Synchrotron sideband spin resonances centered on an integer

Xia Wenhao & Duan Zhe

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CEPC 中的一阶自旋共振

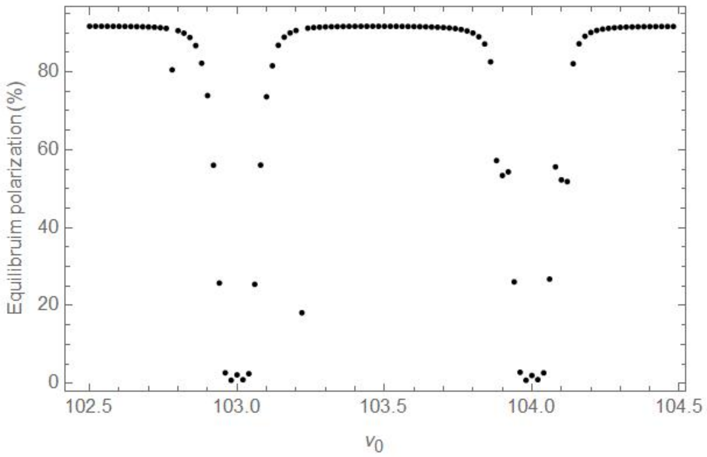
- 含磁铁误差，以**整数自旋共振及其纵向边带共振**为主；
- SLIM模拟结果, $\Delta\nu_0$=0.02;
- 理论解释：

$$P \approx \frac{P_\infty}{1+\frac{\tau_{sk}}{\tau_{dep}}}$$

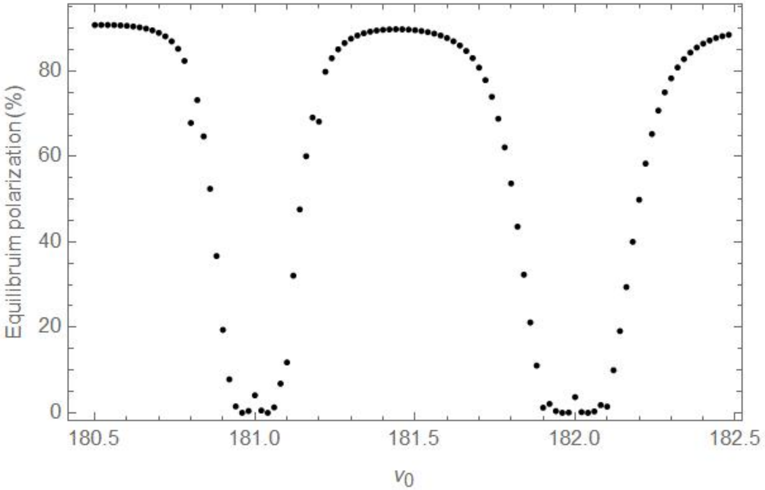
$$P_\infty = \frac{-8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \hat{b} \cdot \left(\hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma}\right) \right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3} \right\rangle}$$

≈92%

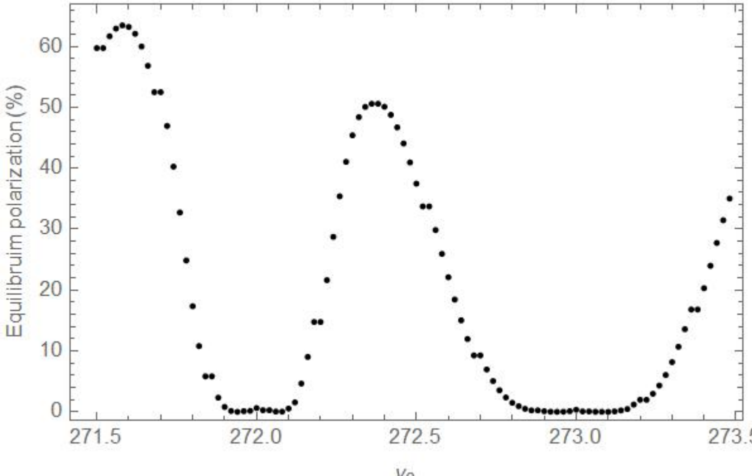
$$\frac{\tau_{sk}}{\tau_{dep}} \approx \frac{11}{18} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \left|\gamma \frac{\partial \hat{n}}{\partial \gamma}\right|^2 \right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3} \right\rangle}$$



Z , vx/ vy/ vz=0.11/0.22/0.028



W , vx/ vy/ vz=0.11/0.22/0.0395



H , vx/ vy/ vz=0.11/0.22/0.065

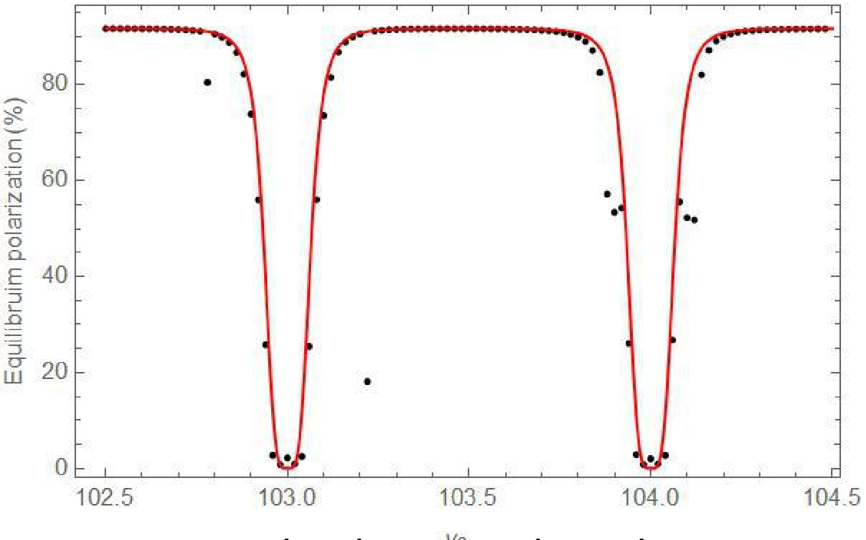
理论解释1

- Derbenev 和Kondratenko的理论，拟合：

Ya.S. Derbenev, A.M. Kondratenko and A.N. Skrinsky, Par. Ace. 1979, Vol. 9, pp. 247-266.

$$\frac{\tau_{sk}}{\tau_{dep}} = \frac{11v^2}{18}\sum_k \frac{|\omega_k|^2}{(\nu - k)^4}$$

- 可以实现上升沿和下降沿的匹配；
- 但是，无法解释整数共振处的“凸起”；



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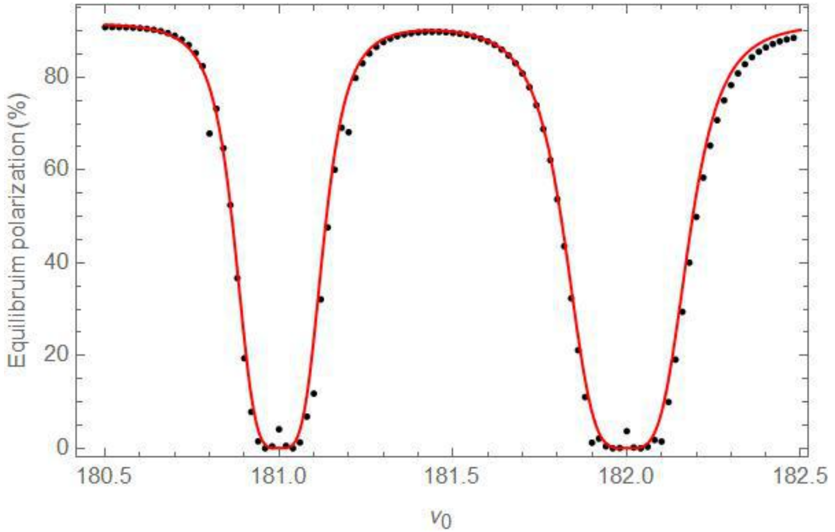

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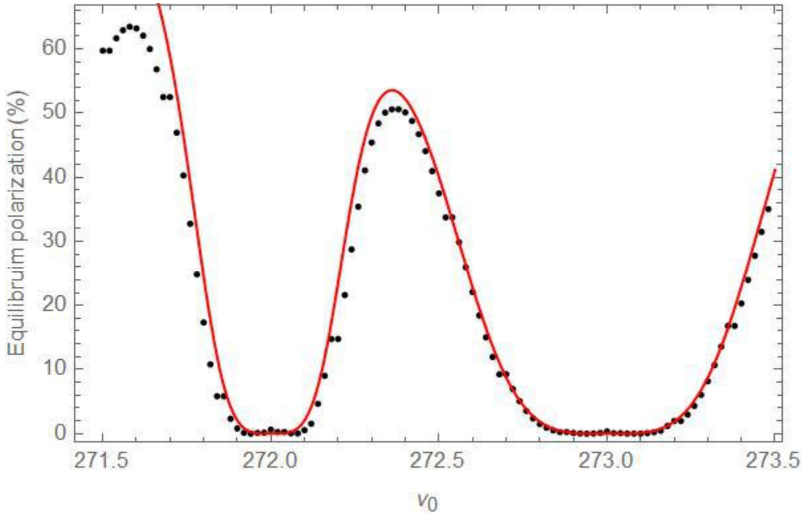


H , vx/ vy/ vz=0.11/0.22/0.065

理论解释2

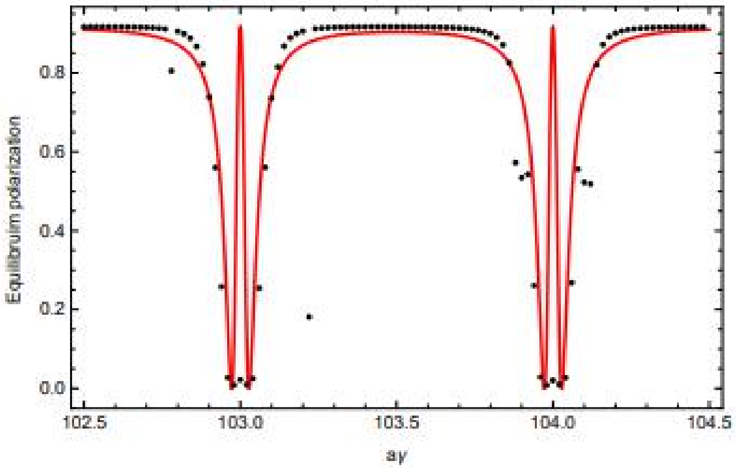
- Yokoya 的理论，拟合：

K. Yokoya, KEK Intenal 85-7, (1985).

$$\frac{\tau_{sk}}{\tau_{dep}} \propto |b_{k,s}|^2 \frac{(\nu-k)^2}{((\nu-k)^2-\nu_s^2)^2}$$

- 整数共振处有“凸起”，但很大；
- 但是，无法实现上升沿，下降沿的匹配；

猜想：边带共振强度|$b_{k,s}$|^2 ,在整数自旋共振附近时，会随着离整数共振之间的距离发生变化，有没有可能消除掉分子项（nu-k）^2。



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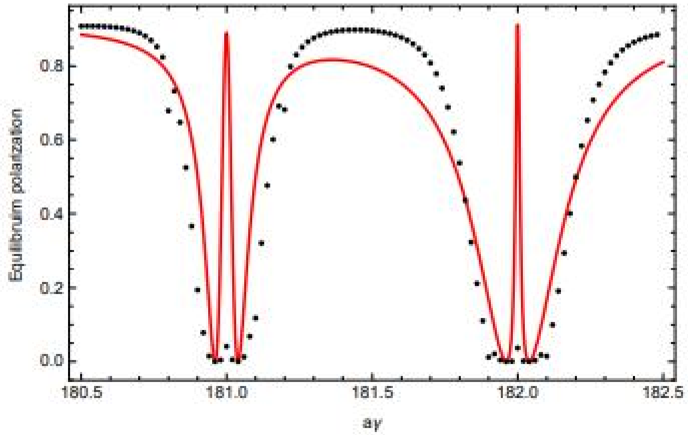

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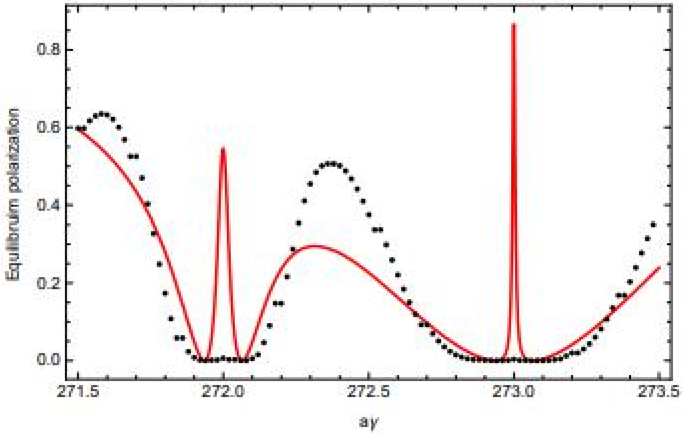


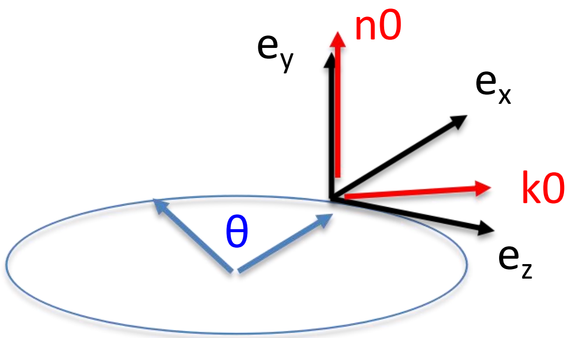
H , vx/ vy/ vz=0.11/0.22/0.065

理论推导：理想水平储存环

$$P \approx \frac{P_\infty}{1+\frac{\tau_{sk}}{\tau_{dep}}} \qquad P_\infty = \frac{-8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \hat{b} \cdot \left(\hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma}\right)\right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3}\right\rangle} \approx 92\% \qquad \frac{\tau_{sk}}{\tau_{dep}} \approx \frac{11}{18} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \left|\gamma \frac{\partial \hat{n}}{\partial \gamma}\right|^2 \right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3}\right\rangle}$$

- 关键在于自旋轨道耦合函数：$\gamma \frac{\partial \hat{n}}{\partial \gamma}$
- 在理想水平储存环中：$\frac{d\vec{S}}{d\theta} = \left[\vec{\Omega}_0 + \vec{\omega}\right] \times \vec{S}$，$\Omega_0$为设计轨道上的自旋进动矢量，w来自于betatron+synchrotron
- w=0时，三个单位正交解n_0, m_0, l_0, 引入$k_0=m_0+il_0$;
- Yokoya给出：

$$\gamma \frac{\partial \vec{n}}{\partial \gamma}(\theta) = \frac{1}{2} Re[\vec{k}_0^\star(\theta) \cdot (D_x + D_{-x} + D_y + D_{-y} + \underline{D_s + D_{-s}})]$$



- 我们只考虑纵向振荡的贡献：

$$\gamma \frac{\partial \vec{n}}{\partial \gamma}(\theta) = \frac{1}{2} Re[\vec{k}_0^\star(\theta) \cdot (D_s + D_{-s})]$$

理论推导：理想水平储存环

- 我们只考虑纵向振荡的贡献：

$$\gamma\frac{\partial\vec{n}}{\partial\gamma}(\theta)=\frac{1}{2}Re[\vec{k}_0^\star(\theta)\cdot(D_s+D_{-s})]$$

$$D_{\pm s}(\theta)=\frac{i}{1-e^{i2\pi(\nu\pm\nu_s)}}e^{\mp i\nu_s\theta}$$
$$\times\int_\theta^{\theta+2\pi}\vec{k}_0(\theta')\left[\frac{\vec{e}_y}{\rho_x}-\frac{\vec{e}_x}{\rho_y}-(1+a\gamma)(\eta_x G_x\vec{e}_y-\eta_y G_y\vec{e}_x)\right]_{\theta'}e^{\pm i\nu_s\theta'}Rd\theta'$$

ν_s :纵向工作点；

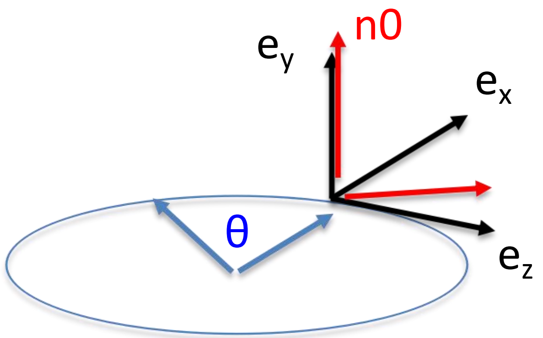
R ：储存环的平均半径；

ρ_x,ρ_y ：局部水平，垂直偏转半径；

η_x,η_y ：局部水平，垂直色散函数；

G_x,G_y ：局部水平，垂直四极铁梯度；

- 由于 $\vec{n}_0\parallel\vec{e}_y$，$\vec{k}_0\perp\vec{e}_{y'}$，$\rho_y\to\infty$，$\eta_y=0$: $D_{\pm s}=0$
- 即在理想水平储存环中，纵向振荡对退极化没有影响。



理论推导：水平储存环+磁铁误差

- 自旋运动方程：

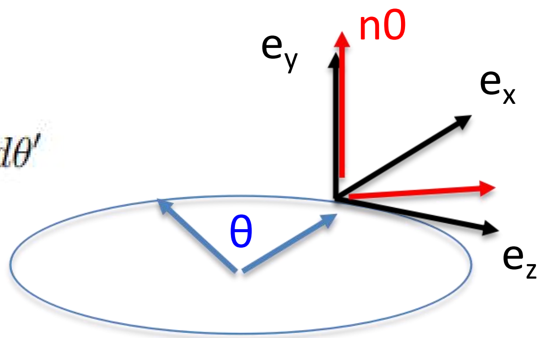
$$\frac{d\vec{S}}{d\theta} = \left[\vec{\Omega}_0 + \Delta\vec{\Omega} + \vec{\omega}\right] \times \vec{S}$$

$$\Delta\vec{\Omega} = -R(1+a\gamma)\left[(x_{COD}\cdot G - t_x)\vec{e}_y + (y_{COD}\cdot G + t_y)\vec{e}_x\right]$$

- w=0时，运动方程的解： $\vec{n} = \vec{n}_0 + \Delta\vec{n}$; $\vec{k} = \vec{k}_0 + \Delta\vec{k} = \vec{m} + i\vec{l}$;
- 扰动视为微扰：

$$\gamma\frac{\partial\vec{n}}{\partial\gamma}(\theta) = \frac{1}{2}Re[\vec{k}^{\star}(\theta)\cdot(\widetilde{D}_s + \widetilde{D}_{-s})|_{\vec{k}_0\to\vec{k}}]$$

$$\widetilde{D}_{\pm s}(\theta) = \frac{i}{1-e^{i2\pi(\nu\pm\nu_s)}}e^{\mp i\nu_s\theta} \times \int_{\theta}^{\theta+2\pi}\vec{k}(\theta')\left[\frac{\vec{e}_y}{\rho_x} - \frac{\vec{e}_x}{\rho_y} - (1+a\gamma)(\eta_x G_x\vec{e}_y - \eta_y G_y\vec{e}_x)\right]_{\theta'} e^{\pm i\nu_s\theta'}Rd\theta'$$



- 对于CEPC: $(1+a\gamma)\gg 1$, $\eta_y \ll \eta_x$:

$$\widetilde{D}_{\pm s}(\theta) \approx \frac{i}{1-e^{i2\pi(\nu\pm\nu_s)}}e^{\mp i\nu_s\theta} \times \int_{\theta}^{\theta+2\pi}[-(1+a\gamma)(\eta_x G_x\vec{e}_y)]_{\theta'}\cdot\vec{k}(\theta')e^{\pm i\nu_s\theta'}Rd\theta'$$

理论推导：水平储存环+磁铁误差

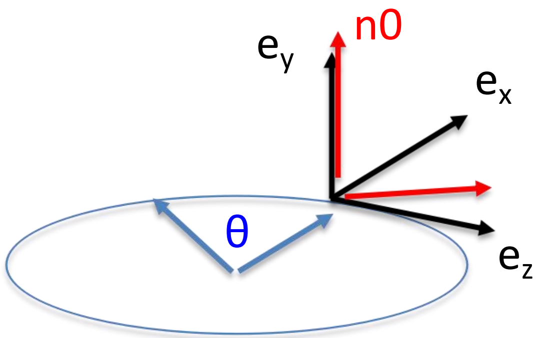
- **k** 在e_y方向的分量：

$$\vec{k} = \vec{k}_0 + \alpha \vec{n}_0 + \beta \vec{k}_0^\star$$

$$\begin{aligned} \vec{e}_y \cdot \vec{k}(\theta') &= -\int_{-\infty}^{\theta'} \Delta\Omega_x e^{i\nu(\Phi(\theta'')-\theta'')} e^{i\nu\theta''} d\theta'' \\ &= -\int_{-\infty}^{\theta'} \sum_k \omega_k e^{-ik\theta''} e^{i\nu\theta''} d\theta'' \\ &= i \sum \frac{\omega_k e^{i(\nu-k)\theta'}}{\nu - k} \end{aligned}$$

$$\Delta\Omega_x e^{i\nu\Phi(\theta'')-\theta''} = \sum_k \omega_k e^{-ik\theta''}$$

- 得到$D_{\pm}$s:



$$\widetilde{D}_{\pm s}(\theta) \approx \frac{e^{\mp i\nu_s\theta}(1 + a\gamma)}{1 - e^{i2\pi(\nu \pm \nu_s)}} \times \sum_k \frac{\omega_k}{\nu - k} \int_\theta^{\theta+2\pi} (R\eta_x G_x) e^{i(\nu - k \pm \nu_s)\theta'} d\theta'$$

$$R\eta_x G_x = \sum_j \xi_j e^{-ij\theta'}$$

理论推导：水平储存环+磁铁误差

- v-k <<1 时：其中经过计算，对于CEPC，$\xi_0 = 0.96 \approx 1$，且j≠0时，$\xi_j \ll 1$

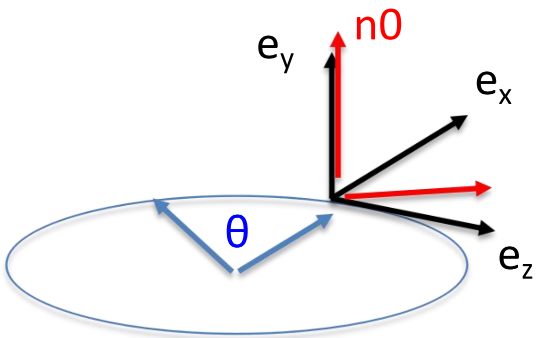
$$\frac{\widetilde{D}_{+s}+\widetilde{D}_{-s}}{2}|_\theta = i(1+a\gamma)\sum_k \frac{\omega_k}{\nu-k}\sum_j \xi_j \frac{\nu-k-j}{(\nu-k-j)^2-\nu_s^2}e^{i(\nu-k-j)\theta}$$

$$\gamma\frac{\partial\vec{n}}{\partial\gamma}(\theta) = Re[\vec{k}^\star\cdot e^{i(\nu-k)\theta}\cdot i(1+a\gamma)\sum_k \underbrace{\frac{\omega_k}{\nu-k}}_{\text{整数共振项}}\xi_0\underbrace{\frac{\nu-k}{(\nu-k)^2-\nu_s^2}}_{\text{边带共振项}}]$$

$$|\gamma\frac{\partial\vec{n}}{\partial\gamma}|^2 \approx (1+a\gamma)^2\left[\sum_k \frac{\omega_k^2\xi_0^2}{((\nu-k)^2-\nu_s^2)^2}\right]$$

- 以整数自旋共振为中心的一阶纵向边带自旋共振：

$$P \approx \frac{P_\infty}{1+\frac{\tau_{sk}}{\tau_{dep}}} \approx \frac{P_\infty}{1+\frac{11}{18}\frac{\oint ds\left\langle\frac{1}{|\rho|^3}|\frac{\partial\vec{n}}{\gamma\partial\gamma}|^2\right\rangle}{\oint ds\left\langle\frac{1}{|\rho|^3}\right\rangle}} \approx \frac{P_\infty}{1+(1+a\gamma)^2\sum_k\frac{\omega_k^2\xi_0^2}{((\nu-k)^2-\nu_s^2)^2}}$$



拟合结果

- CEPC中，(v-k=0) 与 (v-k$\pm$v$_s$=0) **叠加**，其中(v-k$\pm$v$_s$=0)对退极化有主要贡献；

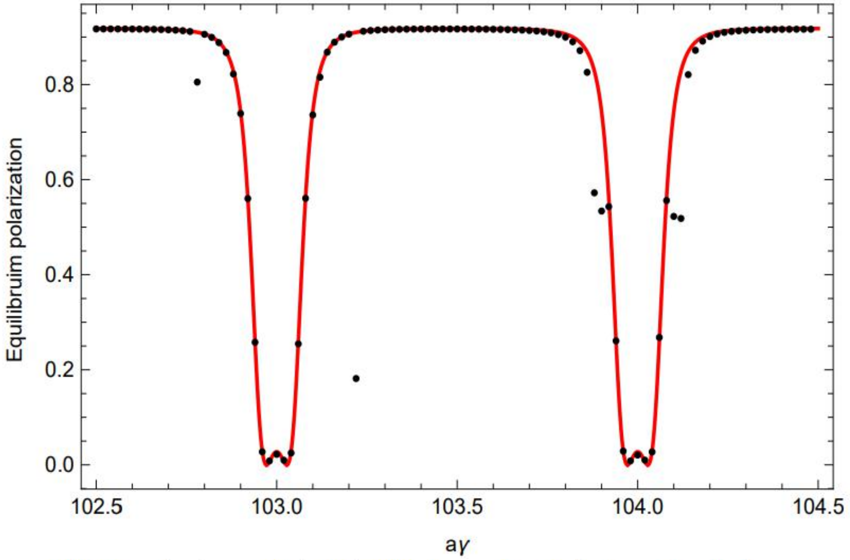
$$P \approx \frac{P_\infty}{1+\frac{\tau_{sk}}{\tau_{dep}}}$$

- |v-k|>>v$_s$, 近似为DK的理论公式，上升，下降沿吻合的原因：

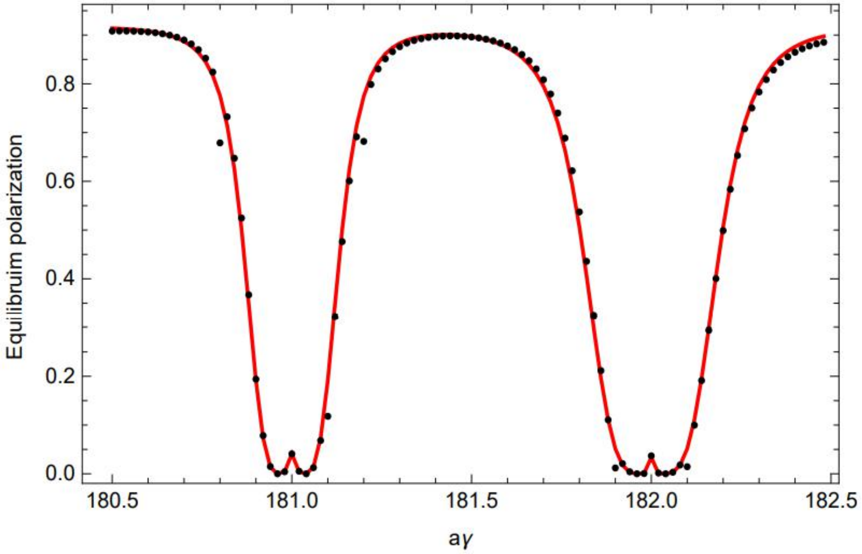
$$\frac{\tau_{sk}}{\tau_{dep}} = \sum_k \frac{A_k}{((\nu-k)^2-\nu_s^2)^2} \longrightarrow \frac{\tau_{sk}}{\tau_{dep}} = \frac{11v^2}{18}\sum_k \frac{|\omega_k|^2}{(\nu-k)^4}$$

- |v-k| < v$_s$时，P∞不再等于92%：

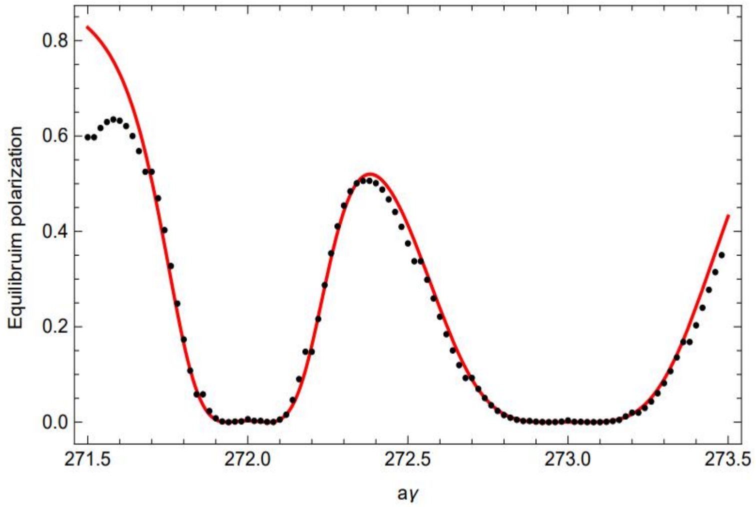
$$P_\infty = \frac{-8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3}\hat{b}\cdot(\hat{n}-\gamma\frac{\partial\hat{n}}{\partial\gamma})\right\rangle}{\oint ds \left\langle \frac{1}{|\rho|^3}\right\rangle}$$



CEPC Z, $\nu_x/\nu_y/\nu_z = 0.11/0.22/0.028$, $A_{103} = 2\times10^{-5}$, $A_{104} = 2\times10^{-5}$



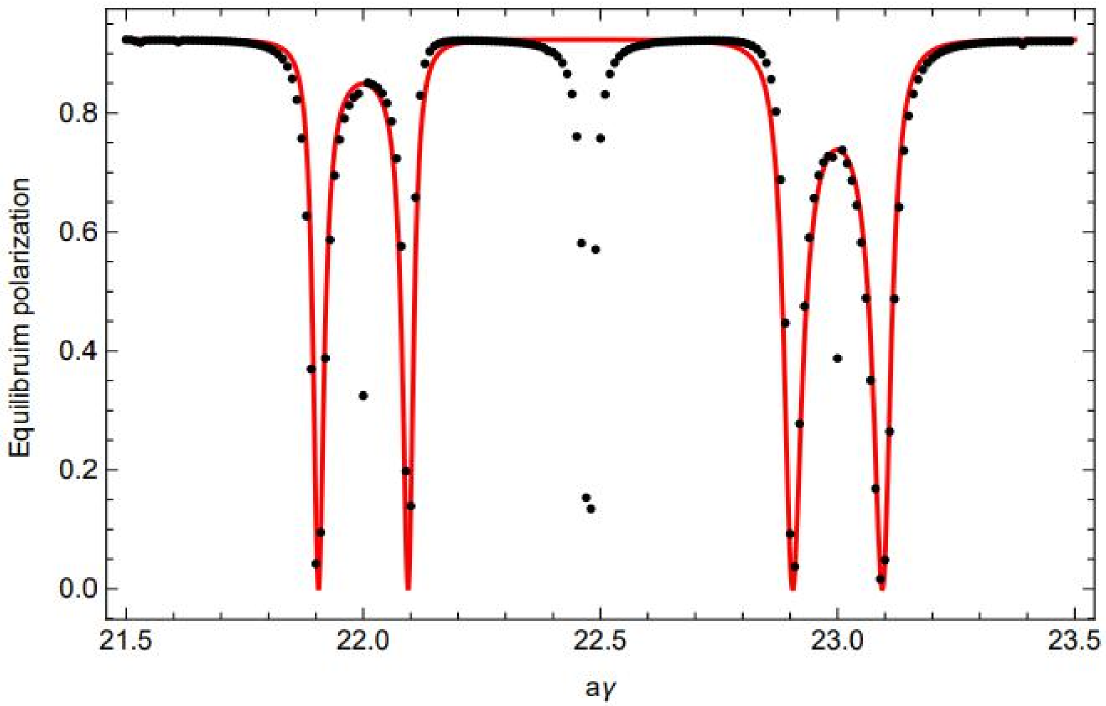
CEPC W, $\nu_x/\nu_y/\nu_z = 0.11/0.22/0.0395$, $A_{181} = 2.7\times10^{-4}$, $A_{182} = 1.2\times10^{-3}$



CEPC H, $\nu_x/\nu_y/\nu_z = 0.11/0.22/0.065$, $A_{272} = 5.8\times10^{-3}$, $A_{273} = 6.8\times10^{-2}$

拟合结果

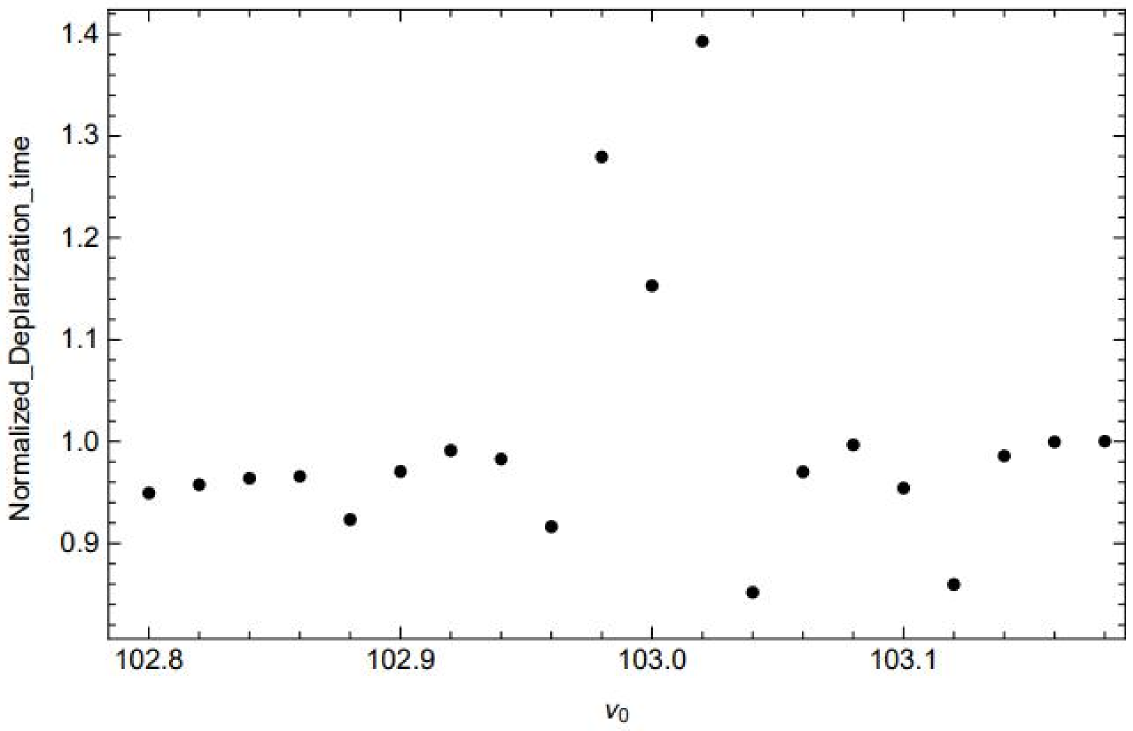
- (v-k=0) 与 (v-k±v_s=0) 不叠加的情况：
  - Other case: C=2112m, E=10GeV, ΔY=10μm;
  - v-k=0 贡献可以忽略， v-k±v_s=0贡献有主要贡献；



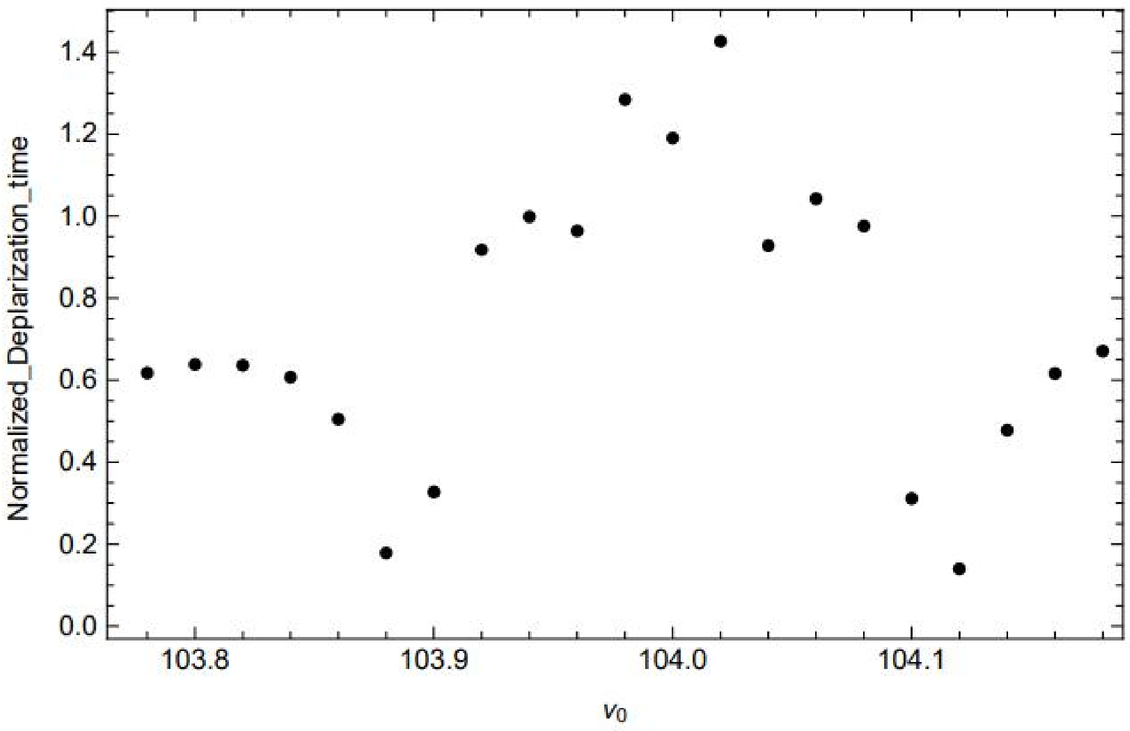
$$\frac{\tau_{sk}}{\tau_{dep}} = \sum_k \frac{A_k}{((\nu - k)^2 - \nu_s^2)^2}$$

$$\nu_x/\nu_y/\nu_z = 0.392/0.475/0.094,\ A_{22} = 0.7 \times 10^{-5}, A_{23} = 2 \times 10^{-5}$$

Back up

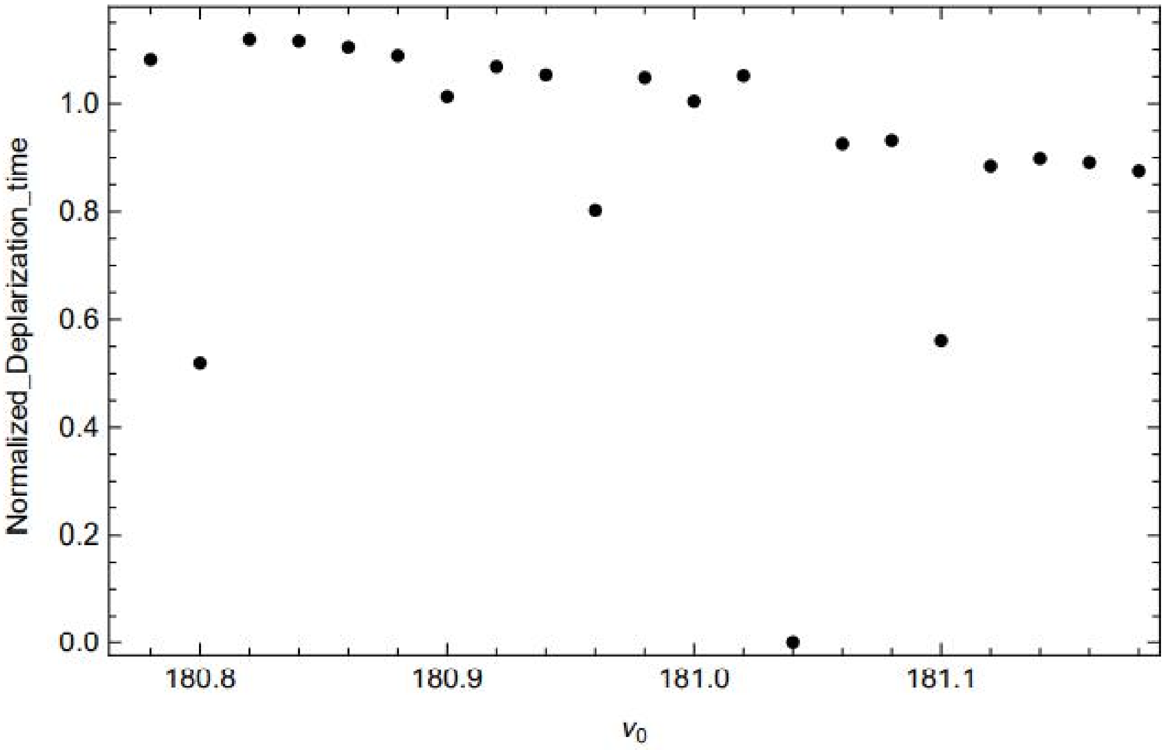


Normalized_Deplarization_time
1.4
1.3
1.2
1.1
1.0
0.9
102.8
102.9
103.0
103.1
v_0



Normalized_Deplarization_time
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
103.8
103.9
104.0
104.1
v_0

Back up


Normalized_Deplarization_time
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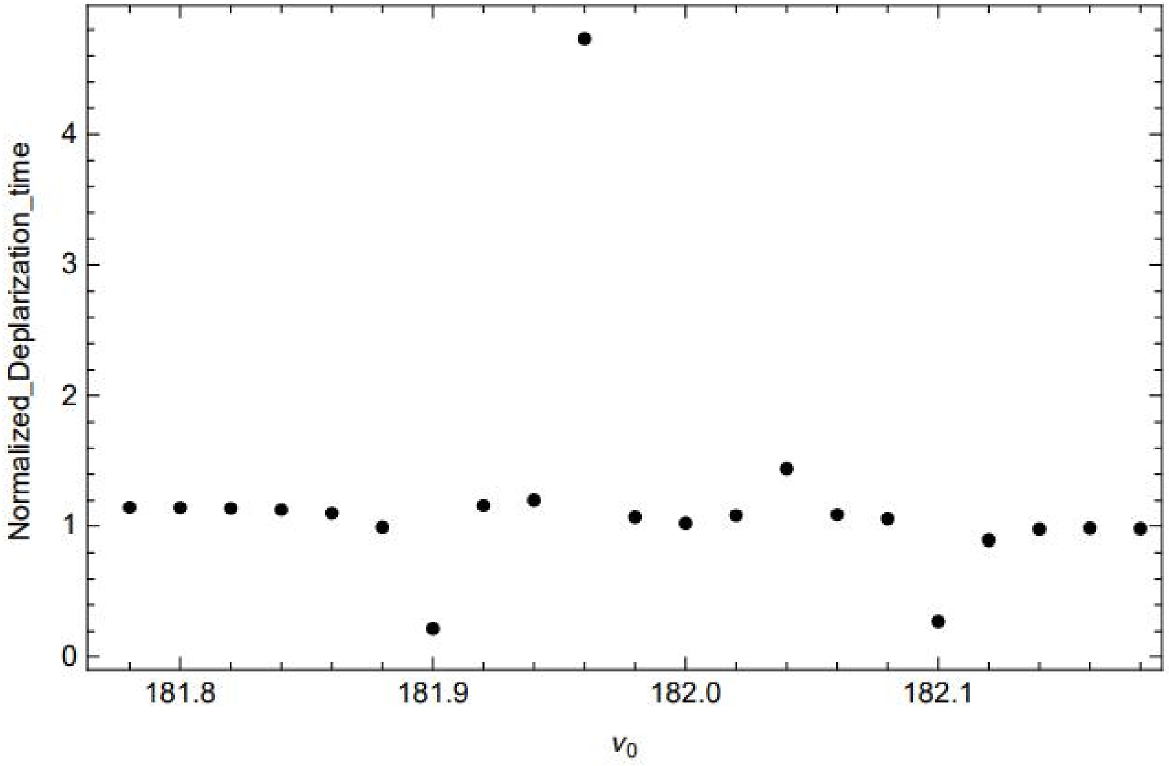
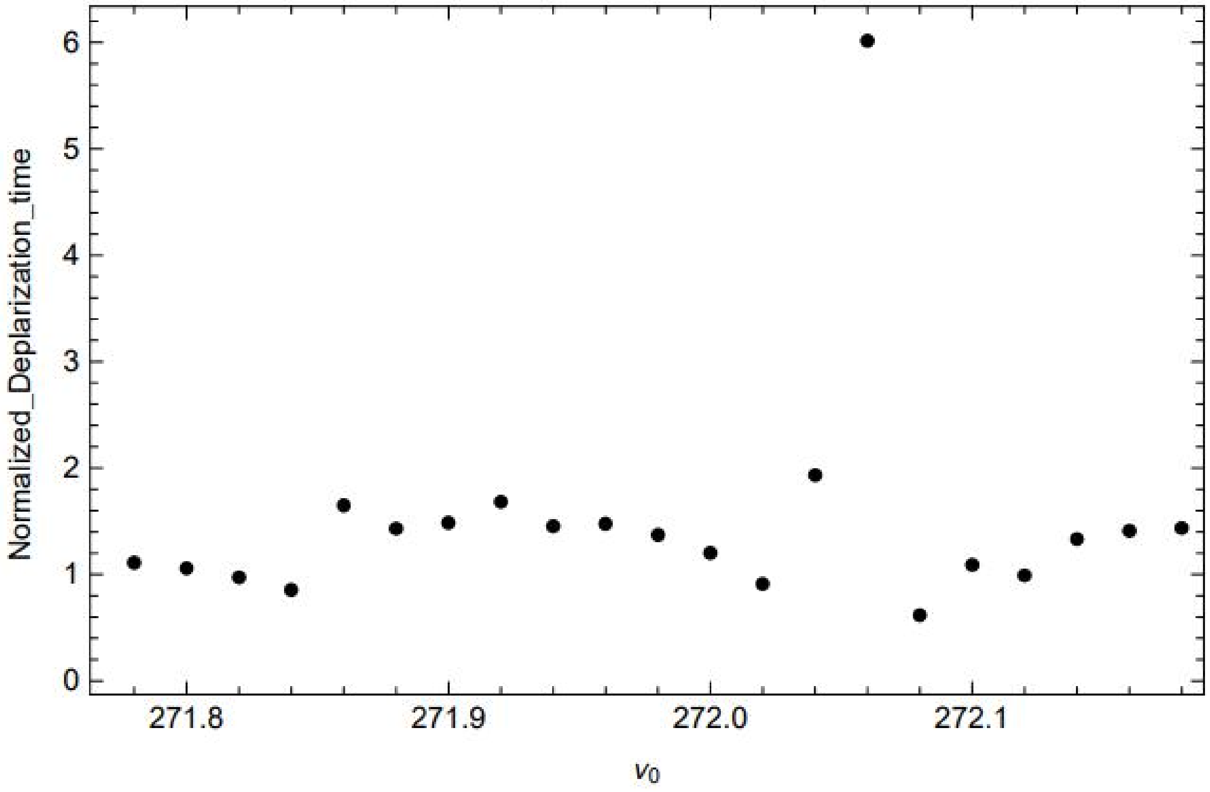
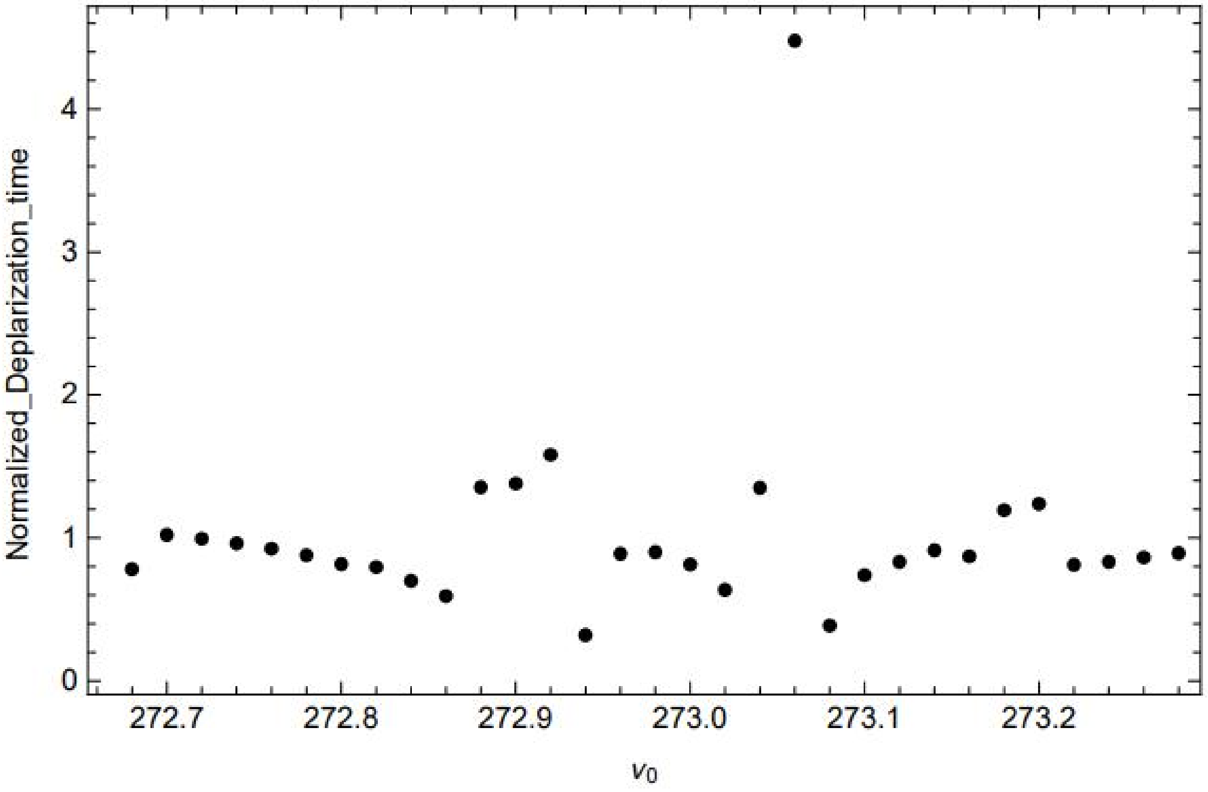
0.8
0.6
0.4
0.2
0.0
180.8
180.9
181.0
181.1
v_0


Normalized_Deplarization_time
4
3
2
1
0
181.8
181.9
182.0
182.1
v_0

Back up



Normalized_Deplarization_time
0
1
2
3
4
5
6
271.8
271.9
272.0
272.1
v_0



Normalized_Deplarization_time
0
1
2
3
4
272.7
272.8
272.9
273.0
273.1
273.2
v_0