# $P \to P, S,^1 A,^3 A$ 跃迁的形状因子

汇　　报　　人　：　　康悦杨

December 14, 2021

# 目录

# 形状因子的定义

- $P \to P$ transition

$$\begin{aligned}\left\langle P''(p'')|\bar{q}_1''\gamma_\mu q_1'|P'(p')\right\rangle &= \left(P^\mu - \frac{M'^2 - M''^2}{q^2}q^\mu\right)F_1(q^2) \\ &+ \frac{M'^2 - M''^2}{q^2}q^\mu F_0(q^2) \qquad (1)\end{aligned}$$

# 形状因子的定义

- $P \to S$ transition

$$\begin{aligned}\left\langle S''(p'')|\bar{q}_1''\gamma_\mu\gamma_5 q_1'|P'(p')\right\rangle &= -\left(P^\mu - \frac{M'^2 - M''^2}{q^2}q^\mu\right)F_1(q^2) \\ &\quad -\frac{M'^2 - M''^2}{q^2}q^\mu F_0(q^2) \qquad (2)\end{aligned}$$

# 形状因子的定义

- $P \to A$ transition

$$\left\langle A''(p'')|\bar{q}_1''\gamma_\mu\gamma_5 q_1'|P'(p')\right\rangle = -\frac{iA(q^2)}{M'-M''}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha q^\beta \quad (3)$$

$$\left\langle A''(p'')|\bar{q}_1''\gamma_\mu q_1'|P'(p')\right\rangle = (M'-M'')\epsilon_\mu^* V_1(q^2) - \frac{\epsilon^*\cdot P}{M'-M''}P_\mu V_2(q^2) + 2M''\frac{\epsilon^*\cdot P}{q^2}q_\mu[V_0(q^2)-V_3(q^2)] \quad (4)$$

$$V_3(q^2) = \frac{M'-M''}{2M''}V_1(q^2) - \frac{M'+M''}{2M''}V_2(q^2) \quad (5)$$

# 拟合公式

$$F(q^2) = \frac{F(0)}{(1-q^2/M_{B,D}^2)[1-a(q^2/M_{B,D}^2)+b(q^2/M_{B,D}^2)^2]} \quad (6)$$

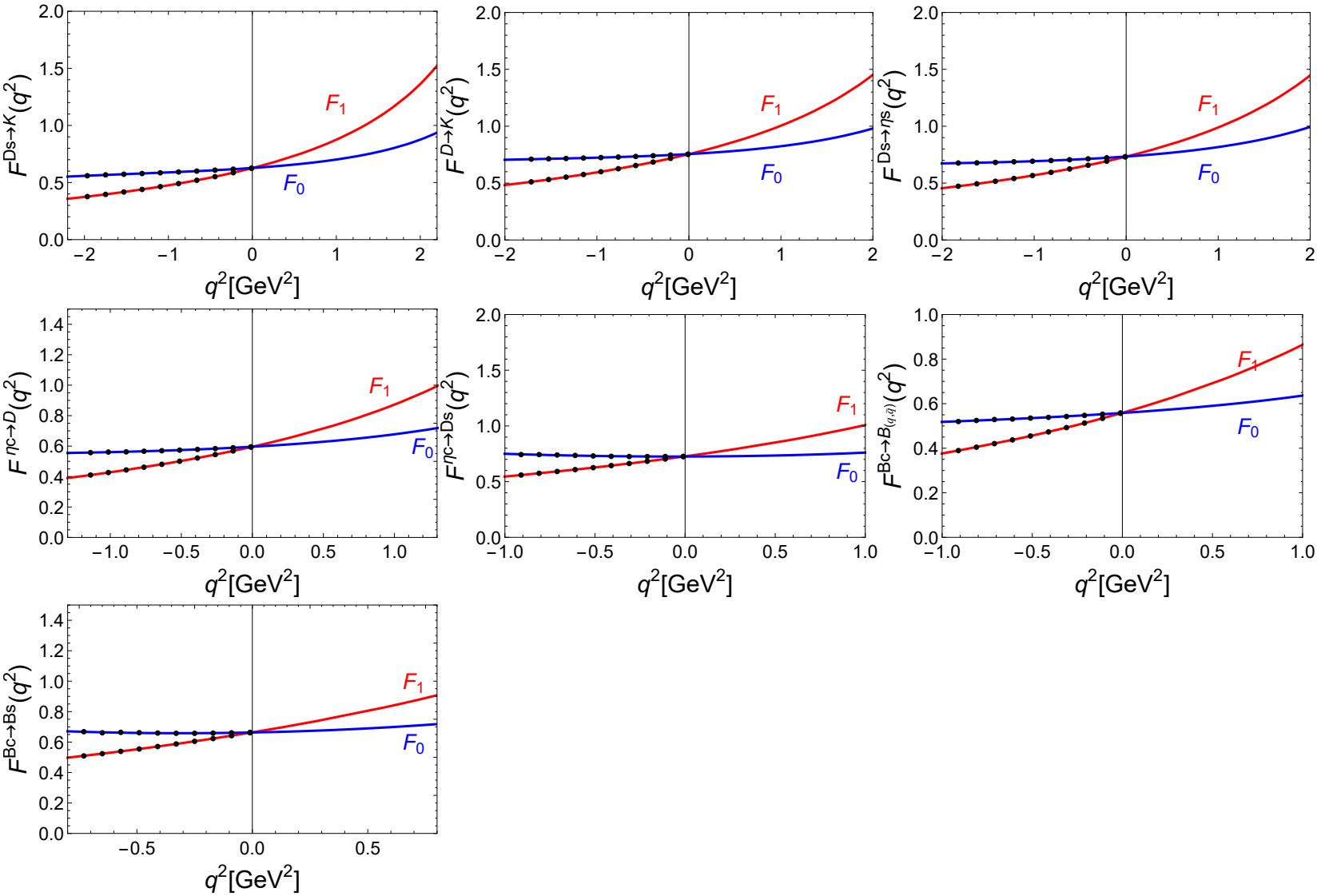
$$F(q^2) = F(0)[1+a(q^2/M_{B,D}^2)+b(q^2/M_{B,D}^2)^2] \quad (7)$$

$$F(q^2) = \frac{F(0)}{1-q^2/M_{B,D}}\{1+\sum_{k=1}^{N} b_k[z(q^2,t_0)^k - z(0,t_0)^k]\} \quad (8)$$

Table: the V-A current form factors of $P \to P$(in unit of GeV)

| | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $F_1^{D_s \to K(q,\bar{s})}$ | $0.63^{+0.09}_{-0.09}$ | $0.21^{+0.25}_{-0.18}$ | $0.07^{+0.11}_{-0.08}$ |
| $F_0^{D_s \to K(q,\bar{s})}$ | $0.63^{+0.09}_{-0.09}$ | $-0.68^{+0.86}_{-0.07}$ | $0.35^{+1.60}_{-0.02}$ |
| $F_1^{D \to K(s,\bar{q})}$ | $0.75^{+0.09}_{-0.09}$ | $0.15^{+0.24}_{-0.18}$ | $0.05^{+0.12}_{-0.08}$ |
| $F_0^{D \to K(s,\bar{q})}$ | $0.75^{+0.09}_{-0.09}$ | $-0.73^{+0.81}_{-0.06}$ | $0.34^{+1.70}_{-0.08}$ |
| $F_1^{D_s \to \eta_s(s,\bar{s})}$ | $0.73^{+0.09}_{-0.10}$ | $0.22^{+0.20}_{-0.15}$ | $0.08^{+0.10}_{-0.07}$ |
| $F_0^{D_s \to \eta_s(s,\bar{s})}$ | $0.73^{+0.09}_{-0.10}$ | $-0.65^{+0.65}_{-0.08}$ | $0.22^{+1.30}_{-0.13}$ |
| $F_1^{\eta_c \to D(q,\bar{c})}$ | $0.60^{+0.11}_{-0.11}$ | $0.42^{+0.14}_{-0.10}$ | $0.19^{+0.13}_{-0.07}$ |
| $F_0^{\eta_c \to D(q,\bar{c})}$ | $0.60^{+0.11}_{-0.11}$ | $-0.64^{+1.40}_{-0.24}$ | $0.18^{+3.80}_{-0.22}$ |
| $F_1^{\eta_c \to D_s(s,\bar{c})}$ | $0.73^{+0.11}_{-0.12}$ | $0.35^{+0.11}_{-0.09}$ | $0.18^{+0.12}_{-0.08}$ |
| $F_0^{\eta_c \to D_s(s,\bar{c})}$ | $0.73^{+0.11}_{-0.12}$ | $-0.98^{+2.40}_{-0.42}$ | $0.19^{+11.00}_{-0.14}$ |
| $F_1^{B_c \to B(q,\bar{b})}$ | $0.56^{+0.10}_{-0.10}$ | $0.66^{+0.21}_{-0.14}$ | $0.40^{+0.02}_{-0.06}$ |
| $F_0^{B_c \to B(q,\bar{b})}$ | $0.56^{+0.10}_{-0.10}$ | $-0.61^{+1.80}_{-0.32}$ | $0.25^{+6.60}_{-0.16}$ |
| $F_1^{B_c \to B_s(s,\bar{b})}$ | $0.66^{+0.10}_{-0.10}$ | $0.67^{+0.28}_{-0.23}$ | $0.39^{+0.01}_{-0.02}$ |
| $F_0^{B_c \to B_s(s,\bar{b})}$ | $0.66^{+0.10}_{-0.10}$ | $-0.80^{+3.40}_{-0.37}$ | $-0.57^{+23.00}_{-0.79}$ |

# 上述过程的图像


$F^{Ds\to K}(q^2)$
$F^{D\to K}(q^2)$
$F^{Ds\to \eta s}(q^2)$
$F^{\eta c\to D}(q^2)$
$F^{\eta c\to Ds}(q^2)$
$F^{Bc\to B_{(q,\bar{q})}}(q^2)$
$F^{Bc\to Bs}(q^2)$
$F_1$
$F_0$
$q^2$[GeV$^2$]

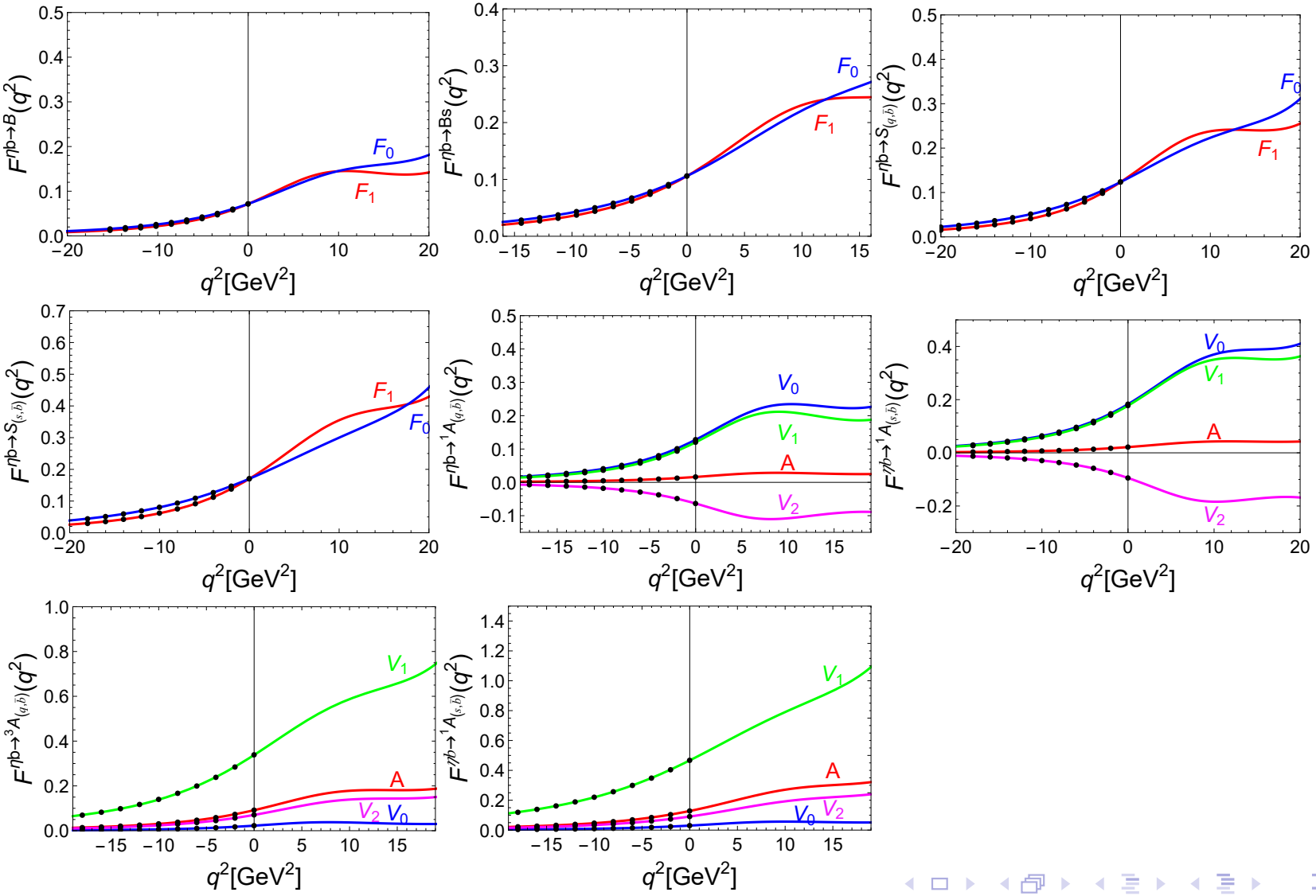
# 出现鼓包的过程

Table: the V-A current form factors of $P \to M$(in unit of GeV)

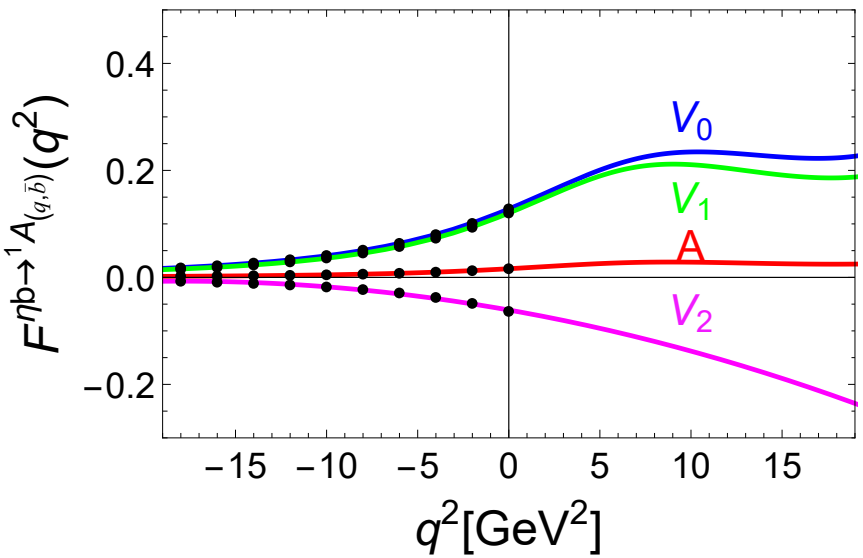
| | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $F_1^{\eta_b \to B_{(q,\bar{b})}}$ | $0.07^{+0.03}_{-0.02}$ | $2.30^{+0.03}_{-0.01}$ | $4.70^{+0.36}_{-0.37}$ |
| $F_0^{\eta_b \to B_{(q,\bar{b})}}$ | $0.07^{+0.03}_{-0.02}$ | $1.80^{+0.05}_{-0.05}$ | $3.30^{+0.36}_{-0.38}$ |
| $F_1^{\eta_b \to B_{s(s,\bar{b})}}$ | $0.11^{+0.05}_{-0.03}$ | $2.20^{+0.02}_{-0.01}$ | $3.90^{+0.32}_{-0.32}$ |
| $F_0^{\eta_b \to B_{s(s,\bar{b})}}$ | $0.11^{+0.05}_{-0.03}$ | $1.70^{+0.05}_{-0.05}$ | $2.70^{+0.31}_{-0.32}$ |
| $F_1^{\eta_b \to S_{(q,\bar{b})}}$ | $0.12^{+0.05}_{-0.04}$ | $2.00^{+0.02}_{-0.00}$ | $4.20^{+0.40}_{-0.42}$ |
| $F_0^{\eta_b \to S(q,\bar{b})}$ | $0.12^{+0.05}_{-0.04}$ | $1.30^{+0.16}_{-0.13}$ | $2.60^{+0.39}_{-0.43}$ |
| $F_1^{\eta_b \to S_{(s,\bar{b})}}$ | $0.17^{+0.06}_{-0.05}$ | $2.00^{+0.01}_{-0.00}$ | $3.40^{+0.35}_{-0.35}$ |
| $F_0^{\eta_b \to S_{(s,\bar{b})}}$ | $0.17^{+0.06}_{-0.05}$ | $1.10^{+0.22}_{-0.18}$ | $1.90^{+0.28}_{-0.34}$ |

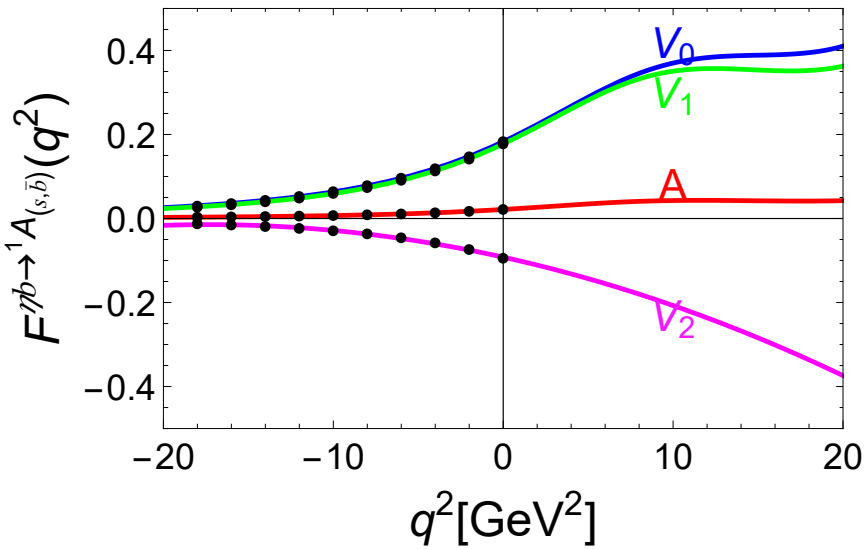
| | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $A^{\eta_b\to{}^1A}(q,\bar{b})$ | $0.02^{+0.01}_{-0.01}$ | $2.30^{+0.00}_{-0.02}$ | $5.70^{+0.69}_{-0.72}$ |
| $V_0^{\eta_b\to{}^1A}(q,\bar{b})$ | $0.13^{+0.05}_{-0.04}$ | $2.10^{+0.02}_{-0.04}$ | $4.70^{+0.65}_{-0.70}$ |
| $V_1^{\eta_b\to{}^1A}(q,\bar{b})$ | $0.12^{+0.07}_{-0.05}$ | $2.30^{+0.03}_{-0.03}$ | $5.50^{+0.72}_{-0.75}$ |
| $V_2^{\eta_b\to{}^1A}(q,\bar{b})$ | $-0.06^{+0.02}_{-0.02}$ | $2.50^{+0.05}_{-0.31}$ | $6.30^{+0.36}_{-1.90}$ |
| $A^{\eta_b\to{}^1A}(s,\bar{b})$ | $0.02^{+0.01}_{-0.01}$ | $2.30^{+0.01}_{-0.01}$ | $4.60^{+0.51}_{-0.49}$ |
| $V_0^{\eta_b\to{}^1A}(s,\bar{b})$ | $0.18^{+0.06}_{-0.05}$ | $2.10^{+0.00}_{-0.01}$ | $3.90^{+0.42}_{-0.42}$ |
| $V_1^{\eta_b\to{}^1A}(s,\bar{b})$ | $0.18^{+0.10}_{-0.07}$ | $2.20^{+0.02}_{-0.02}$ | $4.30^{+0.47}_{-0.47}$ |
| $V_2^{\eta_b\to{}^1A}(s,\bar{b})$ | $-0.09^{+0.02}_{-0.02}$ | $2.40^{+0.06}_{-0.07}$ | $5.20^{+0.16}_{-0.21}$ |
| $A^{\eta_b\to{}^3A}(q,\bar{b})$ | $0.09^{+0.03}_{-0.03}$ | $2.00^{+0.05}_{-0.05}$ | $4.10^{+0.70}_{-0.74}$ |
| $V_0^{\eta_b\to{}^3A}(q,\bar{b})$ | $0.02^{+0.01}_{-0.01}$ | $2.40^{+0.02}_{-0.00}$ | $6.50^{+0.62}_{-0.67}$ |
| $V_1^{\eta_b\to{}^3A}(q,\bar{b})$ | $0.34^{+0.13}_{-0.11}$ | $1.30^{+0.25}_{-0.21}$ | $2.80^{+0.53}_{-0.65}$ |
| $V_2^{\eta_b\to{}^3A}(q,\bar{b})$ | $0.07^{+0.02}_{-0.02}$ | $2.50^{+0.10}_{-0.09}$ | $1.90^{+0.12}_{-0.10}$ |
| $A^{\eta_b\to{}^3A}(s,\bar{b})$ | $0.13^{+0.04}_{-0.04}$ | $2.00^{+0.03}_{-0.03}$ | $3.40^{+0.51}_{-0.48}$ |
| $V_0^{\eta_b\to{}^3A}(s,\bar{b})$ | $0.03^{+0.01}_{-0.01}$ | $2.40^{+0.04}_{-0.02}$ | $5.20^{+0.36}_{-0.38}$ |
| $V_1^{\eta_b\to{}^3A}(s,\bar{b})$ | $0.47^{+0.15}_{-0.13}$ | $1.00^{+0.28}_{-0.22}$ | $2.10^{+0.29}_{-0.37}$ |
| $V_2^{\eta_b\to{}^3A}(s,\bar{b})$ | $0.09^{+0.02}_{-0.02}$ | $2.50^{+0.11}_{-0.09}$ | $2.00^{+0.13}_{-0.10}$ |

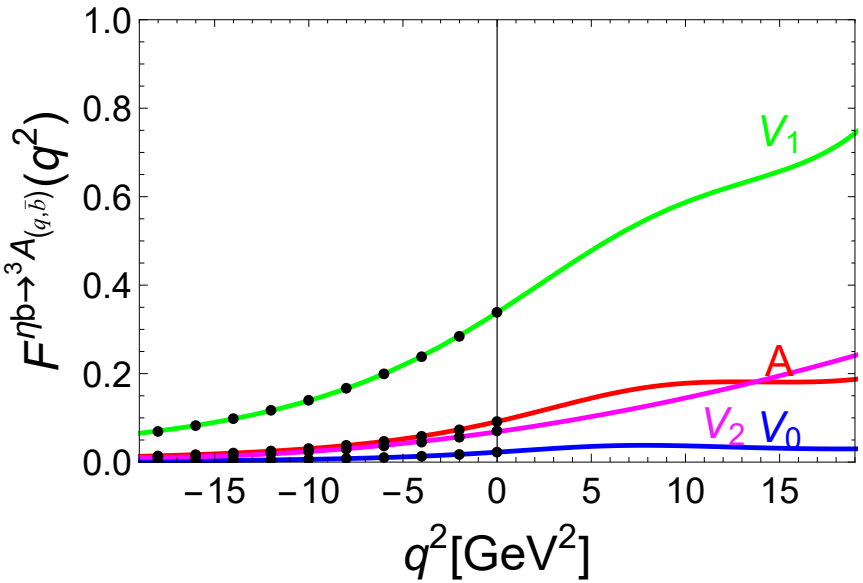
$$F(q^2) = \frac{F(0)}{(1 - q^2/M_{B,D}^2)[1 - a(q^2/M_{B,D}^2) + b(q^2/M_{B,D}^2)^2]} \tag{9}$$

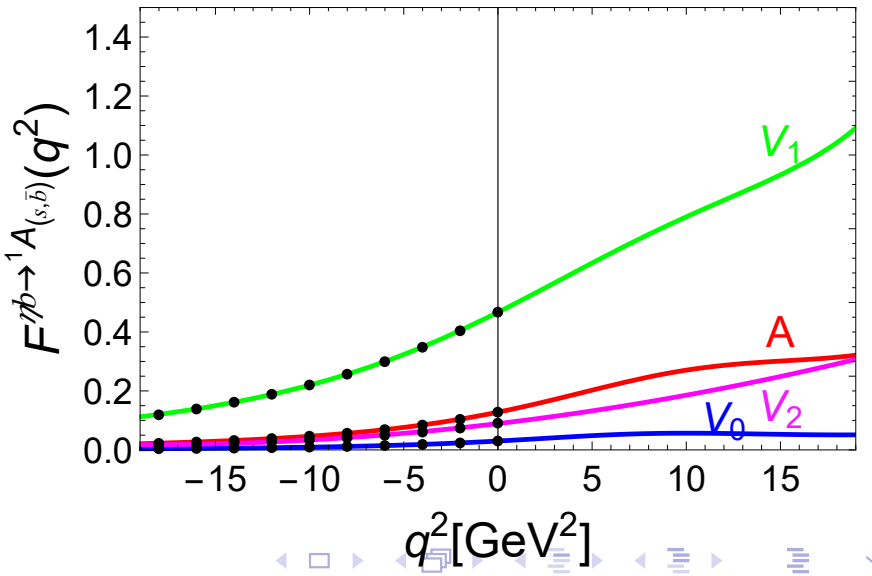


$$F(q^2) \quad = \quad F(0)[1 + a(q^2/M_{B,D}^2) + b(q^2/M_{B,D}^2)^2] \tag{10}$$

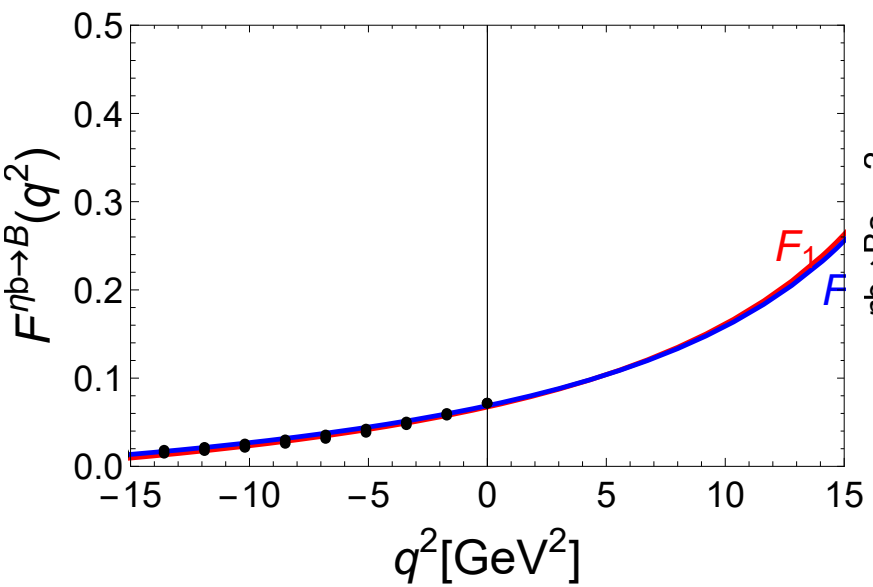


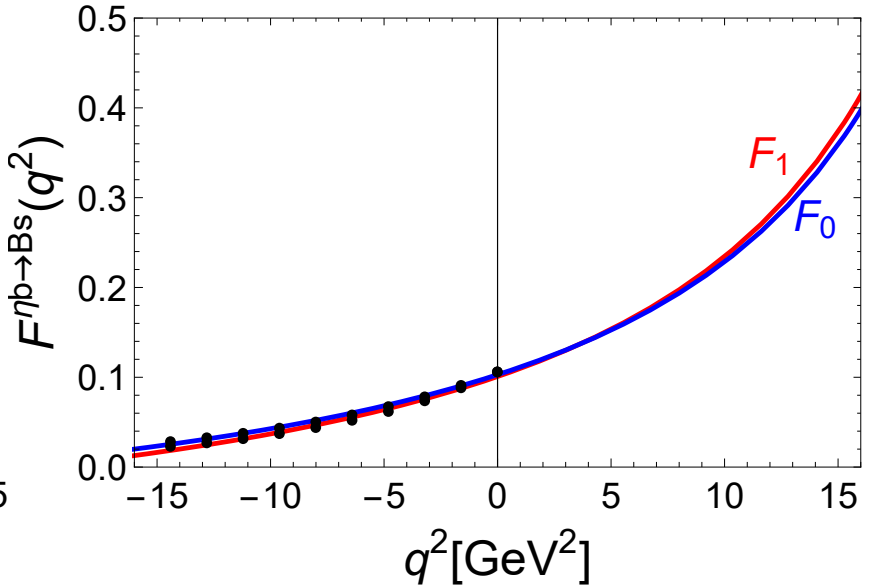



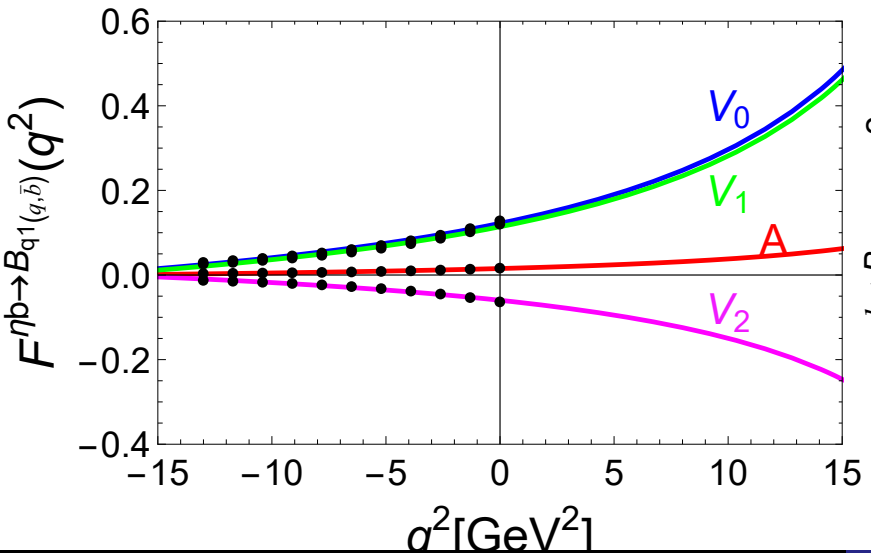



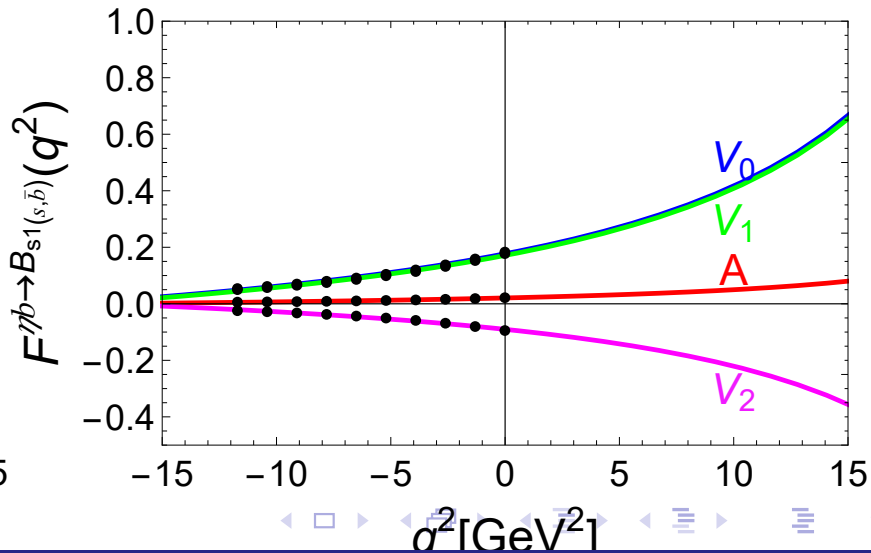



$$F(q^2) = \frac{F(0)}{1 - q^2/M_{B,D}}\{1 + \sum_{k=1}^{N} b_k[z(q^2, t_0)^k - z(0, t_0)^k]\} \quad (11)$$



# 对于$P \to A$ transition 拟合值很大的请况

Table: the V-A current form factors of $P \to^1 A$(in unit of GeV)

| | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $A^{B\to^1 A_{(q,\bar{q})}}$ | $0.12^{+0.03}_{-0.02}$ | $0.75^{+0.11}_{-0.08}$ | $0.18^{+0.05}_{-0.04}$ |
| $V_0^{B\to^1 A_{(q,\bar{q})}}$ | $0.38^{+0.03}_{-0.03}$ | $0.35^{+0.19}_{-0.19}$ | $0.03^{+0.04}_{-0.02}$ |
| $V_1^{B\to^1 A_{(q,\bar{q})}}$ | $0.20^{+0.05}_{-0.04}$ | $-0.01^{+0.04}_{-0.04}$ | $0.14^{+0.01}_{-0.01}$ |
| $V_2^{B\to^1 A_{(q,\bar{q})}}$ | $-0.01^{+0.02}_{-0.02}$ | $-5\times10^{14}$ | $7\times10^{15}$ |

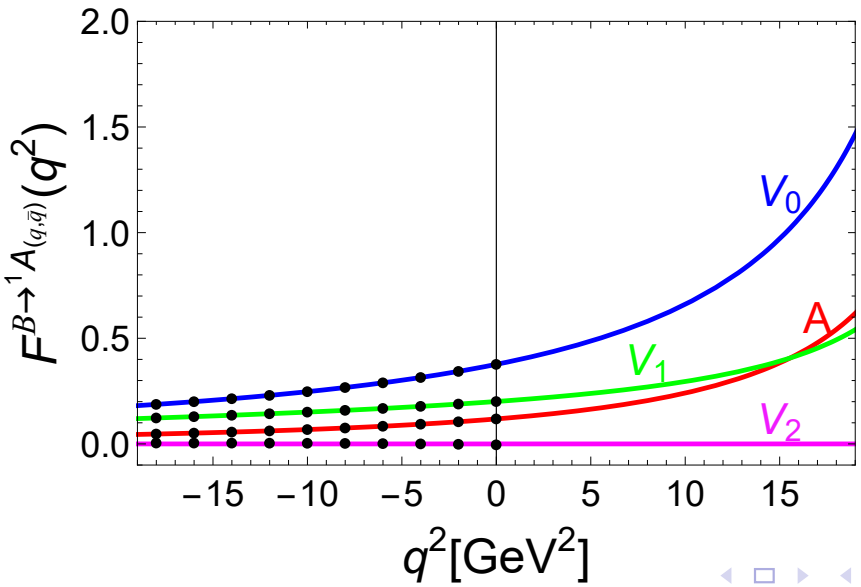
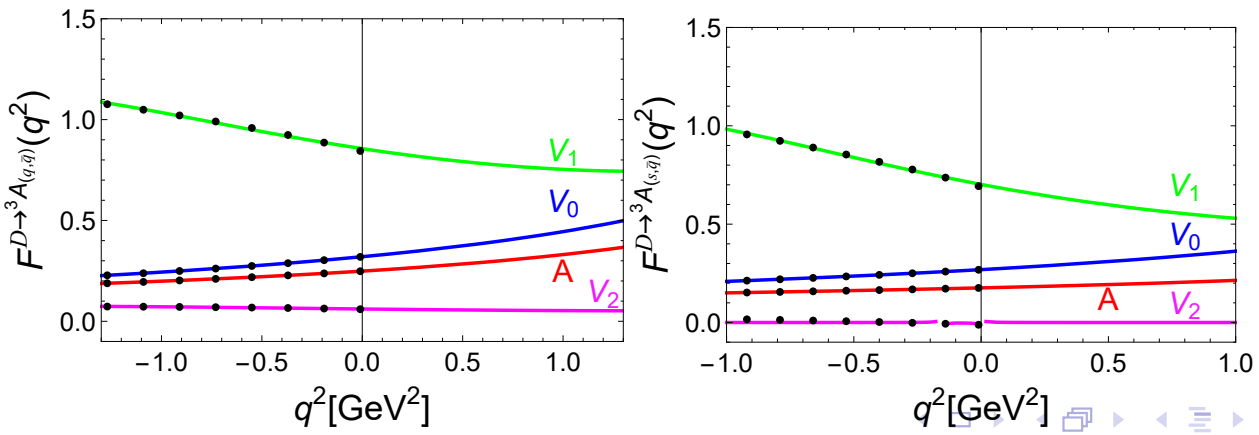


Table: the V-A current form factors of $P \to {}^3A$(in unit of GeV)

| | $F(0)$ | $a$ | $b$ |
|---|---|---|---|
| $A^{D\to {}^3A_{(q,\bar{q})}}$ | $0.25^{+0.05}_{-0.09}$ | $0.00^{+0.12}_{-0.06}$ | $0.02^{+0.10}_{-0.04}$ |
| $V_0^{D\to {}^3A_{(q,\bar{q})}}$ | $0.32^{+0.03}_{-0.04}$ | $0.19^{+0.25}_{-0.17}$ | $0.09^{+0.11}_{-0.07}$ |
| $V_1^{D\to {}^3A_{(q,\bar{q})}}$ | $0.84^{+0.18}_{-0.13}$ | $-1.70^{+0.83}_{-0.30}$ | $1.40^{+1.30}_{-0.32}$ |
| $V_2^{D\to {}^3A_{(q,\bar{q})}}$ | $0.06^{+0.07}_{-0.08}$ | $-1.70^{+0.94}_{-290.00}$ | $1.70^{+1.10}_{-5900.00}$ |
| $A^{D\to {}^3A_{(s,\bar{q})}}$ | $0.18^{+0.09}_{-0.13}$ | $-0.32^{+0.09}_{-2.00}$ | $0.24^{+0.04}_{-2.70}$ |
| $V_0^{D\to {}^3A_{(s,\bar{q})}}$ | $0.27^{+0.03}_{-0.05}$ | $0.09^{+0.21}_{-0.28}$ | $0.13^{+0.07}_{-0.07}$ |
| $V_1^{D\to {}^3A_{(s,\bar{q})}}$ | $0.69^{+0.04}_{-0.06}$ | $-2.40^{+0.69}_{-1.70}$ | $2.70^{+1.10}_{-3.50}$ |
| $V_2^{D\to {}^3A_{(s,\bar{q})}}$ | $-0.01^{+0.04}_{-0.04}$ | $1.7 \times 10^{10}$ | $1.6 \times 10^{16}$ |



# P 介子的盖根鲍尔矩

Table: Gegenbauer moments of pseudoscalar at $\mu$=1GeV .

| | $a_1$ | $a_2$ | $a_3$ | | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|
| $\pi$ | 0 | 0.10 | 0 | $K$ | 0.09 | 0.02 | 0.05 |
| $\eta_s$ | 0 | −0.06 | 0 | $D$ | 0.57 | 0.11 | 0.02 |
| $D_s$ | 0.48 | 0.03 | −0.02 | $\eta_c$ | 0 | −0.33 | 0 |
| $B$ | 1.10 | 0.81 | 0.42 | $B_s$ | 1.06 | 0.72 | 0.31 |
| $B_c$ | 0.69 | 0.09 | −0.20 | $\eta_b$ | 0 | −0.41 | 0 |

# 请各位老师同学批评指正