

HSNJL Model — A New Paradigm

for Dynamical Symmetry Breaking with
Supersymmetry

— talk at 1st LHEP (Nov 2010)

OTTO C. W. KONG

— Nat'l Central U, Taiwan

PAUL HOWARD FRAMPTON'S ACADEMIC GENEALOGY.

Paul Howard Frampton (D.Phil. 1968 Oxford)

Frampton's adviser:

John Clayton Taylor (Ph.D. 1955 Cambridge)

Paul Howard Frampton (Research associate 1968-70 Chicago)

Frampton's mentor:

Yoichiro Nambu (Ph.D. 1952 Tokyo)

Taylor's co-advisers:

Richard Eden (Ph.D. 1948 Cambridge)

Abdus Salam (Ph.D. 1952 Cambridge)

Eden's adviser:

Paul Dirac (Ph.D. 1926 Cambridge)

Salam's adviser:

Nicholas Kemmer (Ph.D. 1935 Zurich)

Kemmer's adviser:

Wolfgang Pauli (Ph.D. 1921 Munich)

Pauli's adviser:

Arnold Sommerfeld (Ph.D. 1891 Konigsberg)

Sommerfeld's adviser:

Ferdinand Lindemann (Ph.D. 1873 Nurenberg)

Lindemann's adviser:

Felix Klein (Ph.D. 1868 Bonn)

Klein's co-advisers:

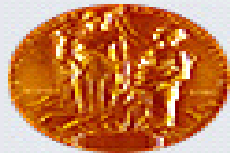
Rudolf Lipschitz (Ph.D. 1853 Berlin)

Julius Plucker (Ph.D. 1823 Marburg)

• Nambu's publications :-

~ 50 × 185 citations

no supersymmetry !



The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"



Photo: University of Chicago

Yoichiro Nambu

🕒 1/2 of the prize



© The Nobel Foundation
Photo: U. Montan

Makoto Kobayashi

🕒 1/4 of the prize



© The Nobel Foundation
Photo: U. Montan

Toshihide Maskawa

🕒 1/4 of the prize

[Previous Story](#) / [Next Story](#) / [Volume 22 archive](#)

[Phys. Rev. **124**, 246](#)
(issue of 1 October 1961)
[Phys. Rev. **122**, 345](#)
(issue of 1 April 1961)
[Phys. Rev. Lett. **4**, 380](#)
(issue of 1 April 1960)
[Titles and Authors](#)

14 October 2008

Nobel Focus: Particle Physics Gets a Break

A Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity II

Y. Nambu and G. Jona-Lasinio

[Phys. Rev. **124**, 246](#)

(issue of 1 October 1961)

A Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity I

Y. Nambu and G. Jona-Lasinio

[Phys. Rev. **122**, 345](#)

(issue of 1 April 1961)

Axial Vector Current Conservation in Weak Interactions

Yoichiro Nambu

[Phys. Rev. Lett. **4**, 380](#)

(issue of 1 April 1960)

References:

[1] Y. Nambu, "Quasi-Particles and Gauge Invariance in the Theory of Superconductivity," [Phys. Rev. **117** 648 \(1960\)](#).



Yoichiro Nambu

The Nobel Prize in Physics 2008

Nobel Lecture

Spontaneous Symmetry Breaking in Particle Physics: a Case of Cross Fertilization



The Nobel Lecture of Yoichiro Nambu was presented by Giovanni Jona-Lasinio, La Sapienza, University of Rome, Italy, 8 December 2008, at Aula Magna, Stockholm University. He was introduced by Professor Joseph Nordgren, Chairman of the Nobel Committee for Physics.

History repeats itself

1960 Midwest Conference in Theoretical Physics, Purdue University

A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)[†]

(In absence of the author the paper was presented by G. Jona-Lasinio.)

The Nambu–Jona-Lasinio (NJL) model

Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

The Lagrangian of the model is

$$L = -\bar{\psi}\gamma_{\mu}\partial_{\mu}\psi + g [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]$$

It is invariant under ordinary and γ_5 gauge transformations

$$\begin{aligned}\psi &\rightarrow e^{i\alpha}\psi, & \bar{\psi} &\rightarrow \bar{\psi}e^{-i\alpha} \\ \psi &\rightarrow e^{i\alpha\gamma_5}\psi, & \bar{\psi} &\rightarrow \bar{\psi}e^{i\alpha\gamma_5}\end{aligned}$$

Other examples of BCS type SSB

- ▶ ^3He superfluidity
- ▶ Nucleon pairing in nuclei
- ▶ Fermion mass generation in the electro-weak sector of the standard model

Nambu calls the last entry

my biased opinion, there being other interpretations as to the nature of the Higgs field

Update of the NJL model



NJL for Electroweak Symmetry Breaking :-

Nambu's Model Resurrected Again, with a Twist

PHYSICAL REVIEW D **81**, 031701(R) (2010)

Holomorphic supersymmetric Nambu—Jona-Lasinio model with application to dynamical electroweak symmetry breaking

Dong-Won Jung,¹ Otto C. W. Kong,¹ and Jae Sik Lee²

¹*Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chung-li, Taiwan 32054*

²*Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 300*

(Received 25 September 2009; published 10 February 2010)

Based on our idea of an alternative supersymmetrization of the Nambu—Jona-Lasinio model for dynamical symmetry breaking, we analyze the resulting new model with a holomorphic dimension-five operator in the superpotential. The approach provides a new direction for modeling dynamical symmetry breaking in a supersymmetric setting. In particular, we adopt the idea to formulate a model that gives rise to the minimal supersymmetric standard model as the low energy effective theory with both Higgs superfields as composites. A renormalization group analysis is performed to establish the phenomenological viability of the scenario, with admissible background scale that could go down to the TeV scale. We give the Higgs mass range predicted.

DOI: 10.1103/PhysRevD.81.031701

PACS numbers: 12.60.Jv, 11.30.Qc, 12.60.Rc

The Modern Perspectives :-

- or my perspectives (Nambu was before SM)
- only effective(quantum field) theories
 - layer after layer (scale)
- symmetry characterizes a theory (completely ?)
- no Dirac fermion / no chiral symmetry
 - nature has no parity symmetry (including space-time ?)
 - chiral (Weyl) fermions
- even scalar fields are chiral, with supersymmetry
 - gauge symmetry forbids (supersymmetric) mass,
keeps symmetry breaking masses (naturally) ‘small’

Why NJL ?

— (simplest/viable) dynamical symmetry breaking

★ original of EW scale, and symmetry breaking ?

- SM with Higgs mechanism : tachyonic Higgs mass
- NJL/DYB : dimensional transmutation
e.g. QCD/proton mass scale (from dynamics/RG-running)
- NJL generates (composite) Higgs with effective tachyonic mass
- NJL has a cutoff scale — effective field theory picture
— fine tuning problem (can be fixed by supersymmetry)

The (supersymmetric) Twist :-

- top mode SM Miransky, W. Bardeen, ... '89/'90 (Nambu)
 - infrared (quasi-)fixed point (IQFP) (Pendleton-Ross), Hill, Marciano, ...
prediction : top mass > 200 GeV ; VEV – top condensate
- supersymmetric NJL (formal – '82, SSM – '90)
 - $m_t = y_t \cdot v$, $m_b = y_b \cdot v'$; NJL predicts y not m ; $y_b < y_t$
 - other not very nice features as MSSM
 - lighter top fine, *but ...* (172.1 GeV top, $\tan\beta < 1.5$)
- our **holomorphic SNJL** (alternative supersymmetrization)
 - non-chiral symmetric 4-superfield interaction *with t and b*
 - **superfield condensate** : both **scalar** and fermion condensate
 - $y_t < y_b$; nice , experimentally viable (LHC)

Nambu–Jona-Lasinio Model :-

- dynamical symmetry breaking
- four-fermion interaction

$$\begin{aligned}\mathcal{L}_\psi &= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + g^2\bar{\psi}_+\bar{\psi}_-\psi_+\psi_- \\ &\longrightarrow \mathcal{L}_\psi - (\mu\phi^\dagger + g\psi_+\psi_-)(\mu\phi + g\bar{\psi}_+\bar{\psi}_-) \\ &= i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- - \mu^2\phi^\dagger\phi - \mu g(\phi^\dagger\bar{\psi}_+\bar{\psi}_- + \phi\psi_+\psi_-)\end{aligned}$$

- auxiliary scalar field ϕ (no kinetic term)
- EL-eq for ϕ^\dagger gives ϕ as composite

$$\phi = -g/\mu\bar{\psi}_+\bar{\psi}_-$$
- $\langle\phi\rangle \neq 0 \implies$ symmetry breaking and fermion mass

→ low energy effective field theory :-

- 1-loop effective potential for ϕ gives gap equation

$$\langle \phi \rangle \neq 0 \text{ solution for } \frac{g^2 \Lambda^2}{8\pi^2} > 1$$

- Dirac fermion mass $m = \mu g \langle \phi \rangle$ for $\psi_+ - \psi_-$

$$g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \quad \Longrightarrow \quad g^2 \langle \bar{\psi}_+ \bar{\psi}_- \rangle \psi_+ \psi_-$$

- **kinetic term for ϕ** through wave-function renormalization
fermion-loop propagator with Yukawa vertices ($m \ll \Lambda$)

$$Z = \frac{N_c \mu^2 g^2}{16\pi^2} \left[\ln \frac{\Lambda^2}{M^2} + O(1) \right]$$

- Higgs with mass $2m$ and a Goldstone boson

$\phi \longrightarrow \phi/\sqrt{Z} :-$

- $$\mathcal{L}_\psi = i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ + i\bar{\psi}_-\sigma^\mu\partial_\mu\psi_- + \partial_\mu\phi^\dagger\partial^\mu\phi$$

$$- \tilde{\mu}^2\phi^*\phi - \frac{\tilde{\lambda}}{2}(\phi^\dagger\phi)^2 - \tilde{y}\phi\psi_+\psi_- + h.c.$$

—
$$\tilde{y} = \frac{\mu g}{\sqrt{Z}} = \frac{4\pi}{\sqrt{N_c}} \frac{1}{\sqrt{\ln(\Lambda^2/M^2)}}$$

—
$$\tilde{\mu}^2 = \left[\frac{8\pi^2}{N_c g^2} - (\Lambda^2 - M^2) \right] \frac{2}{\ln(\Lambda^2/M^2)}$$

—
$$\tilde{\lambda} = \frac{32\pi^2}{N_c \ln(\Lambda^2/M^2)}$$

- condition for $\langle\phi\rangle \neq 0$ gives gap equation result

Supersymmetrizing the NJL Model (Naively):-

- $i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ \longrightarrow \int d^4\theta \bar{\Phi}_+ \Phi_+$
- $g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \longrightarrow \int d^4\theta g^2 \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_-$
- $-\mu g \phi \psi_+ \psi_- \longrightarrow \int d^2\theta \mu g \Phi \Psi_+ \Psi_-$
- $-\mu^2 \phi^* \phi \longrightarrow \int d^2\theta \frac{\mu}{2} \Phi \Phi$

BUT :-

- $\phi = -g/\mu \bar{\psi}_+ \bar{\psi}_-$ implies

$$\mu^2 \phi^* \phi = -\mu g \phi \psi_+ \psi_- = g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \quad (\text{no SUSY !})$$
- **no nontrivial vacuum** without SUSY breaking

The Supersymmetric NJL Model :-

- $i\bar{\psi}_+ \sigma^\mu \partial_\mu \psi_+ \longrightarrow \int d^4\theta \bar{\Phi}_+ \Phi_+ (1 - m^2 \theta^2 \bar{\theta}^2)$
- $g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_- \longrightarrow \int d^4\theta g^2 \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_-$
- $-\mu g \phi \psi_+ \psi_- \longrightarrow \int d^2\theta \mu g \Phi_2 \Psi_+ \Psi_-$
- $-\mu^2 \phi^* \phi \longrightarrow \int d^2\theta \mu \Phi_1 \Phi_2 + \int d^4\theta \bar{\Phi}_1 \Phi_1$

BUT :-

- EL-eq for Φ_2 gives $\Phi_1 = -g \Phi_+ \Phi_-$ implies

$$\int d^4\theta \bar{\Phi}_1 \Phi_1 = \int d^4\theta g^2 \bar{\Phi}_+ \bar{\Phi}_- \Phi_+ \Phi_-$$

- Φ_2 not the composite Φ_1 plays the Higgs superfield $\langle \Phi_1 \rangle = 0$

An Alternative Supersymmetrization ?

$$\bullet \quad i\bar{\psi}_+\sigma^\mu\partial_\mu\psi_+ \quad \longrightarrow \quad \int d^4\theta \quad \bar{\Phi}_+\Phi_+ \quad (1 - m^2\theta^2\bar{\theta}^2)$$

$$\bullet \quad -\mu g \phi\psi_+\psi_- \quad \longrightarrow \quad \int d^2\theta \quad \mu g \Phi_0\Psi_+\Psi_-$$

$$\bullet \quad -\mu^2\phi^*\phi \quad \longrightarrow \quad \int d^2\theta \quad \frac{\mu}{2}\Phi_0\Phi_0$$

$$\begin{aligned} \implies \quad \mathcal{L} = \int d^4\theta \quad [(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)(1 - m^2\theta^2\bar{\theta}^2)] \\ + \int d^2\theta \quad \left[\frac{\mu}{2}\Phi_0^2 + \sqrt{\mu G}\Phi_0\Phi_+\Phi_- \right] + h.c. \end{aligned}$$

$$\bullet \quad \text{consider superpotential} \quad W = \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_-$$

$$\longrightarrow W = \frac{1}{2}(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)$$

With Holomorphic Four-Chiral Superfield Interaction :-

- $W = \frac{G}{2} \Phi_+ \Phi_- \Phi_+ \Phi_-$ contains no $g^2 \bar{\psi}_+ \bar{\psi}_- \psi_+ \psi_-$
- EL-eq for auxiliary superfield Φ_0 gives $\Phi_0 = -\sqrt{G/\mu} \Phi_+ \Phi_-$
 implies $\frac{\mu}{2} \Phi_0^2 = -\frac{\sqrt{\mu G}}{2} \Phi_0 \Phi_+ \Phi_- = \frac{G}{2} \Phi_+ \Phi_- \Phi_+ \Phi_-$
- $\langle \Phi_0 \rangle \implies \frac{G}{2} \langle \Phi_+ \Phi_- \rangle \Phi_+ \Phi_-$ Dirac mass for $\Phi_+ - \Phi_-$
- kinetic term for Φ_0 from wave-function renormalization
 through $\Phi_+ - \Phi_-$ loop with Yukawa vertices

→ low energy effective field theory :-

- (gauged-)kinetic term $\int d^4\theta \ Z_0 \bar{\Phi}_0 e^{2V_\Phi} \Phi_0 \ [1 + (2m^2 + A^2)\theta^2\bar{\theta}^2]$

$$\text{where } Z_0 = \frac{N_c \mu G}{16\pi^2} \left[\ln \frac{\Lambda^2}{M^2} + O(1) \right]$$

- $\tilde{m}_0^2 = -(2m^2 + A^2)$, tachyonic soft mass (cf. radiative EWSB)

$$\text{— } \tilde{y} = \frac{\sqrt{\mu G}}{\sqrt{Z_0}} = \frac{4\pi}{\sqrt{N_c}} \frac{1}{\sqrt{\ln(\Lambda^2/M^2)}}$$

$$\text{— } \tilde{\mu} = \frac{\mu}{Z_0} = \frac{16\pi^2}{N_c G} \frac{1}{\ln(\Lambda^2/M^2)}$$

- $\frac{\mu}{2}\Phi^2$ term $\implies \Phi$ in real representation of symmetry

Condensate/Mass Generation — A Comparison :-

- NJL : $g^2 \langle \bar{\psi}_+ \bar{\psi}_- \rangle \psi_+ \psi_- \longrightarrow -\mu g \langle \phi \rangle \psi_+ \psi_-$

— symmetry breaking with **bi-fermion condensate** $\langle \phi \rangle$

- SNJL : $\int d^4\theta \ g^2 \langle \bar{\Phi}_+ \bar{\Phi}_- \rangle \Phi_+ \Phi_-$

$$\longrightarrow -g \langle F_1^\dagger \rangle [A_+ F_- + A_- F_+ - \psi_+ \psi_-] \quad \left(F_1^\dagger = -\mu A_2 \right)$$

— $\langle F_1 \rangle = -g \langle A_+ F_- + A_- F_+ - \psi_+ \psi_- \rangle$, **sbi-fermion condensate**

- HSNJL : $\int d^2\theta \ -G \langle \Phi_+ \Phi_- \rangle \Phi_+ \Phi_-$

$$\longrightarrow \sqrt{\mu G} \langle A_0 \rangle [A_+ F_- + A_- F_+ - \psi_+ \psi_-] + \sqrt{\mu G} \langle F_0 \rangle A_+ A_-$$

— $\langle A_0 \rangle = -\sqrt{G/\mu} \langle A_+ A_- \rangle$, **a bi-scalar condensate**

Towards
EW Symmetry Breaking

NJL Model \rightarrow SM :-

- four-fermion interaction $g^2 \bar{Q} \bar{t}^c Q t^c$
- Higgs doublet as top-composite

$$\phi = -g/\mu(\bar{Q} \bar{t}^c)$$

- top condensate breaks EW symmetry \rightarrow fermion masses
— gives top quark mass at to infrared quasi-fixed point

- high $m_t \sim 218 \text{ GeV}$ ($\Lambda \sim 10^{19} \text{ GeV}$) Bardeen, Hill, Lindner 90

$$m_t \sim 214 - 202 \text{ GeV} \quad (\Lambda \sim 10^{15} - 10^{19} \text{ GeV}) \quad \text{Marciano 89,90}$$

$$m_t \sim 253 \text{ GeV} \quad \text{Miransky, Tanabashi, Yamawaki 89; King \& Mannan 90,91}$$

- extensions, e.g. two-Higgs-doublet model

SNJL Models \rightarrow MSSM (why SUSY ?):-

- SM \rightarrow MSSM — hierarchy/fine-tuning problem
 scalar field is somewhat sick
- SM fermion spectrum sort of fixed (anomaly cancelation) e.g. OK 96
 scalar content — only part arbitrary (*cf.* gauge symmetry)
- SUSY — **technically natural hierarchy**
 scalar as (part of) **chiral** superfield (**constrained as fermions**)
 Vs Georgi's survival hypothesis
- **BUT μ -problem** — vectorlike pair of Higgs superfields
- SNJL models solve our problems
 — and avoid fine-tuning of four-quark coupling(s)

Towards the MSSM :-

- consider $W = G \varepsilon_{\alpha\beta} \hat{Q}^\alpha \hat{U}^c \hat{Q}'^\beta \hat{D}^c (1 + B\theta^2)$

$$\begin{aligned} W &\longrightarrow W - \mu (\hat{H}_d - \lambda_u \hat{Q} \hat{U}^c) (\hat{H}_u - \lambda_d \hat{Q}' \hat{D}^c) (1 + B\theta^2) \\ &= (-\mu \hat{H}_d \hat{H}_u + y_u \hat{Q} \hat{H}_u \hat{U}^c + y_d \hat{H}_d \hat{Q}' \hat{D}^c) (1 + B\theta^2) \end{aligned}$$

- **two composites** — $\hat{H}_u = \frac{y_d}{\mu} \hat{Q}' \hat{D}^c$ and $\hat{H}_d = \frac{y_u}{\mu} \hat{Q} \hat{U}^c$
- low energy effective theory looks like MSSM ($A = B$)
- symmetric role for \hat{H}_u and \hat{H}_d (also : $\mu \lambda_u \lambda_d = \frac{y_u y_d}{\mu} = G$)
 - numerical lifted through non-universal soft masses
 - expect $\langle h_u \rangle \gtrsim \langle h_d \rangle$ (Vs UBB in D -flat)

Holomorphic Vs Old Model (for MSSM) :-

- **bottom** together with (vs only) **top** mass at quasi-fixed point

★ both (vs one) **Higgs superfields as composites**

- **large** (vs small) **$\tan\beta$**

- $A_t \simeq A_b \simeq B$ (vs $A_t \simeq 0$)

- $m_{H_d}^2 \simeq -(m_Q^2 + m_b^2 + |A_b|^2)$

plus (vs only) $m_{H_u}^2 \simeq -(m_Q^2 + m_t^2 + |A_t|^2)$

★ full W [= $G_{ijkh} Q_i U_j^c Q_k D_h^c (1 + A\theta^2) + G_{ij}^e Q_3 U_3^c L_i E_j^c (1 + A\theta^2)$]

— **non-holomorphic case needs similar holomorphic terms**

for Yukawa couplings of down-type quarks and charged leptons

- **sbottom and stop condensates** for u_i and $d_i + \ell_i$ masses

(vs top condensate and **stop condensates** for u_i and $d_i + \ell_i$ masses)

Numerical (RG analysis) Results :-

- earlier MSSM $t - b - \tau$ quasi-fixed point analysis

Froggatt *et.al* 93

(without background model)

$$m_t = 184.3 \pm 6.8 \text{ GeV}, \quad m_h = 121.8 \pm 4.3 \text{ GeV}$$

- ? $m_t = 171.2 \pm 2.1 \text{ GeV}$

- old SNJL: MSSM t quasi-fixed point analysis

Carena *et.al* 92

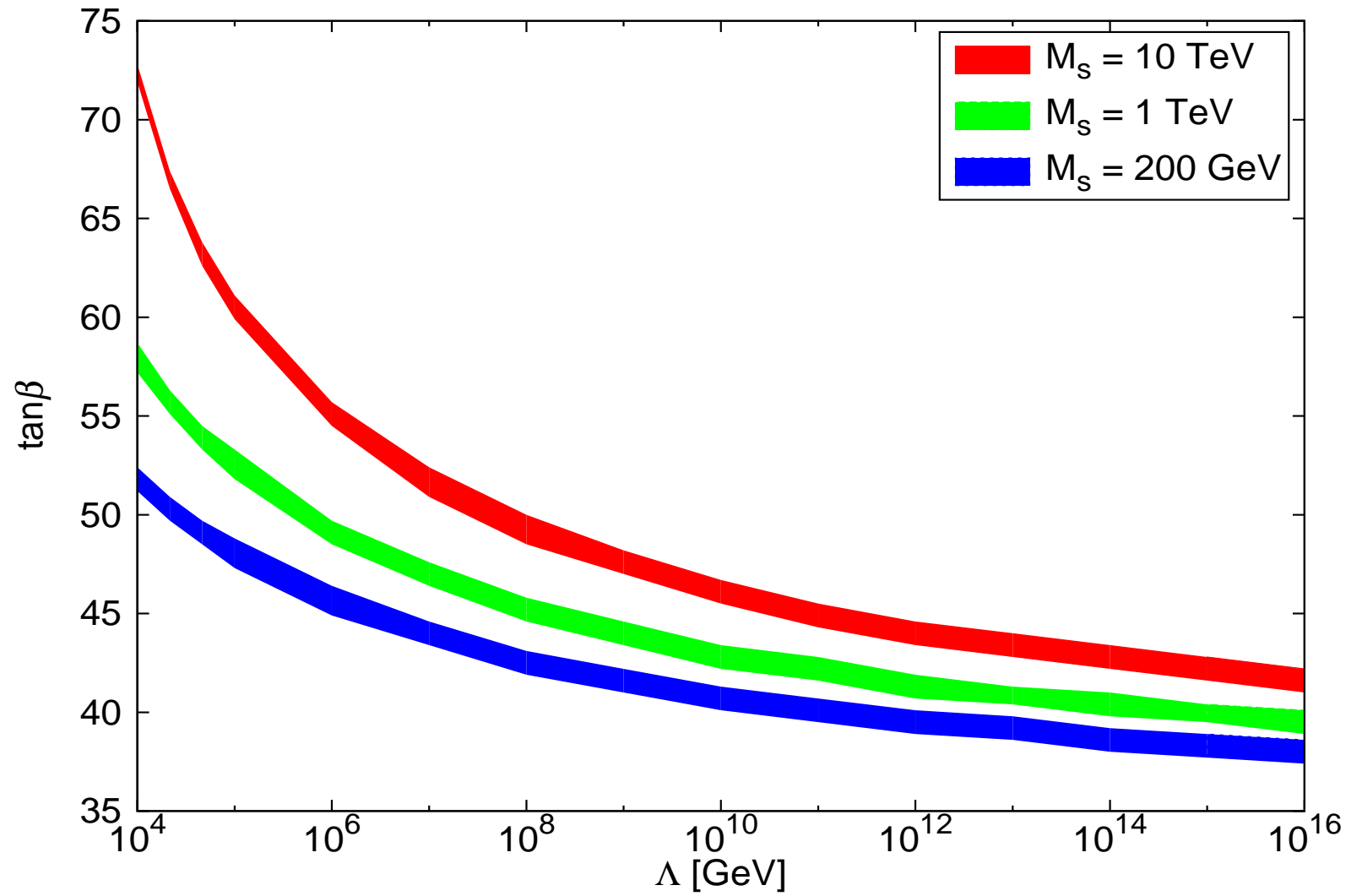
— high Λ and large $\tan\beta$ lower m_t

- infrared quasi-fixed point NOT necessary

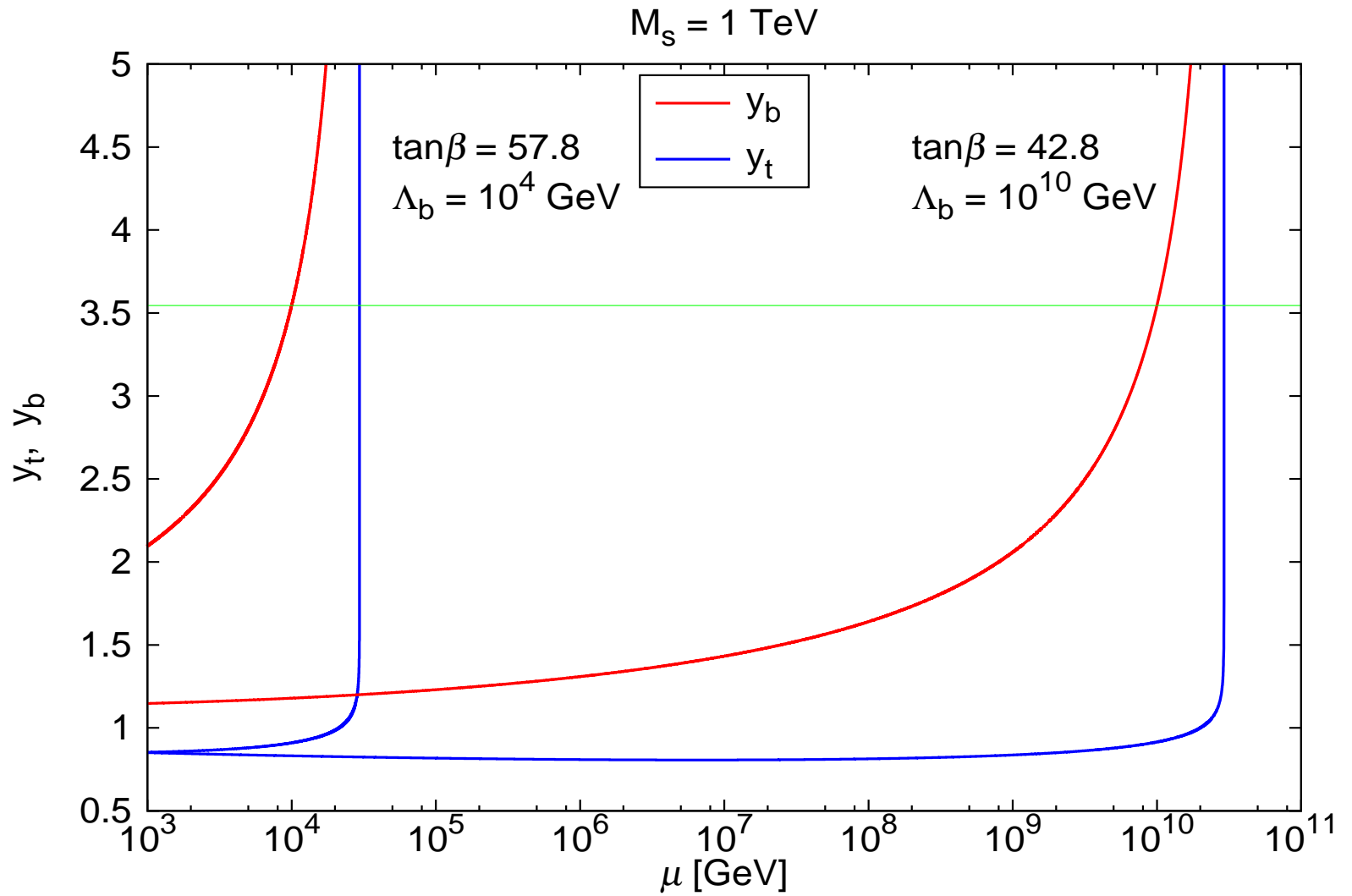
SNJL — y_t blows up at Λ

HSNJL — y_t and y_b blows up at $\sim \Lambda$

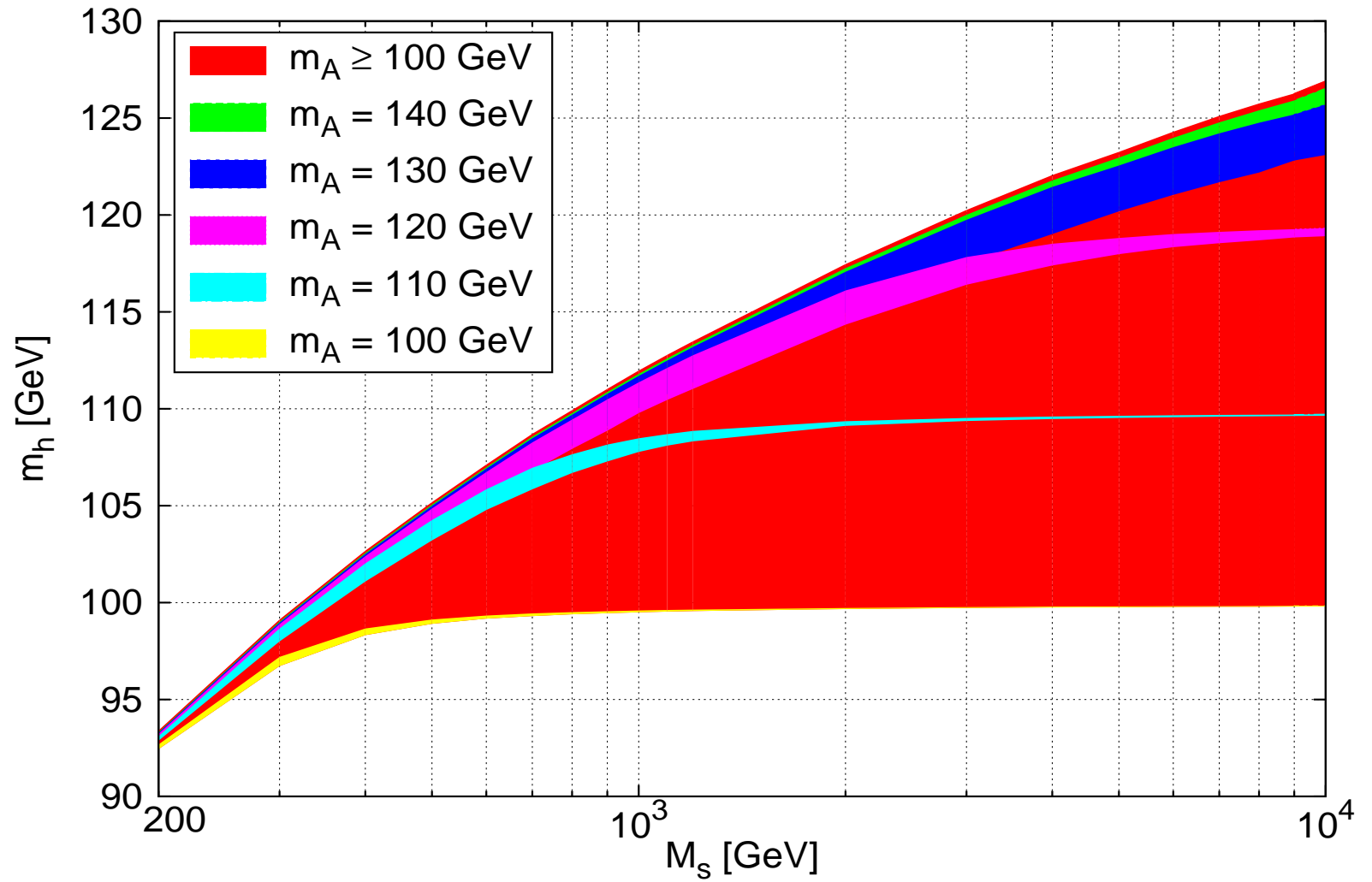
Our Solution :-



Illustrative y_t and y_b :-



Mass of the lightest Higgs boson :-



Final Remarks :-

- SNJL model with holomorphic term works
- may provide more interesting version of MSSM
 - SUSY : scalar \rightarrow chiral superfield
 - *problematic* MSSM superfield spectrum — vectorlike Higgs superfields, turn up as composites
 - four-superfield (G) term from integrated out *heavy* Higgs superfields ?
 - more natural B (and A) term, and all Yukawa coupling
- chiral symmetry explicitly broken

THANK YOU !