Precision Measurements in the LHC Era: Selected Topics

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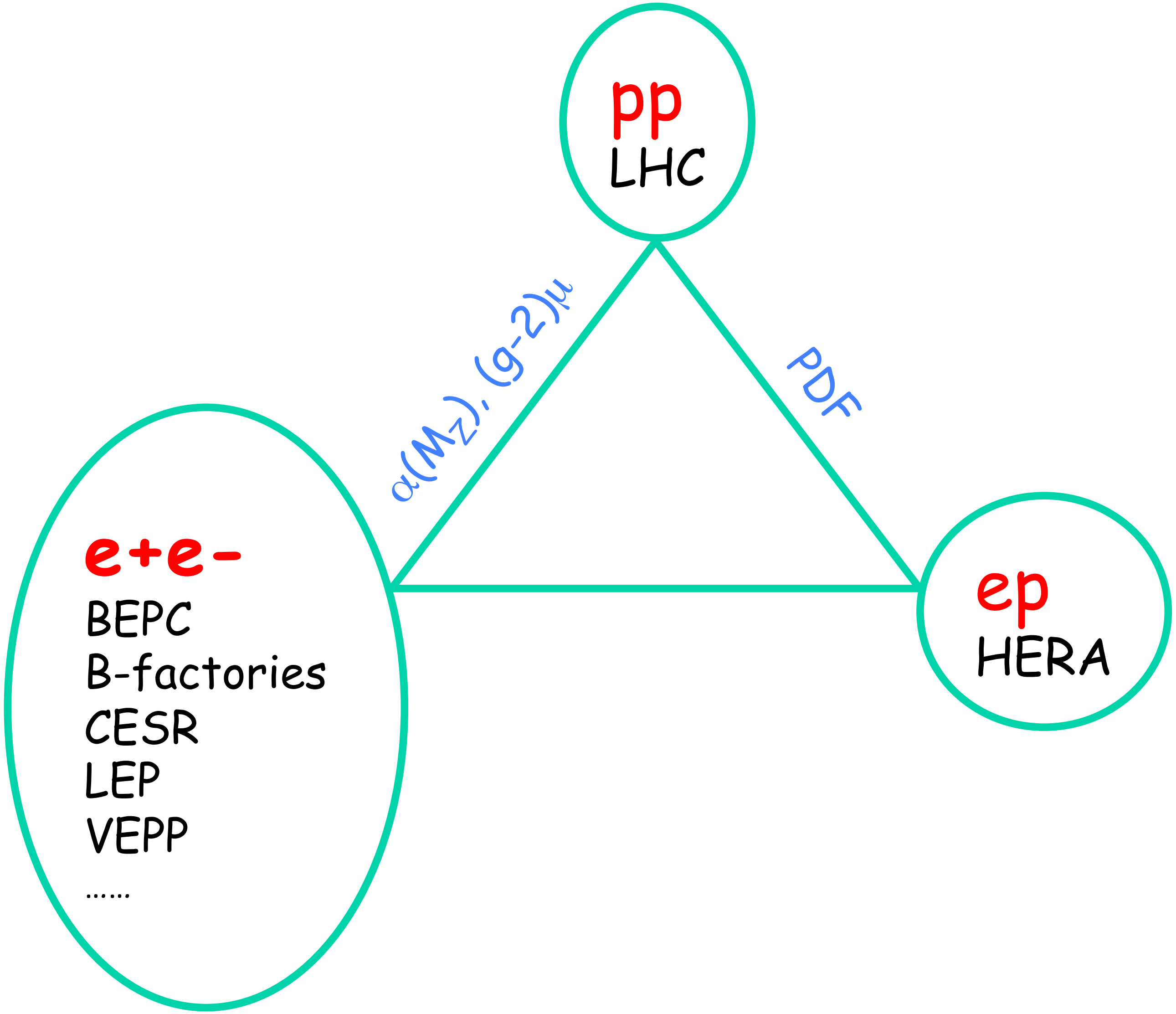
Outline

1. e+e- annihilation data and $\alpha(M_Z)$, $(g-2)_\mu$
2. HERA, PDFs and impact on LHC physics

LHC vs. e+e- & ep colliders



Selected Topic One:

e+e- annihilation data and

QED $\alpha(M_Z)$ & muon magnetic moment anomaly g-2

New results:

- Davier, Hoecker, Malaescu, ZZ, arXiv:1010.4180, submitted to Eur. Phys. J. C

Earlier results:

- Davier et al., Eur. Phys. J. C66 (2010) 127
- Davier, Hoecker, Malaescu, Yuan and ZZ, Eur. Phys. J. C66 (2010) 1
- Davier, Eidelman, Hoecker and ZZ (DEHZ), Eur. Phys. J. C27 (2003) 497
- Davier, Eidelman, Hoecker and ZZ, Eur. Phys. J. C31 (2003) 503

Motivation for a Precise Prediction of $\alpha(M_Z)$

- $\alpha(M_Z)=\alpha/(1-\Delta\alpha(M_Z))$ with $\Delta\alpha(M_Z)=\Delta\alpha_{lep}(M_Z) + \Delta\alpha_{had}(M_Z)$
  - α=1/137.035999084(51) best determined from $(g-2)_e$
  - $\alpha(M_Z)$ less precisely known than G_μ, M_Z

Hanneke, Fogwell, Gabrielse, 2008

- $\Delta\alpha_{lep}$ known up to 3-loop: Steinhauser, 1998

  $\Delta\alpha_{lep}(M_Z)=314.97686\times10^{-4}$

- $\Delta\alpha_{had}=\Delta\alpha_{had}^{(5)} + \Delta\alpha_{had}^{top}$

  $=(276.8\pm2.2)\times10^{-4} + (-0.72\pm0.02)\times10^{-4}$

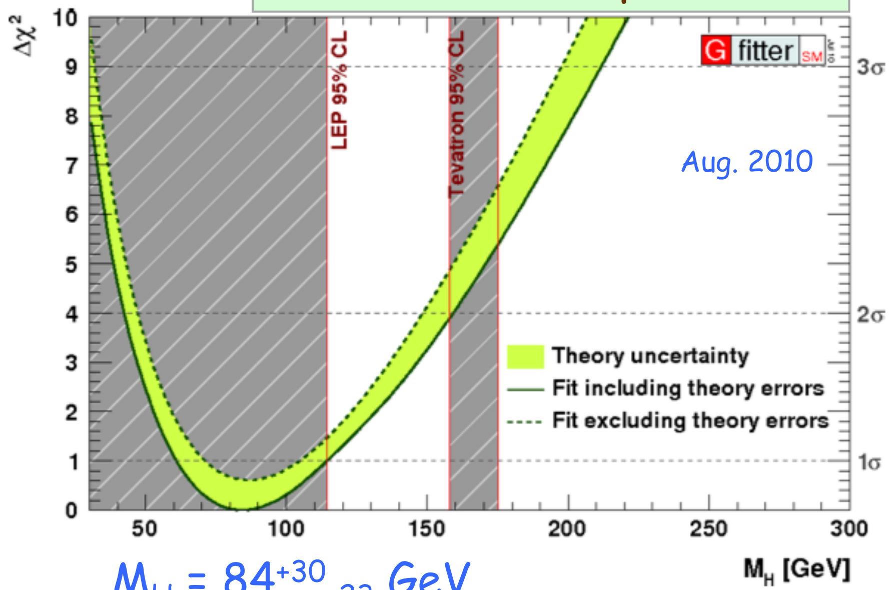
  e.g. HMNT, PLB649, 173 (2007)

  $\rightarrow \alpha^{-1}(M_Z)=128.937\pm0.030$

  i.e. relative precision of 2.3×10^{-4}

  To be compared with G_μ: 0.9×10^{-5} and M_Z: 2.3×10^{-5}

See slide 43 in backup for details



$M_H = 84^{+30}_{-23}$ GeV

$\rightarrow$ Important to improve the precision of $\Delta\alpha_{had}^{(5)}$

Hadronic Vacuum Polarization

Define: photon vacuum polarization function $\Pi_\gamma(q^2)$

$$i\int d^4x\, e^{iqx} \langle 0 | T J^\mu_{\rm em}(x) \left(J^\nu_{\rm em}(x)\right)^\dagger |0\rangle = -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi_\gamma(q^2)$$

Ward identities: only vacuum polarization modifies electron charge

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$$

with: $\Delta\alpha(s) = -4\pi\alpha \mathrm{Re}\left[\Pi_\gamma(s) - \Pi_\gamma(0)\right]$

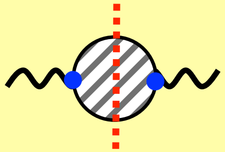
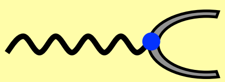
$=\Delta\alpha_{\rm lep}(s)+\Delta\alpha_{\rm had}(s)$

Leptonic $\Delta\alpha_{\rm lep}(s)$ calculable in QED. However, quark loops are modified by long-distance hadronic physics, cannot (yet) be calculated within QCD (!)

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

Way out: Optical Theorem (***unitarity***) ...

$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^{(0)}[e^+e^- \to \text{hadrons}]}{\sigma^{(0)}[e^+e^- \to \mu^+\mu^-]} \equiv R(s)$$

Im[] $\propto$ |  hadrons $|^2$

... and the subtracted dispersion relation of $\Pi_\gamma(q^2)$ (***analyticity***)

$$\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi}\int_0^\infty ds' \frac{\mathrm{Im}\Pi_\gamma(s')}{s'(s'-s)-i\varepsilon} \quad \Rightarrow \quad \Delta\alpha_{\rm had}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_0^\infty ds' \frac{R(s')}{s'(s'-s)-i\varepsilon}$$

… and Muon Anomalous Magnetic Moment g-2

$$\vec{\mu} = g\frac{\pm e}{2m}\vec{s}$$

$$g = 2 + \cdots$$

$a_\mu=(g_\mu-2)/2$

QED Prediction:
Computed up to 4th order **Kinoshita et al.**

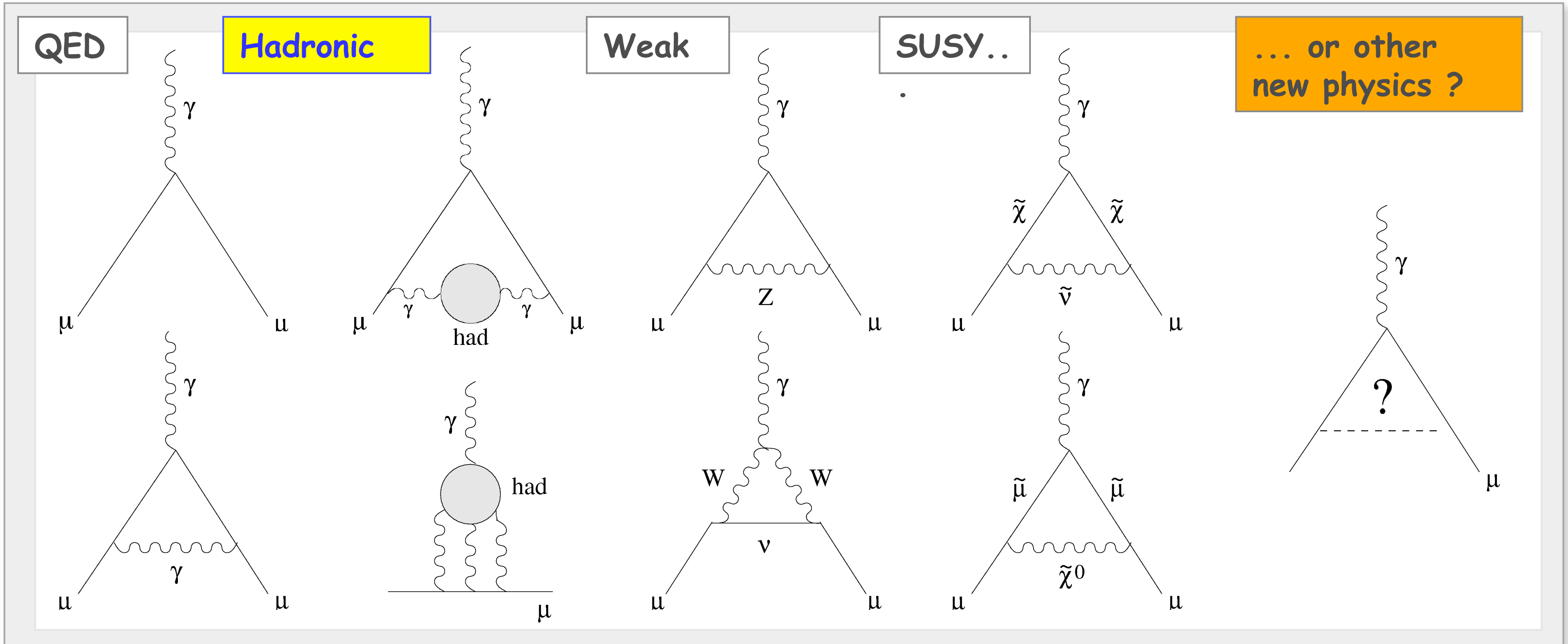
(5th order estimated)

$$\Gamma_\mu = e\gamma_\mu + a_\ell \frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

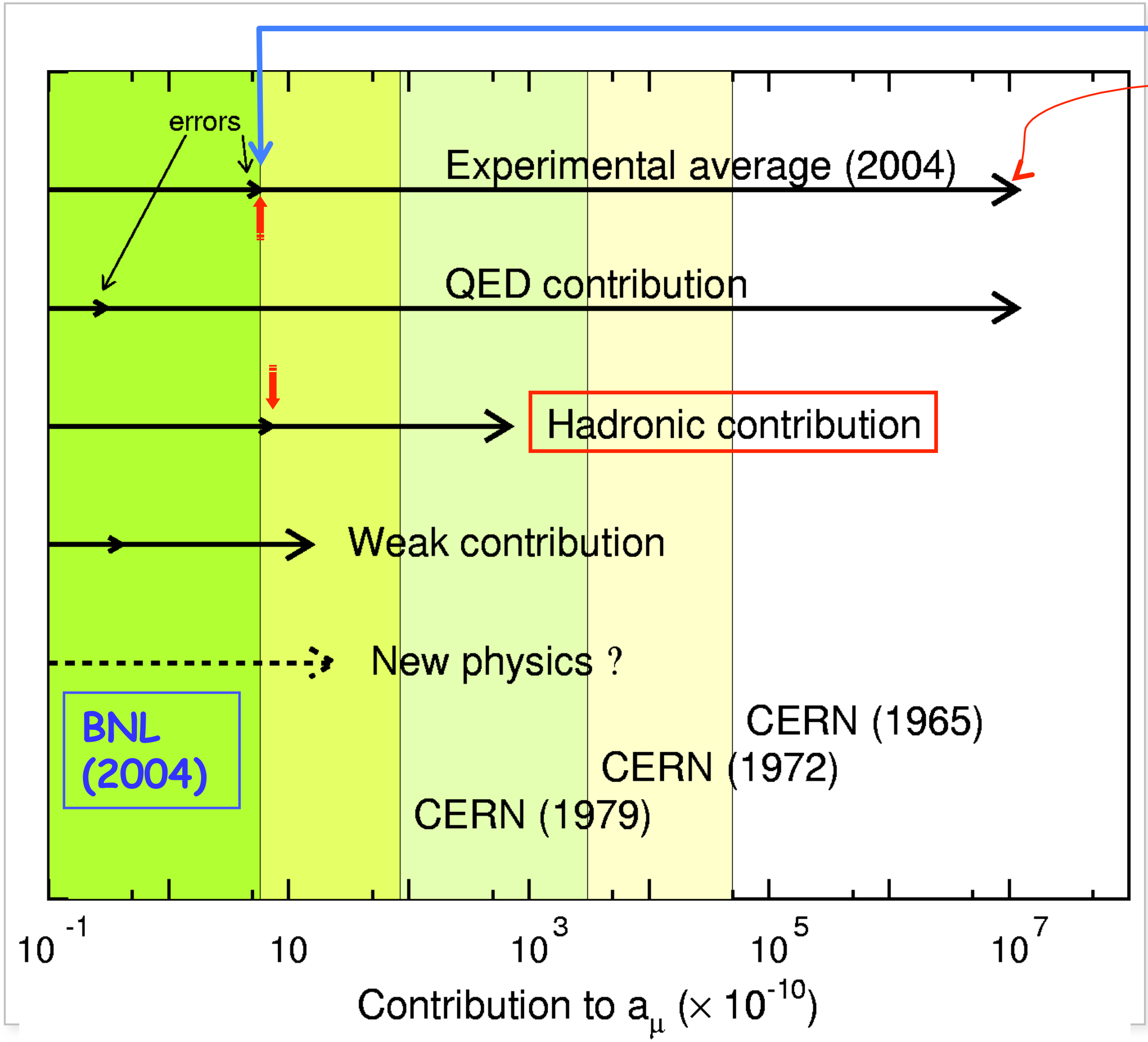
Schwinger 1948

$$a_\ell = \frac{\alpha}{2\pi} = 0.001161\ldots$$

$$a_\mu^{QED} = \sum_{n=1}\left(\frac{\alpha}{\pi}\right)^n A_n \approx \begin{pmatrix} 11614098.1 + 41321.8 \\ + \; 3014.2 + 38.2 + 0.6 \end{pmatrix} \times 10^{-10}$$



Why do we need to know it so precisely?



$a_\mu^{exp} = (11\ 659\ 208.0 \pm 5.8) \times 10^{-10}$

Experimental precision outperforms theory precision on hadronic contribution

Experimental progress on g-2

Miller, de Rafael, Lee Roberts, 2006

Experiment	Beam	Measurement	$\delta a_\mu / a_\mu$	Required th. terms
Columbia-Nevis (57)	μ^+	g=2.00±0.10		g=2
Columbia-Nevis (59)	μ^+	0.001 13(+16)(-12)	12.4%	α/π
CERN 1 (61)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 (62)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 (68)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 (75)	$\mu^\pm$	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3$ + had
CERN 3 (79)	$\mu^\pm$	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (00)	μ^+	0.001 165 919 1(59)	5 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (01)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4$ + had + weak
BNL E821 (02)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
BNL E821 (04)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?

➔ Current world average: $a_\mu^{exp} = 11\ 659\ 208.9 \pm 6.3 \times 10^{-10}$

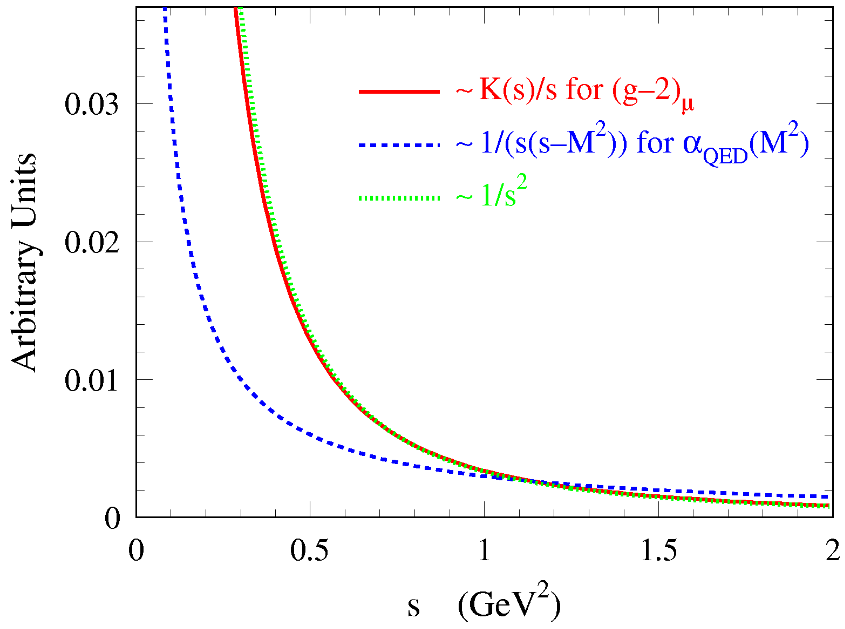
Dominated by by BNL-E821: PRD73, 072003 (2006)

SM Predictions: $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{Weak}$

$a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,HO} + a_\mu^{had,LBL}$ Leading-Order   Higher-Order   Light-By-Light

- Hadronic (q & g) loop contributions cannot be calculated from 1st principles
- Use low energy e^+e^- data to calculate the dominant LO contributions:

$$a_\mu^{had,LO} = \frac{\alpha^2(0)}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s), \qquad R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

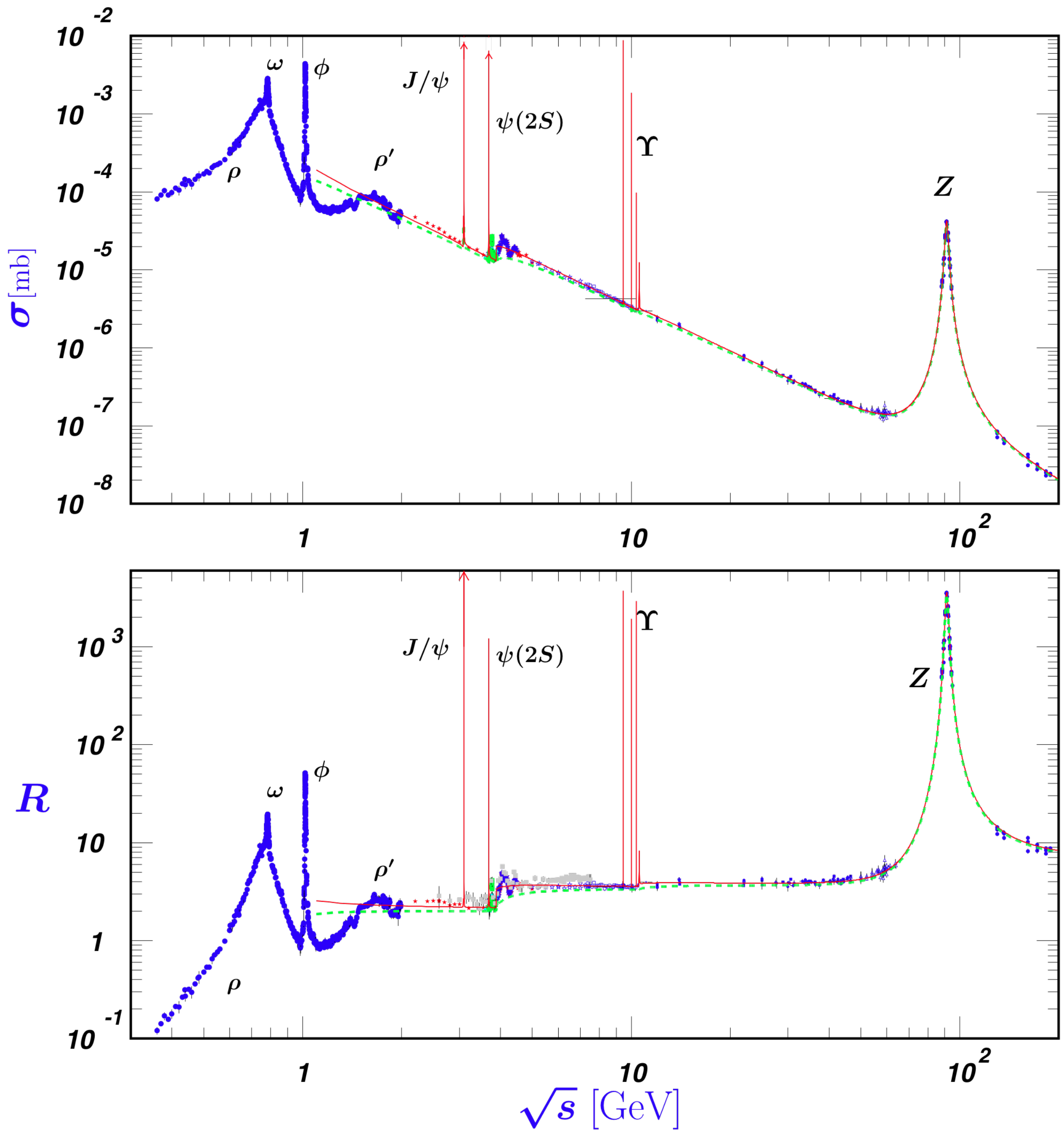


➔ Data driven calculation, its precision depends thus on the input data!

➔ The QED kernel K(s) has such an s dependence that low energy data contribute most

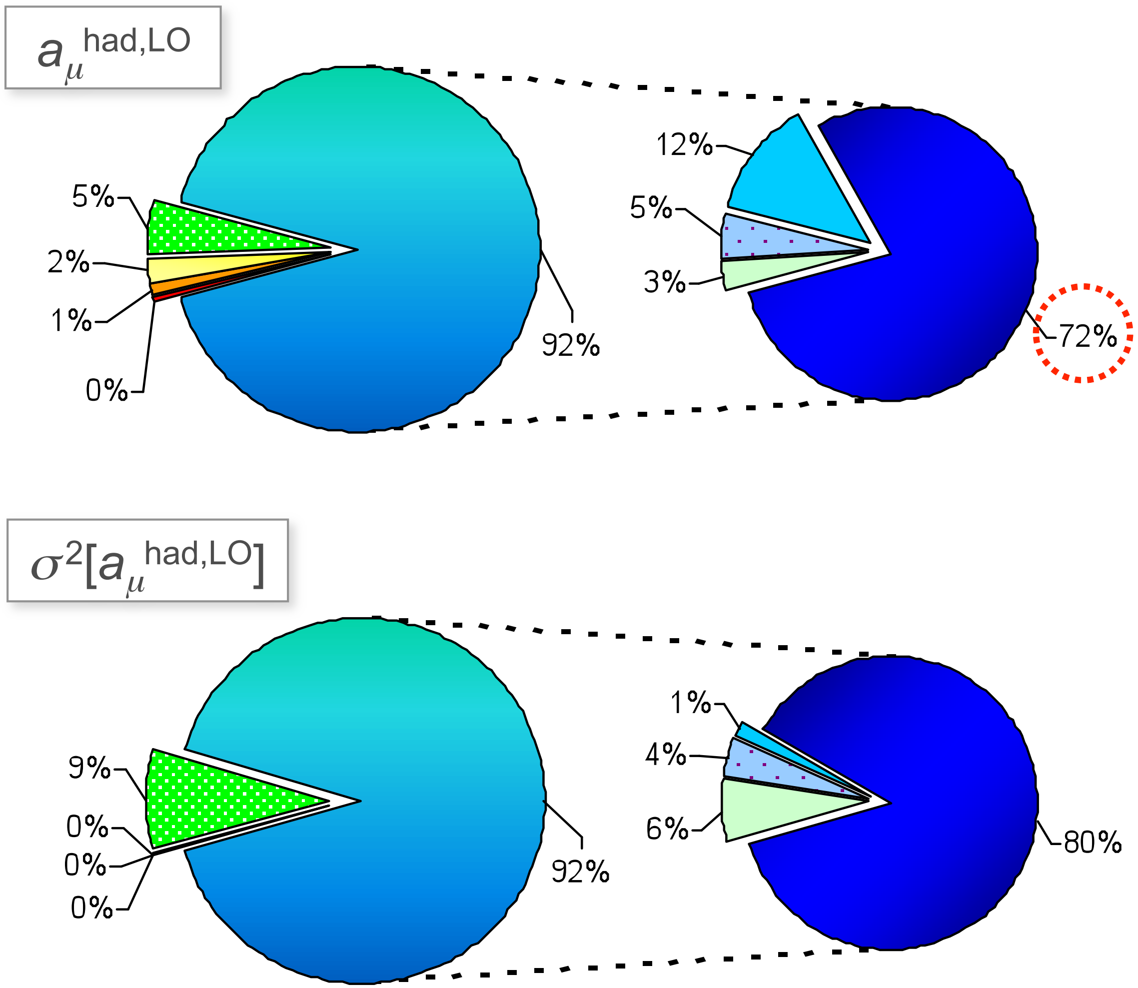
Brodsky, de Rafael, 1968

Low energy e⁺e⁻ annihilation data



Channel
$\pi^0\gamma$
$\eta\gamma$
$\pi^+\pi^-$
$\pi^+\pi^-\pi^0$
$2\pi^+2\pi^-$
$\pi^+\pi^-2\pi^0$
$2\pi^+2\pi^-\pi^0$ (η excl.)
$\pi^+\pi^-3\pi^0$ (η excl., from isospin)
$3\pi^+3\pi^-$
$2\pi^+2\pi^-2\pi^0$ (η excl.)
$\pi^+\pi^-4\pi^0$ (η excl., from isospin)
$\eta\pi^+\pi^-$
$\eta\omega$
$\eta 2\pi^+2\pi^-$
$\eta\pi^+\pi^-2\pi^0$ (estimated)
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)
$\omega\pi^+\pi^-, \omega 2\pi^0$ ($\omega \to \pi^0\gamma$)
ω (non-$3\pi, \pi\gamma, \eta\gamma$)
K^+K^-
$K^0_S K^0_L$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)
$K\overline{K}\pi$ (partly from isospin)
$K\overline{K}2\pi$ (partly from isospin)
$K\overline{K}3\pi$ (partly from isospin)
$\phi\eta$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)
J/ψ (Breit-Wigner integral)
$\psi(2S)$ (Breit-Wigner integral)
$R_{\rm data}$ [3.7 – 5.0 GeV]

Relative Contribution of Input Data vs Energy

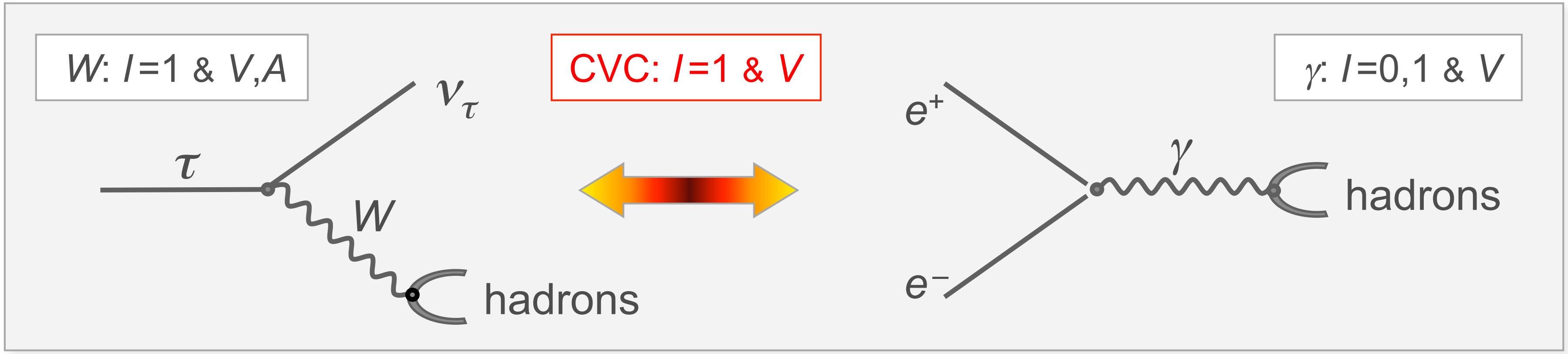


- ➔ 2π channel contributes more than 70% !
- ➔ The e+e- data precision (was) limited

- ➔ Use (complement with) tau data

Alemany, Davier, Hoecker 1998

Connect τ and e^+e^- Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\,\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\,\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions — mass spectrum — kinematic factor (PS)

Recent Development

Davier, Hoecker, Malaescu, Zhang, arXiv: 1010.4180

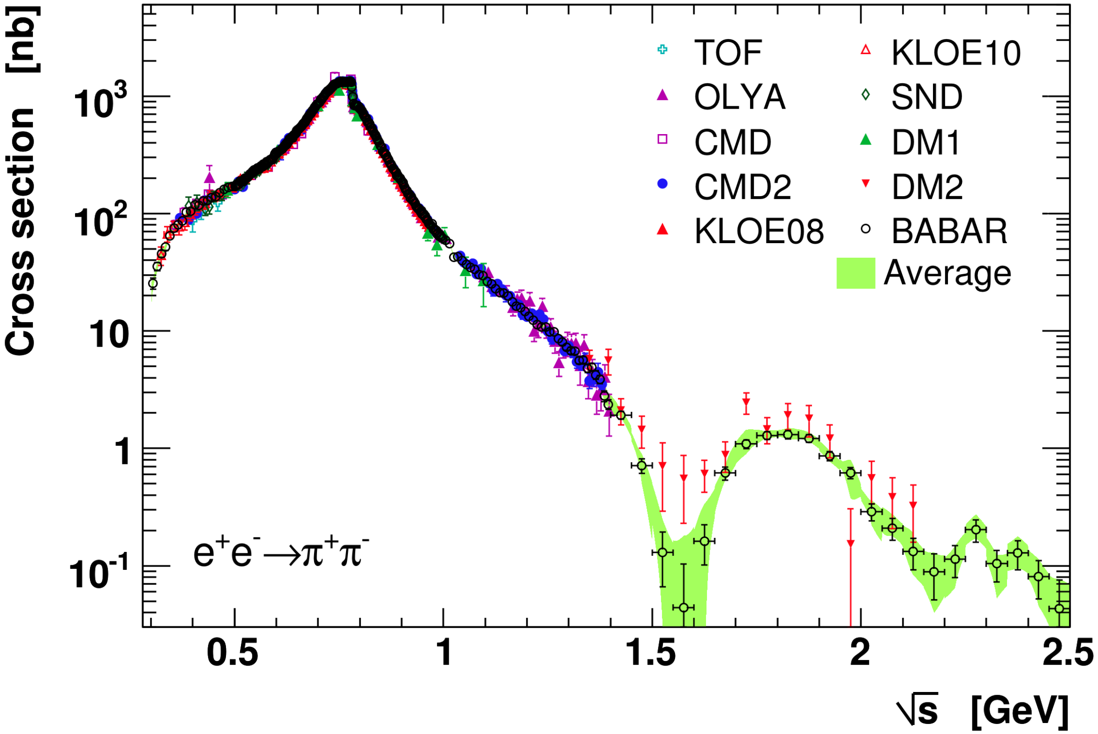
- Use new (precise) e+e- annihilation data e.g. from BABAR, KLOE

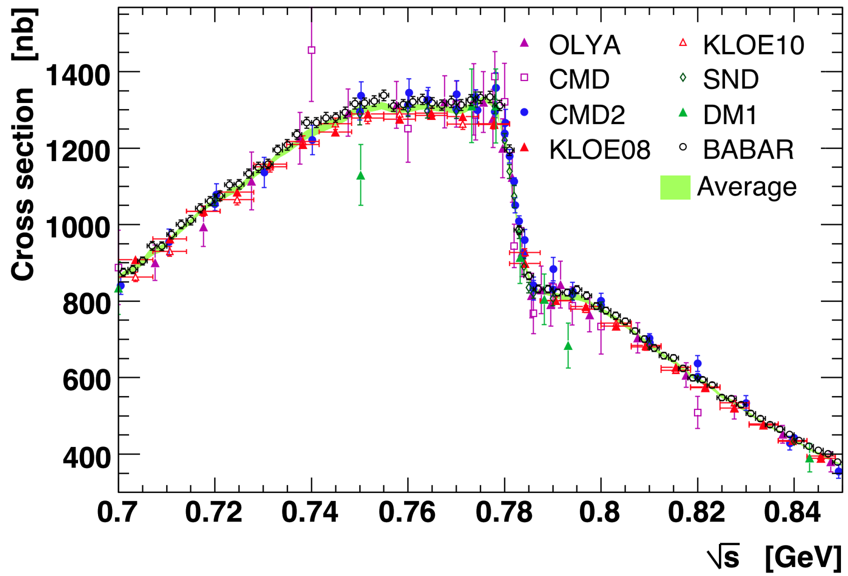
- Develop a new software package HVPTools to
  - perform local average using data from different experiments
  - take into account inter-experiment and inter-channel correlations

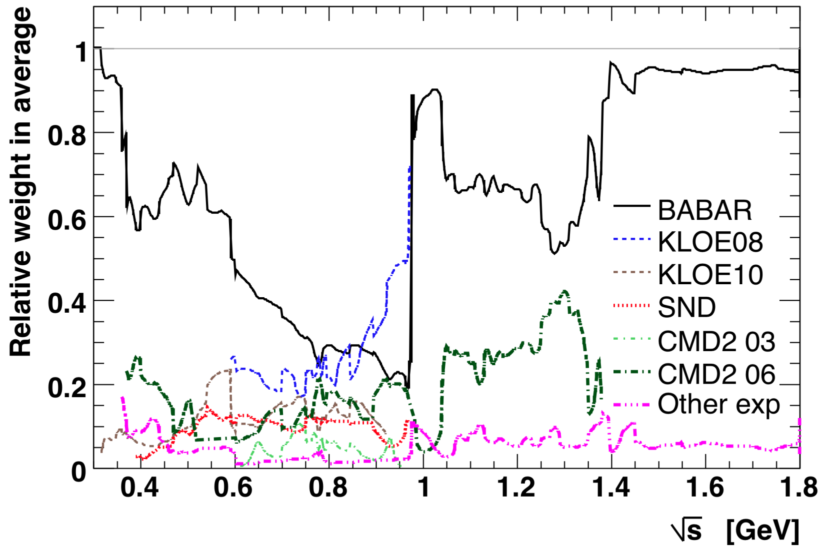
- Perform a comprehensive isospin analysis of small missing channels

- Recompute the continum contributions using pQCD at 4 loops

2π Channel

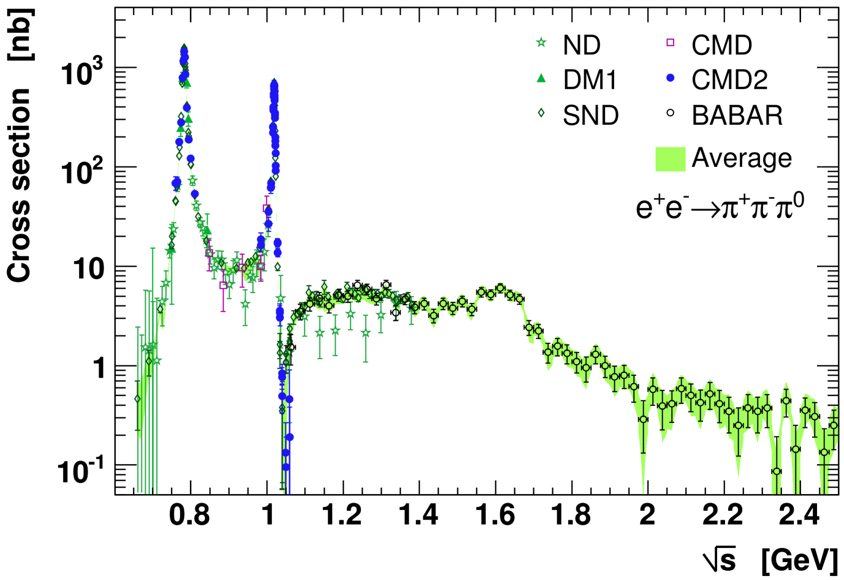


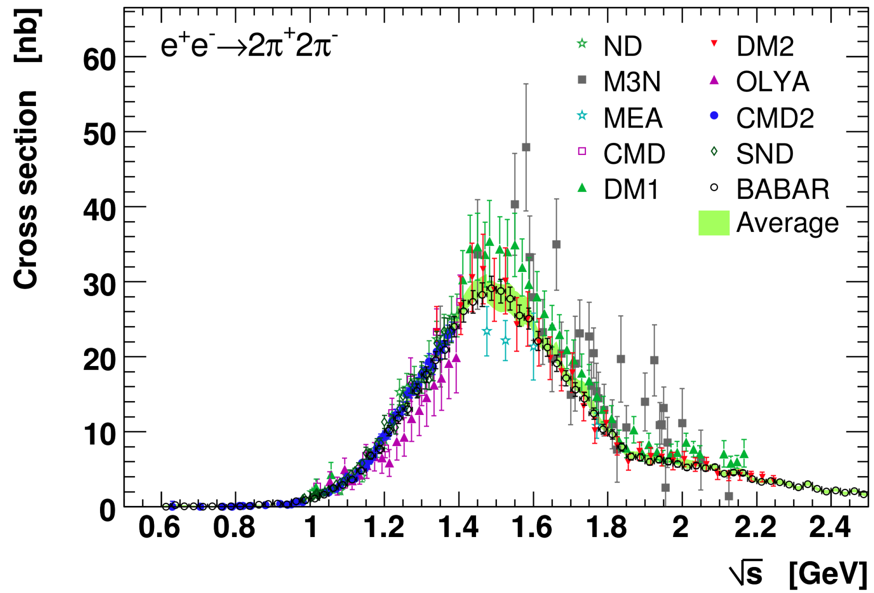


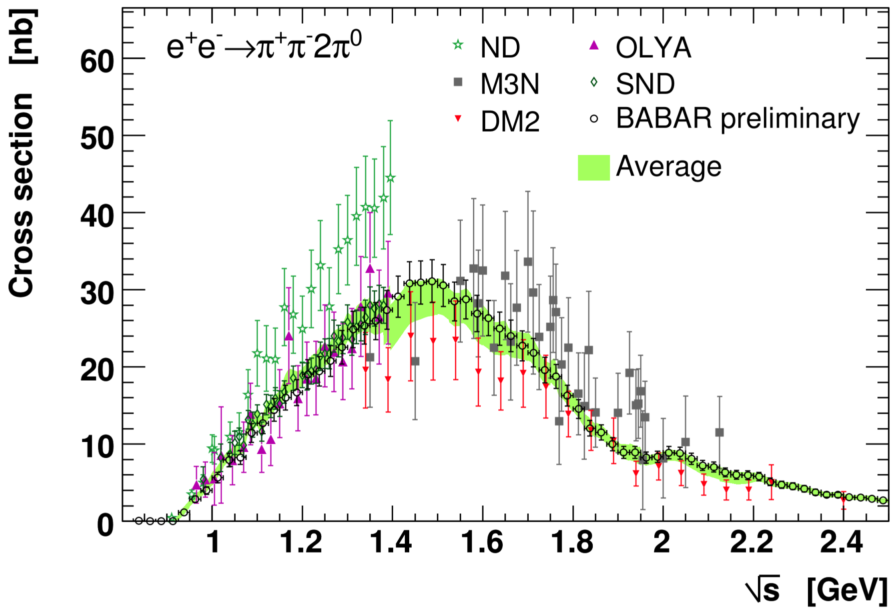


- Measured by many experiments
- BABAR dominates over almost all energy region
- Discrepancy between BABAR and KLOE

$\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$ Channels

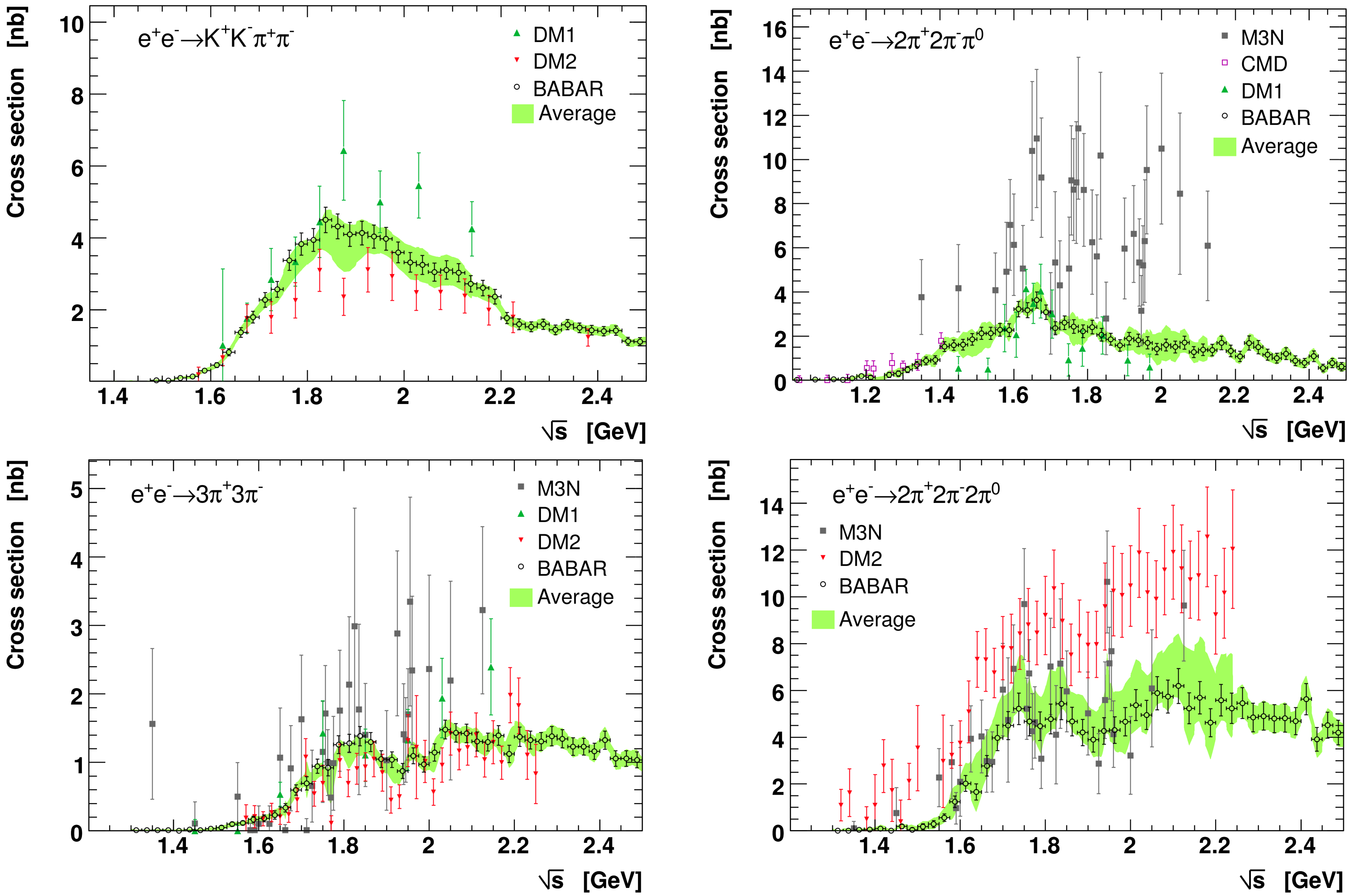




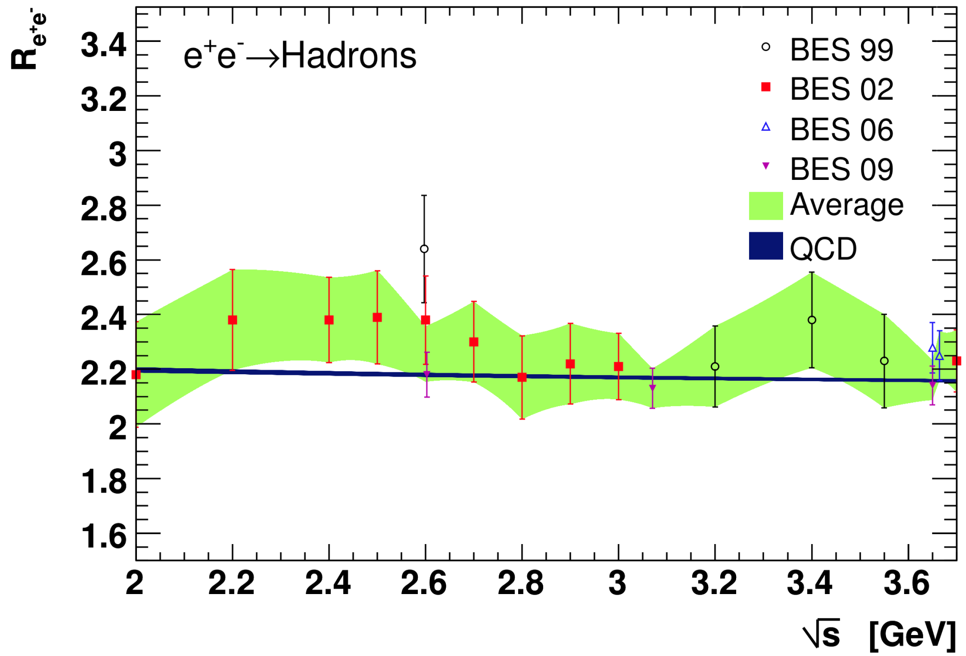


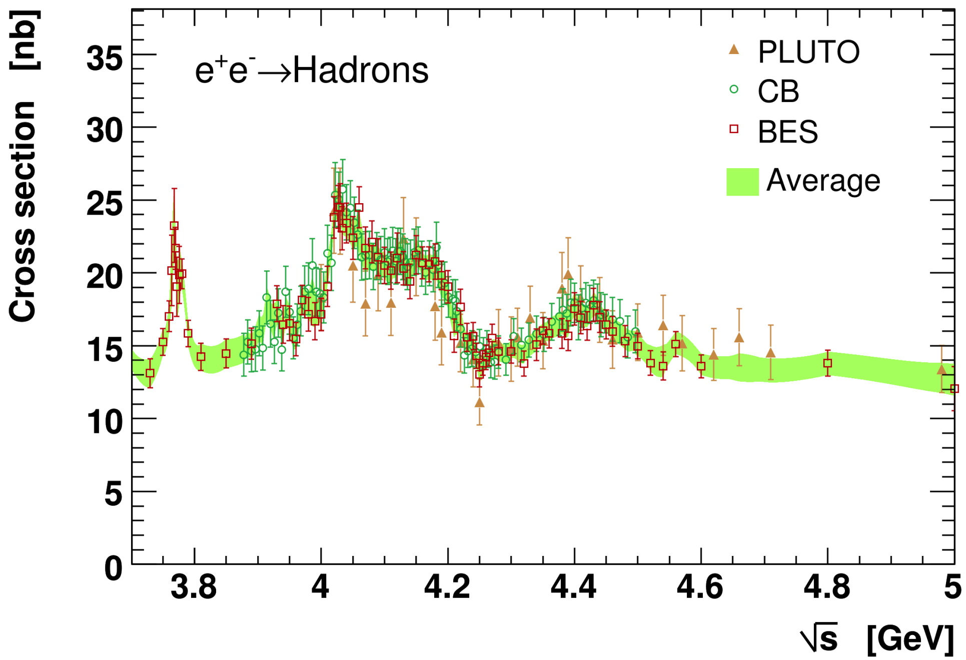
Again BABAR dominates over other experiments though data for $\pi^+\pi^-2\pi^0$ channel still preliminary

Other Multi-hadron Channels



Regions below and above DDbar





pQCD calculation in good agreement with the direct measurements in non-resonance regions and are applied down to 1.8GeV

$R_{\rm QCD}$	$[1.8 - 3.7\ \mathrm{GeV}]_{uds}$
$R_{\rm QCD}$	$[5.0 - 9.3\ \mathrm{GeV}]_{udsc}$
$R_{\rm QCD}$	$[9.3 - 12.0\ \mathrm{GeV}]_{udscb}$
$R_{\rm QCD}$	$[12.0 - 40.0\ \mathrm{GeV}]_{udscb}$
$R_{\rm QCD}$	$[> 40.0\ \mathrm{GeV}]_{udscb}$
$R_{\rm QCD}$	$[> 40.0\ \mathrm{GeV}]_{t}$

New Results on $\alpha(M_Z)$ & Constraint on M_H

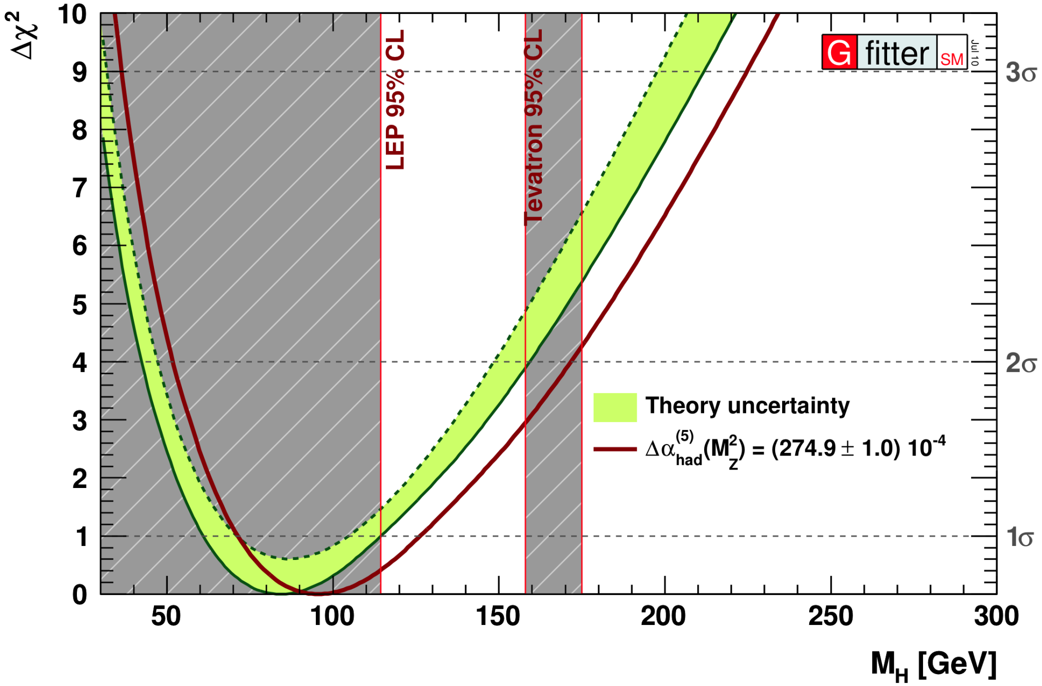
$\Delta\alpha_{had}(M_Z) = 274.21 \pm 0.17_{stat} \pm 0.78_{uncor\text{-}syst} \pm 0.41_{cor\text{-}syst} \pm 0.18\psi \pm 0.52_{QCD}$ $(\times 10^{-4})$

$= 274.21 \pm 1.04_{total}$ $(\times 10^{-4})$

➔ $\Delta\alpha_{had}^{(5)}(M_Z) = 274.9 \pm 1.0$ $(\times 10^{-4})$, $\alpha^{-1}(M_Z) = 128.962 \pm 0.015$

To be compared with

HMNT (06): $\Delta\alpha_{had}^{(5)}(M_Z) = 276.8 \pm 2.2$ $(\times 10^{-4})$, $\alpha^{-1}(M_Z) = 128.937 \pm 0.030$



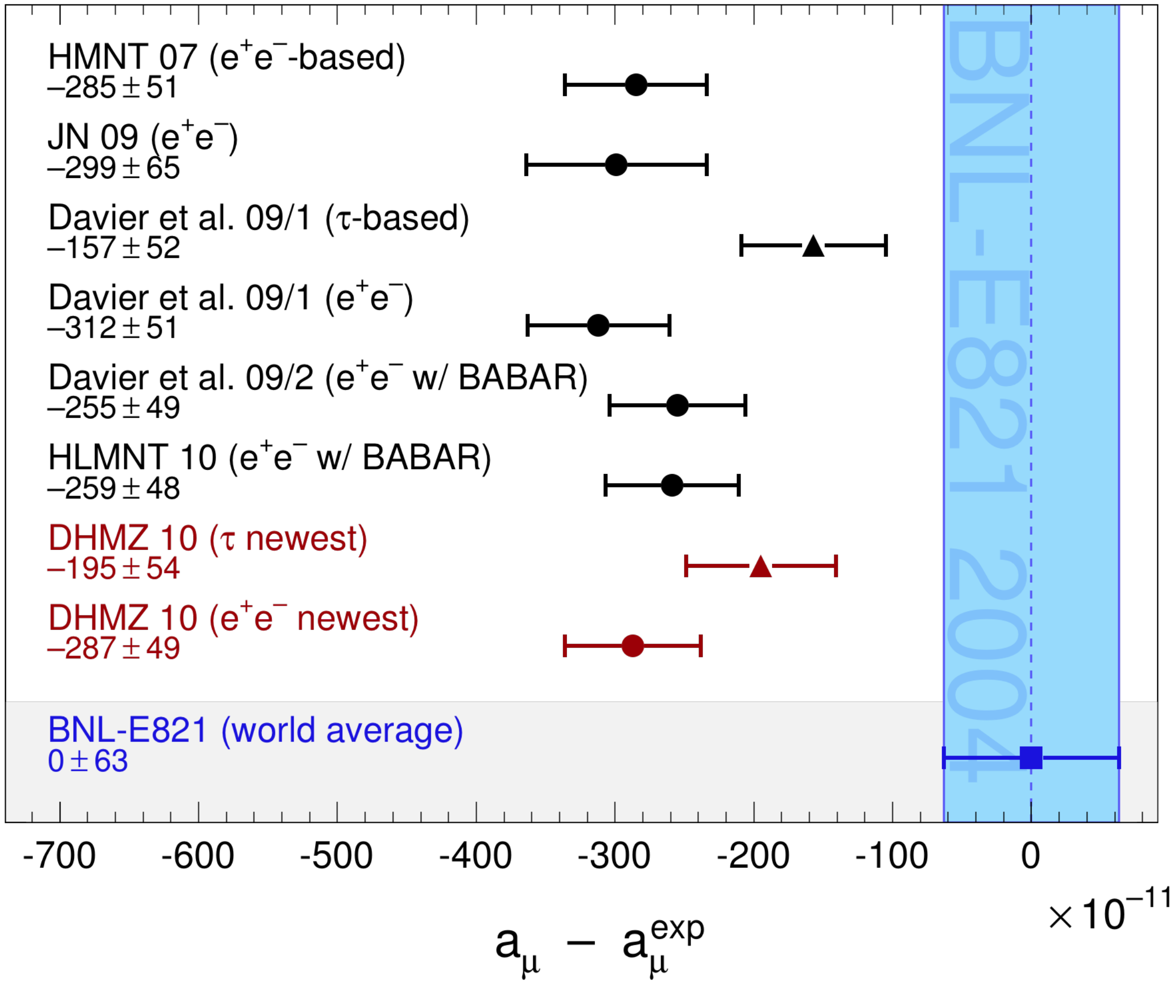
The fitted (Gfitter) Higgs mass shifted from 84^{+30}_{-23} GeV to 96^{+31}_{-24} GeV

The new upper limits are:

170 GeV @90% CL

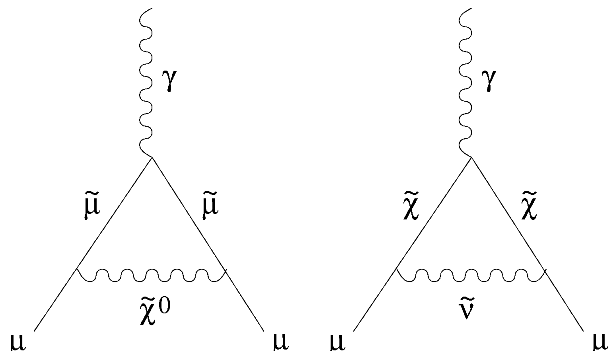
201 GeV @95% CL

Reevaluated $(g-2)_\mu$

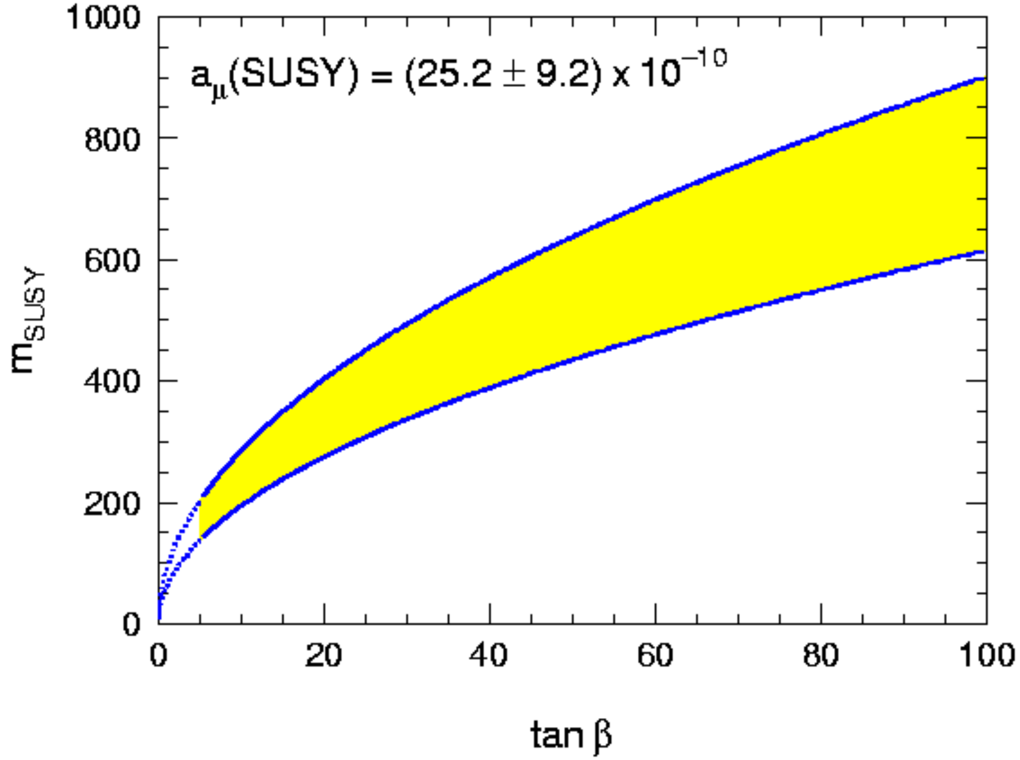


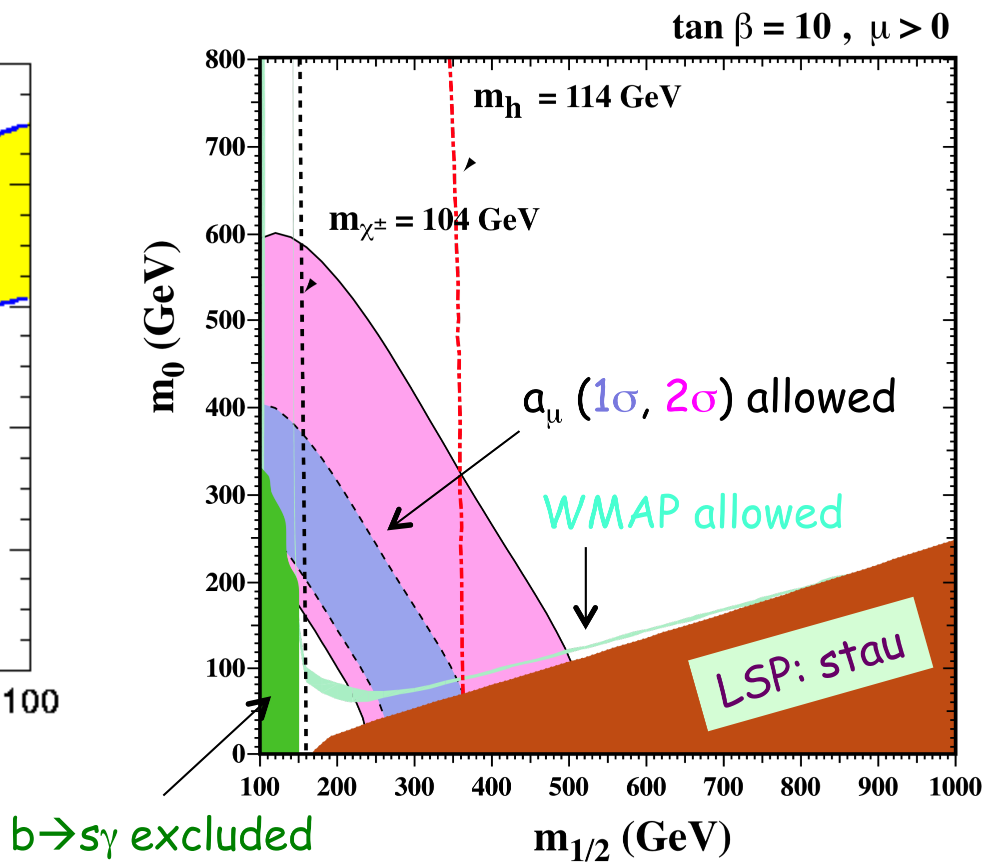
Our e+e- based prediction deviates from the direct measurement by 3.6σ

A SUSY particle could be a good candidate for the discrepancy



Implications on SUSY of the deviation





$$a_\mu^{\rm SUSY} \simeq \pm 130 \times 10^{-11} \cdot \left(\frac{100\ {\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta$$

Ellis, Olive, arXiv: 1001.3651

Selected Topic Two:

HERA, PDF and impact on LHC physics

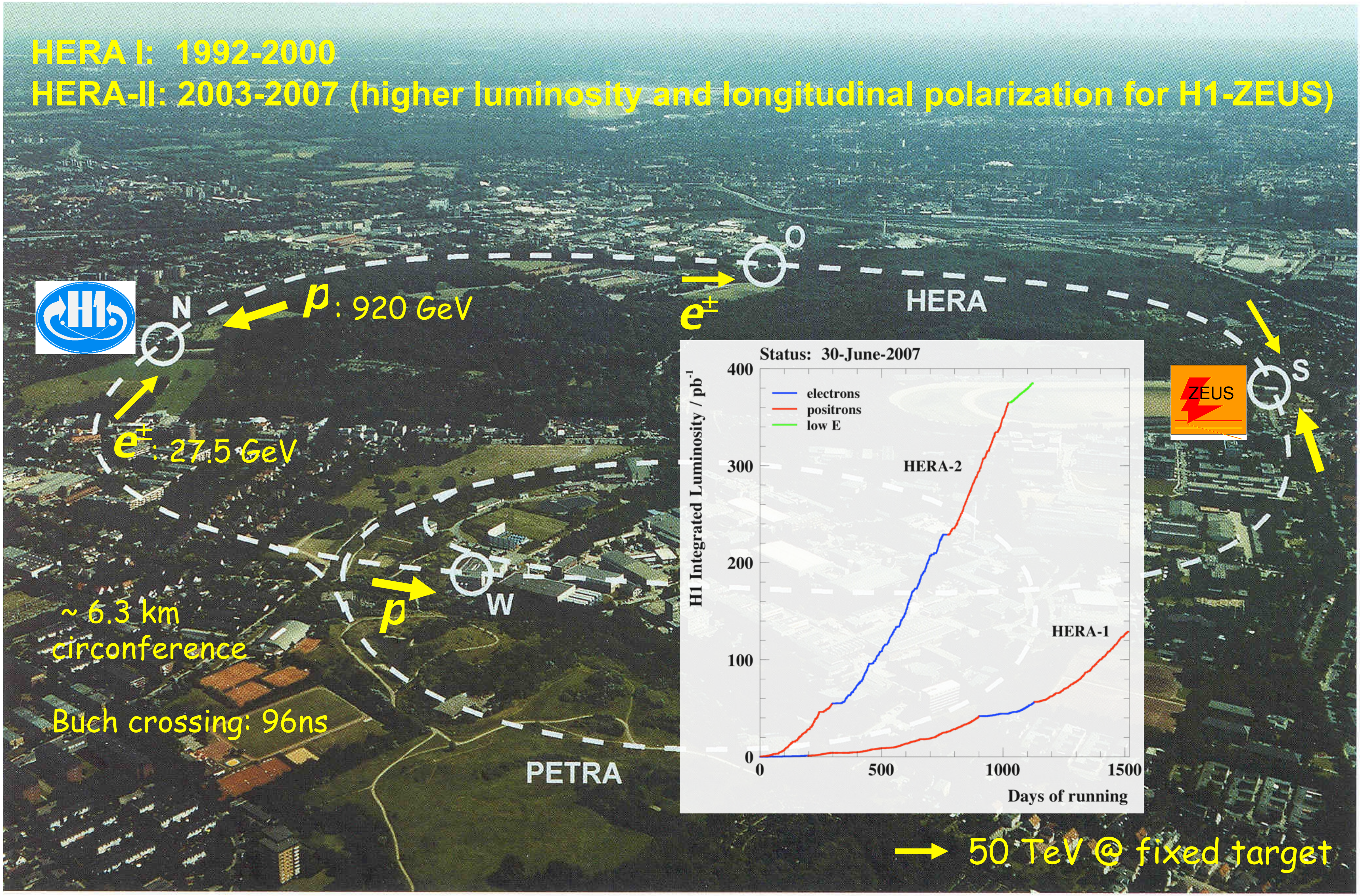
HERAPDF1.0:

- H1 and ZEUS collaborations, JHEP 1001:109 (2010)

Web link:

- https://www.desy.de/h1zeus/combined_results/index.php

HERA (the World's Unique $e^{\pm}p$ Collider)

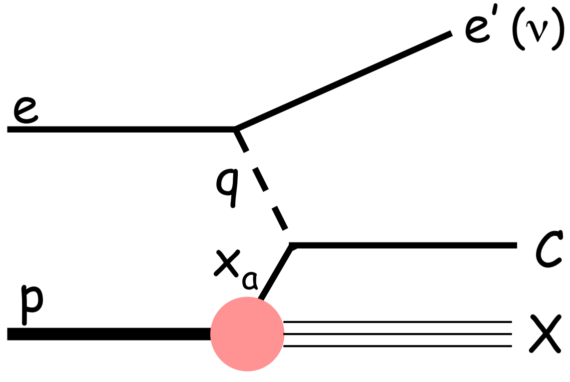


HERA → PDFs → LHC

HERA:

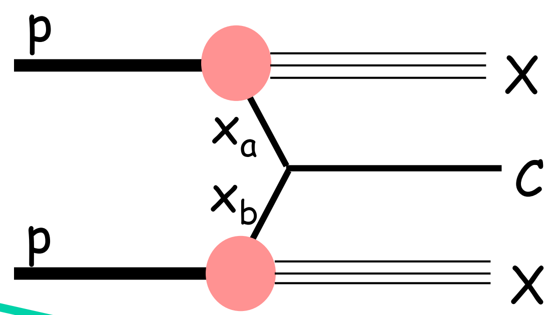
NC (ep→e'X) and CC (ep→νX) or

in general ep→l'+C+X:



$$\sigma_{p\to C}(q,p) = \sum_a \int_x^1 dx_a f_p^a(x_a,\mu)\hat{\sigma}_{ea\to e'C}(q,x_a p,\mu,\alpha_s)$$

→ Universal PDFs (Parton Distribution Functions)

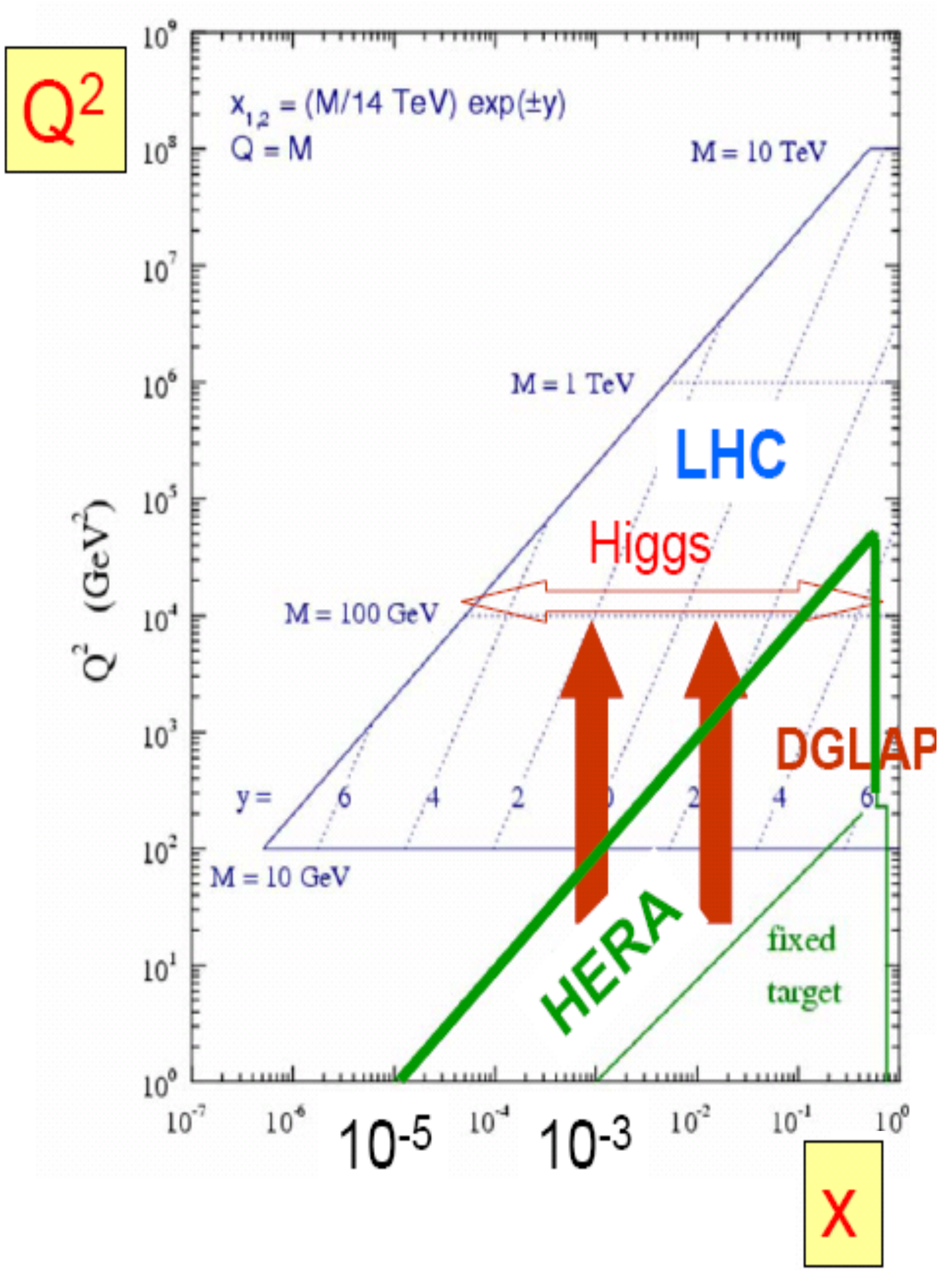


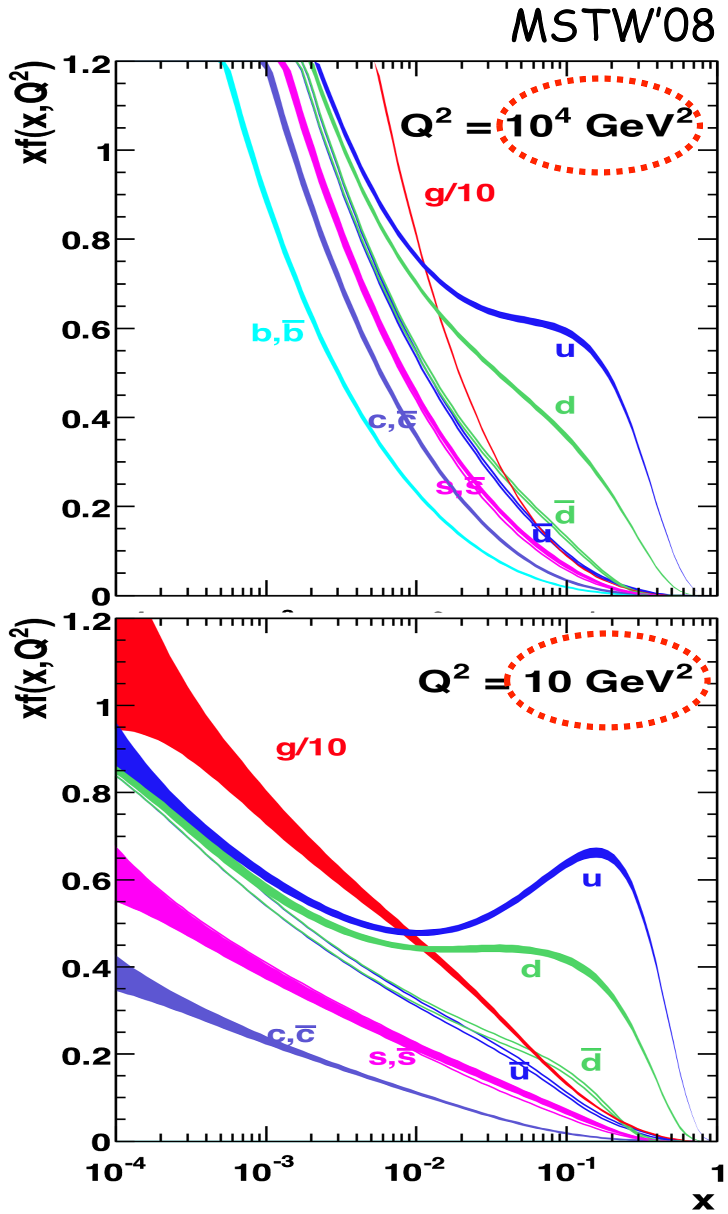
LHC:

pp→C+X:

$$\sigma_{pp\to C}(p_p,p'_p,p_C,\cdots) = \sum_{a,b}\int_{x_a}^1 dx_a \int_{x_b}^1 dx_b f_p^a(x_a,\mu)\hat{\sigma}_{ab\to C}(x_a p_p, x_b p'_p,\mu,\alpha_s) f_p^b(x_b,\mu)$$

Why PDFs?





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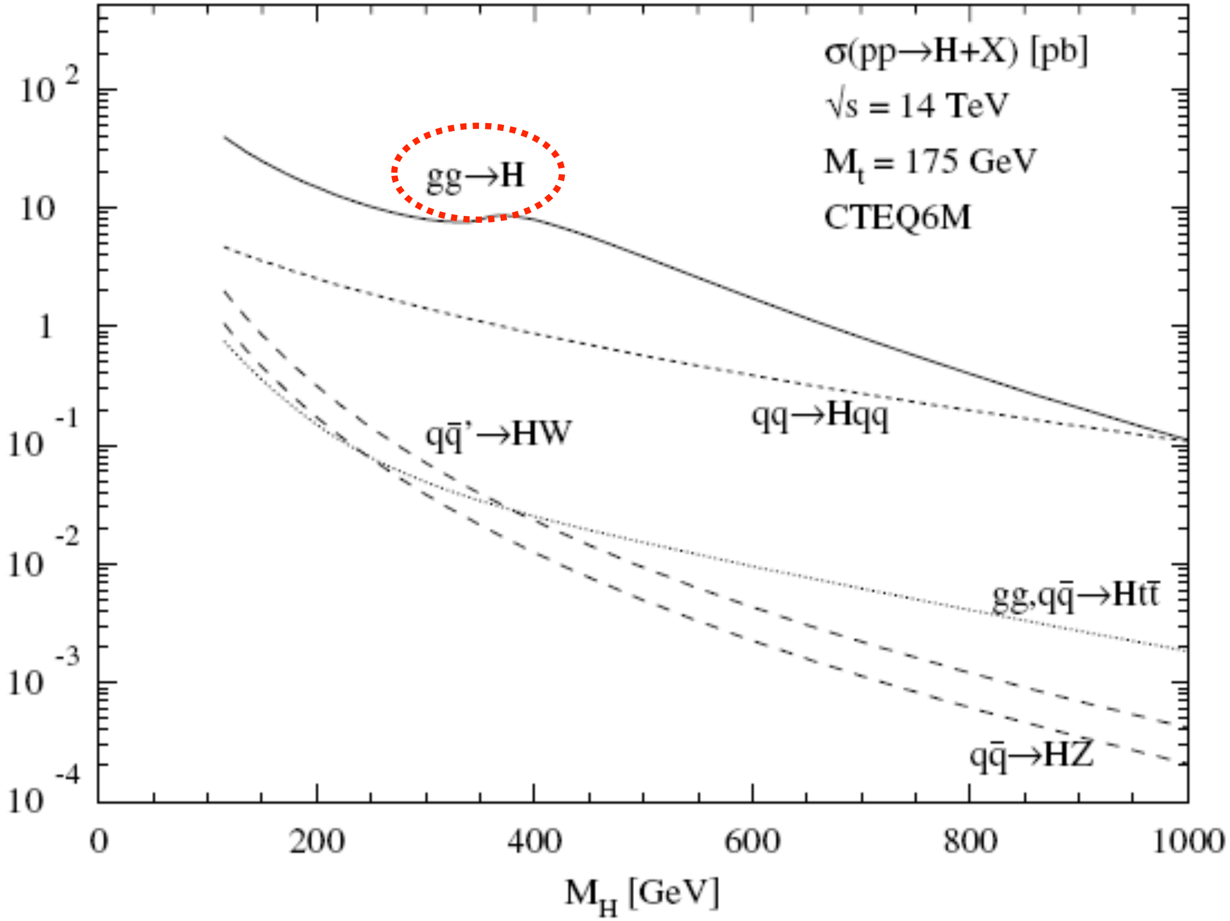
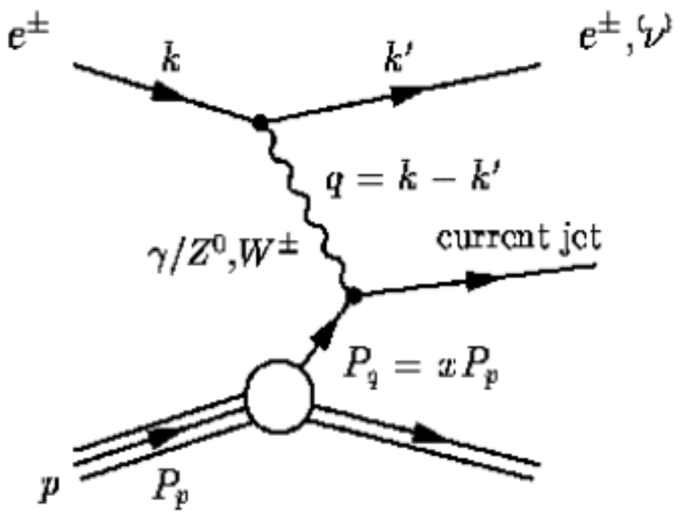
Why PDFs?



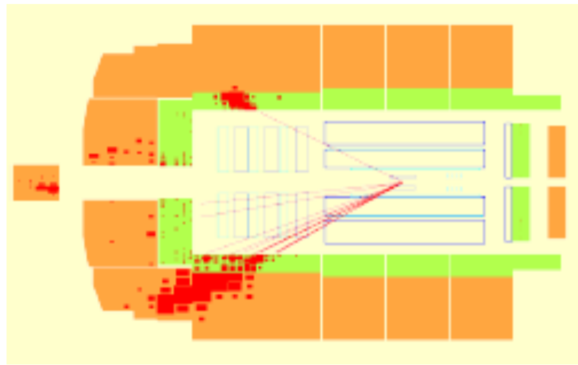
The g(x) density is the 1st necessity at LHC

Precise PDFs needed to provide reliable predictions at LHC

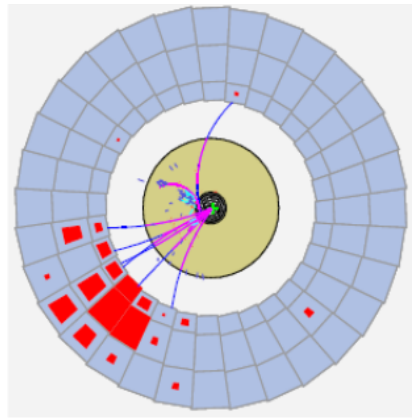
The Dominant SM Processes at HERA



NC in H1



CC in ZEUS



$Q^2 = -q^2 = -(k-k')^2$: spatial resolution
x: Bjorken x
$y=Q^2/xs$: inelasticity
$\sqrt{s}$: center of mass energy

$Q^2 \sim 0$: photoproduction (γp)
$Q^2 \gg 0$: electroproduction (DIS)
$Q^2_{max} > 10^4$ GeV^2 ➔ 1/1000 R_p

➔The two dominant SM Processes for probing proton structure PDFs are Neutral Current (NC) and Charged Current (CC) interactions

Inclusive Cross Sections → PDFs

$$\frac{d^2\sigma^{\pm}}{dx dQ^2} \sim Y_+ F_2 \mp Y_- xF_3 - y^2 F_L$$

with $Y_{\pm} = 1 \pm (1-y)^2$

Structure function terms

NC (LO):

$$F_2 \sim x\left(U + \overline{U} + D + \overline{D}\right) \qquad U = u + c$$

$$xF_3 \sim x\left(U - \overline{U} + D - \overline{D}\right) \qquad D = d + s + b$$

$$F_L = 0$$

NC is quark flavor blind
But larger cross sections
provide better precision

CC (LO):

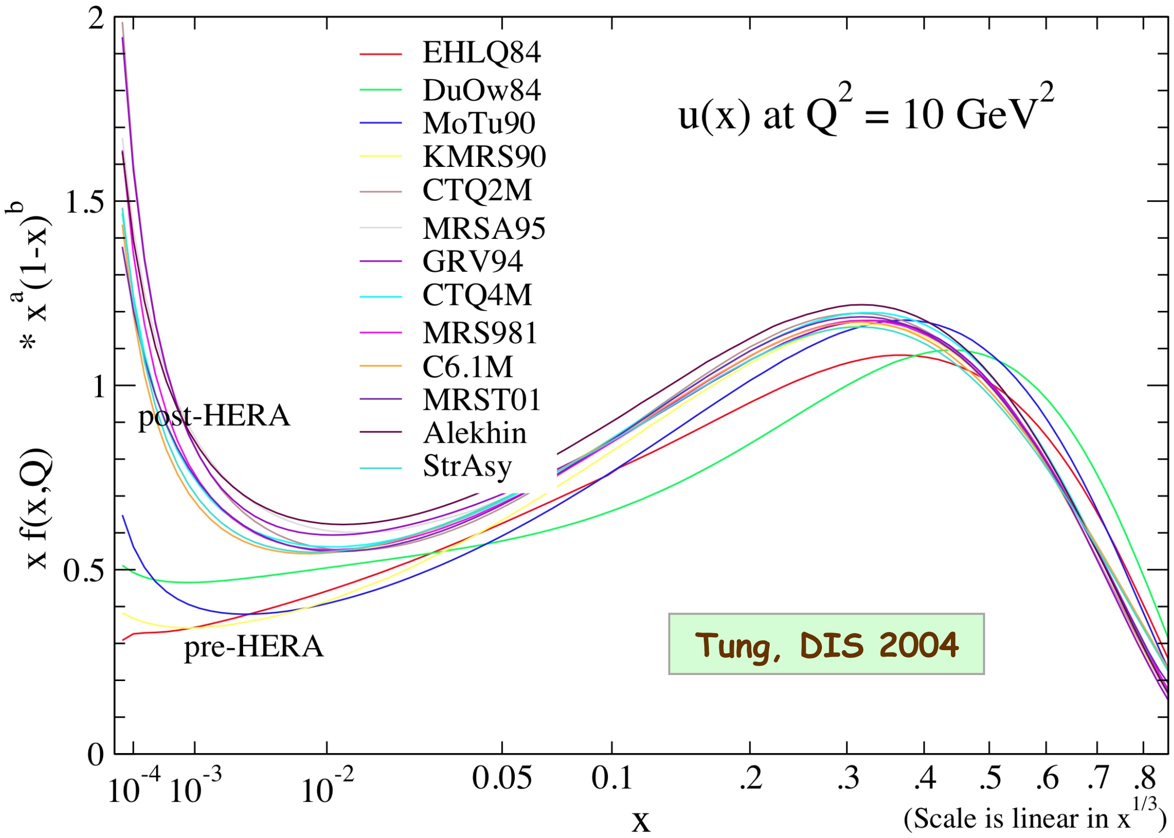
$$F_2^+ \sim x\left(\overline{U} + D\right), \qquad F_2^- \sim x\left(U + \overline{D}\right)$$

$$xF_3^+ \sim x\left(D - \overline{U}\right), \qquad xF_3^- \sim x\left(U - \overline{D}\right)$$

$$F_L = 0$$

CC is quark flavor sensitive
But has smaller cross
sections

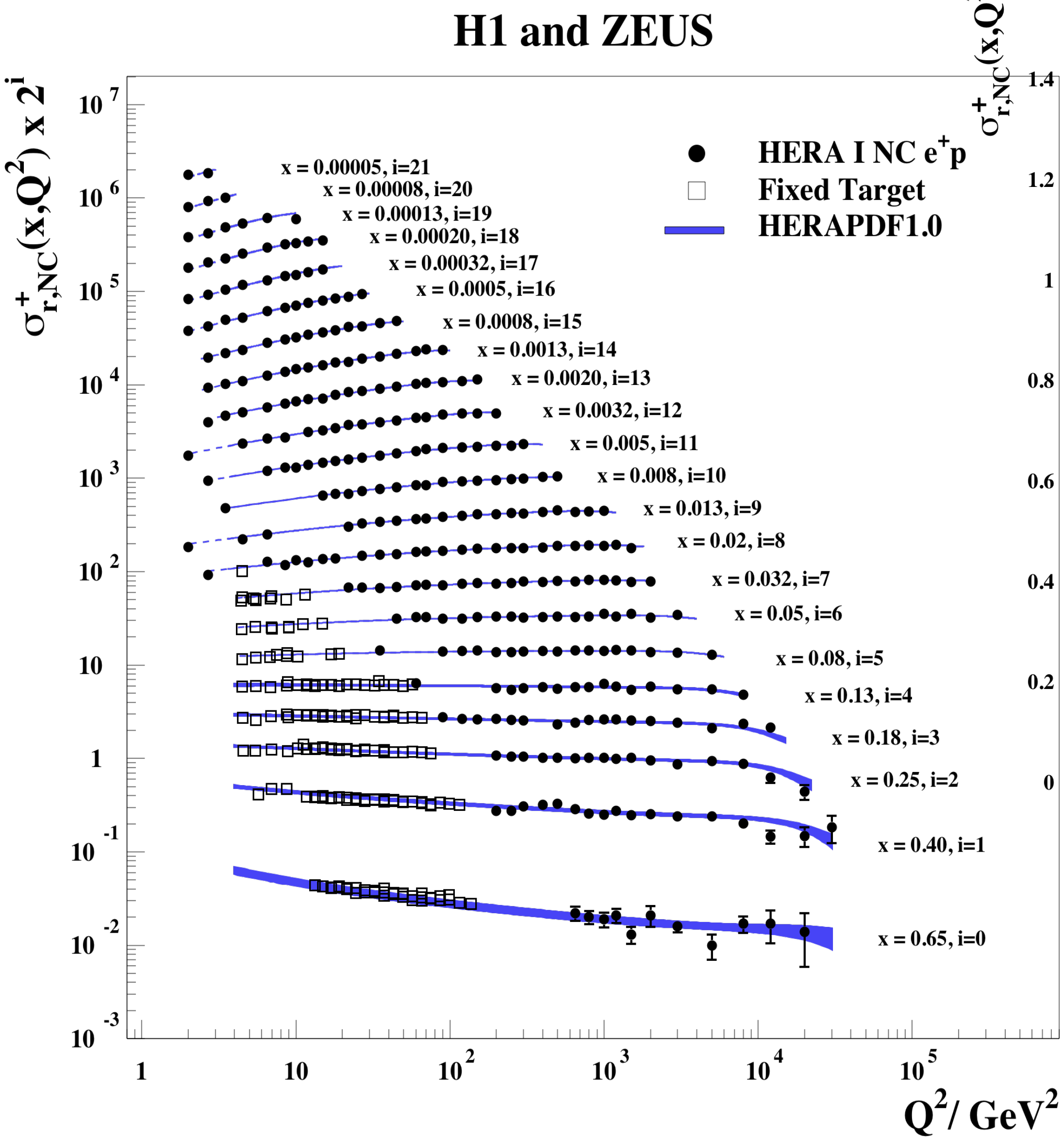
Impact of Earlier HERA-1 Data?

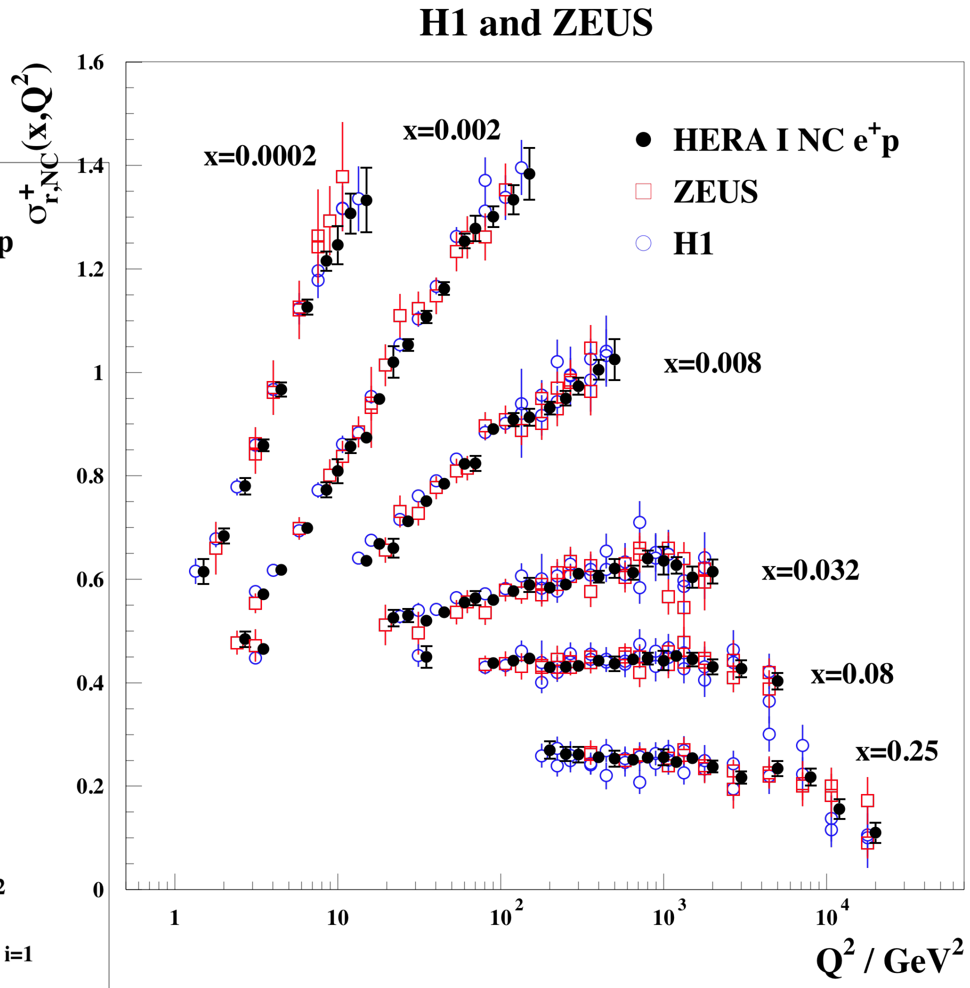


The PDFs $xf(x,Q^2)$ at low x ($<10^{-2}$) before HERA were largely unknown

It is the HERA data which allow PDFs be constrained for x down to 10^{-4}

Combined H1 and ZEUS HERA-1 Data

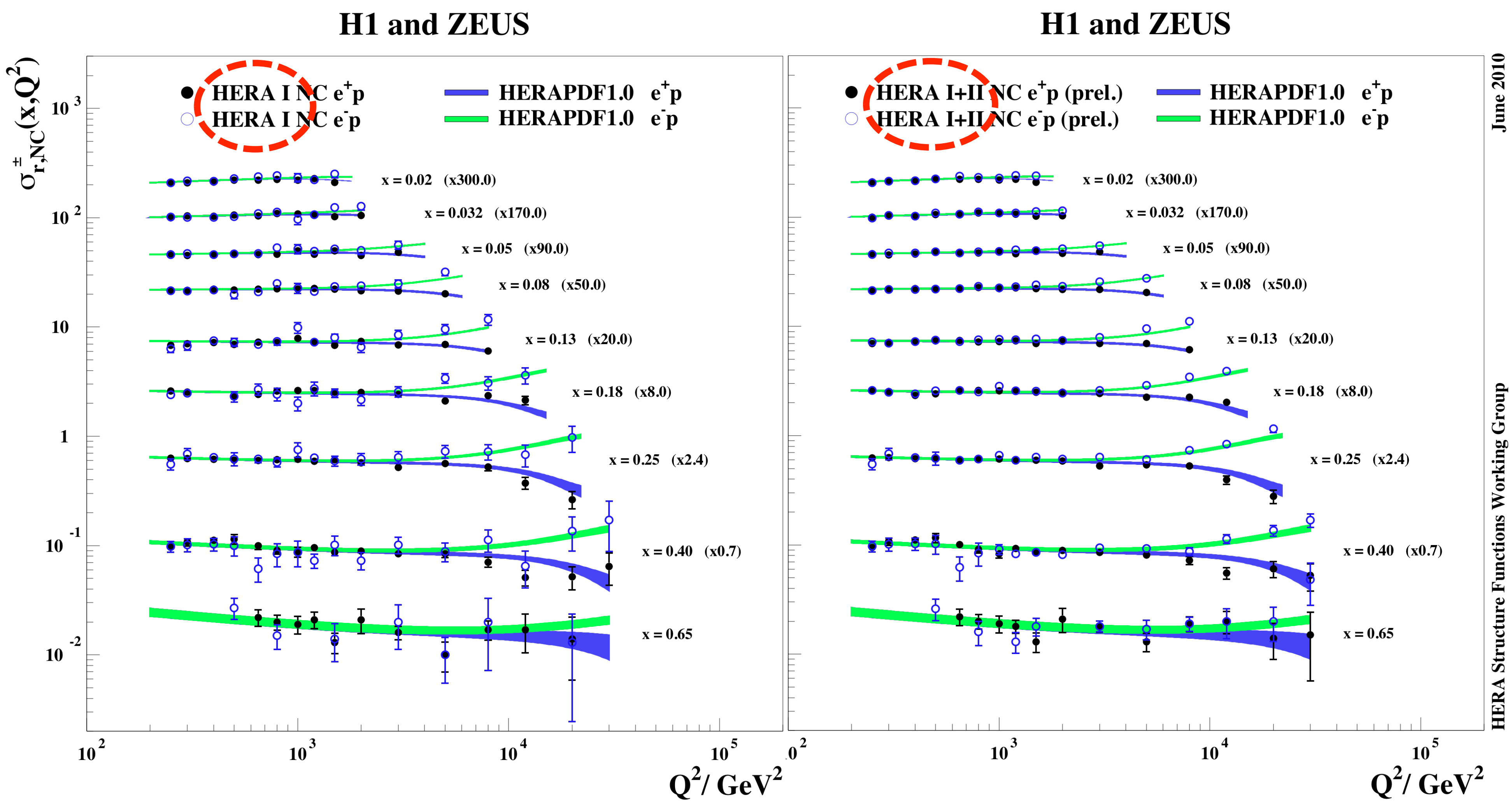




The combination improves both the stat and syst uncertainties!

The strong scaling violation constrains g(x)

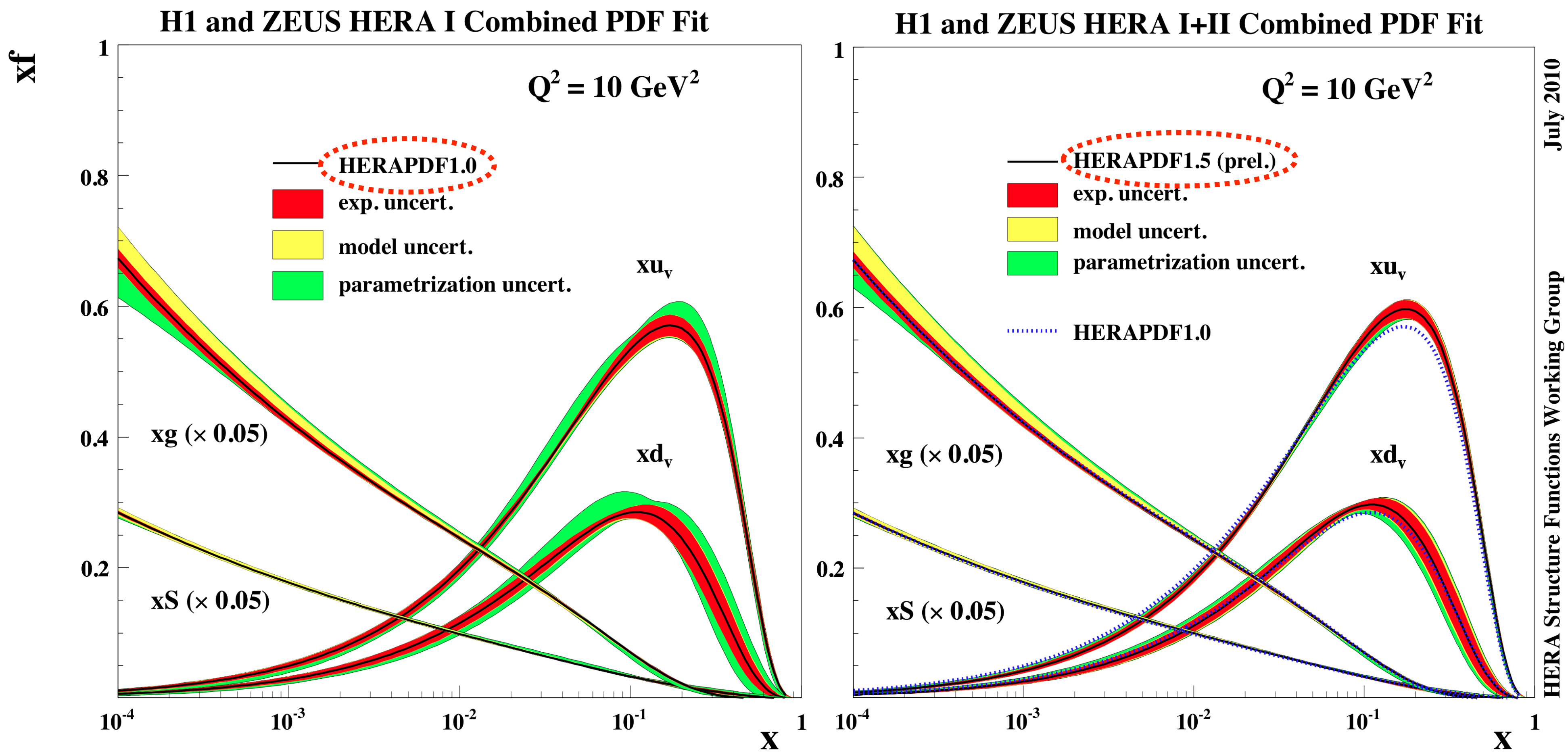
HERA-1 versus HERA-2



HERA-2 data are still preliminary and ZEUS e- data still to be added

Nevertheless the precision of the new combination at high x improved

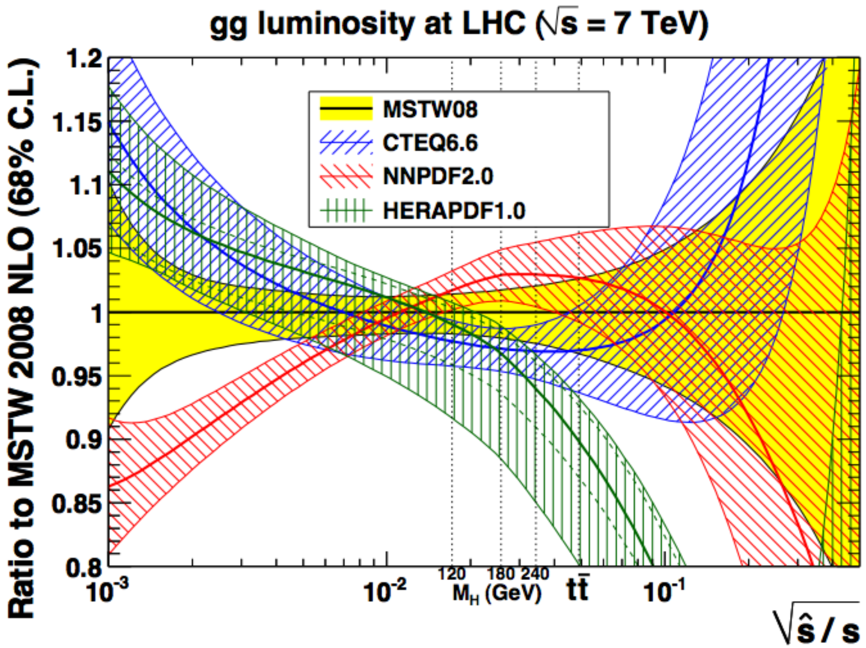
HERAPDF1.0 & HERAPDF1.5

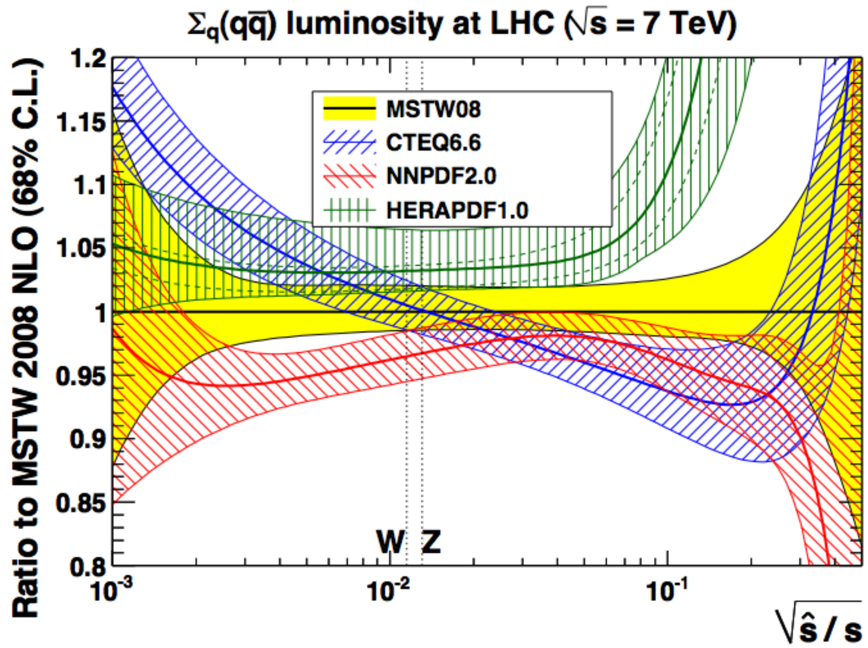


➔ HERAPDF based on HERA data only
is being obtained with steadily improving precision

HERAPDF1.0 versus Other Global PDFs

G.Watt, PDF4LHC, March 2010





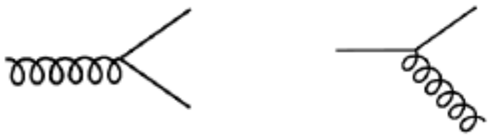
PDFs differences:

- Input data (HERA only vs. including other data)
- PDF parameterization forms or not
- NLO or NNLO, α_s value (fixed or fitted)
- Error treatments
- Heavy quark treatment etc

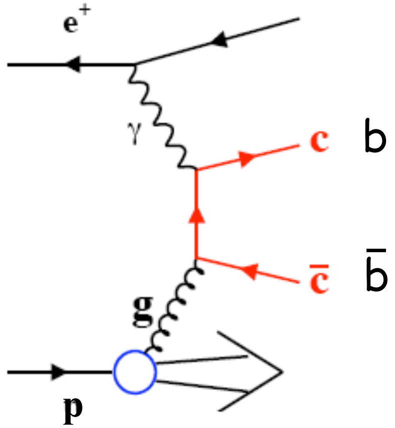
→W (Z) cross sections:
- constrain PDFS
- provide alternative lumi determination

How to Improve Further the g(x) at HERA?

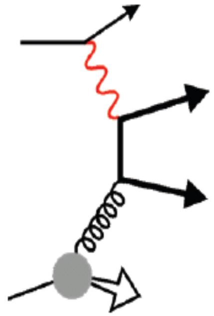
Scaling violation
$dF_2/d\ln Q^2$



Heavy quark production



Jet production at HERA



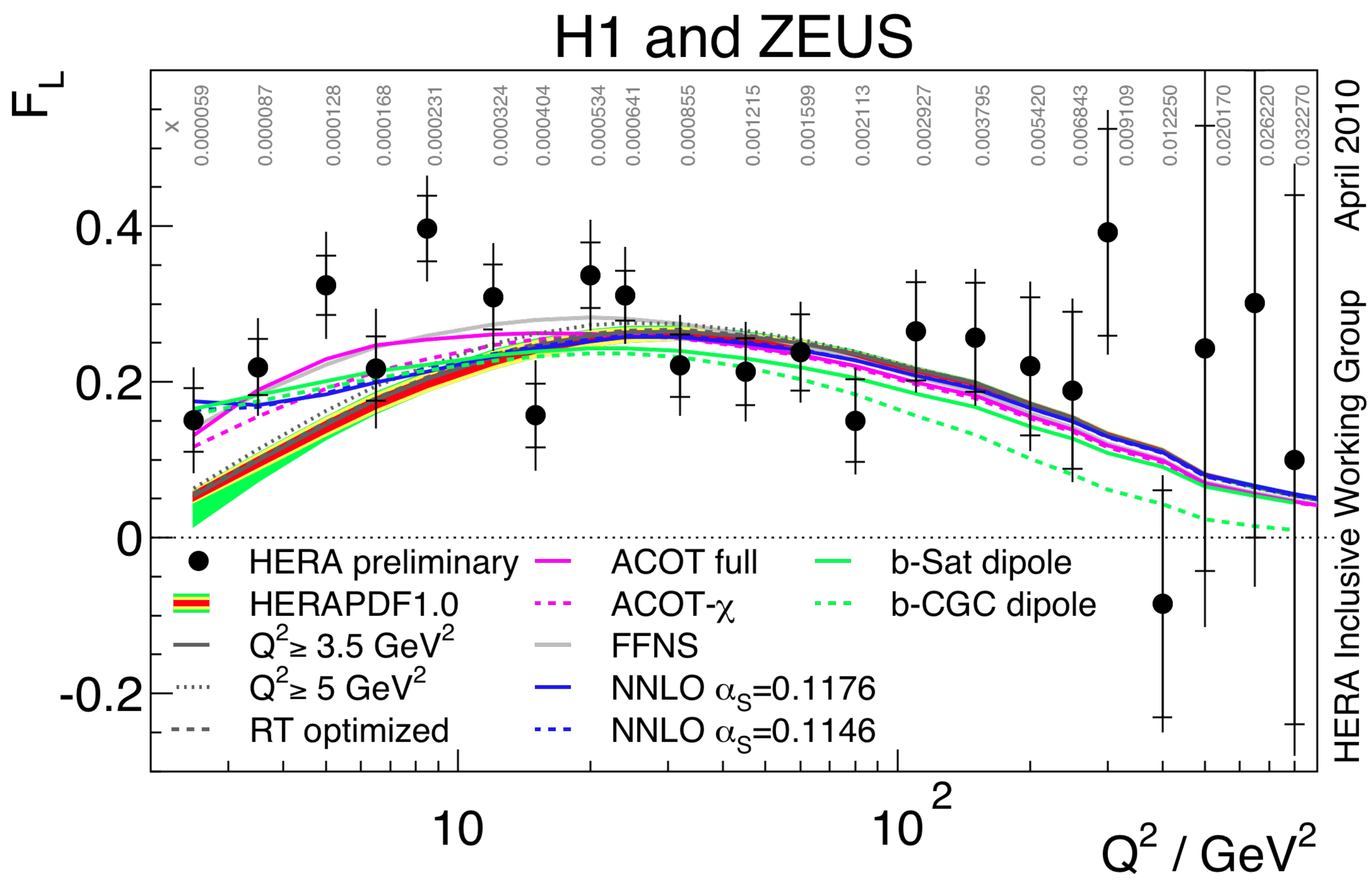
Jets in DIS & γp

g(x)

F_L measurement
$F_L \sim xg(x)$

H1 et ZEUS data combination

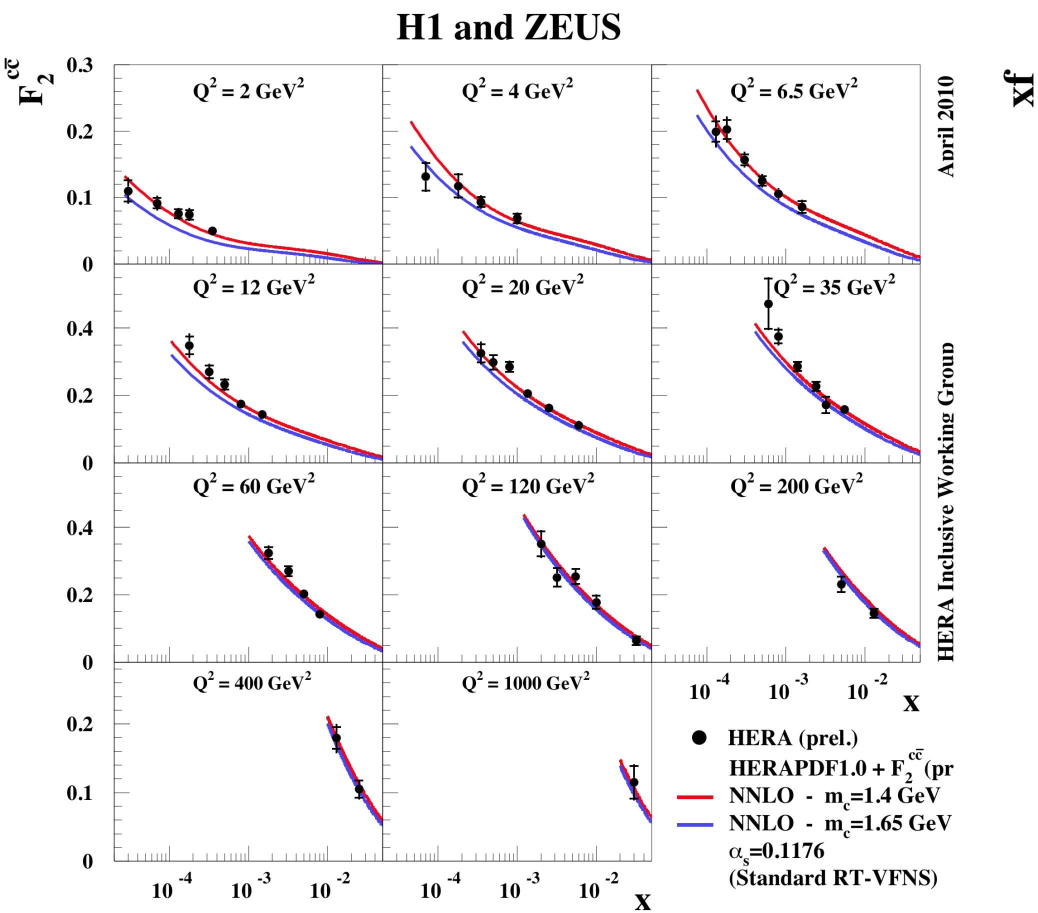
F_L Measurement at HERA

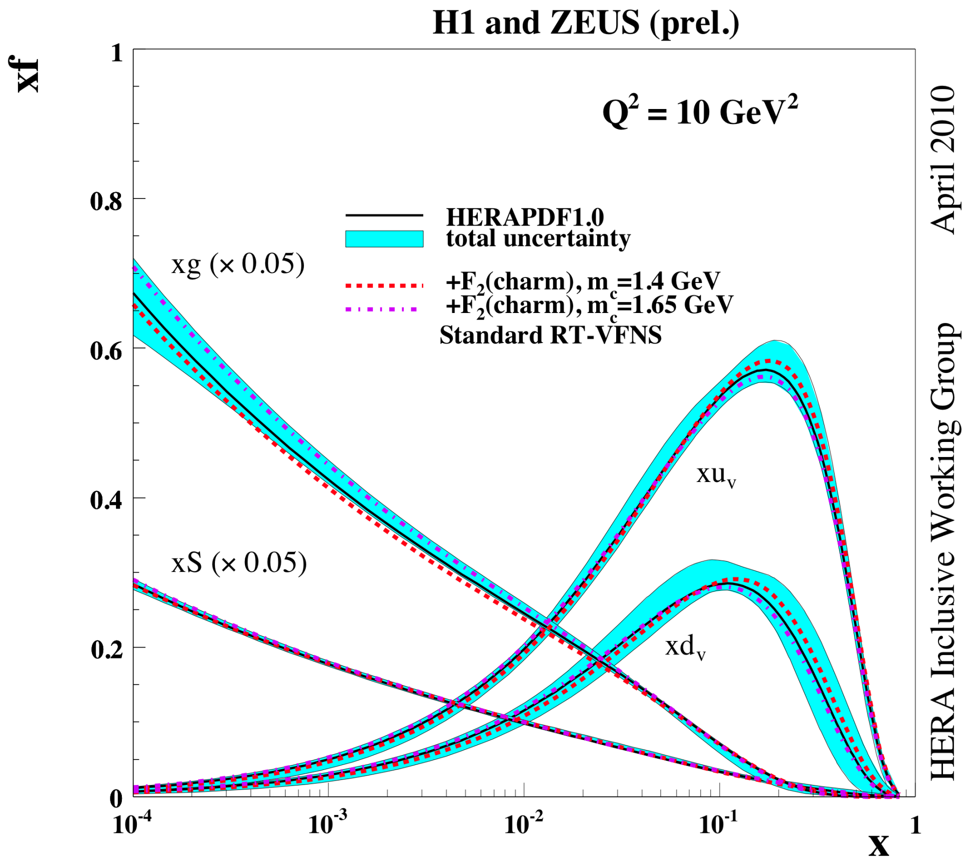


The measurement will be improved & extended in kinematic range

Final F_L can bring further constraint on g(x) at low x

HERA Charm Structure Function F_2^{cc}



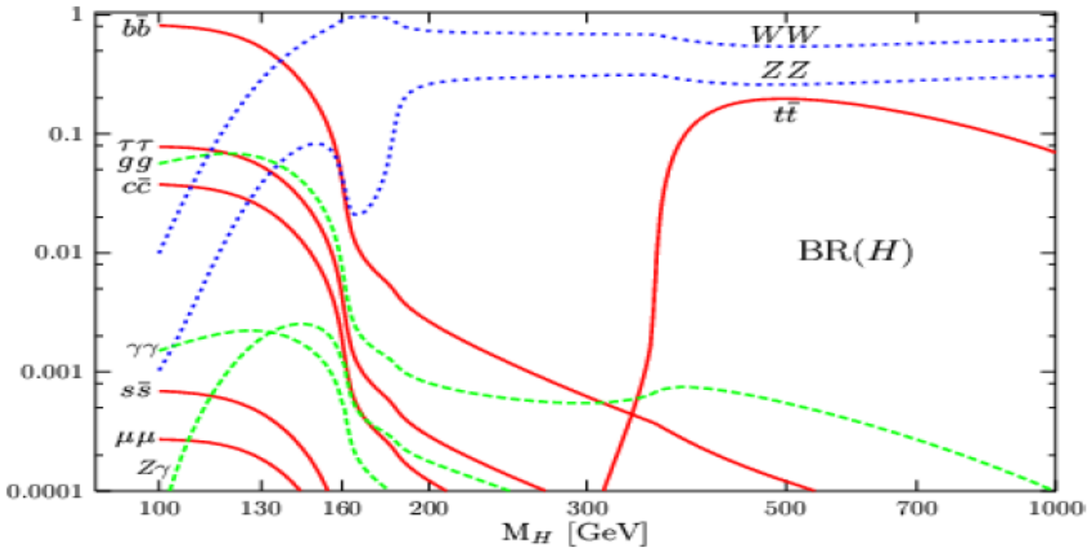


The combined charm SF sensitive to charm mass
It's better described by NNLO than NLO

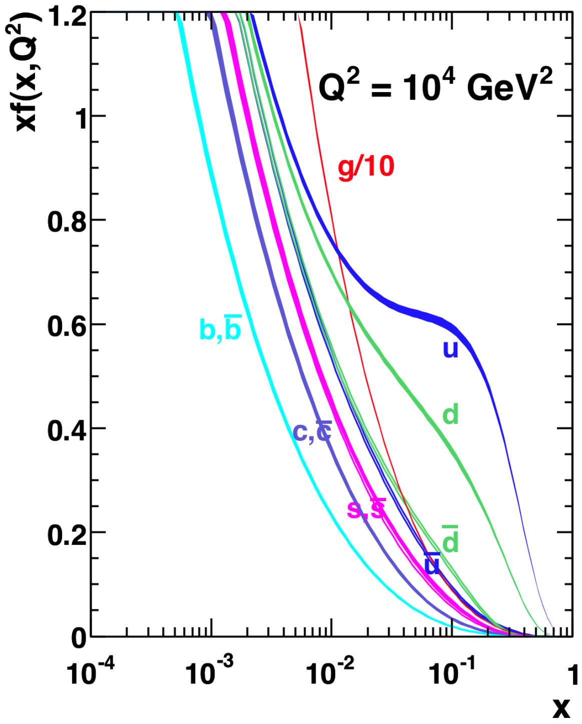
When the charm data included in the fit, the resulting PDFs are compatible with HERAPDF1.0

The Importance of Heavy Quark PDFs

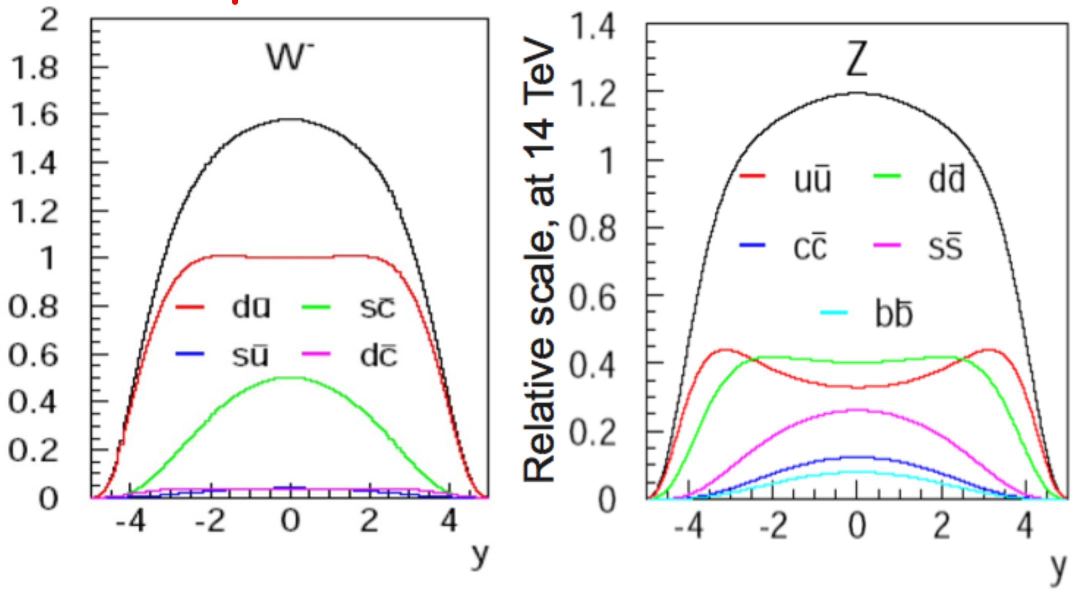
SM Higgs decay branching fractions



PDFs at $Q^2=10^4$ GeV^2

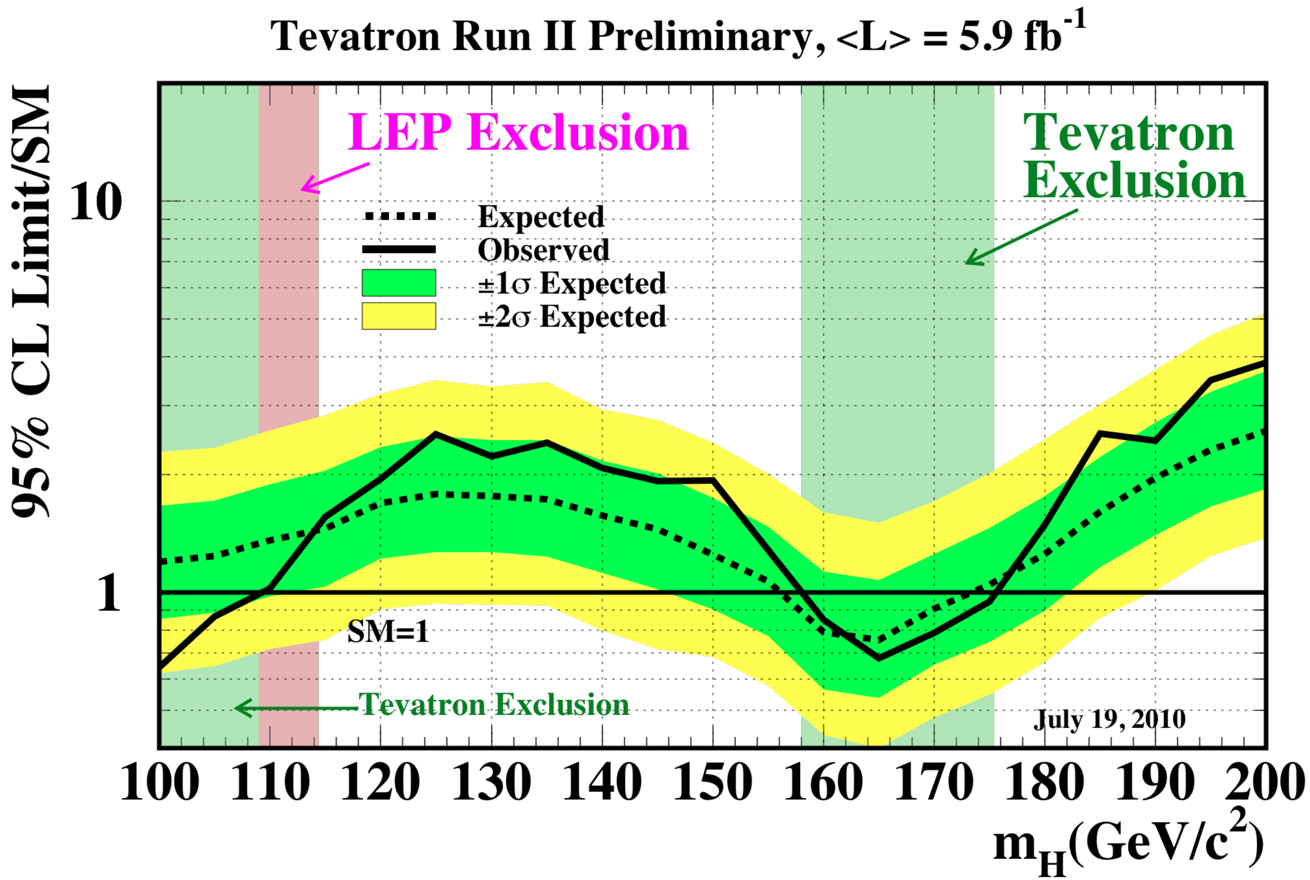


W, Z production at LHC



Precise PDFs of heavy quarks also important at LHC

Relevance of the PDFs for the Higgs Search



The PDFs uncertainties propagate in to the SM Higgs cross section

Thus affect the limit setting

Summary

Low energy precision experiments complementary to high energy machines provide inputs for SM tests & new physics searches and guidelines for Searches at the LHC

* $\alpha(M_Z)$ shift the M_H constraint up by 12GeV
* muon g-2: prediction-measurement @ ~3.6σ

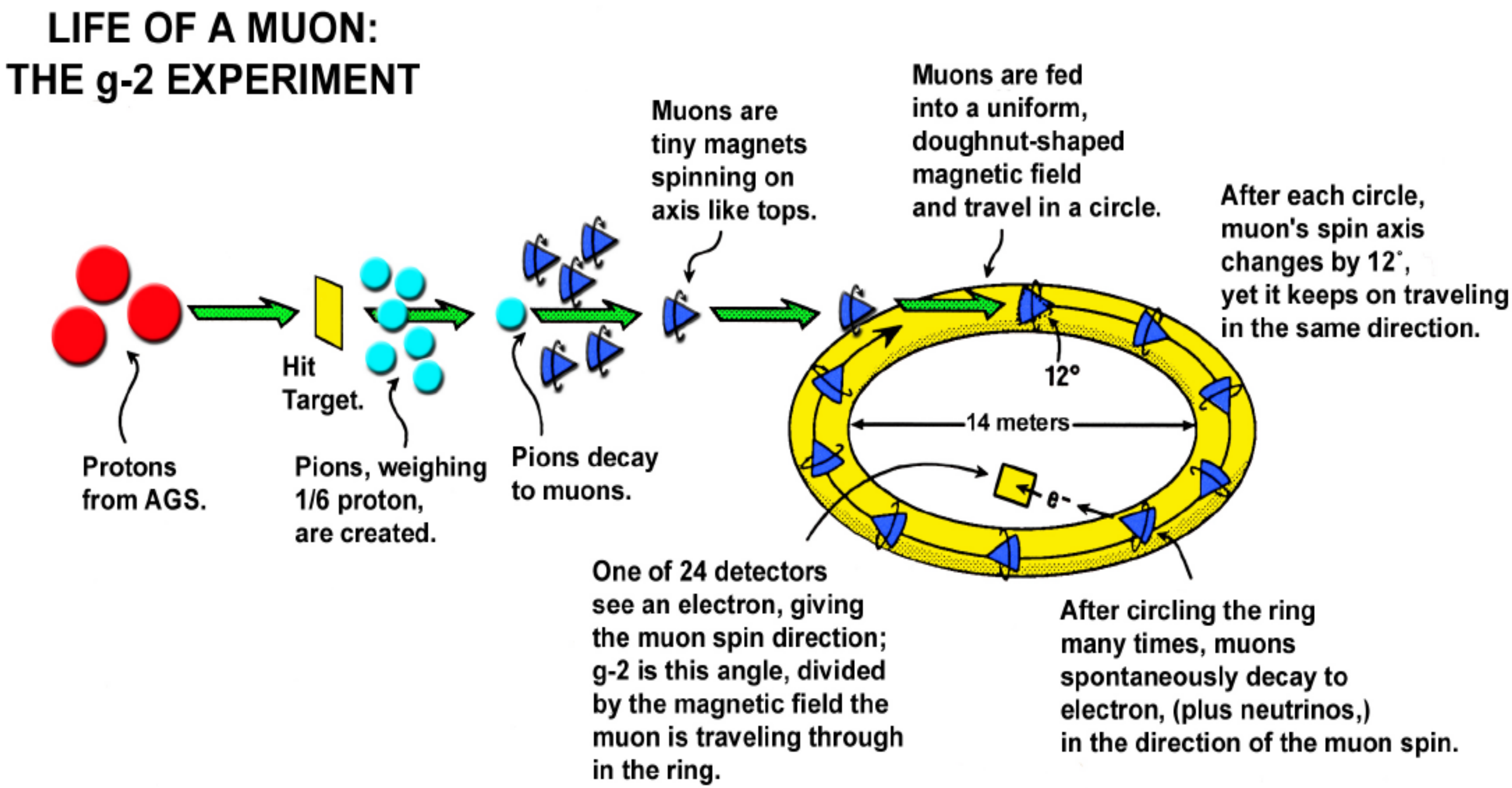
  [The g-2 Fermilab proposal [P-989] aims for an improvement of a factor 4 down to 0.14ppm]

HERA, stopped data taking after 15 years of successful running in 2007, is still providing the dominant input for PDFs with steadily improving precision

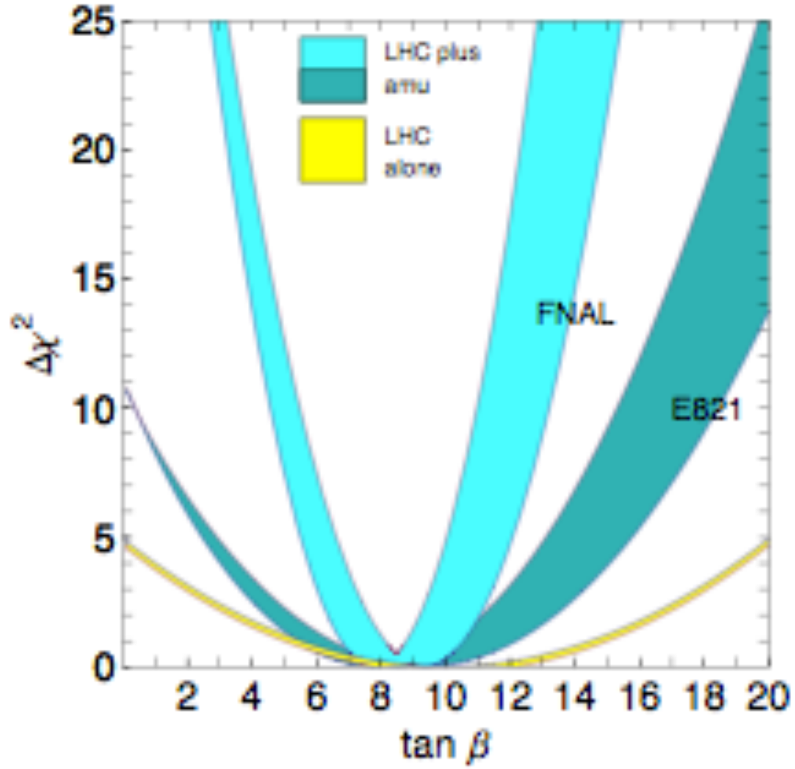
➔ Profound impact on LHC physics programme

[Final HERA-2 data and HERAPDF2.0 will be ready soon]

Overview of a g-2 Experiment



Fermilab g-2 experiment proposal P989



arXiv:1001.2898

Effective proton bunch fill rate: x4 (from 4.4Hz to 18Hz)
Stored muon to pion ratio: x 5-10
Measurement precision: ~4 (from 0.54ppm to 0.14ppm)

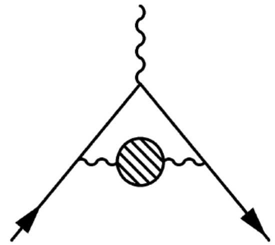
SM Predictions: $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{Weak}$

$$a_\mu^{had} = a_\mu^{had,LO} + a_\mu^{had,HO} + a_\mu^{had,LBL}$$

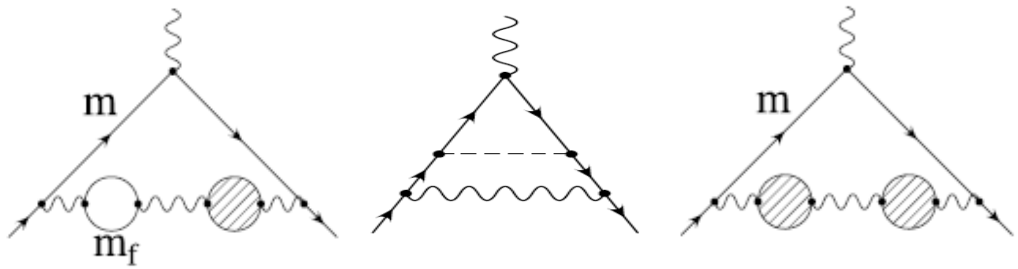
Leading-Order Higher-Order Light-By-Light

$a_\mu^{had,LO} \sim (\,700 \pm 5\,)\times 10^{-10}$

➔ dominant uncertainty
(both e^+e^- and τ based)

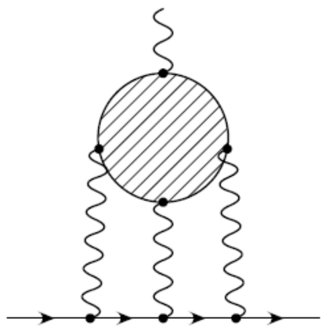


$a_\mu^{had,HO} = (\,-9.8 \pm 0.1\,)\times 10^{-10}$



$a_\mu^{had,LBL} \sim (\,10.5 \pm 2.6\,)\times 10^{-10}$

➔ 2nd leading uncertainty



Isospin Su(2) Breaking Corrections

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contribution:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW}
  - subleading corrections (small)
  - long distance radiative correction $G_{EM}(s)$

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

- Charged/neutral mass splitting:

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

  - $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ-ω mixing (EM $\omega \to \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho^-} \neq m_{\rho^0}$ and $\Gamma_{\rho^-} \neq \Gamma_{\rho^0}$

Flores-Baez-Lopez Castro' 08
Davier et al.'09

- Electromagnetic decays: $\rho \to \pi\pi\gamma$, $\rho \to \pi\gamma$, $\rho \to \eta\gamma$, $\rho \to l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Gfitter

Parameter	Input value	Free in fit	Results from global EW fits: *Standard fit*	*Complete fit*	*Complete fit w/o exp. input in line*
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1874 ± 0.0021	91.1877 ± 0.0021	$91.1981^{+0.0171}_{-0.0162}$
Γ_Z [GeV]	2.4952 ± 0.0023	–	$2.4959^{+0.0016}_{-0.0014}$	2.4956 ± 0.0015	$2.4944^{+0.0025}_{-0.0010}$
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.478 ± 0.014	41.481 ± 0.002	41.469 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.742 ± 0.018	$20.742^{+0.017}_{-0.018}$	20.717 ± 0.027
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01638 ± 0.0002	0.01624 ± 0.0002	$0.01614^{+0.0002}_{-0.0001}$
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1478 ± 0.0010	0.1472 ± 0.0009	–
A_c	0.670 ± 0.027	–	$0.6682^{+0.00045}_{-0.00044}$	$0.6680^{+0.00038}_{-0.00039}$	$0.6679^{+0.00047}_{-0.00040}$
A_b	0.923 ± 0.020	–	0.93468 ± 0.00009	$0.93463^{+0.00006}_{-0.00007}$	$0.93463^{+0.00006}_{-0.00007}$
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	$0.0741^{+0.0006}_{-0.0005}$	0.0737 ± 0.0005	$0.0738^{+0.0004}_{-0.0005}$
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	0.1036 ± 0.0007	0.1031 ± 0.0006	$0.1037^{+0.0004}_{-0.0007}$
R^0_c	0.1721 ± 0.0030	–	0.17225 ± 0.00006	0.17225 ± 0.00006	0.17226 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	$0.21577^{+0.00005}_{-0.00008}$	$0.21577^{+0.00005}_{-0.00008}$	$0.21577^{+0.00004}_{-0.00008}$
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	0.23143 ± 0.00013	$0.23150^{+0.00010}_{-0.00009}$	$0.23152^{+0.00009}_{-0.00010}$
M_H [GeV] $^{(\circ)}$	Likelihood ratios	yes	$84.2^{+30.3[+75.0]}_{-23.3[-41.9]}$	$120.6^{+17.0[+34.3]}_{-5.2[-6.2]}$	$84.2^{+30.3[+75.0]}_{-23.3[-41.9]}$
M_W [GeV]	80.399 ± 0.023	–	$80.384^{+0.014}_{-0.015}$	$80.369^{+0.007}_{-0.009}$	$80.353^{+0.018}_{-0.013}$
Γ_W [GeV]	2.085 ± 0.042	–	2.092 ± 0.001	2.091 ± 0.001	$2.092^{+0.000}_{-0.001}$
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.16}_{-0.07}$	$4.20^{+0.16}_{-0.07}$	–
m_t [GeV]	173.3 ± 1.1	yes	173.4 ± 1.1	173.7 ± 1.0	$178.5^{+9.6}_{-3.7}$
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $^{(\dagger\triangle)}$	2769 ± 22	yes	2772 ± 22	2764^{+21}_{-22}	2729^{+39}_{-55}
$\alpha_s(M_Z^2)$	–	yes	$0.1192^{+0.0028}_{-0.0027}$	0.1193 ± 0.0028	0.1193 ± 0.0028
$\delta_{\rm th} M_W$ [MeV]	$[-4,4]_{\rm theo}$	yes	4	4	–
$\delta_{\rm th}\sin^2\theta^\ell_{\rm eff}$ $^{(\dagger)}$	$[-4.7,4.7]_{\rm theo}$	yes	4.7	−0.3	–
$\delta_{\rm th}\rho^f_Z$ $^{(\dagger)}$	$[-2,2]_{\rm theo}$	yes	2	2	–
$\delta_{\rm th}\kappa^f_Z$ $^{(\dagger)}$	$[-2,2]_{\rm theo}$	yes	2	2	–