A New Entropic Force Scenario and Holographic Entropy

Miao Li

中国科学院理论物理研究所

Institute of Theoretical Physics CAS

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Based on work done with Rong-Xin Miao

and Wei Gu

To appear soon

1. Verlinde's entropic force scenario

Fdx=TdS

$$\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x \qquad F = T\frac{dS}{dx} = 2\pi k_B T\frac{mc}{\hbar}$$

$$k_B T = \frac{1}{2\pi}\frac{\hbar a}{c}$$

$\Rightarrow$ $F = ma.$ Newton's second law

$$N = \frac{Ac^3}{G\hbar} \qquad E = \frac{1}{2}Nk_BT$$

$$\Rightarrow \quad F = G\frac{Mm}{R^2}$$

Newton's law of gravitation

Verlinde's derivation of Einstein Equations

Temperature

$$T = \frac{\hbar}{2\pi} e^{\phi} N^b \nabla_b \phi.$$

Holography, namely the bit number

$$dN = \frac{c^3}{G\hbar} dA$$

Equipartition

$$E = \frac{1}{2} k_B \int_{\mathcal{S}} T dN$$

Thus

$$M = \frac{1}{4\pi G} \int_{\mathcal{S}} e^{\phi} \nabla \phi \cdot dA$$

And

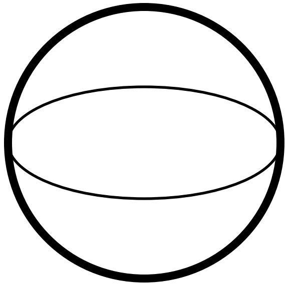
$$2\int_{\Sigma}\left(T_{ab}-\frac{1}{2}Tg_{ab}\right)n^{a}\xi^{b}dV=\frac{1}{4\pi G}\int_{\Sigma}R_{ab}n^{a}\xi^{b}dV$$

From Tolman-Komar mass

From the equipartition theorem

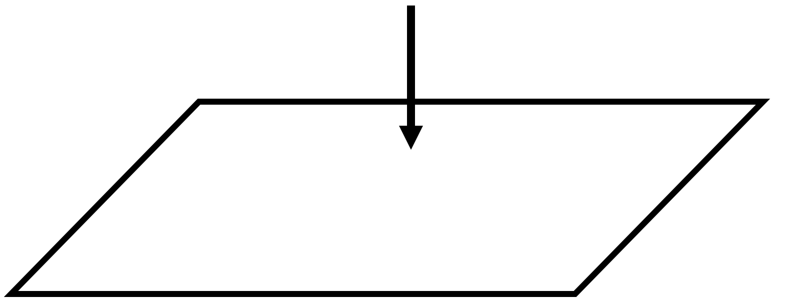
2. Our derivation of Einstein Equations

Verlinde uses a closed holographic screen



We use an open screen

Through the screen, there is an energy flow

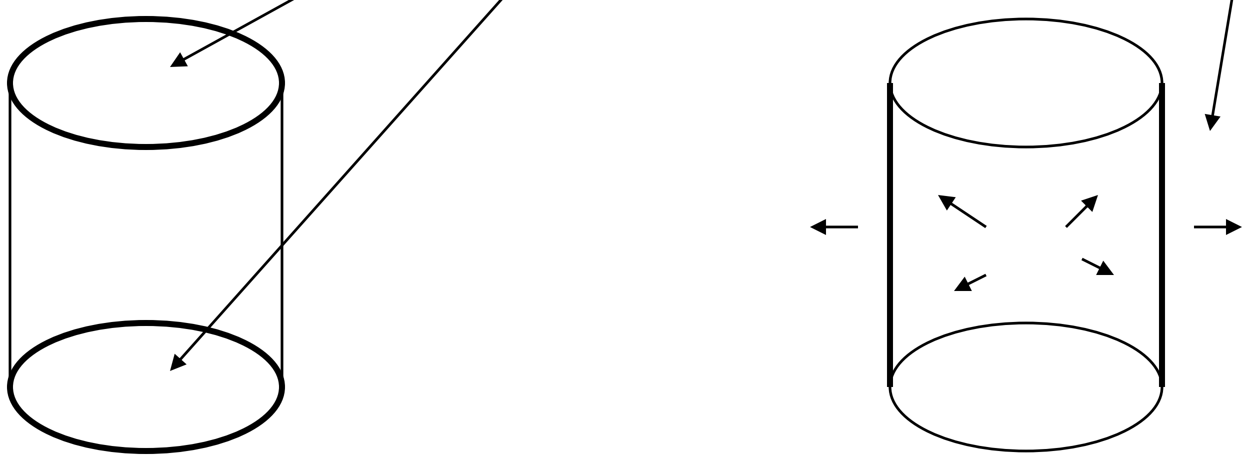


$$\delta E = \int_{\Sigma} T_{ab}\xi^a N^b dA dt$$

This is a bulk flow.

According to holography, this flow can be written using only the physical quantities on the screen

$$\delta E = \int (u_i \tau^{ij} \xi_j) dA\Big|_t^{t+dt} - \int m_i \tau^{ij} \xi_j dy dt$$



Naturally, we assume the surface stress tensor be given by local geometry

$$\tau^{ij} = n(K^{ij} - K\gamma^{ij})$$

Using the Gauss-Codazzi equation

$$(R_{ab} - Rg_{ab}/2)N^a e_i^b = -D_j(K^j{}_i - K\gamma^j{}_i)$$

We have

$$\delta E = \int_{\Sigma} n(R_{ab} - \frac{R}{2} g_{ab}) \xi^i e_i^a N^b dA dt$$

Compare to the bulk flow, we find

$$n(R_{ab} - \frac{R}{2} g_{ab}) e_i^a N^b = T_{ab} e_i^a N^b$$

We almost obtain the Einstein equations.

Note that

$$g_{ab} e_i^a N^b = 0$$

We deduce

$$R_{ab} - \frac{R}{2} g_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}.$$

3. Comparison with Verlinde and Jacobson

Verlinde	Our proposal
Closed holographic screen	Open or closed screen
Temperature T	Without or with T
Tolman-Komar mass	Brown-York Energy
Equipartition	Surface stress tensor

The Brown-York semi-local energy has a form

$$\int \frac{T}{4G\hbar} dA + \int \frac{1}{8\pi G} K \exp(\phi) dA$$

or

$$E = \int \frac{T}{2G\hbar} dA - 2 \int p \, dA,$$

We see that the second term is an extra compared with Verlinde.

The equipartition theorem does not have to be true since it is very peculiar.

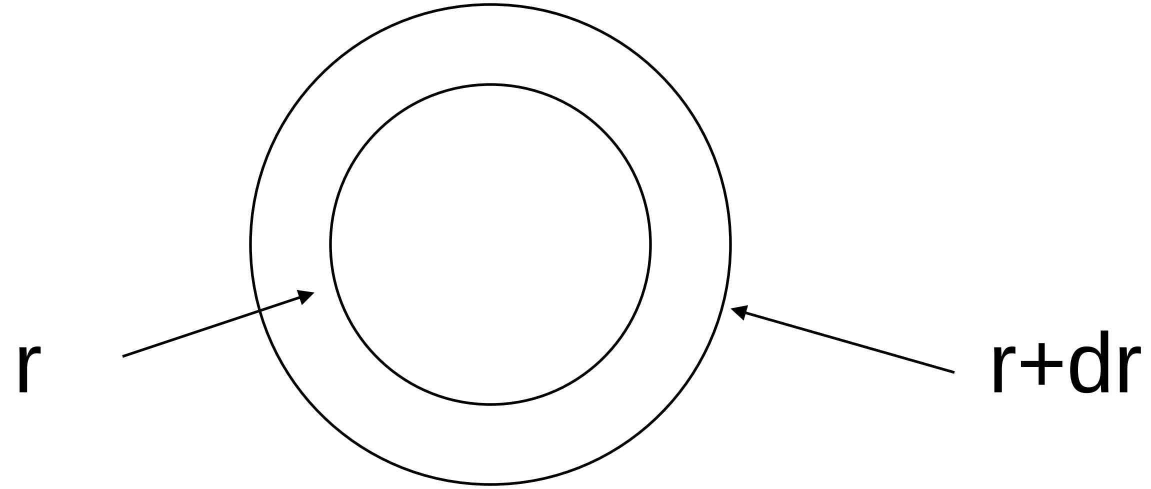
We have extra datum p, which is important in studying thermodynamics.

Jacobson	Our proposal
Open null screen	Open or closed time-like
T only	T, p chemical potential
First law	First law
$dE = TdS$	$dE = TdS - pdA + \mu dN_f$

We have more information.

4. Holographic thermodynamics

Consider a screen adiatically moves in space-time



The first law

$$dE = TdS - pdA + \mu dN_f$$

E and p are defined (to be substracted), we need
To know μ

For a static and spherically symmetric metric

$$ds^2 = -N^2 dt^2 + h^2 dr^2 + r^2 (d\theta^2 + \sin^2\theta, d\phi^2)$$

we have

$$E = \int_B d^2x \sqrt{\sigma}\sigma = \frac{N}{G}(r - \frac{r}{h})$$

$$p = \frac{N}{8\pi G}(\frac{N'}{Nh} + \frac{1}{rh} - \frac{1}{r})$$

and

$$T = \frac{\hbar}{2\pi} e^{\phi} N^b \nabla_b \phi = \frac{\hbar}{2\pi}\frac{N'}{h}$$

We deduce

$$\begin{aligned} dE &= \frac{1}{G}[N'(r - \frac{r}{h}) + N(1 - \frac{1}{h}) + Nr\frac{h'}{h^2}]dr \\ &= -pdA + \frac{1}{8\pi G}(\frac{Nh'}{h^2} + N')dA, \end{aligned}$$

To derive the chemical potential, we notice that for a black hole (or a region of vacuum) Nh=1 and dS=0, so

$$\begin{aligned}
\mu &= \frac{\hbar N'}{8\pi h}(h + \frac{Nh'}{N'h}) \\
&= \frac{T}{4}(h + \frac{Nh'}{N'h}) \\
&= \frac{T}{4}[(h-1) + (1 + \frac{Nh'}{N'h}) - C(Nh-1)] \\
&= \frac{\hbar}{8\pi}(\frac{xN'}{h} - N' - \frac{(1-x)Nh'}{h^2} - \frac{CN'(Nh-1)}{h})
\end{aligned}$$

Assume the above formula be generally true for other N and h, we can compute the holographic entropy for a gas with weak gravity.

$$ds^2 = -(1 + ar^2 + br^4)dt^2 + (1 + cr^2 + dr^4)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

where for example

$$a = \frac{4G\pi(1+3w_1)\rho}{3} \qquad c = \frac{8G\pi\rho}{3}$$

We find in general

$$dS = \frac{1}{4G\hbar}(x(1+\frac{Nh'}{N'h}) + C(Nh-1))dA$$

and for the gas in particular

$$S = x(\frac{3\pi r^2}{G\hbar}\frac{1+w_1}{1+3w_1} + \frac{2\pi^2\rho r^4}{15\hbar}\frac{(3w_1-1)(3(1+w_1)(1+3w_1)-5w_2(5+3w_1))}{(1+3w_1)^2 w_2}) + \frac{C\pi^2\rho(1+w_1)r^4}{\hbar}$$

To make the area term absent, x=0 thus

$$S = \frac{C\pi^2\rho(1+w_1)r^4}{\hbar}$$

This is the same form of the Bekenstein bound

$$(8\pi^2/3)\rho r^4$$

Indeed we also have a bound, when $w_1 = 1$ S reaches its maximum, and agrees with the Bekenstein bound if

$$C = 4/3$$

To conclude:

1. We make a different proposal from Verlinde

2. Our proposal makes derivation of the Einstein equations more complete.

3. Our proposal has a reasonable thermodynamics while Verlinde’s doen’t.

4. We predict a holographic entropy for a gas.

Future work:

1. Derive a general formula for the chemical potential.

2. Discuss various situations such as anti-de Sitter and cosmology (about holographic entropy).

3. Apply it to study dark energy.

We are already working in these directions.

Thank You !