Theory for quarkonium: from NRQCD to soft gluon factorization

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References:
1703.08402: YQM, Chao
1911.05886: Li, Feng, YQM
2005.08786: Chen, YQM
2103.15121: Chen, Jin, YQM, Meng

IHEP
Seminar, 2021/10/27

Outline

I. NRQCD factorization with dominant LDMEs

II. Relativistic corrections

III. Soft gluon factorization

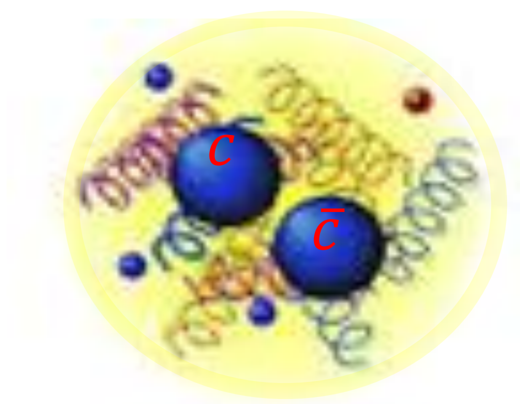
IV. Application to FF: $g \to Q\bar{Q}({}^3S_1^{[8]}) + X$

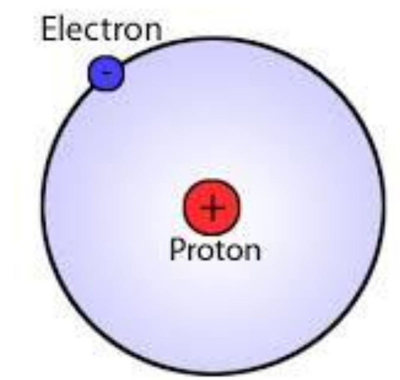
V. Summary and outlook

Heavy quarkonium

- **Bound state of $Q\bar{Q}$ pair under strong interaction**

eg: $J/\psi\ \ \psi', \chi_{cJ}, \Upsilon(nS), \chi_{bJ}(nP) \cdots$





- The simplest system in QCD: two-body problem
- “Hydrogen atom in QCD”, “an ideal laboratory in QCD”
- Example: quarkonium production can be traced by heavy quark pair (vs light hadron), unique to explore hadronization mechanism

NRQCD: factorization

Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle \mathcal{O}_n^H \rangle$$

Production of a heavy quark pair
Expansion in: α_s

Hadronization (LDMEs)
Expansion in: v

- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_J^{[c]}$

A glory history

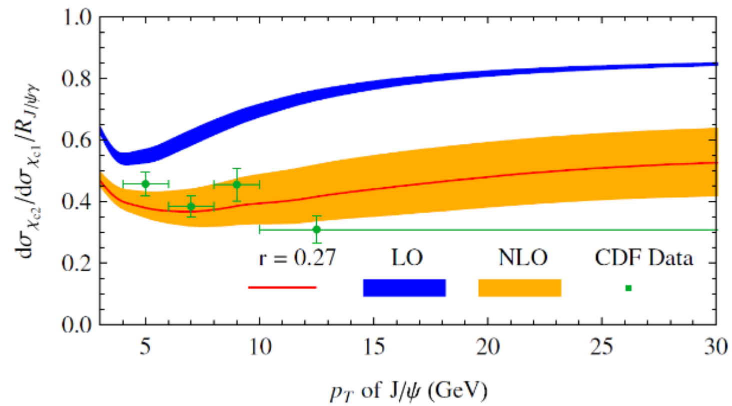
- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus
- Explained χ_{c2}/χ_{c1} production ratio
- ....

Thanks to color-octet mechanism

Achievement: χ_{cJ} production

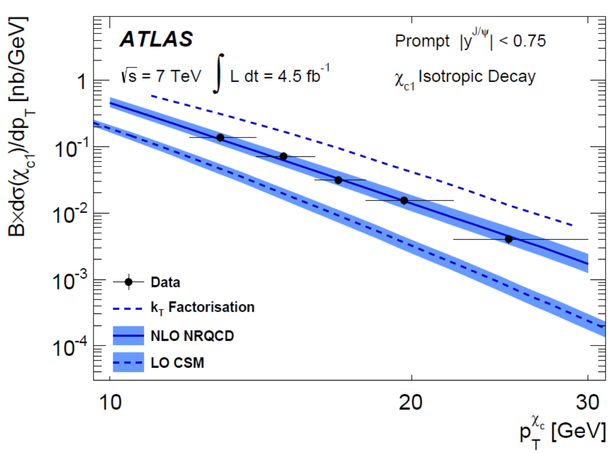
- χ_{cJ} **production:** $d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left({}^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left({}^3S_1^{[8]}\right)\rangle$

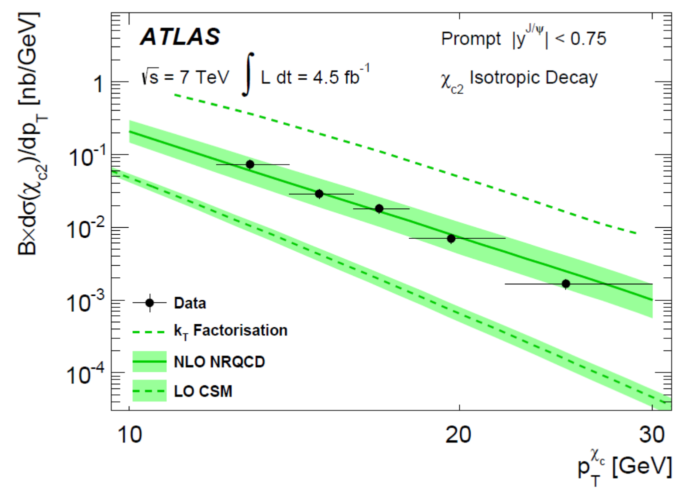
YQM, Wang, Chao, 1002.3987



- **Predictions agree with new data**

ATLAS, 1404.7035



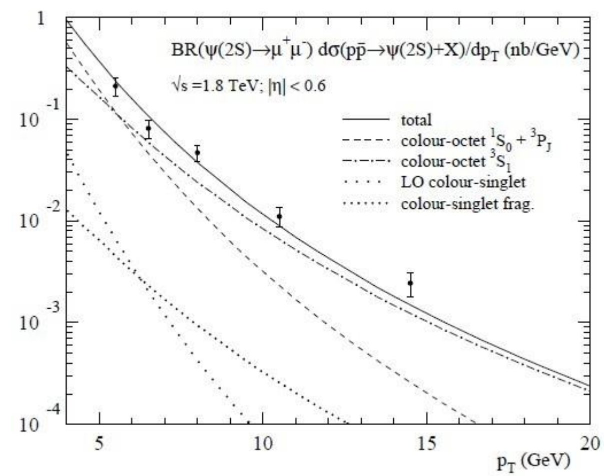


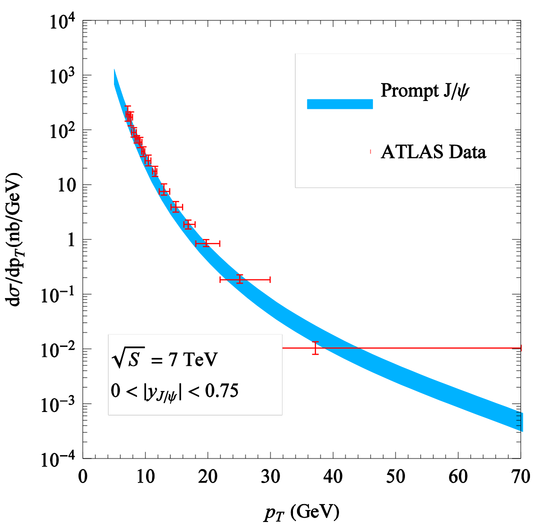
Achievement: explain $\psi(nS)$ surplus

➢ $\psi(nS)$ production in NRQCD

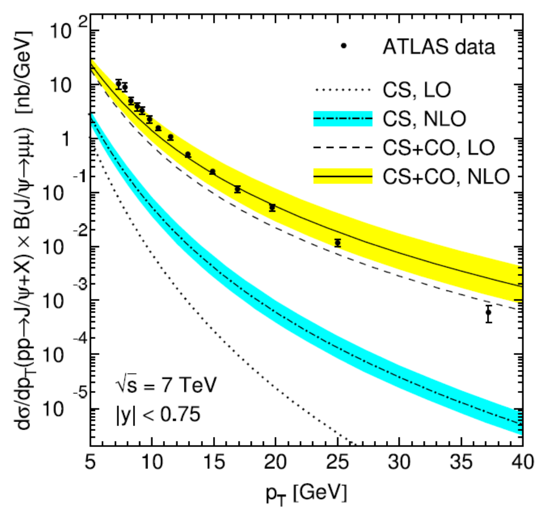
States	Power in v	p_T behavior at LO	p_T behavior at NLO
${}^3S_1^{[1]}$	v^0	p_T^{-8}	p_T^{-6}
${}^3S_1^{[8]}$	v^4	p_T^{-4}	p_T^{-4}
${}^1S_0^{[8]}$	v^3	p_T^{-6}	p_T^{-4}
${}^3P_J^{[8]}$	v^4	p_T^{-6}	p_T^{-4}

Kramer, 0106120

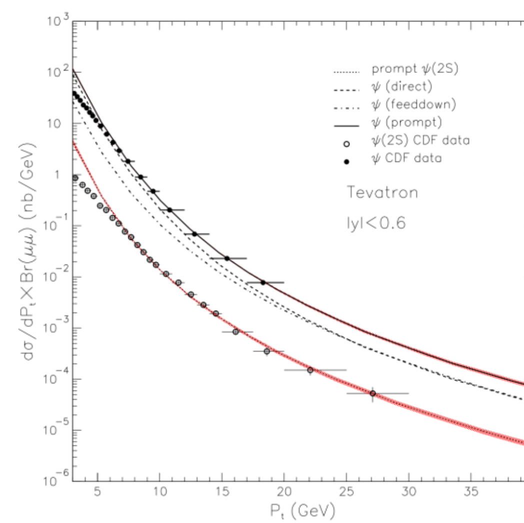




YQM, Wang, Chao, 1012.1030



Butenschoen, Kniehl, 1105.0820



Gong, Wan, Wang, Zhang, 1205.6682

Pheno difficulty: polarization puzzle

➢ LO NRQCD

- **Dominated by $^3S_1^{[8]}$, LO NRQCD predicts transversely polarized $\psi(\mathrm{nS})$ at high p_T, contradicts with Tevatron and LHC data**

CDF, 0704.0638

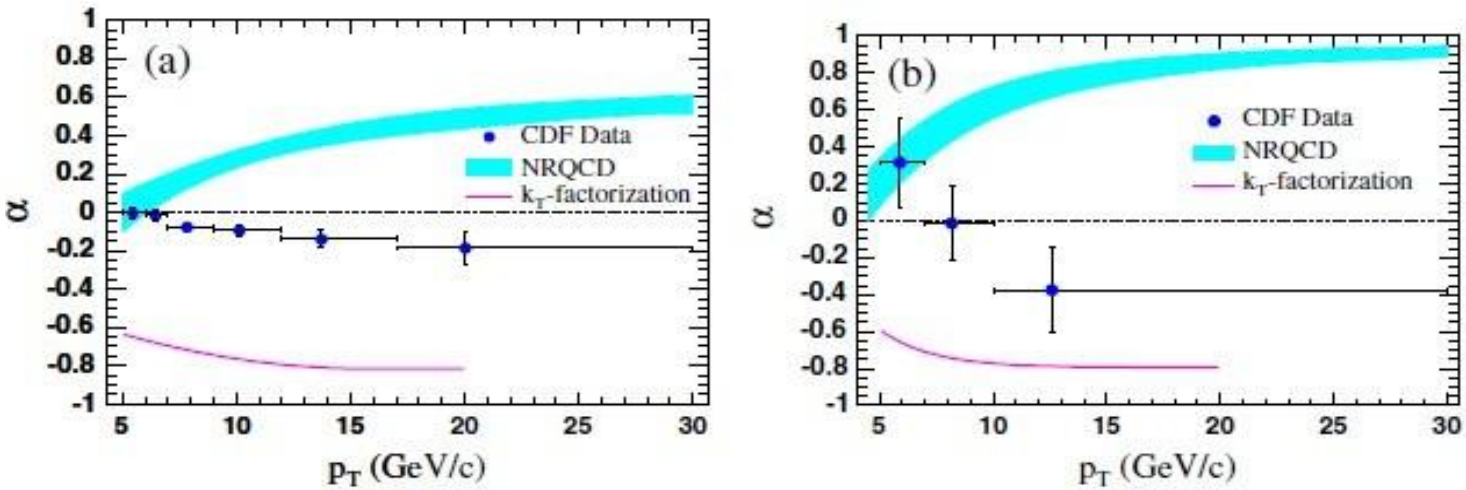
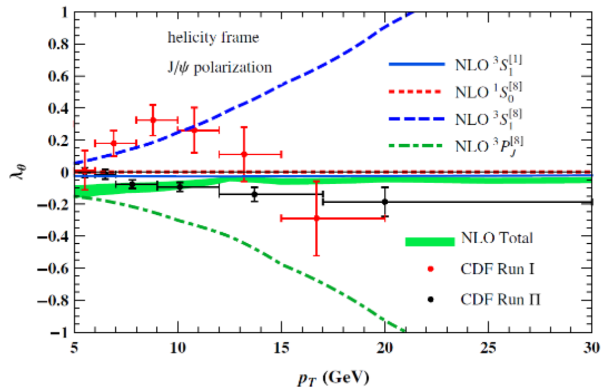


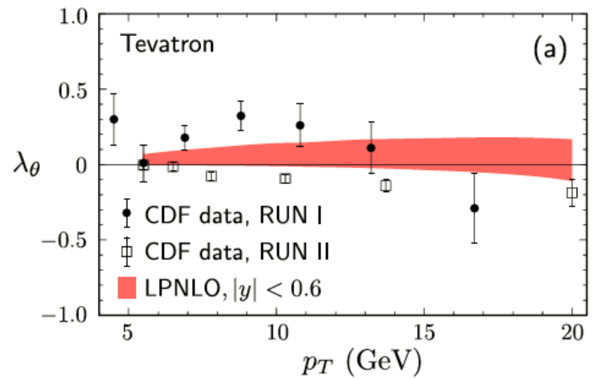
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

Pheno difficulty: polarization puzzle

- J/ψ at NLO: transverse polarization largely canceled (***natural?***) between ${}^3S_1^{[8]}$ and ${}^3\mathrm{P}_J^{[8]}$

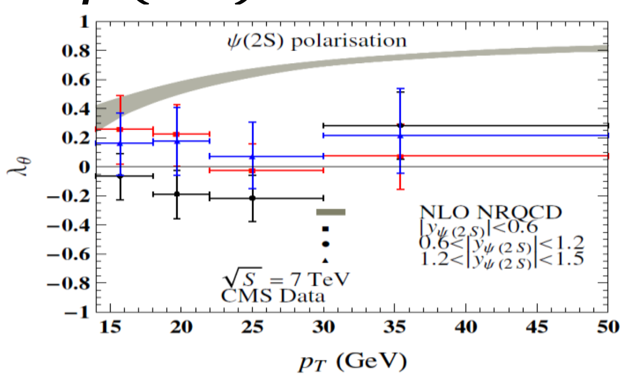


Chao,YQM,Shao,Wang, Zhang,1201.2675

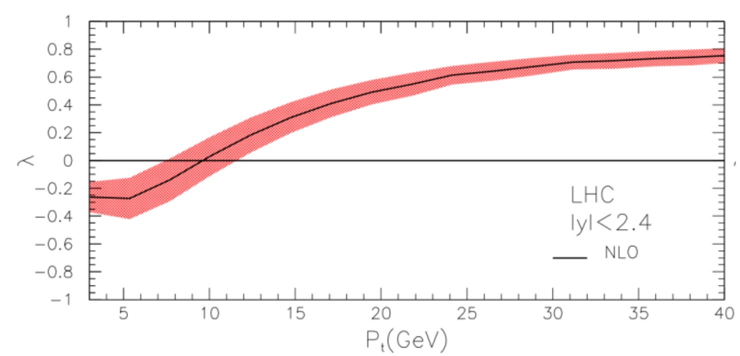


Bodwin, Chung, Kim, Lee, 1403.3612

- $\psi(2S)$: cancelation weak, hard to understand



Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



Gong, Wan, Wang, Zhang, 1205.6682

NLO fit by PKU group

- **Fit J/ψ yield data at Tevatron with $p_T > 7$ GeV**

YQM, Wang, Chao, 1009.3655

  - Due to p_T^{-4} and p_T^{-6} behaviors, constrain two combinations
  - $\mathrm{M}_0 = \langle O\left({}^1\mathrm{S}_0^{[8]}\right)\rangle + 3.9\,\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx (7.4 \pm 1.9)\times 10^{-2}\mathrm{GeV}^3$
  - $\mathrm{M}_1 = \langle O\left({}^3\mathrm{S}_1^{[8]}\right)\rangle - 0.56\,\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx (0.05 \pm 0.02)\times 10^{-2}\ \mathrm{GeV}^3$

- **Two orders difference: hierarchy problem**
  - Velocity scaling rule of NRQCD

$$\langle O\left({}^1\mathrm{S}_0^{[8]}\right)\rangle \sim \langle O\left({}^3\mathrm{S}_1^{[8]}\right)\rangle \sim \langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/\mathrm{m}_c^2$$

  - Thus natural expectation: $M_0 \sim M_1$

- **Upper bound from Belle total cross section**

Zhang, YQM, Wang, Chao, 0911.2166

$$M_0 < 0.02\mathrm{GeV}^3$$

  - No universality of NRQCD LDMEs!

NLO fit by Hamburg group

➢ NLO NRQCD V.S. Global data

Butenschoen, Kniehl, 1105.0820

- Including Belle, LEP, HERA, RHIC, Tevatron, LHC
- Total of 194 data points from 26 data sets
- Exclude $p_T < 3\ GeV$ pp data and $p_T < 1\ GeV$ ep data

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32\ \text{GeV}^3 \qquad \chi^2_{\text{d.o.f.}} = 725/194 = 3.74$$

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}\ \text{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}\ \text{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\ \text{GeV}^5$

- Data are not well described by NLO NRQCD, especially Belle data

Fit by IHEP group

➢ Fit J/ψ yield data at Tevatron and LHC

Gong, Wan, Wang, Zhang, 1205.6682

- **Exclude $p_T < 7\ GeV$ pp data**

$$(\langle\mathcal{O}(^1S_0^{[8]})\rangle, \langle\mathcal{O}(^3S_1^{[8]})\rangle, \frac{\langle\mathcal{O}(^3P_0^{[8]})\rangle}{m_c^2}) \equiv \frac{\mathcal{O}}{100}\ \mathrm{GeV}^3$$

$$\mathcal{O} = (9.7 \pm 0.9, -0.46 \pm 0.13, -0.95 \pm 0.25)$$

➢ Results of the three groups: no universality

- $\mathrm{M}_0 \approx 7.4 \pm 1.9$
- $\mathrm{M}_1 \approx 0.05 \pm 0.02$

PKU

$(\times 10^{-2}\mathrm{GeV}^3)$

- $\mathrm{M}_0 \approx 2.1 \pm 0.56$
- $\mathrm{M}_1 \approx 2.6 \pm 0.59$

Hamburg

- $\mathrm{M}_0 \approx 8.1 \pm 1.0$
- $\mathrm{M}_1 \approx -0.22 \pm 0.14$

IHEP

➢ $\gamma + \gamma^* \to \eta_c$: NNLO fails to describe data

Feng, Jia, Sang, 1505.02665

Current status

➢ Phenomenological: difficulties

- NLO NRQCD theory has difficulty to describe global data: polarization puzzle, hierarchy problem, universality problem

(Warning: Problems with η_c and $\Upsilon(nS)$ production not discussed)

➢ Theoretical: rigorousness

- Based on EFT of QCD: NRQCD
- Factorization has been tested to NNLO

Nayak, Qiu, Sterman, 0509021
Bodwin, Chung, Ee, Kim, Lee, 1910.05497
Zhang, Meng, YQM, Chao, 2011.04905

➢ What is missing?

- Remember: summation over all possible n in NRQCD formula
- Corrections at high powers in v (relativistic corrections)!

Outline

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V. Summary and outlook

Relativistic corrections in NRQCD

➢ Relativistic (power) corrections

- Equations of motion of NRQCD EFT: $\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$
- NRQCD factorization: use EOM to remove ∇_0, leaving operators like:

(Warning: here D replaced by ∇, needs proper gluon fields to make them gauge invariant)

$$\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$$

Type 0: Different colors, spins

$$\chi^\dagger gE^i\psi$$

Type 1: Intrinsic gluons (E, B fields)

$$\chi^\dagger \overleftrightarrow{\nabla}^i\psi$$

Type 2: Orbital angular momentum

$$\chi^\dagger \overleftrightarrow{\nabla}^2\psi$$

Type 3: Relative momentum

$$\nabla^i\left(\chi^\dagger\psi\right)$$

Type 4: Total momentum

Corrections in type 3 widely studied, for charmonium production in pp collision, about 30%-50% corrections

CS-channel:
Fan, YQM, Chao, 0904.4025

CO-channel:
Xu, Li, Liu, Zhang,1203.0207

S-D mixing-channel:
He, Kniehl, 1507.03882

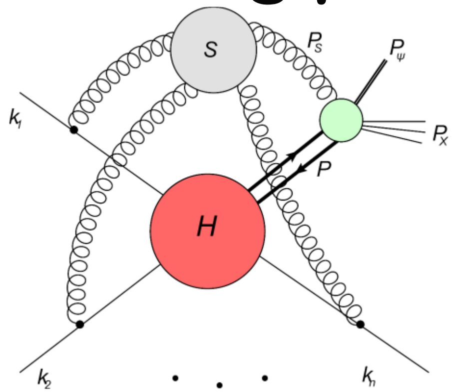
LP in p_T, all order in v:
Li, Chen, Huang, YQM, 1909.03554

However, more relativistic-correction terms may be needed!

Soft gluon emission

➤ Soft gluon emission in color-bleaching process

- P_ψ is different from P, $P = P_\psi[1 + O(\lambda)]$
- NRQCD expand P around P_ψ



YQM, Vogt, 1609.06042

➤ Bad convergence of NRQCD expansion

- Cross section approximately $\propto P^{-4} = P_\psi^{-4}[1 + O(\lambda)]^{-4}$

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

With $\lambda \approx v^2 \approx 0.3$

Mangano, Petrelli, 9610364

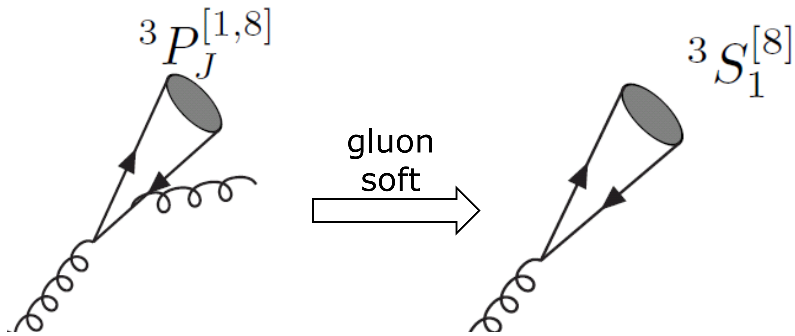
- Solution: soft gluon momentum should be kept but not expanded, which means to resum relativistic corrections (due to kinematic effects) to all powers in v!

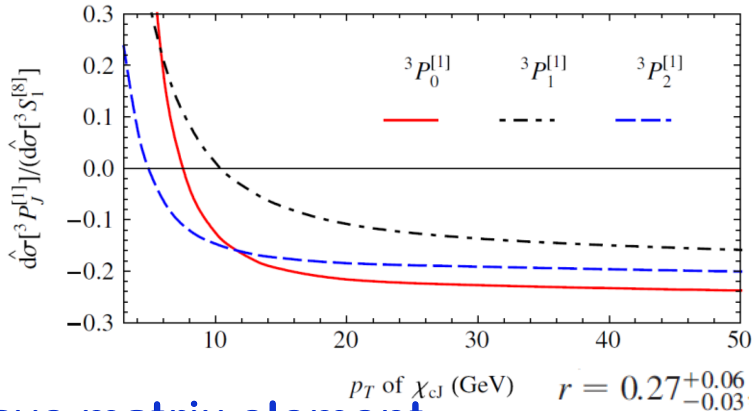
Over subtraction

Braaten, Chen, 9610401
YQM, Wang, Chao, 1002.3987

- **Eg. χ_{cJ} production:**

$$d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left({}^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left({}^3S_1^{[8]}\right)\rangle$$





$r = 0.27^{+0.06}_{-0.03}$

- **Soft gluon in P-wave: factorized to S-wave matrix element**
- **Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted! P-wave negative!**
- **Big cancellation between S-wave and P-wave! Perturbation unstable**
- **Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.**

Threshold region

➢ At threshold region

- Large logarithms appear: can be resummed by introducing shape functions

Beneke, Rothstein, Wise, 9705286
Fleming, Leibovich, Mehen, 0306139
Leibovich, Liu, 0705.3230

- Soft gluon momentum: has leading contribution for quarkonium momentum distribution, cannot be ignored

➢ Combination of logs resummation and powers resummation is needed

  - Keep soft gluon momentum unexpanded is the first step.

Comments

- Relativistic corrections with fixed power in v
  - Bad convergence, too many terms are needed
  - Involves too many LDMEs, very hard to fix them
  - Solution: resum all LDMEs to obtain a function! (Like resum twist-2 local operators to obtain PDFs)
- What do we need to resum?

- Type 0 ($\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$): finite number, can be studied exclusively
- Type 1-2 insertion ($\chi^\dagger gE^i\psi$, $\chi^\dagger \overleftrightarrow{\nabla}^i\psi$): usually not enhanced, less important, do not need to resum
- Type 3 and 4 insertion ($\chi^\dagger \overleftrightarrow{\nabla}^2\psi$, $\nabla^i\left(\chi^\dagger\psi\right)$) : kinematic effects, enhanced if the observable has a steep distribution. E.g., p_T distribution in pp collision, momentum distribution in endpoint region. Need to resum

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Derivation of SGF: exclusive processes

➢ A different way to use EOM

Li, Feng, YQM, 1911.05886
Chen, YQM, 2005.08786

- NRQCD factorization: use EOM to remove ∇_0
- SGF: remove relative derivatives $\overleftrightarrow{\nabla}_0, \overleftrightarrow{\nabla}^2$, leaving only total derivatives:

$$\langle R^H_{nn_1n_2}\rangle = \langle 0|\nabla_0^{n_1}\nabla^{n_2}(\chi^\dagger \mathrm{K}_n\psi)|H\rangle$$

K_n denotes operators for type 0-2 insertions

- Assume: factorization (similar to NRQCD factorization) is valid to all orders:

$$A^H = \sum_{n,n_1,n_2} \hat{A}_{nn_1n_2}(P_H)\langle R^H_{nn_1n_2}\rangle$$

➢ Using integration by parts

- Remove operators unless $n_1 = n_2 = 0$
- H rest frame: matching coefficients are functions of quarkonium mass

➢ Factorization

$$\mathcal{A}^Q = \sum_n \hat{\mathcal{A}}^n \overline{R}_Q^{n*} \qquad \overline{R}_Q^{n*} = \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi](0)|Q\rangle_S$$

"S": field operators are in small momentum regions

Derivation of SGF: inclusive processes

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

➢ Use EOM to remove relative derivatives

Resulting: $\langle O^{H+X}_{nn_1n_2n_3n_4}\rangle = \langle 0\left|\nabla_0^{n_1}\nabla^{n_2}\left(\chi^\dagger\widetilde{\mathrm{K}}_n\psi\right)^\dagger\left(a^\dagger_{H+X}a_{H+X}\right)\nabla_0^{n_3}\nabla^{n_4}\left(\chi^\dagger\mathrm{K}_n\psi\right)\right|0\rangle$

where $a^\dagger_{H+X}a_{H+X} \equiv \sum_{J_z^H}|H+X\rangle\langle H+X|$

- **Assume: factorization is valid at this level**

$$(2\pi)^3 2P_H^0\frac{d\sigma_H}{d^3P_H} = \sum_{n,n_1,n_2,n_3,n_4} d\hat{\sigma}_{nn_1n_2n_3n_4}(P_H)\sum_{X,P_X}\langle O^{H+X}_{nn_1n_2n_3n_4}\rangle$$

➢ Use integration by parts

- **Remove operators unless** $n_1 = n_2 = n_3 = n_4 = 0$
- **Matching coefficients are functions of (H rest frame):** $m_H, E_X, \vec{P}_X^2$

  (Warning: $\vec{P}_X\cdot\langle O^{H+X}_{n0000}\rangle$ **is also possible if polarization is concerned)**
- **In a general frame:** $P_H^2, P_H\cdot P_X, P_X^2$

Derivation of SGF: inclusive processes

- **Factorization becomes**

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_{n,X,P_X} f(P_H^2, P_H \cdot P_X, P_X^2)\langle O_{n0000}^{H+X}\rangle$$

$P = P_H + P_X$: **momentum of** $Q\bar{Q}$

- **Soft gluon factorization**

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

Production of a heavy quark pair Expansion in: α_s, (v)	Hadronization (SGDs) Expansion in: v

- $\mathcal{H}_n$: **perturbatively calculable hard parts**
- $F_{n\to H}$: **nonperturbative soft gluon distributions (SGDs)**
- **UV renormalization scale is suppressed**

$$F_{n\to H}(P, P_H) = \int d^4b e^{-iP\cdot b}\langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_{\mathrm{S}}$$

Soft gluon distributions (SGDs)

➢ Operator definition

- **Expectation values of bilocal operators in QCD vacuum**

$$F_{n\to H}(P,P_H)=\int d^4b e^{-iP\cdot b}\langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_{\rm S}$$

with

$$a_H^\dagger a_H=\sum_X\sum_{J_z^H}|H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb)=\frac{\sqrt{M_H}}{M_H+2m}\frac{M_H+\not{P}_H}{2M_H}\Gamma_n\frac{M_H-\not{P}_H}{2M_H}\mathcal{C}^{[c]}$$

Spin project operators: $\Gamma_n=\sum_{L_z,S_z}\langle L,L_z;S,S_z|J,J_z\rangle\Gamma^o_{LL_z}\Gamma^s_{SS_z}$

Color project operators:

$$\mathcal{C}^{[1]}=\frac{\mathbf{1}_c}{\sqrt{N_c}}\qquad\qquad \mathcal{C}^{[8]}=\sqrt{2}t^{\bar{a}}\,\Phi^{(A)}_{a\bar{a}}(rb)$$

Soft gluon distributions (SGDs)

➢ Gauge link

$$\Phi^{(A)}(rb) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, b_\ell\cdot A^{(A)}(r\,b+\lambda\, b_\ell)\right\}$$

$$b_\ell^\mu = b^\mu + \varepsilon\ell^\mu \qquad\qquad 0 < \varepsilon \ll 1$$

- **When b is finite, gauge link along b direction (avoid gauge-link-collinear divergence)**
- **When $b\to 0$, gauge link unambiguously along l direction (agree with gauge-completed NRQCD matrix elements)**

Nayak, Qiu, Sterman, 0509021

➢ Evaluated in *small* region

- **Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass**

RGEs for SGDs

➢ RGEs

Chen, Jin, YQM, Meng, 2103.15121

$$\frac{d}{d\ln\mu_f}F_{[L'\tilde{L}',\lambda']\to H}(z,M_H,m_Q,\mu_f)=\sum_{L,\tilde{L},\lambda}\int_z^1\frac{dx}{x}\boldsymbol{K}^{[L\tilde{L},\lambda]}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)$$
$$\times F_{[L\tilde{L},\lambda]\to H}(x,M_H,m_Q,\mu_f),$$

- **Evolution kernels**

$$\boldsymbol{K}^{[L\tilde{L},\lambda],LO}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)=\frac{d}{d\ln\mu_f}F^{NLO}_{[L'\tilde{L}',\lambda']\to Q\bar{Q}[L\tilde{L},\lambda]}(\hat{z},M_H/x,m_Q,\mu_f).$$

$$\boldsymbol{K}^{[SS],LO}_{[SS]}(z,M_H,m_Q,\mu_f)=\frac{\alpha_s}{\pi}\Bigg\{N_c\Bigg[\frac{2z}{(1-z)_+}-\ln\frac{\mu^2e^{-1}}{M_H^2}\delta(1-z)$$
$$-2\delta(1-z)\left(\frac{1}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\Bigg]+\frac{1}{N_c}\left(\frac{1+\Delta^2}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\delta(1-z)\Bigg\}.$$

$$\Delta=\frac{\sqrt{M_H^2-4m_Q^2}}{M_H}$$

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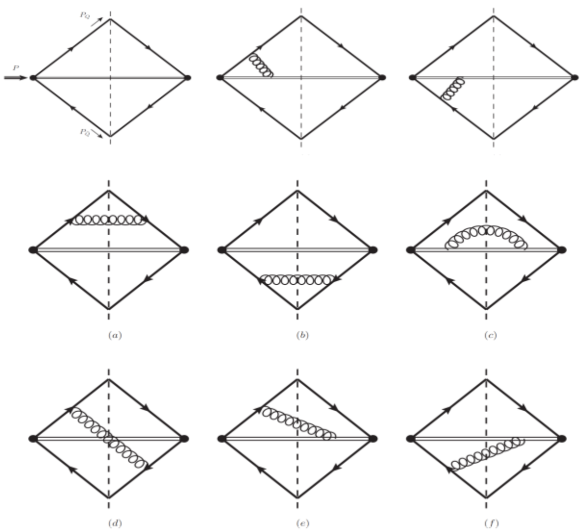
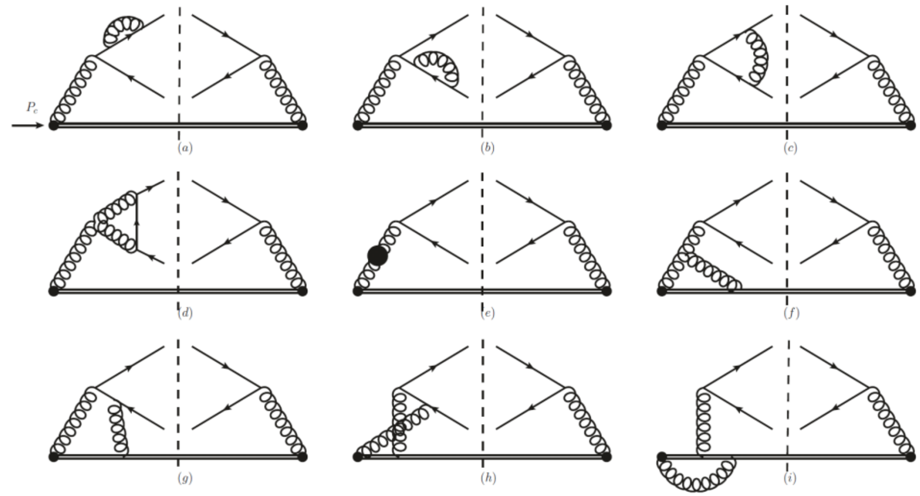
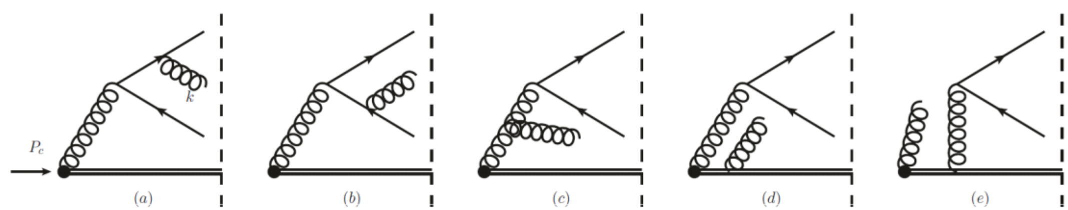
FF: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$ up to NLO

- **Feynman diagrams**

Chen, Jin, YQM, Meng, 2103.15121







Hard part for $g \to Q\overline{Q}({}^3S_1^{[8]}) + X$

➢ NRQCD

$$\hat{d}^{(2)}_{g\to {}^3S_1^{[8]}} = \frac{1}{12C_F}\Big[A(\mu_0)\delta(1-z) + \frac{1}{N_c}P_{gg}(z)\Big(\ln(\frac{\mu_0^2}{4m_Q^2}) - 1\Big) + \frac{2(1-z)}{z} - \frac{4(1-z+z^2)^2}{z}\Big(\frac{\ln(1-z)}{1-z}\Big)_+\Big],$$

Braaten, Lee, 0004228
YQM, Qiu, Zhang, 1311.7078

- **Double logs as $z \to 1$ (threshold logs)**

➢ SGF

Chen, Jin, YQM, Meng, 2103.15121

$$\hat{D}^{LO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8x^3}{M_H^3}\delta(1-\hat{z}), \quad (5.28a)$$

$$\begin{aligned}
&\hat{D}^{NLO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) \\
&= \frac{4\alpha_s^2 N_c x^3}{(N_c^2-1)M_H^3}\Big[\frac{1}{2}\delta(1-\hat{z})\Big(2A(\mu, M_H/x) + \frac{2\beta_0}{N_c}\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) + \ln^2\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) \\
&\quad + \frac{\pi^2}{6} - 1\Big) + \frac{1}{N_c}P^{(0)}_{gg}(\hat{z})\ln\Big(\frac{\mu^2}{\mu_f^2}\Big) + \Big(\frac{2(1-\hat{z})}{\hat{z}} + \hat{z}(4+2\hat{z}^2) + \frac{2\hat{z}^4}{9}(5+\hat{z})\Big) \\
&\quad \times\Big(\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) - 2\ln(1-\hat{z})\Big) + \frac{2(1-\hat{z})}{\hat{z}} - \Big(\frac{4\hat{z}^4}{1-\hat{z}} - \frac{4\hat{z}^4}{9}(5+\hat{z})\Big)\ln\hat{z}\Big]. \quad (5.28b)
\end{aligned}$$

- **No threshold logs in hard part**
- **Logs are factorized to SGDs and then resummed by using REGs**

Nonperturbative models

➢ The first class of models

$$F^{\rm mod}(\omega') = M_H N_H \frac{b^b}{\Gamma(b)} \frac{\omega'^{b-1}}{\bar\Lambda^b} e^{-b\omega'/\bar\Lambda}, \quad \omega' = M_H(1/x-1),$$ Fleming, Leibovich, Mehen, 0306139

$$\text{Model-1:} \quad F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=2}, \qquad \text{Model-2:} \quad F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=1},$$

$$\text{Model-3:} \quad F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=3}, \qquad \text{Model-4:} \quad F^{\rm mod}(\omega')|_{\bar\Lambda=0.5{\rm GeV},b=2},$$

$$\text{Model-5:} \quad F^{\rm mod}(\omega')|_{\bar\Lambda=0.7{\rm GeV},b=2},$$

the zeroth, first and second moments are $M_H N_H$, $M_H N_H \bar\Lambda$ and $M_H N_H \bar\Lambda^2(\frac{1}{b}+1)$

➢ The other models

$$\text{Model-6:} \quad 4M_H N_H[\theta(w' \geq \frac{19}{40}) - \theta(w' > \frac{29}{40})],$$

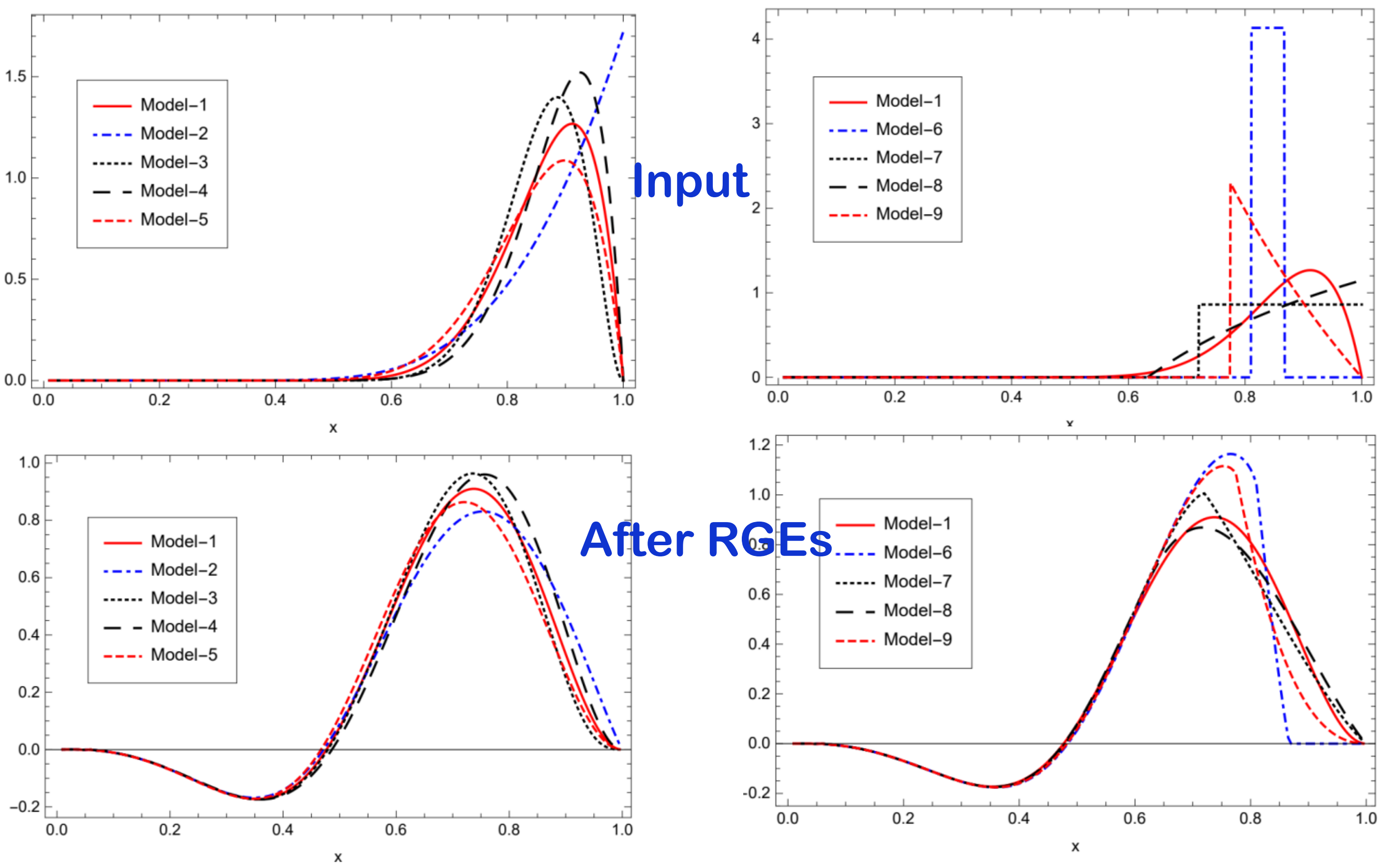
$$\text{Model-7:} \quad \frac{5}{6} M_H N_H[\theta(w' \geq 0) - \theta(w' > \frac{6}{5})],$$

$$\text{Model-8:} \quad \begin{cases} M_H N_H(-\frac{50}{81}w' + \frac{10}{9}), & 0 \leq w' \leq \frac{9}{5}, \\ 0, & w' > \frac{9}{5}, \end{cases}$$

$$\text{Model-9:} \quad \begin{cases} \frac{200}{81} M_H N_H w', & 0 \leq w' \leq \frac{9}{10}, \\ 0, & w' > \frac{9}{10}. \end{cases}$$

RGEs effects

- **Model dependence is significantly reduced after using RGEs**



Numerical: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

➢ Gluon FFs in ${}^3S_1^{[8]}$ channel

Chen, Jin, YQM, Meng, 2103.15121

$\bar{\Lambda}$: average momentum emitted

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$

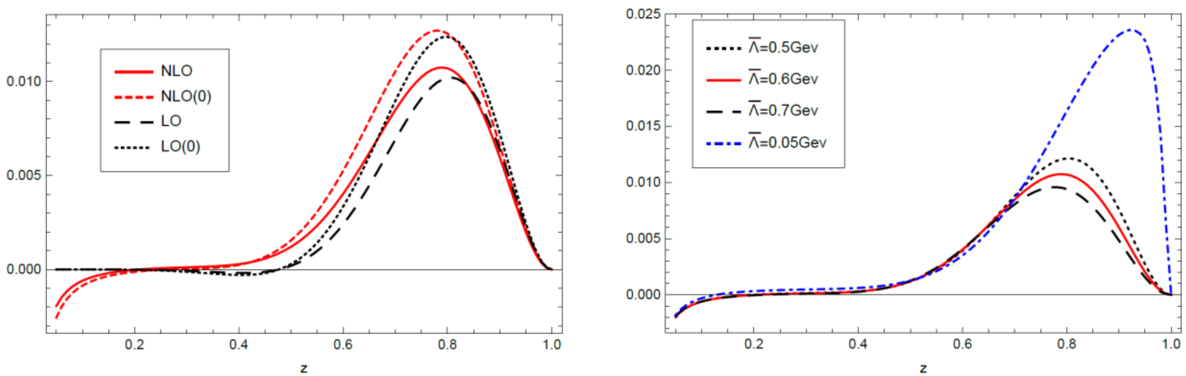
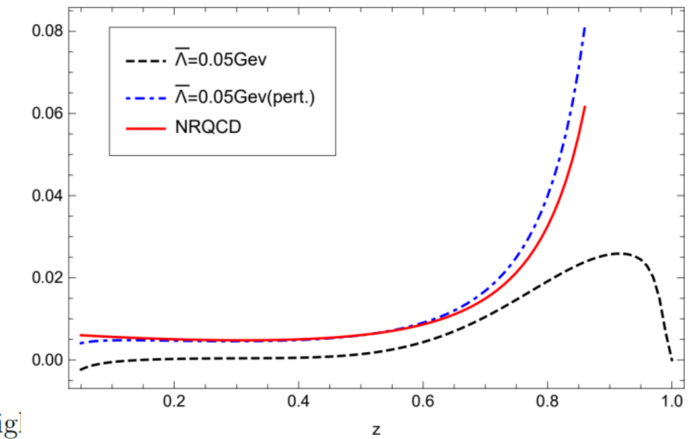


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig[ht]
figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.

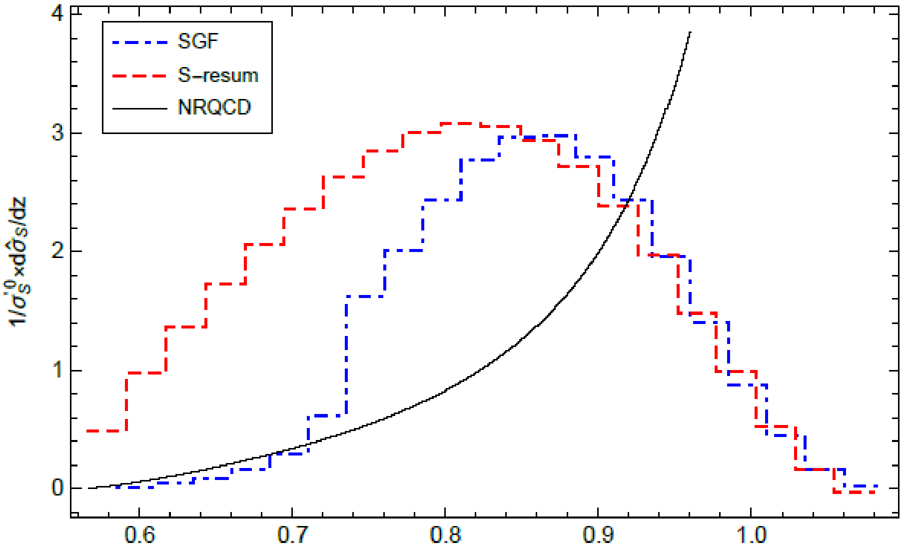


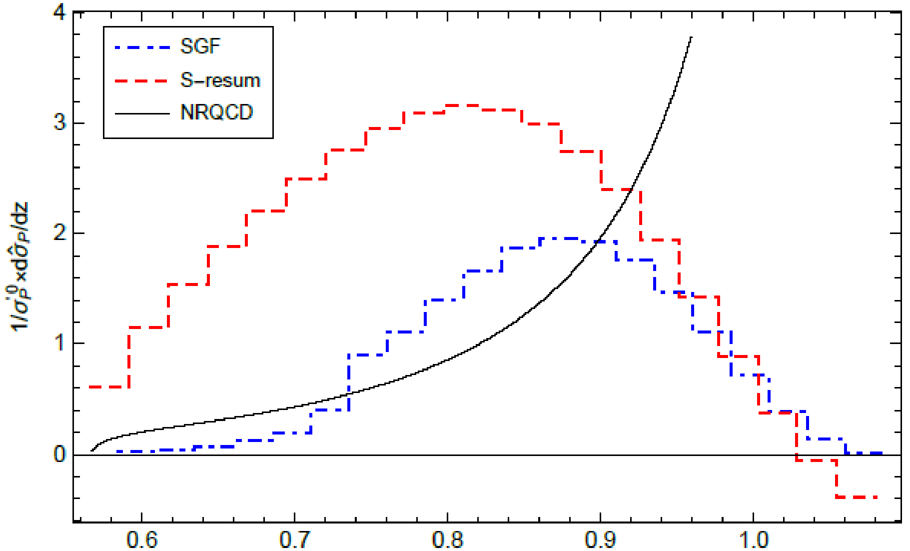
$$R^X(n) \equiv \frac{\int_0^1 dz\, z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz\, z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

$R^{NRQCD} \approx 6$

J/ψ production at B factories

Chen, Jin, YQM, Meng, in preparation





	NRQCD	NLO+NLL	$\mathcal{S}$-resum	SGF
$\hat{\sigma}_S^{\rm NLO}$ (pb/GeV^3)	17.677	7.411	8.246	5.712
$\hat{\sigma}_P^{\rm NLO}$ (pb/GeV^5)	30.370	12.691	14.096	6.375

- **Similar to SCET+NRQCD at large z, different at small z**
- **Has smaller total cross section, easier to reconcile with pp data**

Summary

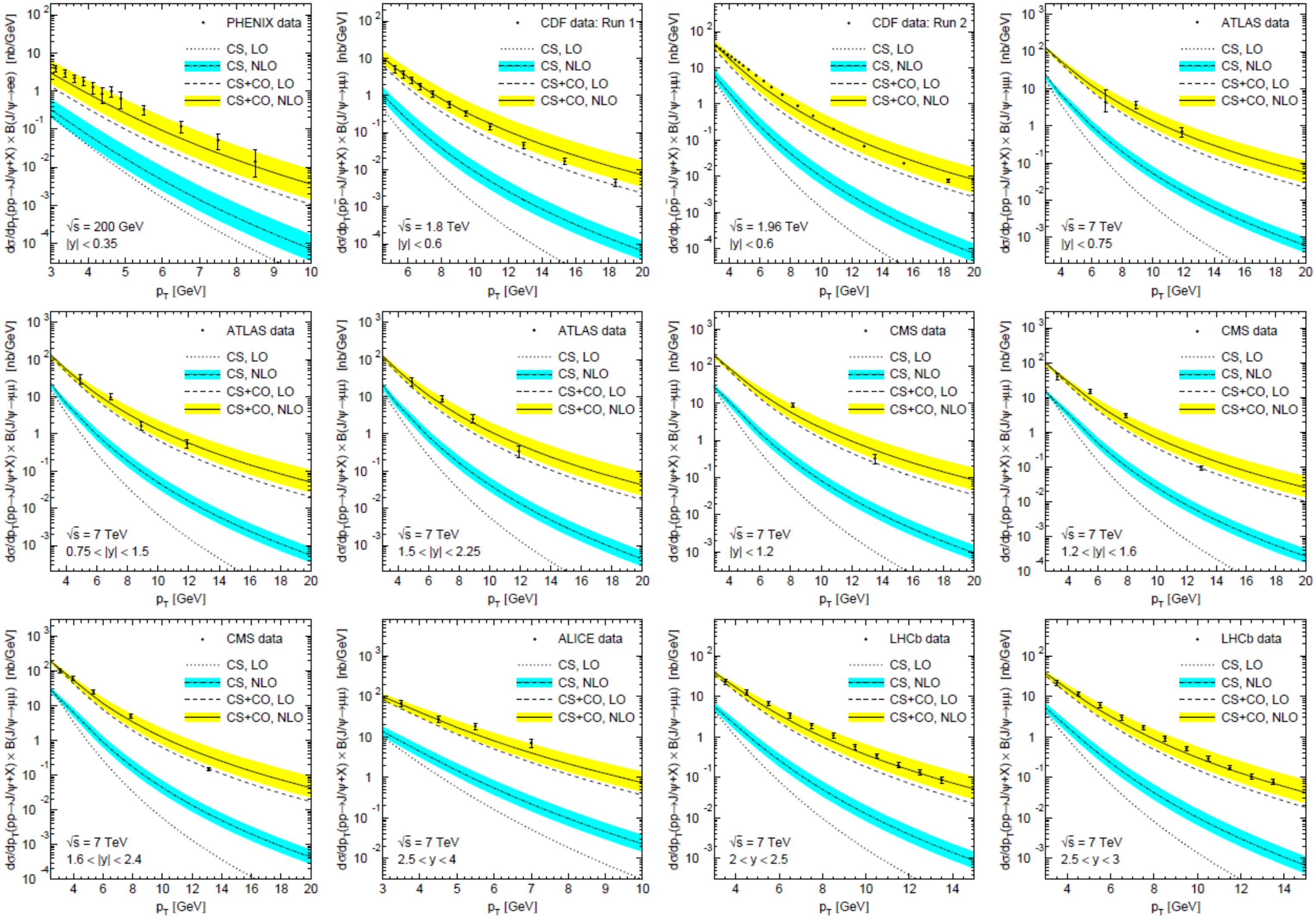
- **NRQCD factorization: polarization puzzle, hierarchy problem, universality problem**
  - Possible reason: convergence of v^2 expansion is bad because of soft gluon emission
- **Soft gluon factorization (SGF)**
  - Soft gluons effects considered; better convergence in v^2 expansion
  - Equivalent to NRQCD, but with power corrections originated from kinematic effects resummed (kinematical factorization), (partial) large logs resummed
  - Phenomenological difficulties encountered in NRQCD: to be studied in SGF
  - Soft gluons effects in other processes, like B production.

Thank you!

Global fit by Butenschoen and Kniehl

➢ NLO NRQCD V.S. RHIC, Tevatro, LHC data

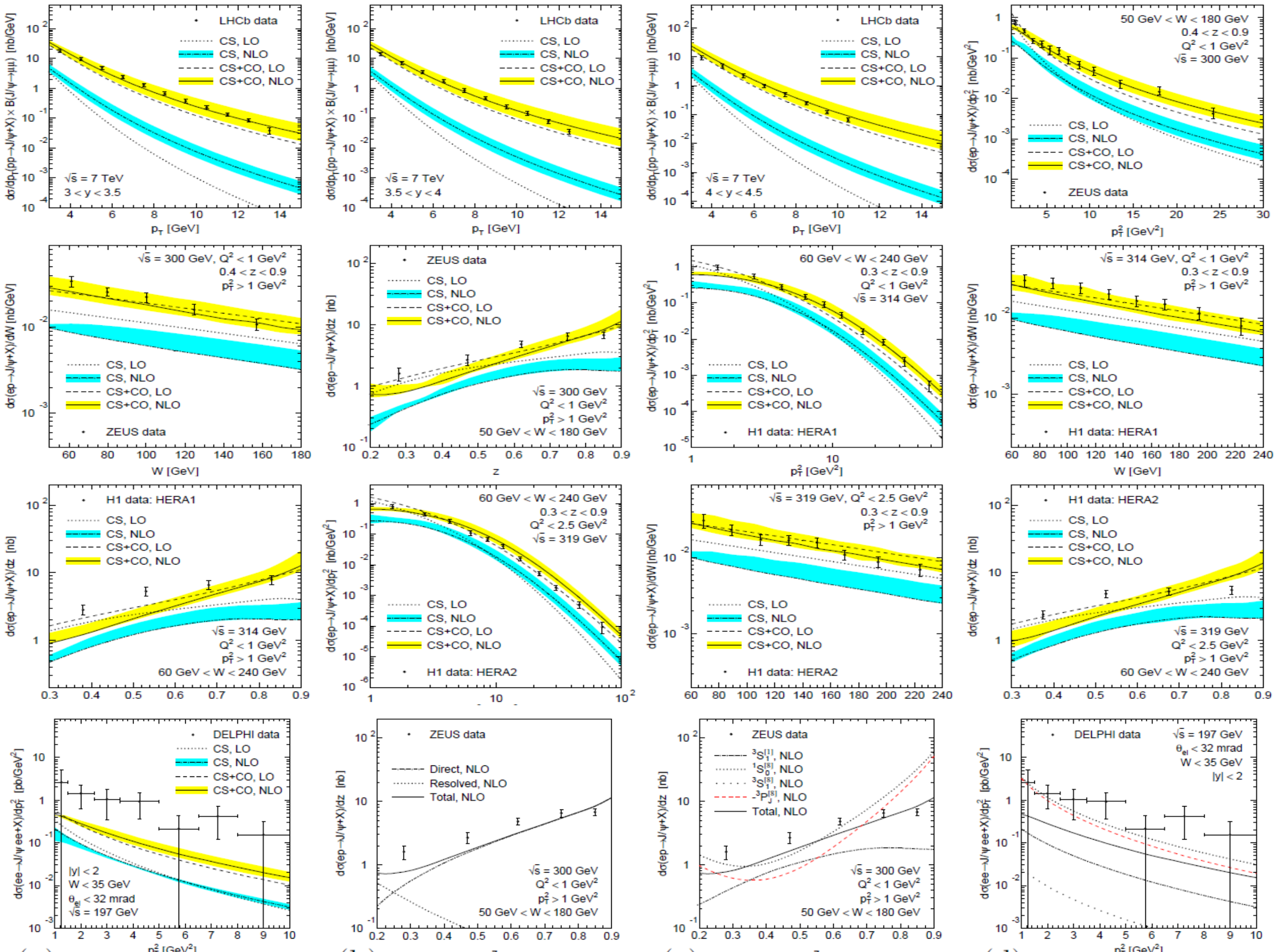
Butenschoen, Kniehl, 1105.0820



Global fit by Butenschoen and Kniehl

➢ NLO NRQCD V.S. LHC, HERA, LEP data

Butenschoen, Kniehl, 1105.0820



(a) (b) (c) (d)

NLO fit by ANL-Korea group

- Fit J/ψ yield data at Tevatron and LHC

Bodwin, Chung, Kim, Lee, 1403.3612

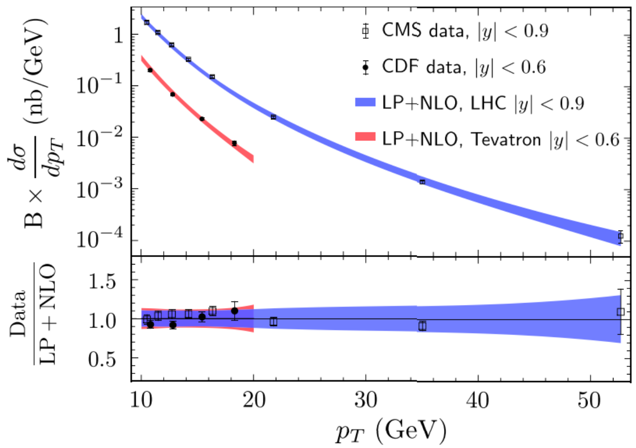
- Exclude $p_T < 10\ GeV$ data
- Large logs at LP in $1/p_T^2$ expansion are resumed

$$\frac{d\sigma^{\mathrm{LP+NLO}}}{dp_T} = \frac{d\sigma^{\mathrm{LP}}}{dp_T} - \frac{d\sigma^{\mathrm{LP}}_{\mathrm{NLO}}}{dp_T} + \frac{d\sigma_{\mathrm{NLO}}}{dp_T}$$

$$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle = -0.030 \pm 0.381\ \mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle = 0.023 \pm 0.057\ \mathrm{GeV}^3$$

$$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle = 0.043 \pm 0.106\ \mathrm{GeV}^5$$



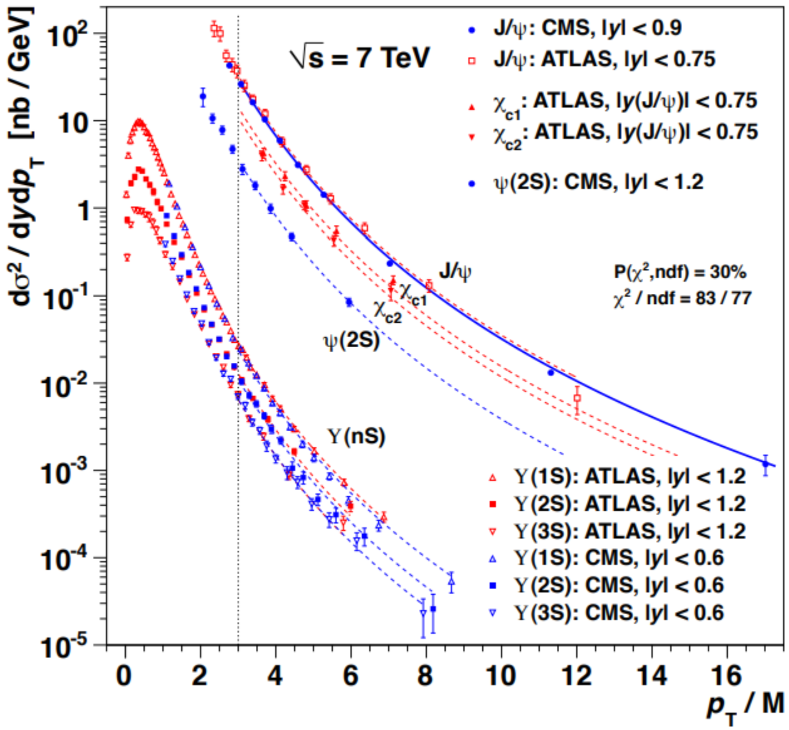
- Good fit, yet another different set of LDMEs

Data driven by CERN et. al. group

➢ Fit J/ψ yield data at Tevatron and LHC

Faccioli, Knunz, Lourenco, Seixas, Wohri, 1403.3970

- Similar shape as functions of p_T/M



- Ignoring ${}^3P_J^{[8]}$ contributions, ${}^1S_0^{[8]}$ dominance