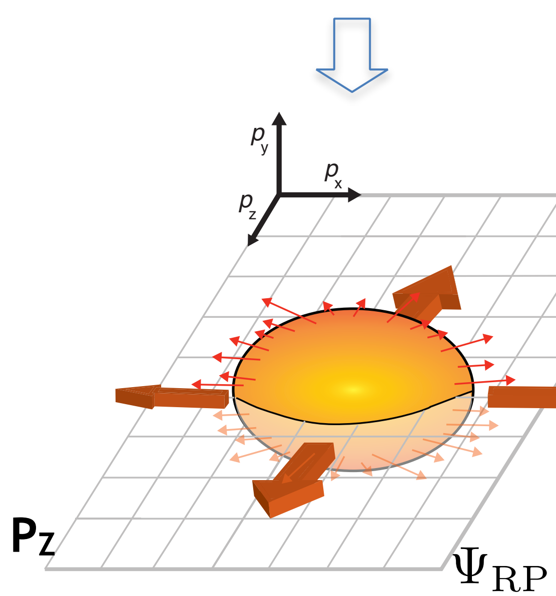
Studying QGP with flow

❖Spatial eccentricity in the initial state converted to momentum anisotropic particle distributions

- known as ***elliptic flow***
- reflect initial **eccentricity** and **transport properties** of QGP

J.Y. Ollitrault, PRD 46 (1992) 229





system expansion

Initial state

$$\varepsilon_2 = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

Initial spatial Eccentricity

Final state

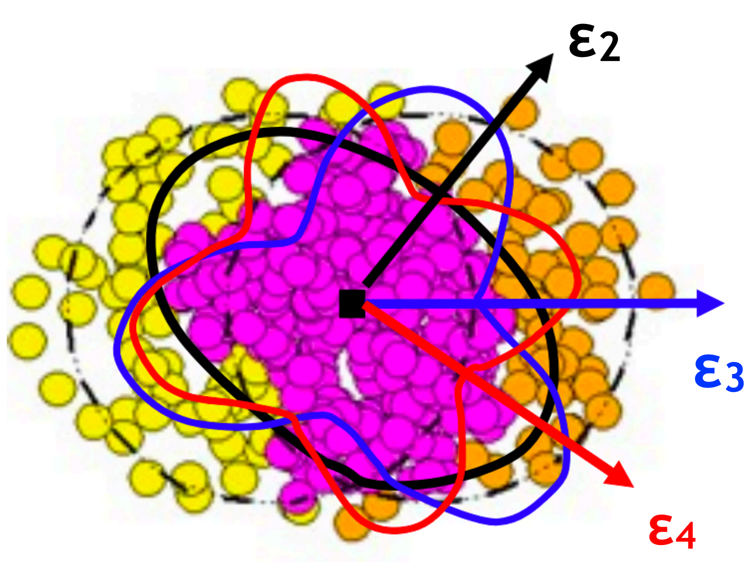
$$v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$$

momentum space Elliptic Flow

Figures by B. Hippolyte

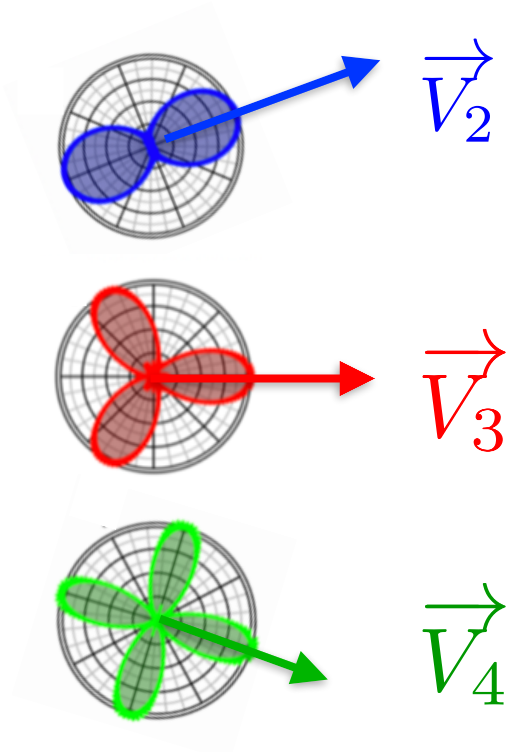
From initial anisotropy to anisotropic flow

Initial state



System expansion

Final state



$$\vec{V}_n = v_n \, e^{-in\Psi_n}$$

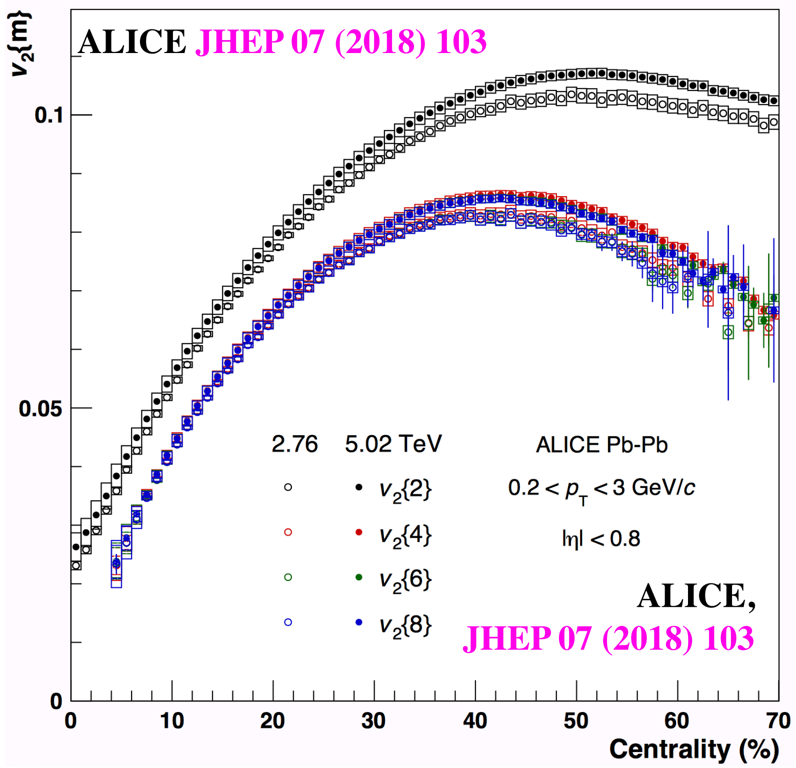
- v_n :Anisotropic flow
- ψ_n : Flow symmetry plane

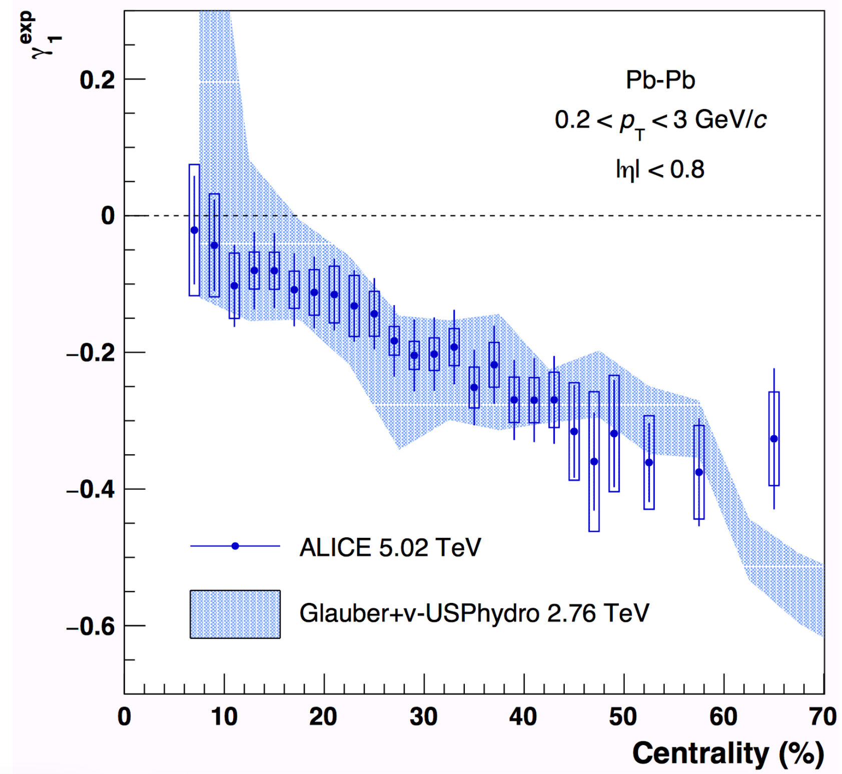
$$P(\varepsilon_m, \varepsilon_n, \varepsilon_k, ..., \Phi_m, \Phi_n, \Phi_k, ...) \Rightarrow P(v_m, v_n, v_k, ..., \Psi_m, \Psi_n, \Psi_k, ...)$$

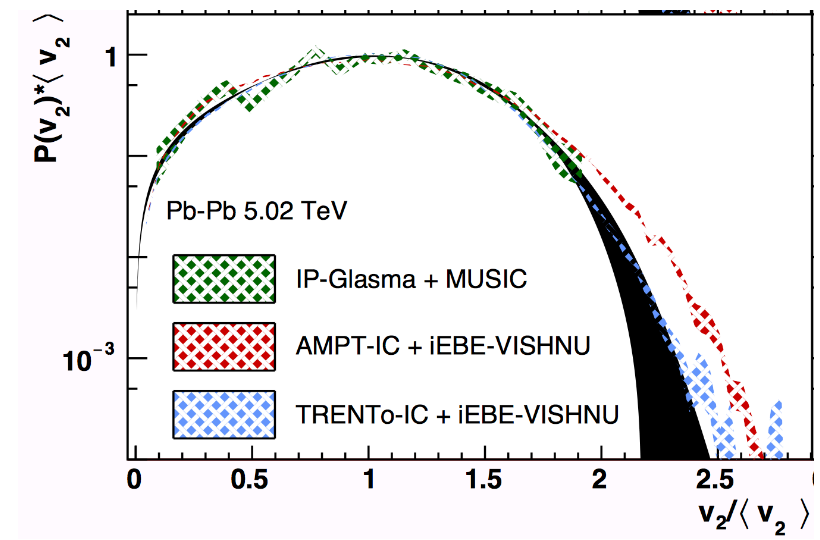
How does v_n fluctuate	How does ψ_n fluctuate	How do v_n and v_m correlate	How do ψ_n and ψ_m correlate
$P(v_n)$	$P(\Psi_n)$	$P(v_m, v_n, v_k, ...)$	$P(\Psi_m, \Psi_n, \Psi_k, ...)$

$P(v_n)$ and $P(\varepsilon_n)$

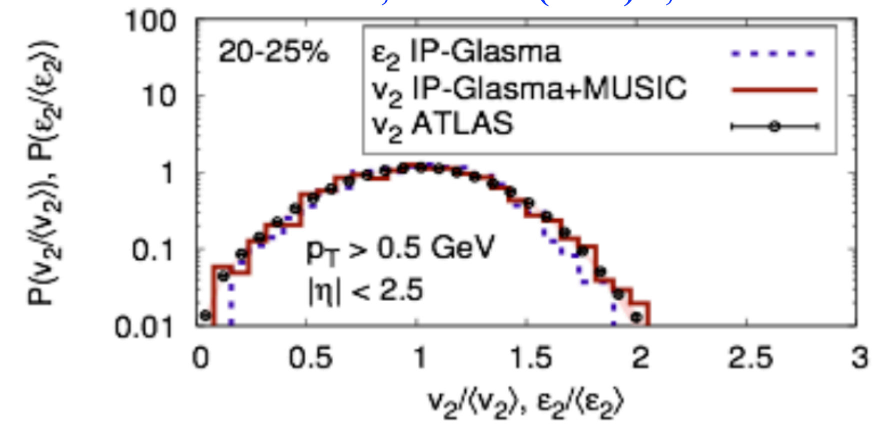
$v_n\{m\}$ → *Moments* → *$p(v_n)$* → *$p(\varepsilon_n)$*







C. Gale etc, PRL110 (2013) 1, 012302



$$v_n\{2\} = \sqrt[2]{\langle v_n^2\rangle},$$
$$v_n\{4\} = \sqrt[4]{2\langle v_n^2\rangle^2 - \langle v_n^4\rangle},$$
$$v_n\{6\} = \sqrt[6]{\langle v_n^6\rangle - 9\langle v_n^2\rangle\langle v_n^4\rangle + 12\langle v_n^2\rangle^3},$$
$$v_n\{8\} = \sqrt[8]{\langle v_n^8\rangle - 16\langle v_n^2\rangle\langle v_n^6\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^2\rangle^2\langle v_n^4\rangle - 144\langle v_n^2\rangle^4}.$$

$$\gamma_1^{\rm exp} = -6\sqrt{2}v_2\{4\}^2\frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}$$

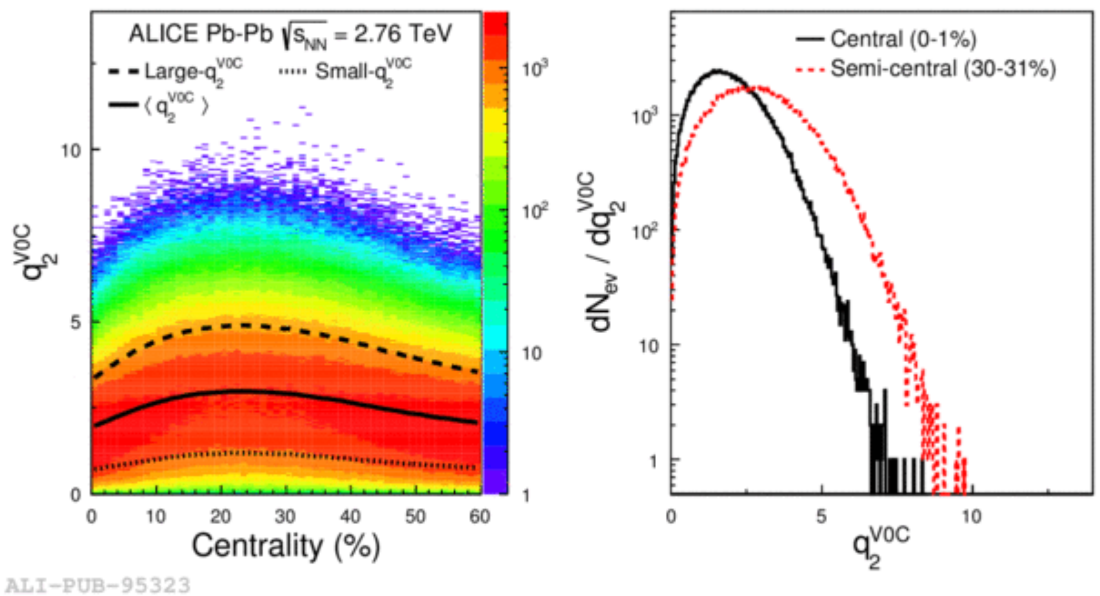
$$\gamma_2 \simeq \gamma_2^{\rm expt} \equiv -\frac{3}{2}\frac{v_2\{4\}^4 - 12v_2\{6\}^4 + 11v_2\{8\}^4}{(v_2\{2\}^2 - v_2\{4\}^2)^2}$$

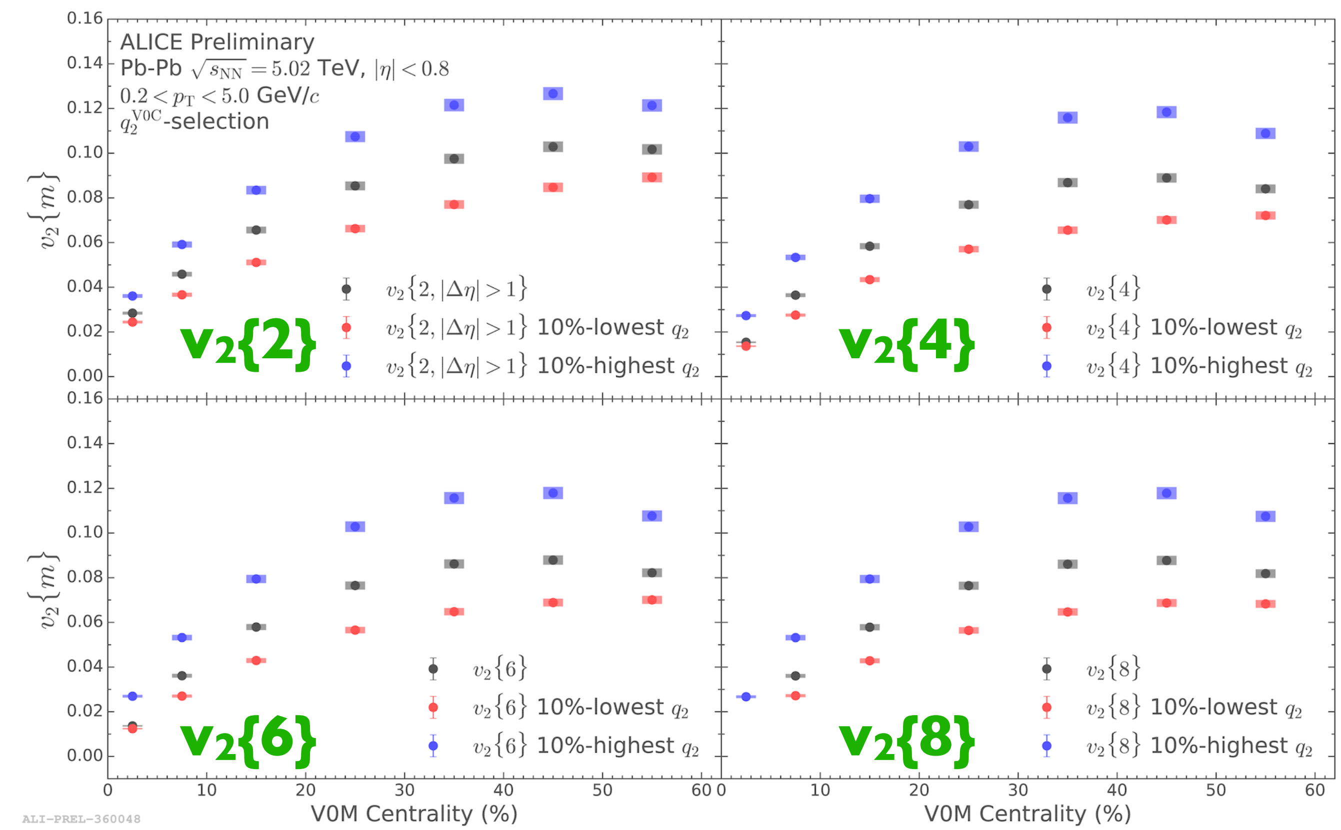
$$v_n \propto \varepsilon_n$$
$$P(v_n/\langle v_n\rangle) \approx P(\varepsilon_n/\langle\varepsilon_n\rangle)$$

- Investigating $p(v_2)$ with multi-particle cumulants
  - Ultra-higher order cumulants e.g. $v_2\{10\}\{12\}\{14\}\{16\}$ is implemented for HL-LHC,
  - Possibility to construct a more precise p.d.f. with higher moments

P(v_n) and ESE

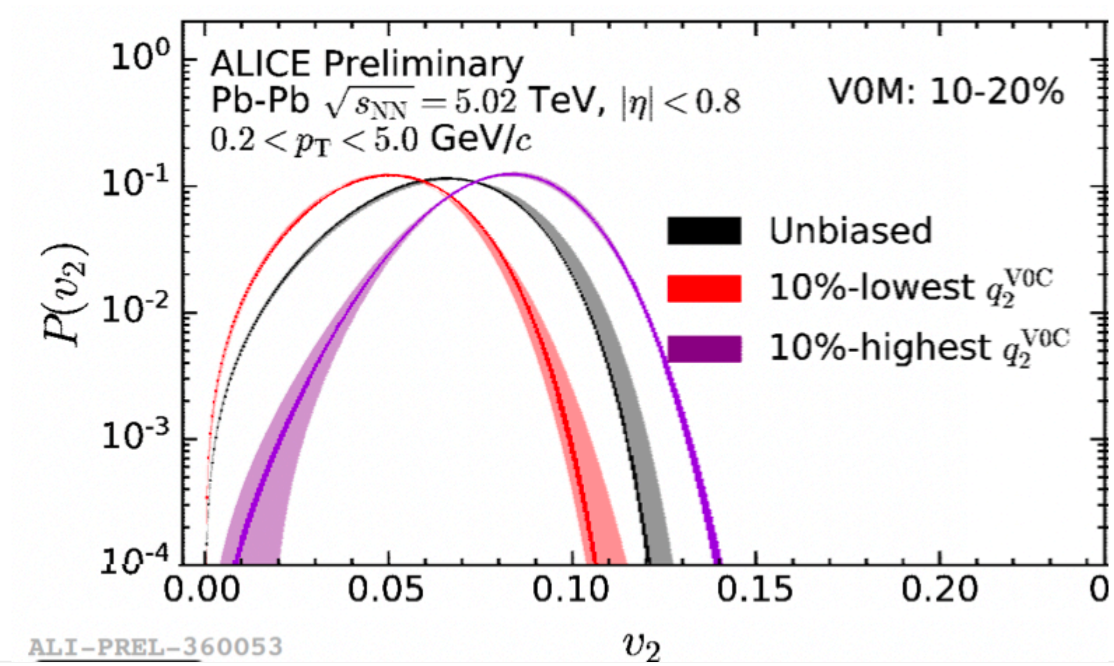
ESE: J. Schukraft etc, PLB719 (2013) 394
ALICE, PRC 93 (2016) 3, 034916



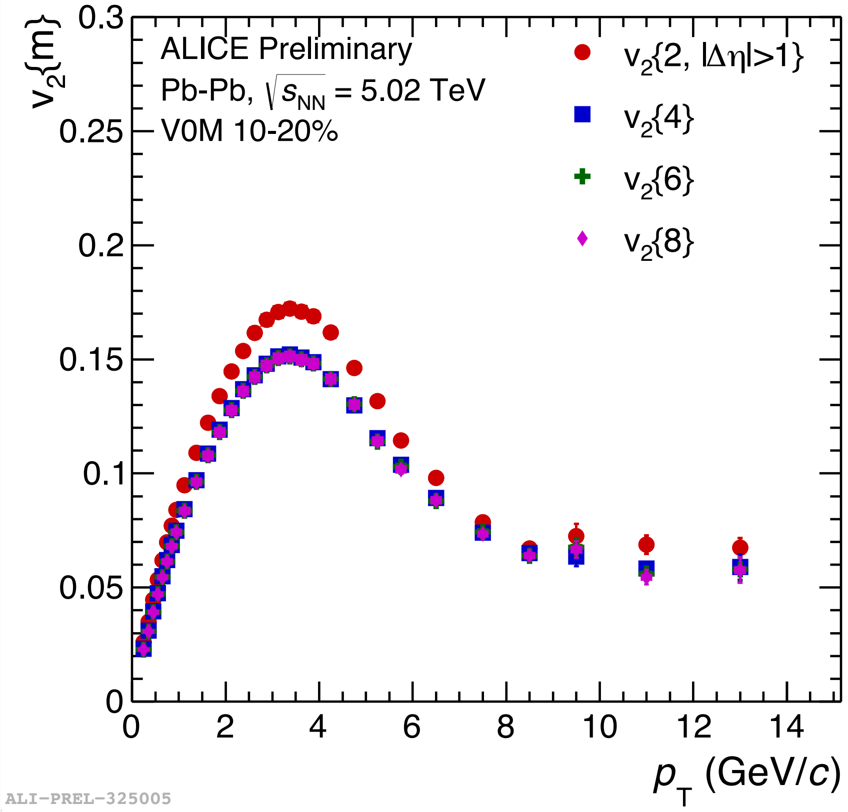


$$q_n = \frac{|Q_n|}{\sqrt{M}} \dashrightarrow \varepsilon_2 \longrightarrow v_2$$

- Using Event-Shape Engineering (ESE) to select high (low) q_2 to get larger (smaller) v_2
- The fluctuation study with ESE reveals that ESE selects not only ε_2 but also its fluctuations, which modifies the p.d.f. (i.e. its skewness)
- One should not compare high 10% q_2 in data to 10% large ε_2 model calculations (common issues in HF studies)

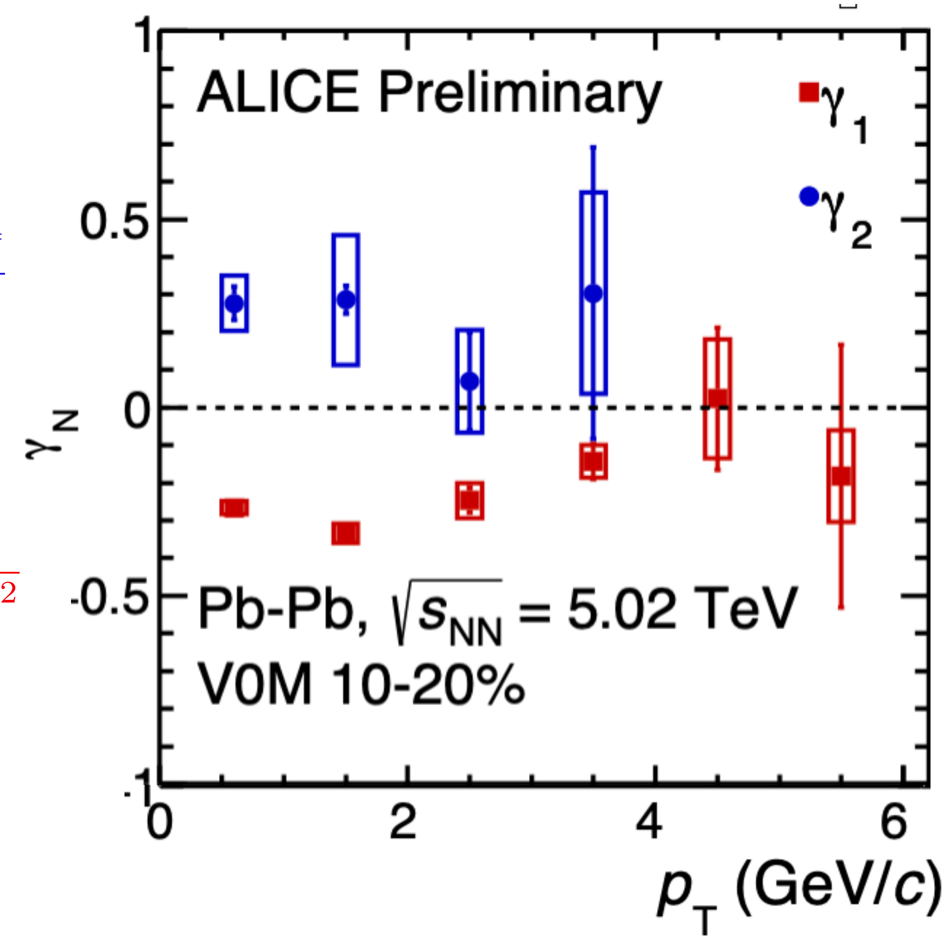


p_T-differential p.d.f.



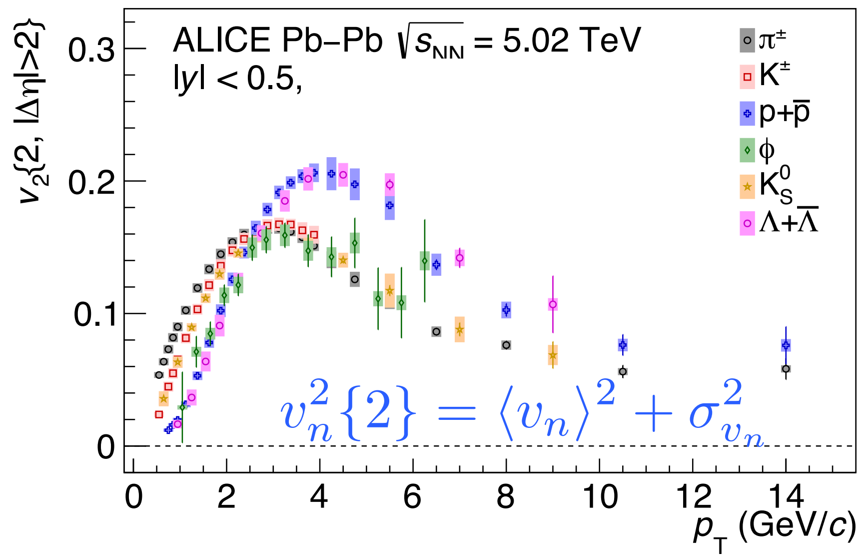
$$\gamma_2 = -\frac{3}{2}\frac{v_2\{4\}^4 - 12\,v_2\{6\}^4 + 11\,v_2\{8\}^4}{(v_2\{2\}^2 - v_2\{4\}^2)^2}$$

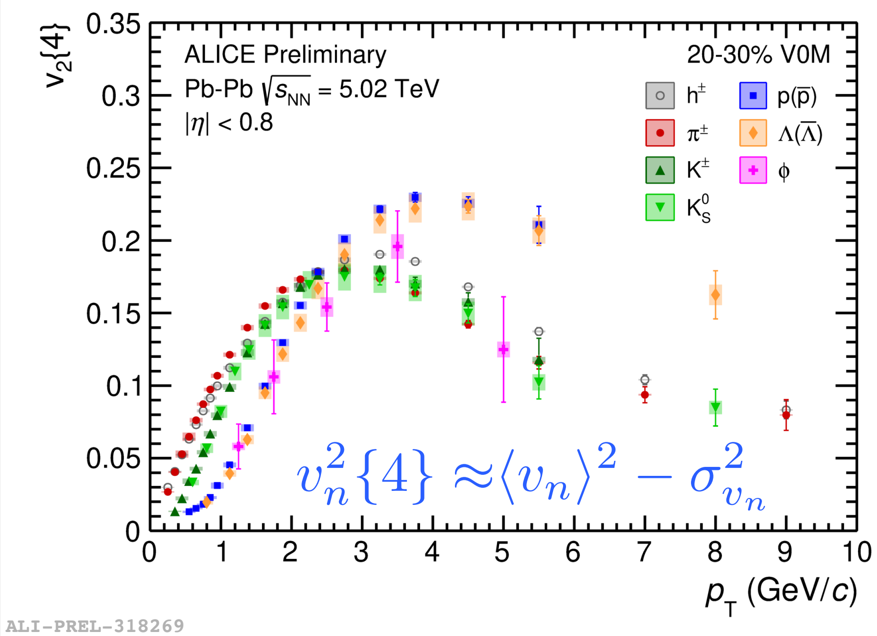
$$\gamma_1 = -6\sqrt{2}\,v_2\{4\}^2\frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}$$



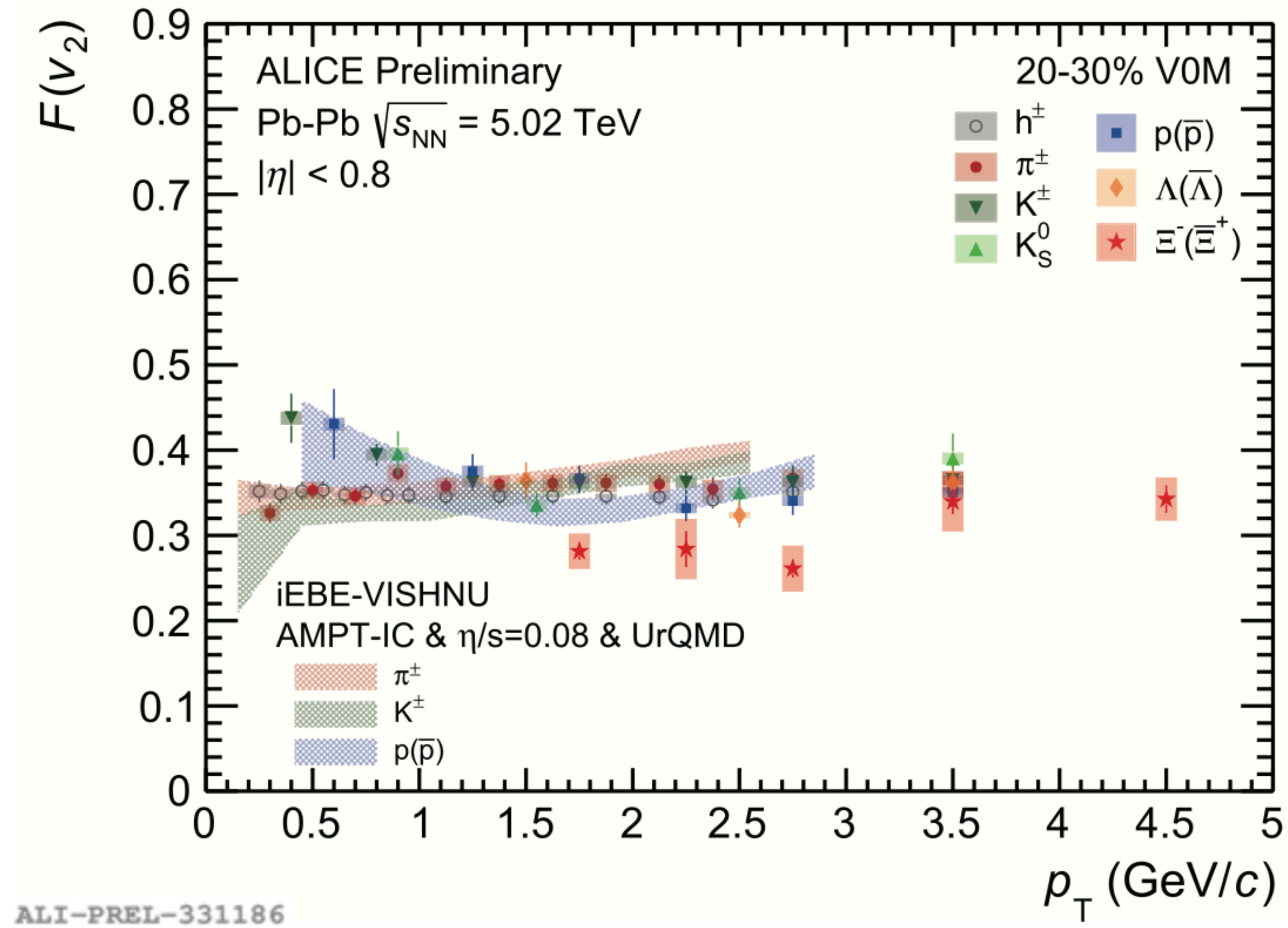
- Study p_T differential p.d.f. of v_2 using multi-particle cumulants
  - Non-trivial p_T dependence of γ_1 and γ_2
    - γ_1 : negative at low p_T and is compatible with 0 at high p_T
    - γ_2: positive for p_T < 2 GeV/c and then consistent with 0 within large uncertainty
  - Different or modification of p.d.f. in differential study?

First PID flow fluctuations





$$F_{v_n} = \frac{\sigma_{v_n}}{\langle v_n \rangle} = \sqrt{\frac{v_n^2\{2\} - v_n^2\{4\}}{v_n^2\{2\} + v_n^2\{4\}}}$$

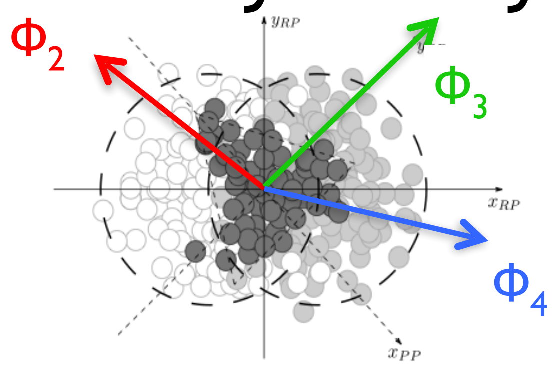


- First study of (relative) flow fluctuations of identified hadrons using $v_2\{2\}$ and $v_2\{4\}$
- Particle species dependence is observed
  - Similar indications from hydro calculations
  - Final state effects modify the p.d.f.?

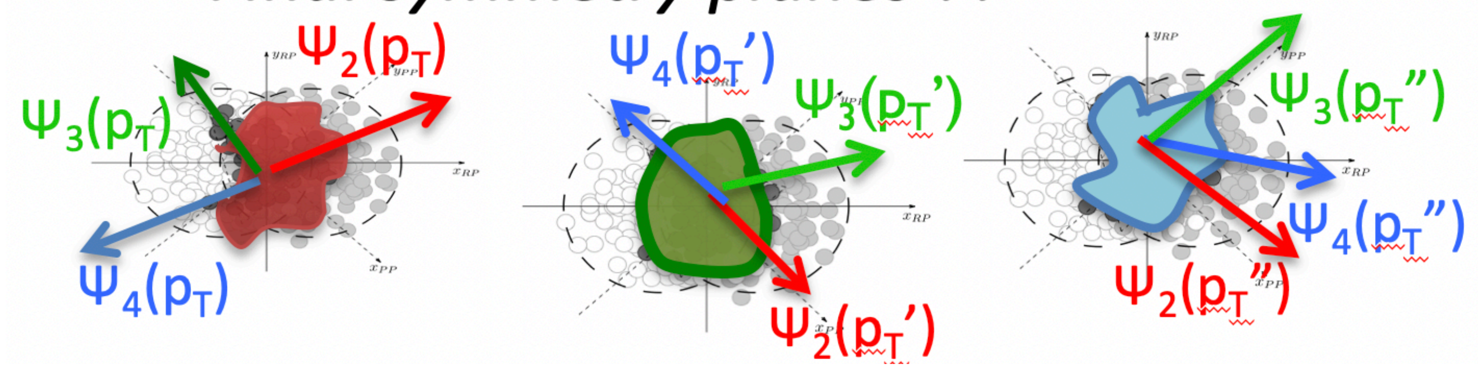
How does ψ_n fluctuate

Flow vector fluctuations

Initial symmetry planes



Final symmetry planes ??



U. Heinz etc, PRC87, 034913 (2013)
F. G. Gardim etc, PRC87, 031901(R) (2013)

$$v_n\{2\} = \frac{\langle v_n(p_{\rm T})\ v_n \cos n[\Psi_n(p_{\rm T}) - \Psi_n]\rangle}{\sqrt{\langle v_n^2\rangle}}$$

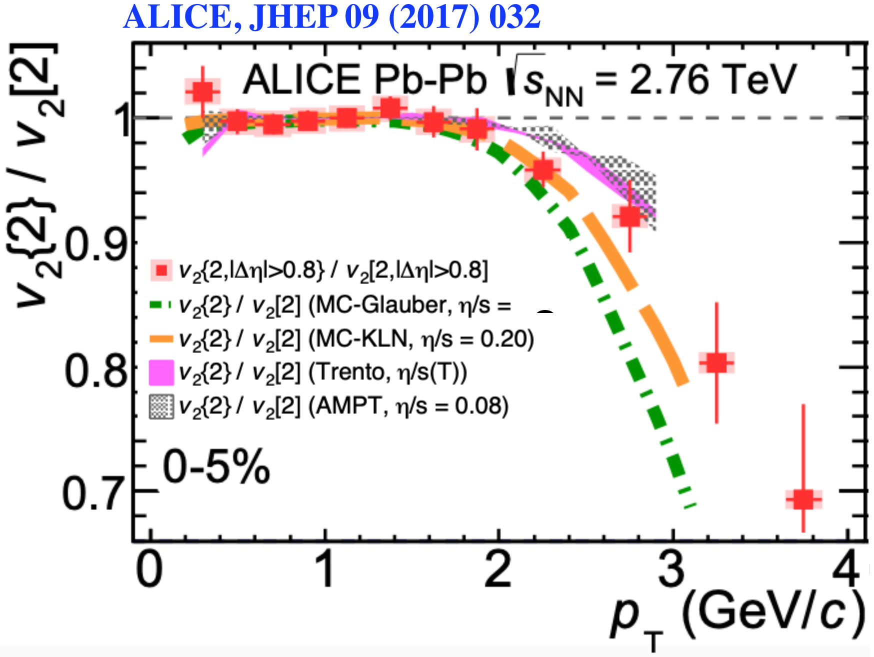
$$v_n[2] = \sqrt{\langle v_n^2(p_{\rm T})\rangle}$$

$$\frac{v_n\{2\}}{v_n[2]} = \frac{\langle v_n(p_{\rm T}^a)\ v_n \cos n[\Psi_n(p_{\rm T}^a) - \Psi_n]\rangle}{\sqrt{\langle v_n^2(p_{\rm T}^a)\rangle}\sqrt{\langle v_n^2\rangle}}$$

→ Flow angle fluctuations

→ Flow magnitude fluctuations

ALICE, JHEP 09 (2017) 032



- ❖ $v_2\{2\}/v_2[2] < 1$, indicates presence of flow angle and magnitude fluctuations
- ❖ How can we disentangle the two contributions and quantify each of them?

Flow angle and magnitude fluctuations

☆ **New observable to measure** **flow angle fluctuations:**

$$F(\Psi_n^a, \Psi_n) = \frac{\langle\langle \cos[n(\varphi_1^a + \varphi_2^a - \varphi_3 - \varphi_4)] \rangle\rangle}{\langle\langle \cos[n(\varphi_1^a + \varphi_2 - \varphi_3^a - \varphi_4)] \rangle\rangle}$$

$$= \frac{\langle v_n^2(p_{\rm T}^a)\ v_n^2 \cos 2n[\Psi_n(p_{\rm T}^a) - \Psi_n] \rangle}{\langle v_n^2(p_{\rm T}^a) v_n^2 \rangle}$$

$$\approx \langle \cos 2n[\Psi_n(p_{\rm T}^a) - \Psi_n)] \rangle$$

$F(\Psi_n^a, \Psi_n) < 1$ indicates $p_{\rm T}$-dependent **flow angle fluctuations**

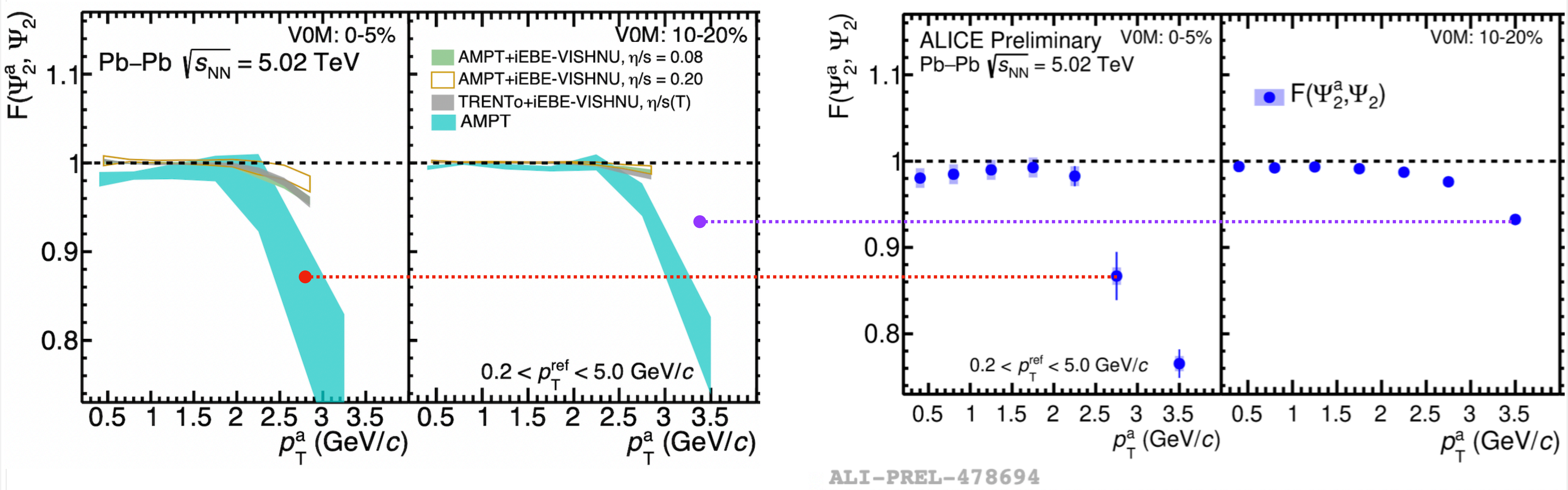
☆ **New observable to measure** **flow magnitude fluctuations:**

$$\frac{\langle\langle \cos n(\varphi_1^a + \varphi_2 - \varphi_3^a - \varphi_4) \rangle\rangle}{\langle\langle \cos n(\varphi_1^a - \varphi_3^a) \rangle\rangle \langle\langle \cos n(\varphi_2 - \varphi_4) \rangle\rangle} = \frac{\langle v_n^2(p_{\rm T}^a)\ v_n^2 \rangle}{\langle v_n^2(p_{\rm T}^a) \rangle \langle v_n^2 \rangle}$$

$p_{\rm T}$-integrated baseline: $\langle v_n^4 \rangle / \langle v_n^2 \rangle^2$

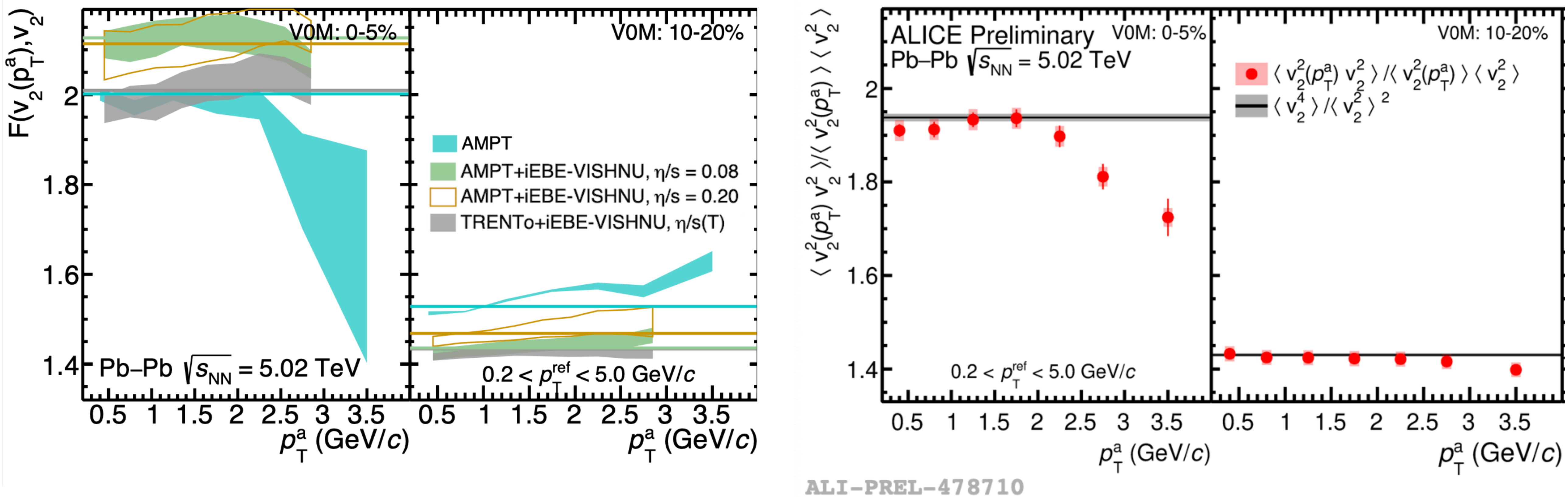
Deviations from baseline indicate the $p_{\rm T}$-dependent **flow magnitude fluctuations**

Flow angle fluctuations



- Probe p_T dependent flow angle fluctuations with $F(\Psi_n^a, \Psi_n) \approx \langle \cos 2n[\Psi_n(p_T^a) - \Psi_n)] \rangle$
- Deviations from unity strongest in central collisions
- More than 5σ significance at high p_T in most centralities
- Comparison with model predictions:
  - iEBE-VISHNU underestimates the deviation in central collisions
  - AMPT works well in central while overestimates the data in semi-central
  - for p_T > 3 GeV/c, hydro calculations/predictions might not be reliable, CoLBT is only available in 10-20% and 40-50% with very limited statistics

Flow magnitude fluctuations



- ❖ Probe p_T dependent flow magnitude fluctuations $\frac{\langle\langle \cos n(\varphi_1^a+\varphi_2-\varphi_3^a-\varphi_4)\rangle\rangle}{\langle\langle\cos n(\varphi_1^a-\varphi_3^a)\rangle\rangle\langle\langle\cos n(\varphi_2-\varphi_4)\rangle\rangle} = \frac{\langle v_n^2(p_T^a)\; v_n^2\rangle}{\langle v_n^2(p_T^a)\rangle\langle v_n^2\rangle}$
- ❖ Deviations from baseline at higher p_T
- ❖ 5σ significance at high p_T in most centralities (~3σ in 30-40%)
- ❖ Comparison with model calculations:
  - iEBE-VISHNU are consistent with their baselines, but fail to reproduce ALICE data
  - AMPT works well in 0-5% centrality, fails in higher centralities
  - Relative deviation from unity by dividing the base line

(Normalized) Symmetric Cumulant

Symmetric cumulants:

$$SC(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

PHYSICAL REVIEW C **89**, 064904 (2014)

Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

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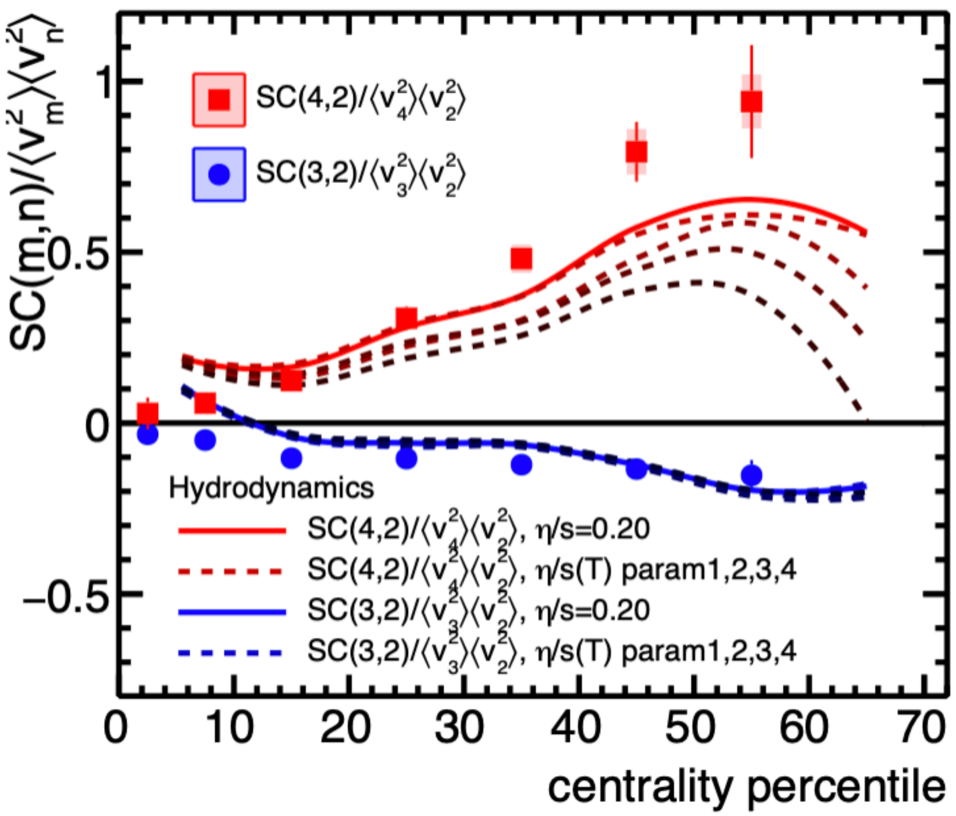
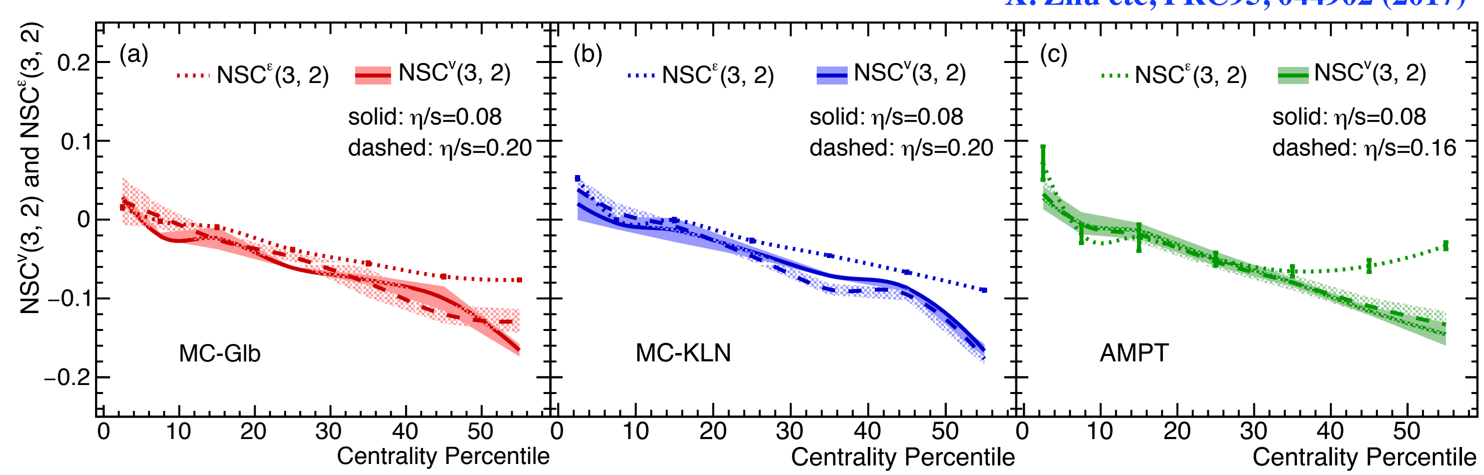
[3]Utrecht University, P.O. Box 80000, 3508 TA Utrecht, The Netherlands

ALICE, PRL117, 182301 (2016)



X. Zhu etc, PRC95, 044902 (2017)



$$v_2 \propto \varepsilon_2 \qquad v_3 \propto \varepsilon_3 \quad \Rightarrow \quad \frac{\langle v_3^2\, v_2^2 \rangle}{\langle v_3^2 \rangle \langle v_2^2 \rangle} \approx \frac{\langle \varepsilon_3^2\, \varepsilon_2^2 \rangle}{\langle \varepsilon_3^2 \rangle \langle \varepsilon_2^2 \rangle}$$

$NSC^v(3,2)$ $\quad NSC^\varepsilon(3,2)$

- Comparison of SC and Normalized SC (NSC) to hydrodynamic calculations
  - Although hydro describes v_n fairly well, there is not a single centrality for which a given η/s parameterization describes simultaneously SC and NSC -> tighter constraints!
  - NSC(3,2) measurements provide direct access into the initial conditions (despite details of systems evolution)
  - what is the general correlation between any order of v_n^k and v_m^p and the correlations among multiple flow coefficients

$P(v_m, v_n, v_k, \ldots)$

PHYSICAL REVIEW C **103**, 024913 (2021)

Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

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❖ Multi-particle mixed harmonic cumulants
- correlation between v_m^k, v_n^l and v_p^q
- correlation between v_m^k and v_n^l

Mixed harmonic cumulants with 4-particles

$$\mathrm{MHC}(v_m^2, v_n^2) = \mathrm{SC}(m,n) = \langle v_m^2\, v_n^2\rangle - \langle v_m^2\rangle\,\langle v_n^2\rangle$$

Mixed harmonic cumulants with 6-particles

$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^2\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^2\rangle \\ &\quad + 4\langle v_2^2\rangle^2\langle v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^4) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3-2\varphi_4-3\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^4\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_3^2\rangle - \langle v_2^2\rangle\langle v_3^4\rangle \\ &\quad +4\langle v_2^2\rangle\langle v_3^2\rangle^2.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^2, v_4^2) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^2\, v_4^2\rangle - \langle v_2^2\, v_3^2\rangle\langle v_4^2\rangle - \langle v_2^2\, v_4^2\rangle\langle v_3^2\rangle \\ &\quad -\langle v_3^2\, v_4^2\rangle\langle v_2^2\rangle + 2\langle v_2^2\rangle\langle v_3^2\rangle\langle v_4^2\rangle.\end{aligned}$$

Mixed harmonic cumulants with 8-particles

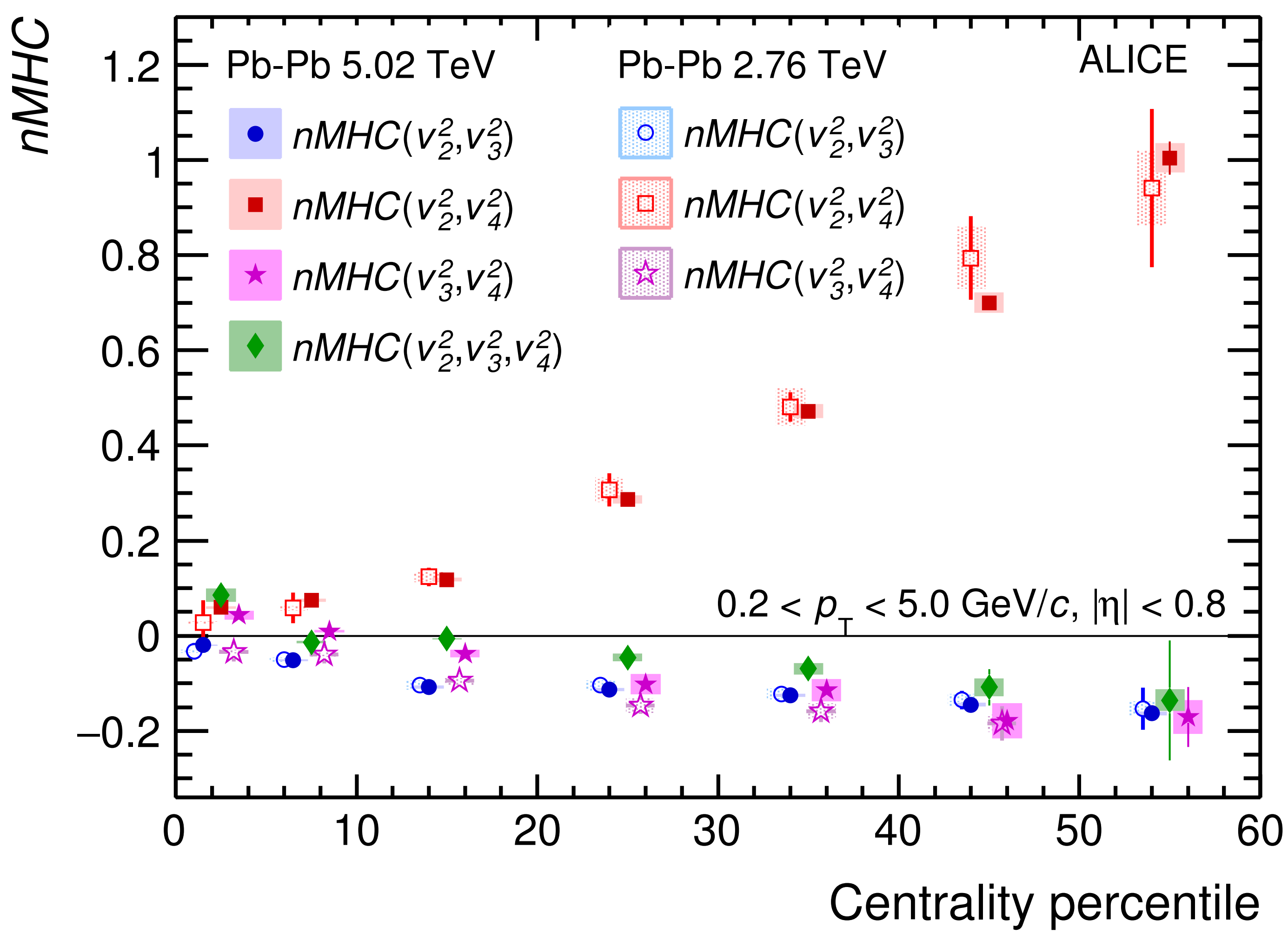
$$\begin{aligned}\mathrm{MHC}(v_2^6, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+2\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-2\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^6\, v_3^2\rangle - 9\langle v_2^4\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^6\rangle\langle v_3^2\rangle \\ &\quad -9\langle v_2^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle^3\langle v_3^2\rangle \\ &\quad +18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^4\rangle + 36\langle v_2^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

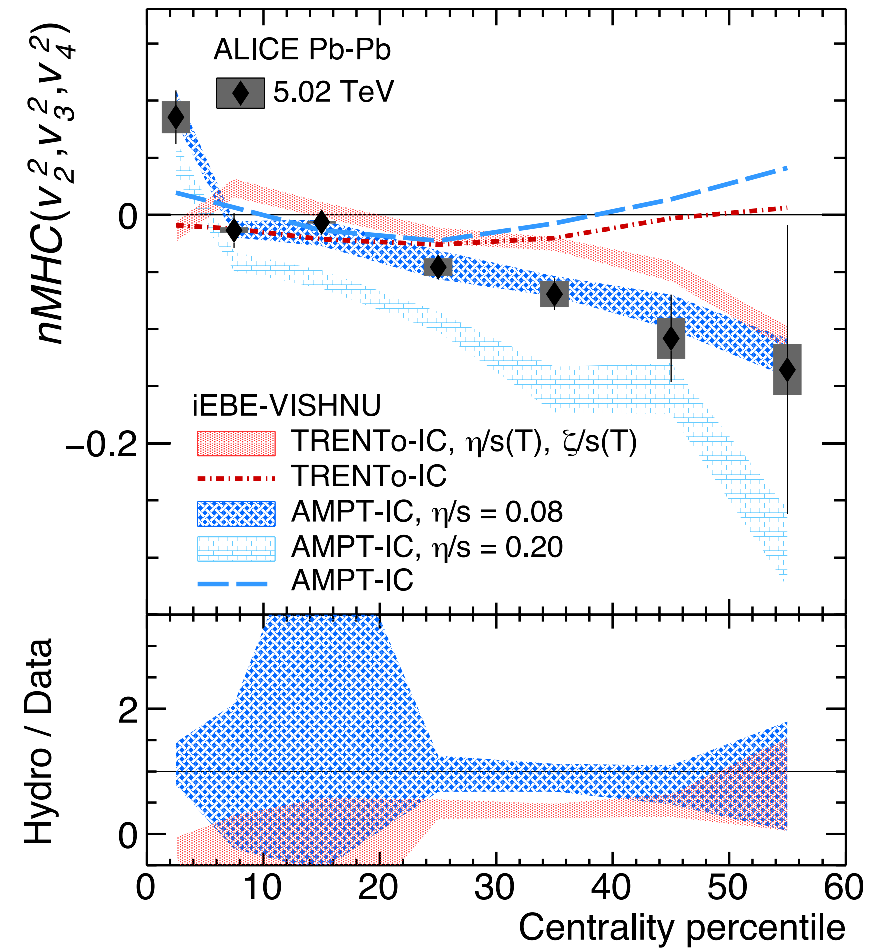
$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^4) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^4\rangle - 4\langle v_2^4\, v_3^2\rangle\langle v_3^2\rangle \\ &\quad -4\langle v_2^2\, v_3^4\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^4\rangle \\ &\quad -8\langle v_2^2\, v_3^2\rangle^2 - 24\langle v_2^2\rangle^2\langle v_3^2\rangle^2 \\ &\quad +4\langle v_2^2\rangle^2\langle v_3^4\rangle + 4\langle v_2^4\rangle\langle v_3^2\rangle^2 \\ &\quad +32\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^6) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-3\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^6\rangle - 9\langle v_2^2\, v_3^4\rangle\langle v_3^2\rangle - \langle v_3^6\rangle\langle v_2^2\rangle \\ &\quad -9\langle v_3^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle\langle v_3^2\rangle^3 \\ &\quad +18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_3^4\rangle + 36\langle v_3^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

Correlations between v_m^2, v_n^2, v_k^2,…

ALICE, PLB818 (2021) 136354

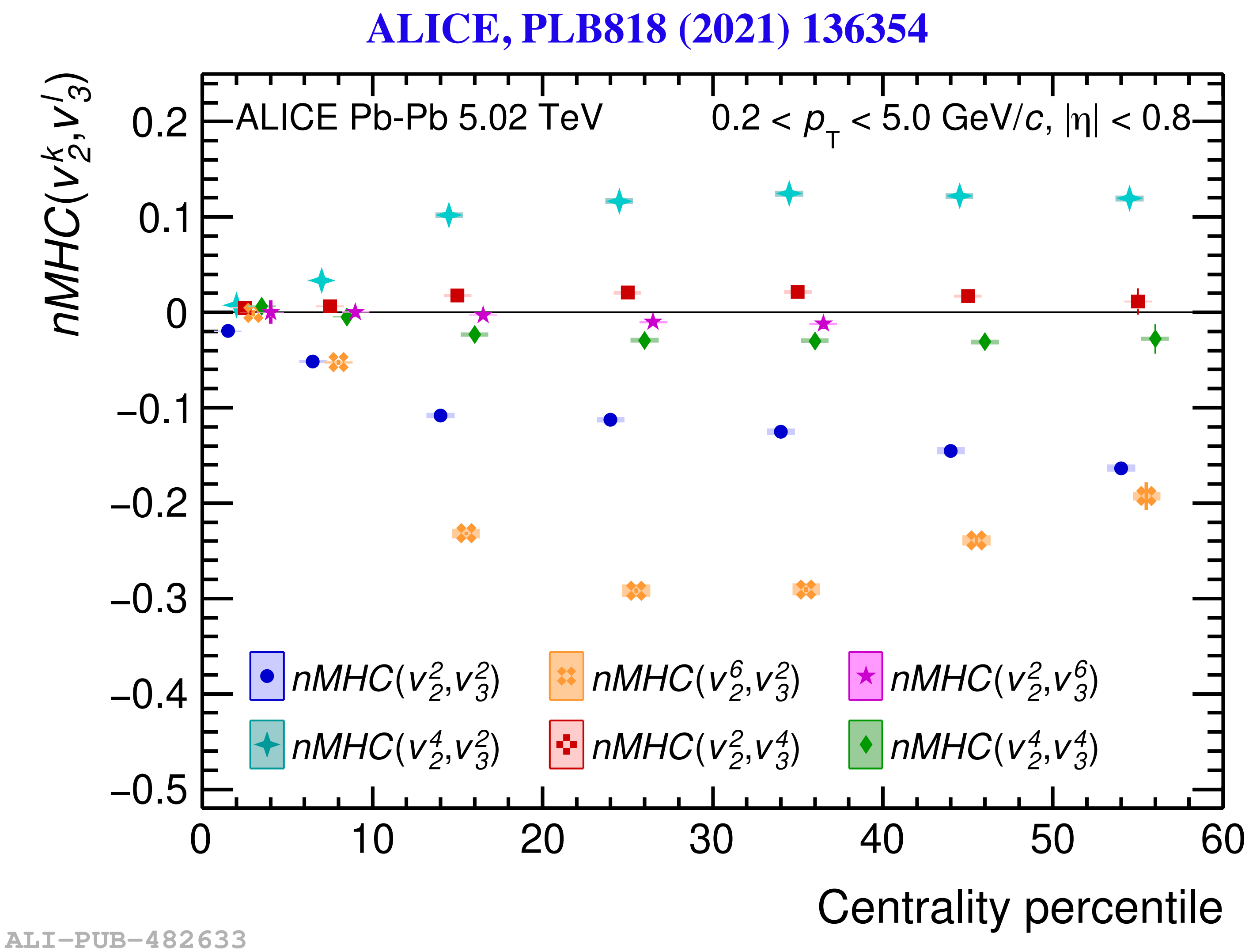


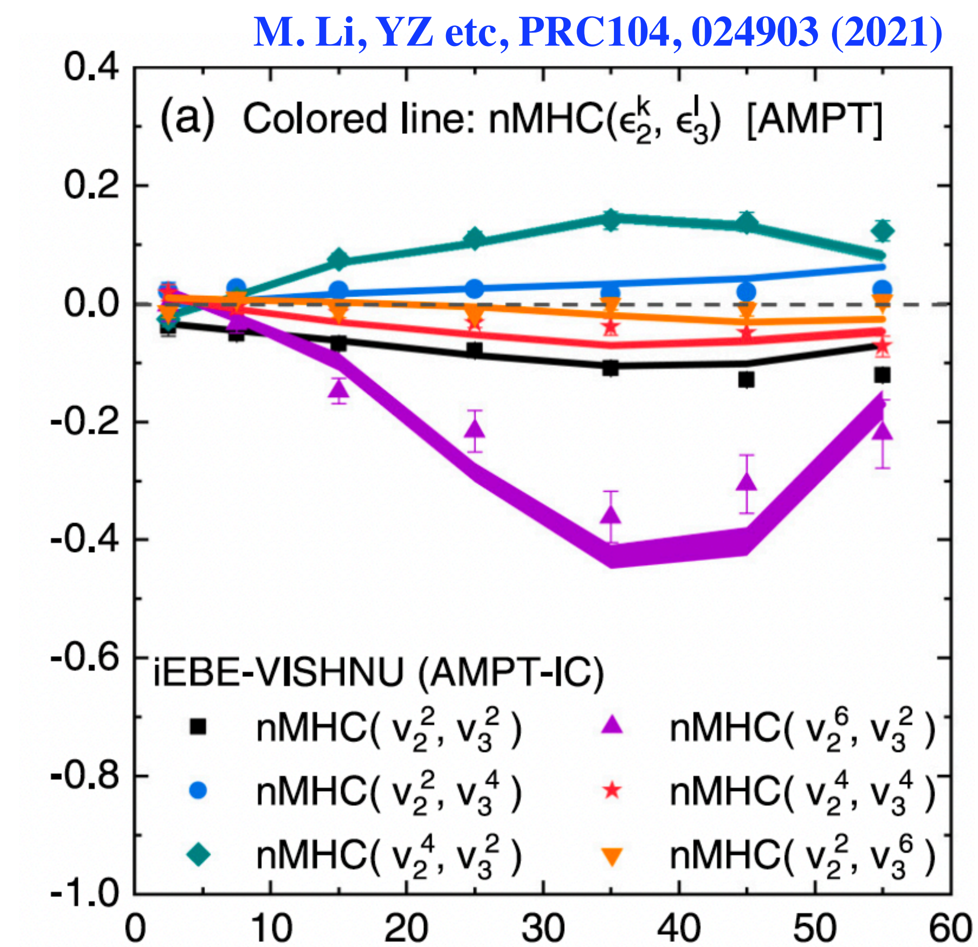


$$
\begin{aligned}
\mathrm{MHC}(v_2^2, v_3^2, v_4^2) &= \langle\langle e^{i\,(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)}\rangle\rangle_c \\
&= \langle v_2^2\, v_3^2\, v_4^2\rangle - \langle v_2^2\, v_3^2\rangle\langle v_4^2\rangle - \langle v_2^2\, v_4^2\rangle\langle v_3^2\rangle \\
&\quad -\langle v_3^2\, v_4^2\rangle\langle v_2^2\rangle + 2\,\langle v_2^2\rangle\langle v_3^2\rangle\langle v_4^2\rangle.
\end{aligned}
$$

- Non-zero value of *nMHC*(v_2^2, v_3^2, v_4^2) in Pb-Pb collisions
  - Highly non-trivial correlations among three flow coefficients

Correlations between v_2^k and v_3^L





❖ First measurement of correlations between higher order moments of v_2 and v_3

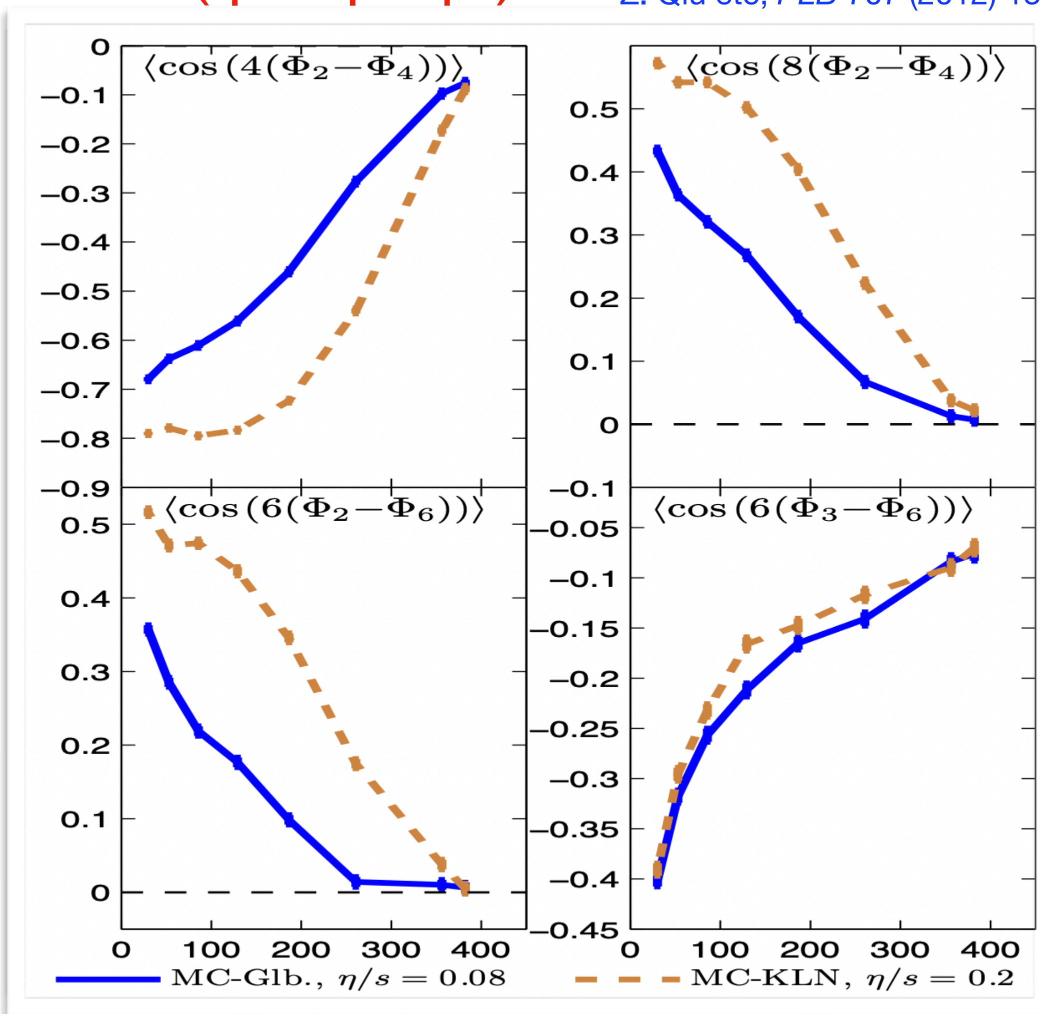
- characteristic -, +, - signs observed for 4-, 6- and 8-particle cumulants of *mixed harmonic*
- Final state results quantitatively reproduced by the initial state correlations
- Experimental data provides direct constraints on the correlations of higher order moments of eccentricity coefficients

How do ψ_n and ψ_m correlate

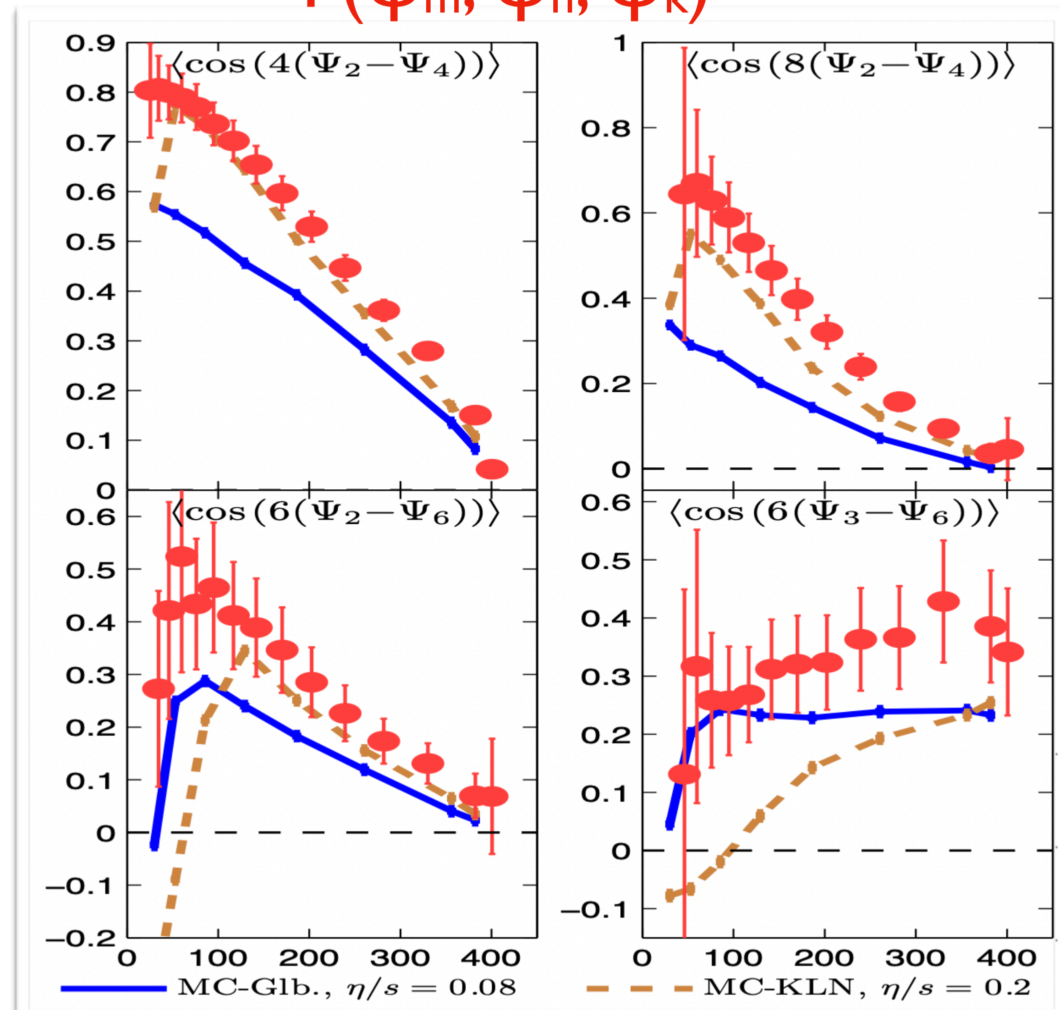
ψ_n correlations: $P(\psi_m, \psi_n, \psi_k)$

$P(\phi_m, \phi_n, \phi_k)$

Z. Qiu etc, *PLB* 707 (2012) 151



$P(\psi_m, \psi_n, \psi_k)$



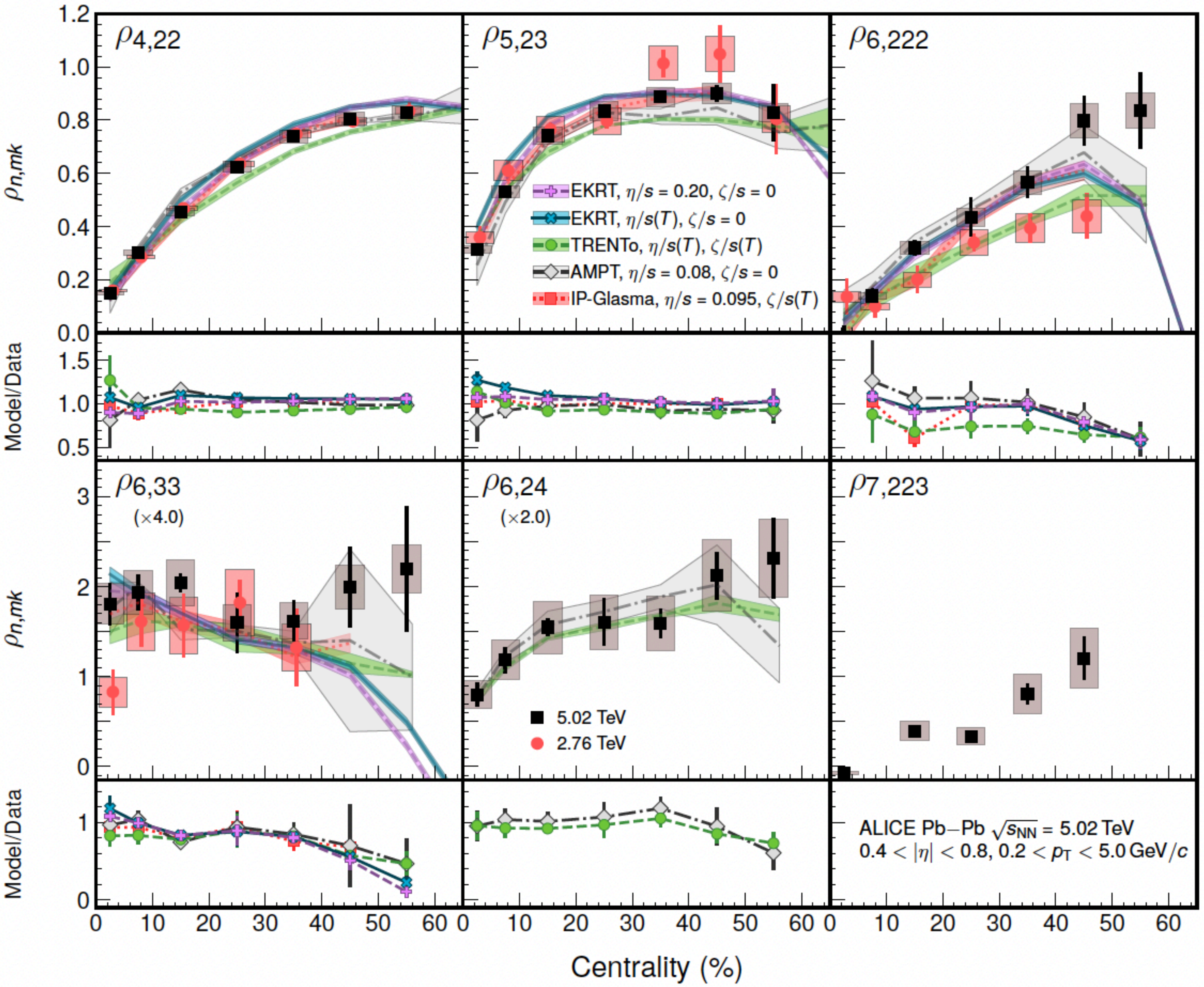
❖ Stronger initial symmetry plane correlations likely results in stronger final state flow symmetry plane correlations

Expectations:

- Central collision:
  - ψ_2, ψ_4 randomly fluctuate, weak correlations
  - <cos4(ψ_2-ψ_4)> is small
- Peripheral collisions:
  - ψ_2, ψ_4 tend to align, strong correlations
  - <cos4(ψ_2-ψ_4)> is large

Ψ_n correlations: $P(\Psi_m, \Psi_n, \Psi_k)$

ALICE, PLB773 (2017) 68, JHEP05 (2020) 085

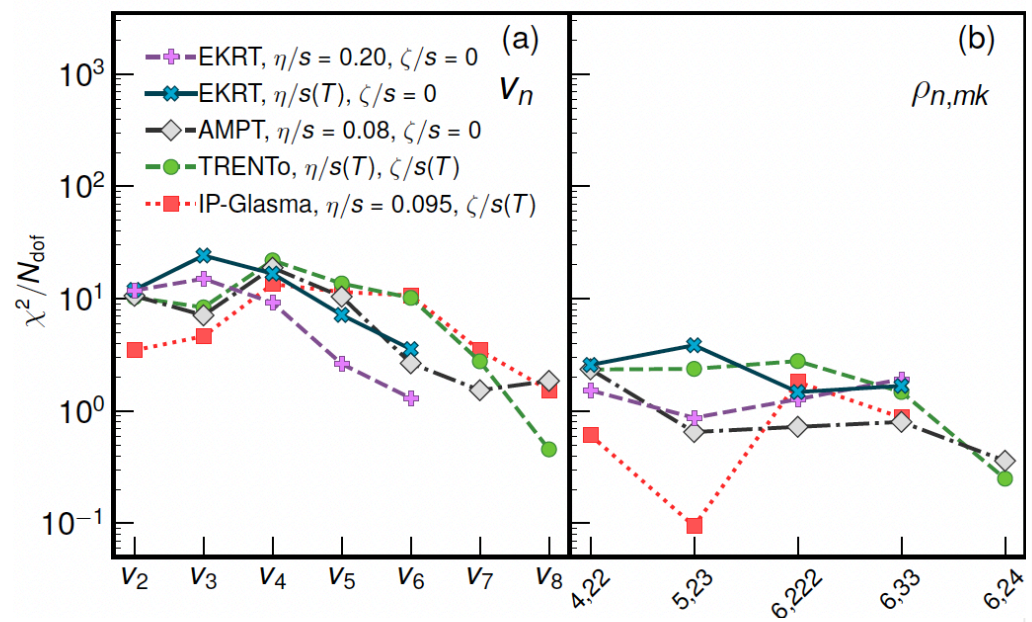


$\rho_{422} \approx \langle\cos(4\Psi_4 - 4\Psi_2)\rangle$

$\rho_{532} \approx \langle\cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2)\rangle$

$\rho_{6222} \approx \langle\cos(6\Psi_6 - 6\Psi_2)\rangle$

$\rho_{633} \approx \langle\cos(6\Psi_6 - 6\Psi_3)\rangle$

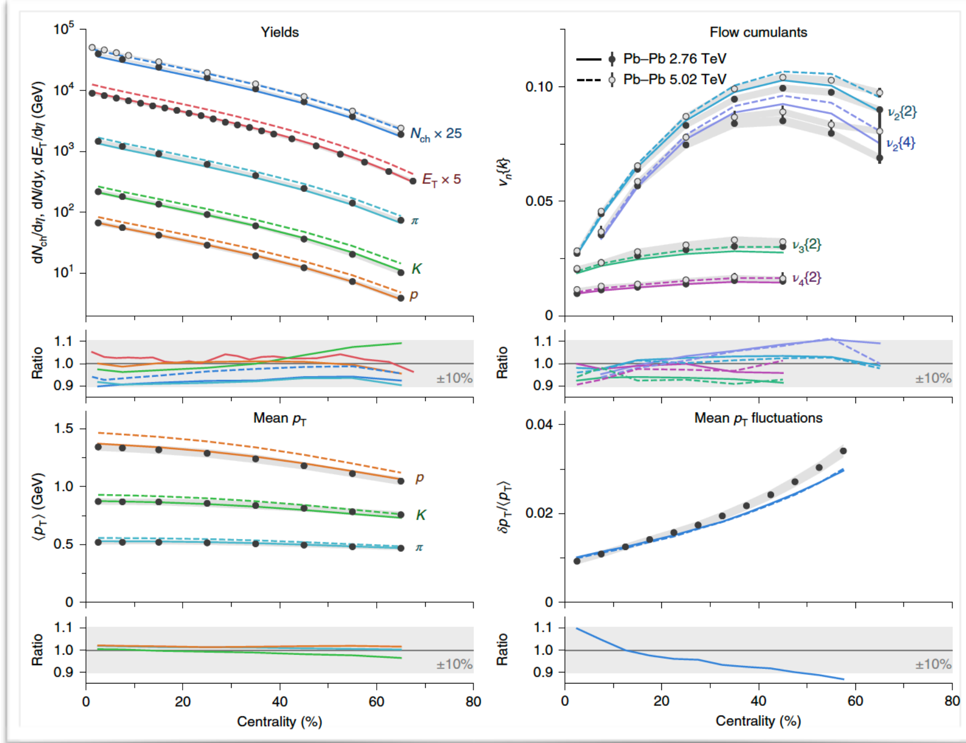


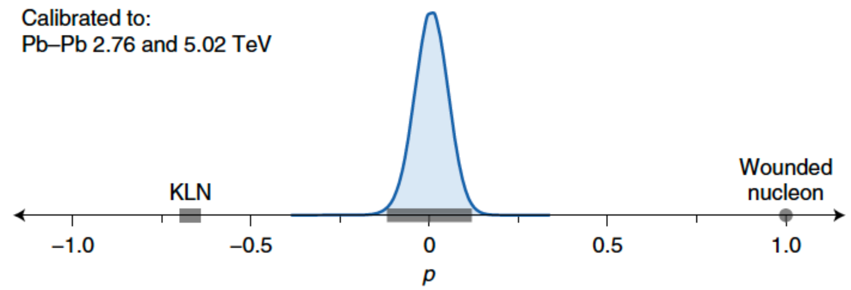
EKRT, PRC93, 024907 (2016)
TRENTo, EPJC77 (2017) 645
AMPT, EPJC77 (2017) 645
IP-Glasma, PRC95, 064913 (2017)

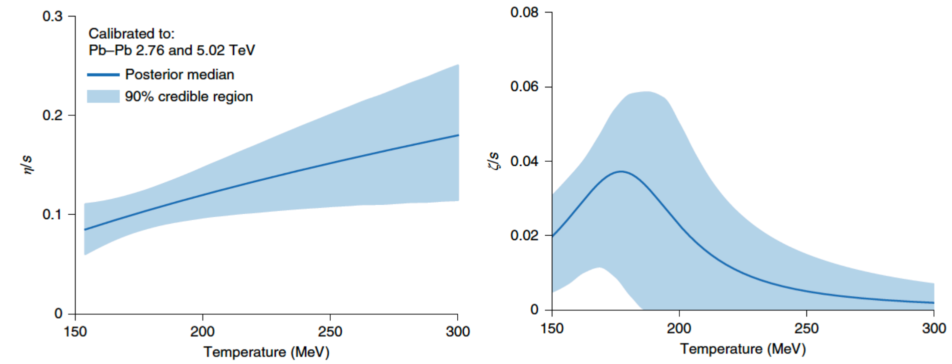
- $\rho_{mn,}$ probes the symmetry plane correlations
  - No energy dependence between measurements except $\rho_{6,222}$
  - Among many models, TRENTo model does not work well in $\rho_{n,mk}$

Bayesian analyses with simple v_n

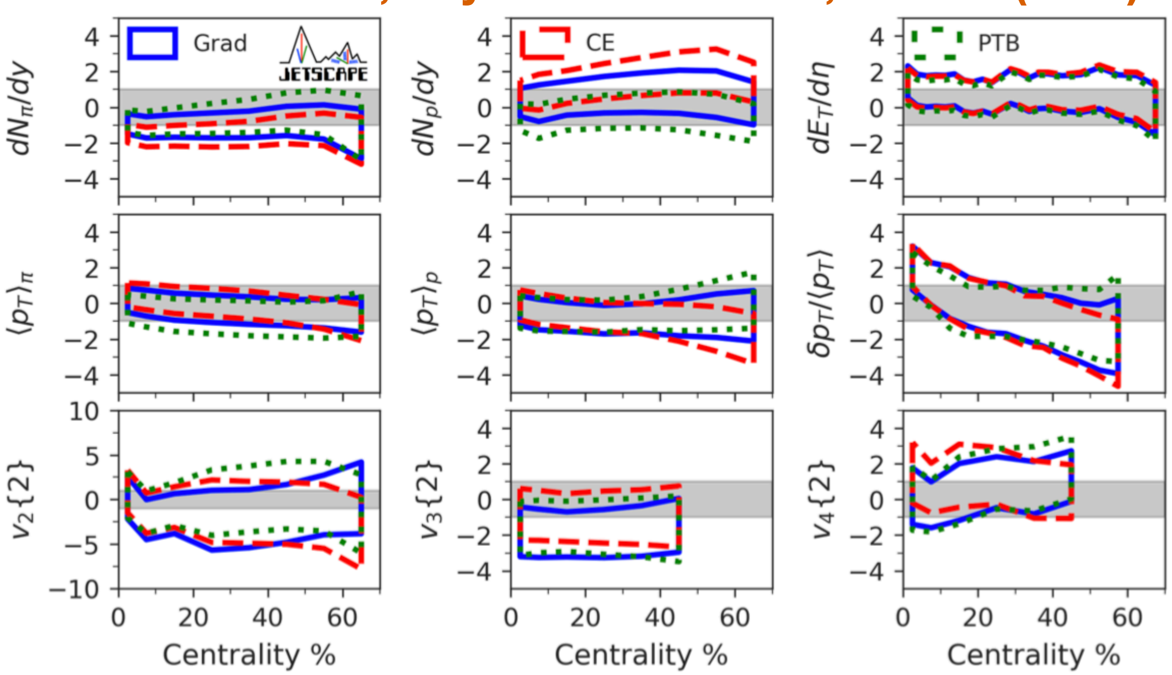
J.E. Bernhard etc, Nature Physics,15, 1113 (2019)

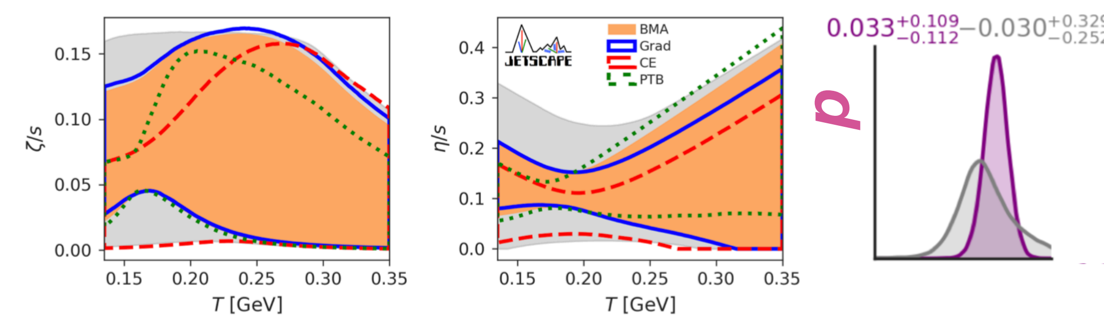






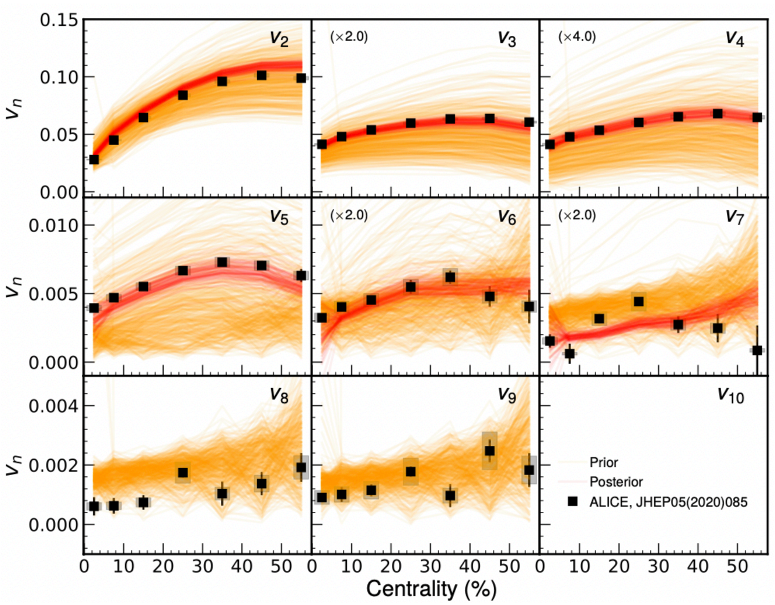
JETSCAPE, Phys. Rev. Lett. 126, 242301 (2021)



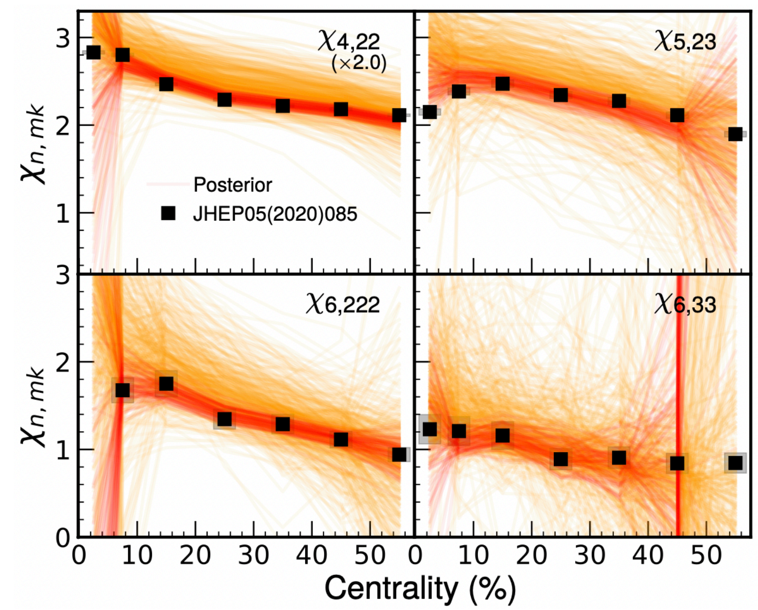


Bayesian analysis with more flow observables

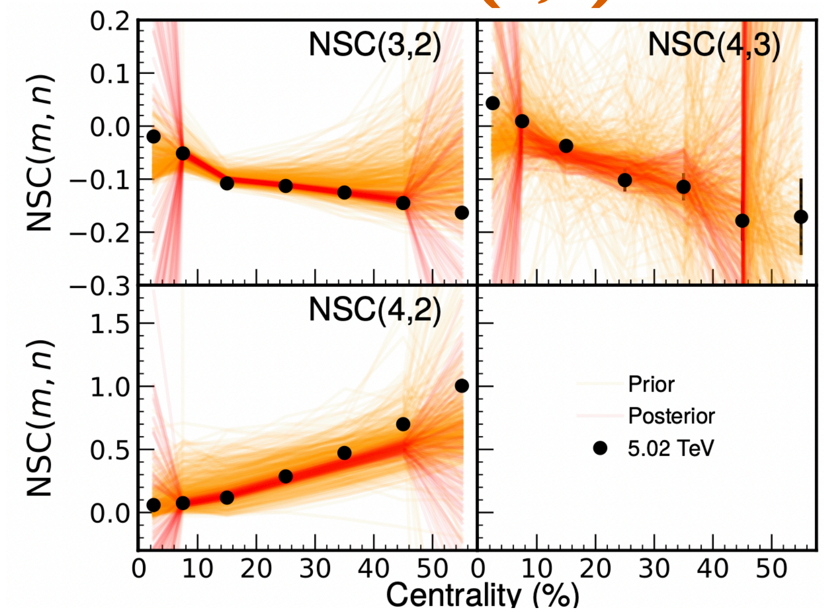
v_n

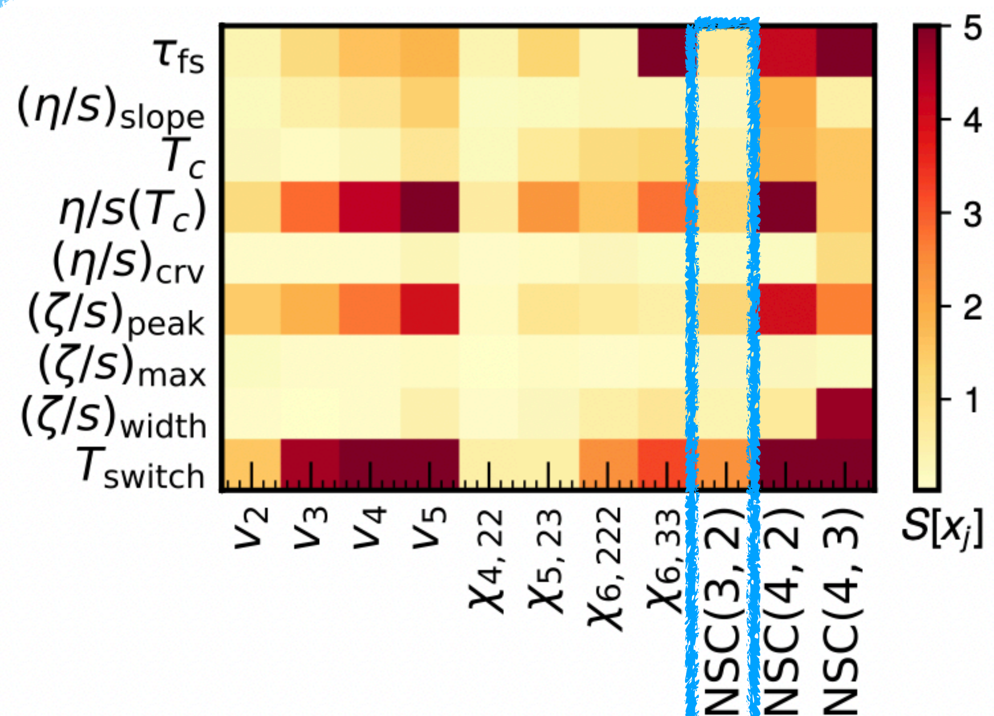


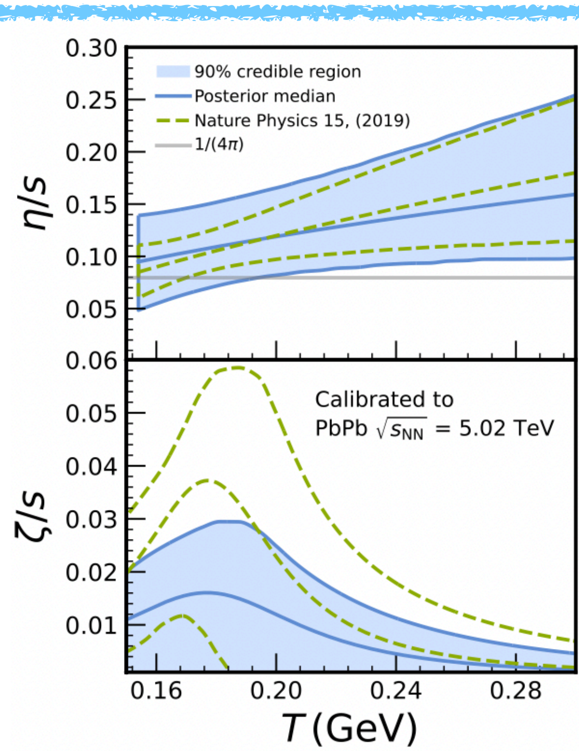
$\chi_{n,mk}$



NSC(3,2)







Initial conditions	
Parameter	MAP value
Norm	21.06
p	0.0077
σ_k	0.881
$d^3_{\min}$	0.975
τ_{fs}	0.901
T_c	0.147
T_{switch}	0.160

J.E. Parkkila etc, arXiv: 2106.05019

Similar study by including flow data in Bayesian analysis, see: G. Nijs, W. Van der Schee, to be appeared on arXiv.

Flow studies -> IC (shape) and QGP properties

How does v_n fluctuate

How do v_n and v_m correlate

$$P(v_m, v_n, v_k, ..., \Psi_m, \Psi_n, \Psi_k, ...)$$

$$P(\varepsilon_m, \varepsilon_n, \varepsilon_k, ..., \Phi_m, \Phi_n, \Phi_k, ...)$$

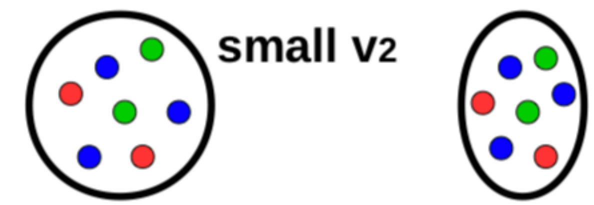
How does ψ_n fluctuate

How do ψ_n and ψ_m correlate

$\langle p_T \rangle$ - v_n correlations

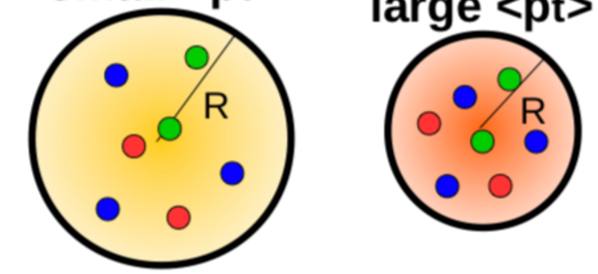
- ❖Shape of the fireball: Anisotropic flow
- ❖Size of the fireball: radial flow, $[p_T]$
- ❖Initial geometry and fluctuations of shape and size
- ❖Final state: correlation between v_n and p_T

$$V_n \propto \mathcal{E}_n$$



$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto \frac{dE}{E}$$

$$\frac{d\langle p_t \rangle}{\langle p_t \rangle} \propto -\frac{dR}{R}$$



$$\rho(v_n^2, [p_T]) = \frac{cov(v_n^2, [p_T])}{\sqrt{var(v_n^2)}\sqrt{var([p_T])}}$$

P. Bozek etc, PRC96 (2017) 014904

$$\rho(v_n^2, [p_T]) \approx \rho(\varepsilon_n^2, E_0^{-1})$$

☆ $cov(v_n^2, [p_T])$: 3-particle correlation (2 azimuthal, 1 $[p_T]$)

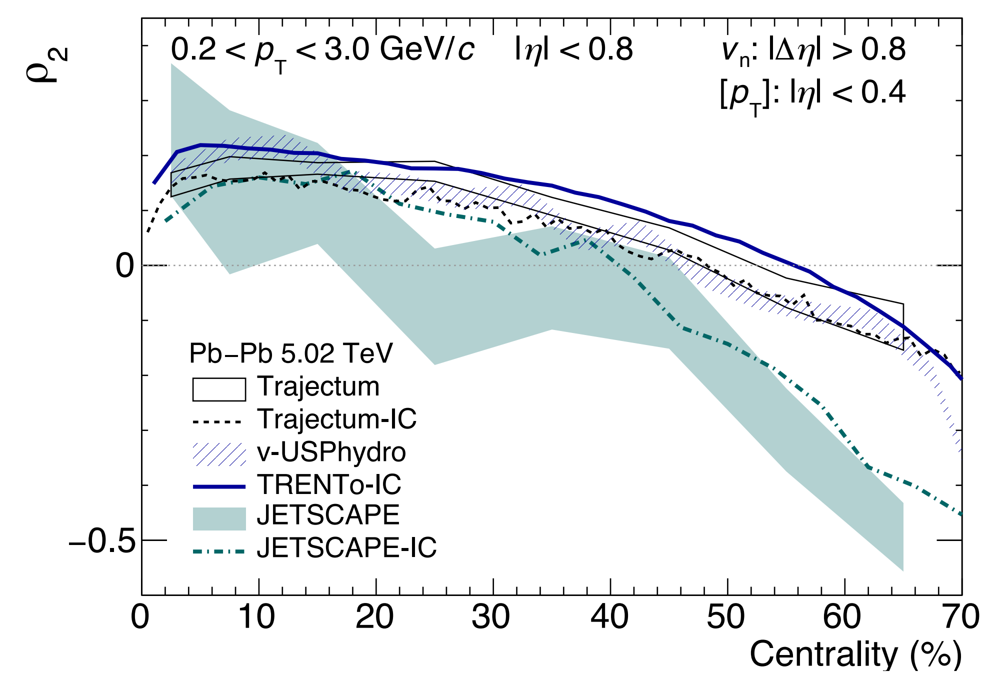
$$\left\langle \frac{\sum_{i\neq j\neq k} w_i w_j w_k e^{in\phi_i} e^{-in\phi_j} (p_{T,k} - \langle\langle p_T \rangle\rangle)}{\sum_{i\neq j\neq k} w_i w_j w_k} \right\rangle_{\mathrm{evt}}$$

☆ $\sqrt{var(v_n^2)}$: 2 and 4-particle azimuthal correlations

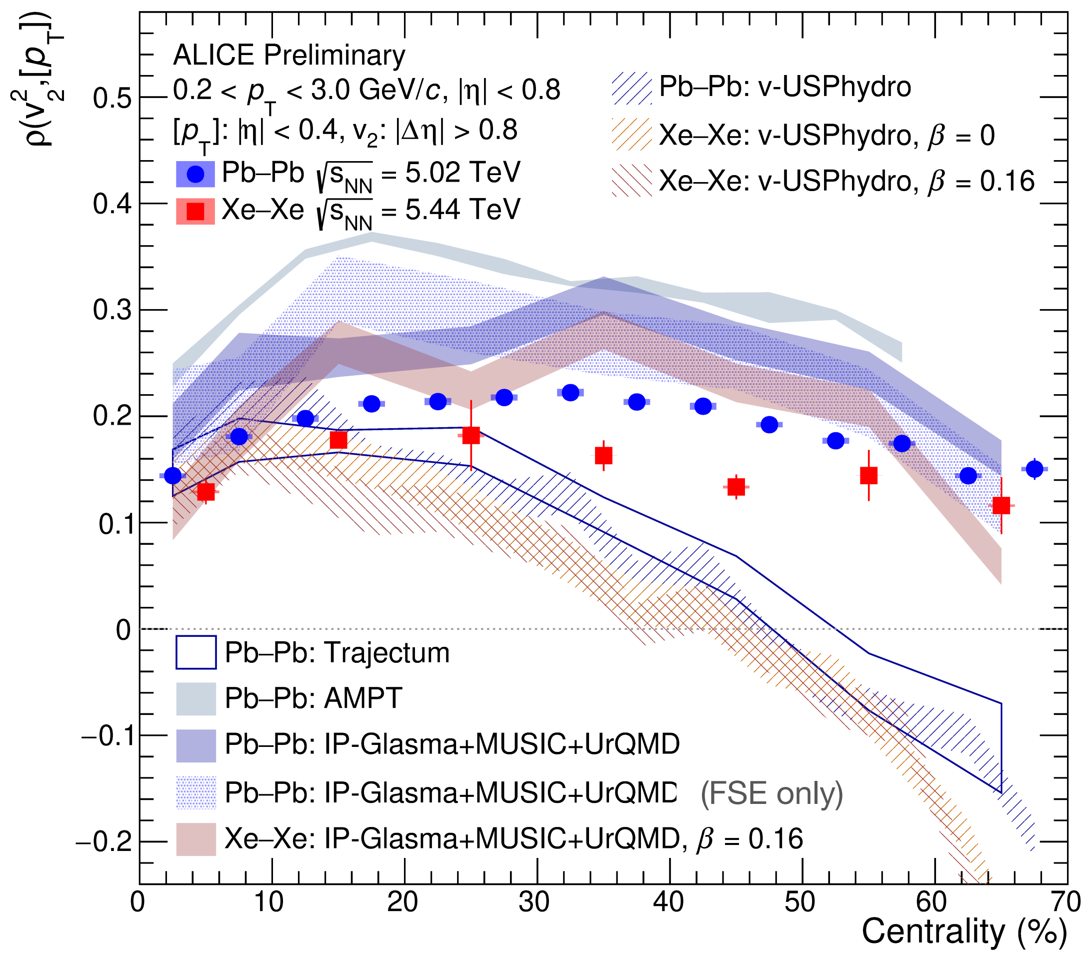
$$= v_n\{2\}^4 - v_n\{4\}^4$$

☆ $\sqrt{var([p_T])}$: 2-particle $[p_T]$ correlations

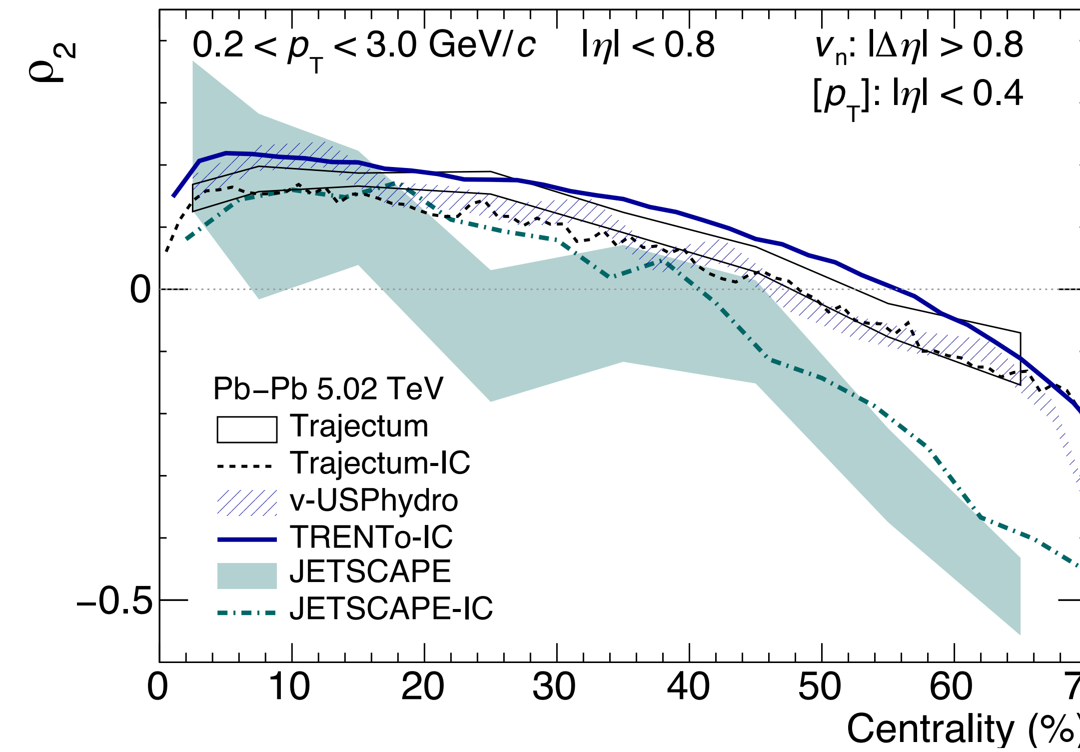
$$\left\langle \frac{\sum_{i\neq j} w_i w_j (p_{T,i} - \langle\langle p_T \rangle\rangle)(p_{T,j} - \langle\langle p_T \rangle\rangle)}{\sum_{i\neq j} w_i w_j} \right\rangle_{\mathrm{evt}}$$



ρ_2 in Pb-Pb



ALI-PREL-494367



JETSCAPE, PRL126, 242301 (2021)
Privation communication

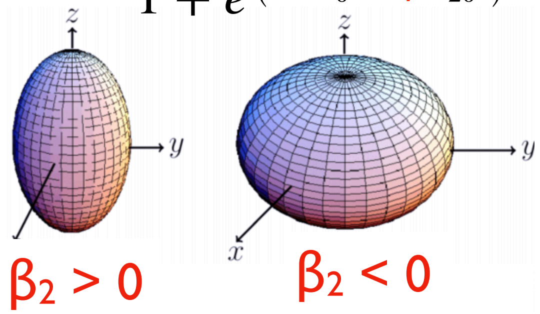
Trajectum, PRL126, 202301 (2021)
Privation communication

v-USPhydro, PRC103 (2021) 2, 024909

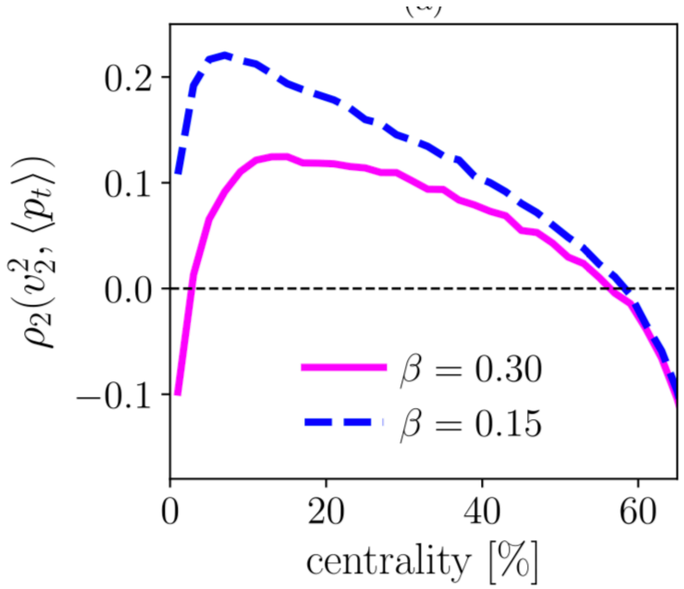
- ❖ IP-Glasma-IC: IP-Glasma+MUSIC+UrQMD slightly overestimate the Pb-Pb data
- ❖ TRENTo-IC based calculations show strong centrality dependence, negative values for centrality >40%
  - v-USPhydro, Trajectum, JETSCAPE
- ❖ The difference is from the initial stage: geometric effects or initial momentum anisotropy (CGC)?
  - No significant difference between the "full IP-Glasma" and "FSE only" for the presented centralities
  - Difference not from initial momentum anisotropy and confirm the different geometric effects

ρ_2 in Xe-Xe

$$D_{WS} = \frac{D_0}{1 + e^{(r - R_0(1+\beta Y_{20}))/a}}$$

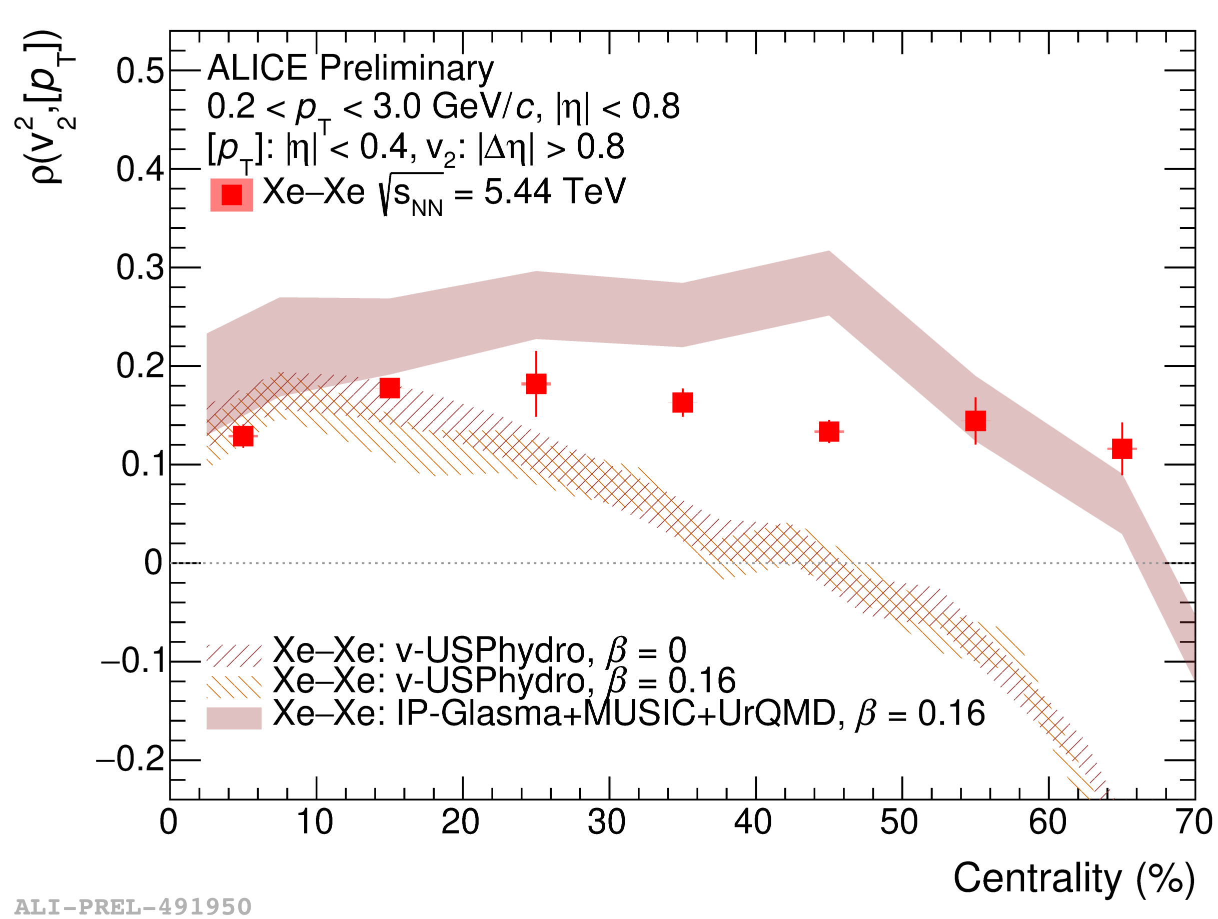


Pb—Pb: $\beta \approx 0$
Xe—Xe: $\beta \approx 0.16$



G.Giacalone, PRC 102 024901 (2020)

v-USPhydro, PRC103 (2021) 2, 024909

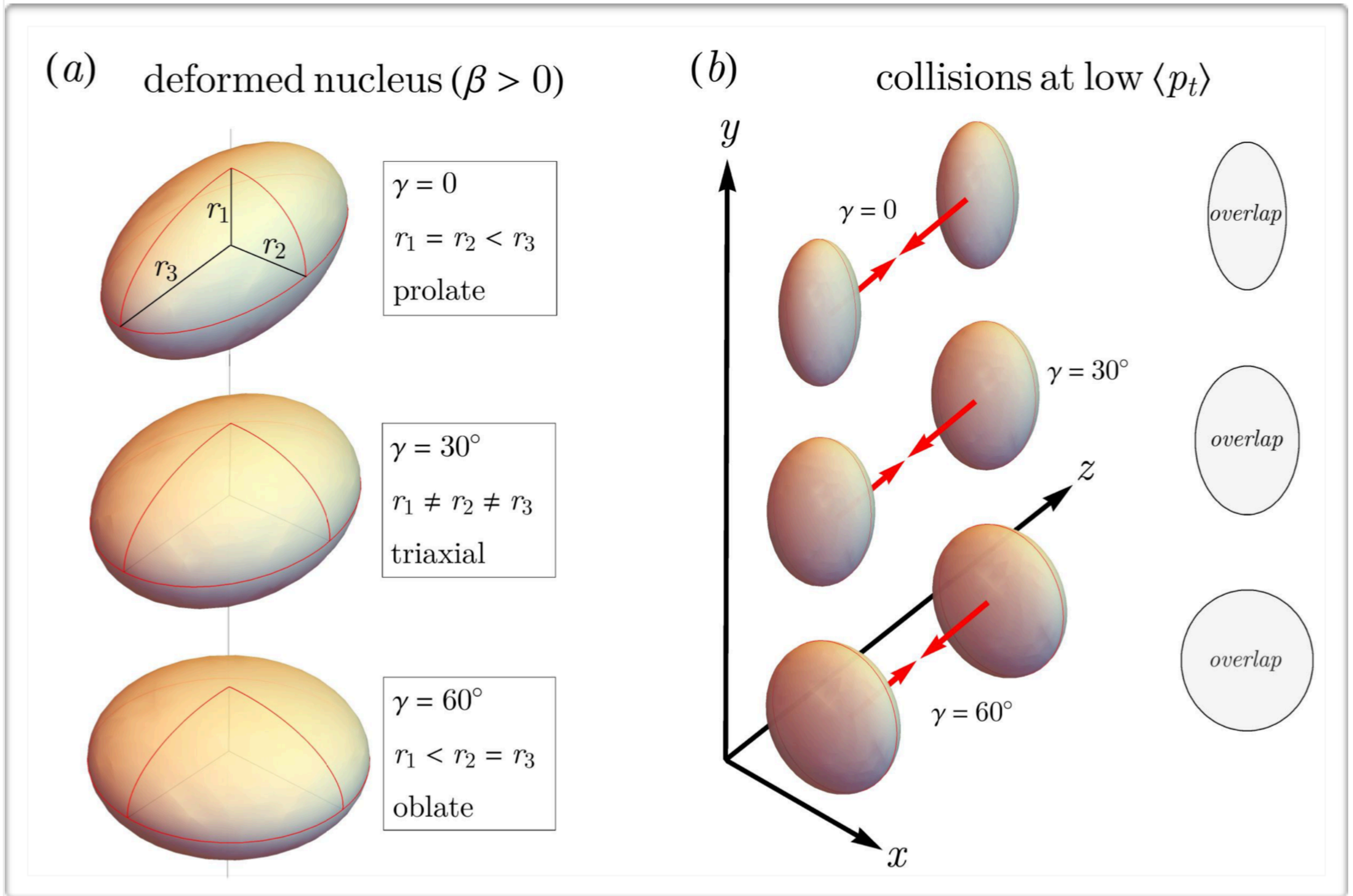


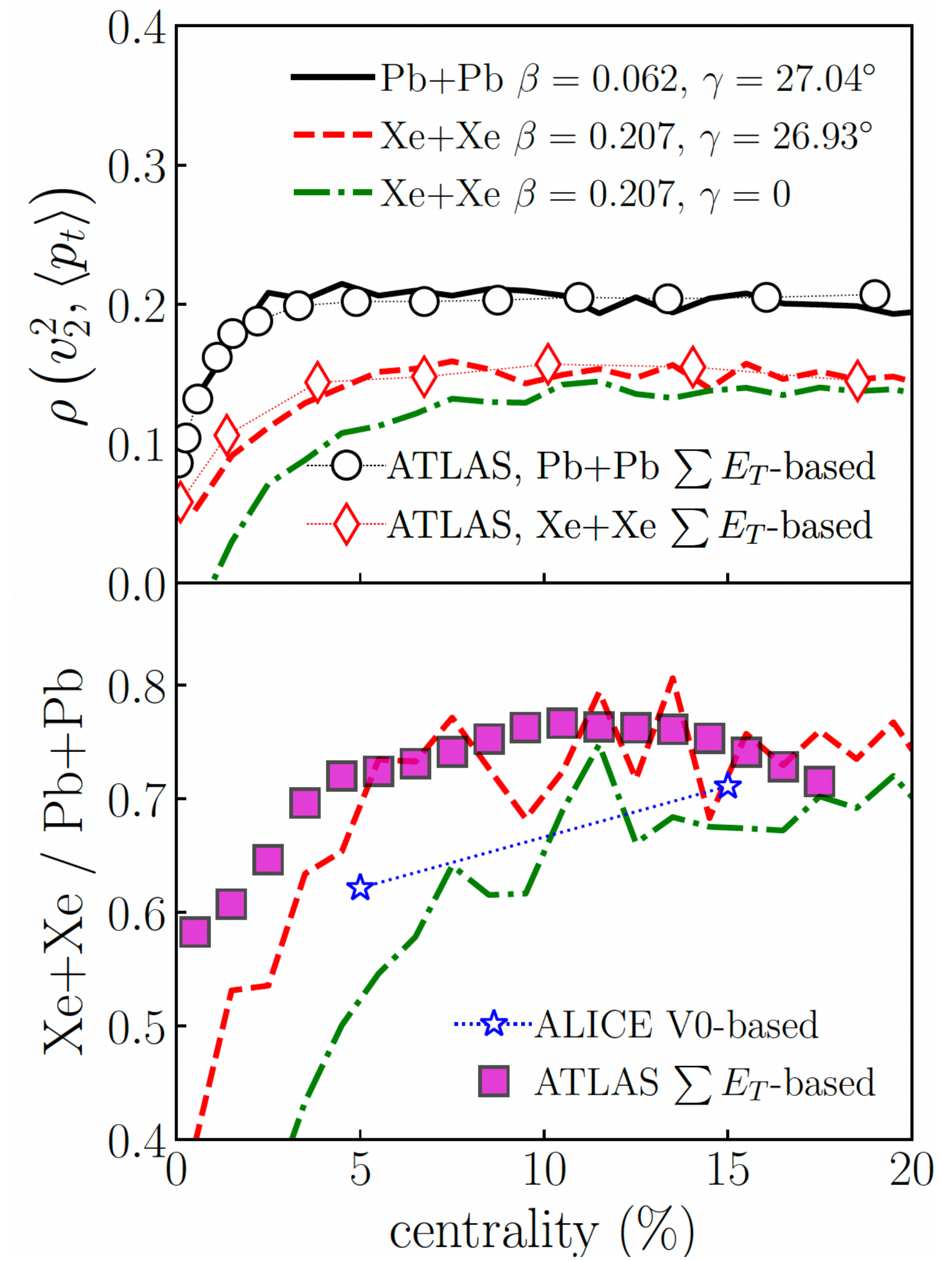
❖ Significant differences of initial state calculations using different deformation parameter in central Xe-Xe collisions

- ρ_2 is sensitivties to β_2
- The uncertainty of current v-USPhydro calculations is too large to draw a confirm conclusions
- Experimental data (in Xe-Xe@LHC and U-U@RHIC) open a new window to study nucleon deformation.

Probe triaxial structure of Xe

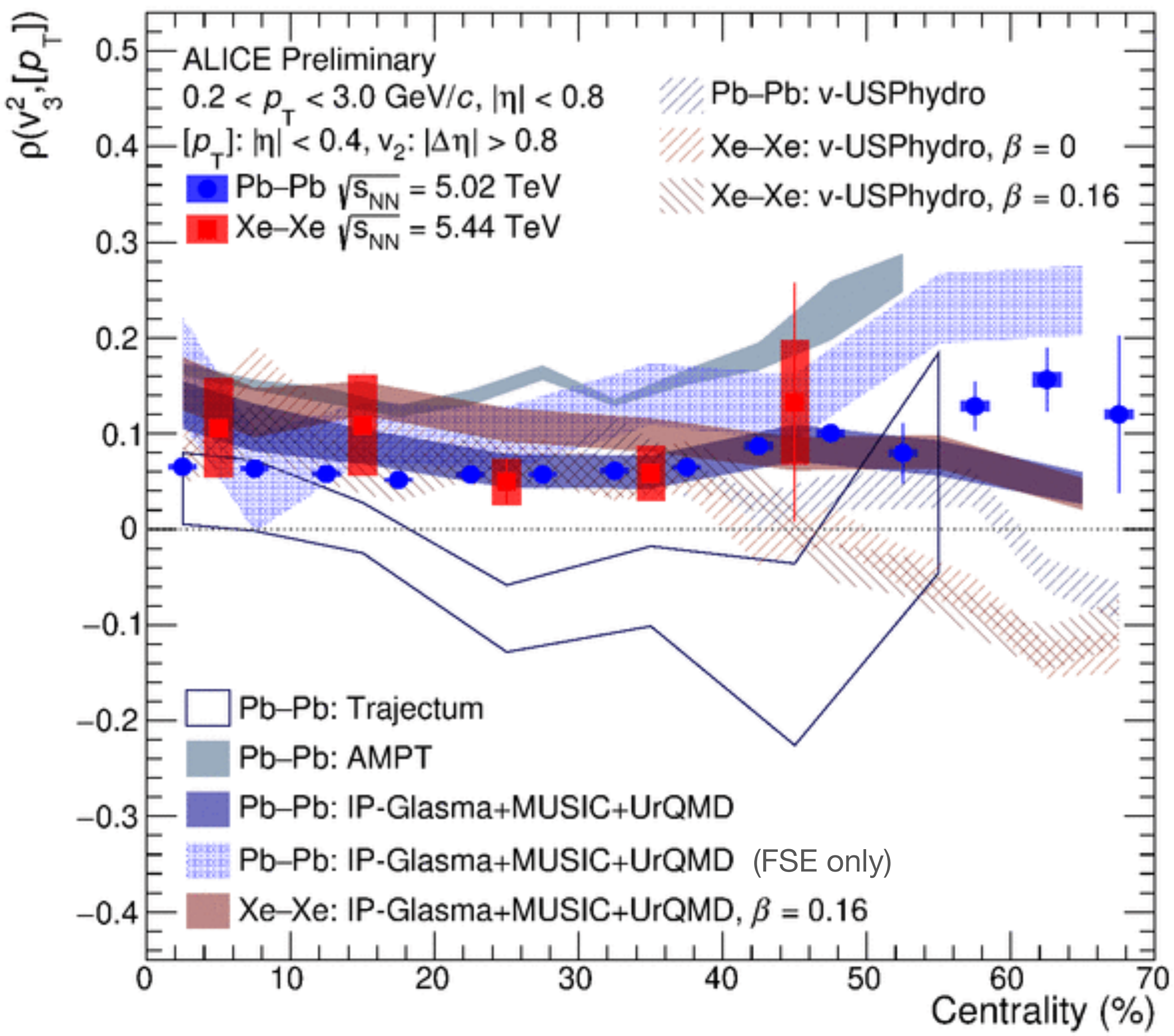
B. Bally etc, arXiv:2108.09578





- Better agreement between LHC data and calculations with γ = 26.93°
  - Indication of triaxial structure of Xe at high energy
  - New connection of high-energy heavy-ion physics to low-energy nuclear (structure) physics

ρ_3 in Pb-Pb and Xe-Xe



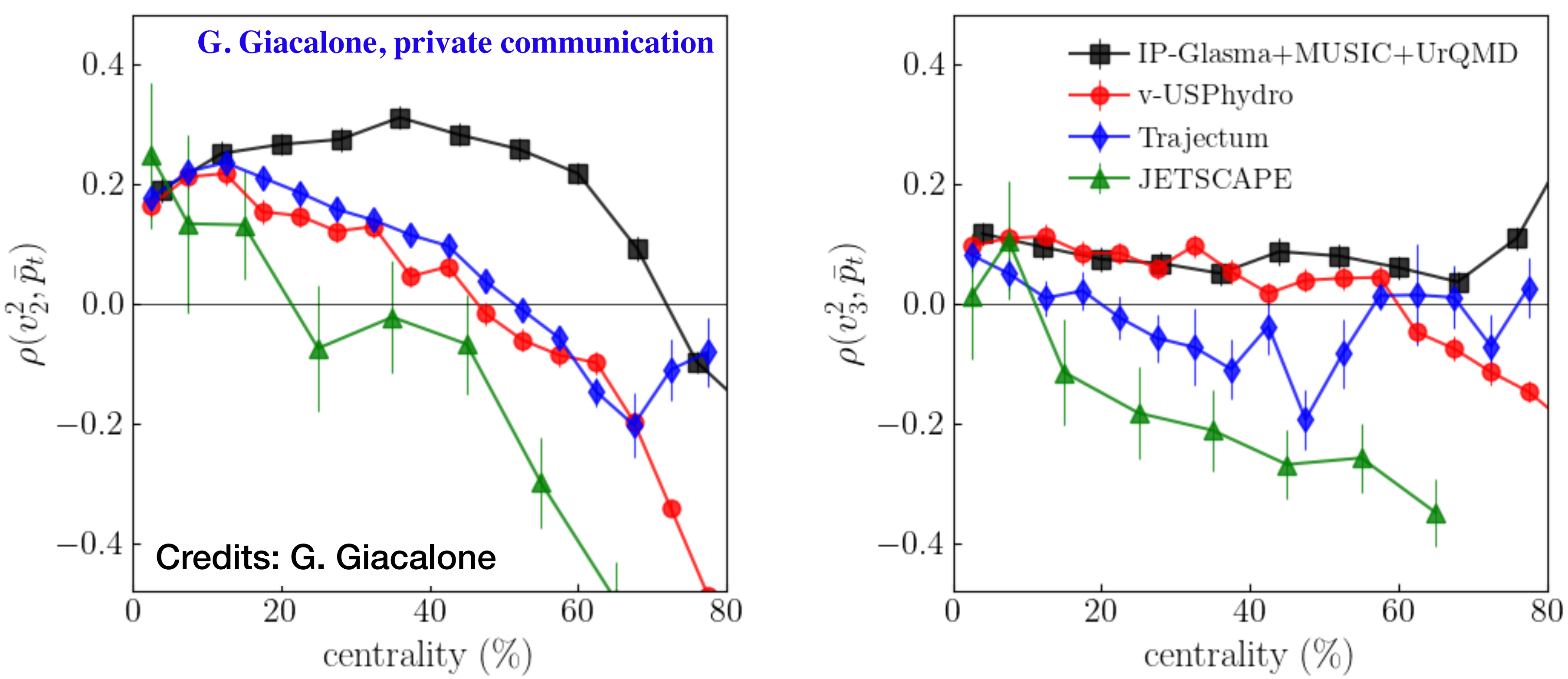
- ρ_3 values:
  - positive
  - have a modest centrality dependence for the presented centralities,
  - better described by IP-Glasma,
  - TRENTo predicts negative ρ_3, getting worse for Trajectum and JETSCAPE calculations
- model shows that ρ_3 is not sensitive to β_2
- Difference of full IP-Glasma and FSE only, indication of potential contributions from IMA in peripheral?

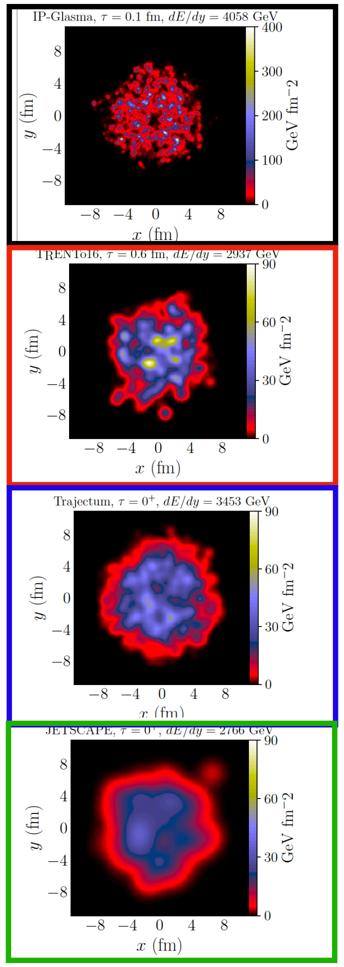
JETSCAPE, PRL126, 242301 (2021)
Privation communication
Trajectum, PRL126, 202301 (2021)
Privation communication
v-USPhydro, PRC103 (2021) 2, 024909

Difference in IP-Glasma and TRENTo: potential explanations

❖ Sensitive to the nucleon width parameter (size of nucleon)

- IP-Glasma ~ 0.3; v-USPhydro ~ 0.5; Trajectum~0.7; JETSCAPE (TRENTo) ~ 1.1
- w(IP-Glasma) < w(v-USPhydro)<w(Trajectum) < w(JETSCAPE)





❖ Different types of thickness functions

- TRENTo $\left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$ with p≈0 $\sqrt{T_A T_B}$, IP-Glasma $T_A T_B$ type

❖ Different contributions from pre-hydrodynamic phase (free streaming) and sub-nucleon structure

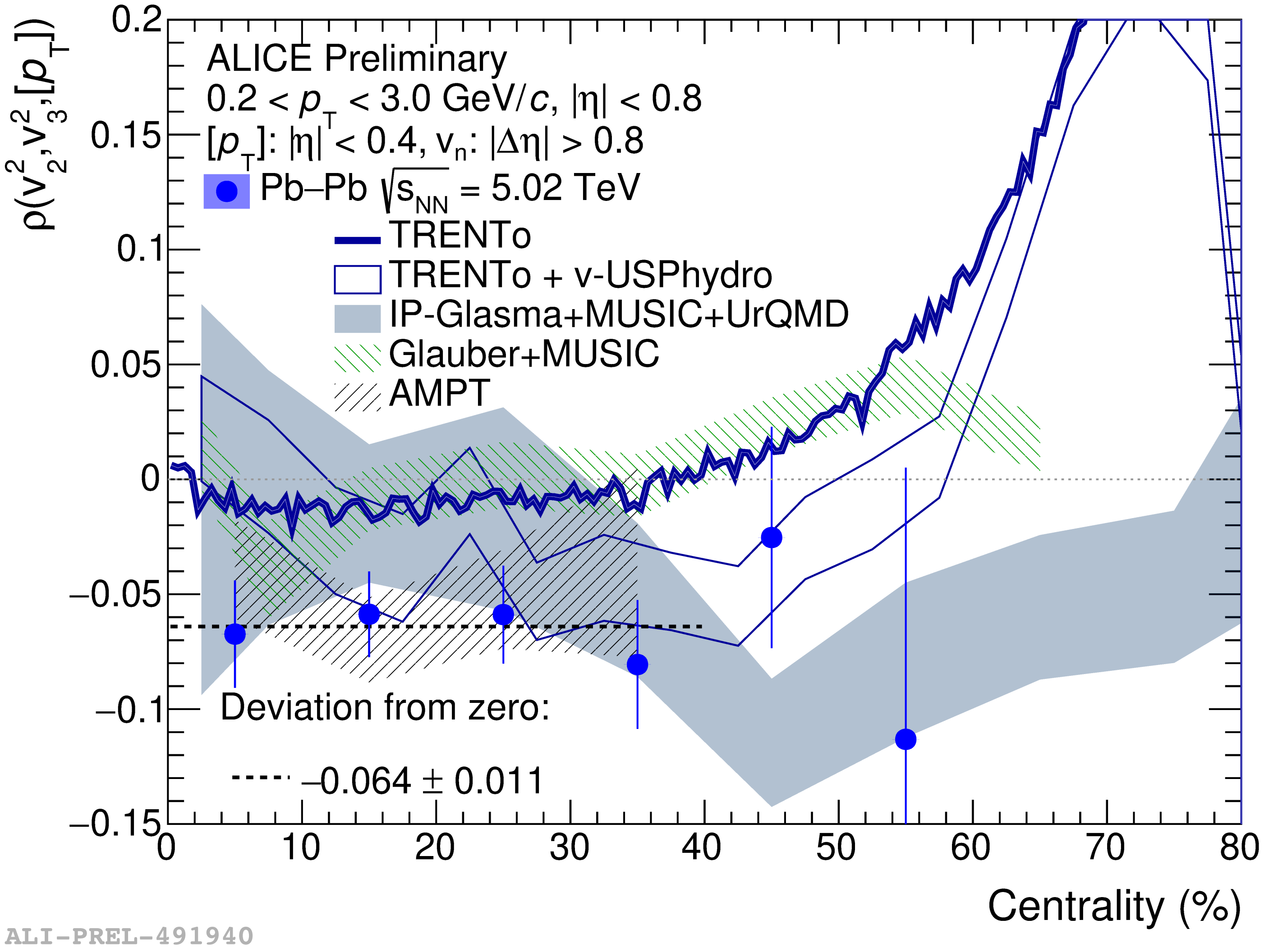
Higher-order correlations

❖ The **first** measurement of higher-order $[p_T]$, v_2 and v_3 correlations

P. Bozek etc, PRC104 (2021) 1, 014905

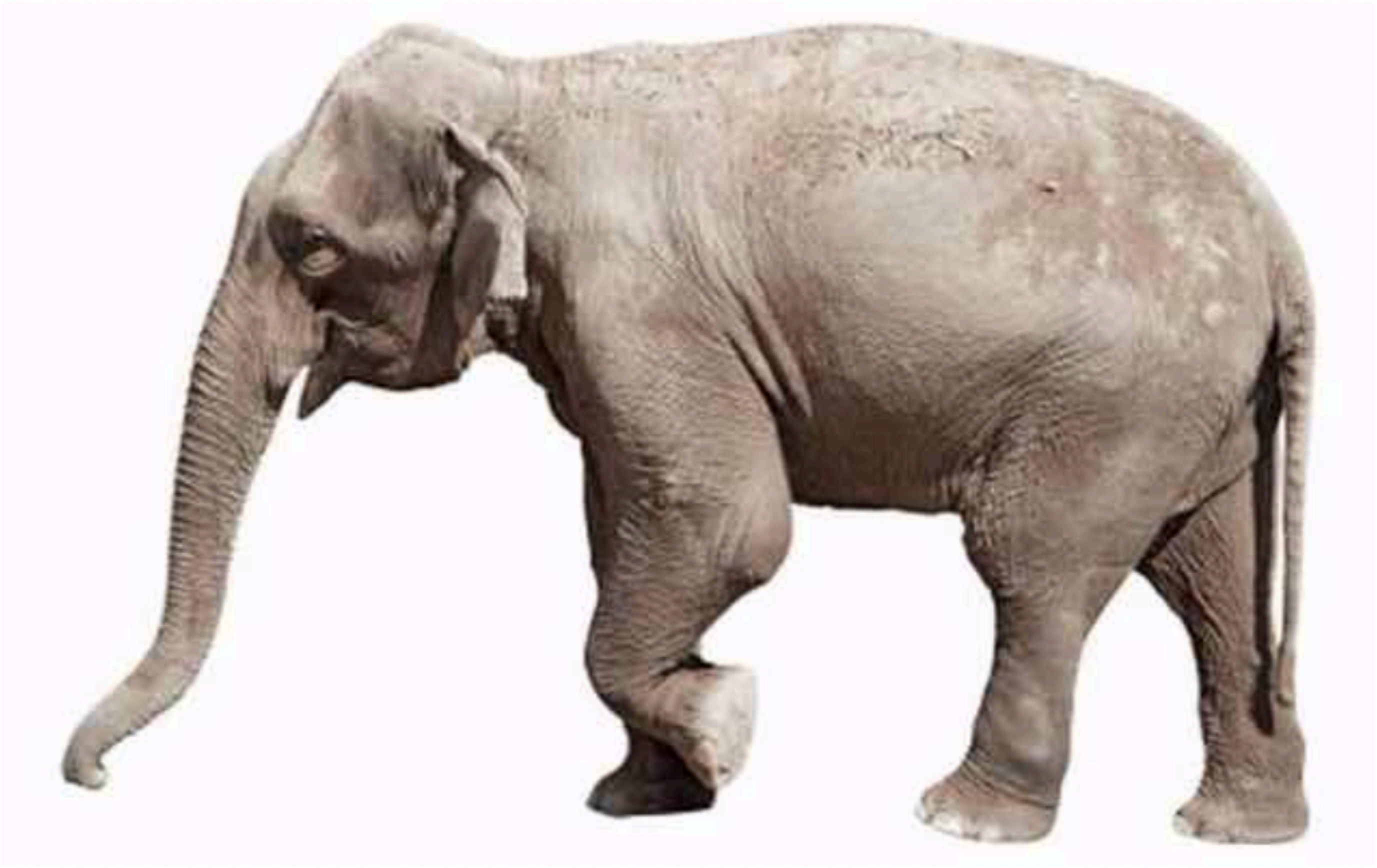
$$\rho(v_m^2, v_n^2, [p_T]) = \frac{C(v_m^2, v_n^2, [p_T])}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}\sqrt{c_k}} - \frac{\langle v_m^2\rangle}{\sqrt{\mathrm{Var}(v_m^2)}}\cdot\rho_n - \frac{\langle v_n^2\rangle}{\sqrt{\mathrm{Var}(v_n^2)}}\cdot\rho_m - \frac{\langle [p_T]\rangle}{\sqrt{c_k}}\cdot\frac{SC(m,n)}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}}$$

- ❖ the first ρ_{23} measurement is non-zero
  - negative for the presented centrality
  - anti-correlations between two flow coefficients and $[p_T]$
- ❖ ρ_{23} from IP-Glasma and v-USPhydro are different for centrality >40%
  - Also difference of full IP-Glasma and FSE only, indication of IMA?
- ❖ Not conclusive on which model works better due to sizeable uncertainties
  - An improved result will be available soon by using the entire Run2 data



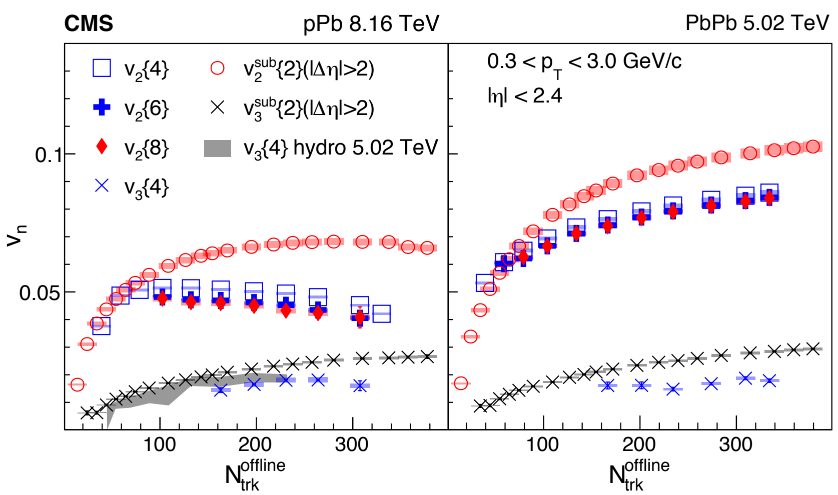
Large

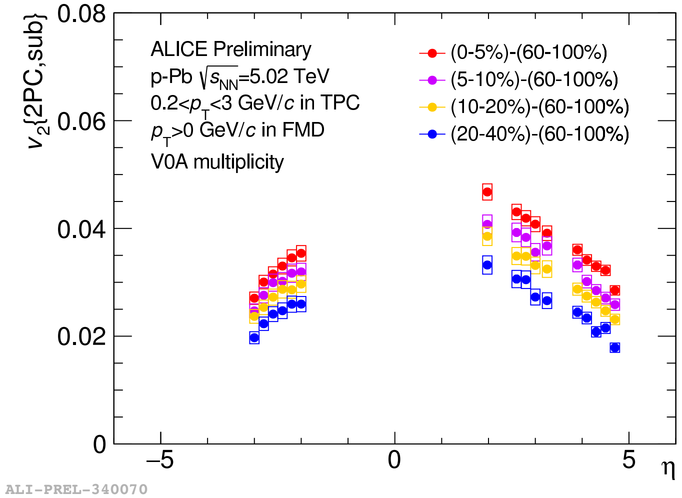
Small

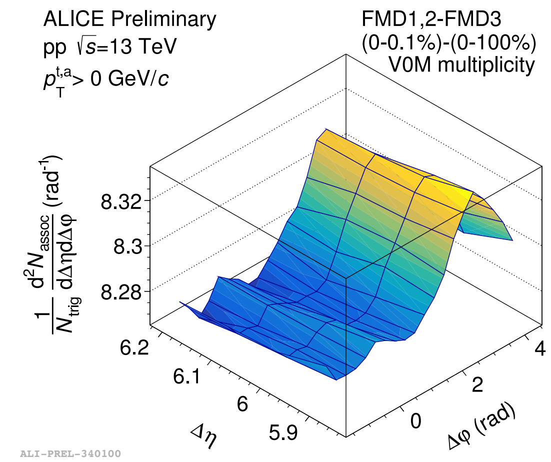


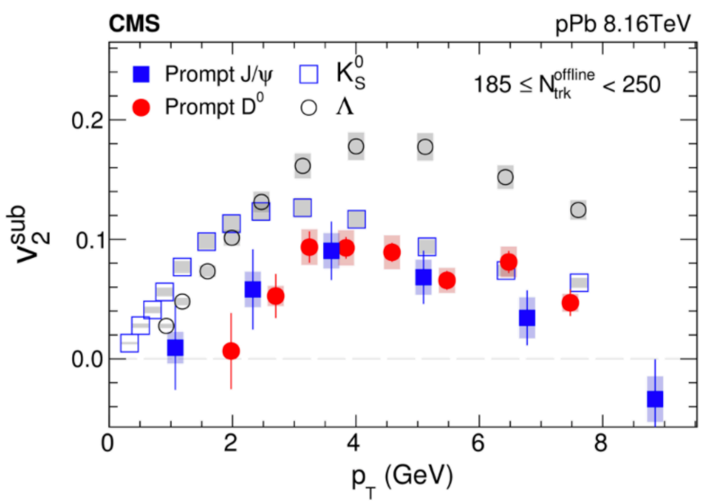
***Large** is **large**, but is **small** really **small** ?*

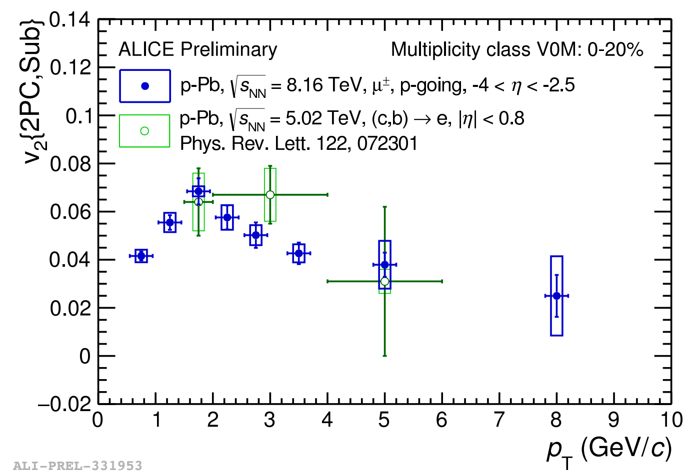
Collective flow in small systems

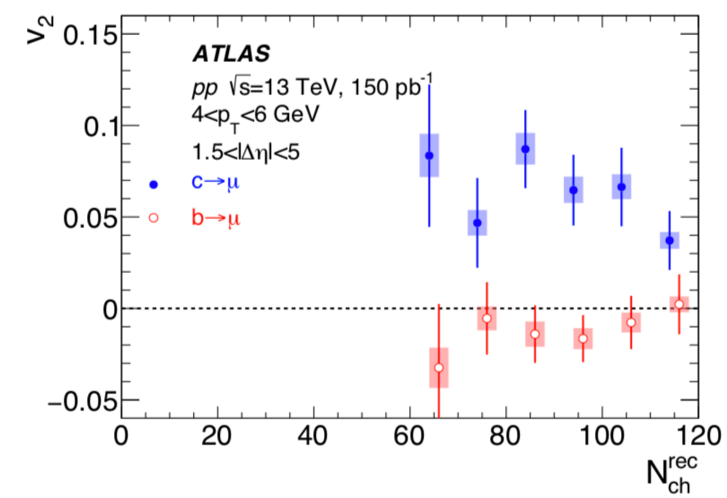


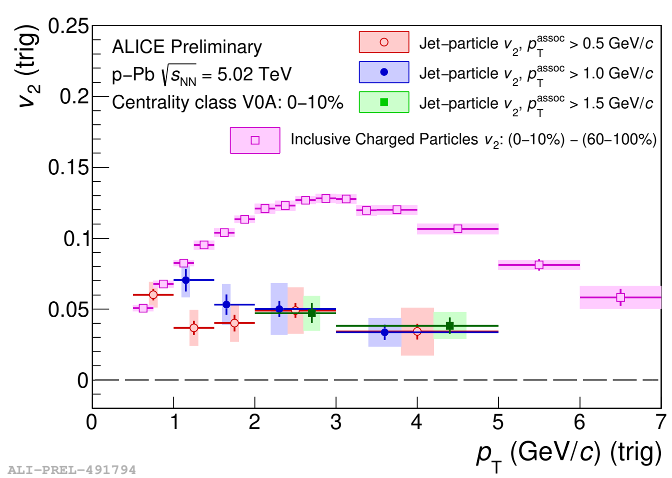










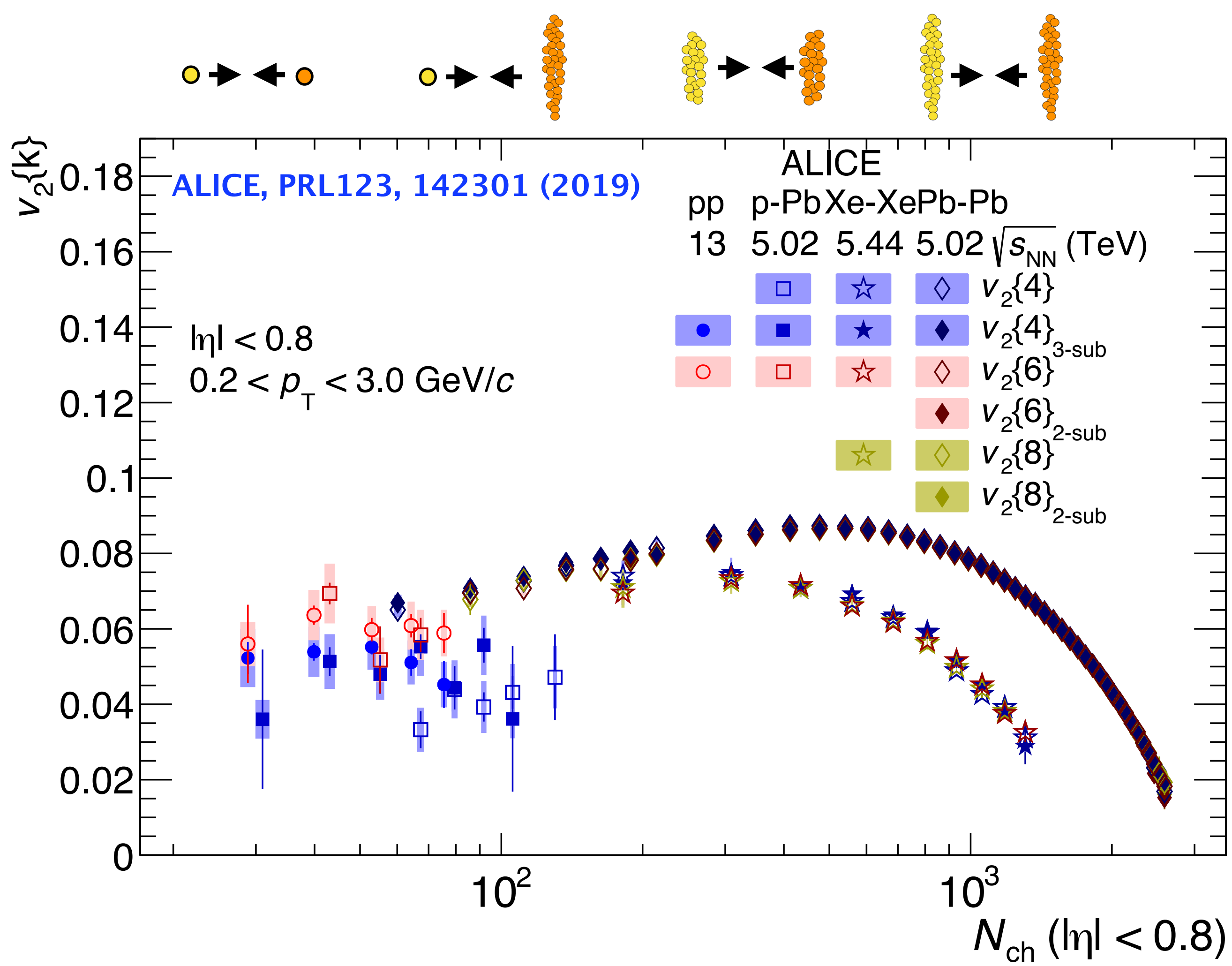


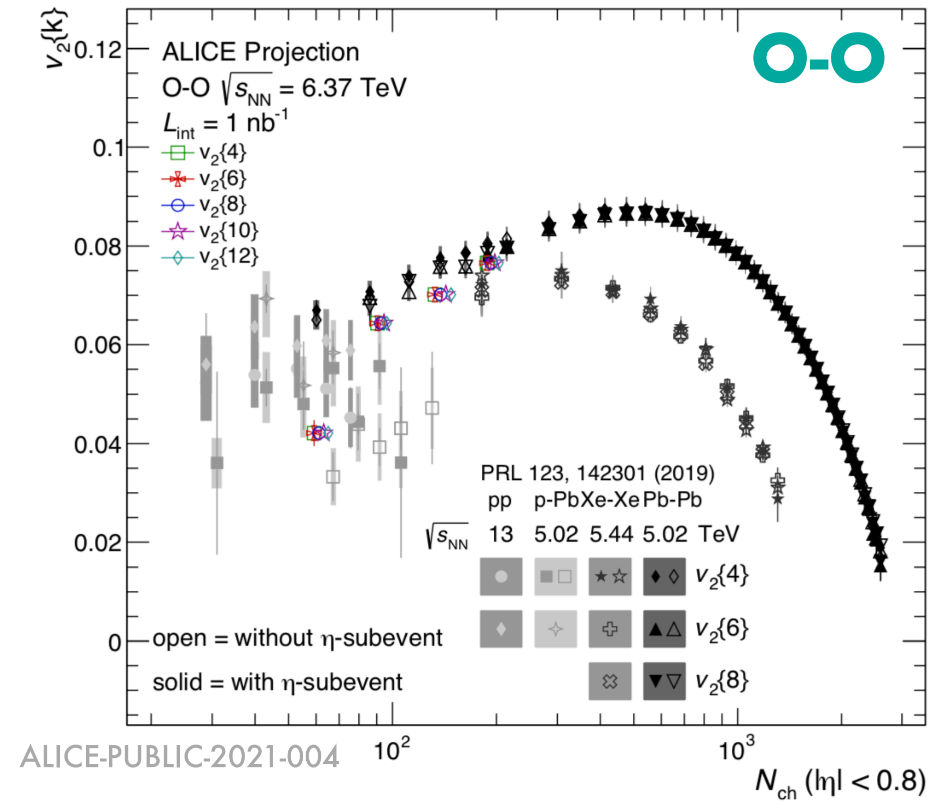
Credits:

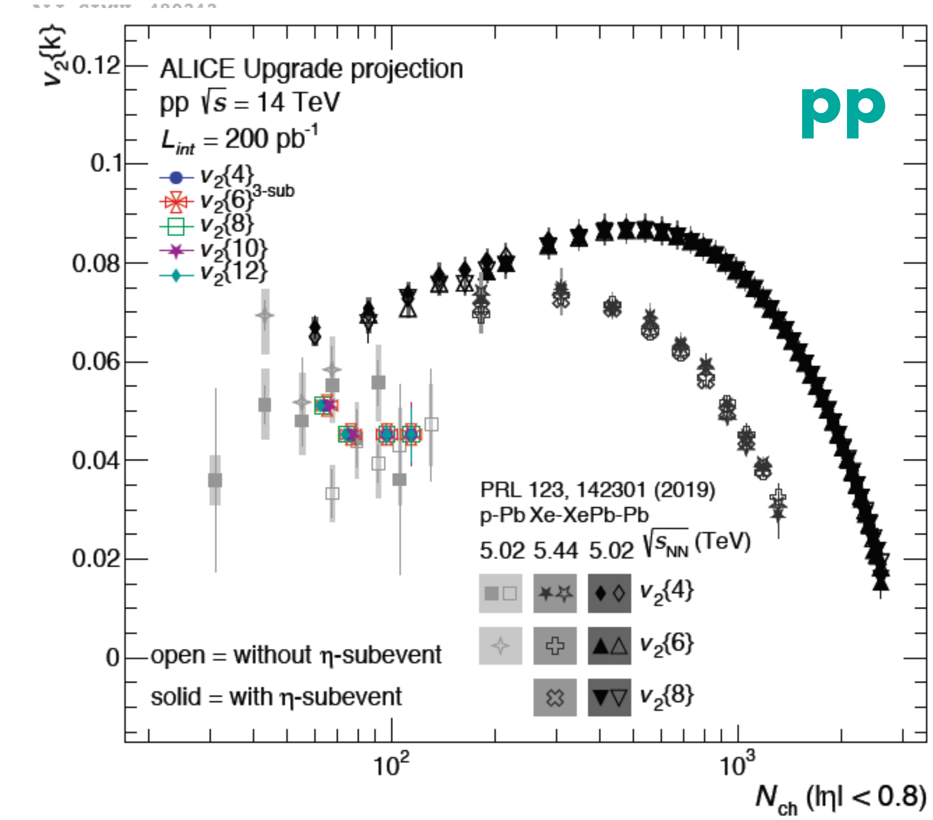
⇨ **hydro-dynamics:**

- **LO** : elliptic (v_2) flow for > 95% of all particles (p_t < few GeV)
- **NLO** : higher harmonics v_n, PID (*m* dependence) of v_n
- **NNLO** : non-linear mode mixing ($v_n \neq \varepsilon_n$), factorization violation r(p_T), EbE P(v_n), …

Multi-particle cumulants



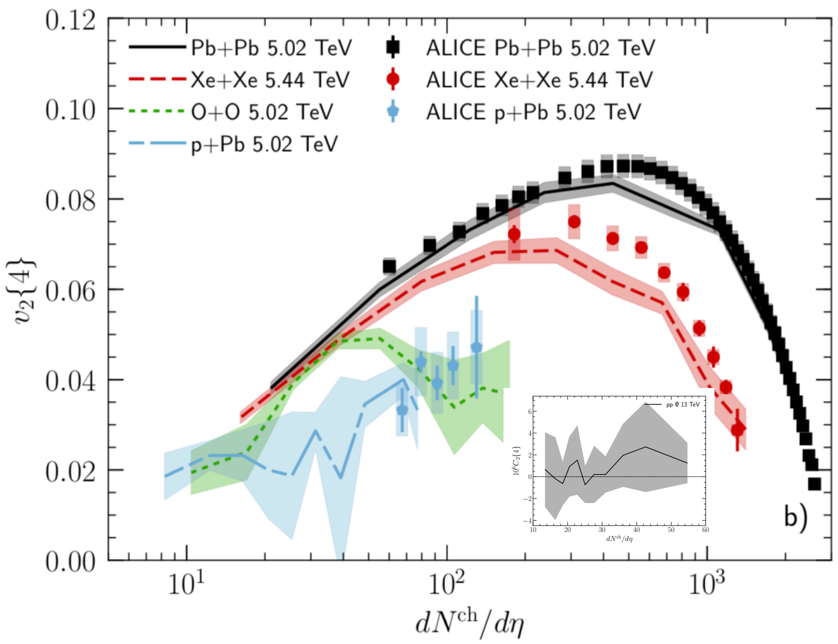




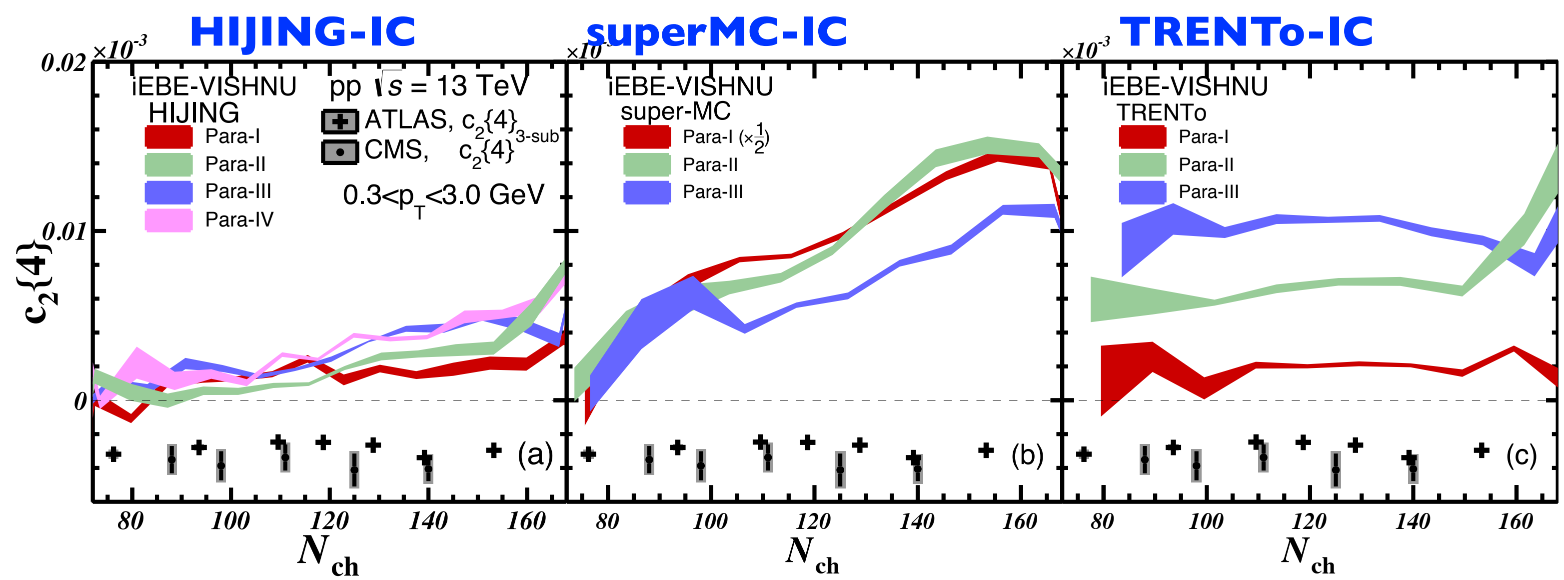
❖ p-Pb and pp collisions

- Non-zero v2 and $v_2\{4\} \approx v_2\{6\}$, strong evidence of flow in small systems!
- Future LHC Run3 & 4 programs will enable the measurements of $v_2\{8\}$, $v_2\{10\}$ and $v_2\{12\}$ in small systems

No negative $c_2\{4\}$ in hydro in pp



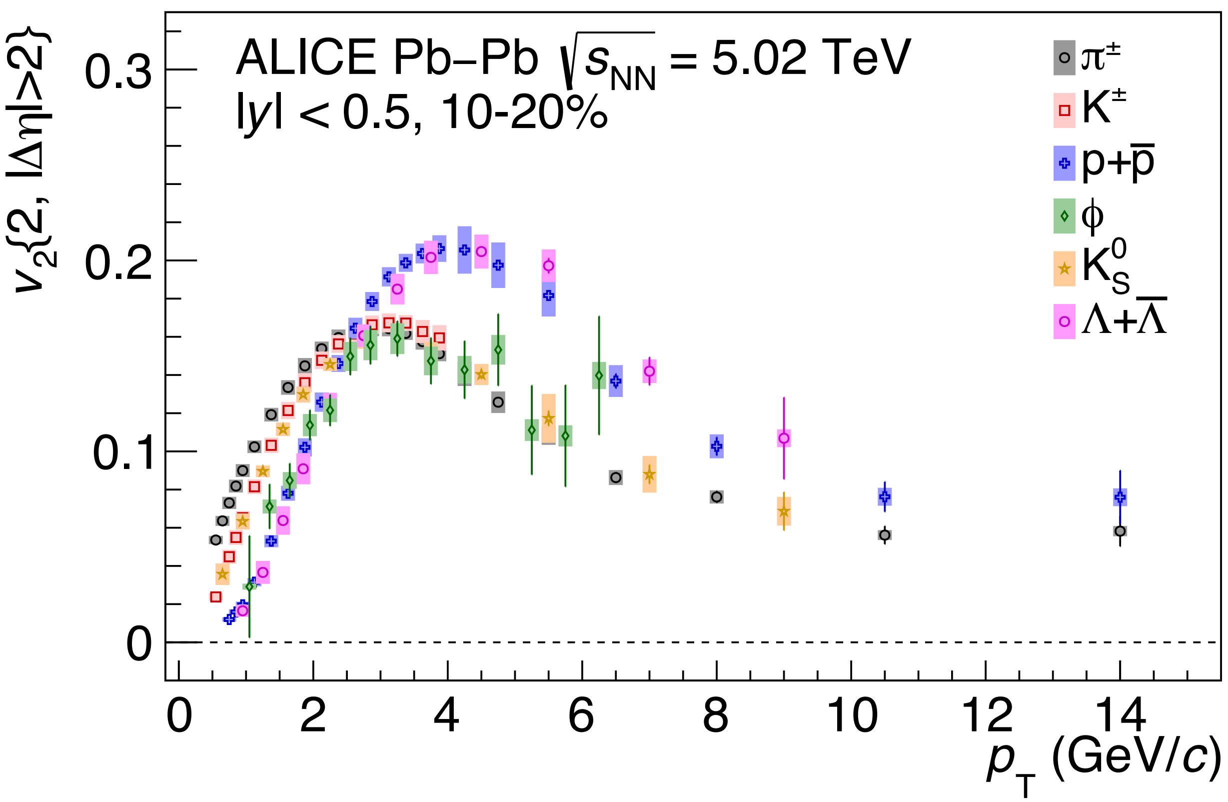
Also see: B. Schenke etc, PRC102, 044905 (2020)

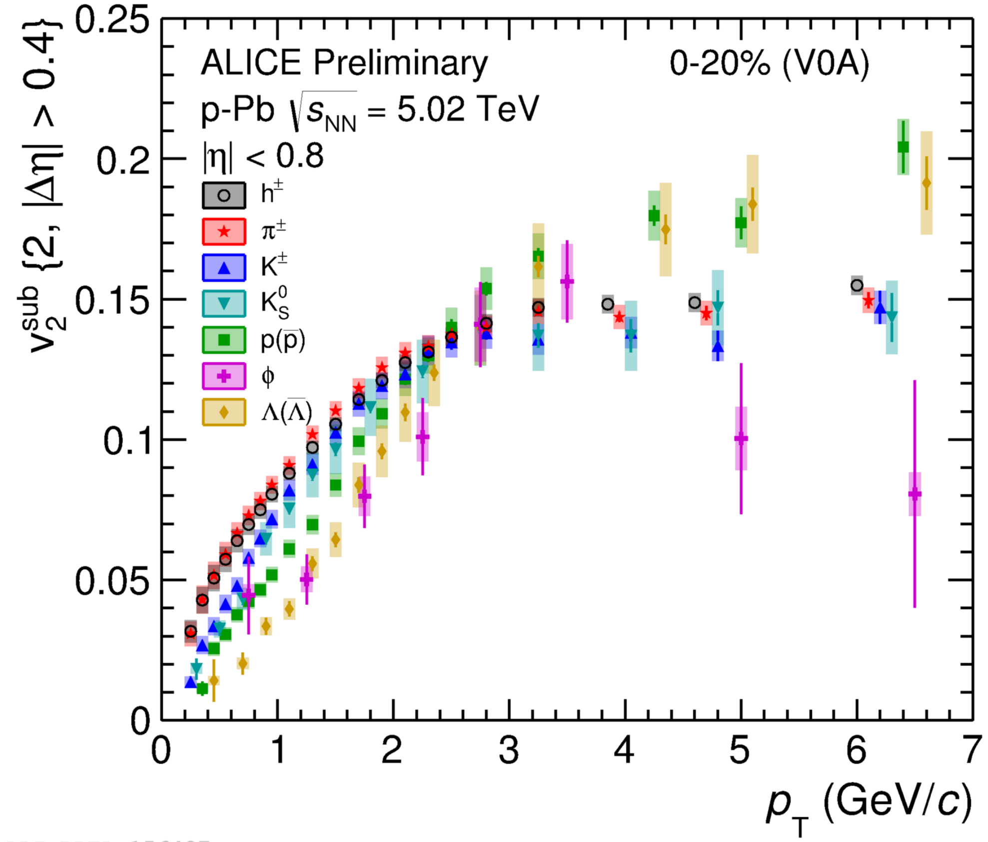


Also see: W. Zhao., etc, EPJC80 (2020) 9, 846
W. Zhao., etc, PLB780 (2018) 495

- Any solution or progress on the wrong sign of $c_2\{4\}$ from hydro?
  - Will 3+1D hydro help?

PID vn in small systems



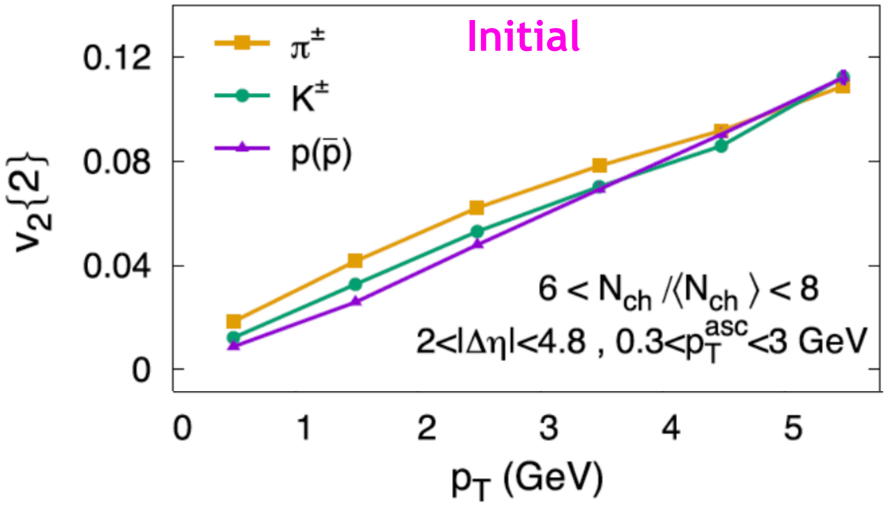


ALI-PREL-156487

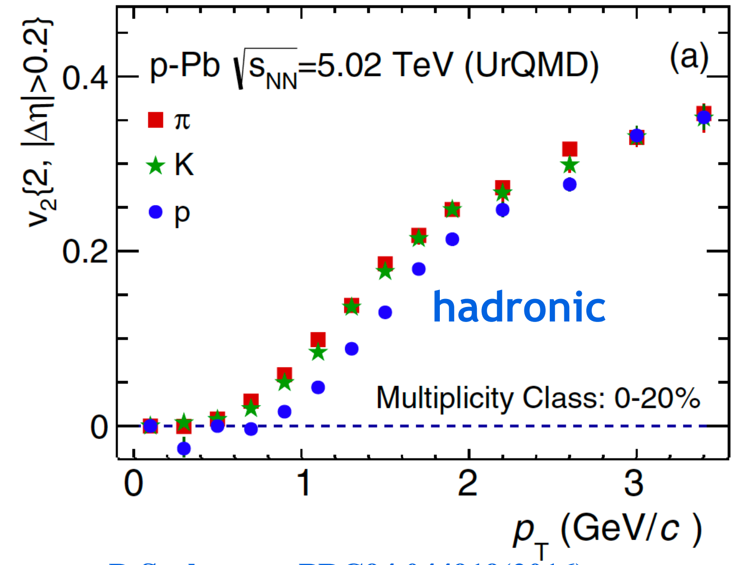
- **What we knew**: v_2 of identified particles in **Pb-Pb**
  - at low p_T: mass ordering, described by hydrodynamic calculations
  - at intermediate p_T: approximate baryon/meson grouping
- **What we also had**: v_2 of identified particles in **p-Pb**
  - at low p_T: most particle species follow mass ordering -> hydrodynamic flow?
  - at intermediate p_T: baryon v_2 > meson v_2 -> partonic collectivity? Indication of QGP?
  - A better experimental treatment on non-flow is in preparation
- **What about pp?**
  - Will similar behaviours remain?

Origin of *mass ordering*

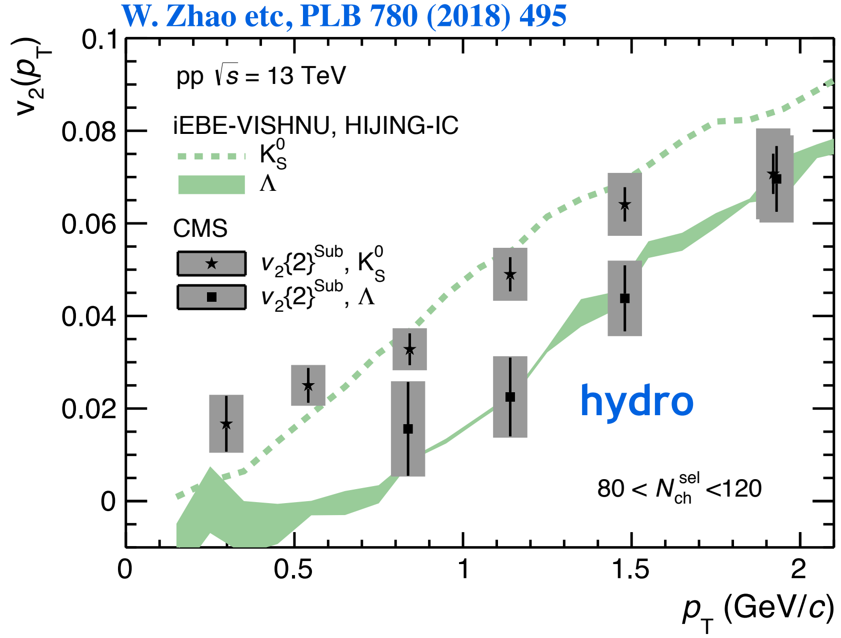
B. Schenke etc, PRL117, 162301 (2016)



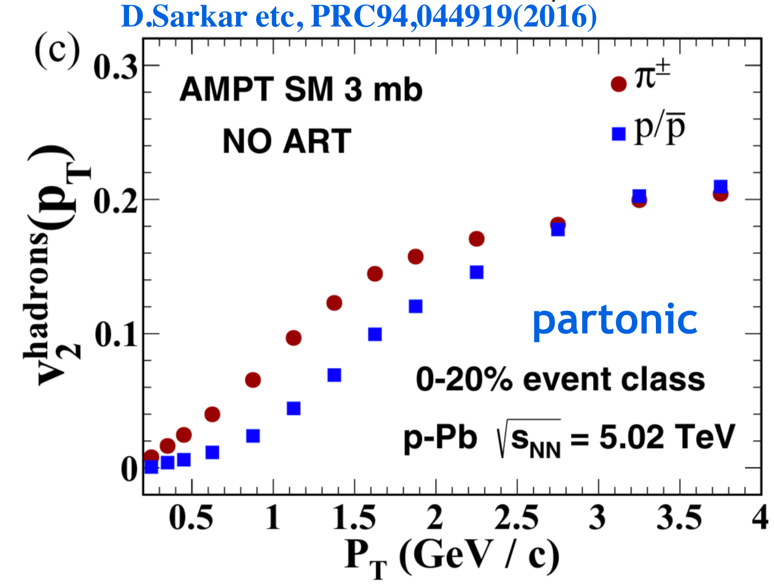
Y. Zhou et al., PRC 91, 064908 (2015)



W. Zhao etc, PLB 780 (2018) 495



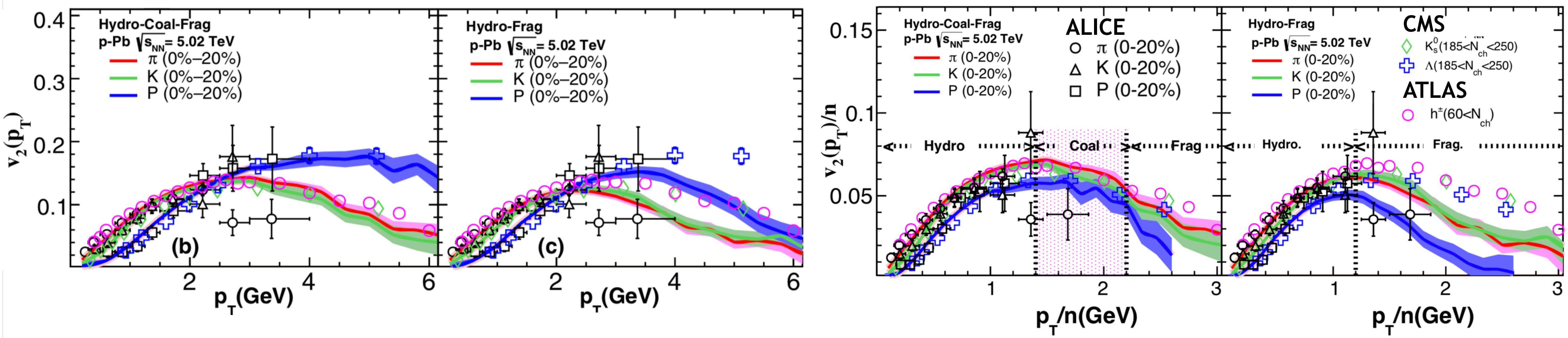
D.Sarkar etc, PRC94,044919(2016)



- Mass ordering of PID v_2 in small collision systems
  - Qualitatively predicted by **initial stage effects** (e.g. CGC+Lund), or **final stage effects**: hydro (iEBE-VISHNU), parton escape (AMPT), hadronic rescatterings (UrQMD)
  - ***Mass ordering at low p_T might not be used as an evidence of hydrodynamic flow***
  - quantitative comparison to non-flow suppressed/subtracted data will be extremely useful

NCQ scaling from coalescence

W. Zhao etc., Phys. Rev. Lett. 125, 072301 (2020)

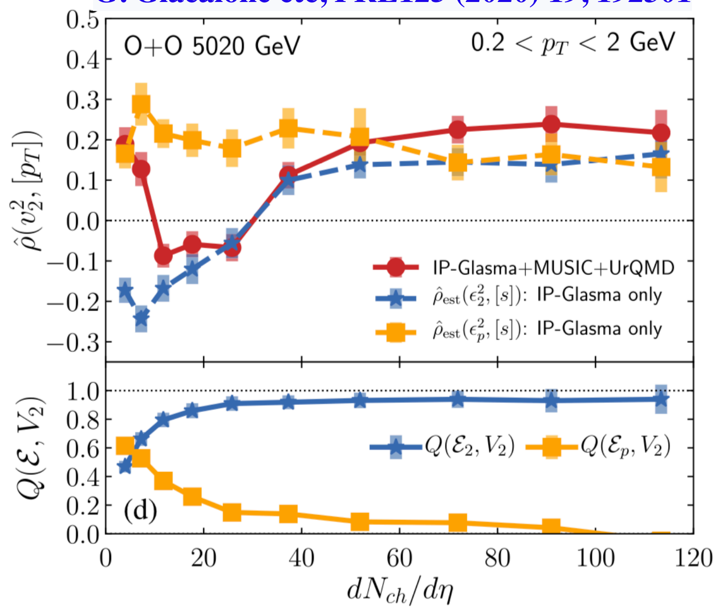


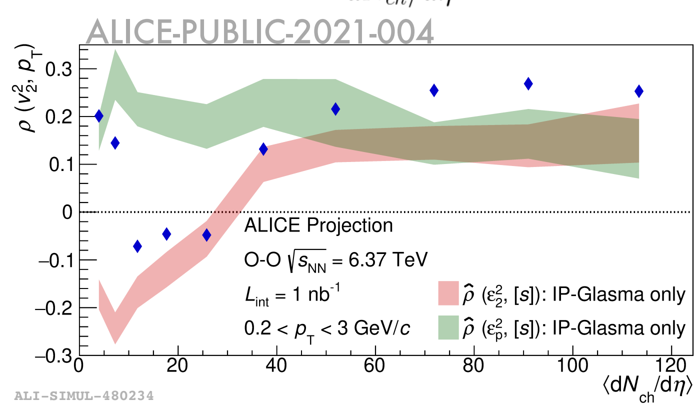
❖ Calculation with quark coalescence gives a better but not a perfect scaling

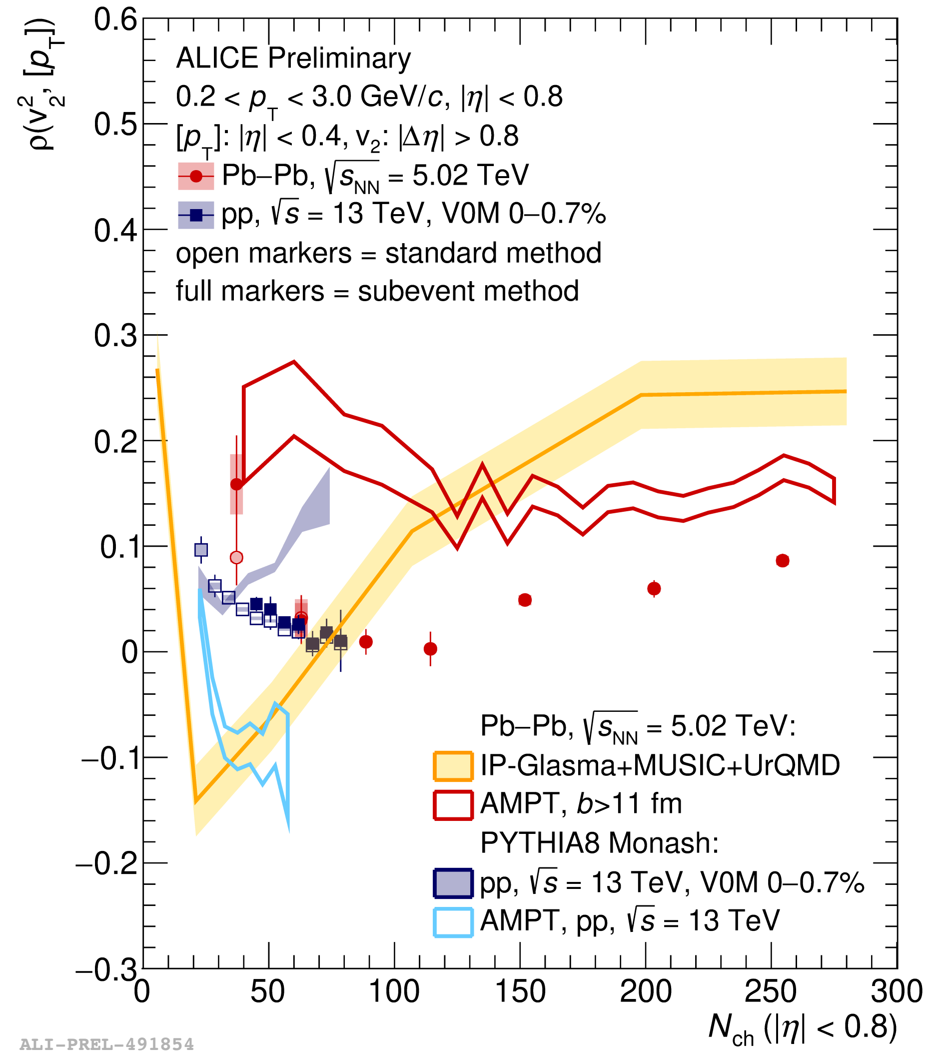
- *A perfect NCQ scaling is not the requirement of partonic collectivity.*
- *Baryon/meson v_2 grouping or v_2(baryons) > v_2(mesons) is not the evidence of partonic collectivity*
  - *Further separation at high p_T might be a better probe of partonic flow?*
  - Special role of ϕ meson (follows meson group if partonic flow and baryon flow if hydro+frag)?
- A future precision data/model comparison will be highly needed
- New ALICE data in both p-Pb and pp with much improved non-flow subtractions at QM2022

More results in smaller colliding systems

G. Giacalone etc, PRL125 (2020) 19, 192301







- Search for the initial momentum anisotropy (IMA) in smaller colliding systems
  - Peripheral Pb-Pb collisions
    - Slope changes for N_{ch} ~ 100 for data and ~ 20 for IP-Glasma calculations
    - Both AMPT and IP-Glasma+hydro predicts slope changes -> not unique signature of IMA?
  - pp collisions:
    - Decreasing trend with increasing N_{ch}, results are consistent with the one in Pb-Pb
    - AMPT generates stronger anti-correlations, PYTHIA predicted a wrong N_{ch} dependence
    - Non-flow is a main challenge, many important studies by J. Jia, C. Zhang, J. Nagle etc

Summary

Collective flow in Large and Small systems

☆ **For *Large* systems:**

- Many flow studies on the joint p.d.f., and new study includes correlations between anisotropic flow and radial flow
  - New constraints on the initial conditions and the properties of QGP
  - New possibility to probe nuclear structure at high energy
  - TRENTo model seems to have a new challenge, which might further affect the current understanding or the properties of QGP, via Bayesian analysis

☆ **For *Small* systems**

- Few selected flow observables have been discussed
  - Wrong sign of $c_2\{4\}$ in hydro remain unsolved
  - PID v_n (at intermediate and high pt) in small system will show more hints of partonic collectivity
  - Probe possible IMA at low multiplicity, where non-flow is still a challenge.

There have been many more exciting new results in the past few years—my apology if I can not cover them all here.

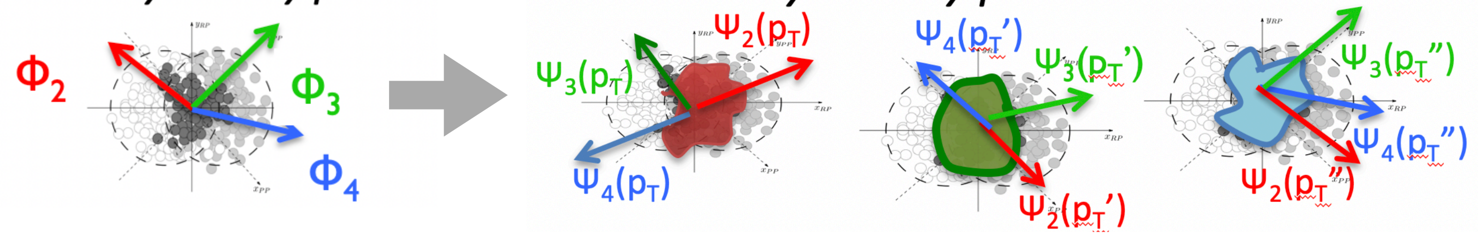
Thanks for your attention!

Backup

Jiangyong Jia, J.Phys.G 41 (2014) 12

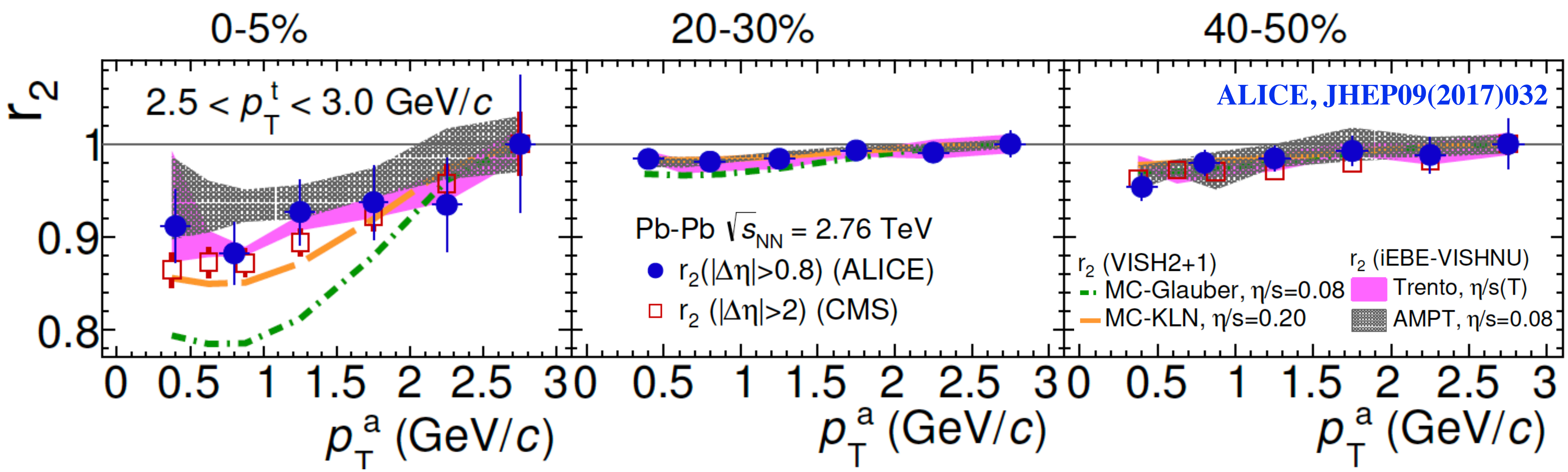
	pdfs	cumulants
Flow-amplitudes	$p(v_n)$	$v_n\{2k\}$, $k = 1,2,...$
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$, $n \neq m$ ...
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v_m^2\rangle\langle v_l^2\rangle -$ $\langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2\rangle\langle v_n^2\rangle - \langle v_l^2 v_n^2\rangle\langle v_m^2\rangle$ $n \neq m \neq l$ ...
	...	Obtained recursively as above
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2\rangle\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$, $n \neq m \neq l ...$

Ψ_n fluctuations $P(\Psi_n)$



$$r_n = \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) \cdot V_{n\Delta}(p_T^b, p_T^b)}}$$

- r_n probes < a, b > ➡ < a, a > & < b, b >
- r_n < 1, Factorization broken



❖ Breakdown of factorization more pronounced in central collisions.

❖ Hydrodynamic reproduce the factorization broken

- Indication of p_T dependent flow angle (and magnitude) fluctuations

❖ Using novel multi-particle correlations, both flow-angle and flow magnitude fluctuations are observed in experiments (see backup for more details)

P(v_n) -> P(ε_n)

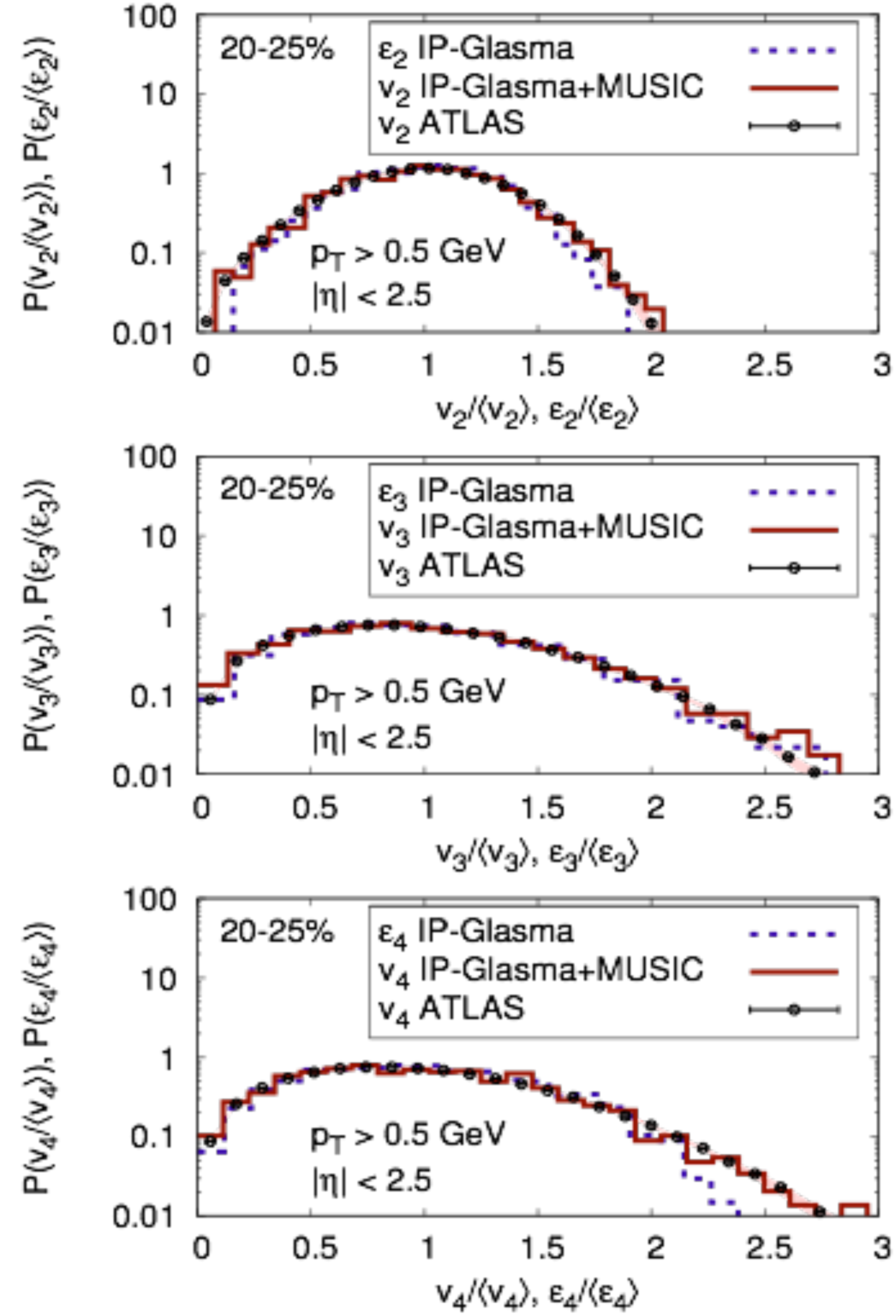
ATLAS, JHEP11, 183 (2013)

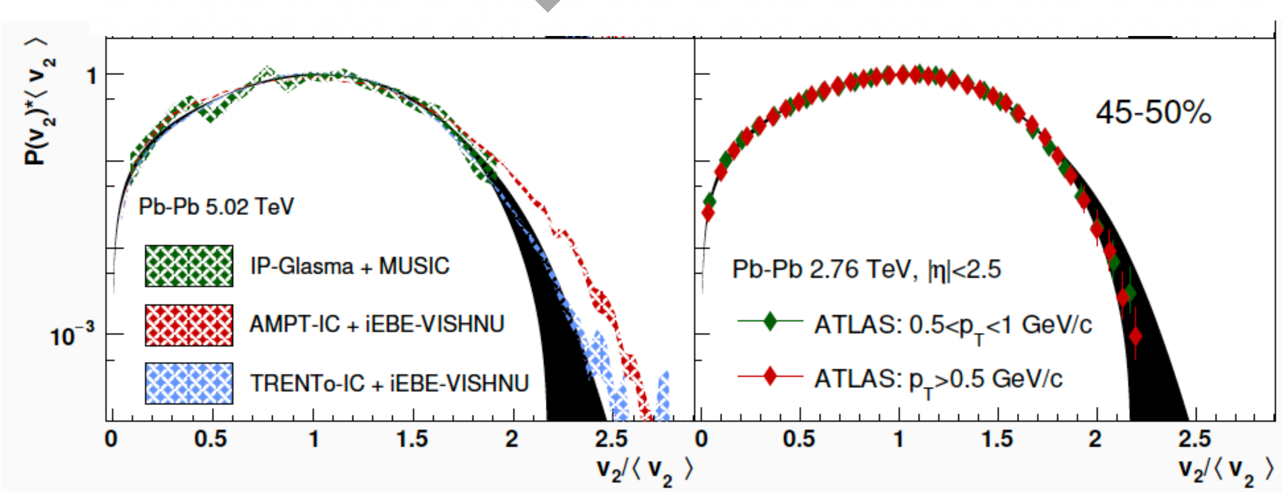
❖ Elliptic-power function:

$$P(v_2) = \frac{\mathrm{d}\varepsilon_2}{\mathrm{d}v_2} P(\varepsilon_2) = \frac{1}{k_2} P\left(\frac{v_2}{k_2}\right) = \frac{2\alpha v_2}{\pi k_2^2}(1-\varepsilon_0^2)^{\alpha+1/2} \int_0^{\pi} \frac{(1-v_2^2/k_2^2)^{\alpha-1}}{(1-v_2\varepsilon_0\cos\varphi/k_2)^{2\alpha+1}}\,\mathrm{d}\varphi$$

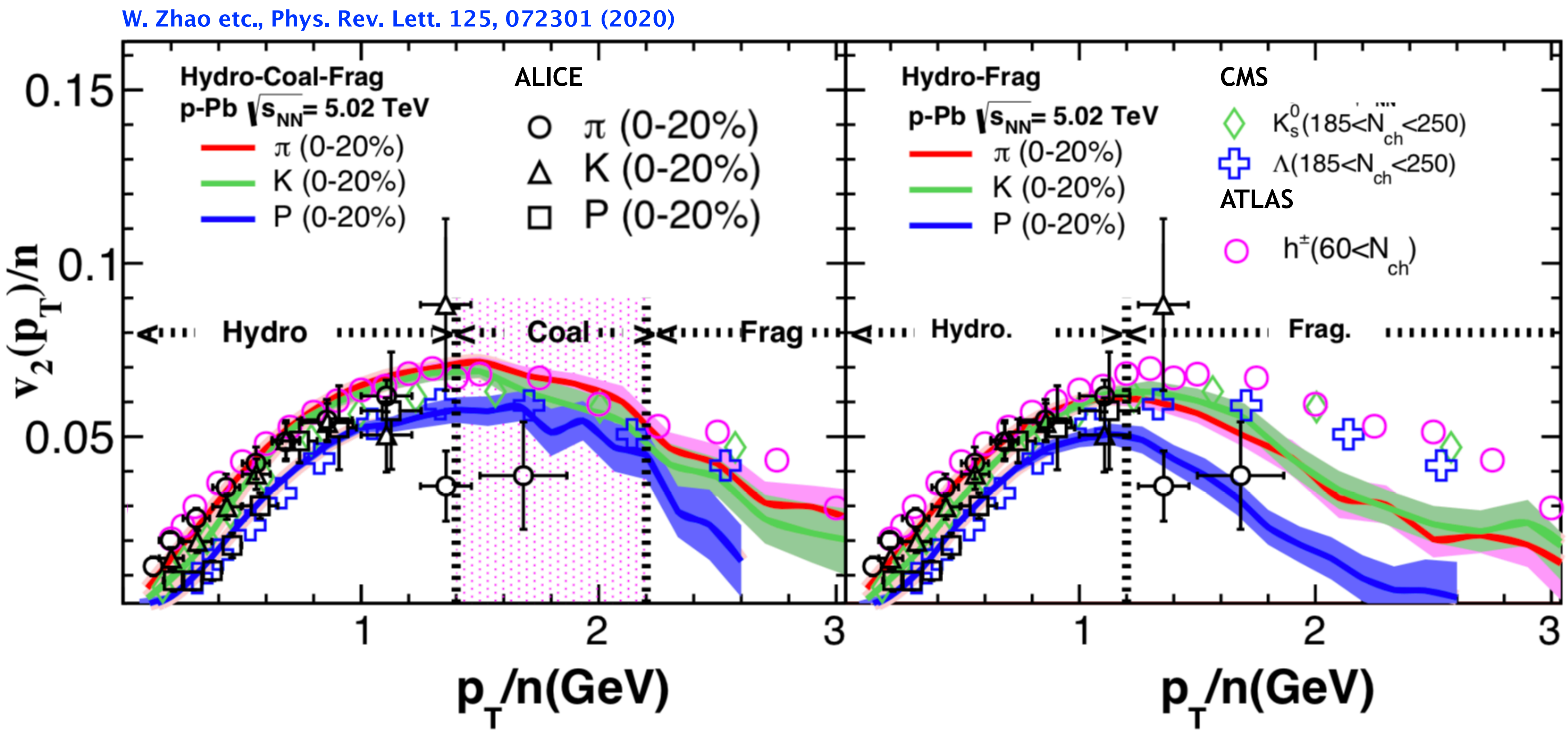
$$\begin{aligned}
c_2\{2\} &= k_2^2\,(1-f_1)\,,\\
c_2\{4\} &= -\,k_2^4\,\left(1-2\,f_1+2\,f_1^2-f_2\right),\\
c_2\{6\} &= k_2^6\,\left(4+18\,f_1^2-12\,f_1^3+12 f_1\,(3 f_2-1)-6\,f_2-\,f_3\right),\\
c_2\{8\} &= -\,k_2^8\,(33-288\,f_1^3+144\,f_1^4-66\,f_2+18\,f_2^2-24\,f_1^2(-11+6\,f_2)\\
&\quad -12\,f_3+4\,f_1(-33+42\,f_2+4\,f_3)-f_4)
\end{aligned}$$

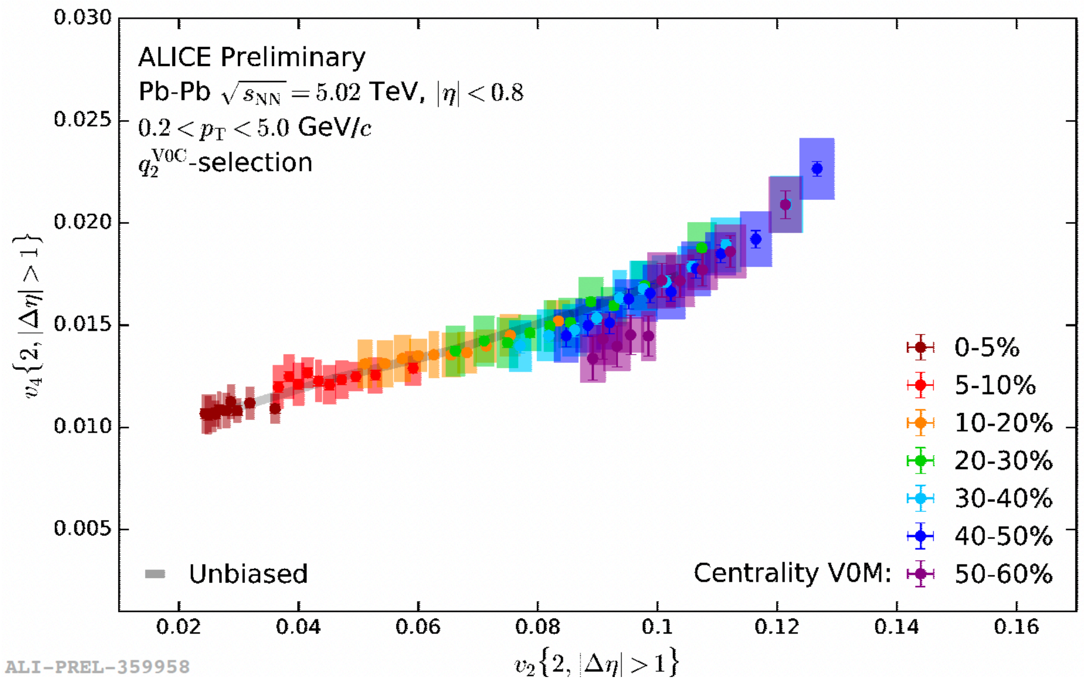
$$f_k \equiv \langle (1-\varepsilon_n^2)^k \rangle = \frac{\alpha}{\alpha+k}\,(1-\varepsilon_0^2)^k\,{}_2F_1\left(k+\frac{1}{2}, k; \alpha+k+1, \varepsilon_0^2\right)$$





NCQ scaling from coalescence



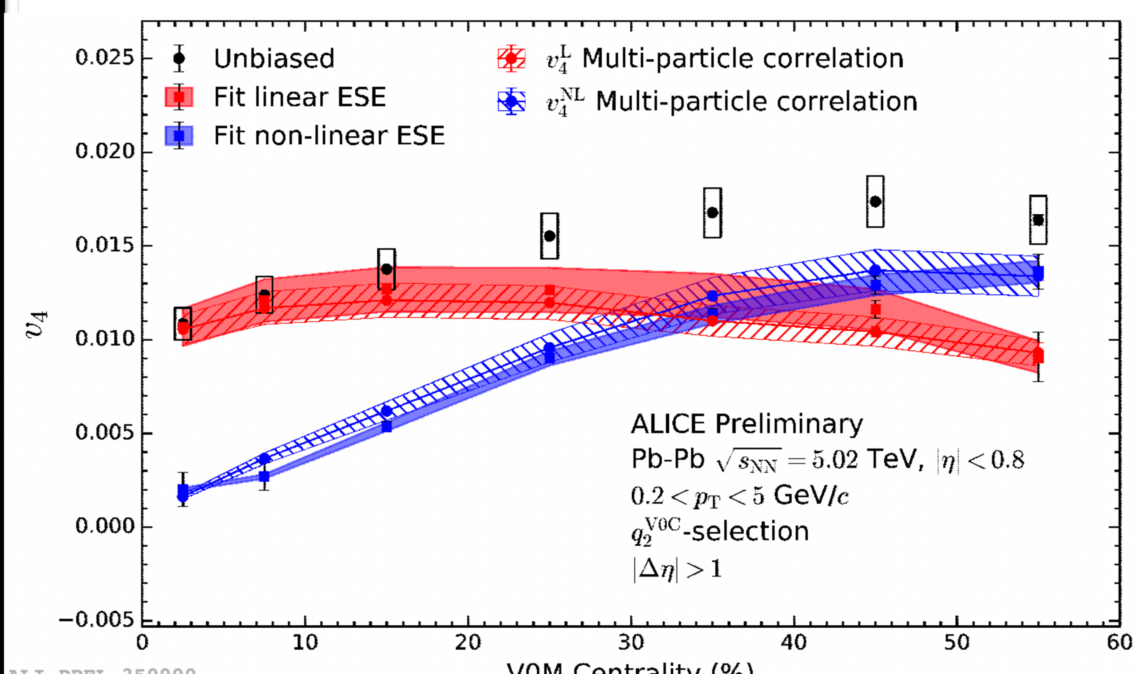


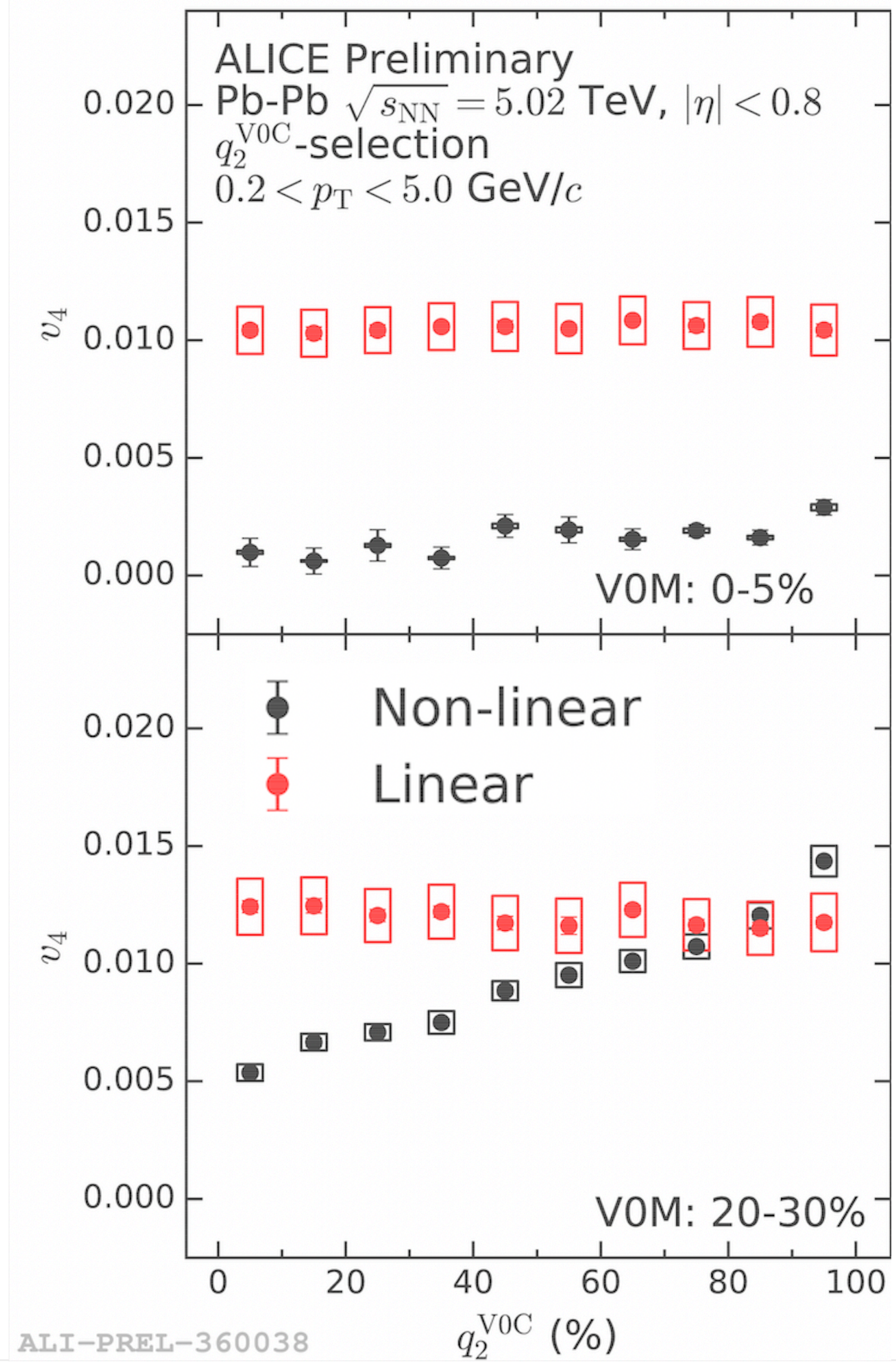
Fit

$$v_4 = \sqrt{c_0^2 + (c_1 v_2^2)^2}$$

$$v_4^L = c_0,$$

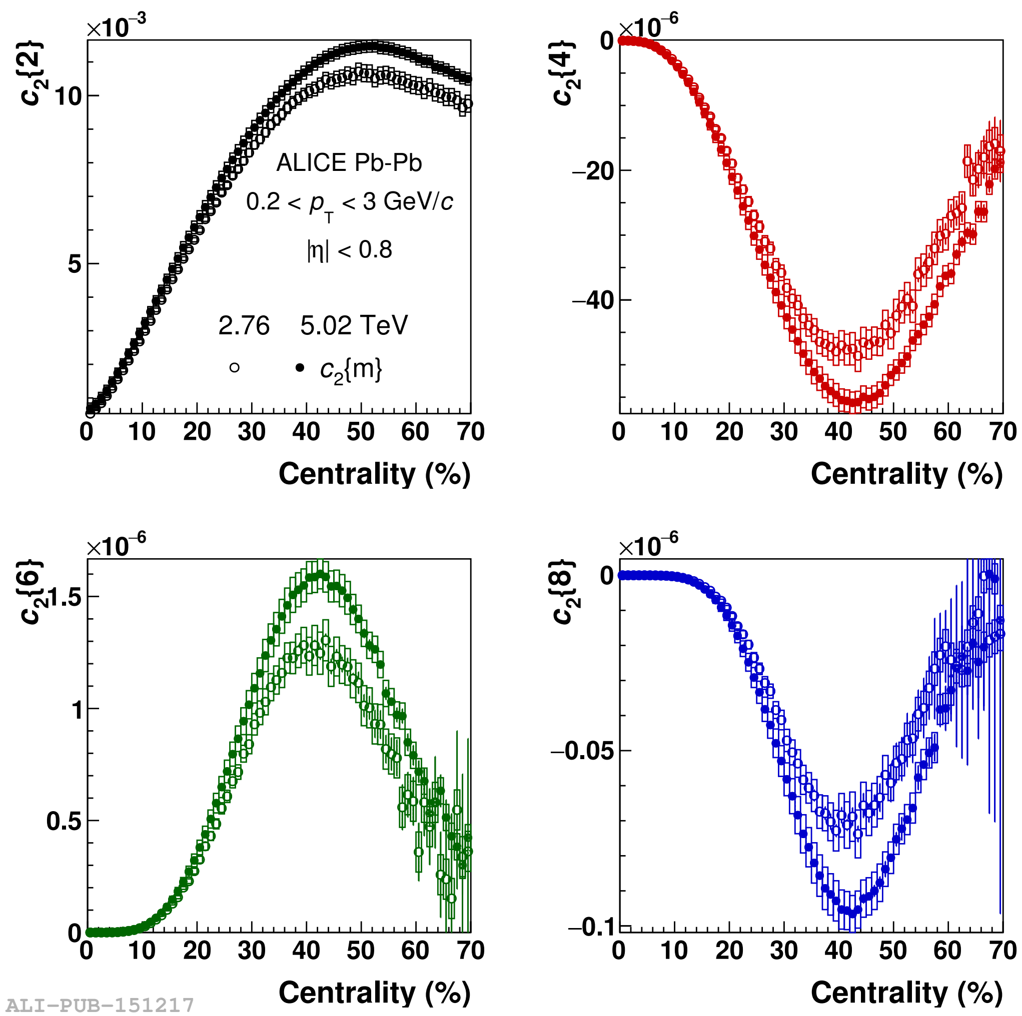
$$v_4^{NL} = \sqrt{(v_4)^2 - (c_0)^2}.$$





$P(v_n)$ from multi-particle cumulants of v_n

ALICE, JHEP 07 (2018) 103



$p(\varepsilon_n)$ → System Expansion → $p(v_n)$

$v_n\{2\}, v_n\{4\}, v_n\{6\}, v_n\{8\}, v_n\{10\}, v_n\{12\}$ …

Multi-particle ***correlations*** of single harmonic v_n

$$\langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle\rangle = \left\langle v_n^4 \cos(n\Phi_n - n\Phi_n + n\Phi_n - n\Phi_n)\right\rangle = \left\langle v_n^4 \right\rangle$$

Multi-particle ***cumulants*** of single harmonic v_n

$$\begin{aligned}\langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle\rangle_c &= \langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle \\ &- \langle\langle \cos(n\phi_1 - n\phi_2)\rangle\rangle \langle\langle \cos(n\phi_3 - n\phi_4)\rangle\rangle \\ &- \langle\langle \cos(n\phi_1 - n\phi_4)\rangle\rangle \langle\langle \cos(n\phi_2 - n\phi_3)\rangle\rangle \\ &= \left\langle v_n^4 \right\rangle - 2\left\langle v_n^2 \right\rangle^2\end{aligned}$$

$$\begin{aligned}v_n\{2\} &= \sqrt[2]{\langle v_n^2\rangle}, \\ v_n\{4\} &= \sqrt[4]{2\langle v_n^2\rangle^2 - \langle v_n^4\rangle}, \\ v_n\{6\} &= \sqrt[6]{\langle v_n^6\rangle - 9\langle v_n^2\rangle\langle v_n^4\rangle + 12\langle v_n^2\rangle^3}, \\ v_n\{8\} &= \sqrt[8]{\langle v_n^8\rangle - 16\langle v_n^2\rangle\langle v_n^6\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^2\rangle^2\langle v_n^4\rangle - 144\langle v_n^2\rangle^4}.\end{aligned}$$