# Yukawa Unification and Sparticle Spectroscopy at the LHC/Tevatron

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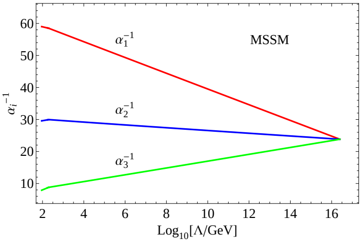
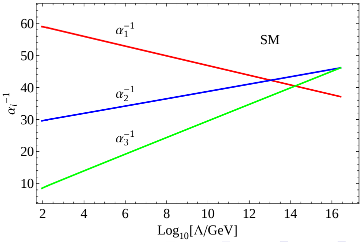
## Low Scale ($\sim$ TeV) Supersymmetry (SUSY):

- Arguably the most compelling extension of the Standard Model;
- Resolves the gauge hierarchy problem;
- Provides cold dark matter candidate (LSP);
- Implements radiative electroweak symmetry breaking;
- Predicts new particles accessible at the LHC, and thereby enables unification of the SM gauge couplings;





## Supersymmetric SO(10):

- Fermion families reside in $\underset{\sim}{16}_i (i{=}1,2,3)$ predicts 'right handed' neutrino ⇒ non-zero neutrino masses;

  (Cf: SU(5) with families in $\overline{10}_i + \overline{5}_i$)
- Yukawa couplings provide masses to SM fermions. They include

$$16_i 16_j 10,\ 16_i 16_j 126, \mathrm{etc.}$$

- $16_3 16_3 10$ yields $t - b - \tau$ unification

$$Y_t = Y_b = Y_\tau = Y_\nu$$

$\longrightarrow$ In the old days it was used to predict the top quark mass![1]

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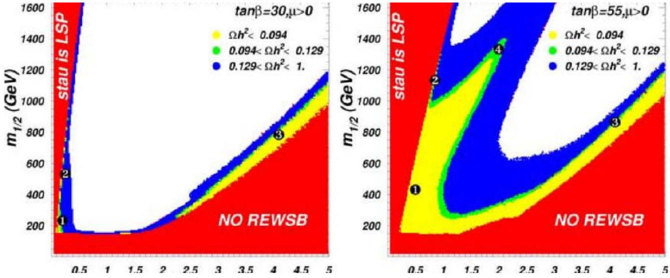
[1] B. Ananthanarayan, George Lazarides, Q. Shafi (1991);

- Nowadays, one employs $t - b - \tau$ unification to make predictions, such as sparticle masses, which can be tested at the LHC/Tevatron (Baer et al.);
- $t - b - \tau$ unification can also be realized in $SU(4)_c \times SU(2)_L \times SU(2)_R$ , a maximal subgroup of SO(10);

# CMSSM (mSUGRA):

- Unbroken $Z_2$ matter parity $\Rightarrow$ stable LSP, typically neutralino;
- Universal soft susy breaking parameters

$$m_0, m_{1/2}, A_0, tan\beta, sign(\mu)$$



Alexander Belyaev, Pramana 72:143-160,2009.

# Supersymmetric SO(10)(Baer et al.)[1]

- $m_{16},\ m_{10},\ M_D,\ M_{1/2},\ A_0,\ \tan\beta,\ sign(\mu)$
- $m_{16} \equiv$ Universal soft SUSY breaking sfermion mass
- $m_{10} \equiv$ Universal soft SUSY breaking MSSM Higgs mass
- $M_D \equiv$ The Higgs mass splitting $M^2_{H_{u,d}} = m^2_{10} \mp 2M^2_D$
- $m_{1/2} \equiv$ Universal SSB gaugino mass
- $A_0 \equiv$ Universal SSB trilinear interaction
- $\tan\beta = \frac{v_u}{v_d}$
- $\mu \equiv$ SUSY bilinear Higgs parameter

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[1] H. Baer, S. Kraml, S. Sekmen and H. Summy, JHEP 0803, 056 (2008)

- Random scans were performed over the parameter space

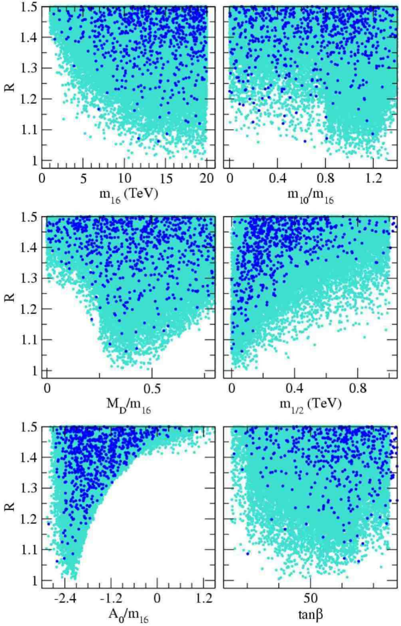
$$\begin{array}{lcl}
m_{16}: & 0 \rightarrow 20\ \mathrm{TeV} & (1-20\ \mathrm{TeV}), \\
m_{10}/m_{16}: & 0 \rightarrow 1.5 & (0.8-1.4), \\
m_{1/2}: & 0 \rightarrow 5\ \mathrm{TeV} & (0-1\ \mathrm{TeV}), \\
A_0/m_{16} & -3 \rightarrow 3 & (-2.5-1.9), \\
M_D/m_{16}: & 0 \rightarrow 0.8 & (0.25-0.8), \\
\tan\beta: & 40 \rightarrow 60 & (46-53).
\end{array}$$

- Quantify Yukawa unification by

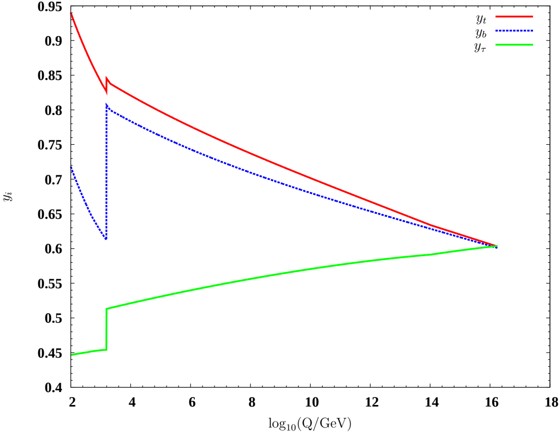
$$R = \frac{\max(y_t, y_b, y_\tau)}{\min(y_t, y_b, y_\tau)}$$

# Constraints

$$
\begin{aligned}
m_{\tilde{\chi}_1^{\pm}}\ \text{(chargino mass)} &\geq 103.5\ \text{GeV},\\
m_h\ \text{(lightest Higgs mass)} &\geq 114.4\ \text{GeV},\\
m_{\tilde{\tau}}\ \text{(stau mass)} &\geq 86\ \text{GeV},\\
m_{\tilde{g}}\ \text{(gluino mass)} &\geq 220\ \text{GeV},\\
BR(B_s \to \mu^+\mu^-) &< 5.8 \times 10^{-8},\\
0.53 < \frac{BR(B_u \to \tau\nu_\tau)MSSM}{BR(B_u \to \tau\nu_\tau)SM} &< 2.03\ (2\sigma),\\
2.85 \times 10^{-4} \leq BR(b \to s\gamma) &\leq 4.24 \times 10^{-4}\ (2\sigma),\\
\Omega_{\rm CDM}h^2 &= 0.111^{+0.028}_{-0.037}\ (5\sigma),\\
3.4 \times 10^{-10} \leq \Delta\alpha_\mu &\leq 55.6 \times 10^{-10}\ \ (3\sigma).
\end{aligned}
$$

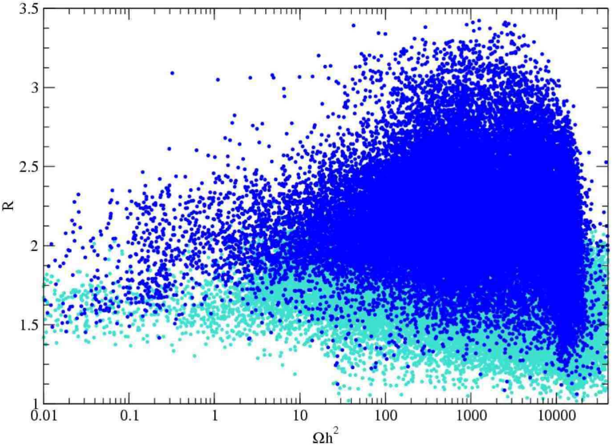

R
1.5
1.4
1.3
1.2
1.1
1
0
5
10
15
20
m$_{16}$ (TeV)
0
0.4
0.8
1.2
m$_{10}$/m$_{16}$
0
0.5
M$_D$/m$_{16}$
0
0.4
0.8
m$_{1/2}$ (TeV)
-2.4
-1.2
0
1.2
A$_0$/m$_{16}$
50
tanβ

- SUSY and $t - b - \tau$ Yukawa coupling unification



Radiative contributions to the bottom quark mass from the gluino and chargino loop

$$\frac{\delta m_b}{m_b} \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}} \tan\beta}{m_{\tilde{b}}^2} - \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_{\tilde{t}}^2} + \dots$$



H. Baer, S. Kraml, S. Sekmen and H. Summy, JHEP 0803, 056 (2008)

| parameter | Pt. A | Pt. D |
|---|---|---|
| $m_{16}$ | 9202.9 | 2976.5 |
| $m_{1/2}$ | 62.5 | 107.0 |
| $A_0$ | $-19964.5$ | $-6060.3$ |
| $m_{10}$ | 10966.1 | 3787.9 |
| $\tan\beta$ | 49.1 | 49.05 |
| $M_D$ | 3504.4 | 1020.8 |
| $f_t$ | 0.51 | 0.48 |
| $f_b$ | 0.51 | 0.47 |
| $f_\tau$ | 0.52 | 0.52 |
| $\mu$ | 4179.8 | 331.0 |
| $m_{\tilde{g}}$ | 395.6 | 387.7 |
| $m_{\tilde{u}_L}$ | 9185.4 | 2970.8 |
| $m_{\tilde{t}_1}$ | 2315.1 | 434.5 |
| $m_{\tilde{b}_1}$ | 2723.1 | 849.3 |
| $m_{\tilde{e}_L}$ | 9131.9 | 2955.8 |
| $m_{\tilde{\chi}_1^\pm}$ | 128.8 | 105.7 |
| $m_{\tilde{\chi}_2}$ | 128.6 | 105.1 |
| $m_{\tilde{\chi}_1}$ | 55.6 | 52.6 |
| $m_A$ | 3273.6 | 776.8 |
| $m_h$ | 125.4 | 111.1 |
| $\sigma$ [fb] | 75579.1 | 89666.1 |
| % $(\tilde{g}\tilde{g})$ | 86.8 | 80.5 |
| % $(\tilde{\chi}_1^\pm \tilde{\chi}_2^\pm)$ | 8.8 | 12.8 |
| % $(\tilde{t}_1 \tilde{t}_1)$ | 0 | 1.1 |

- Lightest colored sparticle is gluino; But $\Omega h^2 >> 1$ !!
- DM: Axions, Axinos.

# Yukawa Unification and Neutralino DM in $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2)

I.G, R. Khalid and Q. Shafi, Phys. Rev. D 79, 115004 (2009) .

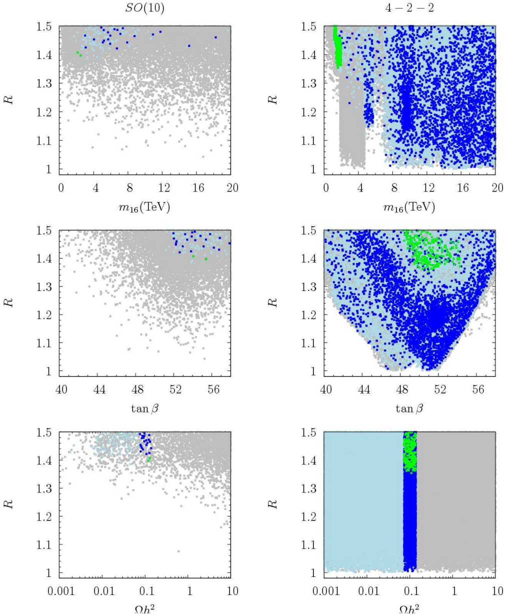
- SM fermions: $\psi_{\mathbf{i}} = (\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $\psi_{\mathbf{i}}^{\mathbf{c}} = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$
- MSSM Higgs: $\mathbf{H} = (\mathbf{1}, \mathbf{2}, \mathbf{2})$
- Third family Yukawa coupling $\psi\,\psi^{\mathbf{c}}\ \mathbf{H}$ yields
$$Y_t = Y_b = Y_\tau = Y_\nu$$
- Asymptotic relation between the three MSSM gaugino masses
$$M_1 = \frac{3}{5}M_2 + \frac{2}{5}M_3$$
- One additional parameter compared to the SO(10) model (from gaugino non-universality)

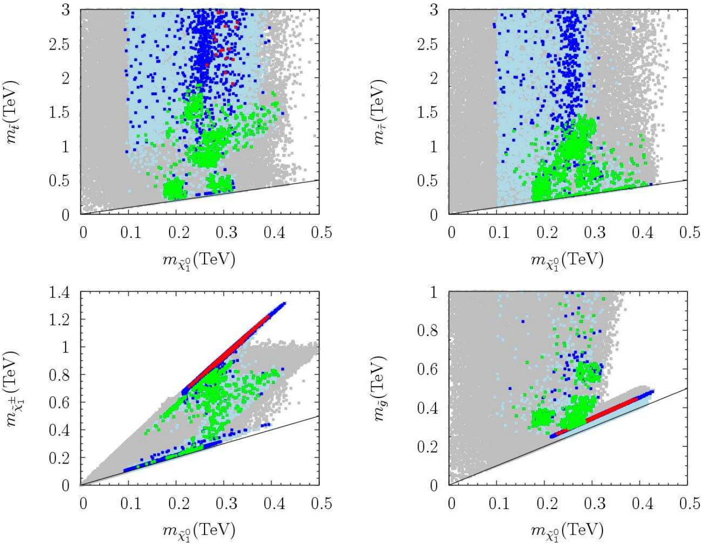
We performed random scans for the following parameter range

$$\begin{array}{rcccl}
0 & \leq & m_{16} & \leq & 20\,\mathrm{TeV}, \\
0 & \leq & M_2 & \leq & 1\,\mathrm{TeV}, \\
0 & \leq & M_3 & \leq & 1\,\mathrm{TeV}, \\
-3 & \leq & A_0/m_{16} & \leq & 0, \\
0 & \leq & M_D/m_{16} & \leq & 0.95, \\
0 & \leq & m_{10}/m_{16} & \leq & 1.5, \\
40 & \leq & \tan\beta & \leq & 58, \\
\mu > 0, & & & & m_t = 172.6\,\mathrm{GeV}.
\end{array}$$

# Constraints

$$
\begin{aligned}
m_{\tilde{\chi}_1^{\pm}}\ (\text{chargino mass}) &\geq 103.5\ \text{GeV}, \\
m_h\ (\text{lightest Higgs mass}) &\geq 114.4\ \text{GeV}, \\
m_{\tilde{\tau}}\ (\text{stau mass}) &\geq 86\ \text{GeV}, \\
m_{\tilde{g}}\ (\text{gluino mass}) &\geq 220\ \text{GeV}, \\
BR(B_s \rightarrow \mu^+\mu^-) &< 5.8 \times 10^{-8}, \\
0.53 < \frac{BR(B_u \rightarrow \tau\nu_\tau)MSSM}{BR(B_u \rightarrow \tau\nu_\tau)SM} &< 2.03\ (2\sigma), \\
2.85 \times 10^{-4} \leq BR(b \rightarrow s\gamma) &\leq 4.24 \times 10^{-4}\ (2\sigma), \\
\Omega_{\text{CDM}} h^2 &= 0.111^{+0.028}_{-0.037}\ (5\sigma), \\
3.4 \times 10^{-10} \leq \Delta\alpha_\mu &\leq 55.6 \times 10^{-10}\ \ (3\sigma).
\end{aligned}
$$


SO(10)
4 − 2 − 2
R
1.5
1.4
1.3
1.2
1.1
1
0
4
8
12
16
20
$m_{16}$(TeV)
40
44
48
52
56
tan β
0.001
0.01
0.1
1
10
$\Omega h^2$



Points in green satisfy all experimental constraints. Red points represent $R \leq 1.1$, but do not satisfy g-2.

| | Point 1 | Point 2 | Point 3 |
|---|---|---|---|
| $m_{16}$ | 14110 | 8429 | 13124 |
| $M_2$ | 832.03 | 1020.2 | 689.4 |
| $M_3$ | 0.7945 | 60.542 | 9.6261 |
| $\tan\beta$ | 50.82 | 46.41 | 51.17 |
| $M_D/m_{16}$ | 0.4543 | 0.5595 | 0.3323 |
| $m_{10}/m_{16}$ | 0.7741 | 1.1584 | 1.3048 |
| $A_0/m_{16}$ | -2.4487 | -2.1527 | -1.8226 |
| $m_h$ | 123 | 126 | 127 |
| $m_H$ | 7569 | 2163 | 9882 |
| $m_A$ | 7520 | 2150 | 9818 |
| $m_{H^\pm}$ | 7571 | 2175 | 9883 |
| $m_{\tilde{\chi}^{\pm}_{1,2}}$ | **887**,13869 | **975**,4047 | **712**,3750 |
| $m_{\tilde{\chi}^{0}_{1,2}}$ | **283**, 885 | **319**,974 | **228**,712 |
| $m_{\tilde{\chi}^{0}_{3,4}}$ | 13879,13879 | 4049,4049 | 3784,3785 |
| $m_{\tilde{g}}$ | **325** | **365** | **265** |
| $m_{\tilde{u}_{L,R}}$ | 14126,13916 | 8435,8361 | 13140,12841 |
| $m_{\tilde{t}_{1,2}}$ | 5337,5726 | **1911**,2640 | 4931,5310 |
| $m_{\tilde{d}_{L,R}}$ | 14126,14203 | 8435,8455 | 13141,13249 |
| $m_{\tilde{b}_{1,2}}$ | 5237,5653 | 2521,2767 | 4115,5146 |
| $m_{\tilde{\nu}_1}$ | 13988 | 8409 | 12926 |
| $m_{\tilde{\nu}_3}$ | 10598 | 6577 | 9535 |
| $m_{\tilde{e}_{L,R}}$ | 13988,14376 | 8408,8514 | 12926,13500 |
| $m_{\tilde{\tau}_{1,2}}$ | 6412,10581 | 4270,6573 | 5580,9559 |
| $\mu$ | 14100 | 4110 | 3840 |
| $\Omega_{LSP}h^2$ | 0.095 | 0.112 | 0.116 |
| $R$ | **1.00** | 1.07 | 1.09 |

# Yukawa unification with negative $\mu$ term

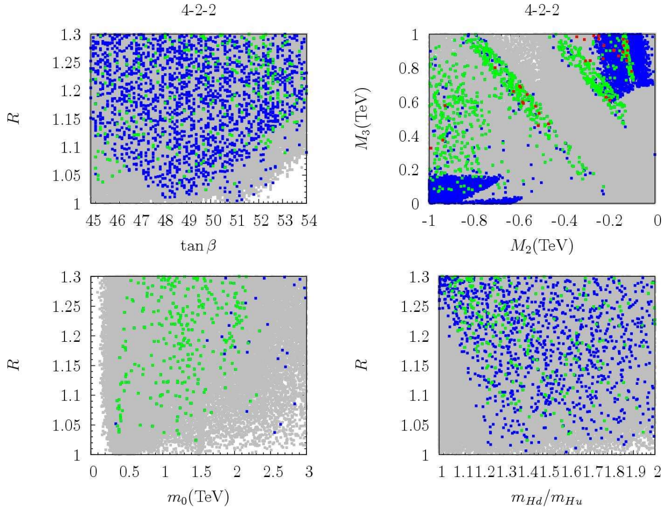
- Yukawa unification prefers $\mu < 0$
- Dominant contributions to the bottom quark mass from the gluino and chargino loop

$$\delta m_b \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}} \tan\beta}{m_{\tilde{b}}^2} - \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_{\tilde{t}}^2} + \ldots$$

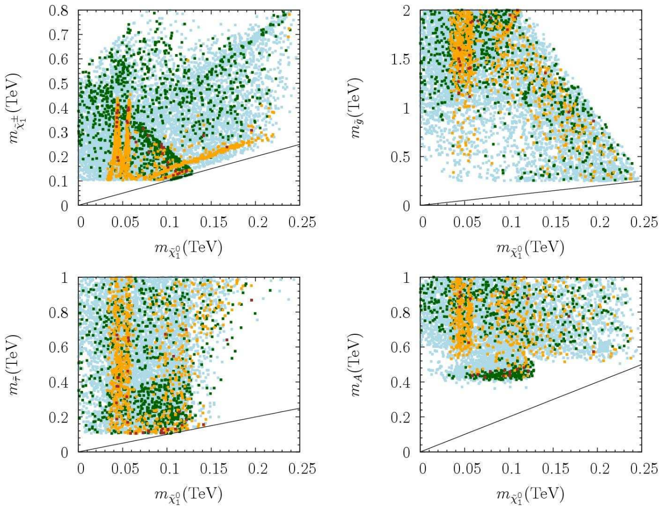
- Dominant contribution to the muon anomalous magnetic moment for large $\tan\beta$ case is $\Delta\alpha_\mu^{SUSY} \propto \mu M_2 \tan\beta/\tilde{m}^4$
- In 4-2-2 model with left-right symmetry, $M_2$ and $M_3$ are free parameters
- We can have $\mu < 0$, $M_2 < 0$ and $M_3 > 0$

We performed random scans for the following parameter range

$$\begin{aligned}
0 &\le m_0, M_{H_u}, M_{H_d} \le 20\,\mathrm{TeV}, \\
1\,\mathrm{TeV} &\le M_2 \le 1\,\mathrm{TeV}, \\
0 &\le M_3 \le 1\,\mathrm{TeV}, \\
-3 &\le A_0/m_{16} \le 3, \\
45 &\le \tan\beta \le 55, \\
\mu > 0, &\quad \mu < 0, \quad m_t = 172.6\,\mathrm{GeV}.
\end{aligned}$$

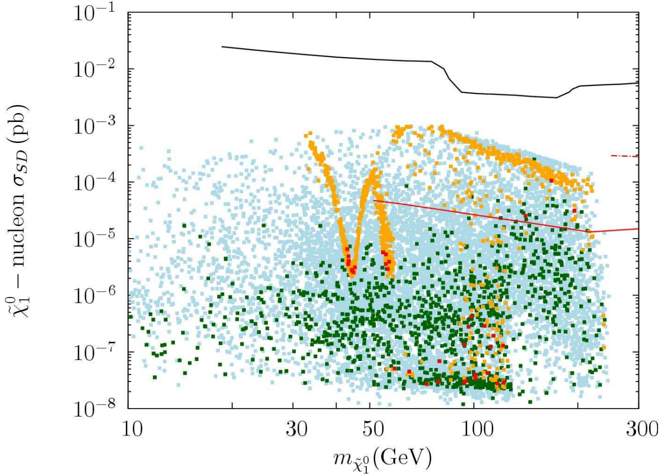


Green points satisfy all constraints. Points in red represent $R \leq 1.1$



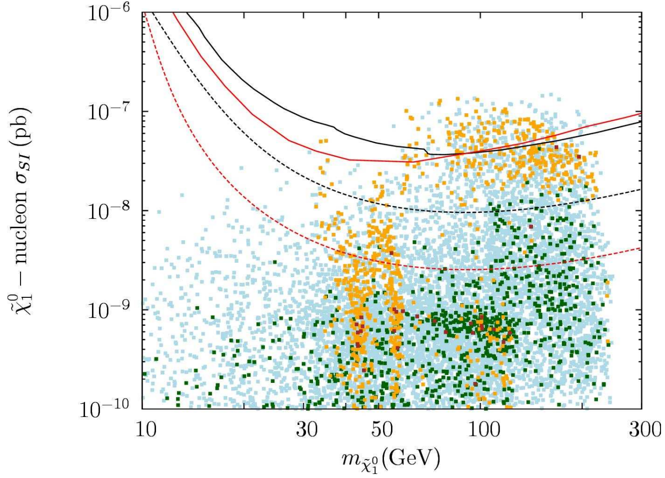
Brown points satisfy all constraints and $R \leq 1.1$

## Dark matter indirect detection



Brown points satisfy all constraints and $R \leq 1.1$

# Dark matter direct detection



Brown points satisfy all constraints and $R \leq 1.1$

|  | Point 1 | Point 2 | Point 3 | Point 4 | Point 5 |
|---|---|---|---|---|---|
| $m_0$ | 1027 | 1800 | 1210 | 980 | 1720 |
| $M_1$ | -665 | -81 | -414 | -126 | -538 |
| $M_2$ | -1475 | -543 | -940 | -517 | -943 |
| $M_3$ | 550 | 611 | 374 | 460 | 70 |
| $\tan\beta$ | 49.1 | 52.8 | 50.6 | 47.0 | 47.6 |
| $A_0/m_0$ | 0.26 | 1.06 | -1.15 | -1.08 | -1.25 |
| $m_{Hu}$ | 743 | 1919 | 1231 | 1090 | 295 |
| $m_{Hd}$ | 1505 | 2395 | 1745 | 1869 | 1729 |
| $m_h$ | 114 | 115 | 114 | 115 | 115 |
| $m_H$ | 847 | 573 | 781 | 1100 | 1006 |
| $m_A$ | 841 | 569 | 776 | 1090 | 1000 |
| $m_{H^\pm}$ | 852 | 581 | 787 | 1100 | 1010 |
| $m_{\tilde{\chi}^0_{1,2}}$ | 280,341 | 43,352 | 168,242 | 56,337 | 233,782 |
| $m_{\tilde{\chi}^0_{3,4}}$ | 352,1236 | 380,513 | 246,795 | 371,476 | 1210,1216 |
| $m_{\tilde{\chi}^\pm_{1,2}}$ | 342,1225 | 355,509 | 239,786 | 338,475 | 782,1217 |
| $m_{\tilde{g}}$ | 1321 | 1470 | 955 | 1110 | 270 |
| $m_{\tilde{u}_{L,R}}$ | 1771,1489 | 2170,2130 | 1550,1410 | 1400,1320 | 1818,1697 |
| $m_{\tilde{t}_{1,2}}$ | 1053,1410 | 1400,1440 | 822,1040 | 826,965 | 1070,1248 |
| $m_{\tilde{d}_{L,R}}$ | 1773,1512 | 2180,2160 | 1550,1440 | 1400,1370 | 1820,1730 |
| $m_{\tilde{b}_{1,2}}$ | 954,1399 | 1350,1430 | 774,1020 | 724,906 | 992,1245 |
| $m_{\tilde{\nu}_1}$ | 1391 | 1810 | 1340 | 1000 | 1807 |
| $m_{\tilde{\nu}_3}$ | 1211 | 1420 | 1100 | 759 | 1550 |
| $m_{\tilde{e}_{L,R}}$ | 1393,1096 | 1820,1820 | 1340,1250 | 1010,1040 | 1809,1763 |
| $m_{\tilde{\tau}_{1,2}}$ | 500,1212 | 885,1420 | 641,1110 | 462,765 | 1170,1554 |
| $\sigma_{SI}(\mathrm{pb})$ | $4.02\times10^{-8}$ | $4.1\times10^{-9}$ | $4.1\times10^{-8}$ | $9.5\times10^{-10}$ | $1.1\times10^{-10}$ |
| $\sigma_{SD}(\mathrm{pb})$ | $8.4\times10^{-5}$ | $7.5\times10^{-6}$ | $1.7\times10^{-4}$ | $8.2\times10^{-6}$ | $2.9\times10^{-8}$ |
| $\Omega_{CDM}h^2$ | 0.08 | 0.11 | 0.09 | 0.08 | 0.11 |
| $R$ | 1.01 | 1.11 | 1.09 | 1.07 | 1.08 |
| $g_3/g_1(M_{\mathrm{GUT}})$ | 0.98 | 0.98 | 0.99 | 0.98 | 1.00 |

## Yukawa Unification & NLSP gluino search at Hadron Colliders

- Yukawa unification predicts light gluino, heavy scalars and is compatible with gluino-bino coannihilation with gluino as NLSP.

- Conventional gluino searches with small SM background

$$\tilde{g}\tilde{g} \quad\rightarrow\quad jets + \tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\pm} \rightarrow jets + l^{\pm}l^{\pm} + \not{E}_T.$$

- For NLSP gluino these channels are absent and we consider the parameter space region with dominant contributions from gluino three body decay $b\bar{b}\tilde{\chi}_1^0$

$$pp, p\bar{p} \rightarrow \tilde{g}\tilde{g} \rightarrow b\bar{b}b\bar{b} + \not{E}_T.$$

- Potential SM backgrounds considered

$$b\bar{b}b\bar{b},\ b\bar{b}b\bar{b}Z \to b\bar{b}b\bar{b}\nu\bar{\nu},\ jjb\bar{b}Z \to jjb\bar{b}\nu\bar{\nu}$$

- We choose two benchmark points from previously described 4-2-2 models

| | $M_{\tilde{g}}$ (GeV) | $M_{\tilde{\chi}_1^0}$ (GeV) | $M_{\tilde{b}_1}$ (GeV) | $\mathrm{Br}(\tilde{g} \to b\bar{b}\tilde{\chi}_1^0)$ |
|---|---|---|---|---|
| Model A ($\mu > 0$) | 329 | 284 | 5294 | 76.3% |
| Model B($\mu < 0$) | 261 | 207 | 950 | 50.8% |

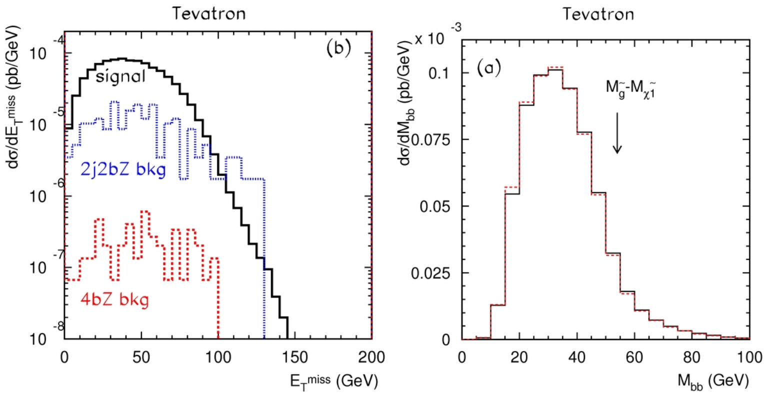
- For Tevatron, we employ the following event selection cuts

$$p_T^j > 15 \text{ GeV}, |\eta_j| < 1.0\ , \Delta R_{jj} > 0.4$$

and b tagging efficiency 50% and $\not{E}_T > 30\,\mathrm{GeV}$ cut for $\tilde{\chi}_1^0$.

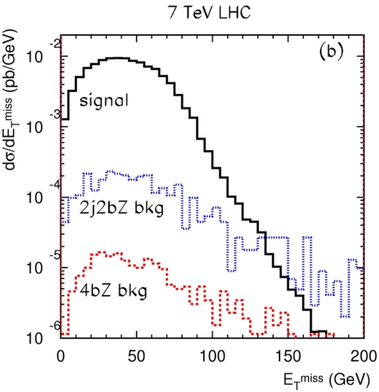
- The production cross section for the two points for Tevatron

| $\sigma$(fb)@ Tevatron | Model A | Model B | $b\bar{b}b\bar{b}$ | $b\bar{b}b\bar{b}Z$ | $jjb\bar{b}Z$ | $S/\sqrt{B}$ |
|---|---|---|---|---|---|---|
| basic cuts and 3b tagging | 2.3 | 4.8 | $2.7 \times 10^3$ | 0.02 | 1 | |
| $\not{E}_T > 30$ GeV | 1.4 | 3.3 | – | 0.019 | 0.95 | 4.5(A)/11(B) |



- The production cross section for the two points for LHC

| $\sigma$(fb)@ 7 TeV LHC | Model A | Model B | | $b\bar{b}b\bar{b}$ | $b\bar{b}b\bar{b}Z$ | $jjb\bar{b}Z$ |
|---|---|---|---|---|---|---|
| basic cuts and 3b tagging | 286 | 541 | | $314 \times 10^3$ | 1.1 | 15 |
| $\not{E}_T > 40$ GeV | 117 | 280 | | – | 0.8 | 12 |



## Summary

- In supersymmetric and L-R symmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ model with gravity mediated supersymmetry breaking, $t - b - \tau$ Yukawa coupling unification is consistent with neutralino dark matter abundance and with all constraints from collider experiments (except $(g-2)_\mu$)for $\mu > 0$. For $\mu < 0$ we can have Yukawa unification satisfying all current constraints.
- The model for $\mu > 0$ predicts a very characteristic sparticle spectrum: very heavy sfermions ($> 5$ TeV ) but relatively light gluinos ($\gtrsim 300$ GeV).
- For $\mu < 0$, Yukawa unification can be achieved with relatively light sparticle spectrum $O(600)$ GeV. NLSP gluino can be tested at LHC/Tevatron.

- NLSP gluino search at Tevatron and 7 TeV LHC through multi-b jets $\tilde{g}\tilde{g} \rightarrow b\bar{b}b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0$ .
- With 10 $\mathrm{fb}^{-1}$ luminosity one can reach $5\sigma$ at Tevatron after selection cuts. At 7 Tev LHC the signal is at least one order of magnitude larger than leading backgrounds.