

# Improve Bs of the Higgs hadronic decays and probe the Higgs CP

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CEPC day @ 2021.12.22

# Outline

## ➤ Part I

- Improve the branch ratios of  $H \rightarrow b\bar{b}/c\bar{c}/gg$  and other hadronic decays in  $e^+e^- \rightarrow \mu\mu H$  --- full simulation

## ➤ Part II

- Differential study in the same process: lepton pair of Z used for Higgs CP study --- fast simulation

## ➤ Summary

# Introduction

- At CDR stage, the branch ratios of  $H \rightarrow b\bar{b}/c\bar{c}/gg$  measured with a 3D-fit method.
- To improve the measurement
  - ✓ Machine learning technique and
  - ✓ Including more decay modes :  $H \rightarrow b\bar{b}/c\bar{c}/gg/ww^*/zz^*$
  - ✓ Matrix method to use more information
- Based on counting analysis, differential study is important extensions
  - ✓ The lepton pair of Z used for Higgs CP study

# Part I

Brs of  $H \rightarrow b\bar{b}/c\bar{c}/gg/ww^*/zz^*$

# Physics processes

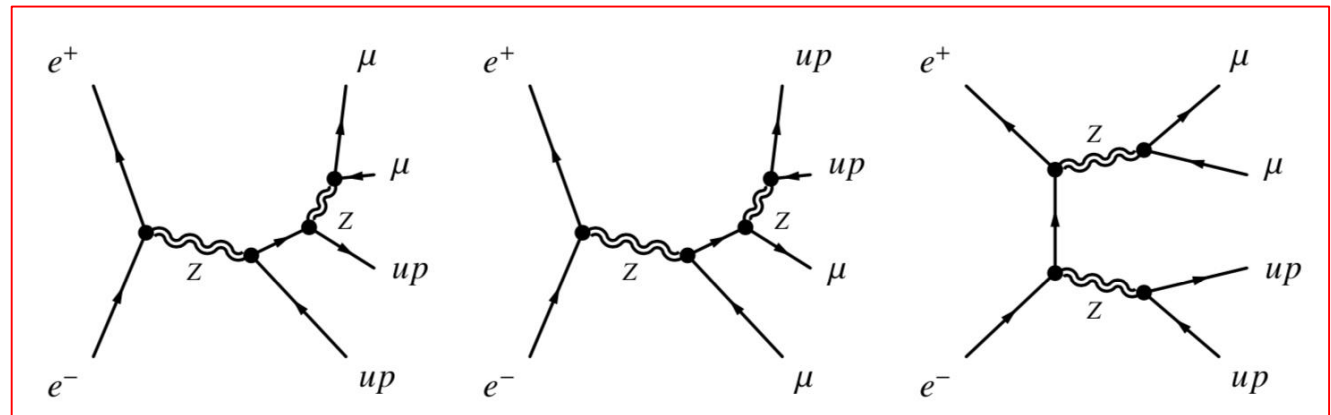
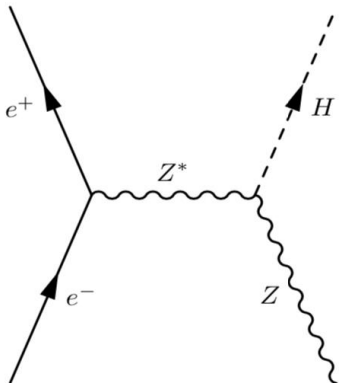
➤ Bs of  $H \rightarrow b\bar{b}/c\bar{c}/gg/ww^*/zz^*$  with  $Z \rightarrow \mu^+\mu^-$

➤ Processes

➤ Signals:  $Z(\mu^+\mu^-) H(bb, cc, gg, WW^*, ZZ^*)$

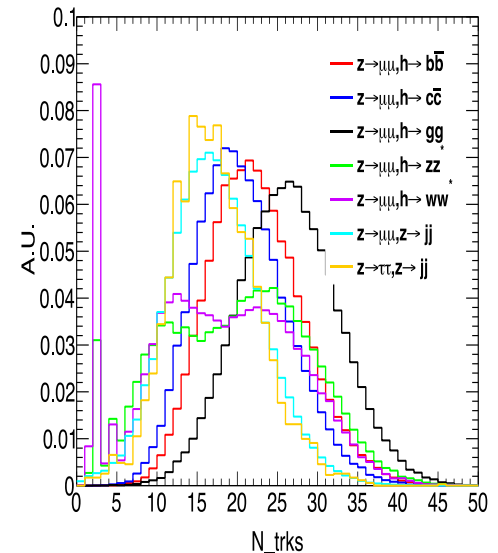
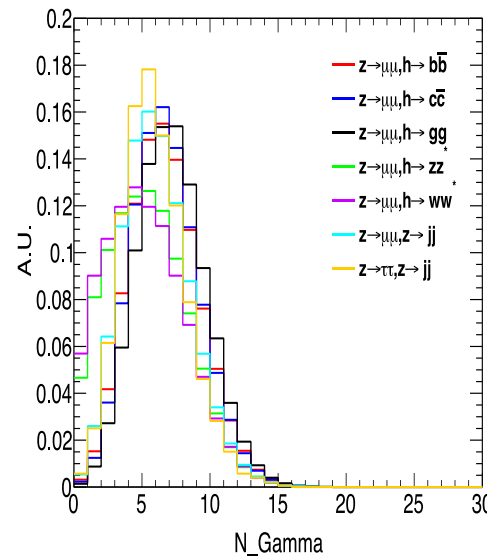
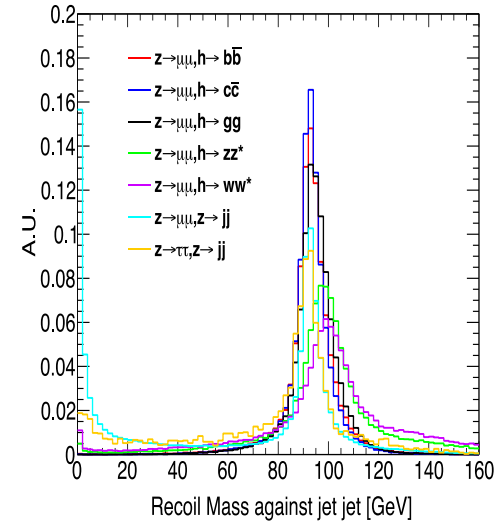
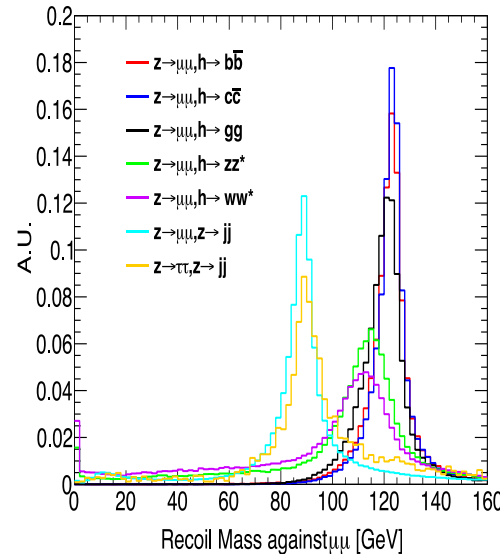
➤ Irreducible Bkgs:  $\mu^+\mu^-qq$  ( $e^+e^- \rightarrow ZZ$ )

➤ Samples simulated with CEPC\_v4 with  $L = 5.6/\text{ab}$



# Pre-selection

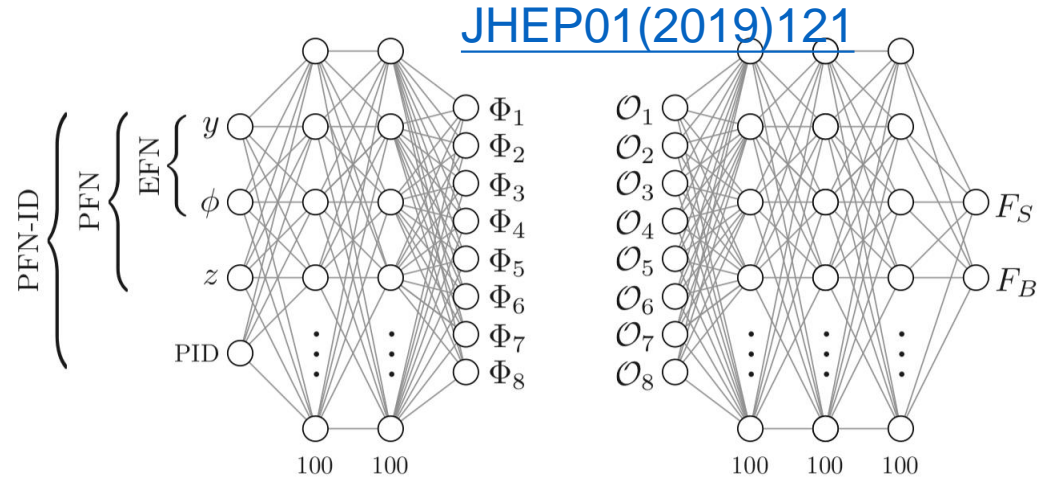
- Cut-based method  
including several variables  
cannot separate the  
different channels very well.
- Machine learning  
technique could help to  
improve the performance  
and make analysis more  
efficient



# Particle Flow Network

- PFN : Particle Flow Networks is an architectures designed for learning from collider events.

$$PFN = F\left(\sum_{i=1}^M \Phi(p_i)\right)$$



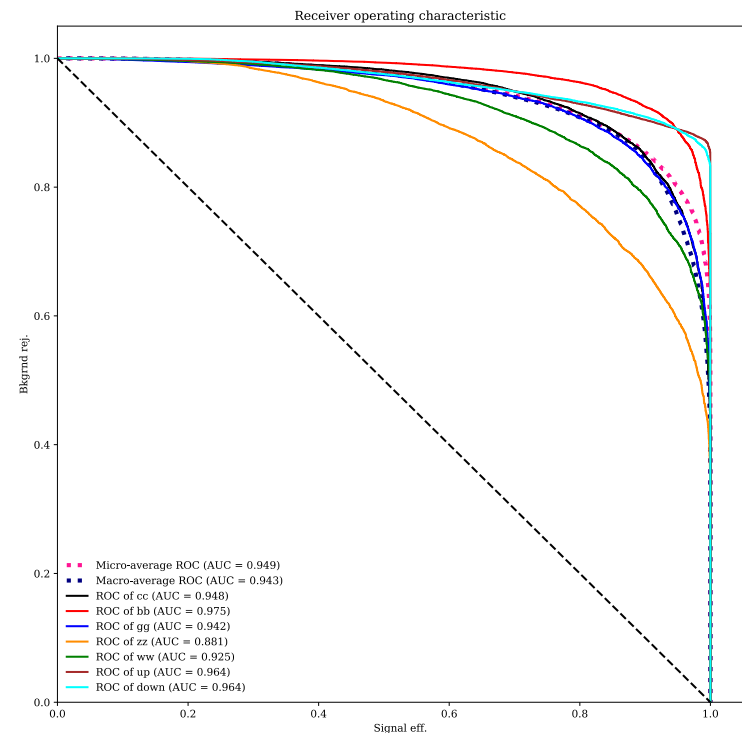
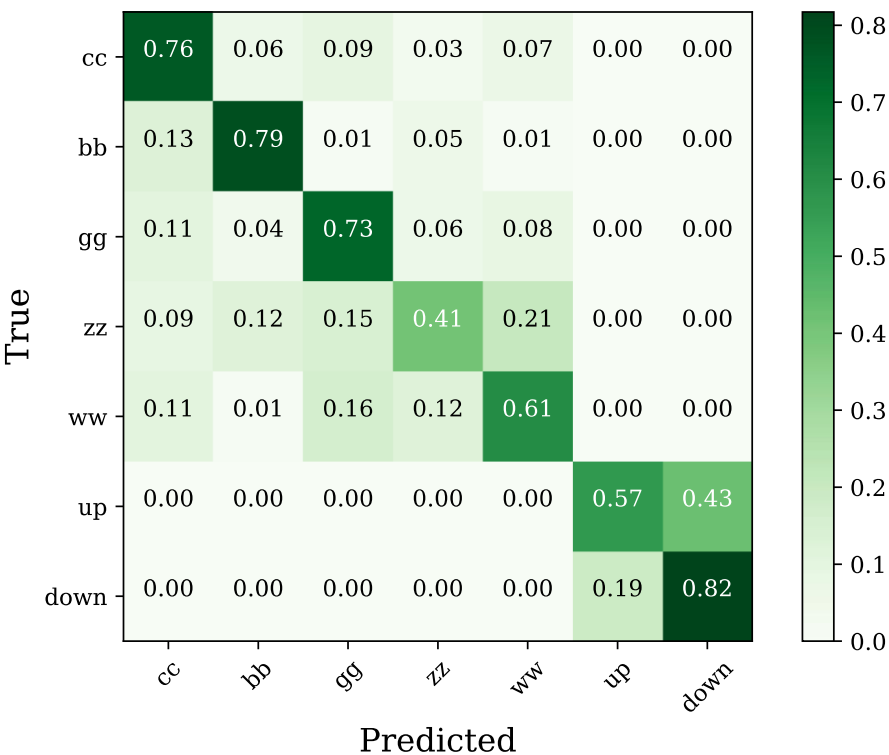
- $p_i$  : the information of particle i, such as four-momentum, charge, or Pid, impact parameter, etc

## ➤ Advantage

- Use all info at particle level,
- W/o impacts of jet clustering and  $e/\gamma$  isolation,
- Multi-classification is possible

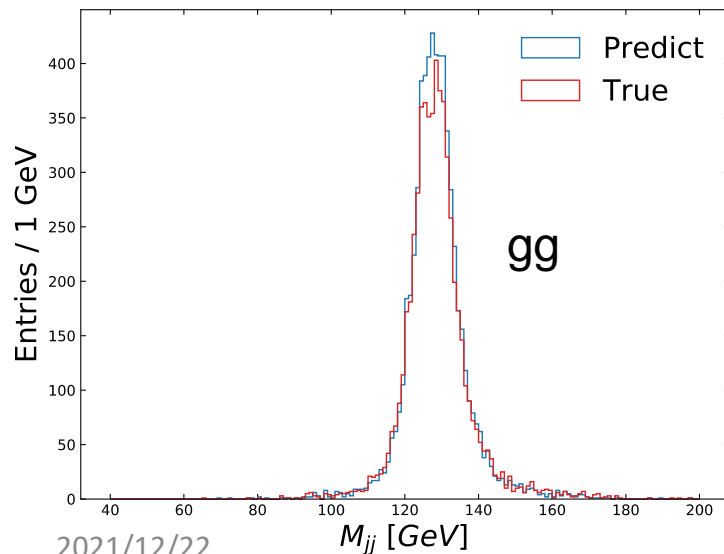
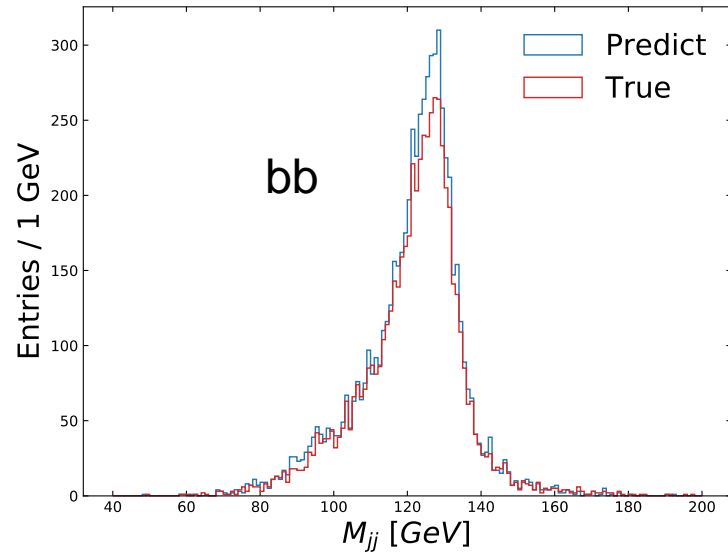
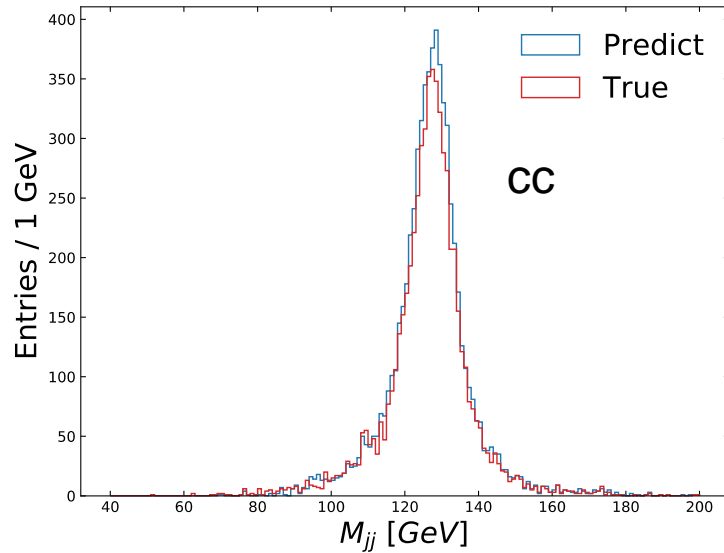
# Results of full simulation

- 5 signals:  $b\bar{b}/c\bar{c}/gg/ww^*/zz^*$
- 2 bkg:  $zz\_sl0mu\_down(down)$  and  $zz\_sl0mu\_up(up)$ .
- Train and validation got consistent losses and accuracies: successful training
- The separation power of  $b\bar{b}$  is the best,  $ZZ^*$  not good as  $bb$



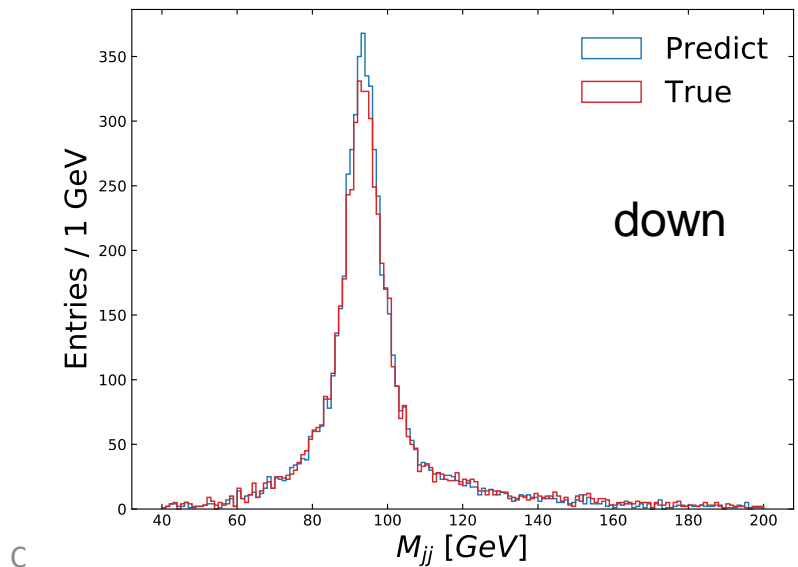
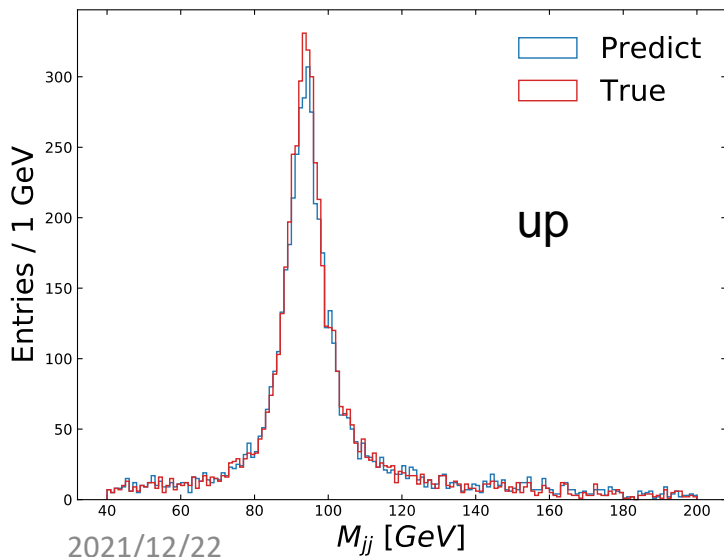
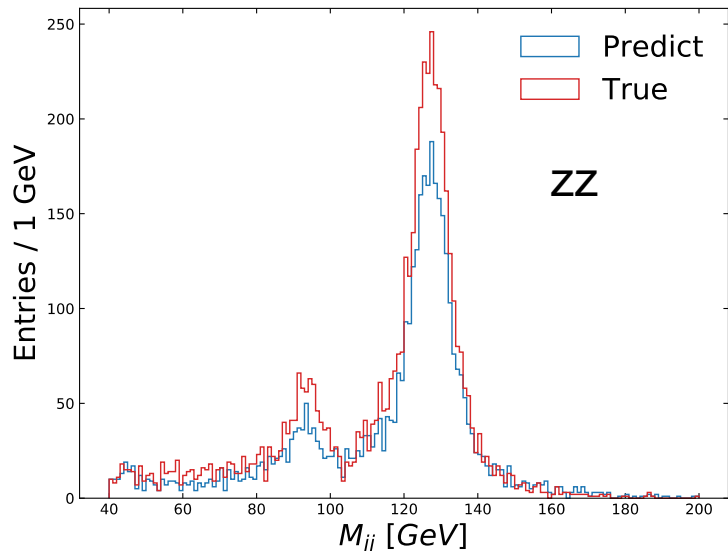
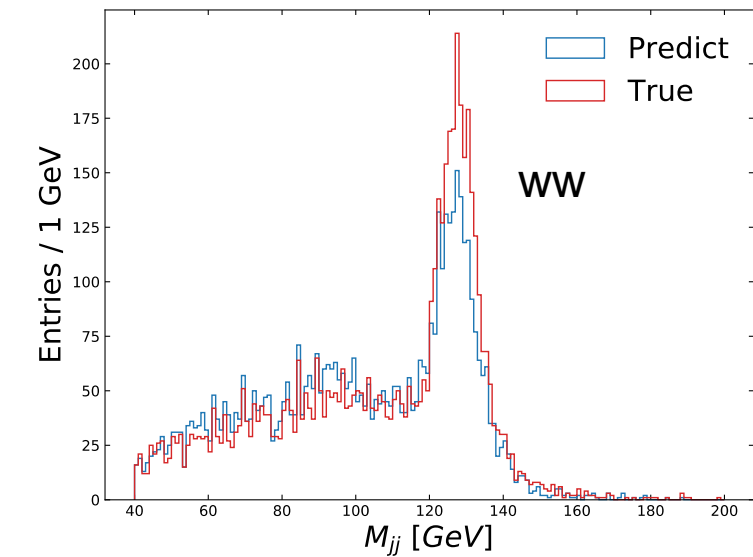


# Performance of PFN



- The comparison between true and prediction on the test sample(10% of the total).
- The predictions of cc, bb and gg are quite good, the differences between predict and true are small.

# Performance of PFN



2021/12/22

# Results

- Use test samples(10% of MC events) to perform the study.
- Scale the MC events according to the cross-section  $\times$  integrated lumi ( $5.6 \text{ ab}^{-1}$ )

	$c\bar{c}$	$b\bar{b}$	$gg$	$zz$	$ww$	up	down
$n$	$1272 \pm 36$	$21435 \pm 146$	$3689 \pm 61$	$8822 \pm 94$	$11709 \pm 34$	$66245 \pm 57$	$105853 \pm 325$
$\hat{N}$	$1079 \pm 33$	$21389 \pm 146$	$3177 \pm 56$	$14189 \pm 19$	$107436 \pm 328$	$72711 \pm 70$	$97784 \pm 13$
$N$	$1089 \pm 33$	$21539 \pm 147$	$3079 \pm 55$	$14430 \pm 119$	$108045 \pm 329$	$72729 \pm 70$	$98448 \pm 14$

$n$  : observed number of events of each channel,

$\hat{N}$  : the true number of events of each channel,

$N$  : the number of events of each channel, calculated from observed number.

# Next to do

- More backgrounds and more statistics
- Optimize the performance of ML model
- Extract the branch ratios with more sophisticated statistical method
- The systematic uncertainties

# Part II

## Probe the Higgs CP

# Introduction

- The SM Higgs:  $m_H = 125.10 \text{ GeV}$ ,  $J^{PC} = 0^{++}$
- Related experiments in LHC:
  - Spin is well determined
  - Study of the CP of the Higgs boson interactions with gauge bosons by the ATLAS and CMS shows no deviations from the SM predictions.
  - Sensitivity need to be improved
- CP could be the mixture of even and odd
- **Any observation of CP odd components of Higgs would be New Physics!**

[Eur. Phys. J. C75 \(2015\) 476](#)

# Theory of $H \rightarrow ZZ^*$

[JHEP 03\(2016\) 050](#)

[JHEP 11\(2014\) 028](#)

The  $H \rightarrow Zll$  matrix element:

$$\mathcal{M}_{HZZ\ell\ell}^\mu = \frac{1}{m_H} \bar{u}(p_3, s_3) \left[ \gamma^\mu (H_{1,V} + H_{1,A} \gamma_5) + \frac{q^\mu \not{p}}{m_H^2} (H_{2,V} + H_{2,A} \gamma_5) + \frac{\epsilon^{\mu\nu\sigma\rho} p_\nu q_\sigma}{m_H^2} \gamma_\rho (H_{3,V} + H_{3,A} \gamma_5) \right] v(p_4, s_4)$$

- Where  $\epsilon_{0123} = +1$  and  $q = p_3 + p_4$ .

And the parameters in the function are following:

$$\begin{aligned} H_{1,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}r}{r-s} g_V \left( 1 + \hat{\alpha}_1^{\text{eff}} - \frac{\kappa}{r} \hat{\alpha}_{ZZ} - \frac{\kappa}{2r} \frac{Q_\ell g_{em}(r-s)}{sg_V} \hat{\alpha}_{AZ} \right) \\ H_{1,A} &= \frac{2m_H(\sqrt{2}G_F)^{1/2}r}{r-s} g_A \left( 1 + \hat{\alpha}_2^{\text{eff}} - \frac{\kappa}{r} \hat{\alpha}_{ZZ} \right), \\ H_{2,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}}{r-s} g_V \left[ 2\hat{\alpha}_{ZZ} + \frac{Q_\ell g_{em}(r-s)}{sg_V} \hat{\alpha}_{AZ} \right] \\ H_{2,A} &= \frac{4m_H(\sqrt{2}G_F)^{1/2}}{r-s} g_A \hat{\alpha}_{ZZ} \\ H_{3,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}}{r-s} g_V \left[ 2\hat{\alpha}_{ZZ} + \frac{Q_\ell g_{em}(r-s)}{sg_V} \hat{\alpha}_{AZ} \right] \\ H_{3,A} &= \frac{4m_H(\sqrt{2}G_F)^{1/2}}{r-s} g_A \hat{\alpha}_{ZZ} \end{aligned}$$

$$\begin{aligned} \hat{\alpha}_1^{\text{eff}} &\equiv \hat{\alpha}_{ZZ}^{(1)} - \frac{m_H(\sqrt{2}G_F)^{1/2}(r-s)}{2\sqrt{r}} \frac{\hat{\alpha}_{\Phi l}^V}{g_V} \\ \hat{\alpha}_2^{\text{eff}} &\equiv \hat{\alpha}_{ZZ}^{(1)} + \frac{m_H(\sqrt{2}G_F)^{1/2}(r-s)}{2\sqrt{r}} \frac{\hat{\alpha}_{\Phi l}^A}{g_A} \end{aligned}$$

   : SM term  
Others : EFT contribution

This process limited by statistics

# cross symmetry: $Z^* \rightarrow ZH$ :

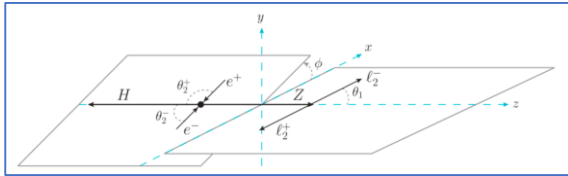
[JHEP 03\(2016\) 050](#)

[JHEP 11\(2014\) 028](#)

Differential cross section for  $e^+e^- \rightarrow Z^* \rightarrow ZH \rightarrow llH$ :

$$\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{\mathcal{N}_\sigma(q^2)}{m_H^2} \mathcal{J}(q^2, \theta_1, \theta_2, \phi),$$

$$\mathcal{N}_\sigma(q^2) = \frac{1}{2^{10}(2\pi)^3} \cdot \frac{1}{\sqrt{r}\gamma_Z} \cdot \frac{\sqrt{\lambda(1,s,r)}}{s^2}$$



$$\begin{aligned} \mathcal{J}(q^2, \theta_1, \theta_2, \phi) = & J_1(1 + \cos^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 + \cos^2 \theta_2) \\ & + J_2 \sin^2 \theta_1 \sin^2 \theta_2 + J_3 \cos \theta_1 \cos \theta_2 \\ & + (J_4 \sin \theta_1 \sin \theta_2 + J_5 \sin 2\theta_1 \sin 2\theta_2) \sin \phi \\ & + (J_6 \sin \theta_1 \sin \theta_2 + J_7 \sin 2\theta_1 \sin 2\theta_2) \cos \phi \\ & + J_8 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi + J_9 \sin^2 \theta_1 \sin^2 \theta_2 \cos 2\phi. \end{aligned}$$

Variables for studying distribution:  $\theta_1, \theta_2, \phi$

Assumption for simplification:

- $\delta_{GF} = \hat{\alpha}_{\phi l}^V = \hat{\alpha}_{\phi l}^A = \hat{\alpha}_{AZ} = \hat{\alpha}_{ZZ} = 10^{-3}$ , others are set to 0, so  $H_{2,V/A} = 0$ .
- $\hat{\alpha}_{AZ}$  and  $\hat{\alpha}_{ZZ}$  have the most contribution for cp-odd.

$$J_1 = 2rs(g_A^2 + g_V^2)(|H_{1,V}|^2 + |H_{1,A}|^2),$$

$$J_2 = \kappa(g_A^2 + g_V^2) [\kappa(|H_{1,V}|^2 + |H_{1,A}|^2) + \lambda \text{Re}(H_{1,V}H_{2,V}^* + H_{1,A}H_{2,A}^*)],$$

$$J_3 = 32rs g_A g_V \text{Re}(H_{1,V}H_{1,A}^*),$$

$$J_4 = 4\kappa\sqrt{rs}\lambda g_A g_V \text{Re}(H_{1,V}H_{3,A}^* + H_{1,A}H_{3,V}^*),$$

$$J_5 = \frac{1}{2}\kappa\sqrt{rs}\lambda(g_A^2 + g_V^2) \text{Re}(H_{1,V}H_{3,V}^* + H_{1,A}H_{3,A}^*),$$

$$J_6 = 4\sqrt{rs}g_A g_V [4\kappa \text{Re}(H_{1,V}H_{1,A}^*) + \lambda \text{Re}(H_{1,V}H_{2,A}^* + H_{1,A}H_{2,V}^*)],$$

$$J_7 = \frac{1}{2}\sqrt{rs}(g_A^2 + g_V^2) [2\kappa(|H_{1,V}|^2 + |H_{1,A}|^2) + \lambda \text{Re}(H_{1,V}H_{2,V}^* + H_{1,A}H_{2,A}^*)],$$

$$J_8 = 2rs\sqrt{\lambda}(g_A^2 + g_V^2) \text{Re}(H_{1,V}H_{3,V}^* + H_{1,A}H_{3,A}^*),$$

$$J_9 = 2rs(g_A^2 + g_V^2)(|H_{1,V}|^2 + |H_{1,A}|^2).$$

6 of these 9 functions are independent

— 0 in assumption  
 □ EFT CP-odd term  
 Others CP-even contribution

More statistics & negligible backgrounds:  $\mu\mu H$  has ~36k signals at CEPC



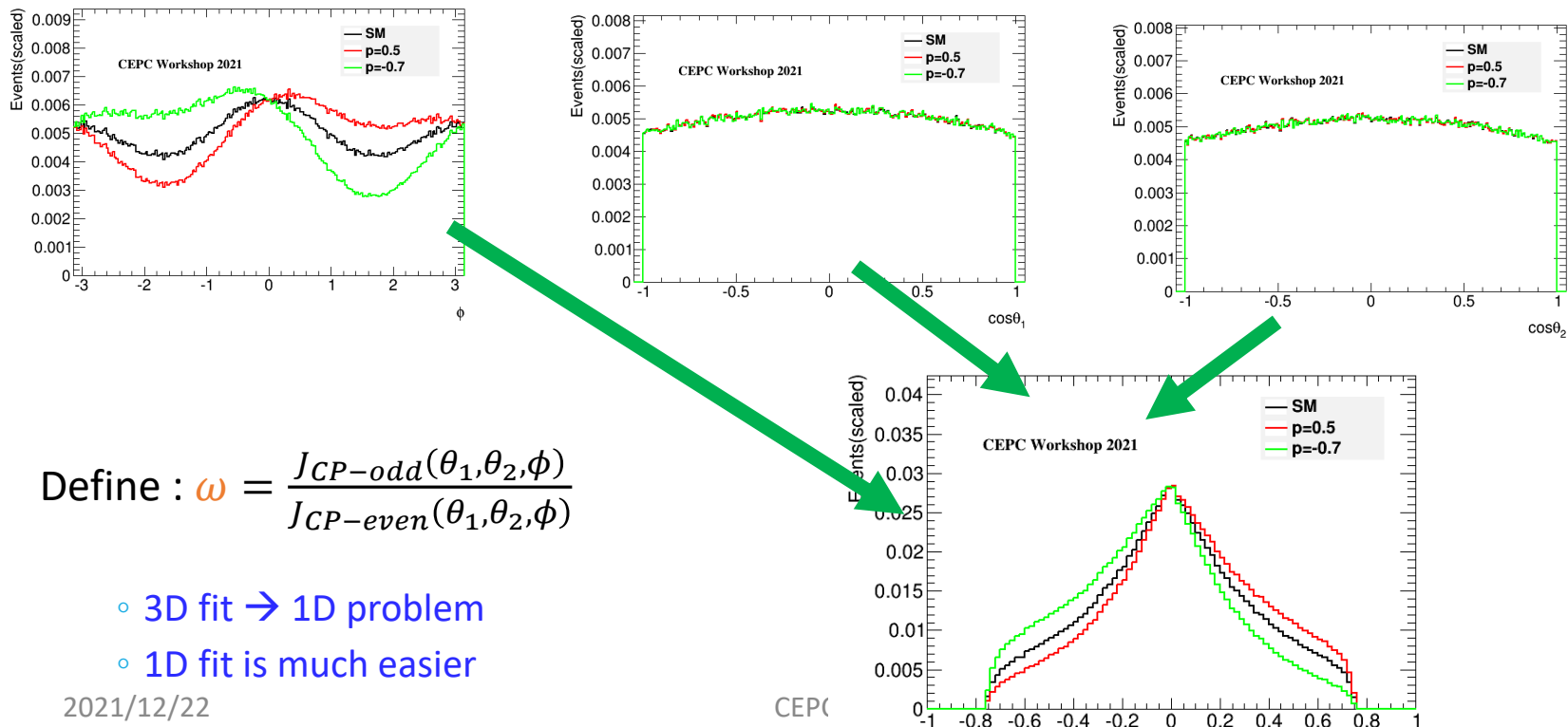
## Dimension reduction: an optimal variable approach

- Differential cross section could be expressed as:

[PLB 306 \(1993\) 411-417](#) By M. Davier

$$\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2 d\phi} = N \times (J_{CP-even}(\theta_1, \theta_2, \phi) + p \times J_{CP-odd}(\theta_1, \theta_2, \phi)).$$

$p$  is an additional global CP-mixing parameter.



# Event selection

- **Signal:**  $e^+e^- \rightarrow ZH \rightarrow \mu^+\mu^-H(\rightarrow jj)$  channel
- **Background:** Irreducible background which contains the same final states.

- Muon pair selection:

$$|\cos\theta_{\mu^+\mu^-}| < 0.81; \quad \text{Mass}_{\mu\mu} \in (77.5\text{GeV}, 104.5\text{GeV}); \quad M_{recoil_{\mu\mu}} \in (124\text{GeV}, 140\text{GeV}).$$

- Jets pair selection:

$$|\cos\theta_{jet}| < 0.96; \quad \text{Mass}_{jj} \in (100\text{GeV}, 150\text{GeV}).$$

# Results of event selection

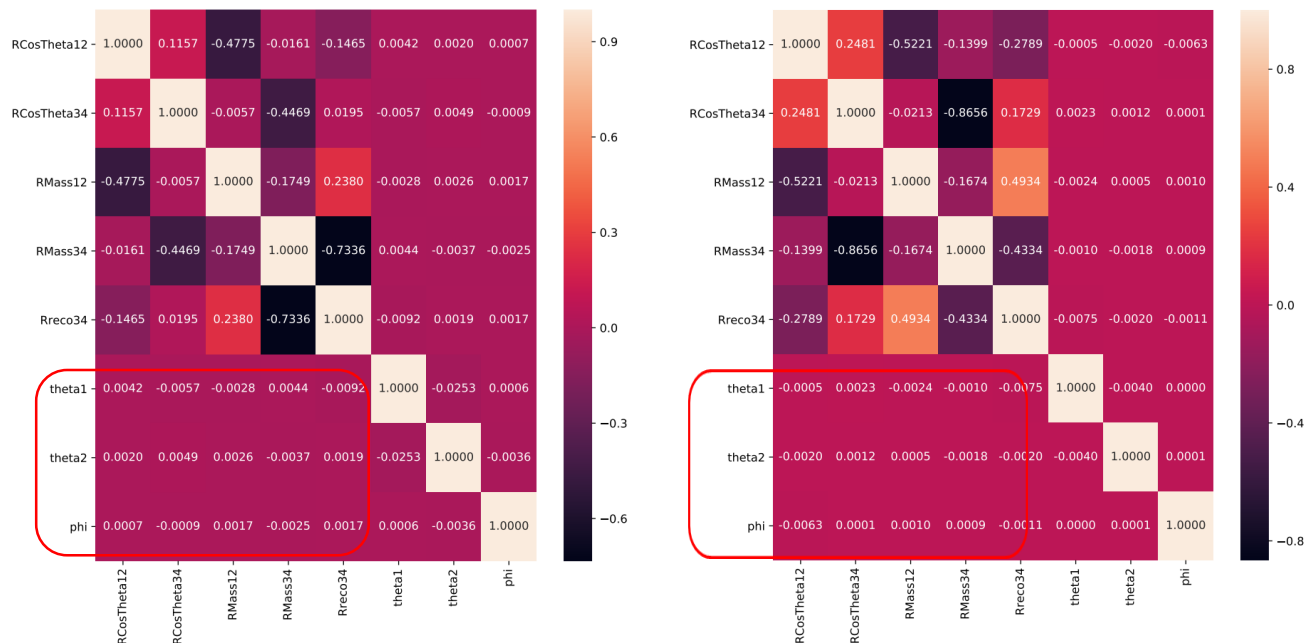
$ZH \rightarrow \mu^+ \mu^- + b\bar{b}/c\bar{c}/gg$ channel		
	Signal	Irreducible Bkg
Original	28627	1251768
Muon pair selection	18555 (efficiency:64.8%)	11198 (efficiency:0.9%)
All selection	13405 (efficiency:46.8%)	3610 (efficiency:0.3%)

# Higgs CP-mixing measurement

- Correlation check

- We can see that  $\theta_1$ ,  $\theta_2$ ,  $\phi$  have negligible correlation with

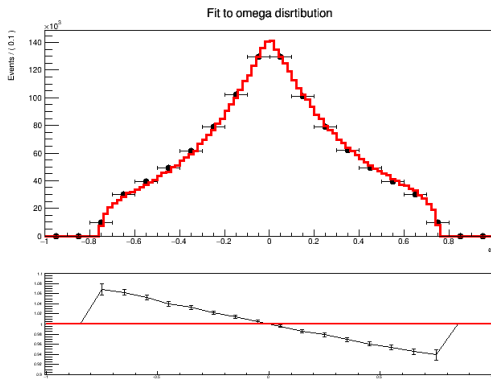
$\cos\theta_{\mu^+\mu^-}$ ,  $\text{Mass}_{\mu\mu}$ ,  $M_{\text{recoil}_{\mu\mu}}$ ,  $\cos\theta_{\text{jet}}$ ,  $\text{Mass}_{jj}$ .



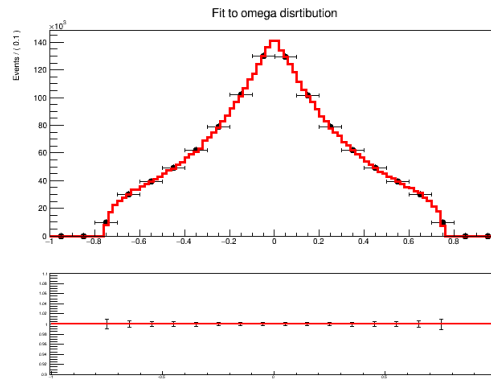
- Impacts of event selections on  $\theta_1$ ,  $\theta_2$ , and  $\phi$  neglected next

# Fitting strategy and result

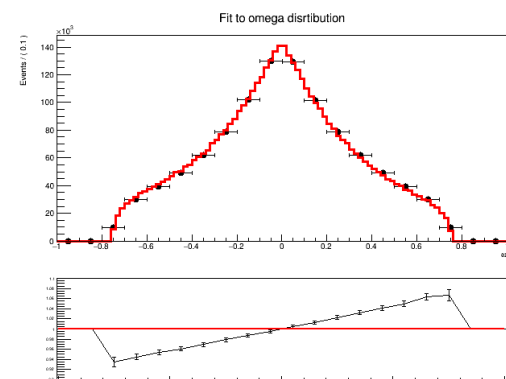
- Fit strategy:
  - Maximum-likelihood of  $f^p(\omega) = N_{sig} * f_{sig}^p(\omega) + N_{bkg} * f_{bkg}^p(\omega)$
  - Fit to  $\omega$  for signal and bkg shape,  $f_{sig}^p(\omega)$  and  $f_{bkg}^p(\omega)$
  - Fit to  $M_{recoil_{\mu\mu}}$  for  $N_{sig}$  and  $N_{bkg}$
  - Evaluate likelihood function for each p value hypothesis, and construct a  $\Delta NLL$  as a function of p.



P= 0.09 vs p= 0



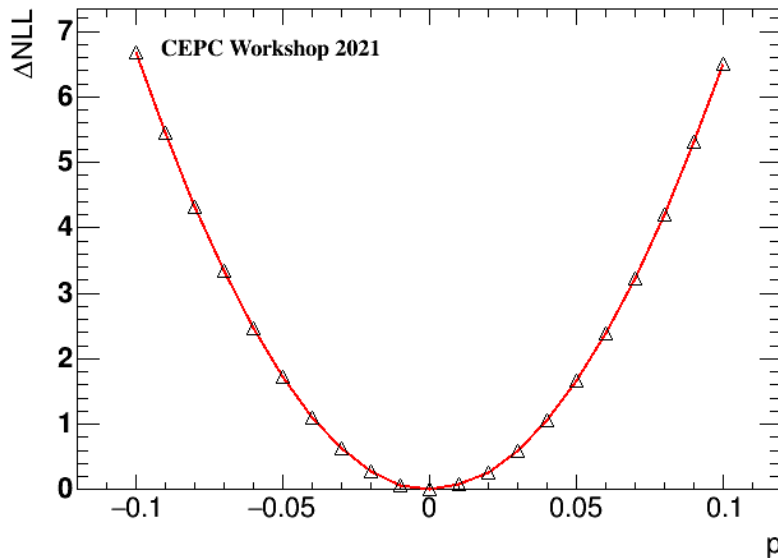
P= 0 vs p= 0



P= -0.09 vs p= 0

# Fitting strategy and result

- Extract maximum-likelihood fit p-value and interval
  - Fit  $\Delta NLL$  curve with a quadratic function  $\Delta NLL(p) = a \cdot (p - p_0)^2$
  - 68%(95%) CL interval corresponds to  $\Delta NLL=0.5(1.96)$ .
  - Assumption:  $\hat{\alpha}_{A\tilde{Z}} = \hat{\alpha}_{Z\tilde{Z}} = 10^{-3}$ .



$$\Delta NLL(p|\omega) = 659.6(p - 5.6 \times 10^{-4})^2$$

For  $p$ :

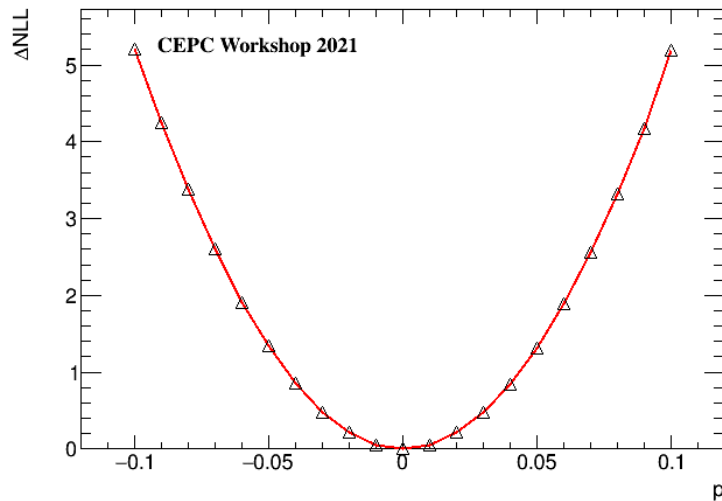
68% CL:  $[-2.79 \times 10^{-2}, 2.70 \times 10^{-2}]$

95% CL:  $[-5.52 \times 10^{-2}, 5.40 \times 10^{-2}]$

$$\hat{\alpha}_{A\tilde{Z}} = \hat{\alpha}_{Z\tilde{Z}} = 10^{-3} \times p$$

# Fitting strategy and result

- Extract maximum-likelihood fit p-value and interval
  - Fit  $\Delta NLL$  curve with a quadratic function  $\Delta NLL(p) = a \cdot (p - p_0)^2$
  - 68%(95%) CL interval corresponds to  $\Delta NLL=0.5(1.96)$ .
  - Assumption:  $\hat{\alpha}_{A\tilde{Z}} = 10^{-3}$ ,  $\hat{\alpha}_{Z\tilde{Z}} = 0$ .



$$\Delta NLL(p|\omega) = 519.53(p - 2.32 \times 10^{-4})^2$$

For  $p$ :

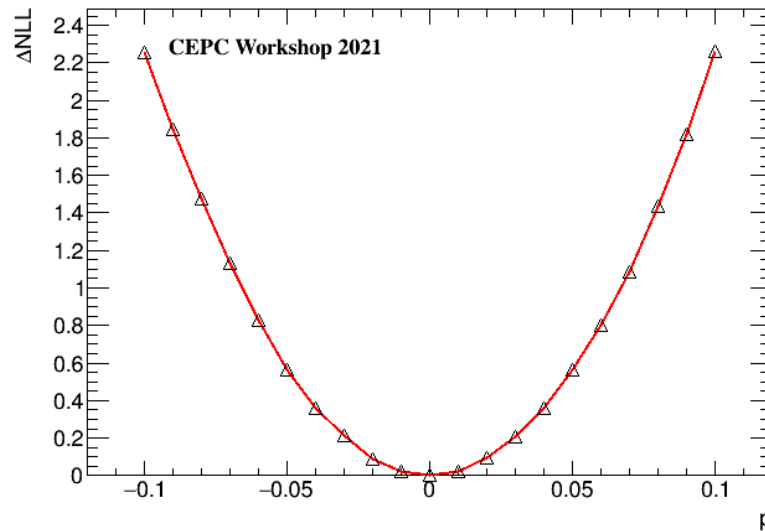
68% CL:  
 $[-3.13 \times 10^{-2}, 3.08 \times 10^{-2}]$

95% CL:  
 $[-6.17 \times 10^{-2}, 6.12 \times 10^{-2}]$

$$\hat{\alpha}_{A\tilde{Z}} = 10^{-3} \times p$$

# Fitting strategy and result

- Extract maximum-likelihood fit p-value and interval
  - Fit  $\Delta NLL$  curve with a quadratic function  $\Delta NLL(p) = a \cdot (p - p_0)^2$
  - 68%(95%) CL interval corresponds to  $\Delta NLL=0.5(1.96)$ .
  - Assumption:  $\hat{\alpha}_{A\tilde{Z}} = 0$ ,  $\hat{\alpha}_{Z\tilde{Z}} = 5 \cdot 10^{-3}$ .



$$\Delta NLL(p|\omega) = 226.22(p - 2.73 \times 10^{-4})^2$$

For  $p$ :

68% CL:  
 $[-4.73 \times 10^{-2}, 4.67 \times 10^{-2}]$

95% CL:  
 $[-9.34 \times 10^{-2}, 9.28 \times 10^{-2}]$

$$\hat{\alpha}_{Z\tilde{Z}} = 5 \cdot 10^{-3} \times p$$



# Result compare

- HL-LHC: ( $1\sigma$ ) [arXiv:1902.00134](https://arxiv.org/abs/1902.00134)

Parameter Analysis	$\tilde{c}_{Z\gamma}$	$\tilde{c}_{ZZ}$	Case
HL-LHC ( $4\ell$ , incl.)	$[-0.22, 0.22]$	$[-0.33, 0.33]$	1P
	$[-0.25, 0.25]$	$[-0.27, 0.27]$	1P <sub>marg.</sub>
HL-LHC ( $4\ell$ , diff.)	$[-0.10, 0.10]$	$[-0.31, 0.31]$	1P
	$[-0.13, 0.13]$	$[-0.22, 0.22]$	1P <sub>marg.</sub>
HE-LHC ( $4\ell$ , incl.)	$[-0.18, 0.18]$	$[-0.17, 0.17]$	1P
	$[-0.23, 0.23]$	$[-0.20, 0.20]$	1P <sub>marg.</sub>
HE-LHC ( $4\ell$ , diff.)	$[-0.05, 0.05]$	$[-0.13, 0.13]$	1P
	$[-0.06, 0.06]$	$[-0.10, 0.10]$	1P <sub>marg.</sub>

This study:

	$\tilde{c}_{Z\gamma}$	$\tilde{c}_{ZZ}$
68% CL( $1\sigma$ )	$[-2.70 \times 10^{-4}, 2.66 \times 10^{-4}]$	$[-1.73 \times 10^{-4}, 1.70 \times 10^{-4}]$
95% CL( $2\sigma$ )	$[-5.32 \times 10^{-4}, 5.28 \times 10^{-4}]$	$[-3.41 \times 10^{-4}, 3.39 \times 10^{-4}]$

# Summary

- A Higgs hadronic decay study is ongoing with ML method and full simulation shows promising performance.
- An EFT based Higgs CP-mixing test is performed.
  - Using ML-fit to optimal variable  $\omega$  and extract  $p$ .
  - Result: 95% CL  $p \in [ - 5.5 \times 10^{-2}, 5.4 \times 10^{-2} ]$ ,
    - Sensitivity :  $\delta G_F, \hat{\alpha}_{\phi l}^V, \hat{\alpha}_{\phi l}^A, \hat{\alpha}_{A\tilde{Z}}, \hat{\alpha}_{Z\tilde{Z}} < 10^{-4}$ ,  
much better than LHC
- Both two studies need more validations and to be finalized in near future

# Backup

# Method to extract production numbers

➤ A simple example, only  $H \rightarrow b\bar{b}$  and  $H \rightarrow c\bar{c}$ .

$$\begin{pmatrix} n_b \\ n_c \end{pmatrix} = \begin{pmatrix} \epsilon_{bb} & \epsilon_{bc} \\ \epsilon_{cb} & \epsilon_{cc} \end{pmatrix} \begin{pmatrix} N_b \\ N_c \end{pmatrix} \quad n = EN$$

- $n_i$  : the observed number of events of i class,
- $N_i$  : the production number of events of i class,
- $\epsilon_{ij}$  : the rate of state i reconstructed to be state j.

➤ If we can measure the matrix E, then  $N = E^{-1}n$

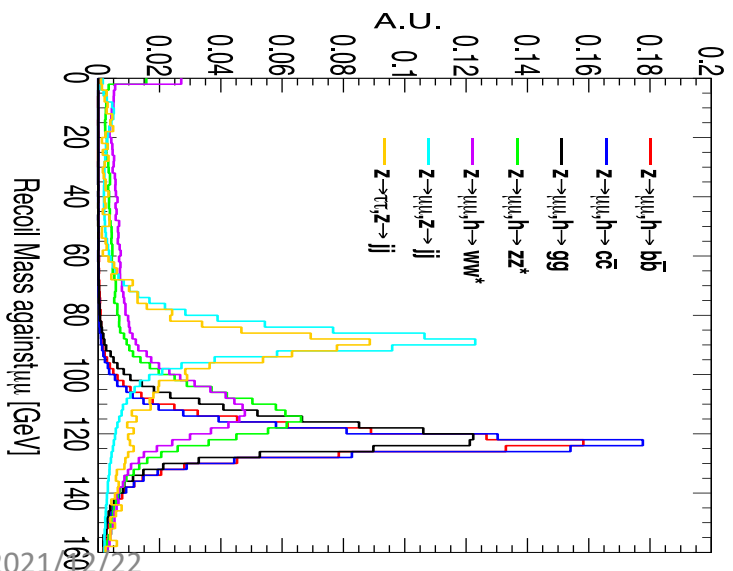
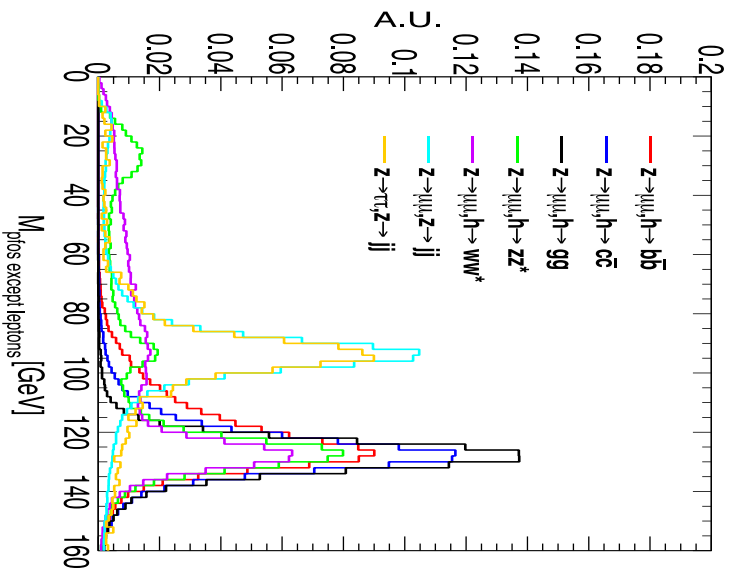
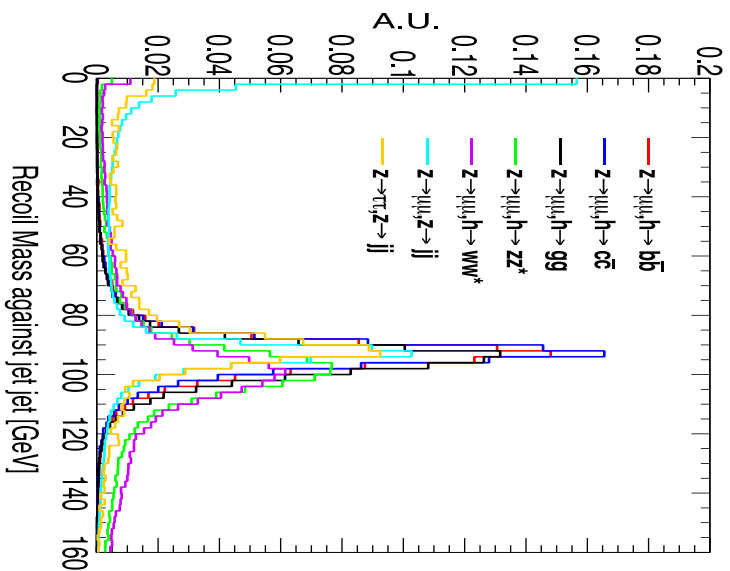
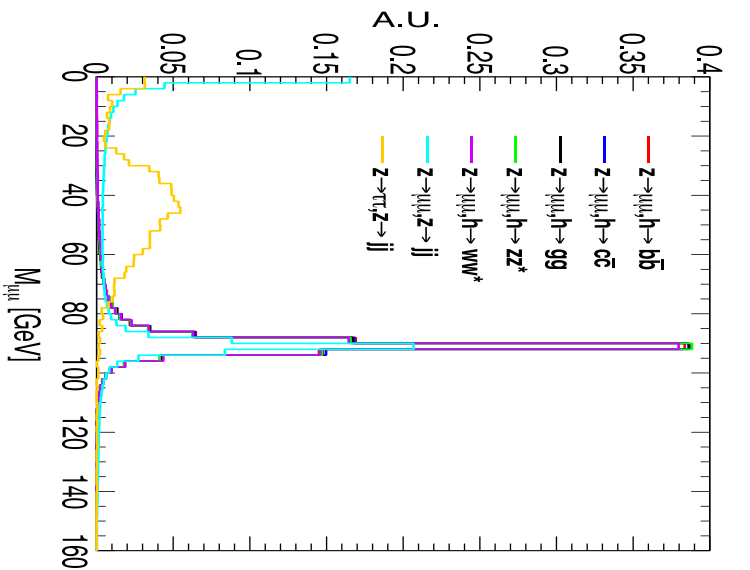
➤ The PFN is used to extract the matrix.

# Backup

Table 2. Uncertainties on  $\sigma_{l^+l^-H}^{b\bar{b}}$ ,  $\sigma_{l^+l^-H}^{c\bar{c}}$  and  $\sigma_{l^+l^-H}^{gg}$ .

Higgs boson production	$\mu^+\mu^-H$			$e^+e^-H$		
Higgs boson decay	$H \rightarrow b\bar{b}$	$H \rightarrow c\bar{c}$	$H \rightarrow gg$	$H \rightarrow b\bar{b}$	$H \rightarrow c\bar{c}$	$H \rightarrow gg$
statistic uncertainty	1.1%	10.5%	5.4%	1.6%	14.7%	10.5%
fixed background	-0.2%	+4.1%	7.6%	-0.2%	+4.1%	7.6%
	+0.1%	-4.2%		+0.1%	-4.2%	
event selection	+0.7%	+0.4%	+0.7%	+0.7%	+0.4%	+0.7%
	-0.2%	-1.1%	-1.7%	-0.2%	-1.1%	-1.7%
flavor tagging	-0.4%	+3.7%	+0.2%	-0.4%	+3.7%	+0.2%
	+0.2%	-5.0%	-0.7%	+0.2%	-5.0%	-0.7%
combined systematic uncertainty	+0.7%	+5.5%	+7.6%	+0.7%	+5.5%	+7.6%
	-0.5%	-6.6%	-7.8%	-0.5%	-6.6%	-7.8%

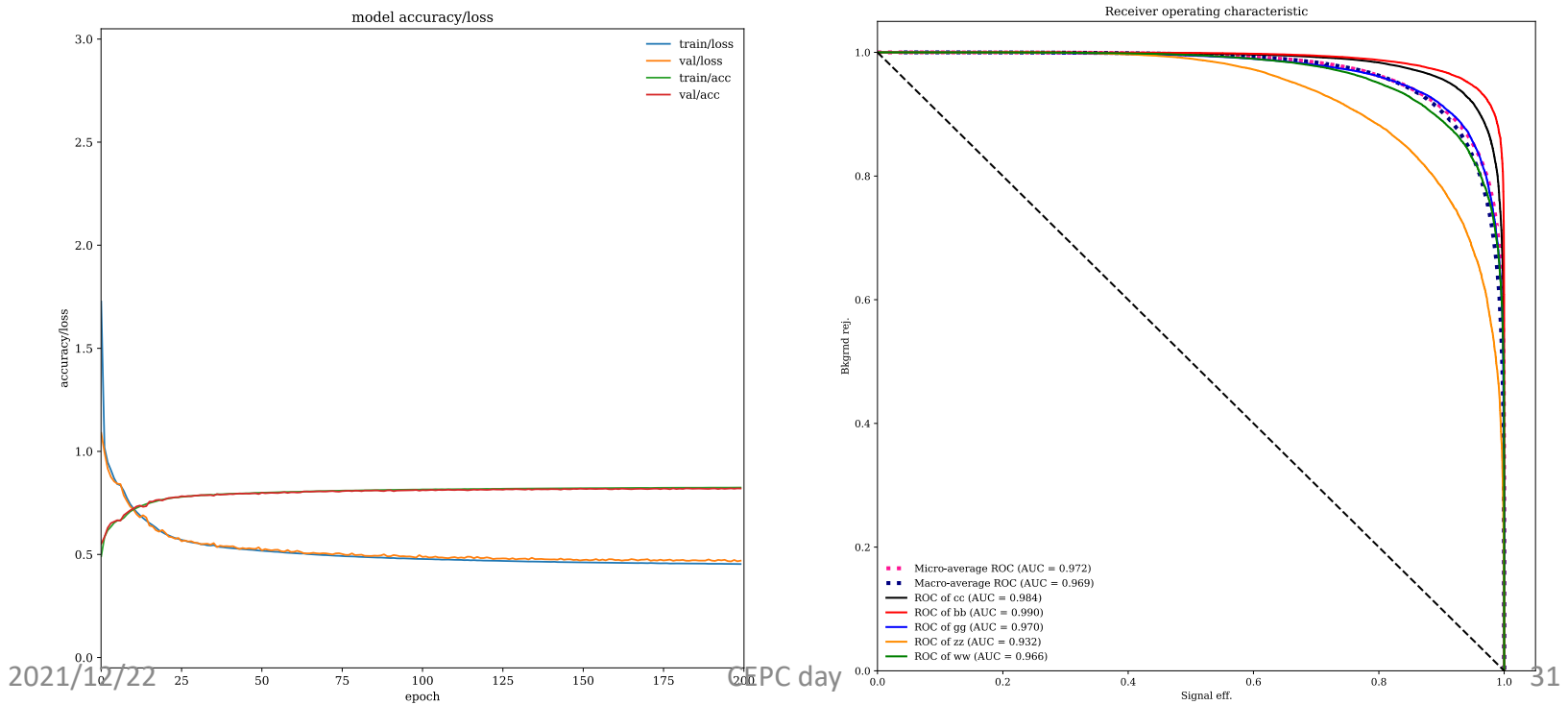
# Pre-selection



# Results of fast simulation

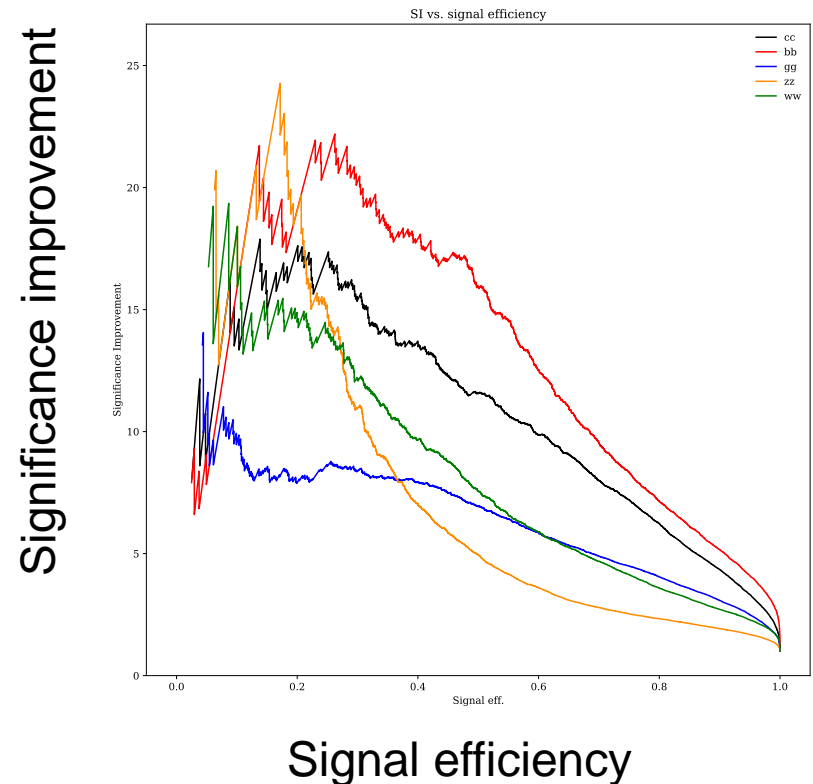
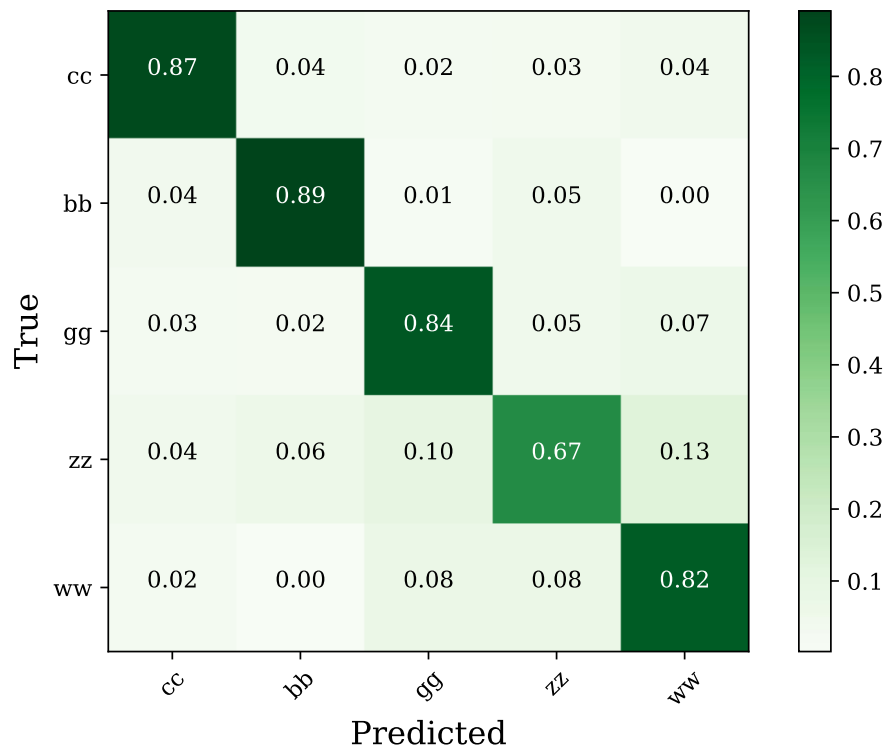
- Fast simulation sample : only has  $b\bar{b}/c\bar{c}/gg/ww^*/zz^*$ .
- Tiny difference at loss between train and validation.
- From the ROC curve, the separation power of  $b\bar{b}$  is highest,  $zz^*$  is lowest.

<https://github.com/Wujinfei/HiggsHadron-PFNs-gpu.git>



# Results of fast simulation

- The performance of PFN on fast simulation is good, except the  $zz^*$  calss.





# Comparison between fast and full simulation

- Why is the performance of full simulation worse than fast simulation:
  - Fast simulation has larger statistic than full simulation.
  - Maybe due to the reconstruction is not perfect.
  - Fewer training epochs of full simulation.
- Possible ways to improve the training performance
  - Include more input variables,
  - Generate more full simulation samples.

# Theory of $H \rightarrow ZZ^*$

[JHEP 03\(2016\) 050](#)

[JHEP 11\(2014\) 028](#)

In a 6-dimension EFT model:  $\mathcal{L}_{eff} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda^2} \sum_{k=1}^{59} \alpha_k \mathcal{O}_k (\mathcal{L}_{BSM})$

- $\mathcal{L}_{eff} \supset c_{ZZ}^{(1)} H Z_\mu Z^\mu + c_{ZZ}^{(2)} H Z_{\mu\nu} Z^{\mu\nu} + c_{Z\tilde{Z}} H Z_{\mu\nu} \tilde{Z}^{\mu\nu} + c_{AZ} H Z_{\mu\nu} A^{\mu\nu} + c_{A\tilde{Z}}^{HZ\mu\nu} \tilde{A}^{\mu\nu} + H Z_\mu \bar{\ell} \gamma^\mu (c_V + c_A \gamma_5) \ell + Z_\mu \bar{\ell} \gamma^\mu (g_V - g_A \gamma_5) \ell - g_{em} Q_\ell A_\mu \bar{\ell} \gamma^\mu \ell$

Where:  $c_{ZZ}^{(1)} = m_Z^2 (\sqrt{2} G_F)^{1/2} (1 + \hat{\alpha}_{ZZ}^{(1)})$ ,  $c_{ZZ}^{(2)} = (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{ZZ}$ ,  $c_{Z\tilde{Z}} = (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{Z\tilde{Z}}$ ,

$$c_{AZ} = (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{AZ}, \quad c_{A\tilde{Z}} = (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{A\tilde{Z}}.$$

- In this base, the  $G_F, m_Z, \alpha_{em}$  could be expressed

$$m_Z = m_{Z0} (1 + \delta_Z), \quad G_F = G_{F0} (1 + \delta_{G_F}), \quad \alpha_{em} = \alpha_{em0} (1 + \delta_A)$$

$$\text{where: } \delta_Z = \hat{\alpha}_{ZZ} + \frac{1}{4} \hat{\alpha}_{\Phi D}, \quad \delta_{G_F} = -\hat{\alpha}_{A l} + 2 \hat{\alpha}_{\Phi l}^{(3)}, \quad \delta_A = 2 \hat{\alpha}_{AA}.$$

# Compared with HL-LHC

- In HL-LHC:

[arXiv:1902.00134](https://arxiv.org/abs/1902.00134)

$$\mathcal{L}_{\text{CPV}} = \frac{H}{v} \left[ \tilde{c}_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} \tilde{A}^{\mu\nu} + \tilde{c}_{Z\gamma} \frac{e\sqrt{g_1^2 + g_2^2}}{2} Z_{\mu\nu} \tilde{A}^{\mu\nu} + \tilde{c}_{ZZ} \frac{g_1^2 + g_2^2}{4} Z_{\mu\nu} \tilde{Z}^{\mu\nu} + \tilde{c}_{WW} \frac{g_2^2}{2} W_{\mu\nu}^+ \tilde{W}^{\mu\nu} \right]$$

Compare theory model in [P5](#), we can get that the value in red frame are same:

$$(g_1=0.358, g_2=0.648, e=0.313, v = 1/\sqrt{\sqrt{2}G_F^0} = 2M_W/g \approx 246.22\text{GeV})$$

$$(\sqrt{2}G_F)^{1/2} \boxed{\hat{\alpha}_{ZZ}} H Z_{\mu\nu} \tilde{Z}^{\mu\nu} = \frac{H}{v} \boxed{\tilde{c}_{ZZ} \frac{g_1^2 + g_2^2}{4}} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \quad \frac{g_1^2 + g_2^2}{4} = 0.137$$

$$(\sqrt{2}G_F)^{1/2} \boxed{\hat{\alpha}_{AZ}} H Z_{\mu\nu} \tilde{A}^{\mu\nu} = \frac{H}{v} \boxed{\tilde{c}_{Z\gamma} \frac{e\sqrt{g_1^2 + g_2^2}}{2}} Z_{\mu\nu} \tilde{A}^{\mu\nu} \quad \frac{e\sqrt{g_1^2 + g_2^2}}{2} = 0.116$$

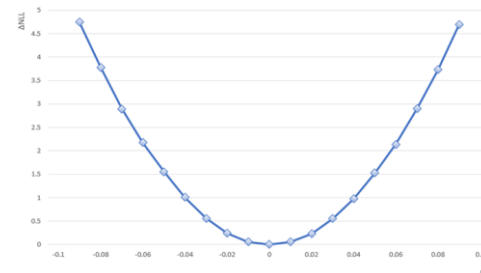
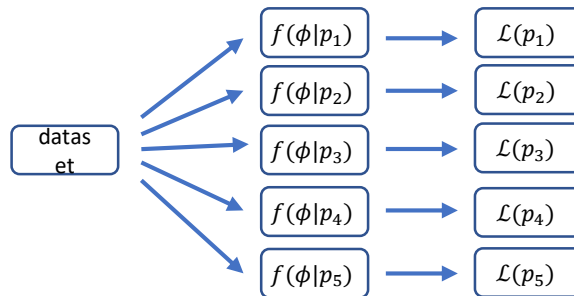
# Maximum likelihood fit

- Construct a likelihood function

- $\mathcal{L}(\vec{x}|p, \vec{\theta}) = \prod_{data} f(x_i | p, \vec{\theta})$

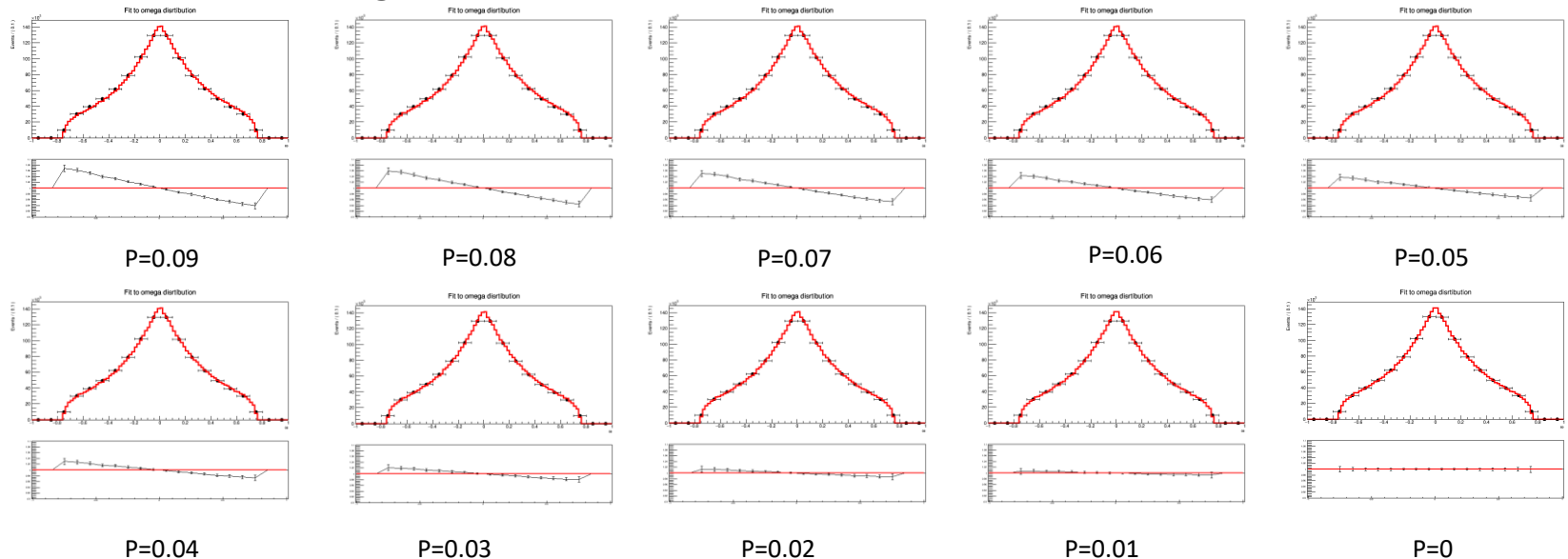
$\vec{\theta}$ : nuisance parameter.  $p$ : POI, CP-mixing parameter.  $x_i$ : dataset ( $\omega$ ).

- When statistics is large enough, we suppose  $\mathcal{L}(\vec{x}|p, \vec{\theta}) \sim \text{Gaus}(\hat{p}, \sigma_p^2)$ , so  $\ln \mathcal{L}(p) = \ln \mathcal{L}_{max} - \frac{1}{2} \left( \frac{p - \hat{p}}{\sigma_p} \right)^2$
- From  $\Delta NLL = NLL - NLL_{min}$  (negative log likelihood) we can extract maximum likelihood estimate  $\hat{p}$  and its CL interval.



# Maximum likelihood fit

- Sample modelling
  - $\omega$  modelling: Histogram pdf. Highly depends on the sample statistics used to build histogram and HistPdf.



# Maximum likelihood fit

- Sample modelling
  - $\omega$  modelling: Histogram pdf. Highly depends on the sample statistics used to build histogram and HistPdf.

