Perturbed spin tune and spin rotator

Xia Wenhao

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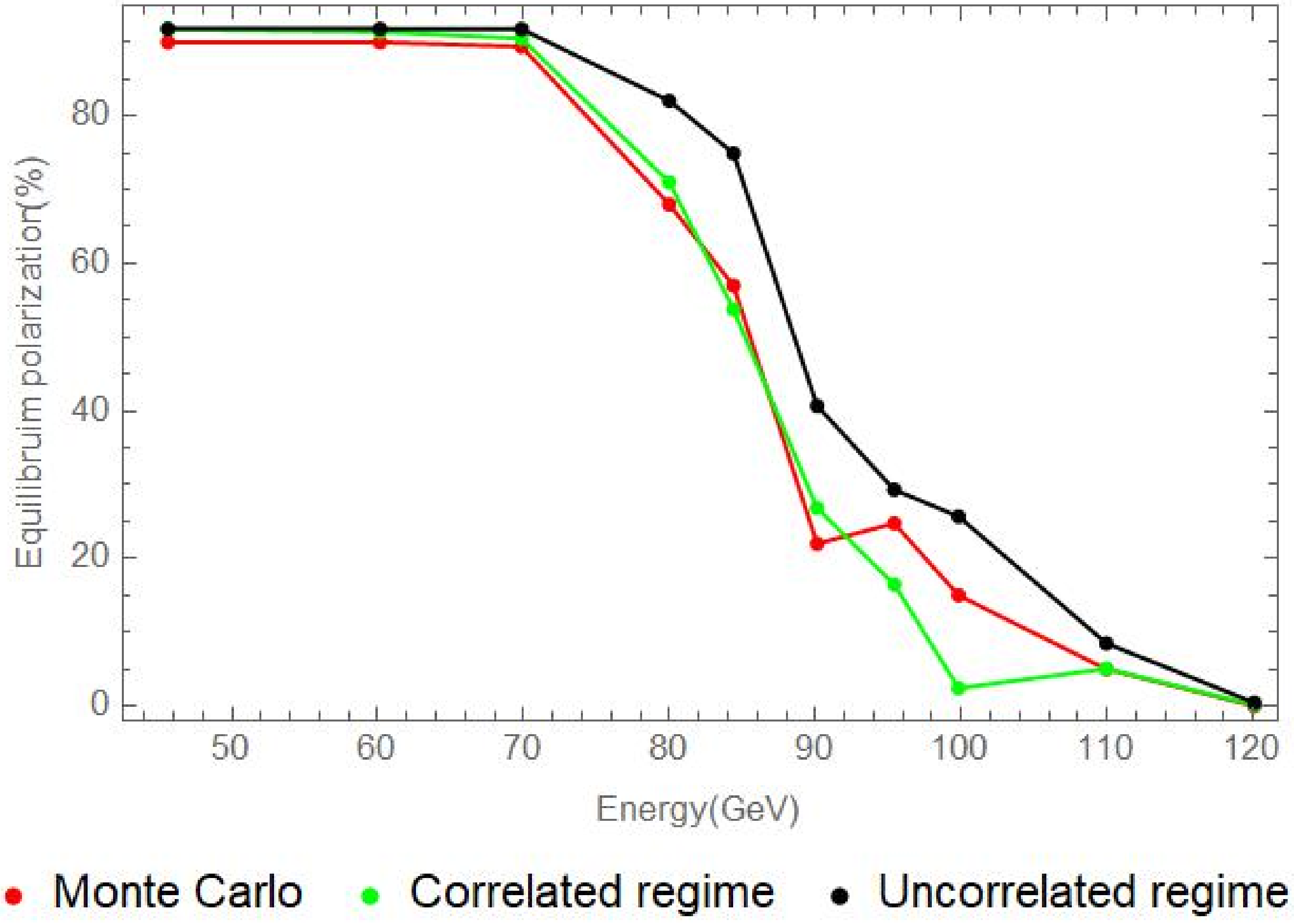
Questions???

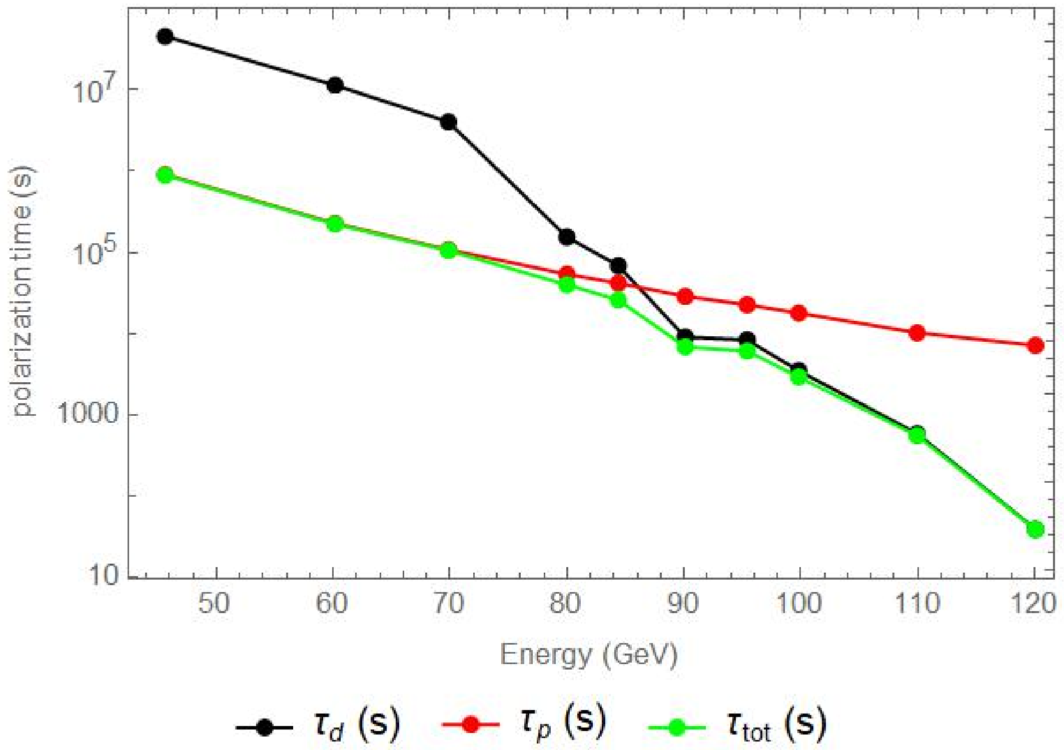
- What's the difference of the correlated and the uncorrelated regime?

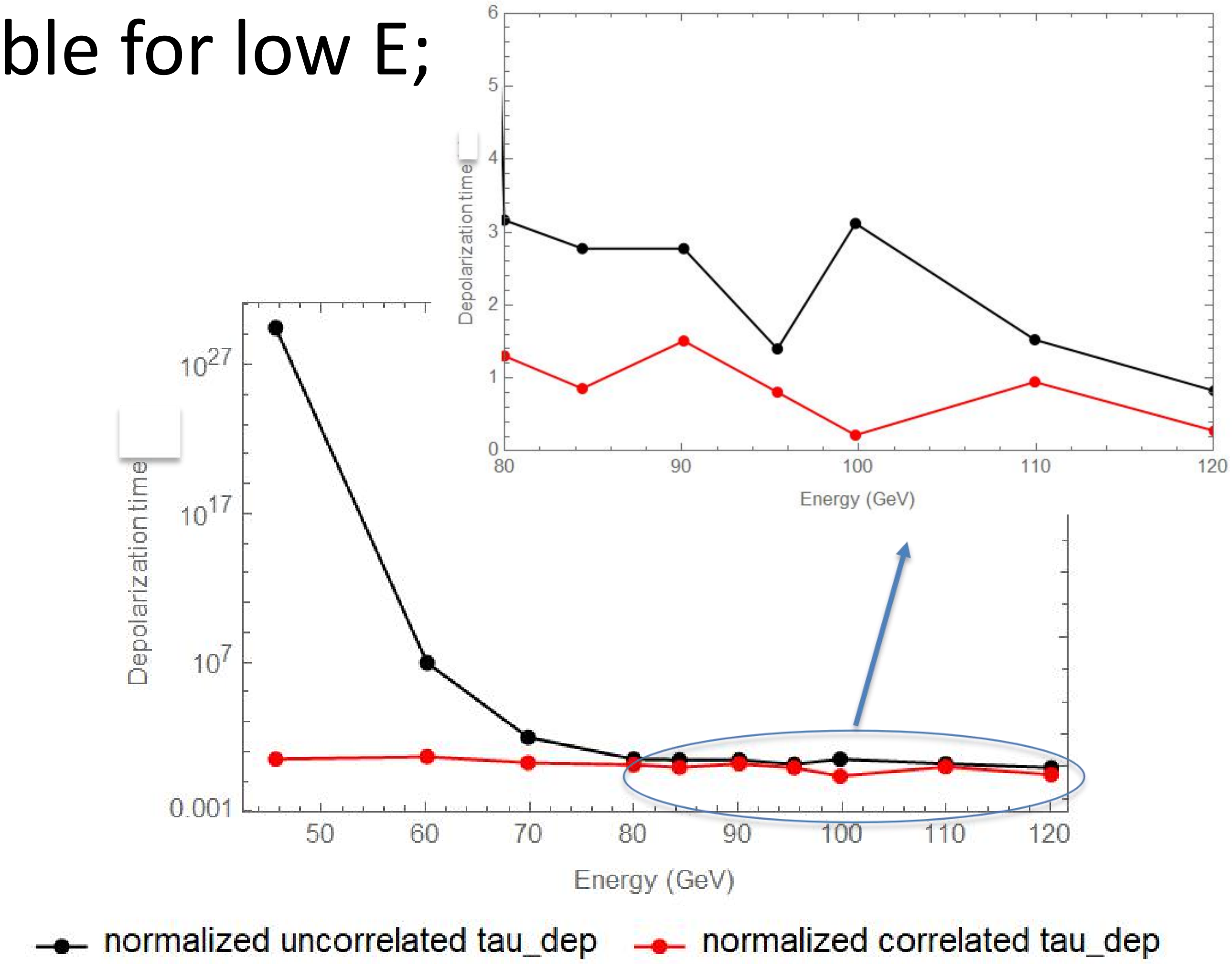


$$P_{eq} \approx \frac{P_\infty}{1 + \tau_p/\tau_d}$$

Depolarization time

- Depolarization time decreases as E↑ , MC ;
- Uncorrelated regime is not suitable for low E;





自旋工作点：实际&理想

- 共振退极化技术：
  - 理想：$E\,[MeV] = \nu \times 440.64843$
  - 实际：$E\,[MeV] = (\nu^{'} - \Delta\nu) \times 440.64843$
- Δν：
  - 水平磁场∝ (1+aγ): Δy (quad, sext), vertical kicker
  - 纵向磁场∝ (1+a)
- 自旋旋转：

$$M = exp\left[-i(\vec{\sigma}\cdot\hat{n})\frac{\phi}{2}\right]$$
$$= I\cos(\frac{\phi}{2}) - i(\vec{\sigma}\cdot\hat{n})\sin(\frac{\phi}{2})$$

$$\vec{\sigma} = (\sigma_x, \sigma_s, \sigma_z)$$

储存环中的自旋运动

- 理想,单圈：
  - $\vec{\sigma}\cdot\hat{\mathrm{n}}=\sigma_z$, ϕ=2πν ， cos(πν)=1/2 trace(M)

$$M = exp\left[-i(\vec{\sigma}\cdot\hat{n})\frac{\phi}{2}\right]$$
$$= I\cos(\frac{\phi}{2}) - i(\vec{\sigma}\cdot\hat{n})\sin(\frac{\phi}{2})$$

- 实际，单圈：

$$M = \prod_i T_i M_i$$

  - 水平磁场： $T_i = I\cos(\chi_i) - i\sigma_x\sin(\chi_i)$ ， $2\chi_i = \nu\alpha_i$
  - 垂直磁场： $M_i = I\cos\left(\frac{\Phi_{i+1,i}}{2}\right) - i\sigma_z\sin\left(\frac{\Phi_{i+1,i}}{2}\right)$ ， $\Phi_{i+1,i} = \Phi(\theta_{i+1}) - \Phi(\theta_i)$ and $\Phi(\theta_i) = \int_0^{\theta_i}\nu K d\theta$
  - 工作点：cos(πν’)=1/2 trace(M)

*A.V. Bogomyagkov,“**Influence of the vertical closed orbit distortions on accuracy of the energy calibration done by resonant depolarization technique**.”

自旋工作点偏移

- 考虑N个自旋扰动：

$$\cos[\pi\nu]-\cos[\pi\nu^{'}]=\cos[\pi\nu]\sum_{i=1}^{N}\frac{\chi_i^2}{2}+\sum_{j>i,i=1}^{N}\chi_i\chi_j\cos[\pi\nu-\Phi_{j,i}]$$

$$=\frac{1}{8}\nu^2\left\{\cos[\pi\nu]\sum_{i=1}^{N}\frac{\alpha_i^2}{2}+2\sum_{j>i,i=1}^{N}\alpha_i\alpha_j\cos[\nu(\pi-\theta_{j,i})]\right\}$$

$$\Delta\nu=\nu^{'}-\nu=\frac{\nu^2}{8\pi\sin[\pi\nu]}\left\{\cos[\pi\nu]\sum\alpha_i^2+2\sum_{j>i}\alpha_j\alpha_i\cos[\nu(\pi-\theta_{j,i})]\right\}$$

- 期望值，假设闭轨充分校正，扰动之间不相关：

$$\overline{\Delta\nu}=E\left[\Delta\nu\right]=\frac{\nu^2}{8\pi}\cot[\pi\nu]E\left[\sum_{i=1}^{N}\alpha_i^2\right]+\frac{\nu^2}{4\pi\sin[\pi\nu]}E\left[\sum_{j>i}\alpha_j\alpha_i\cos[\nu(\pi-\theta_{j,i})]\right]$$

$$E[\alpha_i\alpha_j]\approx E[\alpha_i]\cdot E[\alpha_j]=0\quad with\quad i\neq j.$$

- 标准差：

$$\sigma_{\Delta\nu}=\frac{\nu^2}{8\pi\sin[\pi\nu]}\cdot N\cdot E\left[\alpha_i^2\right]=\frac{E[\Delta\nu]}{\cos[\pi\nu]}$$

Assmann, Ralph.,* ***Optimization of the transverse spin polarization in the LEP storage ring and application for precision measurements on the Z boson**(In German), (1994).*

CEPC 中的自旋工作点偏移

- CEPC lattice: magnets' errors+ vkicker

Component	Misalignment error			Field error
	$\Delta x(\mu m)$	$\Delta y(\mu m)$	$\Delta\theta_z(\mu rad)$	
Dipole	-	-	-	0.01%
Arc quadrupole	100	100	100	0.02%
IR quadrupole	50	50	50	-
Sextupole	100	100	100	-

$$\overline{\Delta\nu} = E\left[\Delta\nu\right] = \frac{\nu^2}{8\pi}\cot[\pi\nu]E\left[\sum_{i=1}^{N}\alpha_i^2\right]$$

$$\sigma_{\Delta\nu} = \frac{\nu^2}{8\pi\sin[\pi\nu]}\cdot N\cdot E\left[\alpha_i^2\right] = \frac{E[\Delta\nu]}{\cos[\pi\nu]}$$

- 垂直速度矢量的偏转与相空间坐标的关系：

$$\alpha_i = \frac{P_y}{P_0}\|_{i,exit} - \frac{P_y}{P_0}\|_{i,entrance}$$

- CEPC Z能区 lattice:

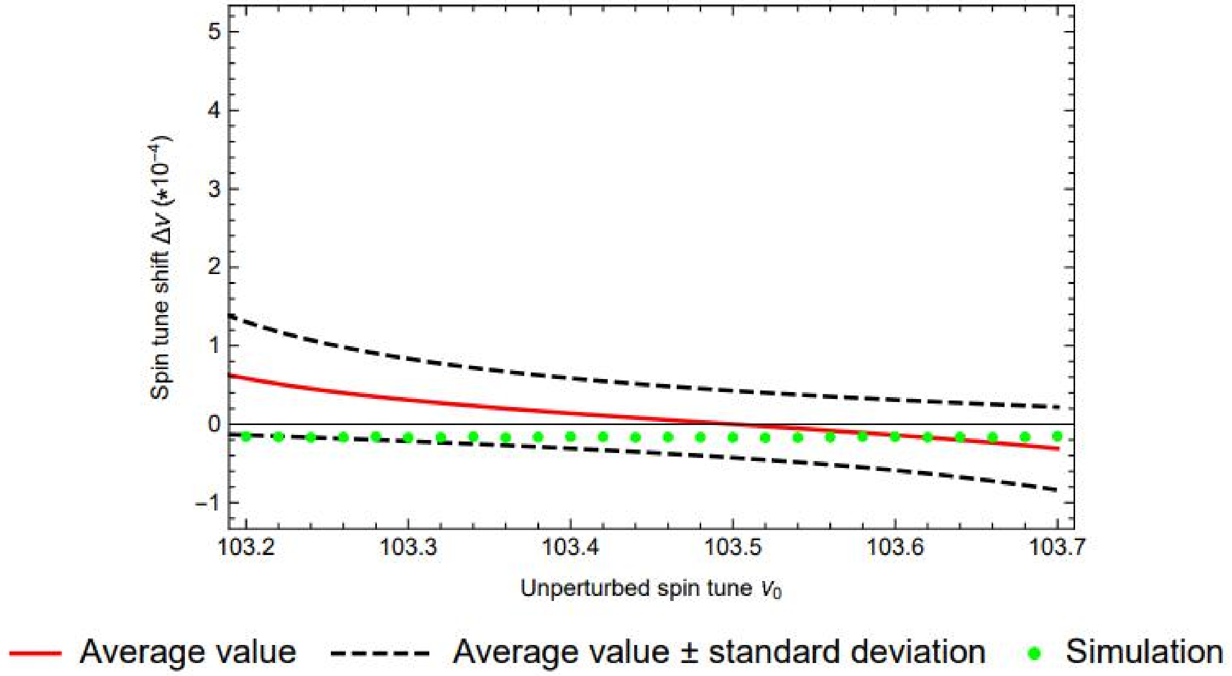
$$\sum\alpha_i^2 = 1\times10^{-7}$$

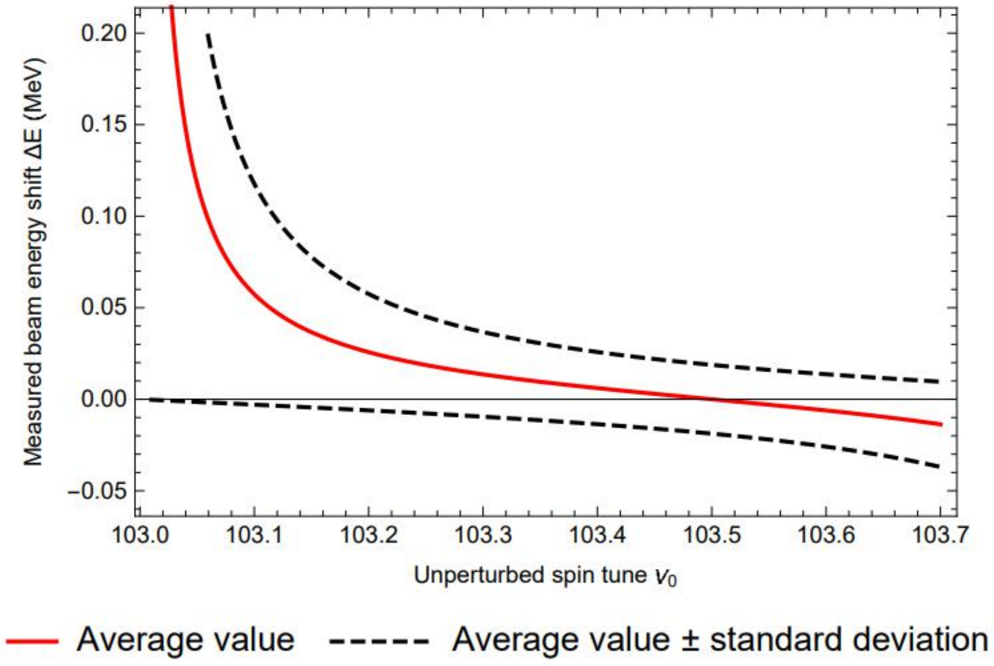
Δν, ΔE V.S ν

- CEPC Z能区：
  - $\Delta\nu \sim 10^{-4}$;
  - $\nu \in [103.3, 103.7]$, ΔE<50keV;

$$\overline{\Delta\nu} = E[\Delta\nu] = \frac{\nu^2}{8\pi}\cot[\pi\nu] E\left[\sum_{i=1}^{N} \alpha_i^2\right]$$

$$\sigma_{\Delta\nu} = \frac{\nu^2}{8\pi\sin[\pi\nu]} \cdot N \cdot E\left[\alpha_i^2\right] = \frac{E[\Delta\nu]}{\cos[\pi\nu]}$$



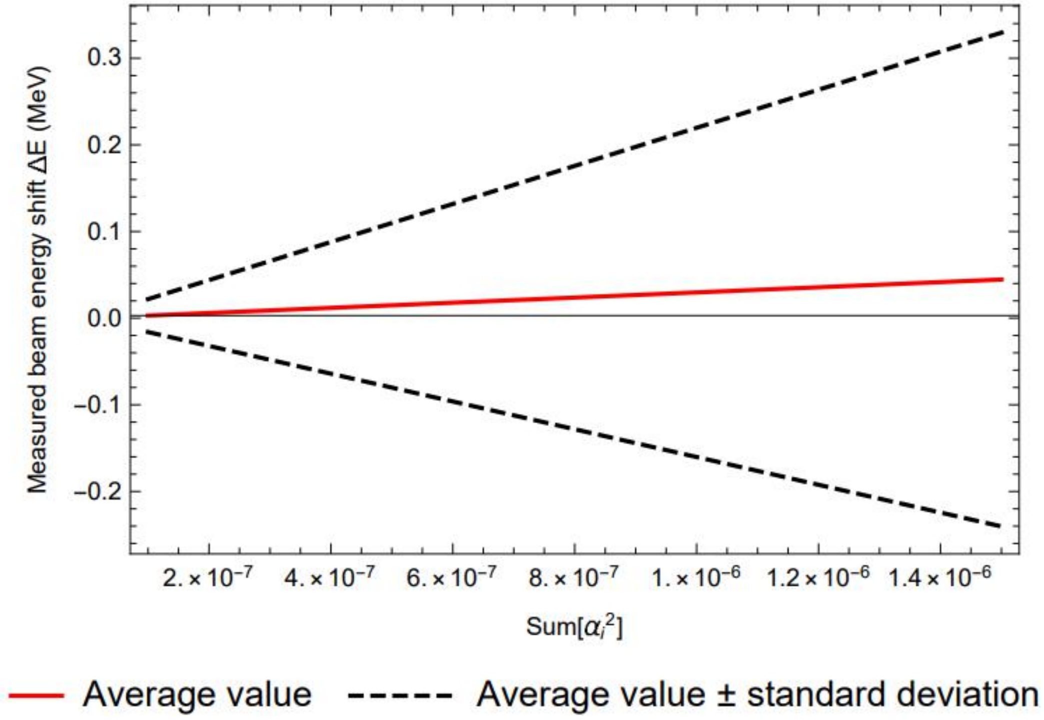


ΔE VS. Sum[$(\alpha_i)^2$]

- CEPC Z能区 aγ=103.45：
  - ΔE<50keV, Sum[$(\alpha_i)^2$] < 3x10 $^{-7}$

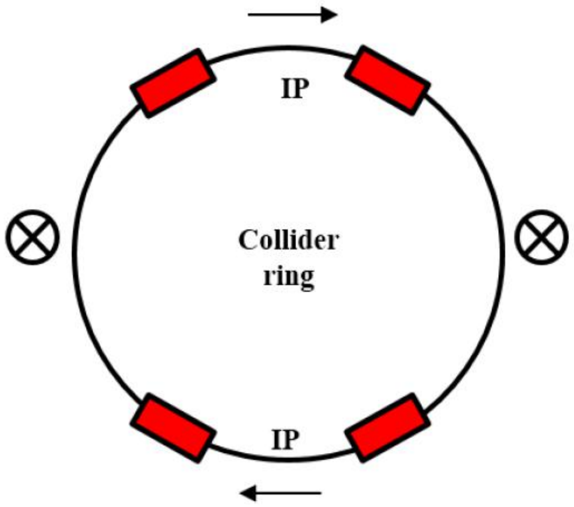
$$\overline{\Delta\nu} = E\left[\Delta\nu\right] = \frac{\nu^2}{8\pi}\cot[\pi\nu]E\left[\sum_{i=1}^{N}\alpha_i^2\right]$$

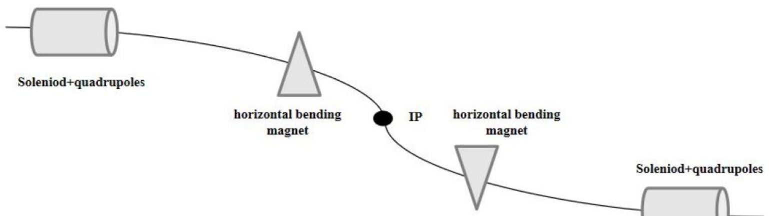
$$\sigma_{\Delta\nu} = \frac{\nu^2}{8\pi\sin[\pi\nu]}\cdot N\cdot E\left[\alpha_i^2\right] = \frac{E[\Delta\nu]}{\cos[\pi\nu]}$$

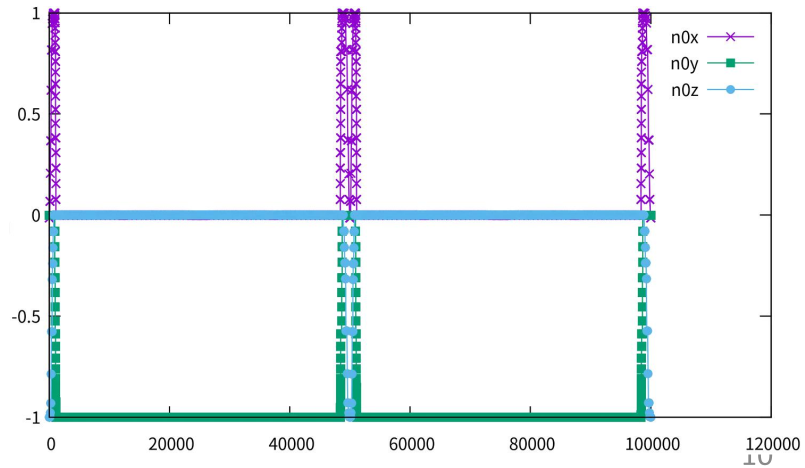


CEPC中的自旋旋转器的设计

- 要求：
  - 可以完成预期的极化方向的偏转；
  - 完成与主环结构的光学和几何匹配；
  - 实现自旋匹配: G_{2x6}=0，提高极化度；







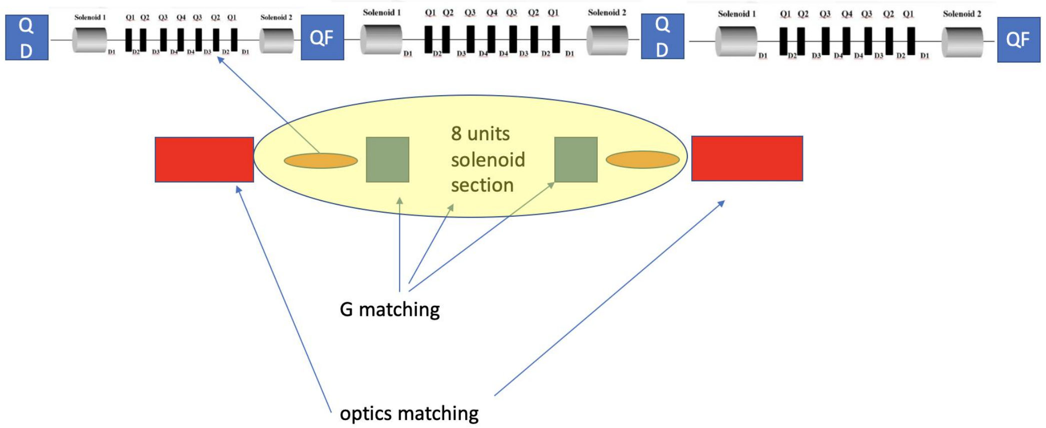
自旋旋转器螺线管区域

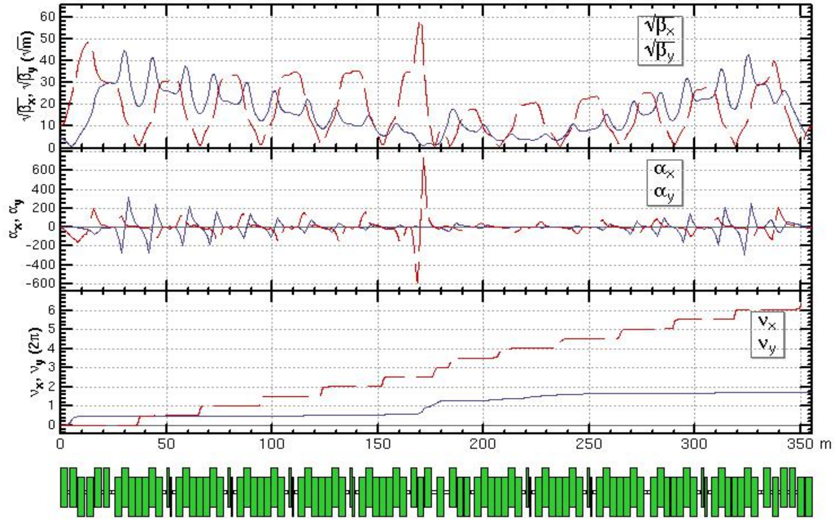
- 螺线管区域:
  - 10*螺线管解耦合结构+ G matching区域 + optics matching 区域;
  - G matching: $G_{2\times6}$=0, 提高极化度;

$$\widetilde{\mathrm{M}}(s1, s2) = \begin{pmatrix} \mathrm{M}_{6\times6} & 0_{6\times2} \\ \mathrm{G}_{2\times6} & \mathrm{D}_{2\times2} \end{pmatrix}$$

$$\begin{pmatrix}
4.01463 & 871.783 & -1.05647\times10^{-8} & -1.89703\times10^{-7} & 0. & 0. & 0. & 0. \\
0.0359296 & 8.05125 & -9.31575\times10^{-11} & 5.34824\times10^{-10} & 0. & 0. & 0. & 0. \\
-4.91162\times10^{-9} & -5.3651\times10^{-7} & 0.681305 & 213.657 & 0. & 0. & 0. & 0. \\
1.31782\times10^{-11} & 6.18793\times10^{-9} & -0.000689292 & 1.25161 & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 1.5708 & 1. & 0. \\
-2.40237\times10^{-9} & -1.86322\times10^{-7} & -1.8352\times10^{-9} & -6.17219\times10^{-8} & 0. & 0. & 0. & 1.
\end{pmatrix}$$

  - Optics matching : betax/betay=20m/100m, alpha=0

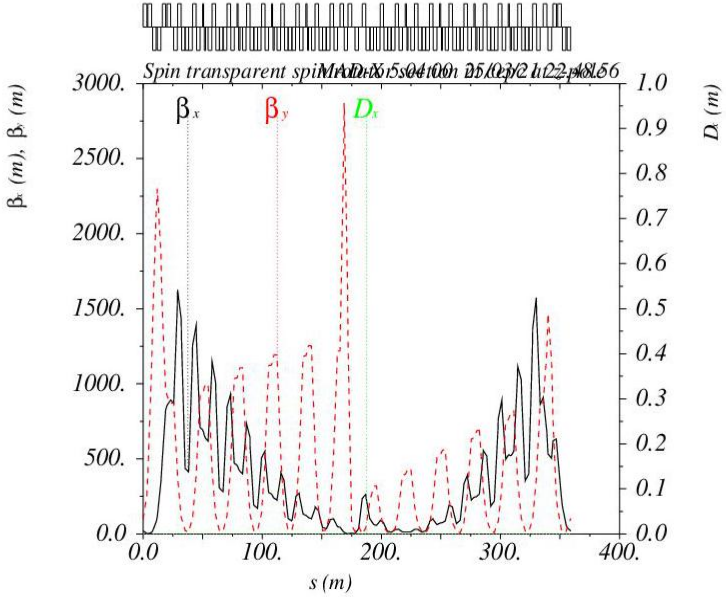




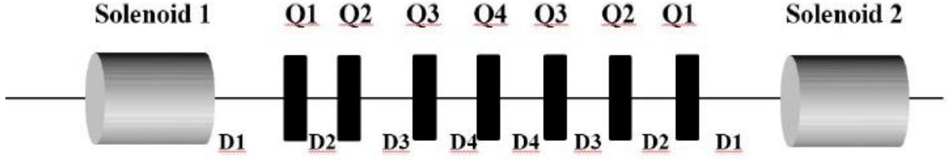
现阶段自旋旋转器的问题

	SPIN ROTATOR 1，3	SPIN ROTATOR 2，4	Final focus Quads
X chromaticity	-2107. 72	-1767. 22	-125. 188
Y chromaticity	-2526. 059	-8016. 724	-3754. 209

- 很强的局部色品，影响DA:
  - 色品∝ K1 *β,
  - 解决方法: (维持现有结构)
    - 拉长解耦合结构，降低K1
    - 增强螺线管强度，减少解耦合单元数量



*Vladimir N Litvinenko and Alexander A. Zholents,
Compensating effect of solenoids with quadrupole lenses



Solenoids		Quadrupoles		Drifts	
Length (m)	Field strength (T)	$\frac{\partial B_y/\partial x}{B\rho}$ (m^{-2})	Length (m)	Length (m)	Total Length (m)
1.48895	8	Q1: -0.83 Q2: 1.35 Q3: -0.90 Q4: -0.82	0.8	D1: 0.2 D2: 0.2 D3: 0.2 D4: 0.1	9.97796

→

Solenoids		Quadrupoles		Drifts	
Length (m)	Field strength (T)	$\frac{\partial B_y/\partial x}{B\rho}$ (m^{-2})	Length (m)	Length (m)	Total Length (m)
1.48898	8	Q1: -7.14502E-2 Q2: 1.17444E-1 Q3: -7.44823E-2 Q4: -6.94446E-2	3	D1: 0.2 D2: 0.2 D3: 0.2 D4: 0.1	25.37796

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- 很强的局部色品，影响DA:
  - 色品∝ K1 *β,
  - 解决方法: (更改结构)
    - SuperkekB, 斜四极铁：尝试减少磁铁的数量和强度
    - 新的结构

**I. Koop,*
Preliminary considerations on the longitudinal polarization at SuperKEKB

Table 1. Spin rotator lattice parameters for beam rigidity $BR = 23.3495$ T·m, L=9.89112 m.

Element type	Length, m	Field/Gradient, T, T/m
Quadrupole 1	0.46227	-23.2503
Drift 1	0.436	
Quadrupole 2	0.46227	24.081
Drift 2	0.25	
Solenoid	2.8	6.54197
Drift 3	0.25	
Quadrupole 3	0.57004	-29.1537

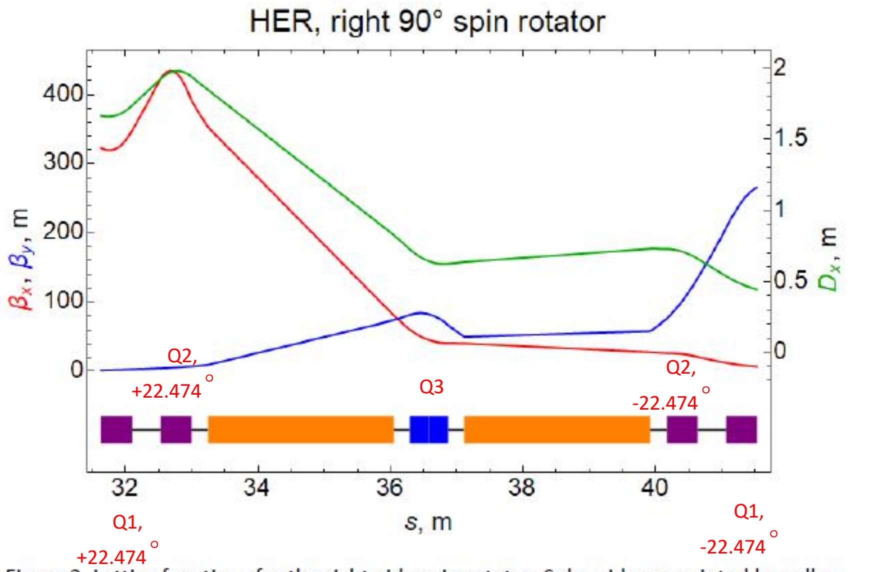

Figure 3. Lattice functions for the right-side spin rotator. Solenoids are painted by yellow, the central quad is normal, while doublets are rolled anti-symmetrically by $\varphi = \pm 22.474^0$.

1.000013	9.891169	7.01266E-6	5.84931E-5
2.64617E-6	1.000013	1.41798E-6	7.01273E-6
-7.0127E-6	-5.8493E-5	-.999995	-9.891107
-1.4180E-6	-7.0127E-6	9.77667E-7	-.999995