Aspects of global spin polarization in heavy ion collisions

Qun Wang
Department of Modern Physics
Univ of Science & Technology of China (USTC)

热烈祝贺清华大学物理系复系40周年！
祝愿清华大学核物理学科在创辉煌！

Symposium on Nuclear Physics Frontiers and Intersections
August 25-28, 2022

Outline

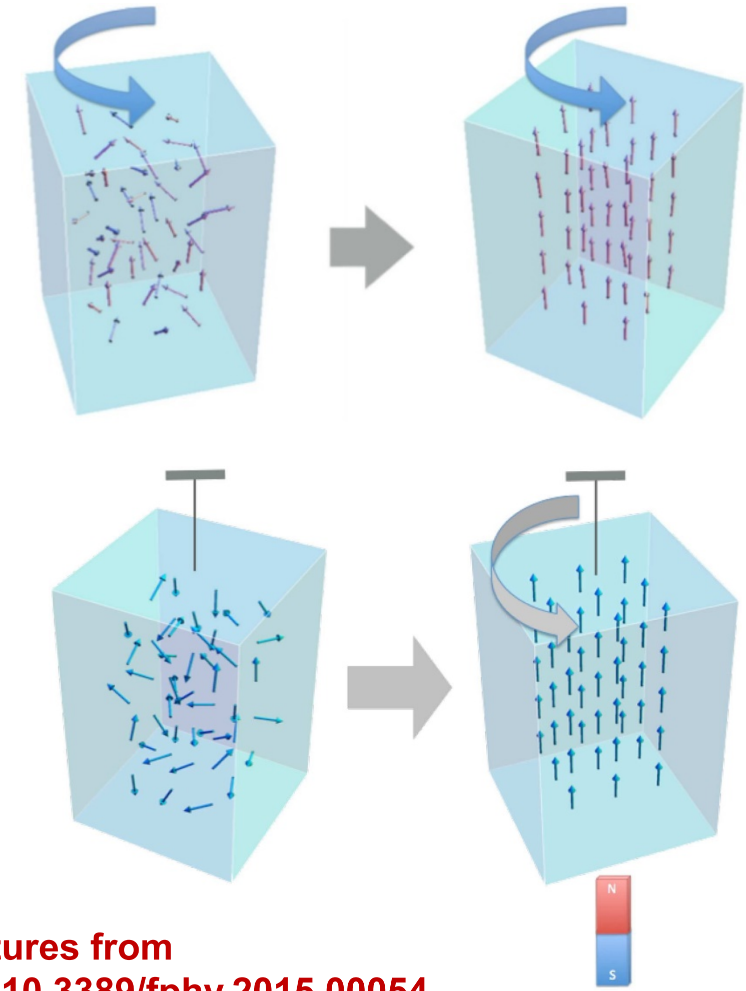
- Introduction
- Global polarization of hyperons in HIC
- Global spin alignment of vector mesons in HIC
- Relativistic Spin Boltzmann Equation from Closed-Time-Path formalism (CTP) or Kadanoff-Baym equation (KBE) for spin alignments of vector mesons
- Questions for discussions

Barnet effects and Einstein-de Hass effects

Barnett effect:

Barnett, Magnetization by rotation, Phys Rev. 6, 239-270 (1915).

Spin-orbit (LS) coupling!

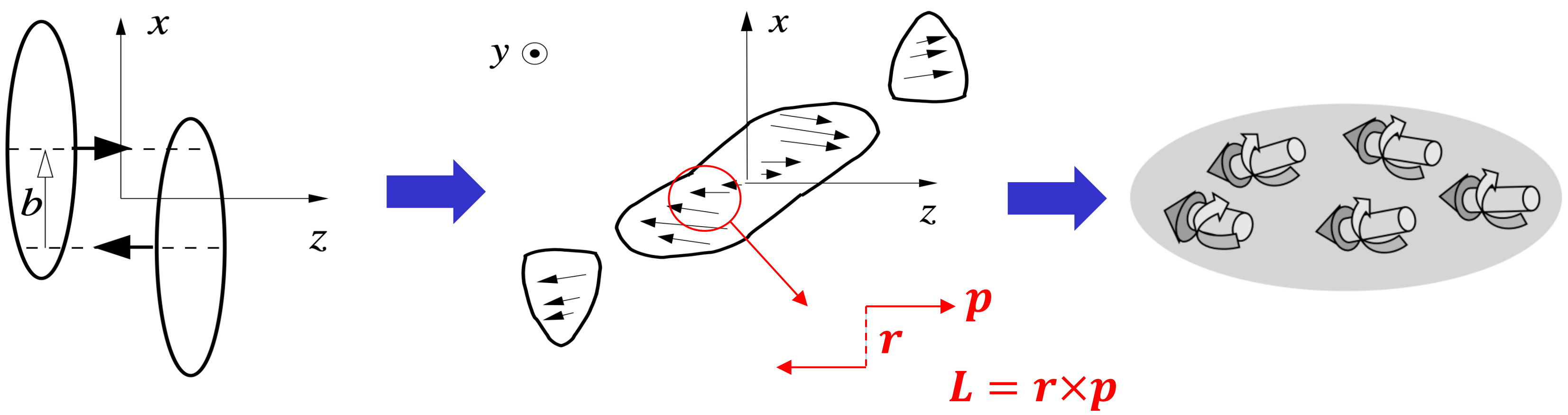


Einstein-de Haas effect:

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents, Verhandl. Deut. Phys. Ges. 17, 152–170 (1915).

Pictures from doi:10.3389/fphy.2015.00054

Global OAM and polarization in HIC

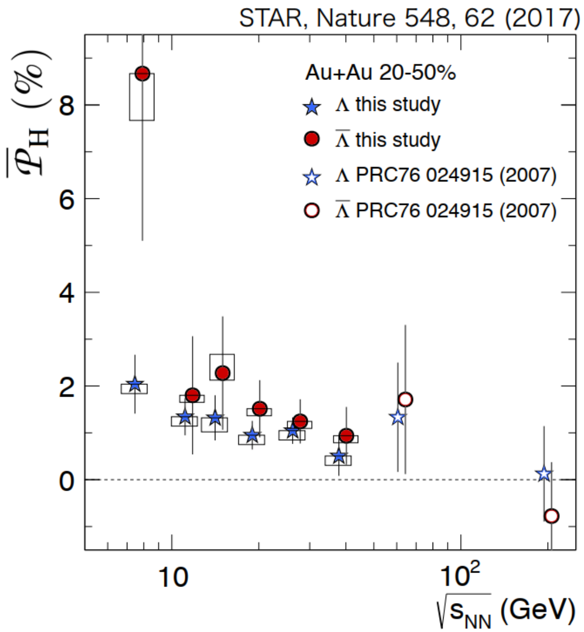


Global OAM leads to global polarization of Λ hyperons and spin alignment of vector mesons through spin-orbit coupling

Liang and Wang, PRL 94,102301(2005); PLB 629, 20(2005)
Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global polarization of hyperons in HIC

STAR results: Hyperon Polarization

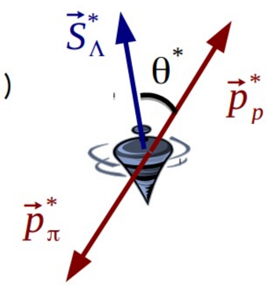


parity-violating decay of hyperons

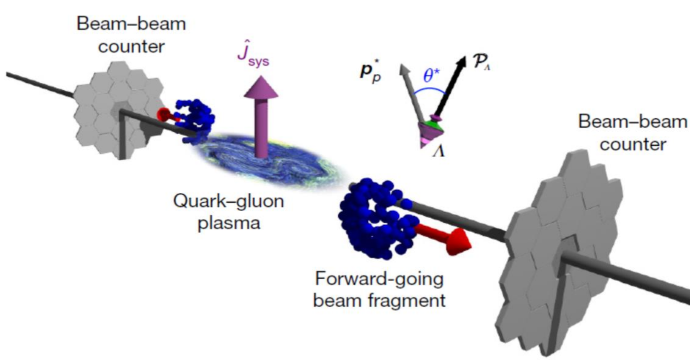
In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



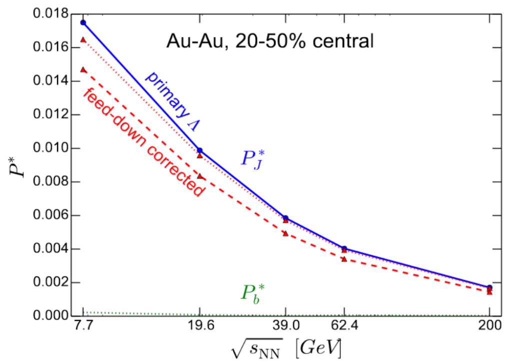
$\Lambda \to p + \pi^+$
(BR: 63.9%, cτ~7.9 cm)



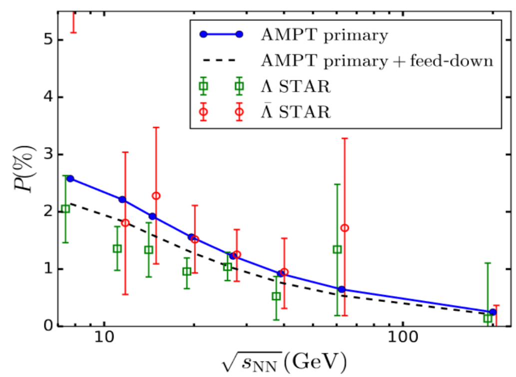
- **The lower energy, the stronger polarization effects.**
- **ω = (9 ± 1)x10^{21}/s, greater than previously observed in any system.**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)

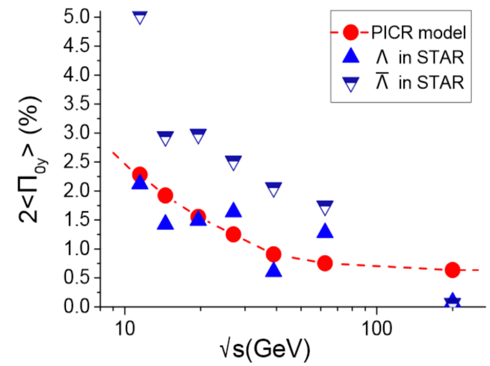
Global polarization in HIC: model calculation



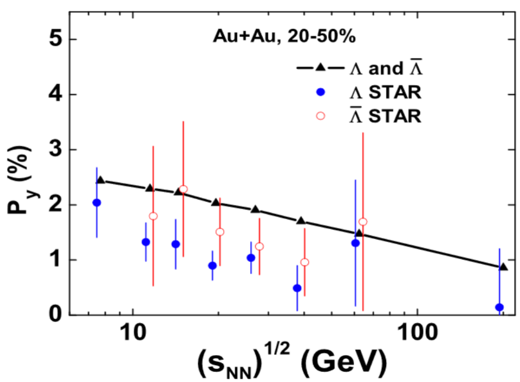
Karpenko, Becattini, EPJC(2017)



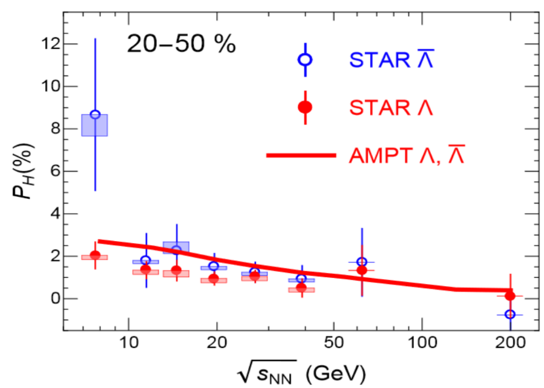
Li, Pang, Wang, Xia PRC(2017)



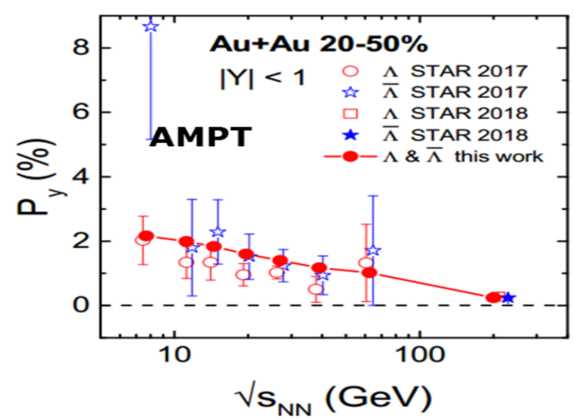
Xie, Wang, Csernai, PRC(2017)



Sun, Ko, PRC(2017)

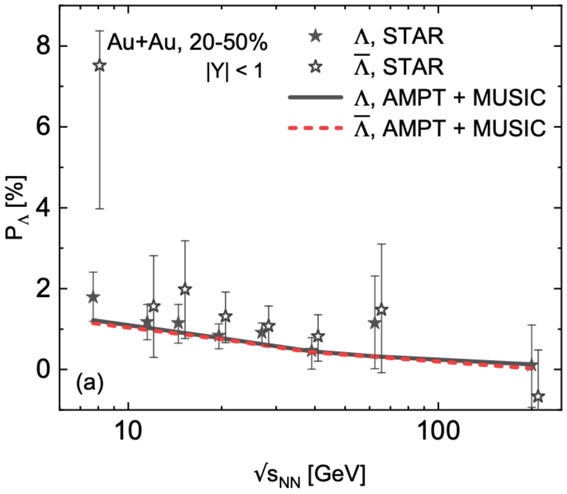


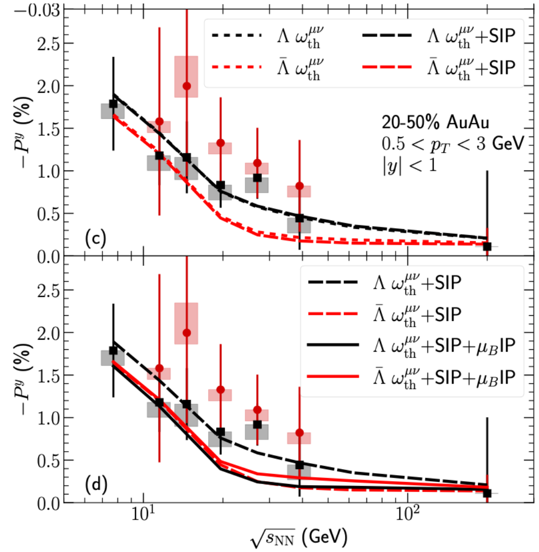
Shi, Li, Liao, PLB(2018)

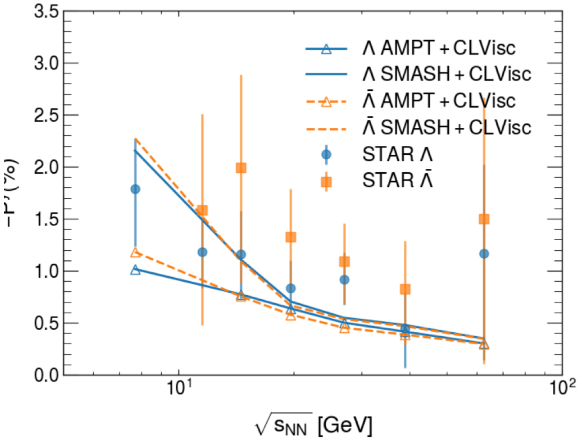


Wei, Deng, Huang, PRC(2019)

Global polarization in HIC: model calculation







B.C. Fu, K. Xu, X.G. Huang, H.C. Song, Phys. Rev. C 103, 024903 (2021)

S. Ryu, V. Jupic, C. Shen, arXiv:2106.08125

Y.X. Wu, C. Yi, G.Y. Qin, S.Pu, arXiv:2204.02218

Local spin polarization from QKT

- **The polarization tensor is connected to the axial current in phase space by modified Cooper-Frye formula [Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)]**

$$\mathcal{S}^{\mu}(\mathbf{p})=\frac{\int d\Sigma\cdot p\mathcal{J}_{5}^{\mu}(p,X)}{2m_{\Lambda}\int d\Sigma\cdot\mathcal{N}(p,X)},$$

- **Inserting all possible contributions to the above equation yields [Hidaka, Pu, Yang, PRD (2018); Yi, Pu, Yang, PRC(2021); Yi, Pu, Gao, Yang, PRC (2022)]**

$$\mathcal{S}_{\mathrm{thermal}}^{\mu}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_{V}^{(0)}(1-f_{V}^{(0)})\boxed{\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T}}$$

Thermal vorticity

$$\mathcal{S}_{\mathrm{shear}}^{\mu}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}\boxed{(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}}\}$$

Shear viscous tensor

$$\mathcal{S}_{\mathrm{accT}}^{\mu}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_{\nu}u_{\alpha}\boxed{(Du_{\beta}-\frac{1}{T}\partial_{\beta}T)},$$

Fluid acceleration

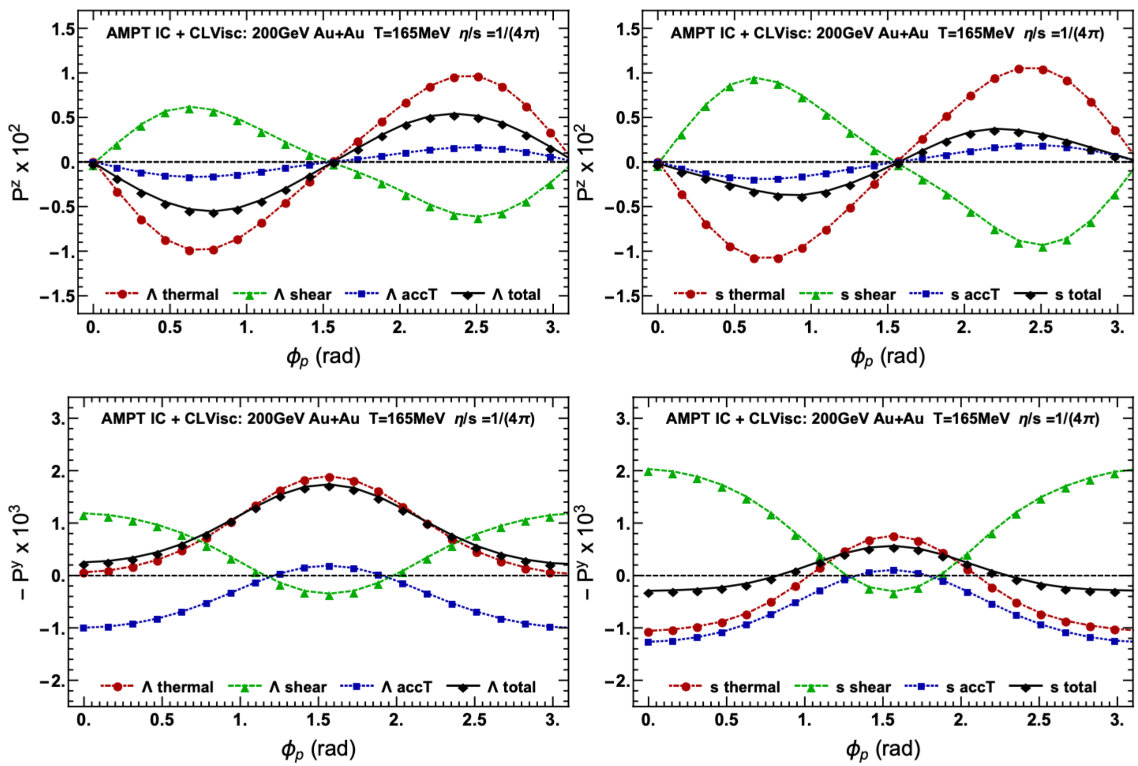
$$\mathcal{S}_{\mathrm{chemical}}^{\mu}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_{V}^{(0)}(1-f_{V}^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$

Gradient of chemical potential

- **Also see from other approaches: Fu, Liu, Pang, Song, Yin (JHEP2021, PRL 2021); Becattini (PRD 2021, PRL 2021);**

Local spin polarization induced by shear tensor

Polarization along beam direction



Polarization along out-of-plane direction

It was shown that

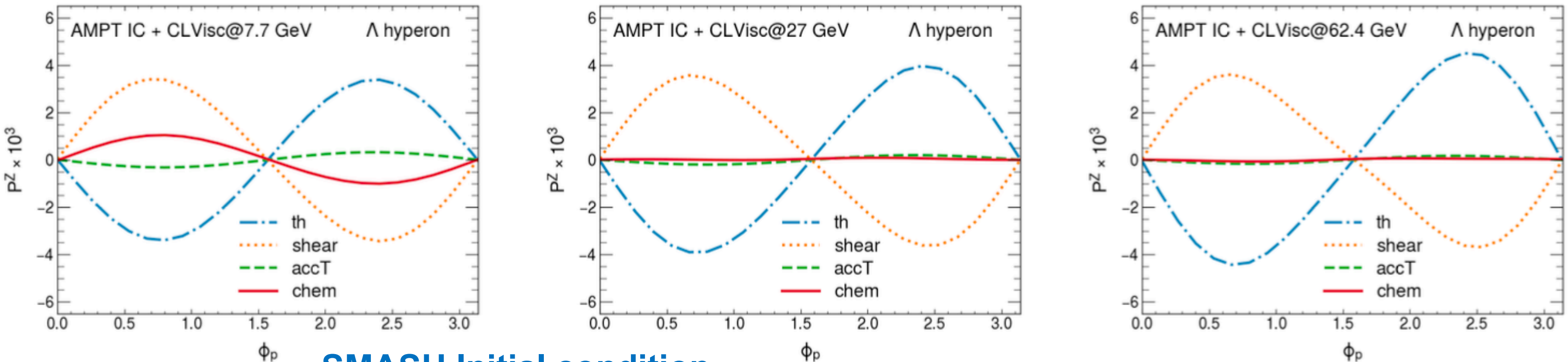
- **Shear induced polarization always give a "correct" sign.**
- **Total local polarization is sensitive to mass of s quark, EoS, freeze out temperature and eta / s.**

The local spin polarization is still an open question. We still need to consider the out-of-equilibrium effects carefully through the spin hydrodynamics and the quantum kinetic theory with collisions.

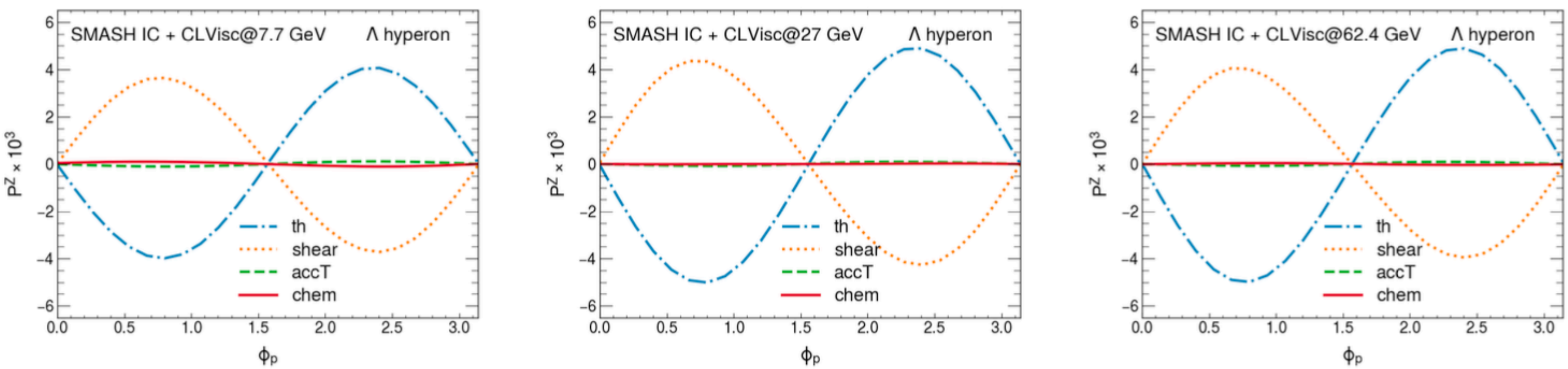
Yi, Pu, Yang, PRC (2021)

Local spin polarization induced by chemical potential

AMPT Initial condition



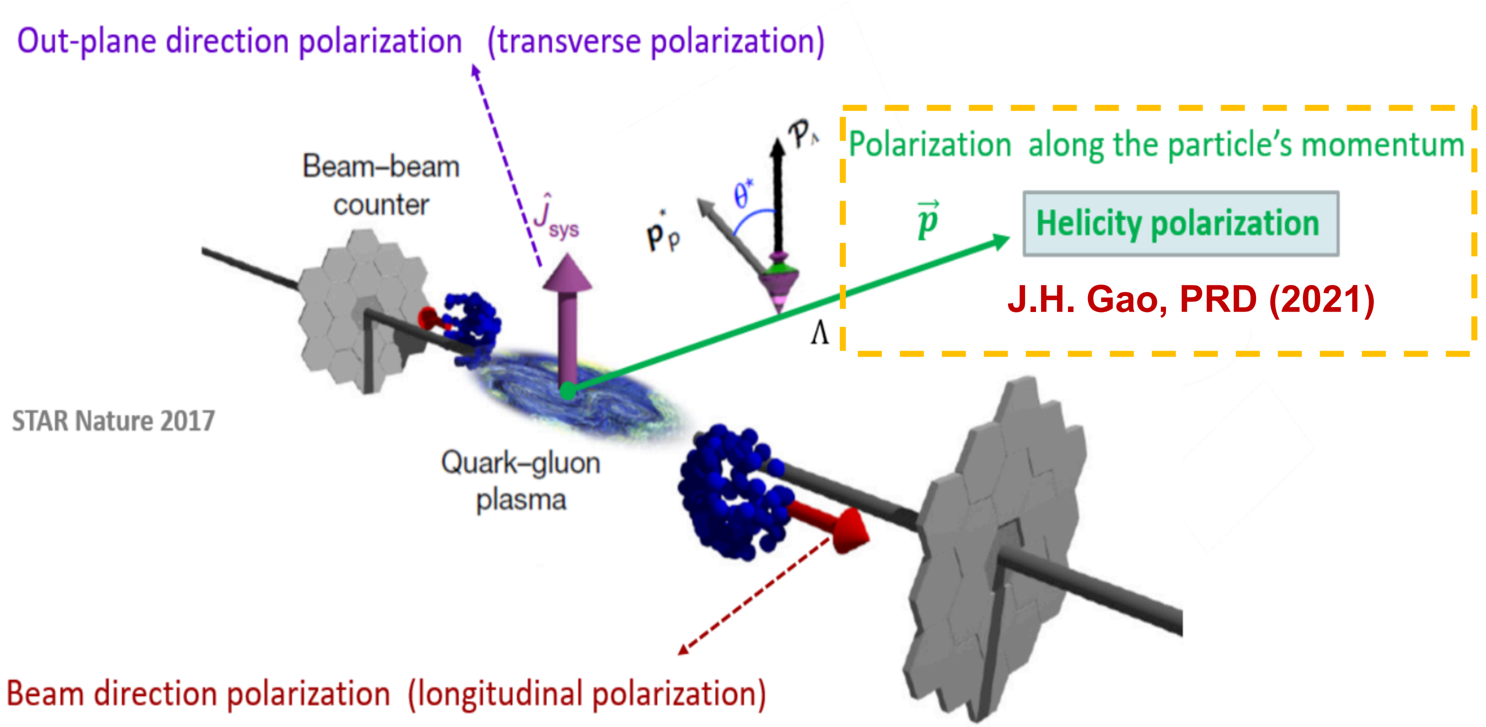
SMASH Initial condition



The polarization at RHIC beam energy scan energies (7.7, 27, 62.4 GeV) via the (3+1)-dimensional CLVisc hydrodynamics model with AMPT and SMASH initial conditions. The results depend on initial condition and baryon diffusion. Wu, Yi, Qin, Pu, PRC (2022)

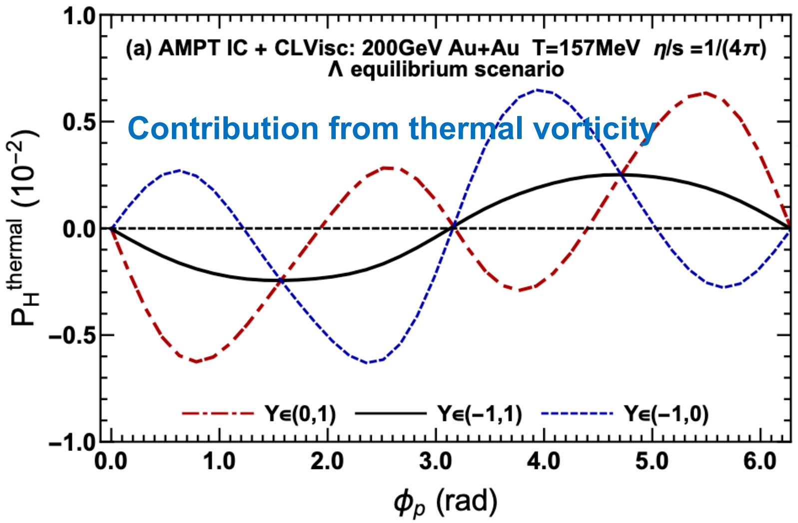
Helicity polarization

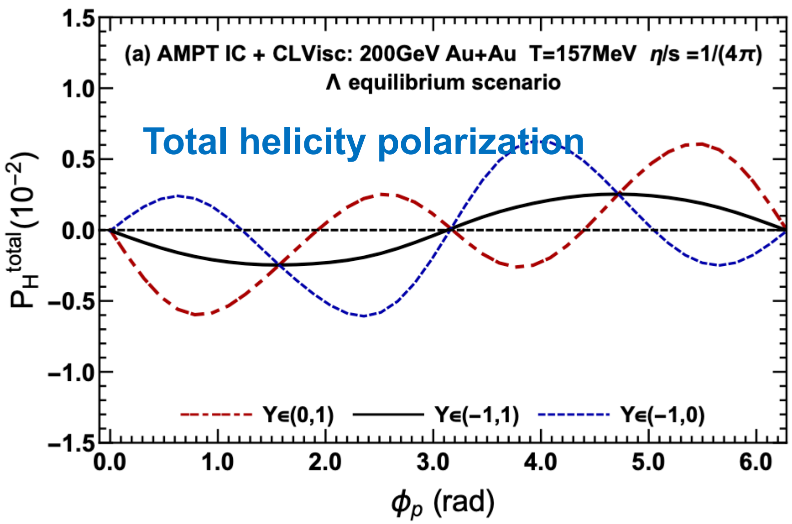
$$S^h = \widehat{\mathbf{p}} \cdot \boldsymbol{S}(\mathbf{p}) = \widehat{p}^x \mathcal{S}^x + \widehat{p}^y \mathcal{S}^y + \widehat{p}^z \mathcal{S}^z,$$



Helicity polarization

- **Helicity polarization polarization can also induced by thermal, shear and fluid acceleration. Yi, Pu, Gao, Yang, PRC (2022)**



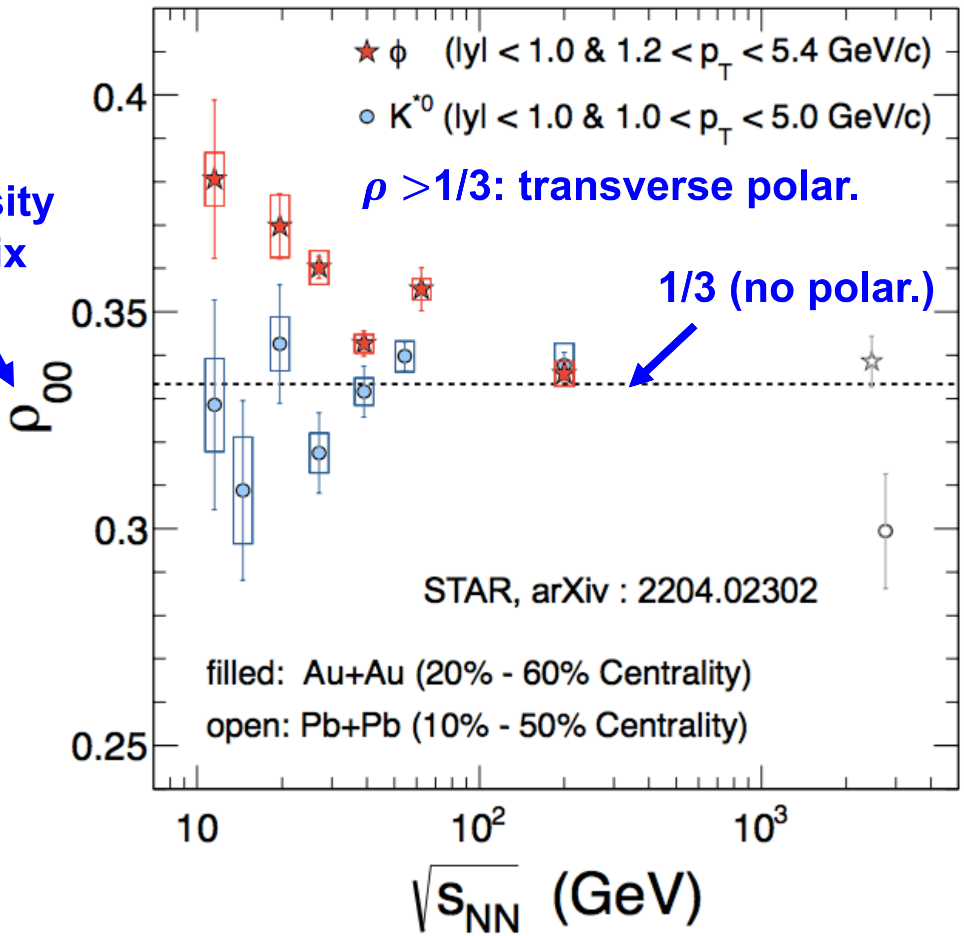


- **Our numerical simulation shows that the thermal vorticity dominates over other contributions in helicity polarization. The helicity polarization can be used to detect the vortical structure in the fireball.**

Global spin alignment of vector mesons in HIC

STAR results: global spin alignments of vector mesons

Spin Density Matrix



STAR Collab., 2204.02302

"the global spin alignment for phi unexpectedly large, while that for K^{*0} is consistent with zero. The observed spin-alignment pattern and magnitude for the phi cannot be explained by conventional mechanisms, while a model with strong force fields [2,3] accommodates the current data. "

[2] Sheng, Oliva, QW (2020, Erratum 2022)
[3] Sheng, QW, Wang (2020)

Possible contributions to ρ_{00}^{ϕ}

$$\rho_{00}^{\phi} = \frac{1}{3} + c_{\varepsilon} + c_{\omega} + c_E + c_B + c_F + c_A + c_L + c_{\phi}$$

E-part of vorticity tensor [1,2]

B-part of vorticity tensor [1,2]

Electric field [1]

Magnetic field [1,3]

Frag. [4]

Turbulent color field [5]

Local+ Helicity [6,7]

ϕ field [1]

cannot explain large positive deviation from 1/3

[1] Sheng, Luica, QW (2019);
[2] Becattini, Csernai, Wang (2013);
[3] Yang, Fang, QW, Wang (2018);
[4] Liang, Wang (2005);
[5] Muller, Yang (2022);
[6] Xia, Li, Huang, Huang (2021);
[7] Gao (2021);

Polarization of strange quarks by ϕ vector fields (non-relativistic model)

- Like electric charges in motion can generate an EM field, s and $\bar{s}$ quarks in motion can generate an effective ϕ vector field [Chiral quark model, Manohar & Giorgi, 1984].
- The ϕ vector field can polarize s and $\bar{s}$ with a large magnitude due to strong interaction, in analogy to how EM field polarize (anti)quarks.

$$
\begin{aligned}
\vec{\mathcal{P}}_{s/\bar{s}} \quad = \quad & \frac{1}{2}\omega + \frac{1}{2m_s}\varepsilon \times \mathbf{p}_{s/\bar{s}} \\
& \pm \frac{Q_s}{2m_s T}\mathbf{B} \pm \frac{Q_s}{2m_s^2 T}\mathbf{E} \times \mathbf{p}_{s/\bar{s}} \\
& \pm \frac{g_\phi}{2m_s T}\mathbf{B}_\phi \pm \frac{g_\phi}{2m_s^2 T}\mathbf{E}_\phi \times \mathbf{p}_{s/\bar{s}}
\end{aligned}
$$

Electric part corresponds to spin-orbit couplings (spin-Hall effects) not accessible via Λ polarization:

$$
\mathbf{E} \times \mathbf{p} \sim -\frac{1}{r}\frac{d\Phi}{dr}(\mathbf{r} \times \mathbf{p})
$$

Sheng, Oliva, QW (2020)

ρ_{00}^{ϕ} from ϕ fields in non-relativistic coalescence model

- The spin density matrix has off-diagonal elements in spin. Assuming the spin quantization direction is y-direction (OAM), we have

$$\rho_{00}^{\phi}(t,\mathbf{x}) \approx \frac{1}{3} - \frac{4}{9}\int \frac{d^3\mathbf{p}}{(2\pi)^3} \underline{\left|\psi_\phi(\mathbf{p})\right|^2}$$

ϕ meson's non-relativistic wave function

$$\times \left\{ P_s^y(\underline{\mathbf{p}}) P_{\bar{s}}^y(\underline{-\mathbf{p}}) - \frac{1}{2}\left[P_s^z(\mathbf{p})P_{\bar{s}}^z(-\mathbf{p}) + P_s^x(\mathbf{p})P_{\bar{s}}^x(-\mathbf{p})\right] \right\}$$

ϕ meson is static

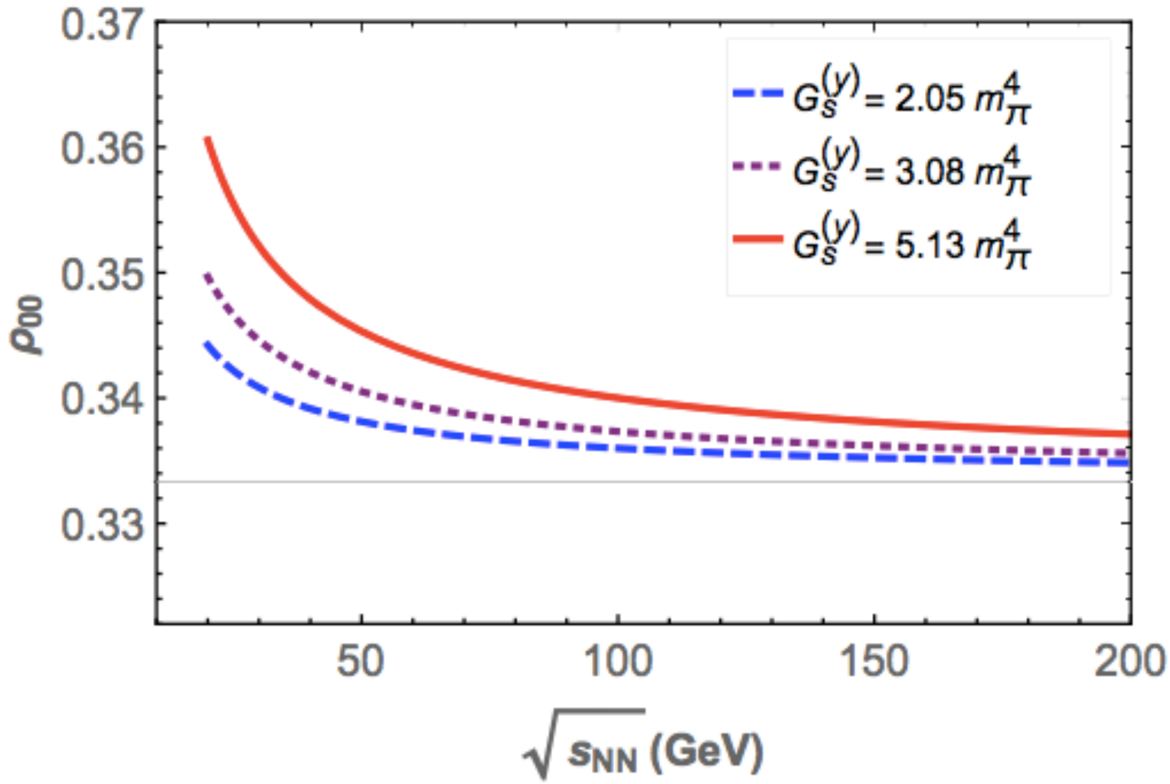
$$\left.\begin{aligned} &\approx \frac{1}{3} + \frac{g_\phi^2}{9m_s^2T_{\text{eff}}^2}\left[\left\langle B_{\phi,y}^2\right\rangle - \frac{1}{2}\left\langle B_{\phi,x}^2 + B_{\phi,z}^2\right\rangle\right] \\ &+ \frac{g_\phi^2\left\langle \mathbf{p}^2\right\rangle_\phi}{27m_s^4T_{\text{eff}}^2}\left[\left\langle E_{\phi,y}^2\right\rangle - \frac{1}{2}\left\langle E_{\phi,x}^2 + E_{\phi,z}^2\right\rangle\right] \end{aligned}\right\} \equiv \frac{1}{27m_s^2T_{\text{eff}}^2}G_s^{(y)}$$

average p^2 for s or $\bar{s}$ in ϕ-meson's WF

- Coalescence is the main mechanism of hadronization in HIC

Greco,Ko,Levai (2003); Fries, Muller, Nonaka, Bass (2003); Hua, Yang (2003)

Prediction for ρ_{00} from ϕ field (non-relativistic coalescence model)



Sheng, Oliva, QW (2020)

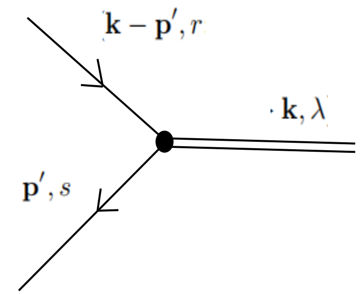
Shortcomings of non-relativistic coalescence model for ρ_{00}^{ϕ}

- **Spins are decoupled from momenta in spin density matrix: too simple to account for spin dynamics. The sign of anti-quark's momentum is not easy to determine (easy to make a mistake)**
- **No Lorentz covariance, only valid for quasi-static ϕ mesons, cannot be applied to ϕ mesons with non-vanishing momenta with confidence**
- **It is not a model based on relativistic quantum field theory**
- **The deeper implication of ϕ field cannot be explored**

- **To solve above problems, it is necessary to develop a relativistic spin transport theory for ϕ mesons, which can describe the relativistic fusion process $s\bar{s} \to \phi$ with spin dof**

Relativistic spin Boltzmann equation for fusion process

- **A phenomenological approach to Relativistic Spin Boltzmann Equation (RSBE) for fusion process**



$$k\cdot\partial_x f_\lambda^V(x,\mathbf{k})$$

$$\Rightarrow \sum_{r,s=\pm 1/2}\int \frac{d^3\mathbf{p}'}{E_{\mathbf{p}'}^{\bar{q}} E_{\mathbf{k}-\mathbf{p}'}^{q}}\delta\left(E_{\mathbf{k}}^V - E_{\mathbf{k}-\mathbf{p}'}^q - E_{\mathbf{p}'}^{\bar{q}}\right)$$
$$\times\left|M(\mathbf{k}-\mathbf{p}',r;\mathbf{p}',s\to\mathbf{k},\lambda)\right|^2$$
$$\times\left\{\underbrace{f_r^q(\mathbf{k}-\mathbf{p}')f_s^{\bar{q}}(\mathbf{p}')\left[1+f_\lambda^V(\mathbf{k})\right]}_{\text{Gain term}} - \underbrace{f_\lambda^V(\mathbf{k})\left[1-f_r^q(\mathbf{k}-\mathbf{p}')\right]\left[1-f_s^{\bar{q}}(\mathbf{p}')\right]}_{\text{Loss term}}\right\}$$

- **It is more rigorous to derive RSBE from CTP (SK) or KBE in terms of Matrix Valued Spin Dependent Distributions (MVSD) for quarks and vector mesons**

$$f_r^q \to f_{r_1 r_2}^q \qquad f_\lambda^V \to f_{\lambda_1\lambda_2}^V$$
$$f_s^{\bar{q}} \to f_{s_1 s_2}^{\bar{q}}$$

MVSD: Sheng, Weickgenannt, Speranza, Rischke, QW (2021); Sheng, QW, Rischke (2022)

Spin density matrix: diagonal ⇒ diagonal + off-diagonal elements

Matrix Valued Spin Dependent Distributions (MVSD)

- **MVSD in phase space**

$$p^\mu \equiv \frac{1}{2}(p_1^\mu + p_2^\mu) \qquad q^\mu \equiv p_1^\mu - p_2^\mu$$

$$f_{rs}(x,p) \equiv \int \frac{d^4q}{2(2\pi)^3} \exp\left(-\frac{i}{\hbar} q \cdot x\right) \delta(\underline{p \cdot q}) \langle a^\dagger(\underline{s, \mathbf{p}_2}) a(\underline{r, \mathbf{p}_1}) \rangle$$

- **MVSD can be parameterized in terms un-polarized distributions and polarization distributions**

$$f_{rs}^{(+)}(x,\mathbf{p}) = \frac{1}{2}\underline{f_q(x,\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^q(x,\mathbf{p})}\,\underline{n_j^{(+)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

$$f_{rs}^{(-)}(x,-\mathbf{p}) = \frac{1}{2}\underline{f_{\overline{q}}(x,-\mathbf{p})}\left[\delta_{rs} - \underline{P_\mu^{\overline{q}}(x,-\mathbf{p})}\,\underline{n_j^{(-)\mu}(\mathbf{p})}\tau_{rs}^j\right],$$

Pauli matrices in spin space (rs-space)

un-polarized dist. **polarization dist.**

Four-vectors of three basis directions in rest frame of q and $\overline{q}$ (one is the spin uantization direction)

MVSD:
Sheng, Weickgenannt, et al. (2021); Sheng, QW, Rischke (2022)

RSBE in MVSD from CTP or KBE

- **A general RSBE based on relativistic quantum field theory (CTP or KBE) for fusion process**

$$k\cdot\partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{16}\sum_{\lambda_1',\lambda_2'}\left[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_1',\mathbf{k})\delta_{\lambda_2\lambda_2'} + \delta_{\lambda_1\lambda_1'}\epsilon^*_\mu(\lambda_2',\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\right]\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k}),$$

polarization vector for vector meson

collision kernel

collision kernel $\mathcal{C}^{\mu\nu}_{\lambda_1'\lambda_2'}(x,\mathbf{k})$

$$= \sum_{r_1,s_1,r_2,s_2}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar q}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}} - E^{\bar q}_{\mathbf{p}'} - E^q_{\mathbf{p}-\mathbf{p}'}\right)$$

gain term $\times\left\{f^{\bar q}_{r_1s_1}(\mathbf{p}')f^q_{r_2s_2}(\mathbf{k}-\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'} + f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right]\right.$

loss term $\left.-\left[\delta_{r_1s_1} - f^{\bar q}_{r_1s_1}(\mathbf{p}')\right]\left[\delta_{r_2s_2} - f^q_{r_2s_2}(\mathbf{k}-\mathbf{p}')\right]f^V_{\lambda_1'\lambda_2'}(\mathbf{k})\right\}$

$$\times\,\mathrm{Tr}\left[\Gamma^\nu v_{s_1}(\mathbf{p}')\bar v_{r_1}(\mathbf{p}')\Gamma^\mu u_{r_2}(\mathbf{k}-\mathbf{p}')\bar u_{s_2}(\mathbf{k}-\mathbf{p}')\right], \qquad (2)$$

$$\boxed{\Gamma^\alpha \approx g_V B(\mathbf{p}-\mathbf{p}',\mathbf{p}')\gamma^\alpha}$$

Bethe-Salpeter wave function for vector meson [Roberts et al (2019, 2021)]

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Fusion and dissociation process

- **In the dilute gas limit**

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

$$f^V_{\lambda_1\lambda_2} \sim f^q_{rs} \sim f^{\overline{q}}_{rs} \ll 1.$$

- **RSBE for fusion (coalescence) and dissociation process $q\overline{q} \leftrightarrow V$ can be simplified as**

$$k \cdot \partial_x f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) = \frac{1}{8}\Big[\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\underbrace{\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})}_{\text{Coalescence collision kernel}}$$
$$-\underbrace{\mathcal{C}_{\text{diss}}(\mathbf{k})}_{\text{Dissociation collision kernel}} f^V_{\lambda_1\lambda_2}(x,\mathbf{k})\Big],$$

$$\epsilon^\mu(\lambda,\mathbf{k}) = \left(\frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V}, \epsilon_\lambda + \frac{\mathbf{k}\cdot\epsilon_\lambda}{m_V(E^V_{\mathbf{k}}+m_V)}\mathbf{k}\right) \Rightarrow k_\mu\epsilon^\mu(\lambda,\mathbf{k}) = 0$$

n_x, n_y, n_z are three basis directions in rest frame of vector meson

$$\begin{aligned}
\epsilon_0 &= \mathbf{n}_y \\
\epsilon_{+1} &= -\frac{1}{\sqrt{2}}(\mathbf{n}_z + i\mathbf{n}_x) \\
\epsilon_{-1} &= \frac{1}{\sqrt{2}}(\mathbf{n}_z - i\mathbf{n}_x)
\end{aligned}$$

- **The fusion part depends on MVSDs of q and $\overline{q}$, while the dissociation part does not.**

MVSD or spin density matrix element for vector mesons

- Forml solution to MVSD (spin density matrix) for vector mesons

$$f^V_{\lambda_1\lambda_2}(x,\mathbf{k}) \sim \frac{1}{\mathcal{C}_{\text{diss}}(\mathbf{k})}\left[1-e^{-\mathcal{C}_{\text{diss}}(\mathbf{k})\Delta t}\right] \times \epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k})$$

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

- where the coalescence collision kernel $C^{\mu\nu}_{coal}$ is given by

$$\begin{aligned}\mathcal{C}^{\mu\nu}_{\text{coal}}(x,\mathbf{k}) = \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^2}\frac{1}{E^{\bar{q}}_{\mathbf{p}'}E^q_{\mathbf{k}-\mathbf{p}'}}\delta\left(E^V_{\mathbf{k}}-E^{\bar{q}}_{\mathbf{p}'}-E^q_{\mathbf{k}-\mathbf{p}'}\right)\\ \times \text{Tr}\left\{\underline{\Gamma^\nu}\left(p'\cdot\gamma - m_{\bar{q}}\right)\left[1+\gamma_5\gamma\cdot \underline{P^{\bar{q}}(x,\mathbf{p}')}\right]\right.\\ \left.\times \underline{\Gamma^\mu}\left[(k-p')\cdot\gamma + m_q\right]\left[1+\gamma_5\gamma\cdot \underline{P^q(x,\mathbf{k}-\mathbf{p}')}\right]\right\}\\ \times \underline{f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{k}-\mathbf{p}')},\end{aligned}$$

BS wave function for vector meson

polarization dist. in phase space for q and $\bar{q}$

Un-polarized quark distribution functions

Spin density matrix element for vector mesons

- **Spin density matrix (normalized MVSD) for vector mesons**

$$f^V_{\lambda_1\lambda_2} \propto \rho^V_{\lambda_1\lambda_2} = \frac{\epsilon^*_\mu(\lambda_1,\mathbf{k})\epsilon_\nu(\lambda_2,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}}{\sum_{\lambda=0,\pm1}\epsilon^*_\mu(\lambda,\mathbf{k})\epsilon_\nu(\lambda,\mathbf{k})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}}$$

- **Focus on ϕ meson, the polarization vector for s and $\bar{s}$ appear in the collision kernel**

$$P^\mu_s(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_{\mathrm{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

$$P^\mu_{\bar{s}}(x,\mathbf{p}) \approx \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_{\mathrm{eff}}}F^\phi_{\rho\sigma}\right)p_\nu$$

Field strength tensor of ϕ field

Sheng, Lucia, Liang, QW, Wang, 2205.15689, 2206.05868

Spin density matrix element for vector mesons

- **The fusion collision kernel $C_{coal}^{\mu\nu}$ can be evaluated in the rest frame of ϕ meson, which gives ρ_{00}^{ϕ}**

$$\rho_{00}(x,\mathbf{0}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2\right] + C_2\left[\frac{1}{3}\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_{\mathrm{eff}}^2}C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right],$$

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2(m_\phi^2 + 2m_s^2)},$$

$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120 m_s^2(m_\phi^2 + 2m_s^2)}.$$

All fields with prime are defined in the rest frame of ϕ meson

spin quantization direction

- **Features: (1) Perfect factorization of x and p dependence; (2) Perfect cancellation for mixing terms (protected by symmetry): all ϕ fields appear in squares, i.e. ρ_{00}^{ϕ} measures fluctuations of fields. Suprising results!**

Lorentz transformation for ϕ fields

- **We can express ρ_{00}^{ϕ} in terms of ϕ fields in the lab frame and obtain the dependence on momenta of ϕ mesons**

$$\mathbf{B}'_{\phi} = \gamma \mathbf{B}_{\phi} - \gamma \mathbf{v} \times \mathbf{E}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_{\phi}}{v^2}\mathbf{v},$$

$$\mathbf{E}'_{\phi} = \gamma \mathbf{E}_{\phi} + \gamma \mathbf{v} \times \mathbf{B}_{\phi} + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_{\phi}}{v^2}\mathbf{v},$$

- **where** $\gamma = E_{\mathbf{p}}^{\phi}/m_{\phi}$ **and** $\mathbf{v} = \mathbf{p}/E_{\mathbf{p}}^{\phi}$

- **In terms of lab-frame fields we obtain (factorization of x and p)**

$$\overline{\rho}_{00}^{\phi}(x,\mathbf{p}) \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} I_{B,i}(\mathbf{p})\left[\boldsymbol{\omega}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{B}_i^{\phi})^2\right] + \frac{1}{3}\sum_{i=1,2,3} I_{E,i}(\mathbf{p})\left[\boldsymbol{\varepsilon}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\text{eff}}^2}(\mathbf{E}_i^{\phi})^2\right],$$

three basis directions in lab frame

Momentum functions

Parameters and comparison with data

- **Two parameters (transverse and longitudinal field squares)**

$$\left\langle (g_\phi \mathbf{B}^\phi_{x(y)})^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_{x(y)})^2 \right\rangle = F^2$$

$$\left\langle (g_\phi \mathbf{B}^\phi_z)^2 \right\rangle = \left\langle (g_\phi \mathbf{E}^\phi_z)^2 \right\rangle = r_z F^2 < F^2$$

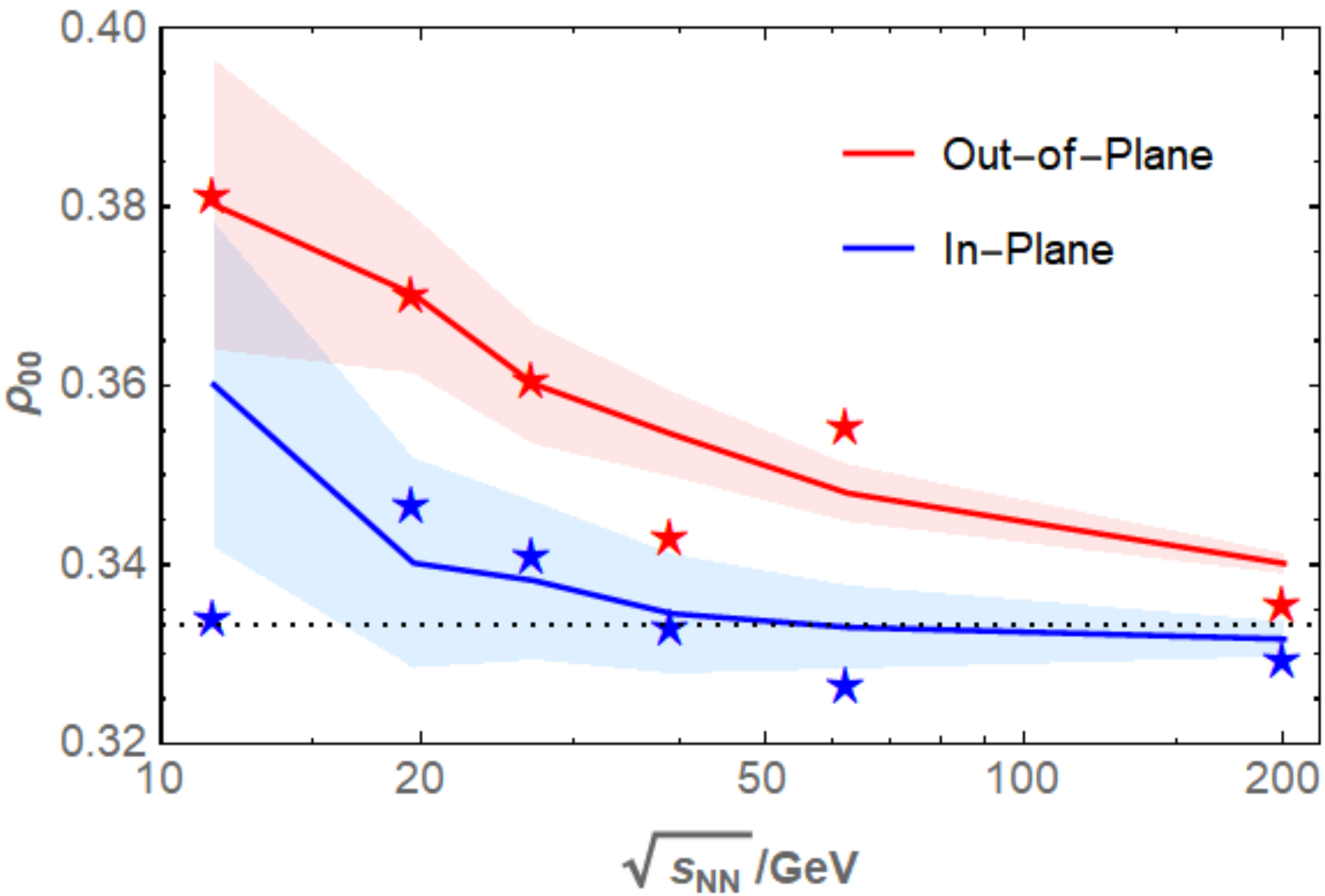
- **Two sets of values for parameters give the same result**

$$F^2 = 0.45 m_\pi^4,\ m_s = 170\,\mathrm{MeV}$$
$$F^2 = 5.02 m_\pi^4,\ m_s = 530\,\mathrm{MeV}$$

$$r_z = 0.79$$

- **The magnitude of electric field's contribution decreases with increasing m_s**

Collision energy dependence



Transverse momentum dependence

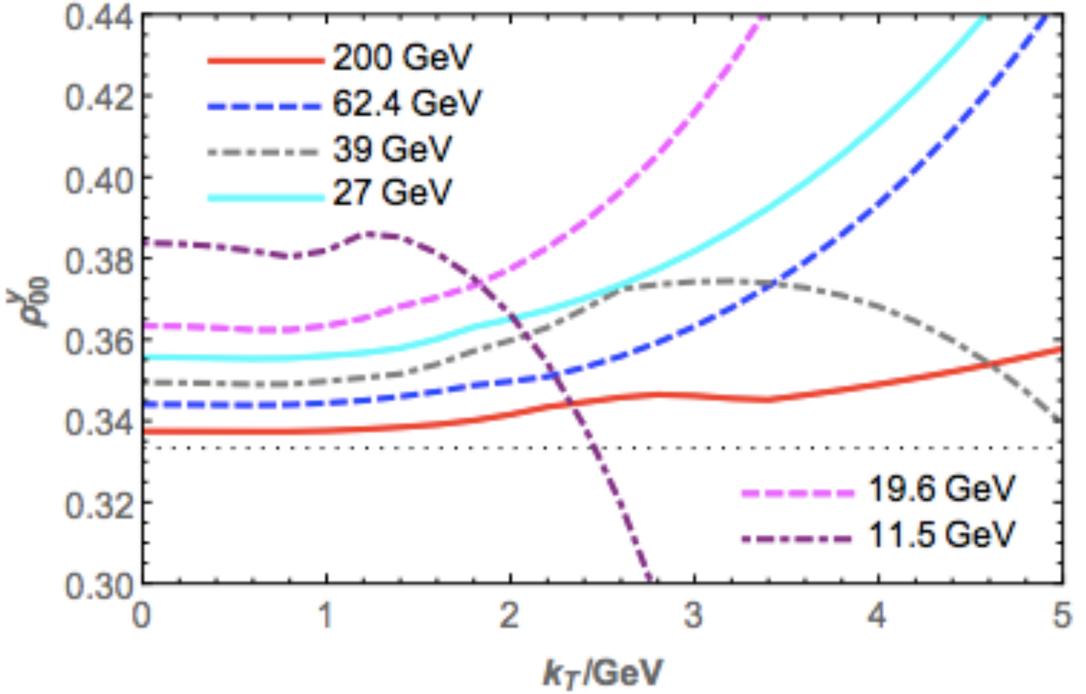


Figure 2. The ϕ meson's ρ_{00}^{y} as functions of transverse momenta at different collision energies.

Centrality dependence

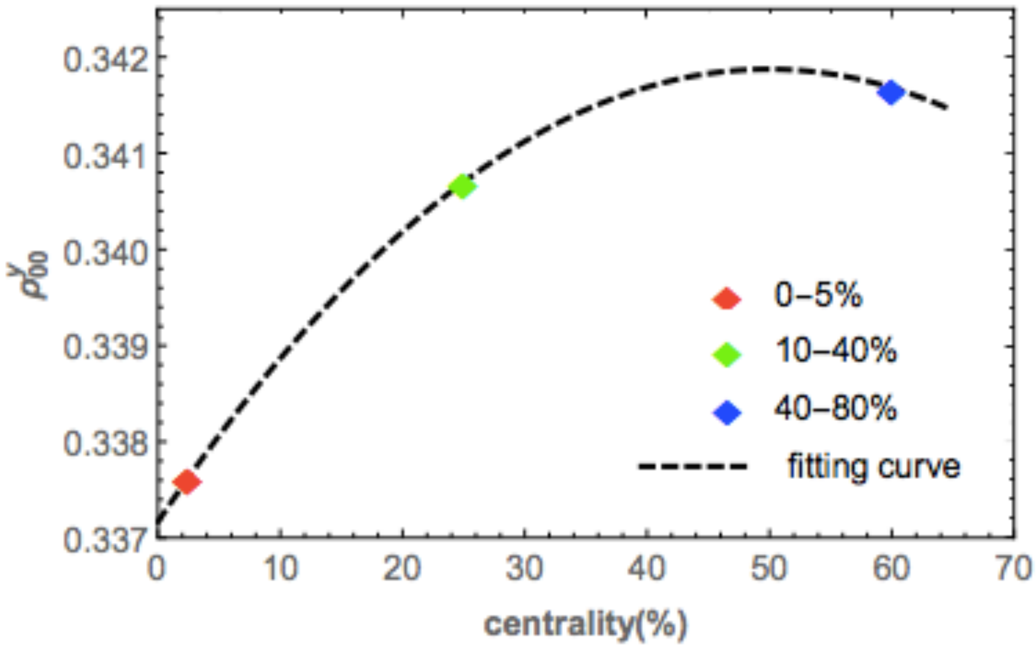


Figure 4. The ϕ meson's ρ_{00}^{y} as a function of centrality at 200 GeV. Red, green, and blue diamond points are our results for centrality ranges 0-5%, 10-40%, and 40-80%, respectively. The dashed line is the fitting curve using a second order polynomial.

Take-home message and Questions for discussions

- **Take-home message: P_Λ measures the fields (net mean field), ρ_{00}^{ϕ} measures field squares (field fluctuation).**

- **Questions for dicussions**
- **What are particles? What are fields? Particle-field duality?**
- **What is the nature of vector meson fields? Are they real entities? Can we calculate field squares on Lattice?**
- **Any connection with QCD sum rules and QCD vacuum properties? Any connection with quark or gluon condensates (trace anomaly)?**
- **Any implication for J/Psi polarization (gluon fields)?**