



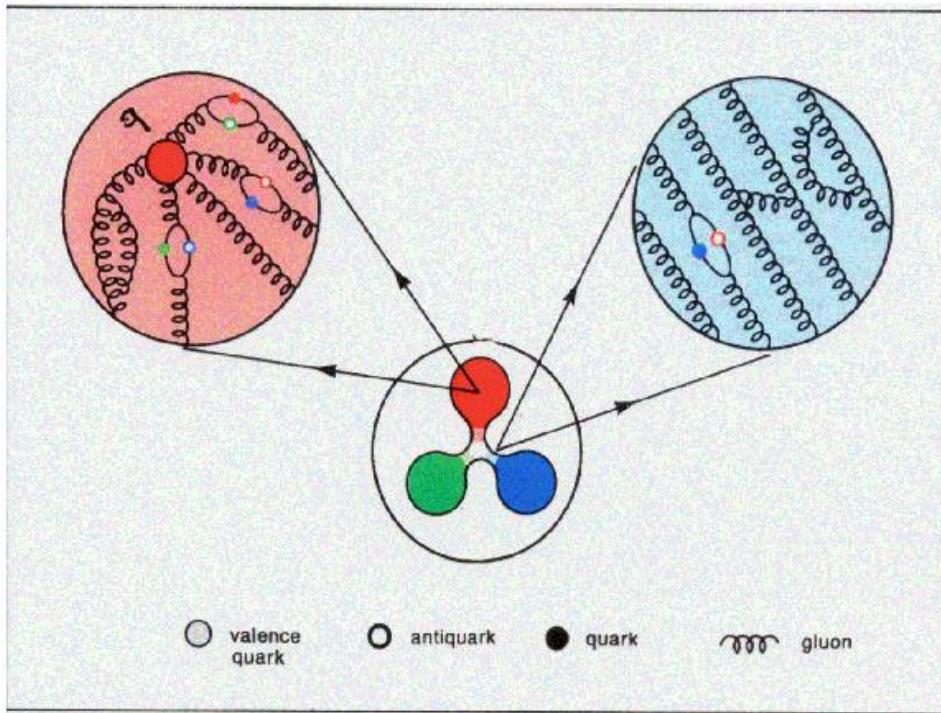
核子的自旋和味道结构

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核物理前沿与交叉研讨会—清华大学物理系主办
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Tianbo Liu, Xiannan Liu, Yingda Han,

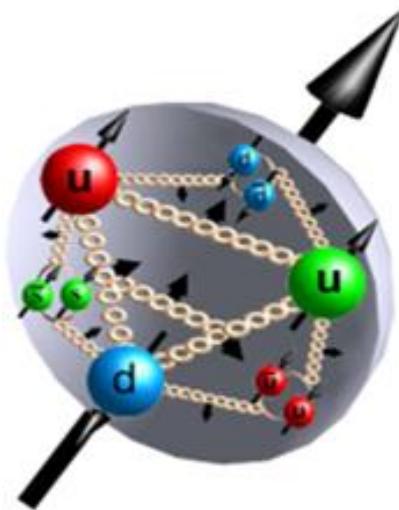
Nucleons: Building Block of Matter



- Nucleon anomalous magnetic moment (*Stern, Nobel Prize 1943*)
- Electromagnetic form factor from electron scattering (*Hofstadter, Nobel Prize 1961*)
- Deep-in-elastic scattering, quark underlying structure of the nucleon (*Freedman, Kendall, Feldman, Nobel Prize 1990*)

Understanding the underlying nucleon structure

The structure of the nucleon



- The most abundant piece of matter in our world.
- A very mysterious object with many puzzles: proton spin crisis, sea content of the nucleon, 3-dimentional structure of the nucleon.

Surprises & Anomalies about the Quark Structure of Nucleon: Sea

- Spin Structure: $\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$

spin “crisis” or “puzzle”: where is the proton’s missing spin

- Flavor Asymmetry $\bar{u} \neq \bar{d}$

- Strange Content $\Delta s \neq 0 \quad s(x) \neq \bar{s}(x)$?

Brodsky & Ma, PLB381(96)317

- Isospin Symmetry Breaking $\bar{u}_p \neq \bar{d}_n \quad \bar{d}_p \neq \bar{u}_n$?
or Charge Symmetry Violation

Ma, PLB 274 (92) 111

Boros, Londergan, Thomas, PRL81(98)4075

The Proton “Spin Crisis”

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark
model expectation:

Naïve Quark Model:

$$\Delta u = \frac{4}{3}; \quad \Delta d = -\frac{1}{3}; \quad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

The Proton “Spin Crisis”

These results could be argued to imply that the sum of the spins carried by the quarks in a proton was consistent with zero, rather than with $1/2$ as given in the quark model, suggesting a “Spin Crisis” in the parton model.

M.Anselmino, A.Efremov, E.Leader
Physics Reports 261 (1995) page 4

质子“自旋危机”

这表明质子中的夸克实际上对其自旋没有贡献，这一事实即为人们所称的“自旋危机”。

《低能及中高能原子核物理》 415页

程檀生 钟毓树 编著

北京大学出版社 1997 年

Why there is the proton spin puzzle/crisis?

- The quark model is very successful for the classification of baryons and mesons
- The quark model is good to explain the magnetic moments of octet baryons
- The quark model gave the birth of QCD as a theory for strong interaction

**So why there is serious problem with spin of the proton
in the quark model?**

Many Theoretical Explanations

- The sea quarks of the proton are largely negatively polarized
- The gluons provide a significant contribution to the proton spin

It was thought that the spin “crisis” cannot be understood within the quark model: “ the lowest uud valence component of the proton is estimated to be of only a few percent.” R.L. Jaffe and Lipkin, PLB266(1991)158

How to get a clear picture of nucleon?

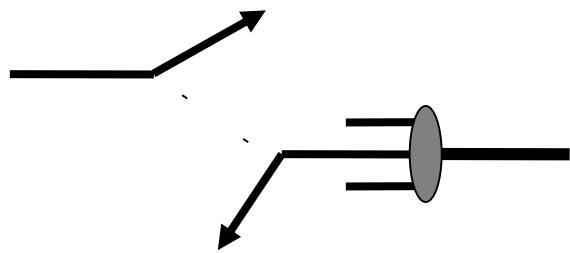
- PDFs are physically defined in the IMF (infinite-momentum frame) or with space-time on the light-cone.
- Whether the physical picture of a nucleon is the same in different frames?

A physical quantity defined by matrix element is frame-independent, but its physical picture is frame-dependent.

The parton model (Feynman 1969)

- photon scatters incoherently off massless, pointlike, spin-1/2 **quarks**
- probability that a quark carries fraction ξ of parent proton's momentum is $q(\xi)$, ($0 < \xi < 1$)

Infinite
Momentum
Frame



$$\begin{aligned} F_2(x) &= \sum_{q,q} \int_0^1 d\xi \ e_q^2 \xi q(\xi) \delta(x - \xi) = \sum_{q,q} e_q^2 x q(x) \\ &= \frac{4}{9} x u(x) + \frac{1}{9} x d(x) + \frac{1}{9} x s(x) + \dots \end{aligned}$$

- the functions $u(x)$, $d(x)$, $s(x)$, ... are called **parton distribution functions** (pdfs) - they encode information about the proton's deep structure
- **Parton model is established under the collinear approximation:** The transversal motion of partons is neglected or integrated over.

The improvement to the parton model?

- What would be the consequence by taking into account the transversal motions of partons?
- It might be trivial in unpolarized situation. However it brings significant influences to spin dependent quantities (helicity and transversity distributions) and transversal momentum dependent quantities (TMDs or 3dPDFs).

The most simple case:

The Pion Spin Structure

Based on collaborated works with T.Huang and Q.-X.Shen

- [1] T. Huang, B.Q. Ma, and Q.X. Shen, Phys. Rev. D **49**, 1490 (1994).
- [2] B. Q. Ma, Z. Phys. A **345**, 321 (1993).
- [3] B.Q. Ma and T.Huang, J. Phys. G **21**, (765) (1995).

Fu-Guang Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 53 (1996) 6582-6585.

Fu-Guang Cao, Jun Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 55 (1997) 7107-7113.

Jun Cao, Fu-Guang Cao, Tao Huang, Bo-Qiang Ma, Phys. Rev. D 58 (1998) 113006.

Analysis of the pion wave function in the light-cone formalism

Tao Huang, Bo-Qiang Ma, and Qi-Xing Shen

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(Received 22 January 1991; revised manuscript received 12 August 1993)

We analyze several general constraints on the pionic valence-state wave function. It is found that the present model wave functions used in the light-cone formalism of perturbative quantum chromodynamics have failed to reproduce the Chernyak-Zhitnitsky (CZ) distribution amplitude which is required to fit the pionic form factor data and the reasonable valence-state structure function which does not exceed the pionic structure function data for $x \rightarrow 1$ simultaneously. A possible model wave function which can satisfy all the general constraints has been suggested and analyzed.

PACS number(s): 12.38.-t, 12.39.-x, 13.60.-r

calculation. Also, we have shown that there are two higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components in the light-cone wave function for the pion as a natural consequence from the Melosh rotation and it is speculated that these components should be incorporated into the perturbative quantum chromodynamics. Some progress has been

Pion Spin-Space Wave Function in Rest Frame

In the pion rest frame, the instant-form spin space wave-function of pion is

$$\chi_T = (\chi_1^\uparrow \chi_2^\downarrow - \chi_2^\uparrow \chi_1^\downarrow) / \sqrt{2},$$

in which $\chi_i^{\uparrow,\downarrow}$ are the two-component Pauli spinors.

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame
and infinite momentum frame

Or between spin states in the conventional equal time
dynamics and the light-front dynamics

$$\chi^\uparrow(T) = w[(q^- + m)\chi^\uparrow(F) - q^R\chi^\downarrow(F)];$$

$$\chi^\downarrow(T) = w[(q^- + m)\chi^\downarrow(F) + q^L\chi^\uparrow(F)].$$

The Notion of Spin

- Related to the space-time symmetry of the Poincaré group
- Generators $P^\mu = (H, \vec{P})$, space-time translator

$J^{\mu\nu}$ infinitesimal Lorentz transformation

$$\vec{J} \quad J^k = \frac{1}{2} \varepsilon_{ijk} J^{ij} \quad \text{angular momentum}$$

$$\vec{K} \quad K^k = J^{k0} \quad \text{boost generator}$$

$$\text{Pauli-Lubanski vector } w_\mu = \frac{1}{2} J^{\rho\sigma} P^\nu \varepsilon_{\nu\rho\sigma\mu}$$

Casimir operators: $P^2 = P^\mu P_\mu = m^2$ mass

$$w^2 = w^\mu w_\mu = s^2 \quad \text{spin}$$

The Wigner Rotation

for a rest particle $(m, \vec{0}) = p^\mu \quad (0, \vec{s}) = w^\mu$

for a moving particle $L(p)p = (m, \vec{0}) \quad (0, \vec{s}) = L(p)w/m$

$L(p)$ = ratationless Lorentz boost

Wigner Rotation

$$\vec{s}, p_\mu \rightarrow \vec{s}', p'_\mu$$

$$\vec{s}' = R_w(\Lambda, p)\vec{s} \quad p' = \Lambda p$$

$$R_w(\Lambda, p) = L(p')\Lambda L^{-1}(p) \quad \text{a pure rotation}$$

E. Wigner,
Ann. Math. 40(1939)149

The Lowest Valence State Wave Function in Light-Cone

$$\begin{aligned} |\psi_{q\bar{q}}^{\pi}\rangle = & \psi(x, \mathbf{k}_{-}, \uparrow, \downarrow) | \uparrow\downarrow \rangle + \psi(x, \mathbf{k}_{-}, \downarrow, \uparrow) | \downarrow\uparrow \rangle \\ & + \psi(x, \mathbf{k}_{-}, \uparrow, \uparrow) | \uparrow\uparrow \rangle + \psi(x, \mathbf{k}_{-}, \downarrow, \downarrow) | \downarrow\downarrow \rangle, \end{aligned}$$

where

$$\psi(x, \mathbf{k}_{-}, \lambda_1, \lambda_2) = C_0^F(x, \mathbf{k}_{-}, \lambda_1, \lambda_2) \varphi(x, \mathbf{k}_{-}).$$

Here $\varphi(x, \mathbf{k}_{-})$ is the momentum space wave function in the light-cone formalism.

The Spin Component Coefficients

The spin component coefficients C_0^F have the forms,

$$C_0^F(x, q, \uparrow, \downarrow) = w_1 w_2 [(q_1^- + m)(q_2^- + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x, q, \downarrow, \uparrow) = -w_1 w_2 [(q_1^- + m)(q_2^- + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x, q, \uparrow, \uparrow) = w_1 w_2 [(q_1^- + m)q_2^L - (q_2^- + m)q_1^L]/\sqrt{2};$$

$$C_0^F(x, q, \downarrow, \downarrow) = w_1 w_2 [(q_1^- + m)q_2^R - (q_2^- + m)q_1^R]/\sqrt{2}.$$

C_0^F satisfy the relation

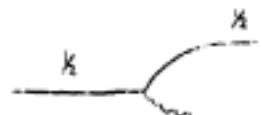
$$\sum_{\lambda_1, \lambda_2} C_0^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) C_0^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = 1.$$

From field theory vertex calculation

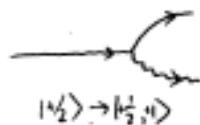
$$\begin{aligned}
 & \frac{\bar{v}(p_2^+, p_2^-, -\mathbf{k}_\perp)}{\sqrt{p_2^+}} \gamma_5 \frac{u(p_1^+, p_1^-, \mathbf{k}_\perp)}{\sqrt{p_1^+}}, \\
 \left\{ \begin{array}{lcl}
 \frac{\bar{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} & = & -\frac{2mP^+}{4mx(1-x)P^{+2}}, \\
 \frac{\bar{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} & = & +\frac{2mP^+}{4mx(1-x)P^{+2}}, \\
 \frac{\bar{v}_\uparrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\uparrow}{\sqrt{p_1^+}} & = & +\frac{2(k_1+ik_2)P^+}{4mx(1-x)P^{+2}}, \\
 \frac{\bar{v}_\downarrow}{\sqrt{p_2^+}} \gamma_5 \frac{u_\downarrow}{\sqrt{p_1^+}} & = & +\frac{2(k_1-ik_2)P^+}{4mx(1-x)P^{+2}},
 \end{array} \right.
 \end{aligned}$$

A QED Example of Relativistic Spin Effect

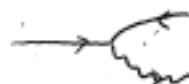
S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311



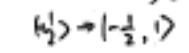
what are the helicities of each particle?



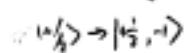
$$\psi_{\frac{1}{2},+}^{\uparrow}(x, k_0) = -\sqrt{2} \frac{-k_1 + k_2}{x(1-x)} \phi$$



$$\psi_{\frac{1}{2},-}^{\downarrow}(x, k_0) = -\sqrt{2} \left(M - \frac{m}{x} \right) \phi$$



$$\psi_{\frac{1}{2},+}^{\uparrow}(x, k_0) = -\sqrt{2} \frac{k_1 + k_2}{1-x} \phi$$



$$\psi_{\frac{1}{2},-}^{\downarrow}(x, k_0) = 0$$

The lowest spin states of a composite system must contain the orbital angular momentum contribution.

$$\Delta S_{\text{non-rel}} + L_{\text{non-rel}} = \Delta S_{\text{rel}} + L_{\text{rel}}$$

What is Δq measured in DIS

- Δq is defined by $\Delta q \ s_\mu = \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle$
$$\Delta q = \langle p, s | \bar{q} \gamma^+ \gamma_5 q | p, s \rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 dx \left[q^\uparrow(x) - q^\downarrow(x) \right]$$

- Using conventional Dirac spinors

$$\Delta q = \int d^3 \vec{p} M_q \left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p}) \right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^2}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin, or quark spin in IMF, not that in the rest frame of the proton

The proton spin crisis

& the Melosh-Wigner rotation

- It is shown that the proton “spin crisis” or “spin puzzle” can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame

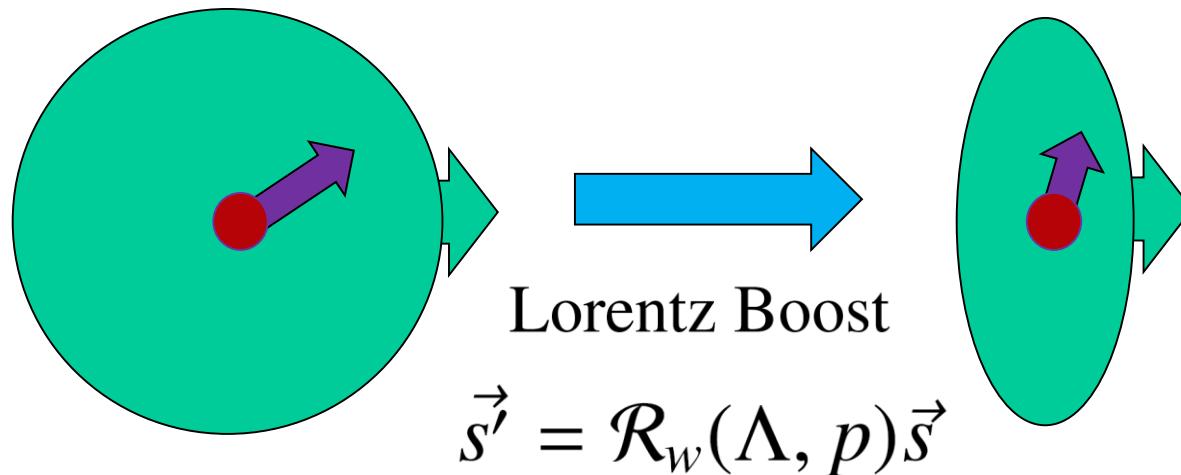
$$\sum_q \vec{s}_q = \vec{S}_p \text{ in the rest frame}$$

does not mean that

$$\sum_q \vec{s}_q = \vec{S}_p \text{ in the infinite momentum frame}$$

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

An intuitive picture to understand the spin puzzle



Rest Frame

$$\sum \vec{S} = \vec{S}_p$$

Infinite Momentum Frame

$$\sum \vec{s}' \neq \vec{S}_p$$

A general consensus

The quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

H.-Y.Cheng, hep-ph/0002157,
Chin.J.Phys.38:753,2000

Other approaches with same conclusion

**Contribution from the lower component
of Dirac spinors in the rest frame:**

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

D.Qing, X.-S.Chen, F.Wang, Phys.Rev.D58:114032,1998.

P.Zavada, Phys.Rev.D65:054040,2002.

Baryon properties from light-front holographic QCD

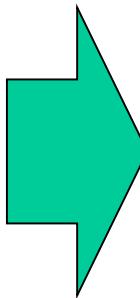
Tianbo Liu^{1,*} and Bo-Qiang Ma^{1,2,3,†}

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$$\psi_+(x, \mathbf{k}_\perp) \sim \frac{4\pi}{\sqrt{\lambda x(1-x)}} e^{-\frac{1}{2\lambda} \left(\frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x} \right)},$$



$$\psi_-(x, \mathbf{k}_\perp) \sim \frac{4\pi |\mathbf{k}_\perp|}{\lambda x(1-x)} e^{-\frac{1}{2\lambda} \left(\frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x} \right)}.$$

$$\psi_{L=0}(x, \mathbf{k}_\perp) = N \frac{4\pi [m_1 + \sqrt{\lambda x(1-x)}]}{\lambda x(1-x)}$$

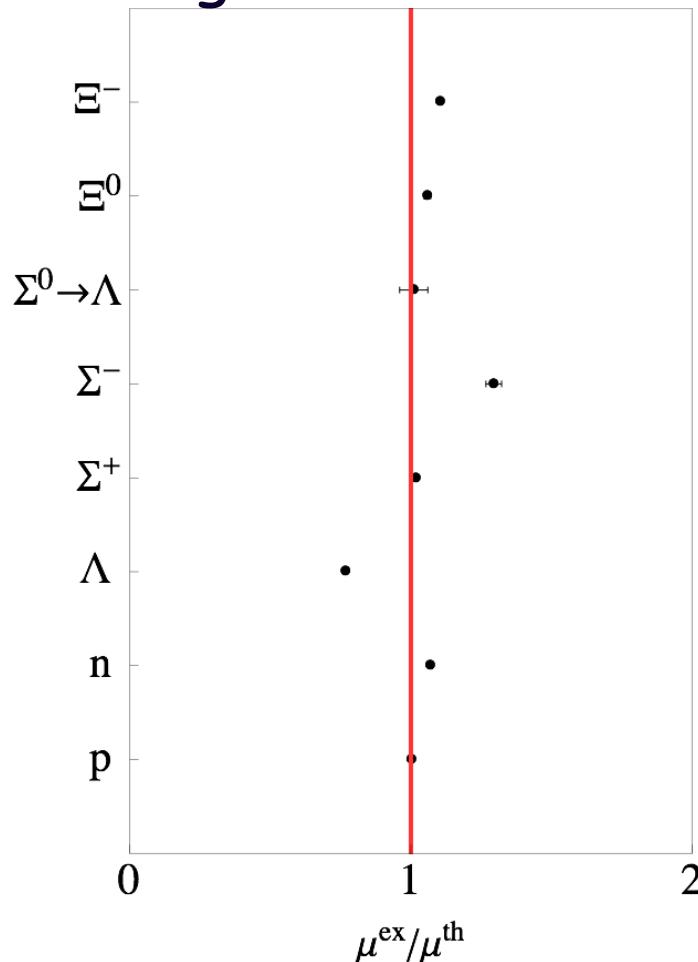
$$\times e^{-\frac{1}{2\lambda} \left(\frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x} \right)},$$

$$\psi_{L=1}(x, \mathbf{k}_\perp) = N \frac{-4\pi(k^1 + ik^2)}{\lambda x(1-x)}$$

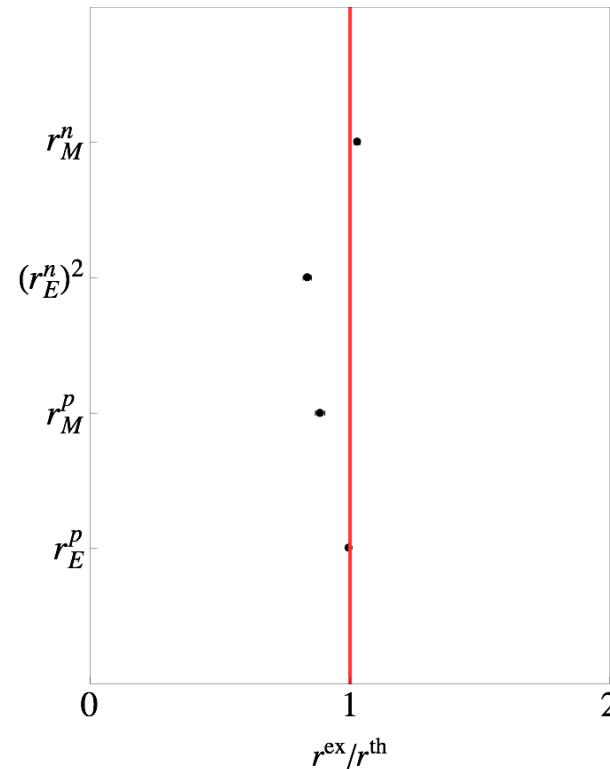
$$\times e^{-\frac{1}{2\lambda} \left(\frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x} \right)},$$

Magnetic moments, Radii

Magnetic moments



Radii



Axial charge: 0.308

$0.330 \pm 0.011(\text{theo.}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol.})$

The Spin of Nucleon from Holographic QCD

Axial charge: 0.308

$0.330 \pm 0.011(\text{theo.}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol.})$

The light-front holographic model with nonzero quark mass is essential to understand the spin structure with other low energy properties reproduced.

The Melosh-Wigner rotation is not the whole story

- **The role of sea is not addressed**
- **The role of gluon is not addressed**

Gluons are hidden in the spectators in our quark-diquark model

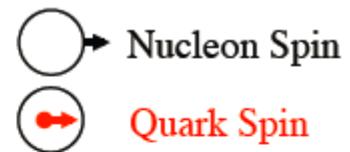
It is important to study the roles played by the sea quarks and gluons. Thus more theoretical and experimental researches can provide us a more completed picture of the nucleon spin structure.

Chances : New Research Directions

- New quantities: Transversity, Generalized Parton Distributions, Collins Functions, Sivers Functions, Boer-Mulders Functions, Pretzelosity, Wigner Distributions
- Hyperon Physics: The spin structure of Lambda and Sigma hyperons

B.-Q. Ma, I. Schmidt, J.-J. Yang, PLB 477 (2000) 107, PRD 61 (2000) 034017
B.-Q. Ma, J. Soffer, PRL 82 (1999) 2250

Leading-Twist TMD PDFs



		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1		h_1^\perp - Boer-Mulders
	L		g_1 - Helicity	h_{1L}^\perp - Long-Transversity
	T	 f_{1T}^\perp Sivers	 g_{1T} Trans-Helicity	 h_{1T}^\perp Transversity Pretzelosity

The Melosh-Wigner Rotation in Transversity

$$2\delta q = \langle p, \uparrow | \bar{q}_\lambda \gamma^\perp \gamma^+ q_{-\lambda} | p, \downarrow \rangle$$

$$\delta q(x) = \int [d^2 k_\perp] \tilde{M}_q(x, k_\perp) \Delta q_{\text{RF}}(x, k_\perp)$$

$$\tilde{M}_q(x, k_\perp) = \frac{(k^+ + m)^2}{(k^+ + m)^2 + k_\perp^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i \left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

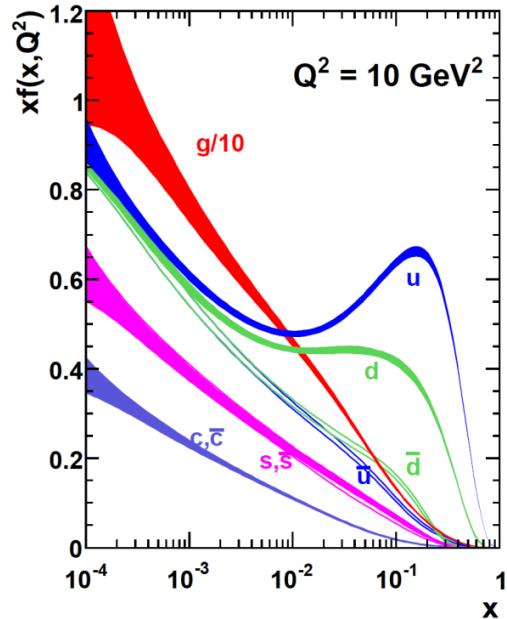
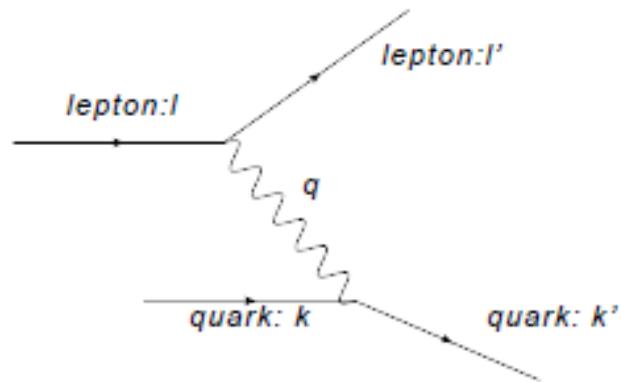
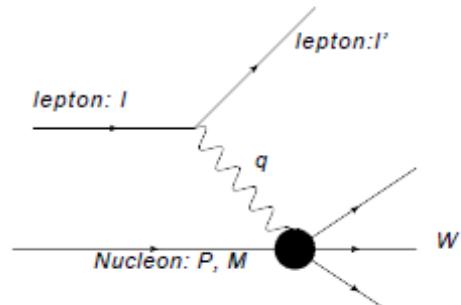
Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子 ($W_D(D = V, S)$)
g_{1L}	$[(x\mathcal{M}_D + m_q)^2 - p_\perp^2]/[(x\mathcal{M}_D + m_q)^2 + p_\perp^2]$
g_{1T}	$2M_N(x\mathcal{M}_D + m_q)/[(x\mathcal{M}_D + m_q)^2 + p_\perp^2]$
h_1	$(x\mathcal{M}_D + m_q)^2/[(x\mathcal{M}_D + m_q)^2 + p_\perp^2]$
h_{1L}^\perp	$-2M_N(x\mathcal{M}_D + m_q)/[(x\mathcal{M}_D + m_q)^2 + p_\perp^2]$
h_{1T}^\perp	$-2M_N^2/[(x\mathcal{M}_D + m_q)^2 + p_\perp^2]$

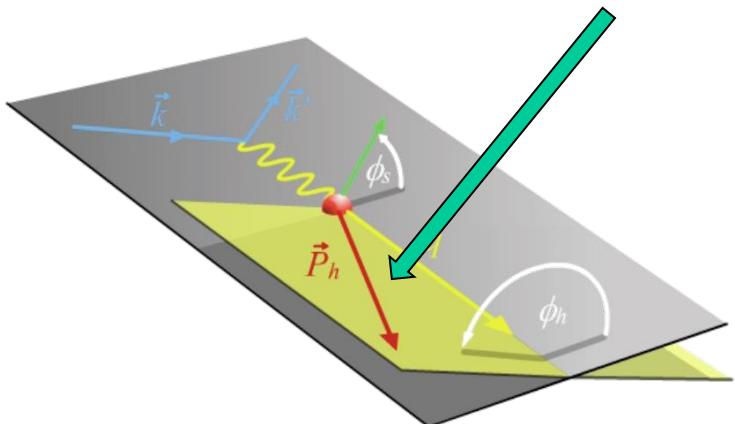
$\mathcal{M}_D^2 = \frac{m_q^2 + p_\perp^2}{x} + \frac{m_D^2 + p_\perp^2}{1-x}$ 是旁观双夸克的不变质量。

Lepton Scattering ----- A powerful tool



$x=1$ dimensional longitudinal momentum

tagging the struck quark through
leading hadrons (semi-inclusive
DIS)
to image in 3-momentum space



8 New TMD PDFs
 $f_1(x, kT), \dots h_1(x, kT)$

Names for New (tmd) PDF: g_{1T} and h_{1L}^\perp

g_{1T}

trans-helicity

横纵度

h_{1L}^\perp

longi-transversity / heli-transversity 纵横度

Physics Letters B 696 (2011) 246–251



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Proposal for measuring new transverse momentum dependent parton distributions g_{1T} and h_{1L}^\perp through semi-inclusive deep inelastic scattering

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^b Center for High Energy Physics, Peking University, Beijing 100871, China

Probing Pretzelosity in pion p Drell-Yan Process

COMPASS pion p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

Physics Letters B 696 (2011) 513–517



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Single spin asymmetry in πp Drell-Yan process

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Boer–Mulders function of the pion and pretzelosity distribution of the proton in the polarized pion-proton Drell–Yan process at COMPASS

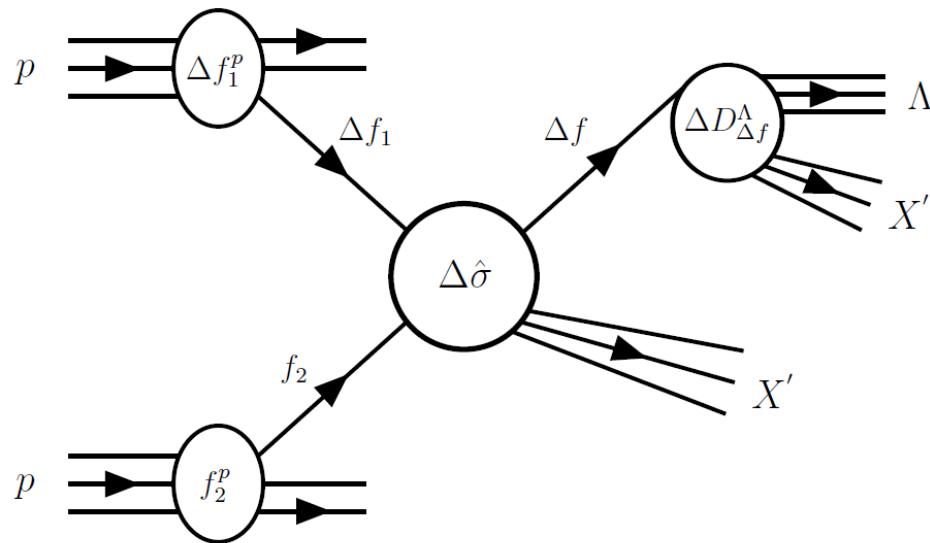
Xiaonan Liu¹, Bo-Qiang Ma^{1,2,3,a}

$$\pi^- p^\uparrow \rightarrow \mu^-\mu^+ X$$

The generic q_T -weighted TSA

$$A_T^{XW_X} = \frac{\int d^2\mathbf{q}_T W_X F_T^X}{\int d^2\mathbf{q}_T F_U^1}$$

- The COMPASS Collaboration at CERN adopts a π^- beam with $P_\pi = 190$ GeV colliding on a NH_3 target which provides a great opportunity to **explore the Boer-Mulders function of the pion.**



Providing information about

- the inclusive production of hadrons
- the strange and antistrange quark polarizations of the proton.

Results from fitting STAR data

Table: Fitting results of α_i and calculated results of Δs and $\Delta \bar{s}$.

	value	Δs	$\Delta \bar{s}$	χ^2_{min}
α_1	-1.20 ± 1.31	-0.014 ± 0.015		0.37
α_2	-0.24 ± 0.49		-0.003 ± 0.005	2.48
α_3	-2.17 ± 1.65	-0.025 ± 0.019		0.42
α_4	-0.087 ± 1.08		-0.001 ± 0.012	2.24

Two options: with/without gluon polarization

最新研究进展：探索核子的六维结构

Physics Letters B 830 (2022) 137127

Six-dimensional light-front Wigner distribution of hadrons

Yingda Han^a, Tianbo Liu^b, Bo-Qiang Ma^{a,c}

A B S T R A C T

We propose a six-dimensional light-front Wigner distribution for the complete description of partonic structures of a hadron such as pion and proton, taking advantage of the recently proposed light-front variable \tilde{z} by Miller and Brodsky. Quantities derived from the Wigner distribution contain the most general information of partonic structures, including also new quantities correlating longitudinal coordinate with transverse momenta or transverse coordinates, together with spins. The new Wigner distribution can be viewed as a relativistic version of the original Wigner distribution in hadron physics and an extension of widely utilized five-dimensional light-front Wigner distribution.

探索核子的六维结构

洛伦兹不变的纵向空间坐标

- ▶ 纵向空间坐标 x^- 不是洛伦兹不变的，且在 IMF 中 $x^- \rightarrow 0$ ，需要找到洛伦兹不变的纵向空间坐标。
- ▶ 纵向动量 k^+ 与纵向坐标 x^- 共轭 \Rightarrow 纵向动量分数 $x = \frac{k^+}{P^+}$ 与 $\tilde{z} = P^+ x^-$ 共轭 $\Rightarrow \tilde{z}$ 为洛伦兹不变的纵向空间坐标。⁵
- ▶ 对动量空间的波函数做傅里叶变换可以得到坐标空间的波函数

$$\psi_n(\tilde{z}, \mathbf{b}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx \psi_n(x, \mathbf{b}) e^{i\tilde{z}x}$$

$$\psi_n(\tilde{z}, \mathbf{k}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx \psi_n(x, \mathbf{k}) e^{i\tilde{z}x}$$

⁵G. A. Miller and S. J. Brodsky, Phys. Rev. C102, no.2, 022201(2020)

探索核子的六维结构

六维 Wigner 分布函数

通过洛伦兹不变的纵向空间坐标 \tilde{z} 定义六维 Wigner 分布函数

- ▶ 六维 Wigner 算符

$$\begin{aligned} \hat{W}^{[\Gamma]}(b^-, \vec{b}_\perp, k^+, \vec{k}_\perp) = & \int \frac{dy^- d^2 \vec{y}_\perp}{2(2\pi)^3} e^{ik^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \\ & \times \bar{\psi} \left(b - \frac{y}{2} \right) \Gamma \mathcal{L}_{[b-y/2, b+y/2]} \psi \left(b + \frac{y}{2} \right) \Big|_{y^+=0} \end{aligned}$$

- ▶ 六维 Wigner 分布函数

将算符置于初末态间便得到六维 Wigner 分布函数

$$\begin{aligned} \rho^{[\Gamma]}(\tilde{z}, x, \vec{b}_\perp, \vec{k}_\perp, S) = & \int \frac{d\xi d^2 \vec{\Delta}_\perp}{4\pi^3} \\ & \times \left\langle P + \frac{\Delta}{2}, S \middle| \hat{W}^{[\Gamma]}(b^-, \vec{b}_\perp, k^+, \vec{k}_\perp) \middle| P - \frac{\Delta}{2}, S \right\rangle \end{aligned}$$

其中 $b = (0, b^-, \vec{b}_\perp)$, $\tilde{z} = b^- P^+$, $x = \frac{k^+}{P^+}$, $\xi = \frac{\Delta^+}{2P^+}$, \mathcal{L} 为 Wilson line

Enable a complete description of the structure of a hadron

强子物理的研究

- 人类对核子结构的研究进入新的时期, 将揭示核子结构的三维乃至六维图像。
- EIC和EICC实验设施对探测核子的三维及六维结构有着独特的优势。
- 在国际大型实验室的大型加速器上开展实验研究：
JLab@美国, RHIC@美国, COMPASS@cern, FAIR@德国,
JPARC@Japan, AFTER@CERN, EIC@美国, EICC@中国
- 实验和理论的结合可以寻找探测核子新物理量的最佳物理条件和过程, 从而实现对新物理量的具体测量。
- 新物理量的测量必将为加深人类对物质结构的认识提供新的机遇和挑战。