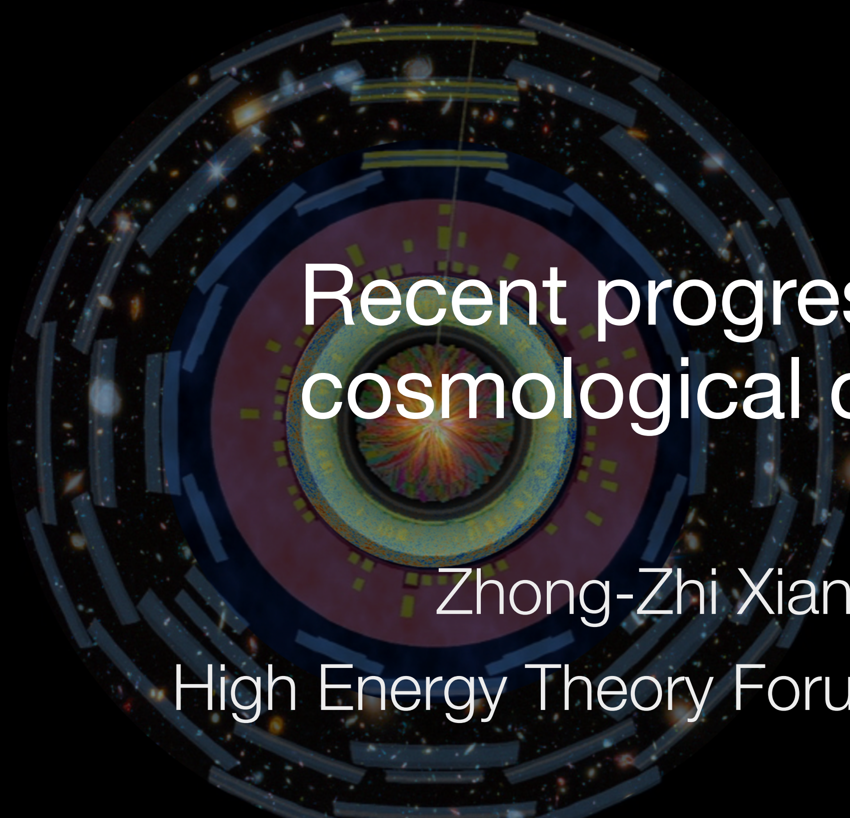


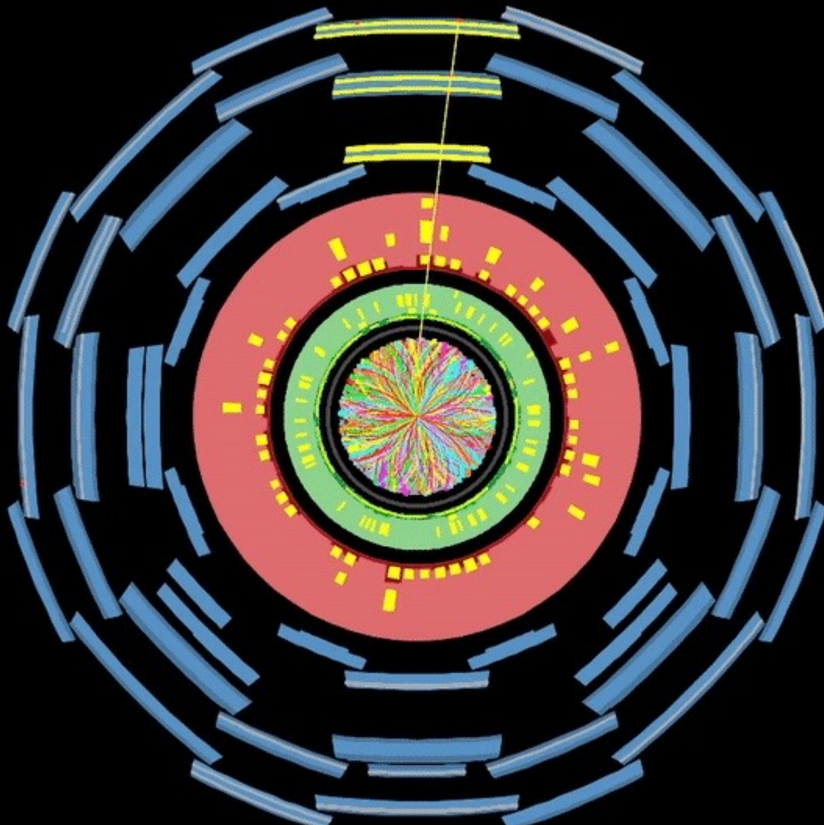
Recent progress in cosmological collider physics

Zhong-Zhi Xianyu (Tsinghua)

High Energy Theory Forum, IHEP | Nov 3, 2021

w/ Haipeng An, Xingang Chen, Yanou Cui, Tao Liu, Qianshu Lu, Shiyun Lu, Matthew Reece, Xi Tong, Yi Wang, Liantao Wang, Chen Yang, Yiming Zhong

JHEP 08 (2016) 051; PRL 118 (2017) 261302; JHEP 04 (2017) 058; JCAP 12 (2017) 006; JCAP 05 (2018) 049; JHEP 09 (2018) 022; JHEP 02 (2020) 011; JHEP 02 (2020) 044; JHEP 04 (2020) 189; JHEP 11 (2020) 082; 2108.11385; 2109.14635, ongoing projects

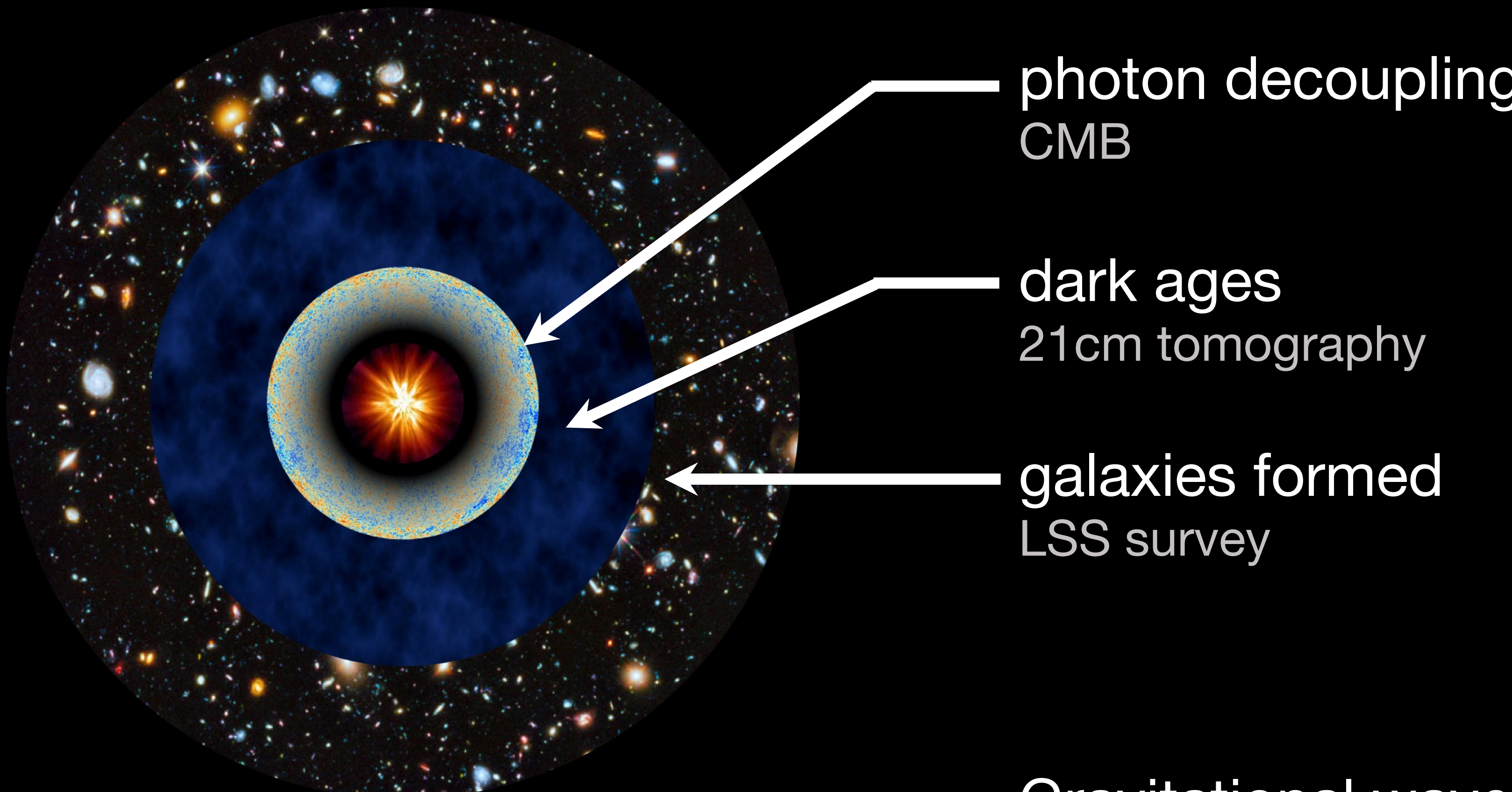


Large Hadron Collider

ATLAS detector

Cosmological Collider

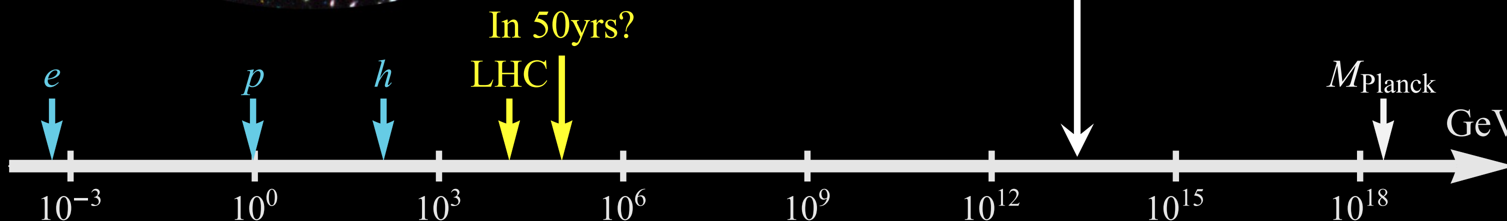
The universe



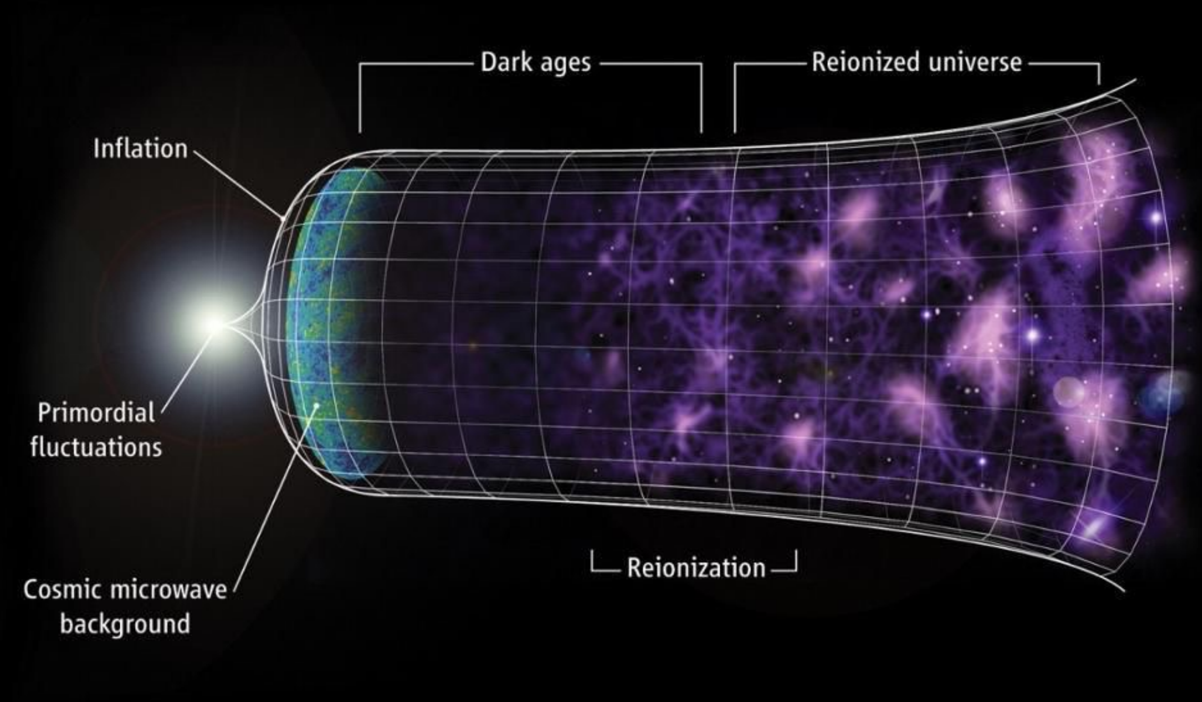
Gravitational waves

Cosmological Collider

The universe



Cosmic inflation: the engine of cosmic collider

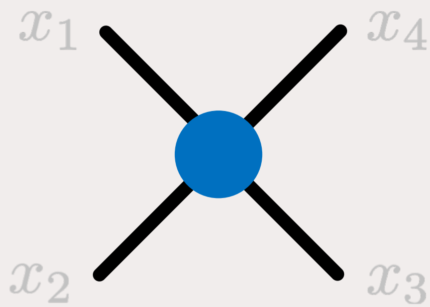


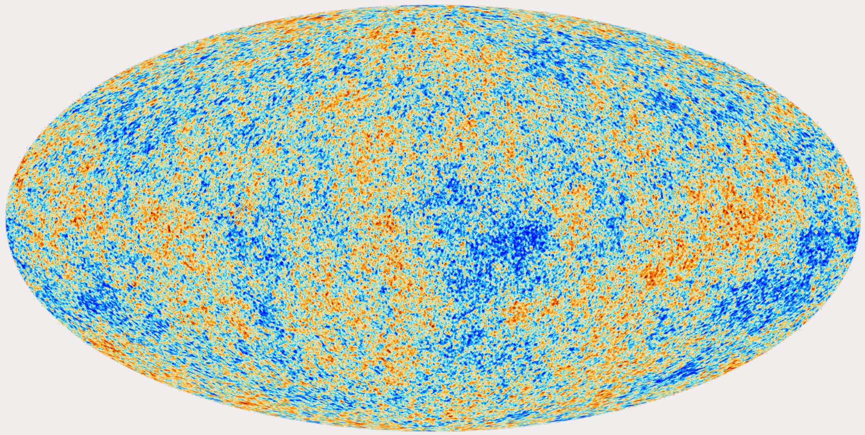
A period of exponentially fast expansion
Within down to 10^{-36} s, the size increased by up to 10^{26}

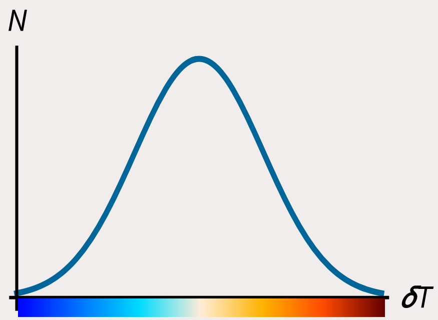
The quantum fluctuations of spacetime shape us all

How to extract more information from CMB map?

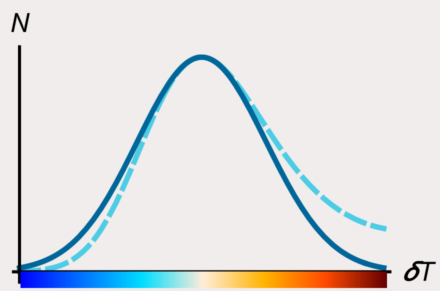
$$\langle \delta T(x_1) \cdots \delta T(x_n) \rangle$$



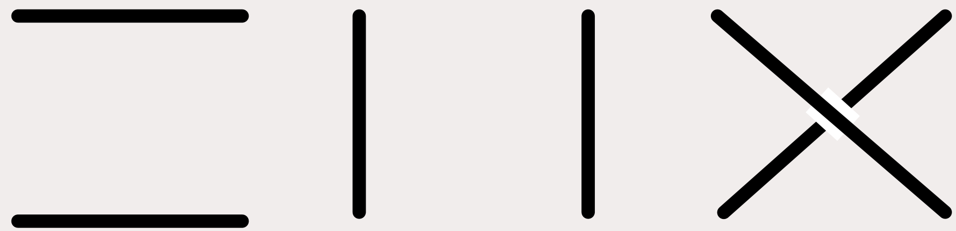


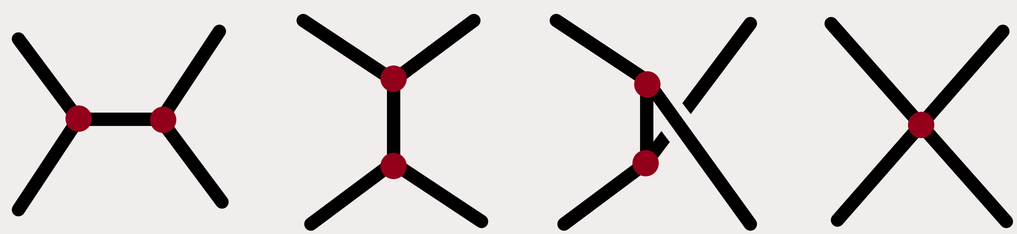


Gaussian

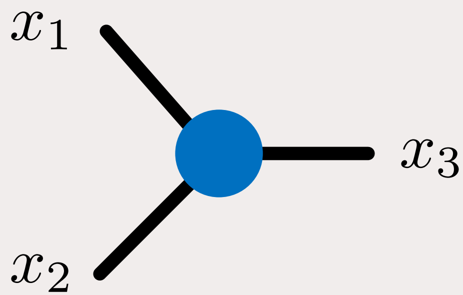


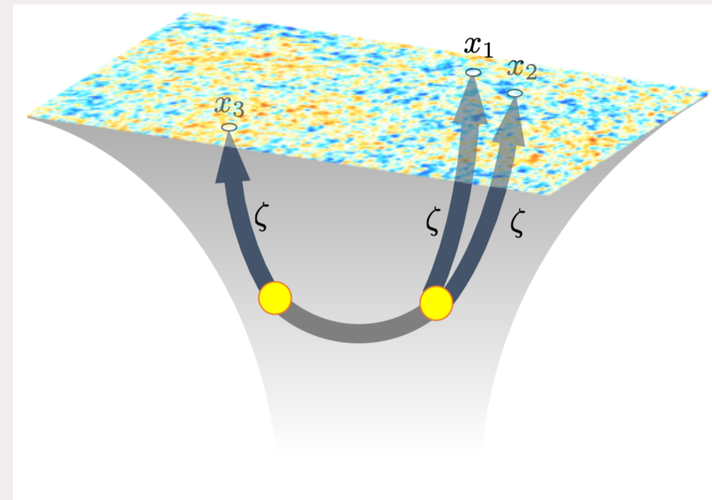
Non-Gaussian





“Non-Gaussianity”

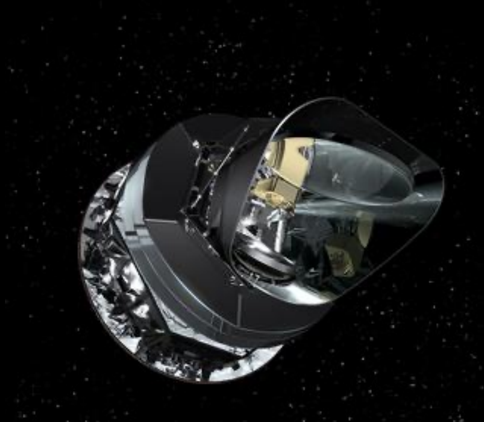




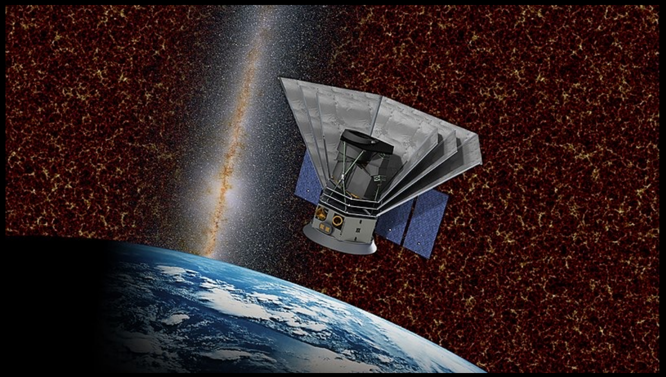
Non-Gaussianity ~ interaction

The size measured by a dimensionless number $f_{\rm NL}$

Observational prospects



Planck: final data release in 2018



SPHEREx: selected by NASA in 2019, launching in ~2024

Planck 2018

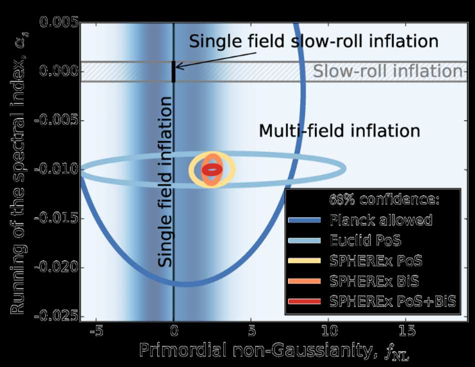
1905.05697

$$f_{\rm NL}^{\rm (local)} = -0.9 \pm 5.1$$

$$f_{\rm NL}^{\rm (equil)} = -26 \pm 47$$

$$f_{\rm NL}^{\rm (ortho)} = -38 \pm 24$$

O(1) in ~10yrs?



SPHEREx, 1412.4872

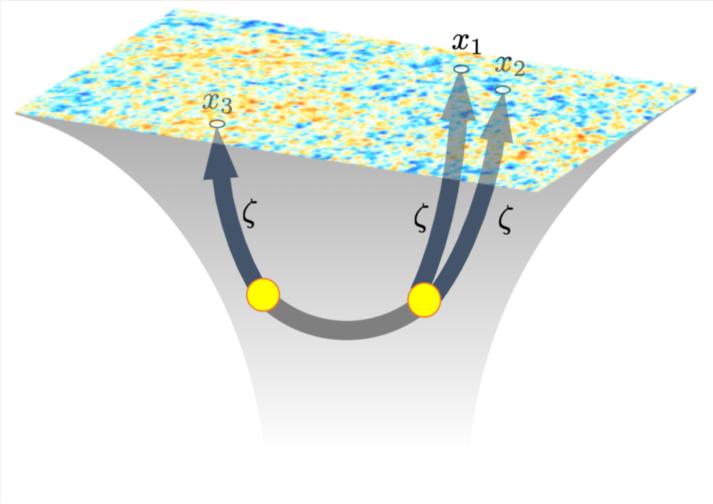
O(0.01) ultimately
21cm tomography

Meerburg, Muñoz, Ali-Haïmoud, Kamionkowski, 1506.04152; Münchmeyer, Muñoz, Chen, 1610.06559; Dizgah, Lee, Muñoz, Dvorkin 1801.07265;

Discover new heavy particles

When massive particles are produced, the inflation did two things:

1. Dilute the number density
2. Exhaust the momentum, so that the particle quickly becomes nonrelativistic



$$\sigma(t) \sim \left(e^{-imt} + e^{-\pi m/H} e^{+imt}\right) e^{-\frac{3}{2}Ht}$$

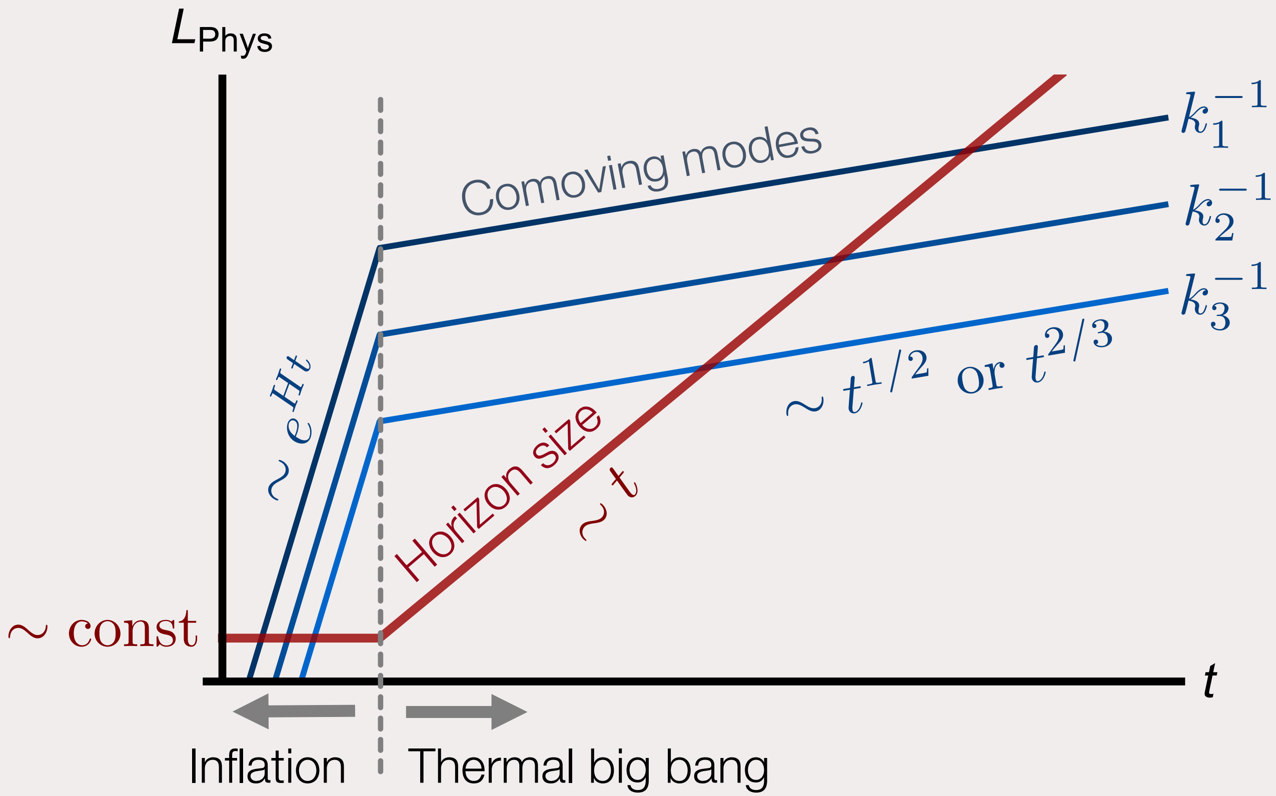
Boltzmann factor

Negative frequency mode; particle production

Comoving dilution

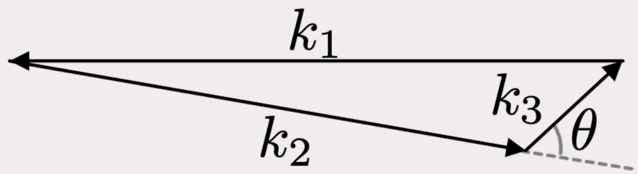
We would be able to measure the mass if we can trace the time dependence.
But we can't. We observe only the final state (CMB)

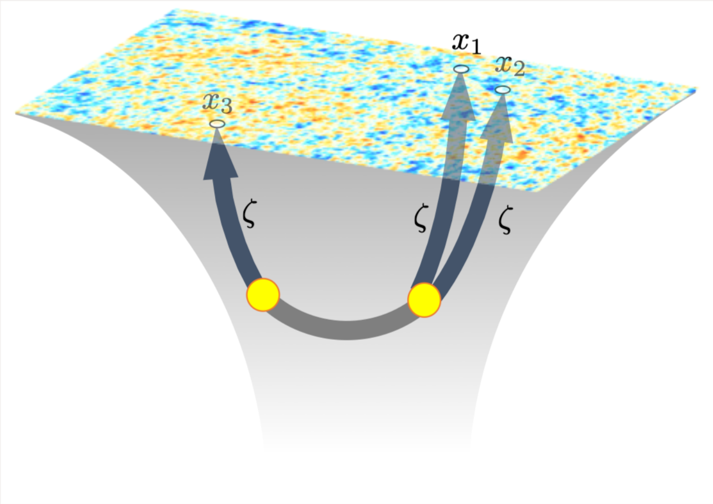
Discover new heavy particles



Discover new heavy particles

A solution: we try to measure the 3-point correlation in the **squeezed limit**





Small-momentum mode redshifts earlier, and oscillates like a nonrelativistic particle when the other two large-momentum modes are still deeply inside the horizon.

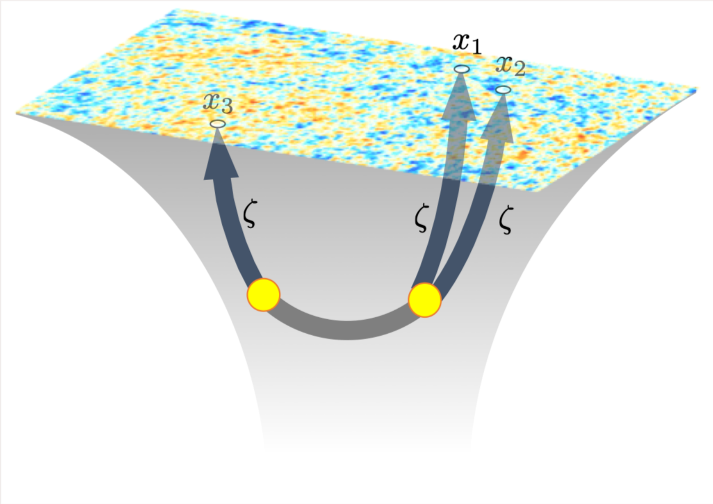
The ratio of long and short momenta is actually a measure of time difference. => Measure the 3pt function at different k ratio ~ measure the mode at different time

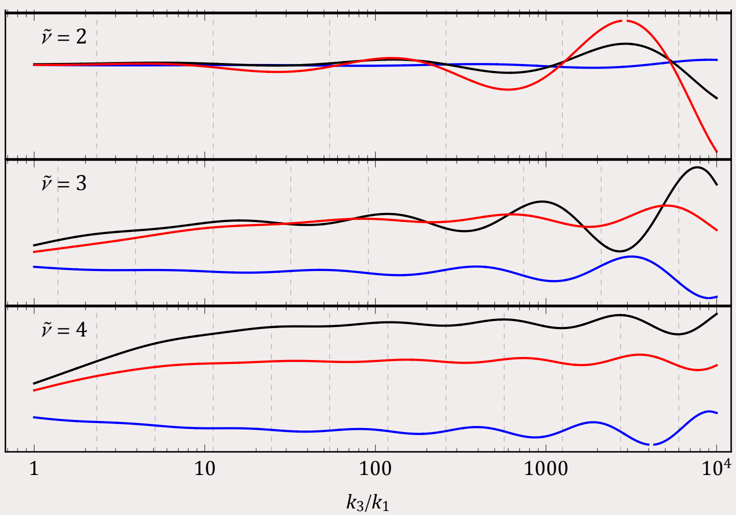
Discover new heavy particles

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m)\left(\frac{k_3}{k_1}\right)^{1/2 \pm \nu} P_s(\cos\theta)$$

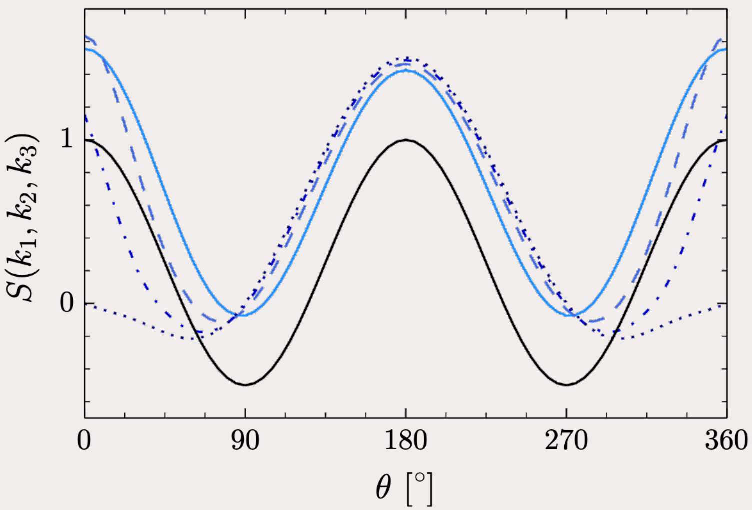
$$\nu = \begin{cases} \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} & s = 0 \\ \sqrt{\left(s - \frac{1}{2}\right)^2 - \frac{m^2}{H^2}} & s \neq 0 \end{cases}$$

Chen, Wang, 0911.3380;1205.0160
Arkani-Hamed, Maldacena, 1503.08043





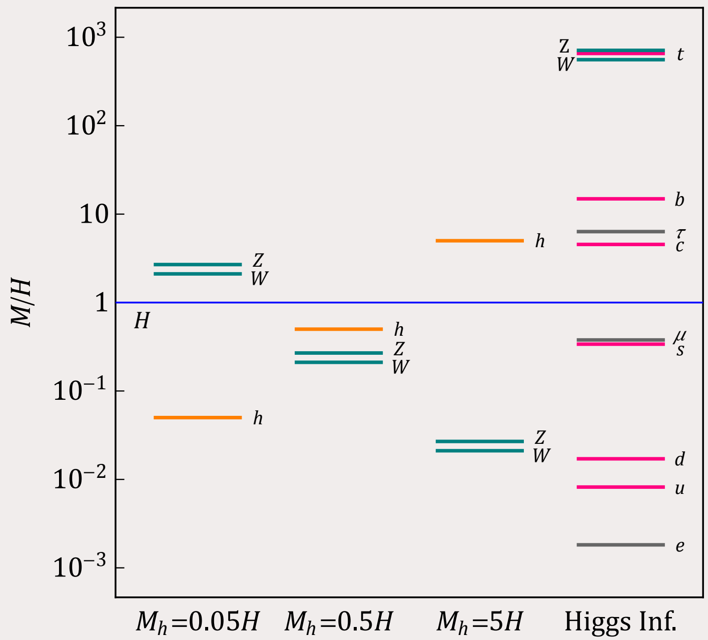
Chen, Chua, Guo, Wang, ZZX, Xie, 1803.04412



Lee, Baumann, Pimentel, 1607.03735

How NOT to use the cosmological collider

$$S(\mathbf{k}_1, \mathbf{k}_3) = A(\lambda, m)\left(\frac{k_3}{k_1}\right)^{1/2\pm\nu} P_s(\cos\theta)$$



Example: "SM background"

"Thermal" mass ~ Hubble

All in loops: spin info lost

Signal size: tiny unless tuned

Xingang Chen, Yi Wang, ZZX, JHEP 1608 (2016) 051; PRL 118 (2017) 261302; JHEP 1704 (2017) 058

Signal size

Challenging to get visible signals in “minimal” scenario

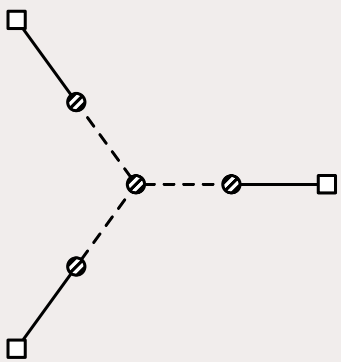
1. Standard slow-roll inflation
2. Scale invariance (up to slow-roll correction)
3. No further spacetime symmetry breaking
4. Dimensionless parameter being O(1)
5. No tree-level tuning

Signal size

$$\frac{1}{\Lambda^2}(\partial_\mu\phi)^2\sigma^2 \longrightarrow \frac{\dot{\phi}_0^2}{\Lambda^2} \sim H^2 \longrightarrow \Lambda \simeq 3600H$$

No Boltzmann supression $\quad \dot{\phi}_0 \simeq (60H)^2$

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \lambda\sigma^4 + \frac{1}{\Lambda^2}(\partial_\mu\phi)^2\sigma^2$$



$$f_{\rm NL} \sim 3600 \cdot \left(\frac{\dot{\phi}_0}{\Lambda^2}\langle\sigma\rangle\right)^3 \cdot \lambda\langle\sigma\rangle \sim 10^{-7} \cdot \lambda\langle\sigma\rangle^4$$

QSFI: a very shallow potential is needed

$$\langle\sigma\rangle^2 \sim H^2/\lambda \longrightarrow \lambda\langle\sigma\rangle^4 \sim 1/\lambda \longrightarrow \lambda \lesssim 10^{-7}$$

$f_{\rm NL} \gtrsim 1$

Where are large signals from?

$$\frac{1}{\Lambda}(\partial_\mu\phi)\mathcal{J}^\mu \longrightarrow \frac{1}{\Lambda}\dot{\phi}_0\mathcal{N}$$

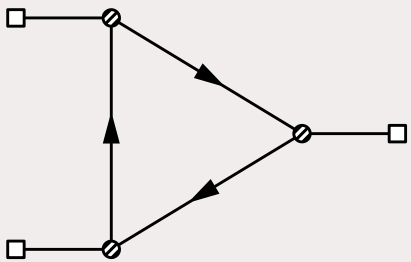
A new source of particle production

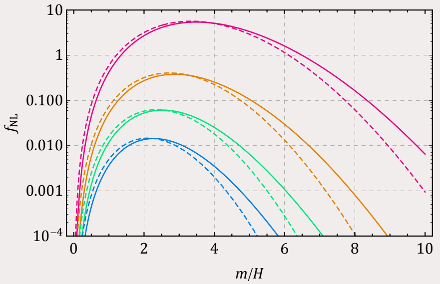
L. Wang, ZZX, 1910.12876

Fermion $(\partial_\mu\phi)\overline{\Psi}\gamma^\mu\gamma^5\Psi$

Probing heavy neutrinos

Chen, Wang, ZZX, 1805.02656



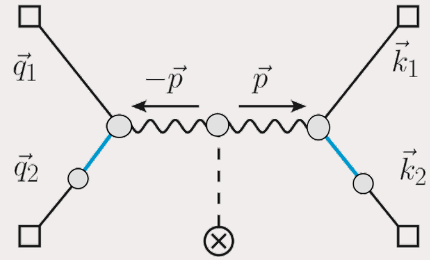


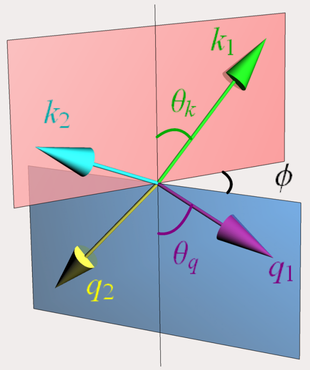
Gauge boson $\phi F\widetilde{F}$

Liantao Wang, ZZX, 2004.02887

CP-breaking in trispectrum

Liu, Tong, Wang, ZZX, 1909.01819



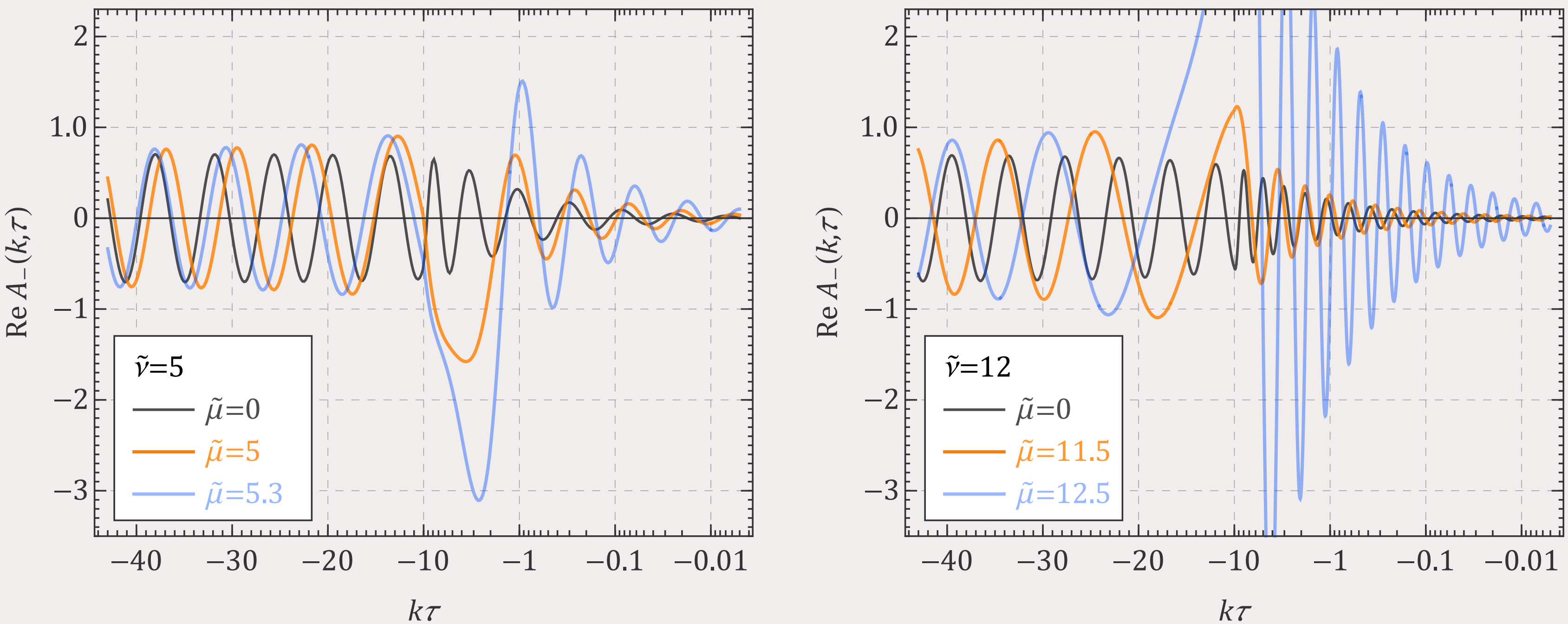


Helical GWs $\phi R\widetilde{R}$

Lue, Wang, Kamionkowski, astro-ph/9812088

dim-5 operators: chemical potential

$$e^{\pi\mu/H}e^{-\pi m/H}$$



Liantao Wang, ZZX, 2004.02887

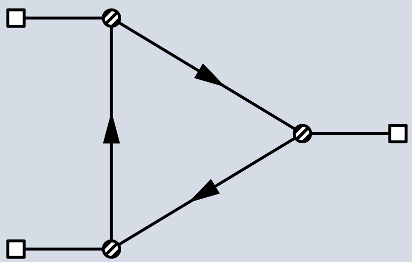
Probing heavy neutrinos

A rare chance to see right-handed neutrinos

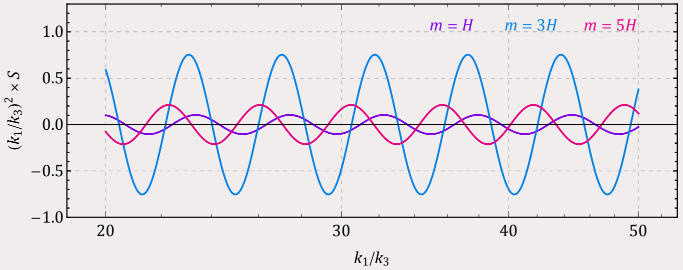
$$m \sim 10^{13}\text{GeV} \sim H$$

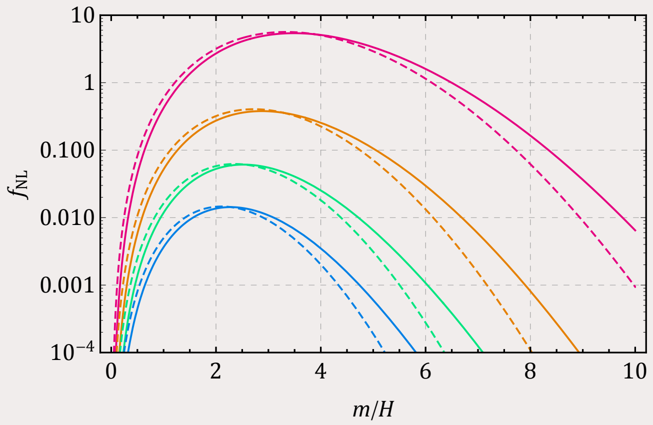
Inflaton background as a neutrino source

$$\frac{1}{\Lambda}(\partial_\mu \phi) N^\dagger \bar{\sigma}^\mu N \longrightarrow \frac{\dot{\phi}}{\Lambda} N^\dagger \bar{\sigma}^0 N$$



$$\lambda = \frac{\dot{\phi}_0}{\Lambda} \qquad \mu = \sqrt{m^2 + \lambda^2}$$





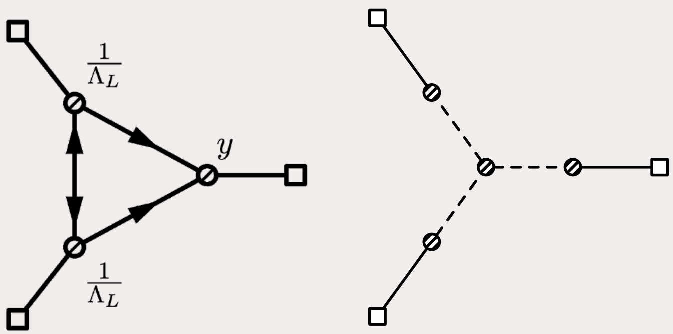
$$f_{NL}(\text{clock}) \simeq \frac{3\pi^2}{2} P_\zeta \widetilde{\lambda}^5 \widetilde{m}^3 e^{-5\pi \widetilde{m}^2/(4\widetilde{\lambda})}$$

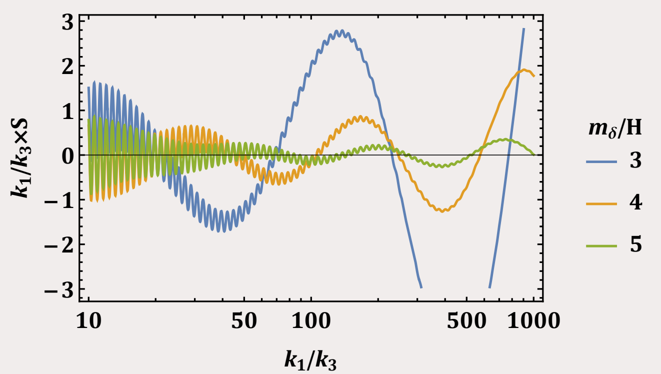
Chen, Wang, ZZX, JHEP **1809** (2018) 022

Probing heavy neutrinos

Probing seesaw mechanisms

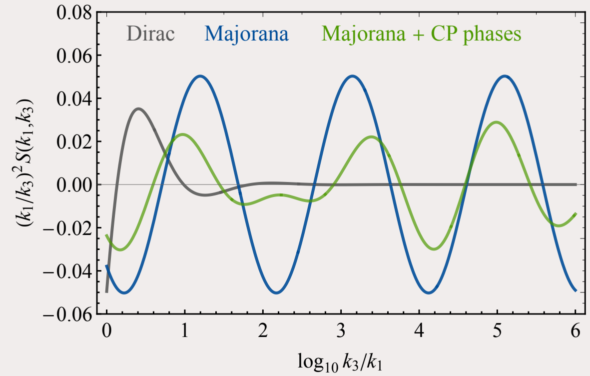
$$\mathcal{L}_\phi = \frac{\partial_\mu \phi}{\Lambda_L} L^\dagger \bar{\sigma}_\mu L - \frac{\partial_\mu \phi \partial^\mu \phi}{\Lambda_\Delta^2} \mathbf{s} \cdot \mathbf{s}^*$$

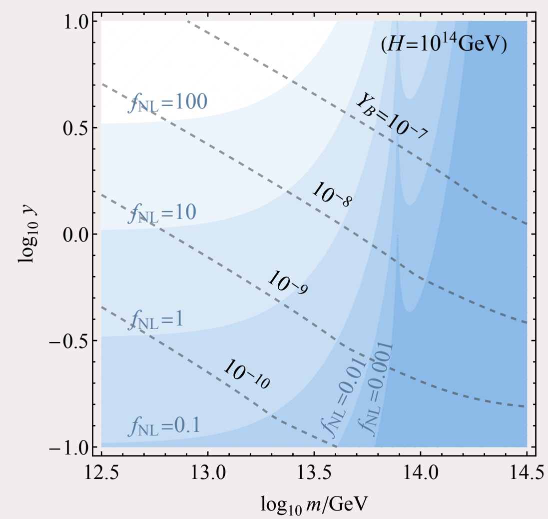




Haipeng An, ZZX, Chen Yang, to appear

Majorana mass and CP phases (Probing leptogenesis?)





Yanou Cui, ZZX, to appear

CP violation

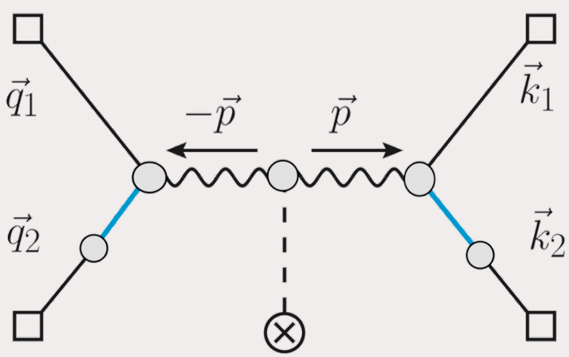
$$\Delta\mathscr{L} = \frac{c_1}{\Lambda}\partial_\mu\phi(\mathcal{H}^\dagger D^\mu\mathcal{H}) + \frac{c_2}{\Lambda^2}(\partial\phi)^2\mathcal{H}^\dagger\mathcal{H} - \frac{c_0}{4}\theta(t)Z_{\mu\nu}Z_{\rho\sigma}\mathcal{E}^{\mu\nu\rho\sigma}$$

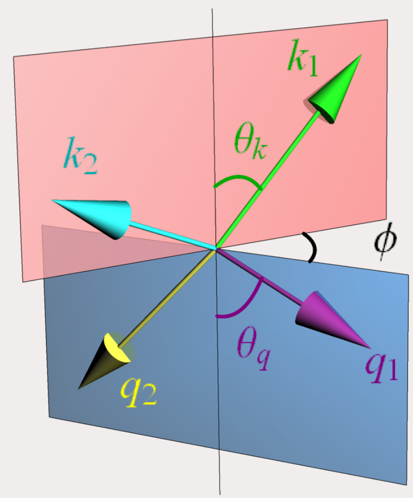
Two types of external legs needed

Odd-angular dependence in imaginary part

No local CP-odd correlations in dS limit

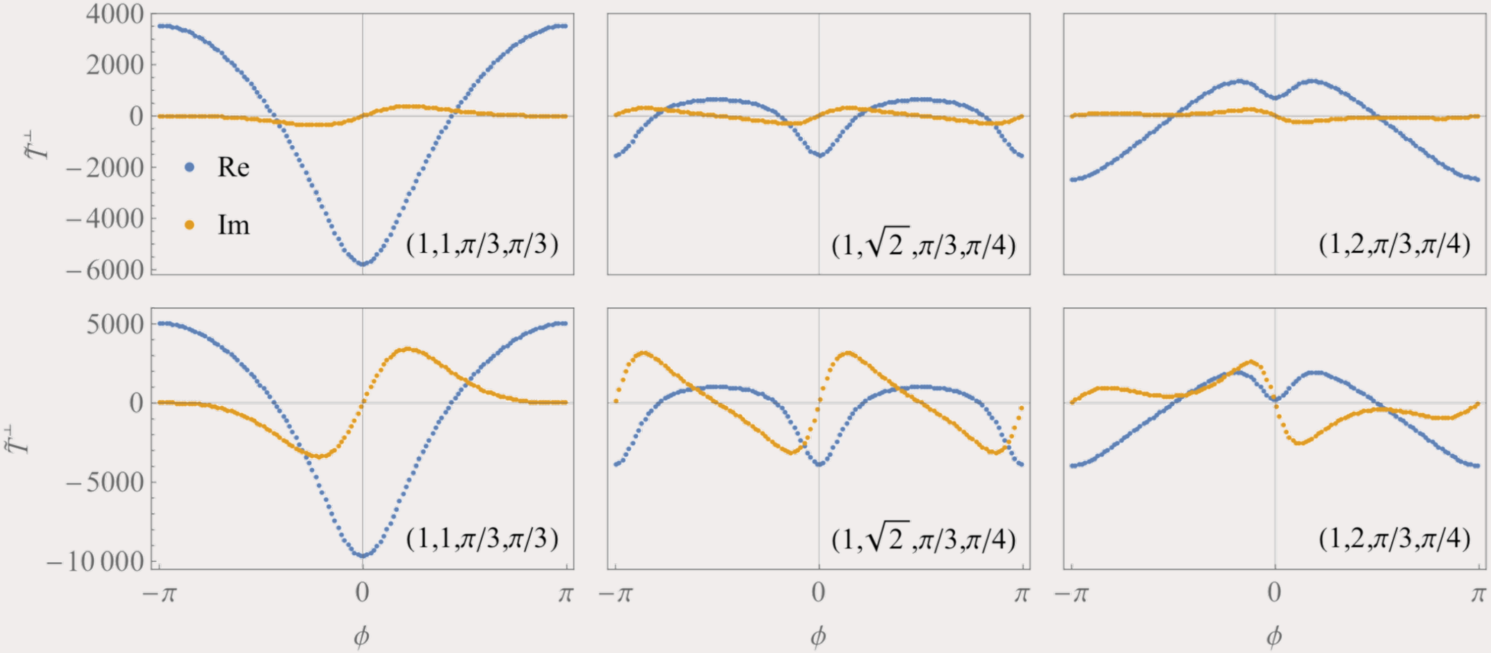
Chemical potential helps

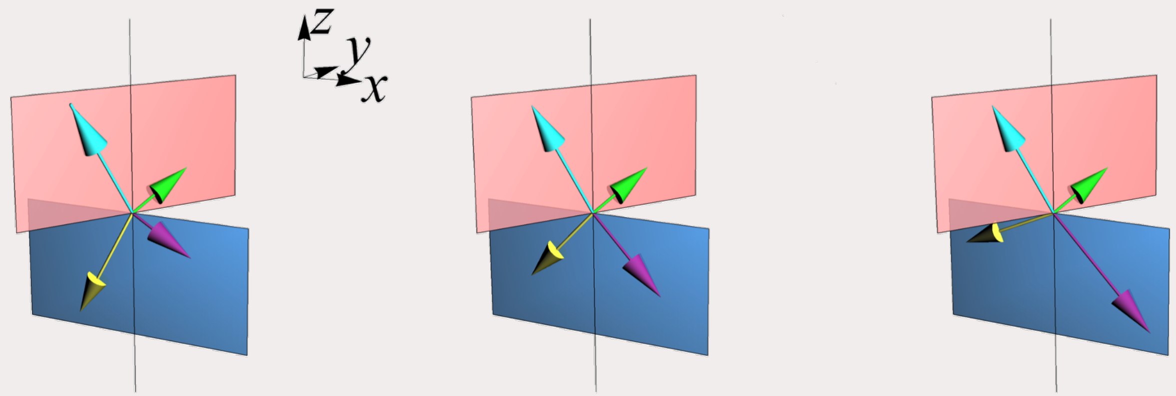




Liu, Tong, Wang, ZZX, 1909.01819

CP violation





Liu, Tong, Wang, ZZX, 1909.01819

Signal size

Beyond the “minimal” scenario

1. Standard slow-roll inflation
2. Scale invariance (up to slow-roll correction)
3. No further spacetime symmetry breaking
4. Dimensionless parameter being O(1)
5. No (tree-level) tuning

Signal size

Beyond the "minimal" scenario

~~1. Standard slow-roll inflation~~

Providing vacuum energy to expand; Generating inhomogeneities

Can separate

Vacuum energy from inflaton / fluctuations from a different source

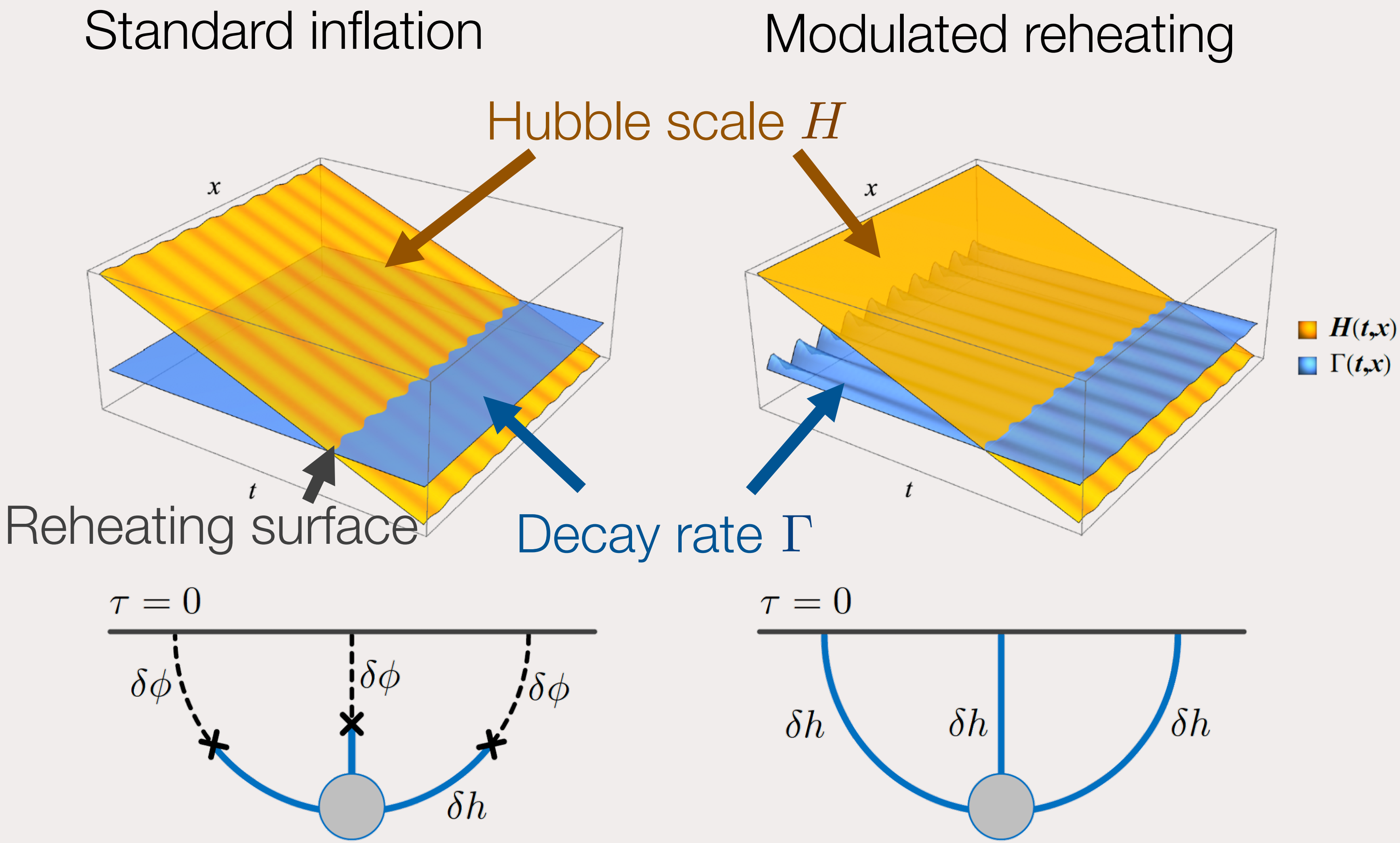
Modulated reheating (Dvali, Gruzinov, Zaldarriaga, astro-ph/0303591)

CHC: A cosmological Higgs collider Lu, Wang, ZZX, 1907.07390

Curvaton Kumar, Sundrum, 1908.11378

Modulated reheating

Shiyun Lu, Yi Wang, ZZX, JHEP 02 (2020) 011



Modulated reheating

Shiyun Lu, Yi Wang, ZZX, JHEP 02 (2020) 011

t_1 end of inflation — reheating $t_{\rm reh}$ — t_2

matter — radiation

$$\zeta(t_2, \mathbf{x}) = \zeta(t_1, \mathbf{x}) + \delta N(t_1, t_2, \mathbf{x})$$

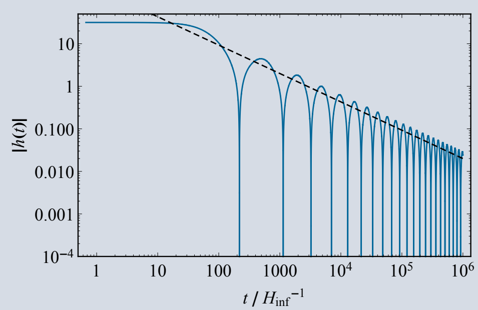
Standard inflation ($\zeta(t_1, \mathbf{x})$) — Modulated reheating ($\delta N(t_1, t_2, \mathbf{x})$)

$$\delta N = \frac{1}{6}\frac{\delta t_{\rm reh}}{t_{\rm reh}} = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma}$$

$$\mathcal{O} = \phi h \cdot \text{something}$$

$$\rho_h \sim \lambda h^4 \sim a^{-4} \sim t^{-8/3}$$

$$\Rightarrow \Gamma(\phi \to \text{somthing}) \propto h^2 \propto t^{-4/3}$$



$$\Delta\mathscr{L} = -\frac{1}{2}(\partial_\mu S_i)^2 - \frac{1}{2}m_{S0}^2 S_i^2 - \alpha S_i^2|\mathbf{H}|^2 + \frac{1}{\Lambda_S}(\partial_\mu\phi)S_i\partial^\mu S_i$$

$$\Gamma(\phi \to SS) = \frac{m_\phi^3}{16\pi\Lambda_S^2}\left(1 - \frac{4m_S^2}{m_\phi^2}\right)^{1/2} \qquad m_S^2(h_0) = m_{S0}^2 + \alpha h_0^2$$

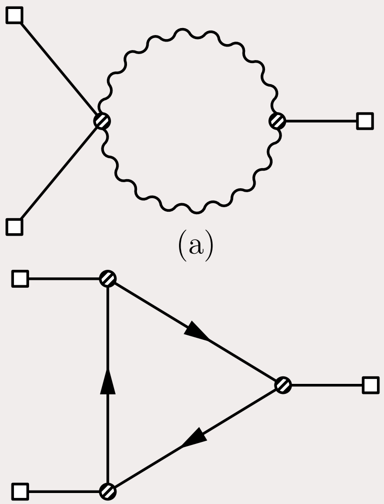
A Cosmological Higgs Collider

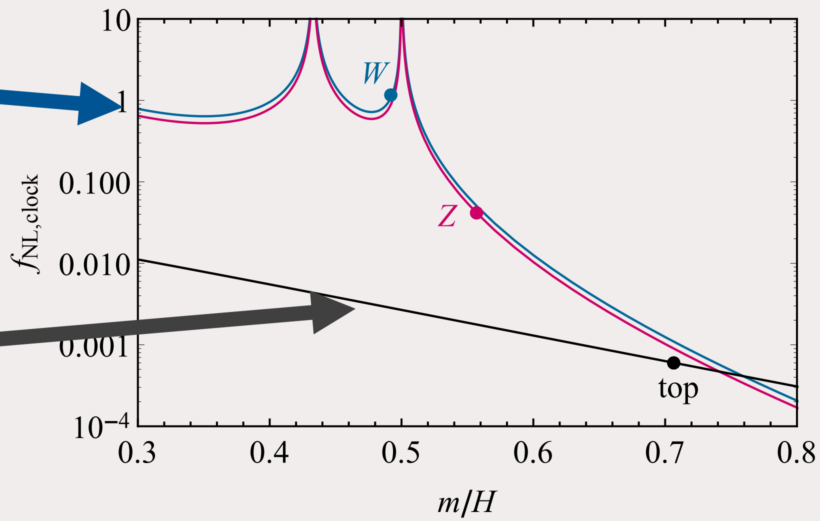
Shiyun Lu, Yi Wang, ZZX, JHEP 02 (2020) 011

Constraint from local non-G

$$f_{\rm NL}(\text{local}) \sim -\mathcal{O}(1)\frac{R_h^3}{2\pi P_\zeta^{1/2}}\lambda N_e + \mathcal{O}(1)\frac{R_h^3}{(2\pi)^6 P_\zeta}\frac{2\alpha N}{(m_\phi/H_{\rm inf})^2}$$

$$R_h \lesssim 0.14\left(\frac{\lambda}{0.01}\right)^{-1/3}\left(\frac{N_e}{50}\right)^{-1/3}$$

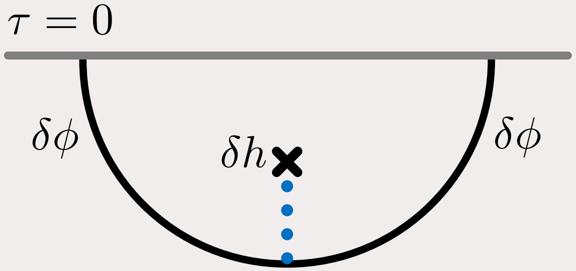


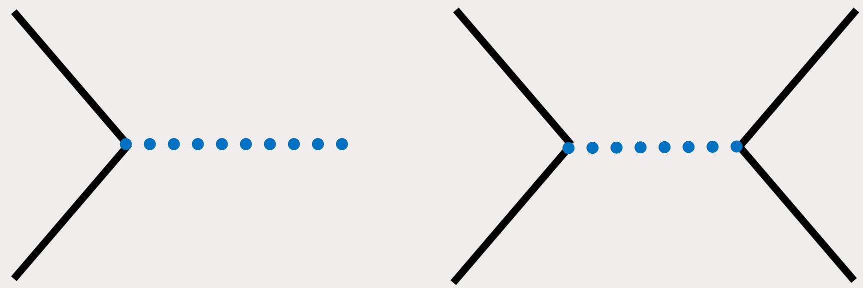


Missing energy

Qianshu Lu, Matt Reece, ZZX, 2108.11385

What if a light scalar does not modulate the reheating?



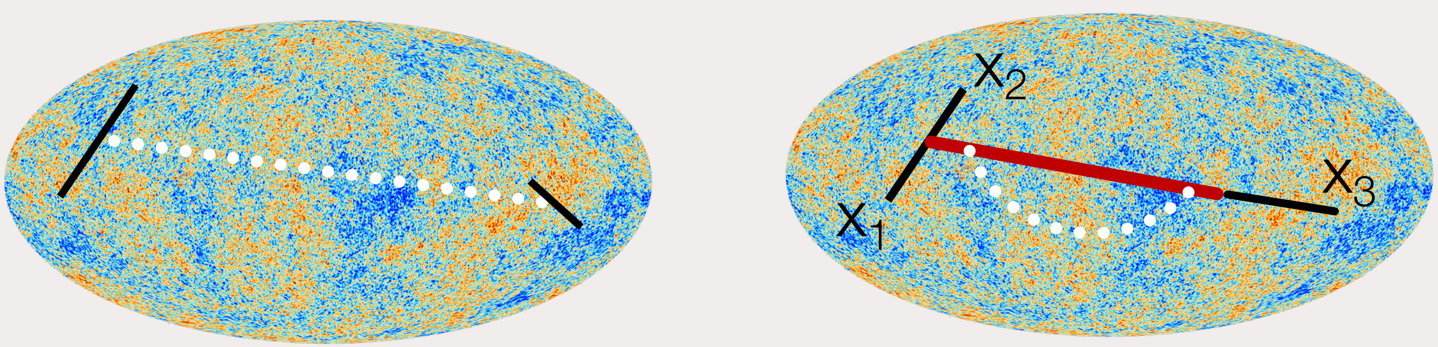


Missing energy. How to probe it?

momentum non-conservation doesn't work

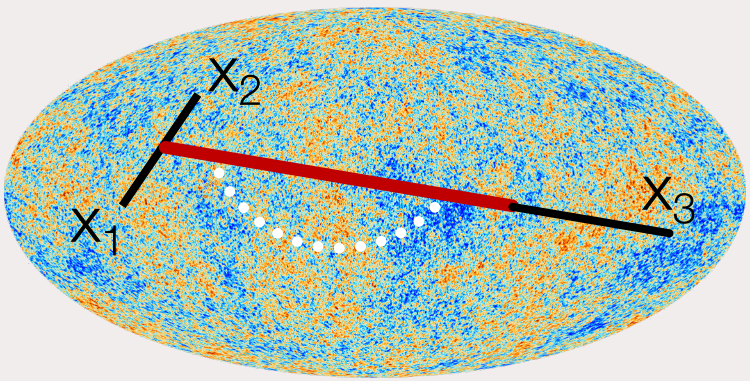
"Cosmic fossils," but only in trispectrum

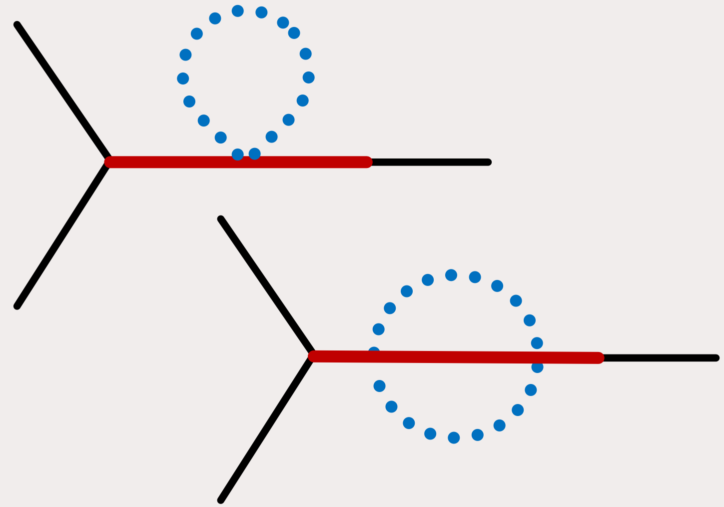
Dai, Jeong, Kamionkowski 1302.1868



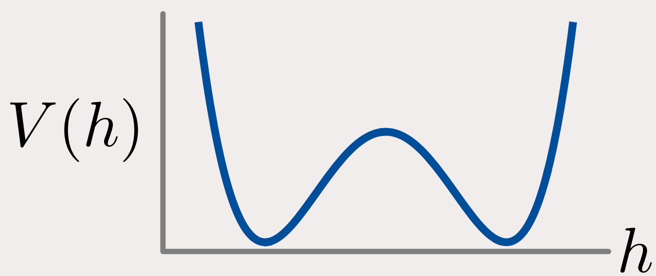
Missing energy

Qianshu Lu, Matt Reece, ZZX, 2108.11385

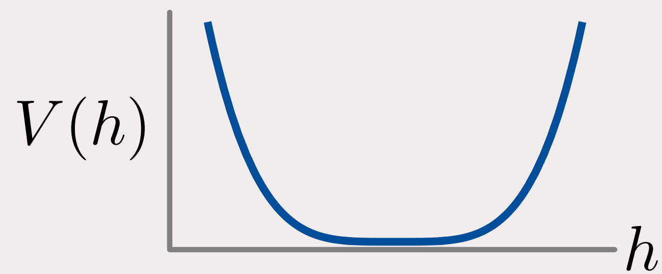




Telling thermal mass from symmetry-breaking mass



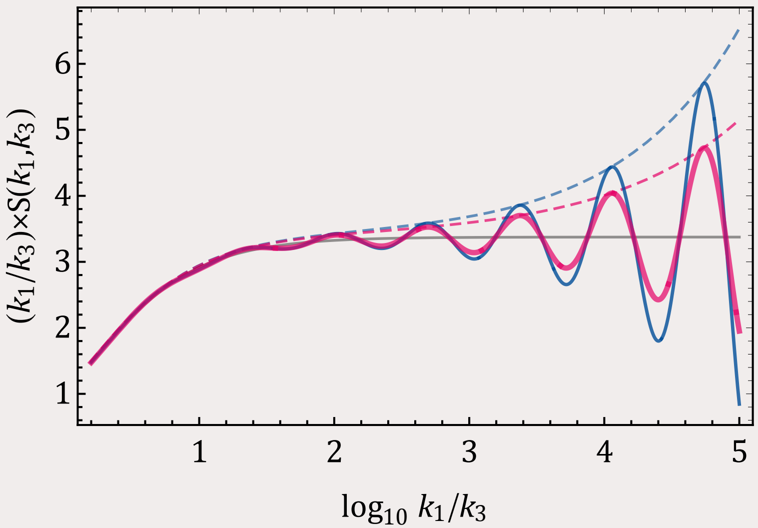
$$m^2 \sim m_0^2 + g^2 \langle h \rangle^2$$



$$m^2 \sim m_0^2 + g^2 \langle h^2 \rangle$$

Missing energy

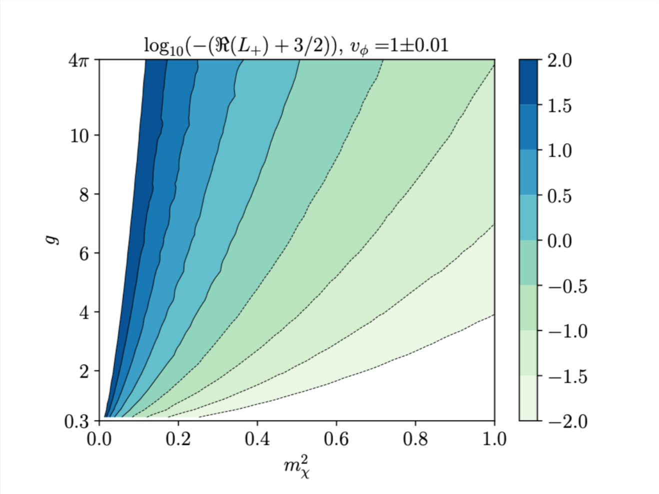
Qianshu Lu, Matt Reece, ZZX, 2108.11385

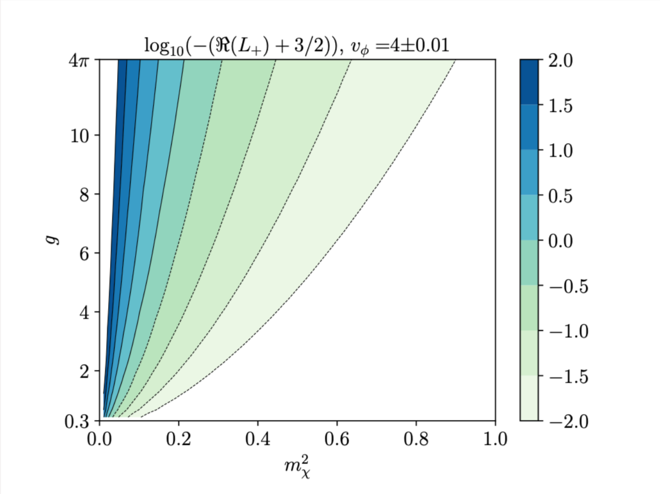


$\alpha = 0$

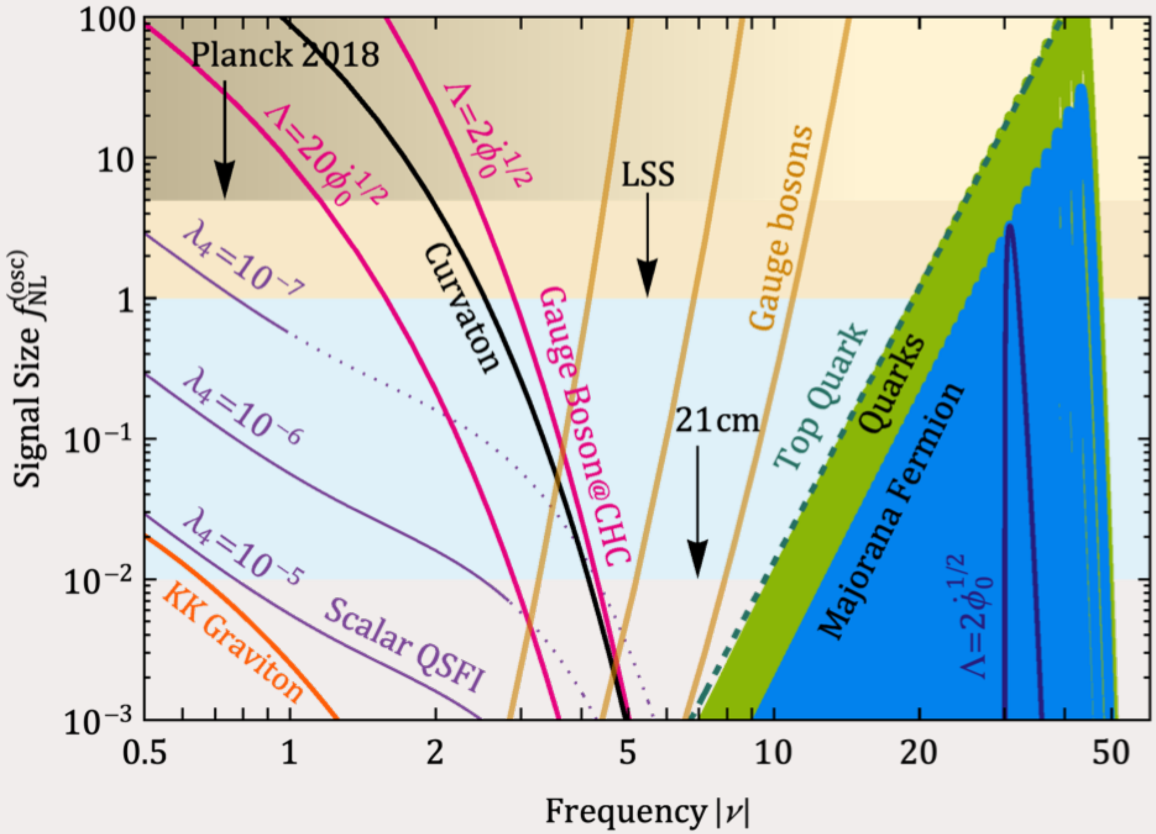
$\alpha = 0.05$

$$S = A(\lambda, m)\left(\frac{k_3}{k_1}\right)^{1/2-\alpha\pm \mathrm{i}|\nu|}$$



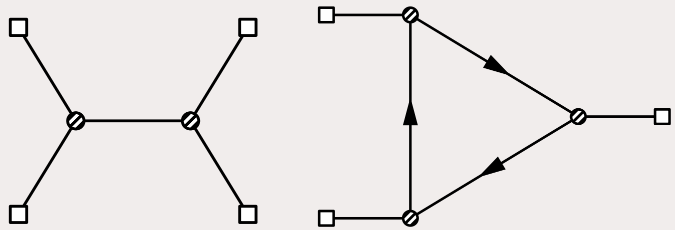


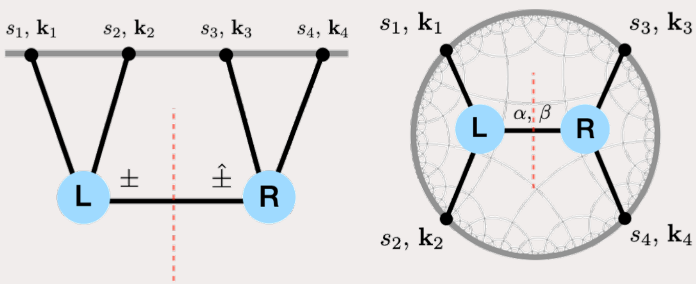
A status summary

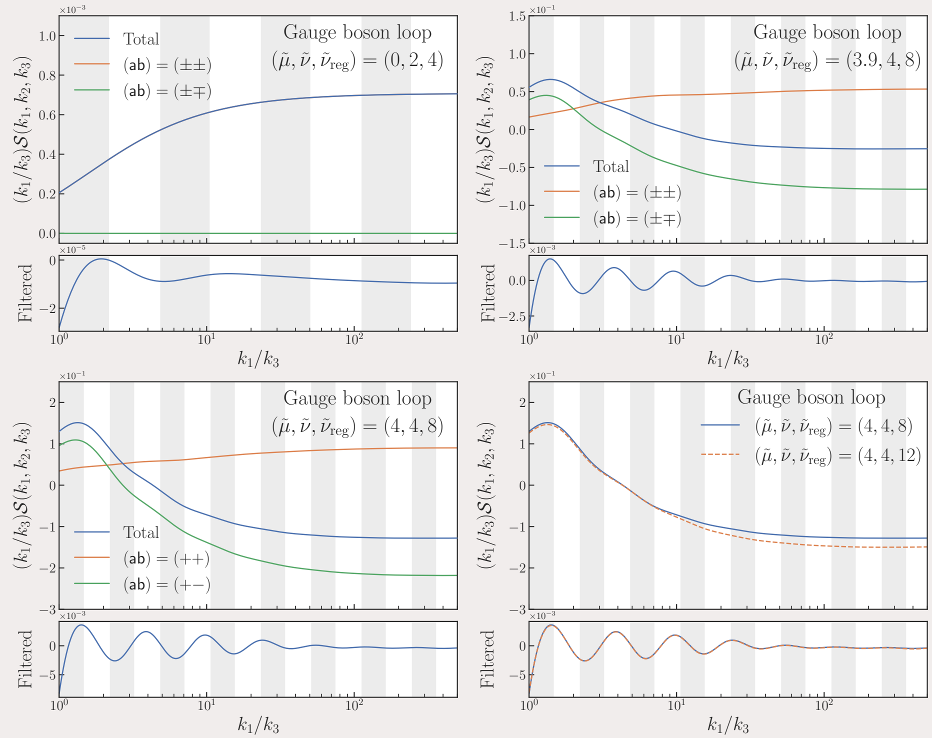


L. Wang, ZZX, 1910.12876, 2004.02887

Theory challenges







Recent development in formal techniques

Conformal Bootstrap

Arkani-Hamed, Baumann, Joyce, Lee, Pimentel, 1811.00024, 1910.14051, 2005.04234

Mellin-Barnes representation

Sleight, Taronna, 1906.12302, 1907.01143, 2109.02725, etc

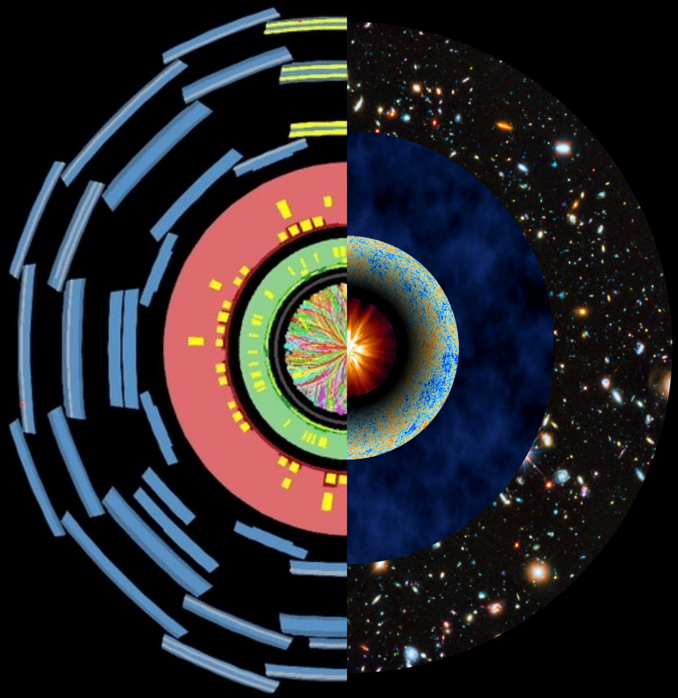
More pragmatic approaches

Schwinger-Keldysh diagrammatics

Chen, Wang, ZZX, 1703.10166

“brute force” computation

Wang, ZZX, Zhong, 2109.14635



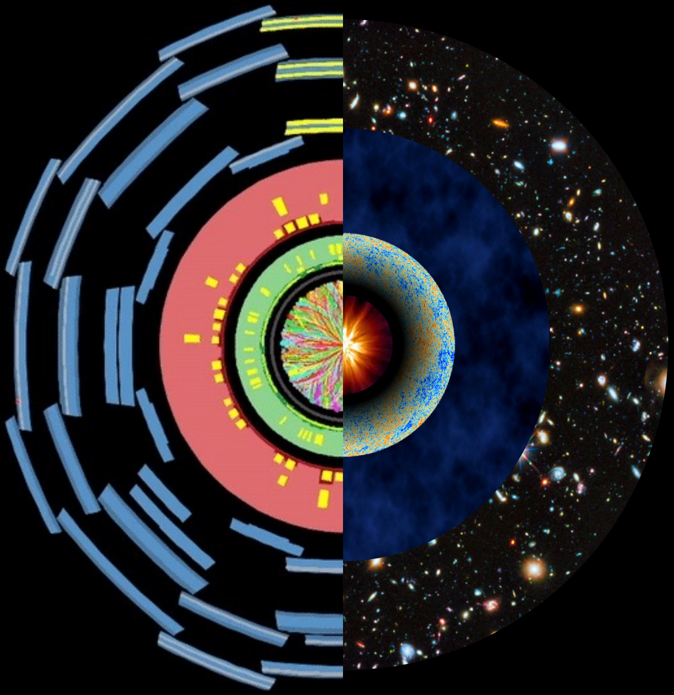
Take-home

Observational progress ahead

1 order of magnitude improvement in next decade / Can already test some interesting scenarios / another 1-2 orders ultimately, can reach gravity floor

New opportunities to particle physics and dS QFT

Our ultimate hope for probing ultra high energy physics in the real world / More theoretical efforts called for / Not the sort of “1000 inflation models to fit 2 parameters n_s and r” thing



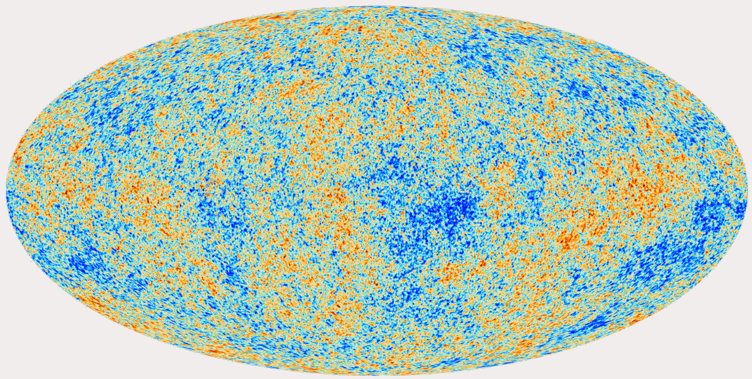
Thank you

Basic picture

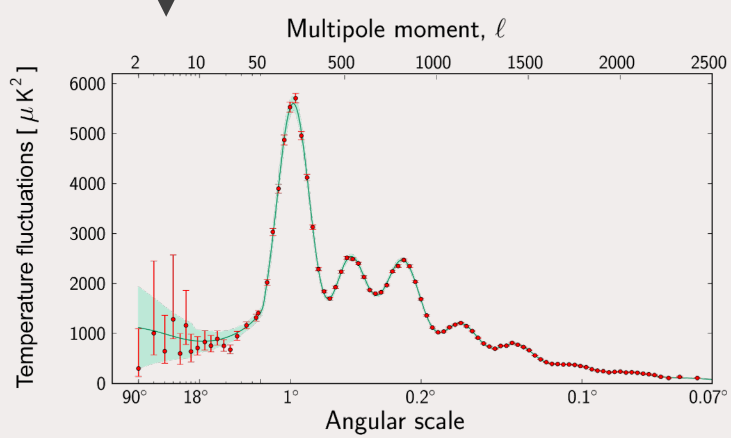
Two puzzles of big-bang cosmology:

1 - Why so uniform?

2 - Where were these fluctuations from?

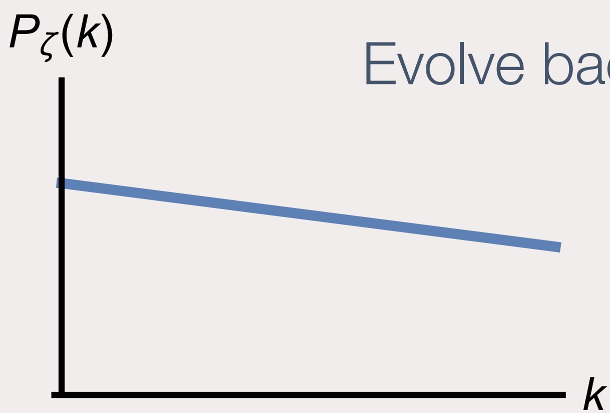


“Fourier transform”

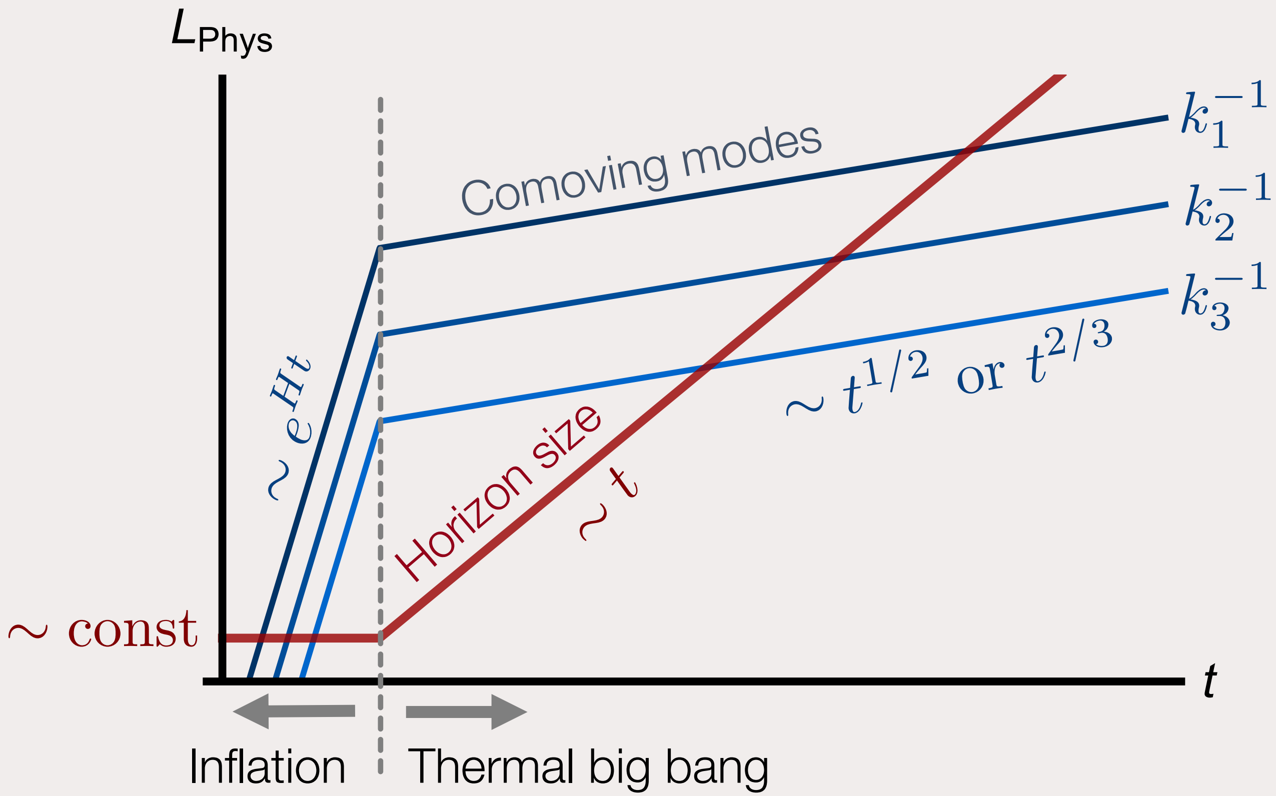


Planck 2018

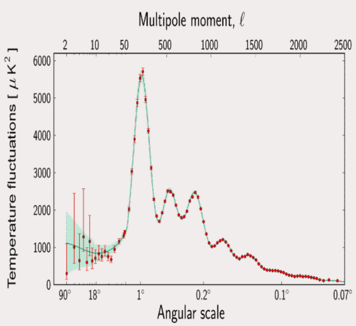
Evolve back in time

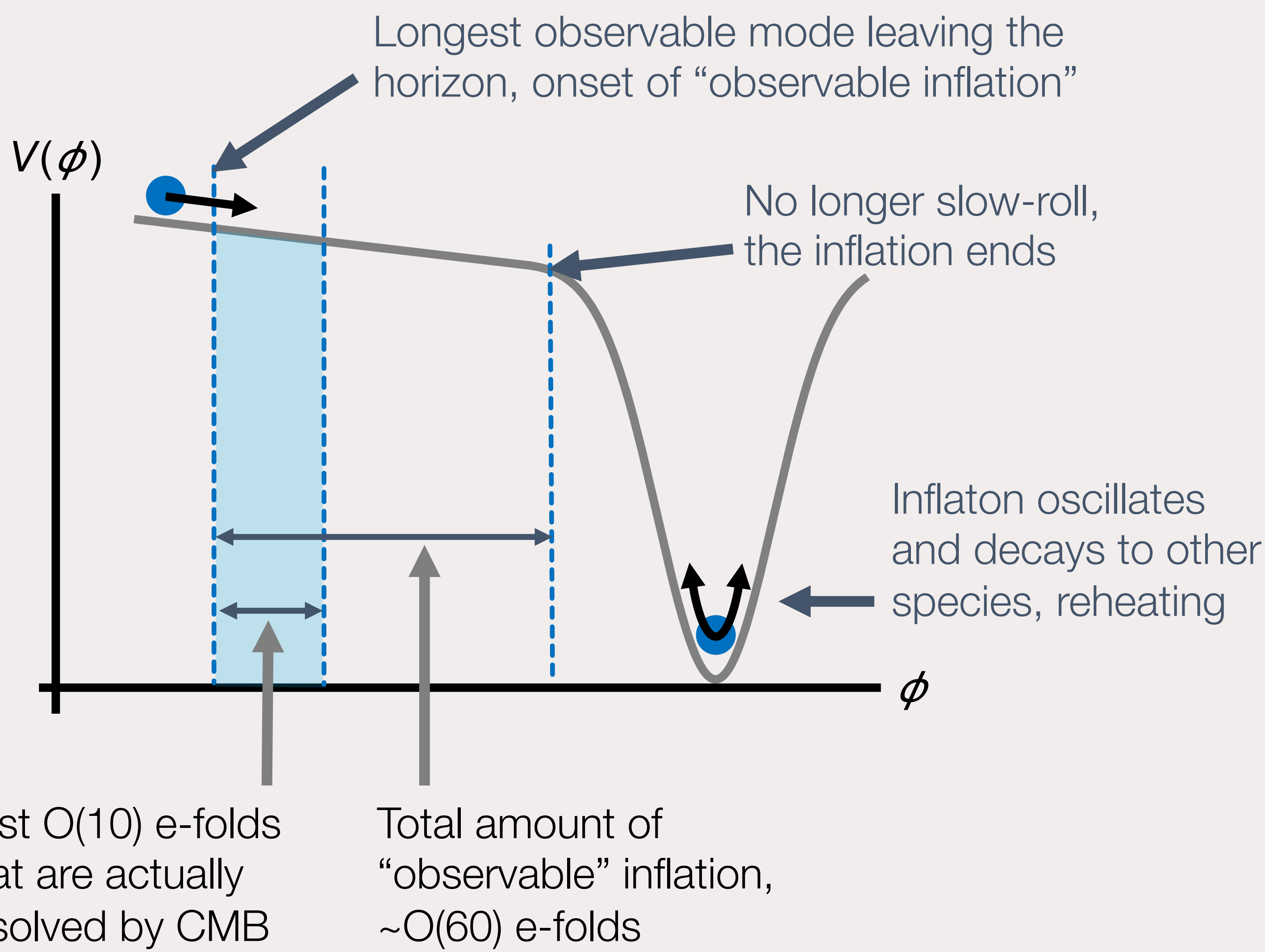


Basic picture

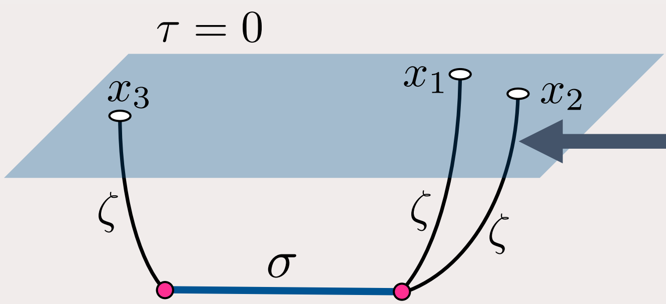


Basic picture





Basic picture



massive mode

long-lived mode ζ

$$\zeta_k = \frac{H}{M_{\rm Pl}\sqrt{4\epsilon k^3}}(1+ik\tau)e^{-ik\tau}$$

$$\langle \zeta^2 \rangle' \equiv \frac{2\pi^2}{k^3} P_\zeta(k)$$

$$P_\zeta(k) = \frac{H^2}{8\pi^2 \epsilon M_{\rm Pl}^2} \simeq 2 \times 10^{-9}$$

$$\langle \sigma_k(\tau_1)\sigma_{-k}(\tau_2)\rangle'$$

$$\sim \frac{H^2}{4\pi k^3}\left[\Gamma^2(-\nu)\left(\frac{k^2\tau_1\tau_2}{4}\right)^{3/2+\nu} + (\nu \to -\nu)\right] + \text{local} \qquad \nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

Boltzmann factor $\propto e^{-\pi m/H}$

comoving dilution

EFT $\propto 1/m$

Chen, Wang, 0911.3380;1205.0160; Pi, Sasaki, 1205.0161; Arkani-Hamed, Maldacena, 1503.08043

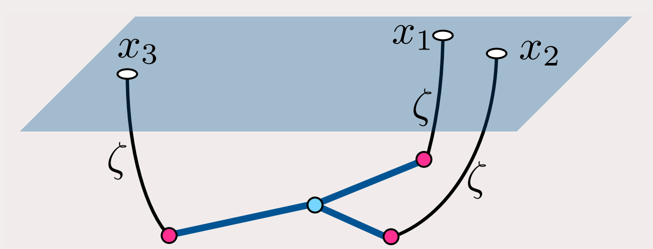
Signal size

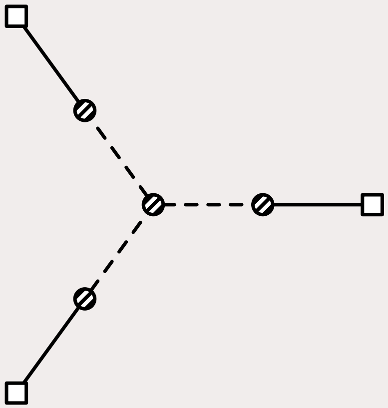
$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle' \equiv (2\pi)^4 P_\zeta^2 \frac{1}{(k_1k_2k_3)^2} S(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

In the unit of Hubble: $\zeta = -\frac{H}{\dot\phi_0}\delta\phi = -2\pi P_\zeta^{1/2}\delta\phi$

$$f_{\rm NL} \sim (2\pi P_\zeta^{1/2})^{-1}\langle\delta\phi^3\rangle$$

$\sim 3.6\times 10^3$ · (vertices) · (propagators)



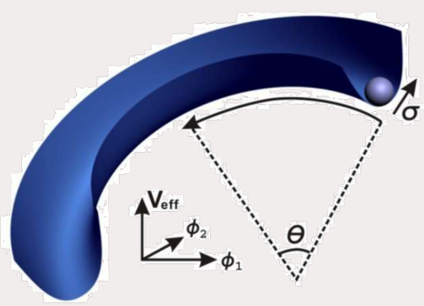


how to estimate

An example of QSFI

Chen, Wang, 0911.3380

$$\mathscr{L} = \sqrt{-g}\left[-\frac{1}{2}(\widetilde{R}+\sigma)^2(\partial_\mu\theta)^2 - \frac{1}{2}(\partial_\mu\sigma)^2 - V_{\rm sr}(\theta) - V(\sigma)\right]$$

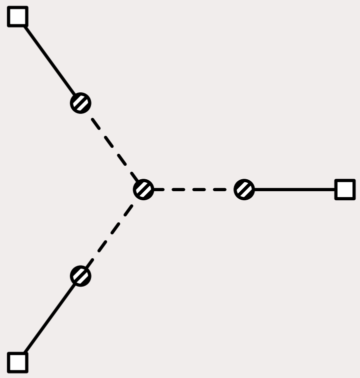


$$\frac{a^2}{2}\left[(\delta\phi')^2 - (\partial_i\delta\phi)^2 + (\delta\sigma')^2 - (\partial_i\delta\sigma)^2\right]$$
$$- \frac{1}{2}a^4m^2\delta\sigma^2 + a^3\kappa_1\delta\sigma\delta\phi' - \frac{1}{6}a^4\lambda_3\delta\sigma^3$$

$$f_{NL} \sim P_\zeta^{-1/2} \cdot \left(\frac{\kappa_1}{H}\right)^3 \cdot \left(\frac{\lambda_3}{H}\right)$$

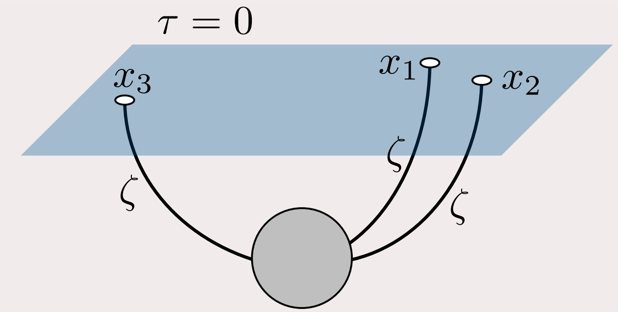
· (propagators)

$\kappa_1 < m$ $\qquad m \lesssim H$



“in-in formalism”

S-matrix = $\langle \text{out} | \text{in} \rangle$ ➞ Feynman diagrams



$$\langle \text{in} | \phi_1 \cdots \phi_n | \text{in} \rangle = \sum_{\text{out}} \langle \text{in} | \text{out} \rangle \langle \text{out} | \phi_1 \cdots \phi_n | \text{in} \rangle$$

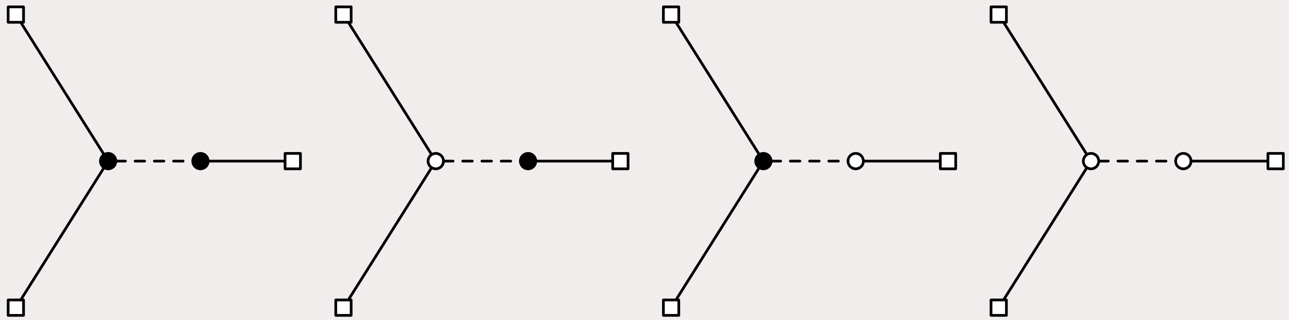
$$\int \mathcal{D}\phi_+ \mathcal{D}\phi_- e^{\mathrm{i}S[\phi_+] - \mathrm{i}S[\phi_-]} \delta\big[\phi_+(\tau = 0) - \phi_-(\tau = 0)\big]$$

Still Feynman diagrams, but with 2 sets of fields

2 types of vertices & 4 types of propagators

"in-in formalism"

Decorated Feynman diagrams



"Schwinger-Keldysh diagrammatics"

Chen, Wang, ZZX, arXiv:1703.10166

— A recipe for particle physicists

“in-in formalism”

$$(1) = -12u^*_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^3c_2v^*_{p_1}u'_{p_1}(\tilde\tau_1)\int_{-\infty}^{\tilde\tau_1} d\tilde\tau_2\; a^4c_3v_{p_1}v_{p_2}v_{p_3}(\tilde\tau_2) \times \int_{-\infty}^{0} d\tau_1\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_2)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.2}$$

$$(2) = -12u^*_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^4c_3v^*_{p_1}v_{p_2}v_{p_3}(\tilde\tau_1)\int_{-\infty}^{\tilde\tau_1} d\tilde\tau_2\; a^3c_2v_{p_1}u'_{p_1}(\tilde\tau_2) \times \int_{-\infty}^{0} d\tau_1\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_2)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.3}$$

$$(3) = 12u_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^4c_3v_{p_1}v_{p_2}v_{p_3}(\tilde\tau_1) \times \int_{-\infty}^{0} d\tau_1\; a^3c_2v^*_{p_1}u'^*_{p_1}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_2)\int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_3)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.4}$$

$$(4) = 12u^*_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^3c_2v_{p_1}u'_{p_1}(\tilde\tau_1) \times \int_{-\infty}^{0} d\tau_1\; a^4c_3v^*_{p_1}v_{p_2}v_{p_3}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_2)\int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_3)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.5}$$

$$(5) = 12u^*_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^3c_2v_{p_1}u'_{p_1}(\tilde\tau_1) \times \int_{-\infty}^{0} d\tau_1\; a^3c_2v_{p_2}u'^*_{p_2}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^4c_3v^*_{p_1}v^*_{p_2}v_{p_3}(\tau_2)\int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_3)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.6}$$

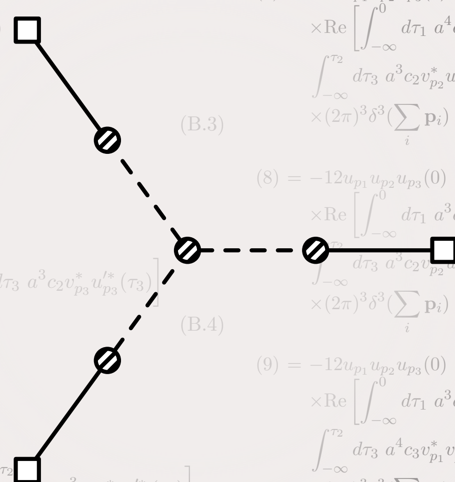
$$(6) = 12u^*_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tilde\tau_1\; a^3c_2v_{p_1}u'_{p_1}(\tilde\tau_1) \times \int_{-\infty}^{0} d\tau_1\; a^3c_2v_{p_2}u'^*_{p_2}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v_{p_3}u'^*_{p_3}(\tau_2)\int_{-\infty}^{\tau_2} d\tau_3\; a^4c_3v^*_{p_1}v^*_{p_2}v^*_{p_3}(\tau_3)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.7}$$

$$(7) = -12u_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tau_1\; a^4c_3v_{p_1}v_{p_2}v_{p_3}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v^*_{p_1}u'^*_{p_1}(\tau_2) \int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_3)\int_{-\infty}^{\tau_3} d\tau_4\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_4)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.8}$$

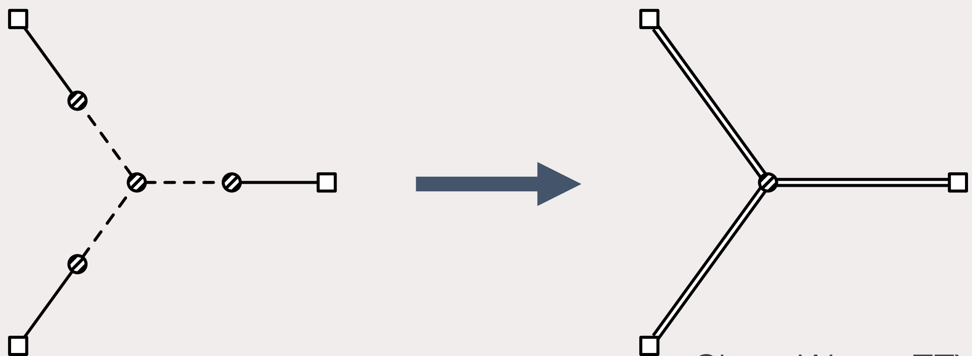
$$(8) = -12u_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tau_1\; a^3c_2v_{p_1}u'^*_{p_1}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^4c_3v^*_{p_1}v_{p_2}v_{p_3}(\tau_2) \int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v^*_{p_2}u'^*_{p_2}(\tau_3)\int_{-\infty}^{\tau_3} d\tau_4\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_4)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.9}$$

$$(9) = -12u_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tau_1\; a^3c_2v_{p_1}u'^*_{p_1}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v_{p_2}u'^*_{p_2}(\tau_2) \int_{-\infty}^{\tau_2} d\tau_3\; a^4c_3v^*_{p_1}v^*_{p_2}v_{p_3}(\tau_3)\int_{-\infty}^{\tau_3} d\tau_4\; a^3c_2v^*_{p_3}u'^*_{p_3}(\tau_4)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.10}$$

$$(10) = -12u_{p_1}u_{p_2}u_{p_3}(0) \times \mathrm{Re}\Big[\int_{-\infty}^{0} d\tau_1\; a^3c_2v_{p_1}u'^*_{p_1}(\tau_1)\int_{-\infty}^{\tau_1} d\tau_2\; a^3c_2v_{p_2}u'^*_{p_2}(\tau_2) \int_{-\infty}^{\tau_2} d\tau_3\; a^3c_2v_{p_3}u'^*_{p_3}(\tau_3)\int_{-\infty}^{\tau_3} d\tau_4\; a^4c_3v^*_{p_1}v^*_{p_2}v^*_{p_3}(\tau_4)\Big] \times(2\pi)^3\delta^3(\sum_i \mathbf{p}_i) + 5\text{ perm.} \tag{B.11}$$

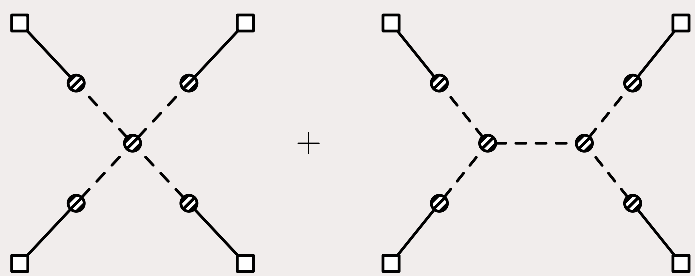


“in-in formalism”



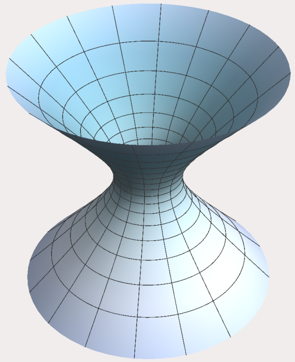
Chen, Wang, ZZX, 1703.10166

$$\langle \delta\phi^3 \rangle' = \frac{\pi^3 \lambda_2^3 \lambda_3}{256 H k_2^3 k_3^3} \, \mathrm{Im} \int_0^\infty \frac{\mathrm{d}z}{z^4} I_+(z) I_+(\frac{k_2}{k_1} z) I_+(\frac{k_3}{k_1} z)$$

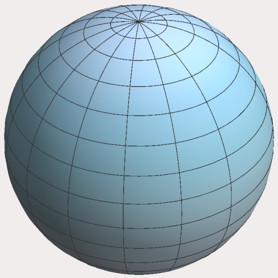


Chen, Chua, Guo, Wang, ZZX, Xie, 1803.04412

fun with spherical harmonics



Wick rotation



$$\Box Y_{\vec{L}}(x) = -H^2 L(L+d) Y_{\vec{L}}(x)$$

$$(\Box - m^2)\phi = 0$$

$$G(x,x') = \sum_{\vec{L}} \frac{H^{d+1}}{\lambda_L} Y_{\vec{L}}(x) Y^*_{\vec{L}}(x')$$

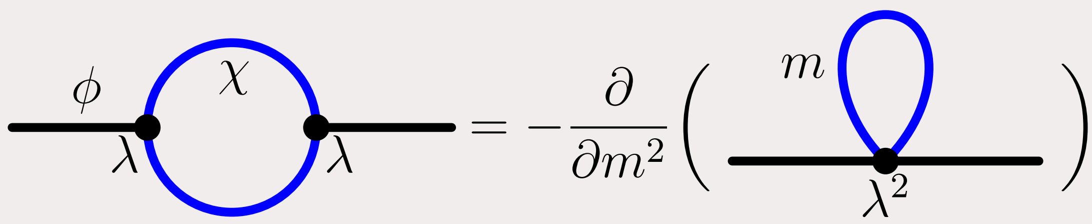
$$\lambda_L = L(L+d) + (m/H)^2$$

Zero mode $\dfrac{H^{d+3}}{m^2} Y_{\vec{0}}^2$

fun with spherical harmonics

$$\int_{x,x'} G(x,x')^2 = \sum_{L,M} \int_{x,x'} \frac{1}{\lambda_L \lambda_M} Y_{\vec{L}}(x) Y^*_{\vec{L}}(x') Y_{\vec{M}}(x') Y^*_{\vec{M}}(x)$$

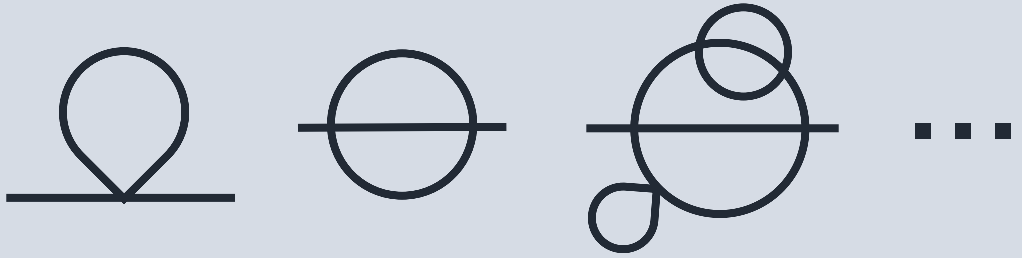
$$= \sum_{L} \int_{x} \frac{1}{\lambda_L^2} Y_{\vec{L}}(x) Y^*_{\vec{L}}(x) = -\frac{\partial}{\partial m^2} \int_x G(x,x)$$



Small mass limit $m_\chi \ll H$ ➡ $\delta m_\phi^2 = \frac{3\lambda^2 H^4}{8\pi^2 m_\chi^2}$

Higgs mass

Loop expansion breaks down



$\propto \sqrt{\lambda}$

The zero-mode path integral to all orders

non-vanishing in the classically massless limit

$$M_H^2 = \sqrt{\frac{6\lambda}{\pi^3}} H^2$$

Rajaraman, 1008.1271

Chen, Wang, ZZX, 1612.08122

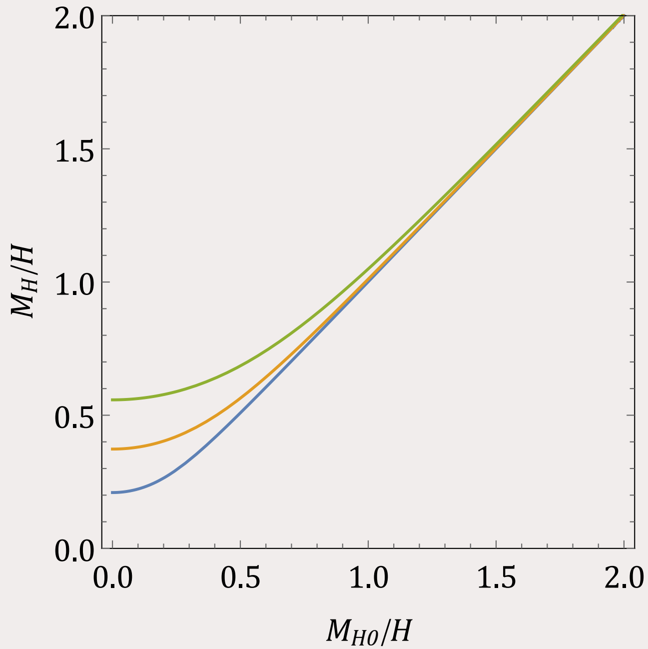
Higgs mass

The zero-mode path integral to all orders

$$\frac{H^D}{M_H^2}\left|Y_{\vec{0}}\right|^2 = \langle h_0^2 \rangle = \frac{\int \mathrm{d}^N h_0\, h_0^2 e^{-V_D\left(\frac{1}{2}M_{H0}^2 h_0^2 + \frac{1}{4}\lambda h_0^4\right)}}{\int \mathrm{d}^N h_0\, e^{-V_D\left(\frac{1}{2}M_{H0} h_0^2 + \frac{1}{4}\lambda h_0^4\right)}}$$

non-vanishing in the classically massless limit

$$M_H^2 = \sqrt{\frac{6\lambda}{\pi^3}}H^2$$



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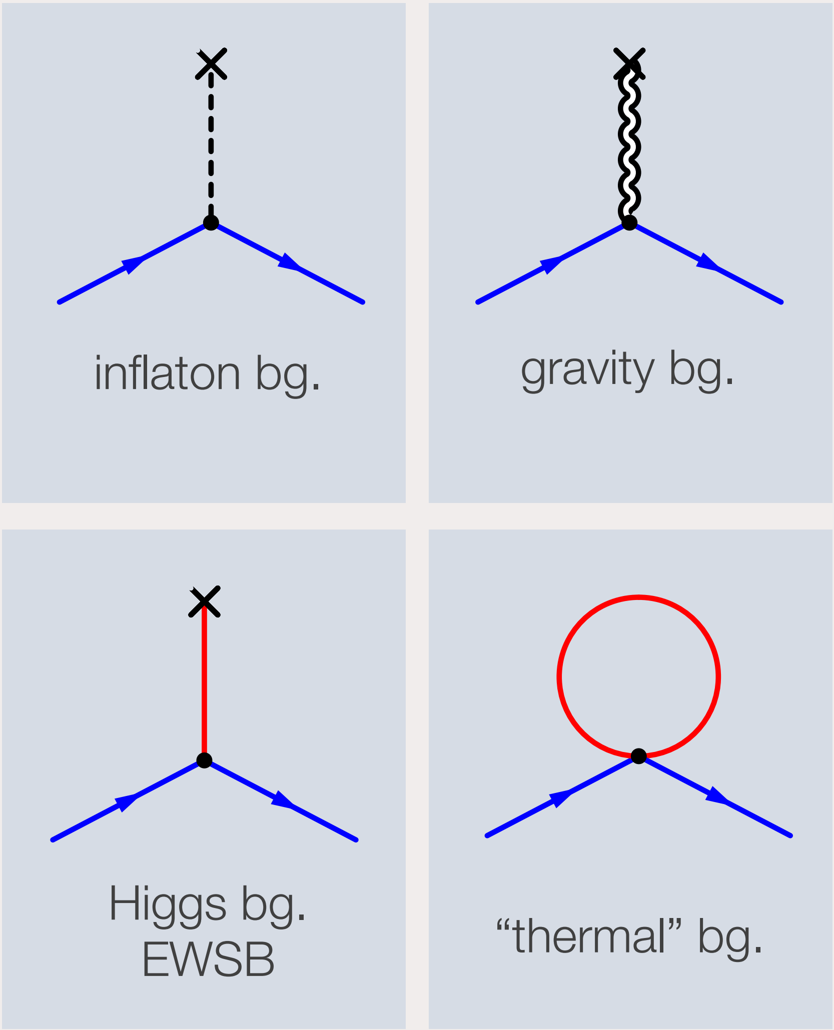
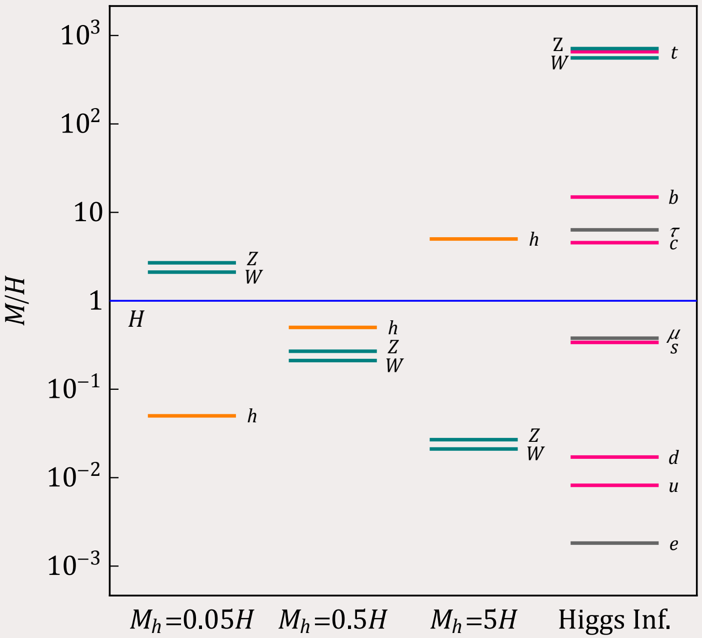
SM spectrum





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