核物理前沿与交叉研讨会,2022年8月25日-28日

BEIHANG UNIVERSITY

High Precision Relativistic Chiral Nuclear Force: Present and Future

Li-Sheng Geng（耿立升）@ Beihang U.

Phys.Rev.Lett. 128 (2022)142002, **Jun-Xu Lu, Yang Xiao, Chun-Xuan Wang, LSG*, Jie Meng, and Peter Ring**

热烈祝贺清华大学物理系复系四十周年！

预祝清华大学核物理学科取得更大的成绩！

Contents

Why relativistic/covariant chiral nuclear forces

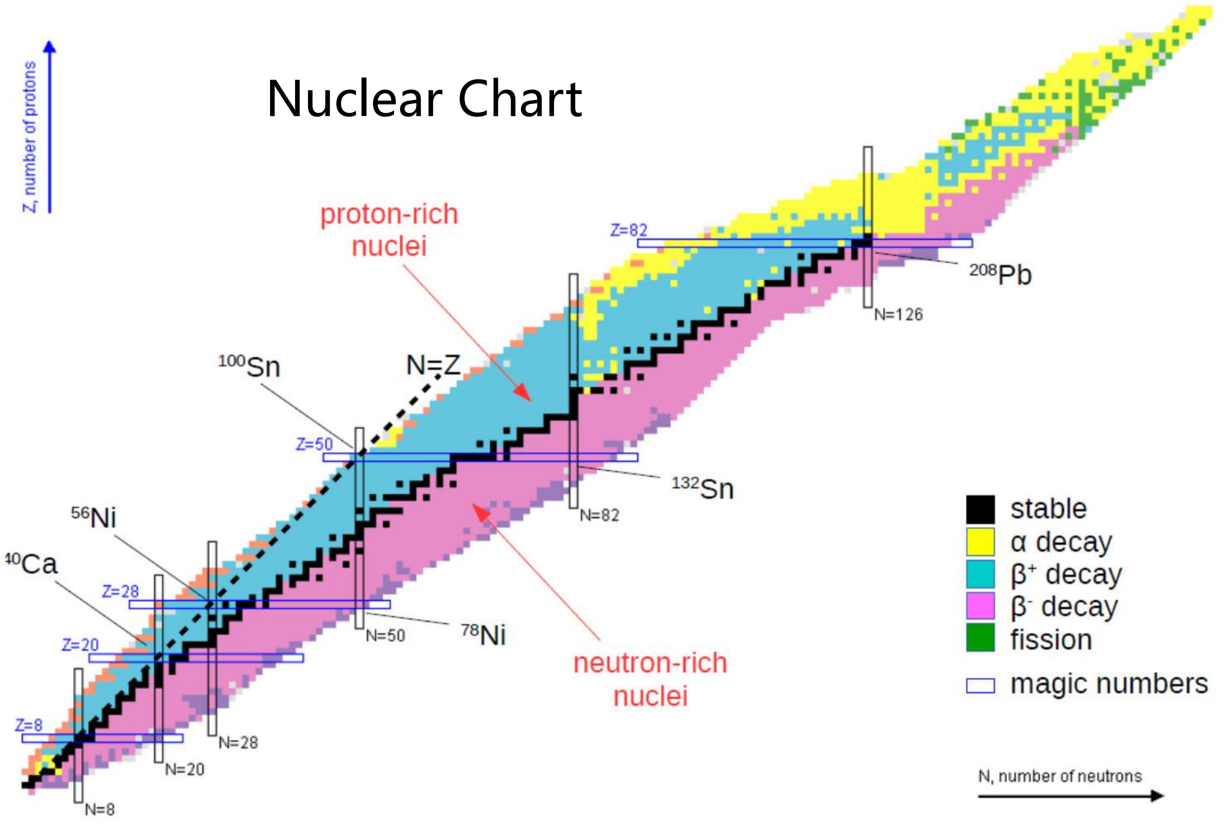
Our purpose, and where we are

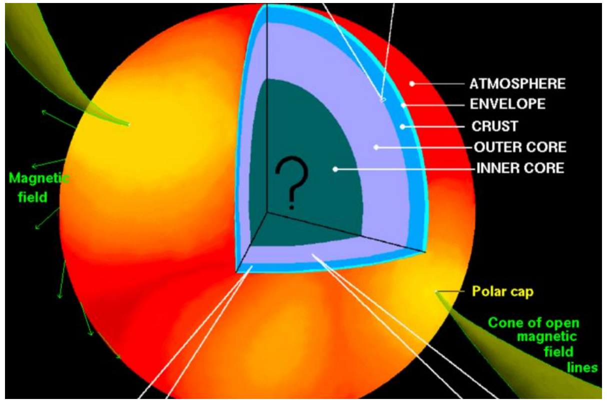
First relativistic high-precision chiral nuclear force

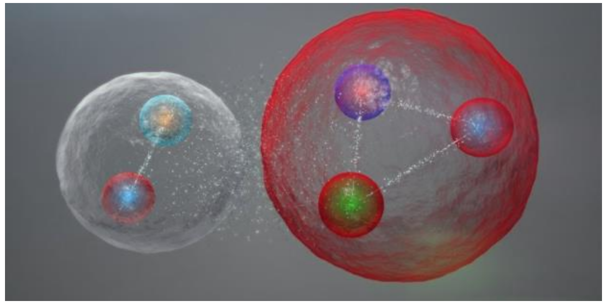
Summary and outlook

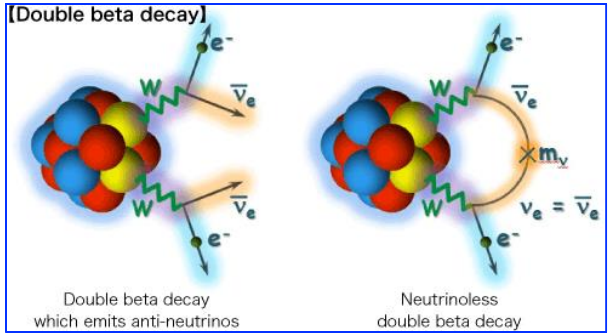
Contents

NN—most important input for microscopic understanding of nuclei







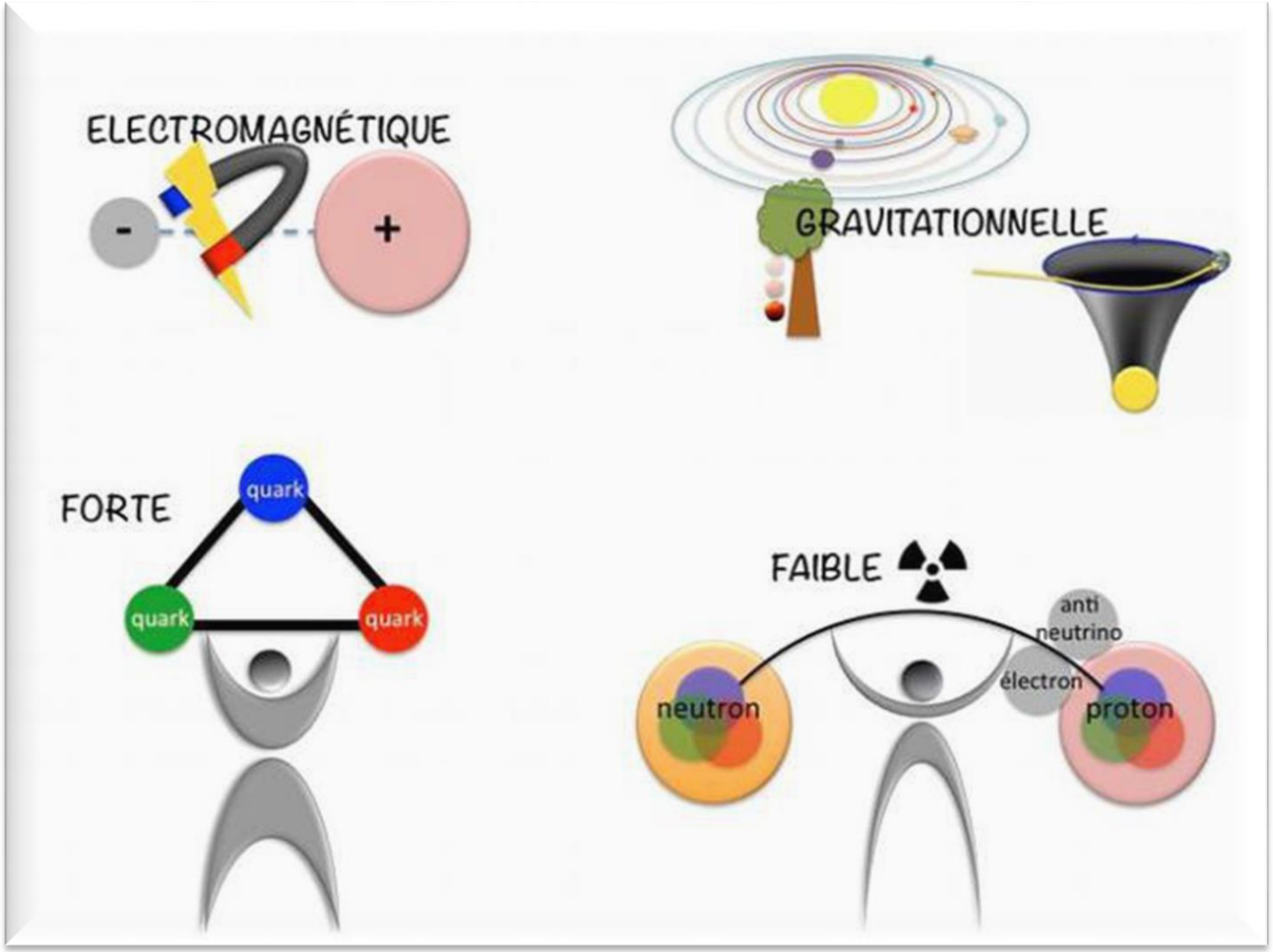


- ☐ **Nuclear structure, reactions**
- ☐ **Nuclear astrophysics**
- ☐ **Exotic hadrons~deuteron**
- ☐ **Searches for BSM physics**

One of the four interactions in Nature

F= $\frac{kq_1q_2}{r^2}$

F=q$\vec{v} \times \vec{B}$



F= $\frac{Gm_1m_2}{r^2}$

F=..............

....

F=G_F

One of the most difficult scientific problems

SCIENTIFIC AMERICAN, September 1953

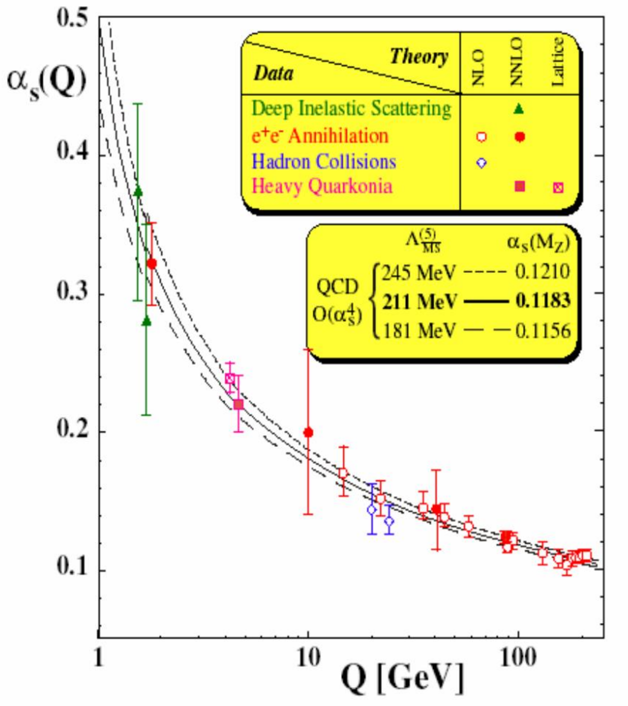
What Holds the Nucleus Together?

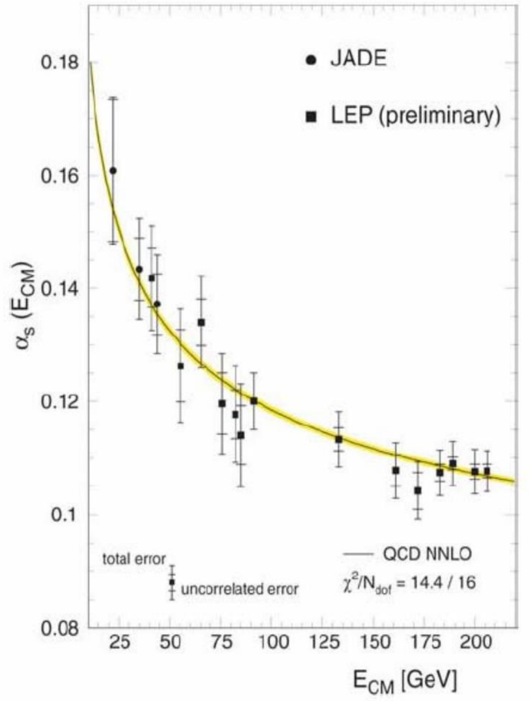
by Hans A. Bethe

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind.

- Hans Bethe
- Nobel Prize in Physics 1967

After QCD, even more difficult





Asymptotic Freedom

The Nobel Prize in Physics 2004

Photo from the Nobel Foundation archive.	Photo from the Nobel Foundation archive.	Photo from the Nobel Foundation archive.
David J. Gross	H. David Politzer	Frank Wilczek
Prize share: 1/3	Prize share: 1/3	Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction"

CMI

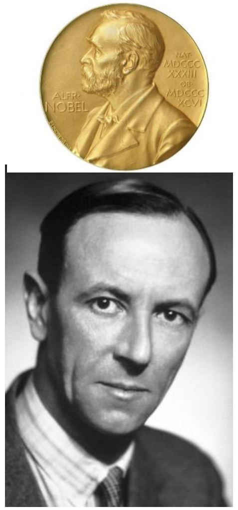
Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Color confinement

A brief account of the long history: three milestones



1932 Chadwick

1935 Yukawa

Meson theo.

1950-60

Pheno.

1974 Wilson

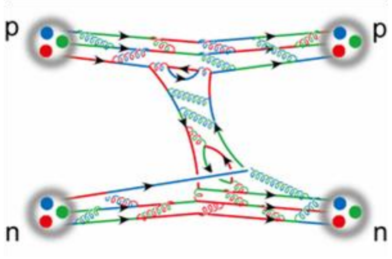
Lattice QCD

1990/91/92 Weinberg

HB ChEFT

2006/07

Lattice QCD

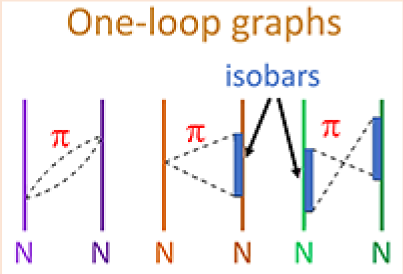


Central to developments in the field of nuclear physics

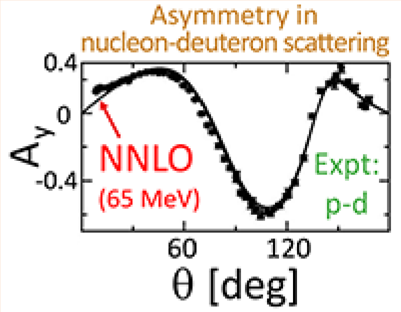
PHYSICAL REVIEW C — 50 YEARS

Remain central to developments in the field of nuclear physics
Announcing major discoveries/opening up new avenues of research

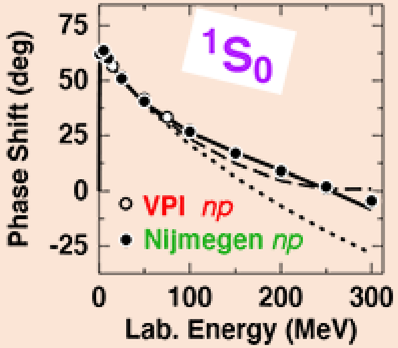
chiral EFT



Ordóñez, Ray, van Kolck
Phys. Rev. C **53**, 2086 (1996)

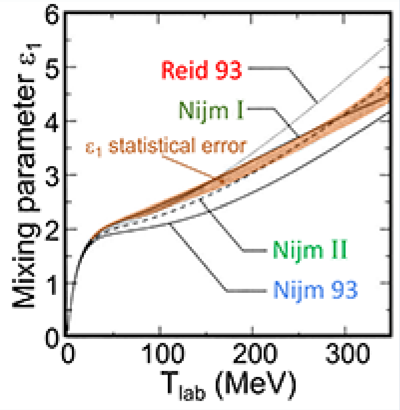


Epelbaum, Nogga, Glöckle, Kamada, Meißner, Witała
Phys. Rev. C **66**, 064001 (2002)

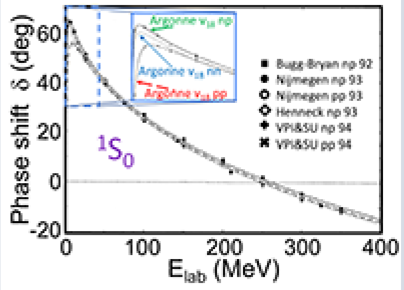


Entem, Machleidt
Phys. Rev. C **68**, 041001 (2003)

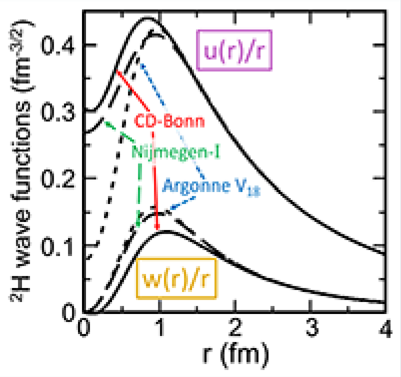
realistic



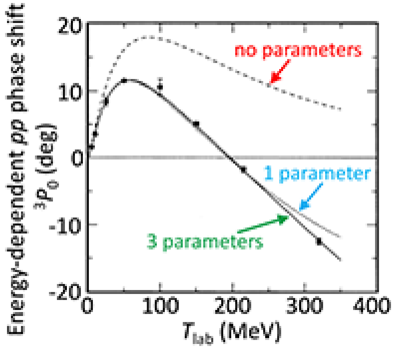
Stoks, Klomp, Terheggen, Swart
Phys. Rev. C **49**, 2950 (1994)



Wiringa, Stoks, Schiavilla
Phys. Rev. C **51**, 38 (1995)



R. Machleidt
Phys. Rev. C **63**, 024001 (2001)



Stoks, Klomp, Rentmeester, Swart
Phys. Rev. C **48**, 792 (1993)

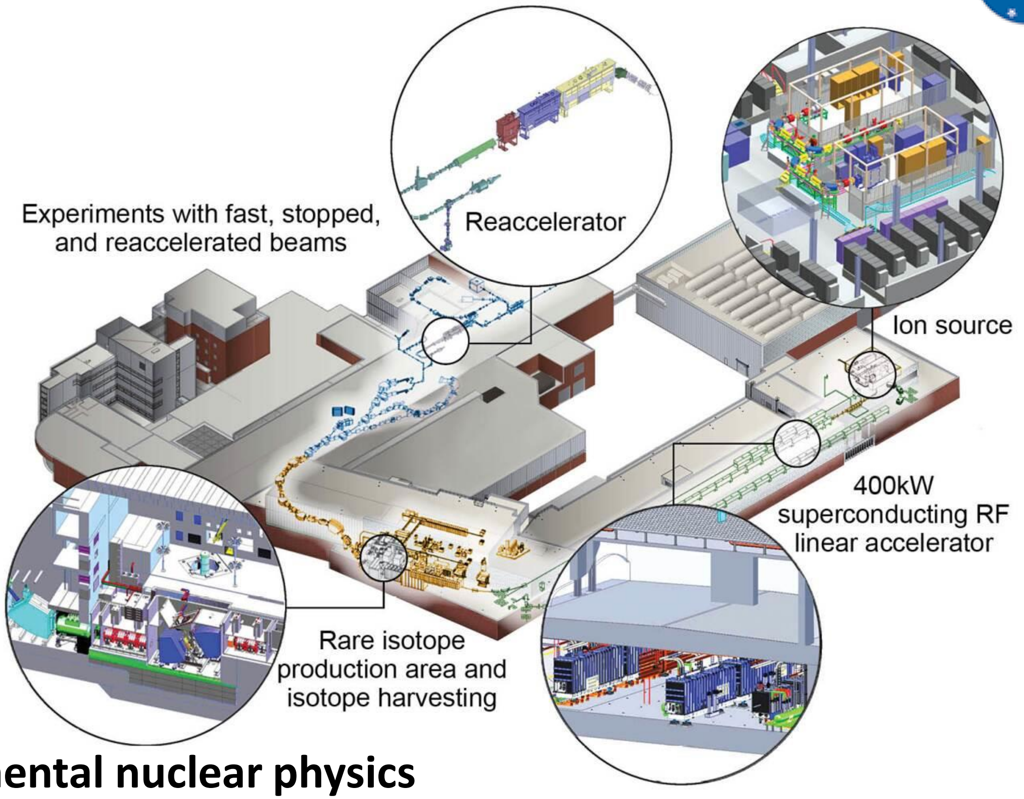
Always at the frontiers of nuclear physics

十二五"国家重大科技基础设施
"强流重离子加速器装置"|impcas.ac.cn)

- 认识原子核内有效相互作用。
- 探索宇宙中从铁到铀元素的来源
- 粒子辐照应用研究

杨建成、孙志宇

Facility for Rare Isotope Beams at Michigan State University



Experimental nuclear physics

- map the nuclear landscape,
- **understand the forces that bind nucleons into nuclei,**
- answer questions about the astrophysical origin of nuclear matter,
- address societal needs related to nuclear science and technology.

Solute to Chinese Pioneers

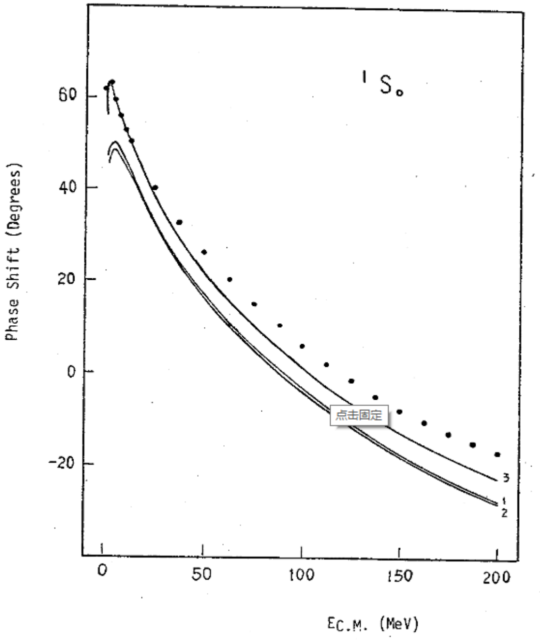


Fig.3

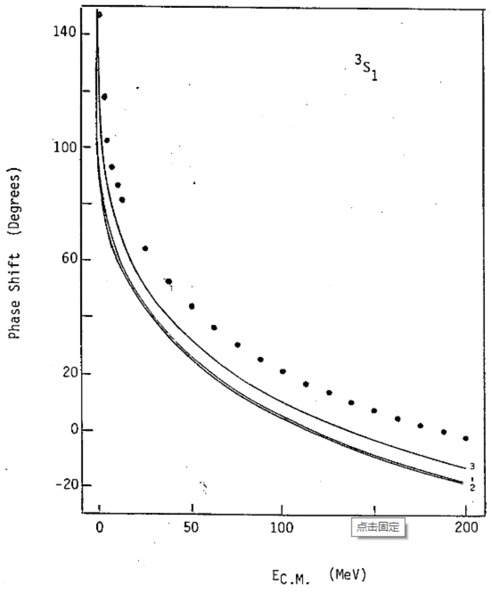
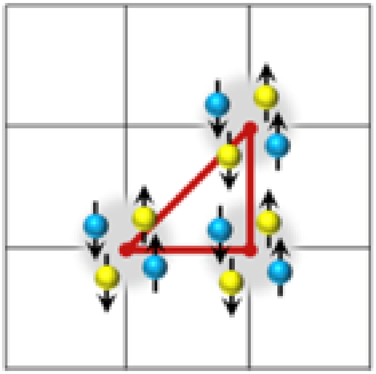


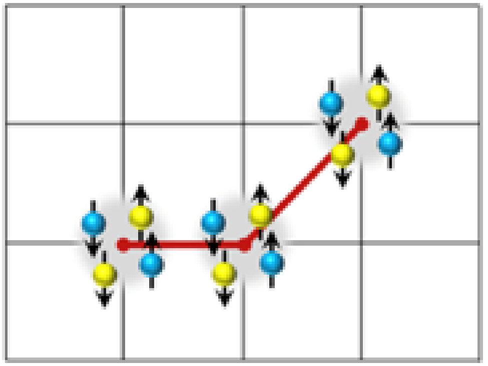
Fig.4

1988	Quark model	CPL 5 (1988) 297
1991	Quark-antiquark pair creation model	NPA 528 (1991) 513
1993	Quark potential model	NPA 561 (1993) 595
2003	Extended chiral SU(3) quark model	NPA 727 (2003) 321

Quark Delocalization Color Screening Model (QDCSM)
1980, F. Wang, and Y. He, Chin. J. Nucl. Phys. 2,261 (1980)
1992, F. Wang, G. H. Wu, L. J. Teng and T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992)

Why high precision—we are entering the era of first principles





Hoyle state of Carbon

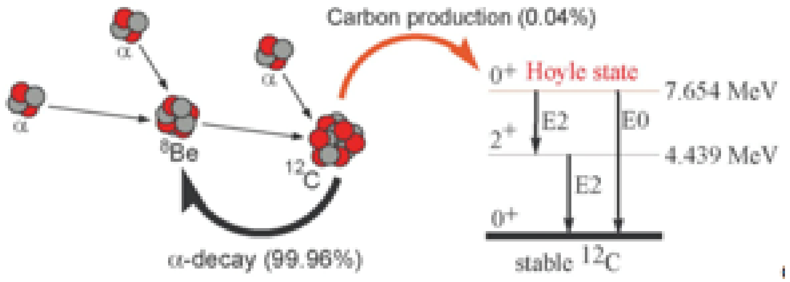


TABLE II. Lattice and experimental results for the energies of the low-lying even-parity states of ^{12}C, in units of MeV.

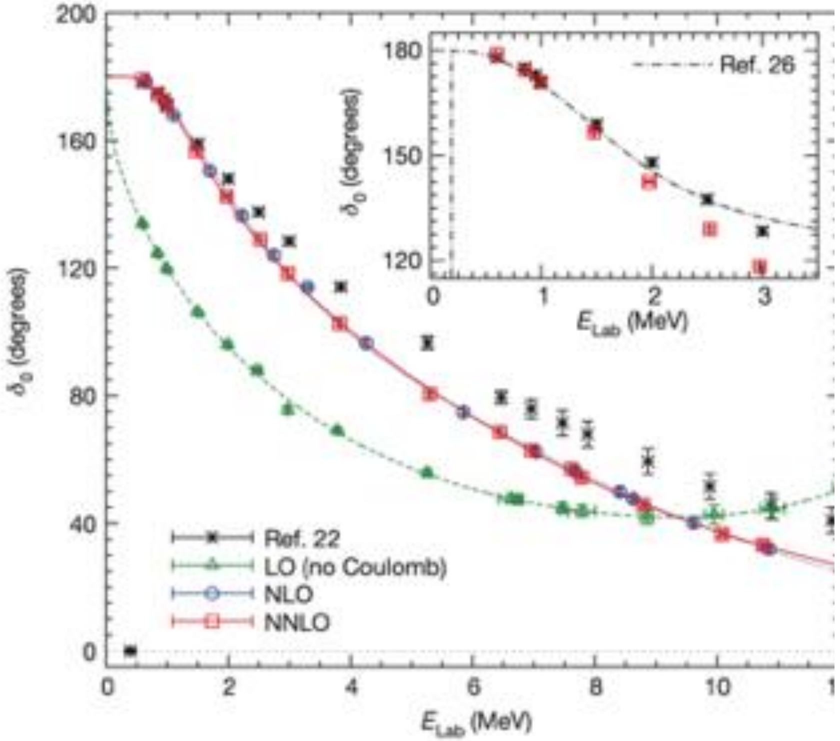
	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	−96(2)	−94(2)	−89(2)	−88(2)
NLO	−77(3)	−74(3)	−72(3)	−70(3)
NNLO	−92(3)	−89(3)	−85(3)	−83(3)
Expt.	−92.16	−87.72	−84.51	−82.6(1) [8,10]
				−81.1(3) [9]
				−82.32(6) [11]

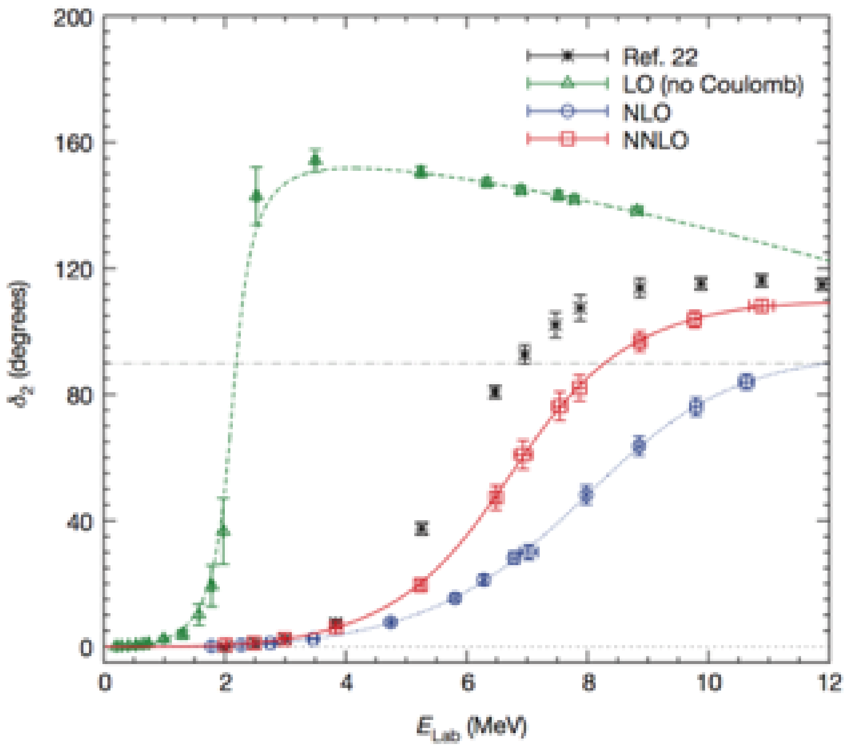
PRL 109, 252501 (2012) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2012

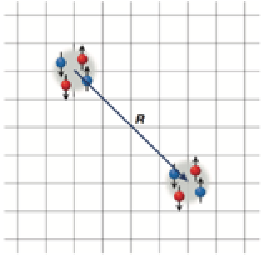
Structure and Rotations of the Hoyle State

Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]

Why high precision—we are entering the era of first principles







LETTER

doi:10.1038/nature16067

Ab initio alpha–alpha scattering

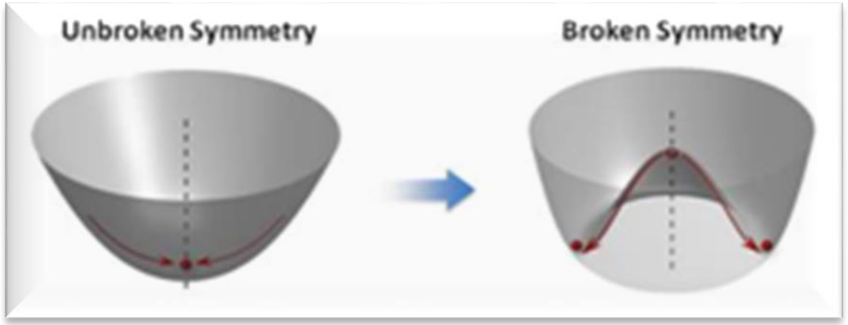
Serdar Elhatisari[1], Dean Lee[2], Gautam Rupak[3], Evgeny Epelbaum[4], Hermann Krebs[4], Timo A. Lähde[5], Thomas Luu[1,5] & Ulf-G. Meißner[1,5,6]

Nature 16067

Why chiral (effective field theory)

☐ Chiral perturbation theory—low energy EFT of QCD

- ✓ Because of **quark confinement and asymptotic freedom**, low energy QCD can not be solved perturbatively
- ✓ **Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons**
- ✓ **Allows a perturbative formulation of low energy QCD in powers of external momenta and light quark masses, by utilizing chiral symmetry and its breaking pattern (the third feature of QCD)**



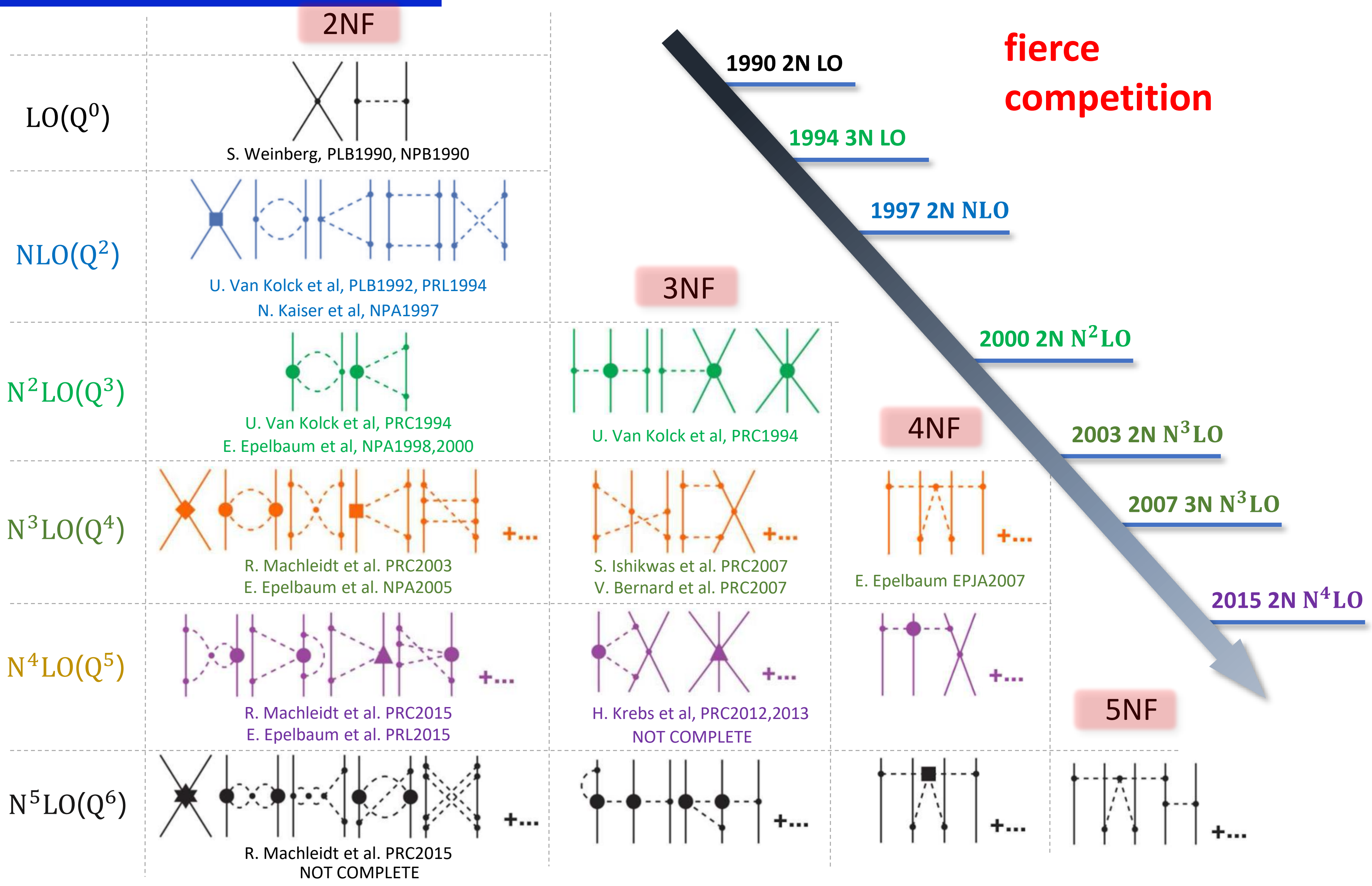
☐ Development—Trilogy

- ✓ 1979, pion-pion, Weinberg
- ✓ 1989, to the one-baryon sector, Gasser, Sainio, Svarc
- ✓ 1990/91/92, to NN/NNN, Weinberg—very successful

Steven Weinberg
Nobel Prize in Physics in 1979

Rev. Mod. Phys. 81(2009)1773; Phys. Rept. 503(2011)1; Rev. Mod. Phys. 92 (2020) 025004

Many scientists contributed: 30 years of endeavor



fierce competition

Adopted from Front.in Phys. 8 (2020) 57

van Kolck

Kaiser

Epelbaum

Machleidt

Philipps

Savage

Why chiral nuclear forces

☐ Fewer parameters, similar precision

	AV-18	Chiral		
		N3LO	NNLO	NLO
No. of parameters	40	24	9	9
Description of 2402 np data	1.04	1.10	10.1	36.2

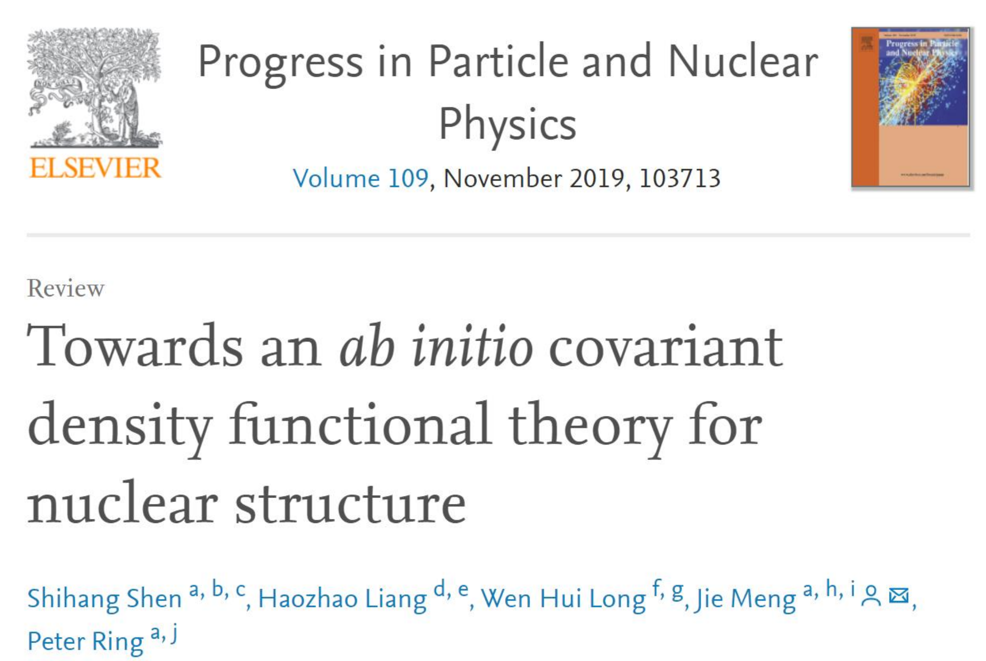
☐ More importantly, they are derived from EFTs

- ✓ **Closer link with QCD**
- ✓ **Systematic/order-by-order improvements**
- ✓ **Consistent descriptions of two/three/four body interactions on the same footing**
- ✓ **Quantifiable uncertainties-- PRA83(2011)040001**

Why relativistic/covariant

孟杰

- **Lorentz invariance is one of the most important symmetries in Nature.**
- Both kinematical and dynamical relativistic corrections self-consistently included
- **Relativistic approaches successful in explaining fine structures**
  - ✓ Atomic and molecular systems: why gold is yellow
  - ✓ **Nuclear system： spin-orbit splitting, pseudospin symmetry, covariant DFT** 孟杰、龙文辉、郭建友、蒋维洲
  - ✓ One-baryon sector： magnetic moments、 masses、 sigma terms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Progress in Particle and Nuclear Physics

Volume 109, November 2019, 103713

Review

Towards an *ab initio* covariant density functional theory for nuclear structure

Shihang Shen [a, b, c], Haozhao Liang [d, e], Wen Hui Long [f, g], Jie Meng [a, h, i], Peter Ring [a, j]

Einstein Dirac Mayer Jensen Arima

Why covariant/relativistic

- Lorentz invariance or relativistic corrections might play an important role in **speeding up** the convergence of ChEFT

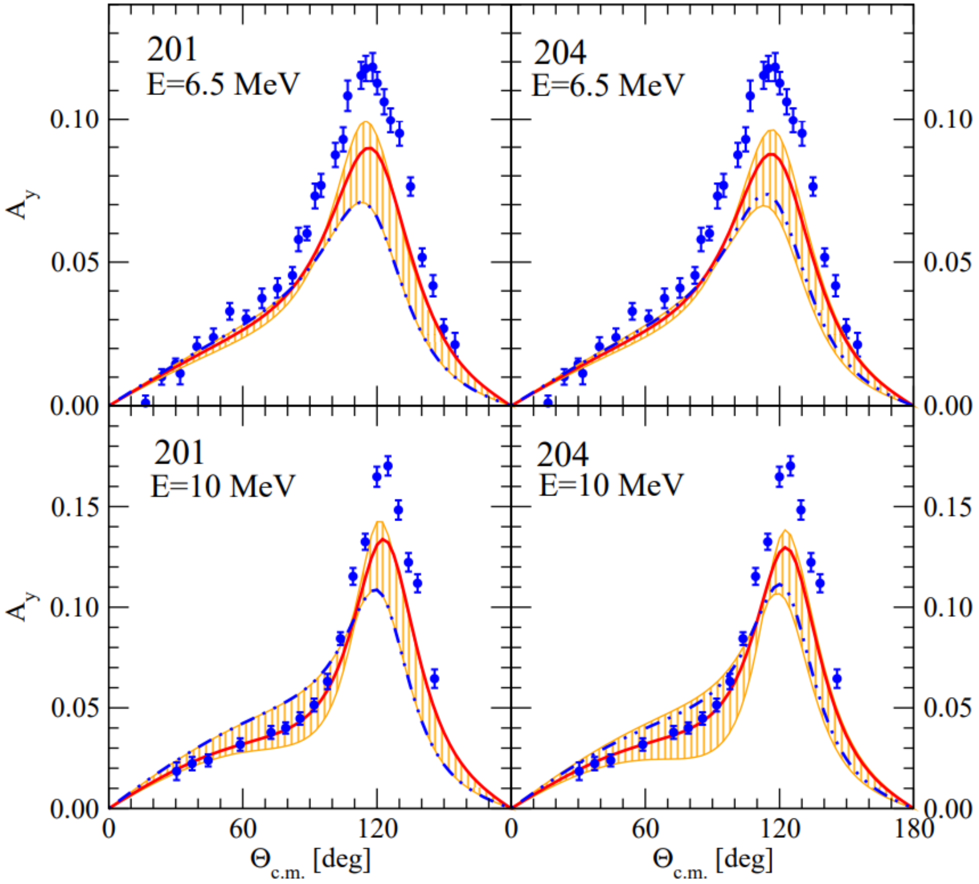
T_{lab} [MeV]	1	50	100	150	200	250	300
P_{cm} [MeV/c]	21.67	153.22	216.68	265.38	306.43	342.60	375.30
P_{cm}/M_N	0.023c	0.16c	0.23c	0.28c	0.33c	0.36c	0.40c

In comparison m_π/m_N=138/939~0.15

Possible solution to puzzles in NR chiral nuclear forces

A_y puzzle in N-d scattering

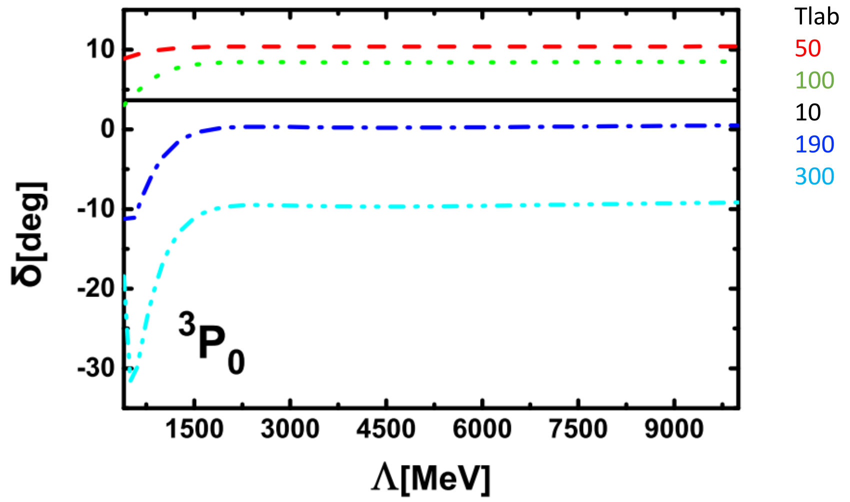
J. Golak et al. EPJA50,177

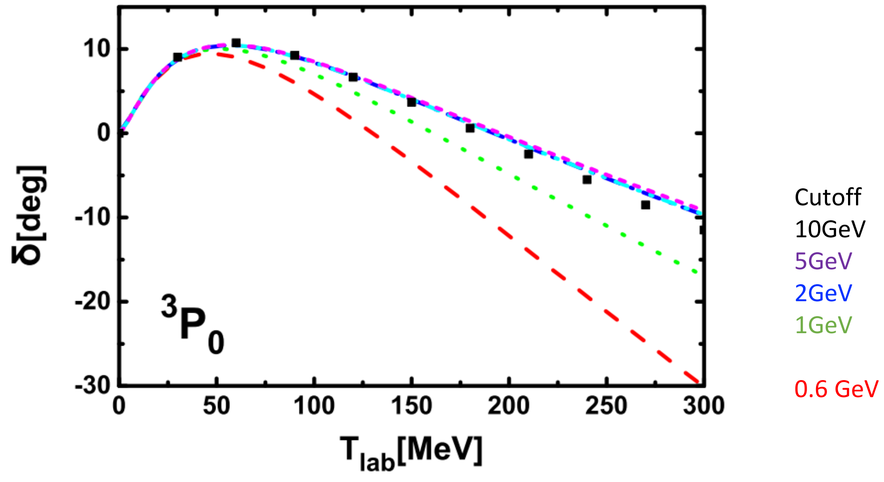


Red: N3LO chiral NN potential Blue: N3LO chiral NN + full 3N potential

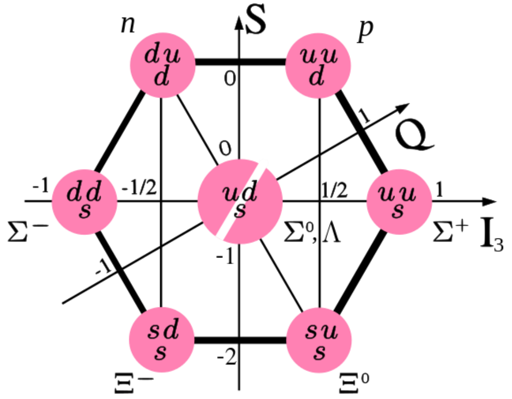
Renormalization group invariance in 3P0

C.X. Wang et al. CPC45.054101

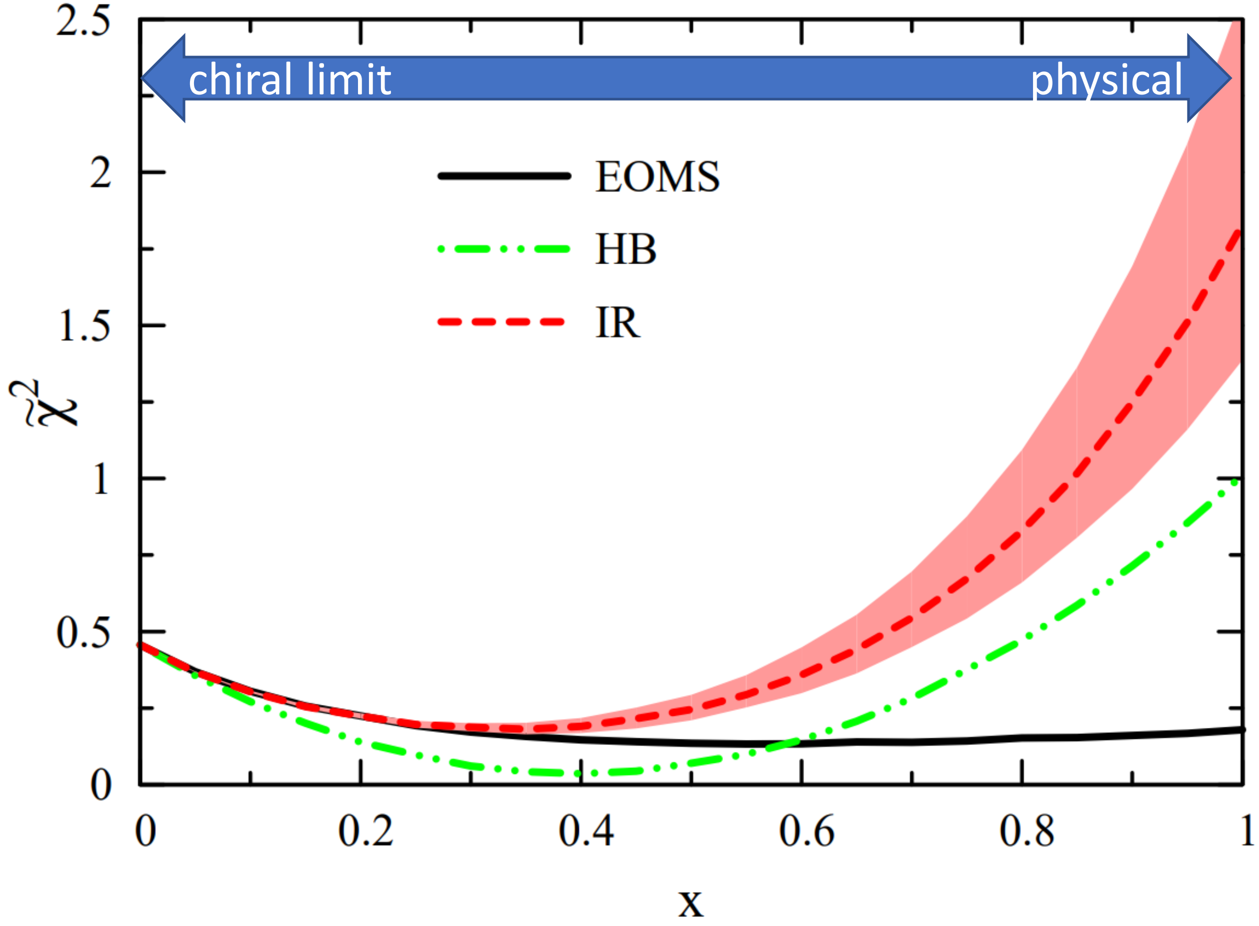


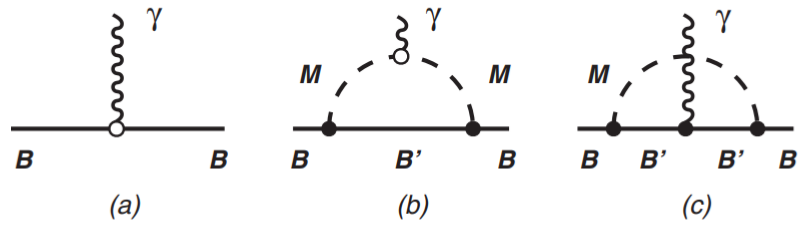


One-Baryon sector: covariant BChPT is essential



Li-Sheng Geng et al., PRL101 (2008) 222002





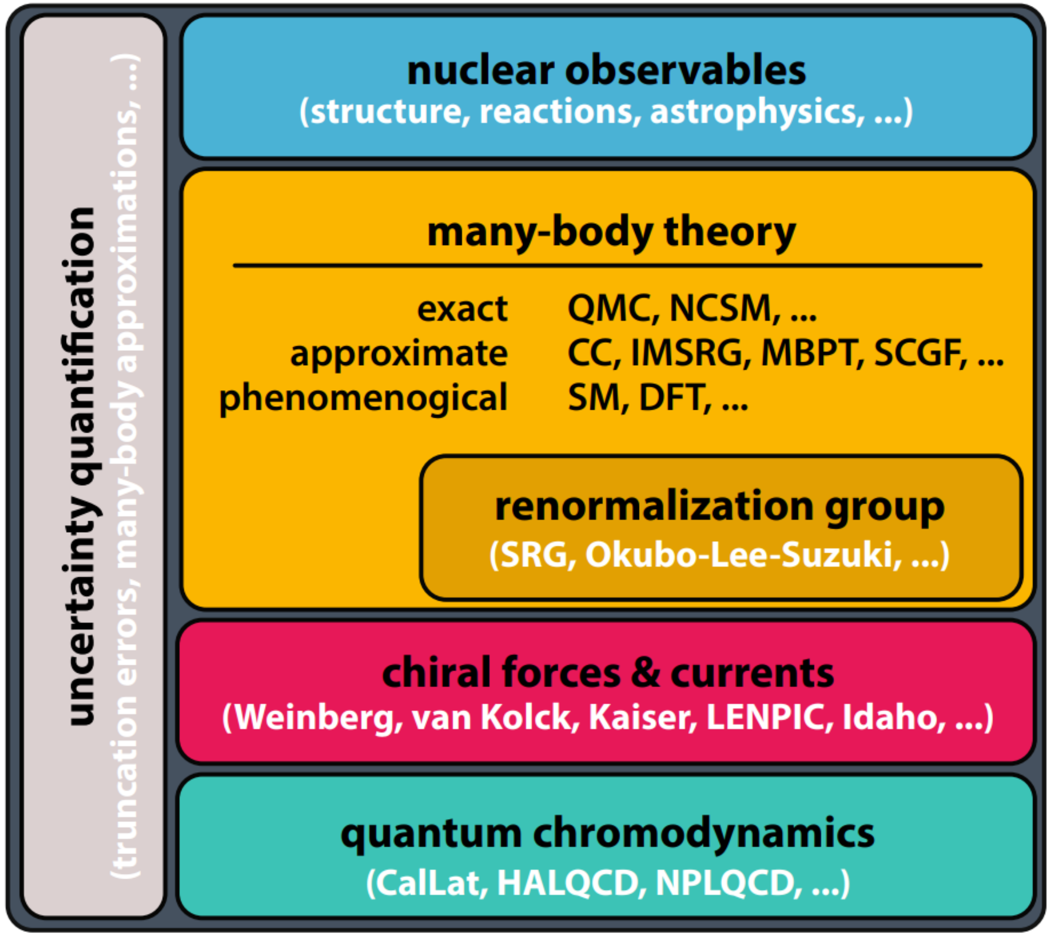
	p	n	Λ	Σ^-	Σ^+	Σ^0	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{b}_6^D$	$\tilde{b}_6^F$	$\tilde{\chi}^2$
					$\mathcal{O}(p^2)$							
Tree level	2.56	−1.60	−0.80	−0.97	2.56	0.80	−1.60	−0.97	1.38	2.40	0.77	0.46
					$\mathcal{O}(p^3)$							
HB	3.01	−2.62	−0.42	−1.35	2.18	0.42	−0.70	−0.52	1.68	4.71	2.48	1.01
IR	2.08	−2.74	−0.64	−1.13	2.41	0.64	−1.17	−1.45	1.89	4.81	0.012	1.86
EOMS	2.58	−2.10	−0.66	−1.10	2.43	0.66	−0.95	−1.27	1.58	3.82	1.20	0.18
Expt.	2.793(0)	−1.913(0)	−0.613(4)	−1.160(25)	2.458(10)	⋯	−0.651(3)	−1.250(14)	±1.61(8)		⋯	

It solves a longstanding problem in the understanding of the baryon magnetic moments

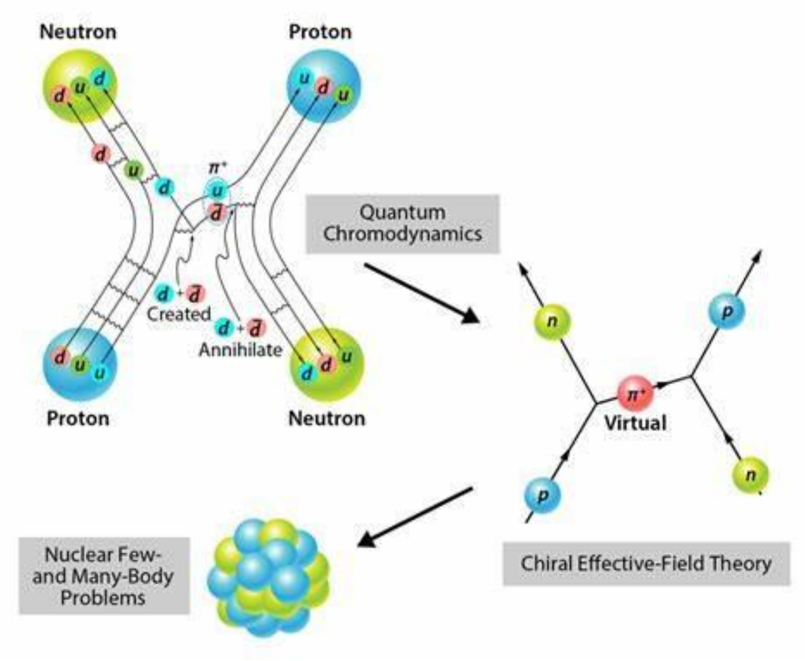
Contents

P1: inputs for ab initio covariant nuclear physics studies

Idealized workflow for ab initio many-body calculations in modern nuclear theory



Christian Drischler · Scott K. Bogner, Few-Body Syst (2021) 62:109



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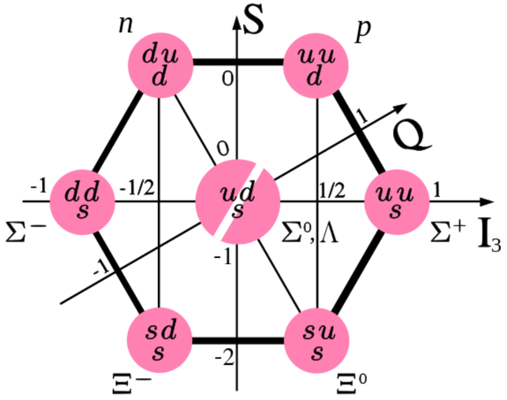
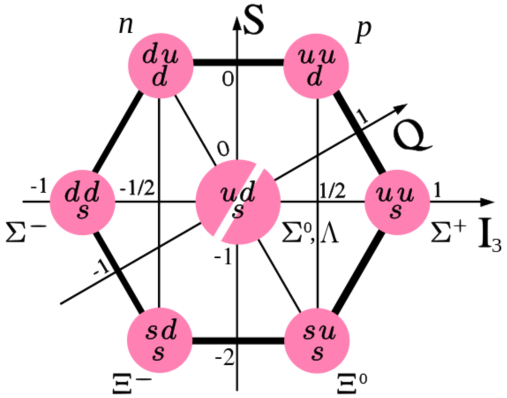
P2: inputs for studies of hypernuclei and neutron stars

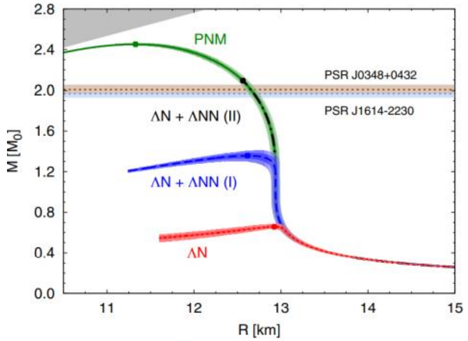
- ☐ **Nucleon-nucleon**
- ☐ **Hyperon-nucleon**
- ☐ **Hyperon-hyperon**





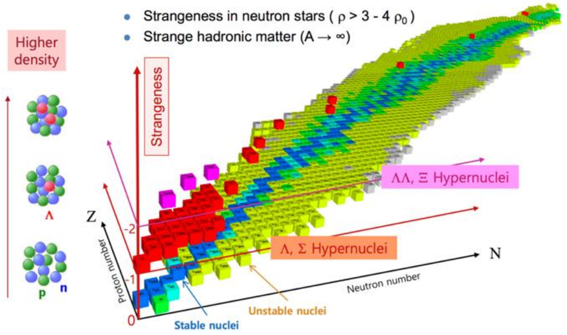
A bound H-dibaryon?

Inoue PRL 106 (2011) 162002



Hyperon puzzle

Lonardoni PRL 114 (2015) 092301



Three D nuclear chart

Kaneta M, Tohoku University, Japan)

NN

leading order

- *Leading order relativistic chiral NN interaction* — CPC42(2018)014103
- *Pion-mass dependence of the NN interaction* — PLB809(2020)135745
- *Relativistic Chiral Description of the 1S0 NN Scattering* — CPL38(2021)062101
- *Renormalizability of leading order covariant chiral NN interaction* — CPC45(2021)054101
- *Nucleon-nucleon interaction in the 3S1-3D1 coupled channel for a pion mass of 469 MeV* — PLB833(2022)137347

next-to-next-to-leading order

- *Covariant chiral NN contact Lagrangians up to N3LO* — PRC99(2019)024004
- *Meson-baryon scattering up to one-loop* — PRD99(2019)054024
- *Perturbative two-pion exchange contributions* — PRC102(2020)054001
- *Non-perturbative two-pion exchange contributions* — PRC105(2022)014003
- *Relativistic high precision chiral nuclear forces* — PRL128(2022)142002

YN&YY

- *Strangeness $S=-1$ YN scattering in covariant ChEFT* — PRD94(2016) 014029
- *Leading order relativistic YN interactions* — CPC42(2018)014105
- *Strangeness $S=-1$ YN interactions* — PRC98(2018)065203
- *Strangeness $S=-2$ BB interactions* — PRC98(2018)065203
- *Strangeness $S=-3$ and $S=-4$ BB interactions* — PRC103(2021)025201
- *Test of the YN interaction of the leading order covariant ChEFT* — PRC105(2022)035203
- *Strangeness $S=-2$ BB interactions and femtoscopic correlation functions* — 2201.04997

A systematic study from scratch

1 PRL, 9 PRC/D, 2 PLB, 5CPC/CPL

Contents

How to become relativistic/covariant

- **Dirac spinors and algebra (instead of non-relativistic wave functions and Pauli matrices)**

$$u(\boldsymbol{p}, s) = N_p \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{\epsilon_p} \end{pmatrix} \chi_s, N_p = \sqrt{\frac{\epsilon_p}{2M_N}}$$

	$\mathbb{1}$	γ_5	γ_μ	$\gamma_5\gamma_\mu$	$\sigma_{\mu\nu}$	$\epsilon_{\mu\nu\rho\sigma}$	$\overleftrightarrow{\partial}_\mu$	∂_μ
$\mathcal{P}$	+	−	+	−	+	−	+	+
$\mathcal{C}$	+	+	−	+	−	+	−	+
h.c.	+	−	+	+	+	+	−	+
$\mathcal{O}$	0	1	0	0	0	−	0	1

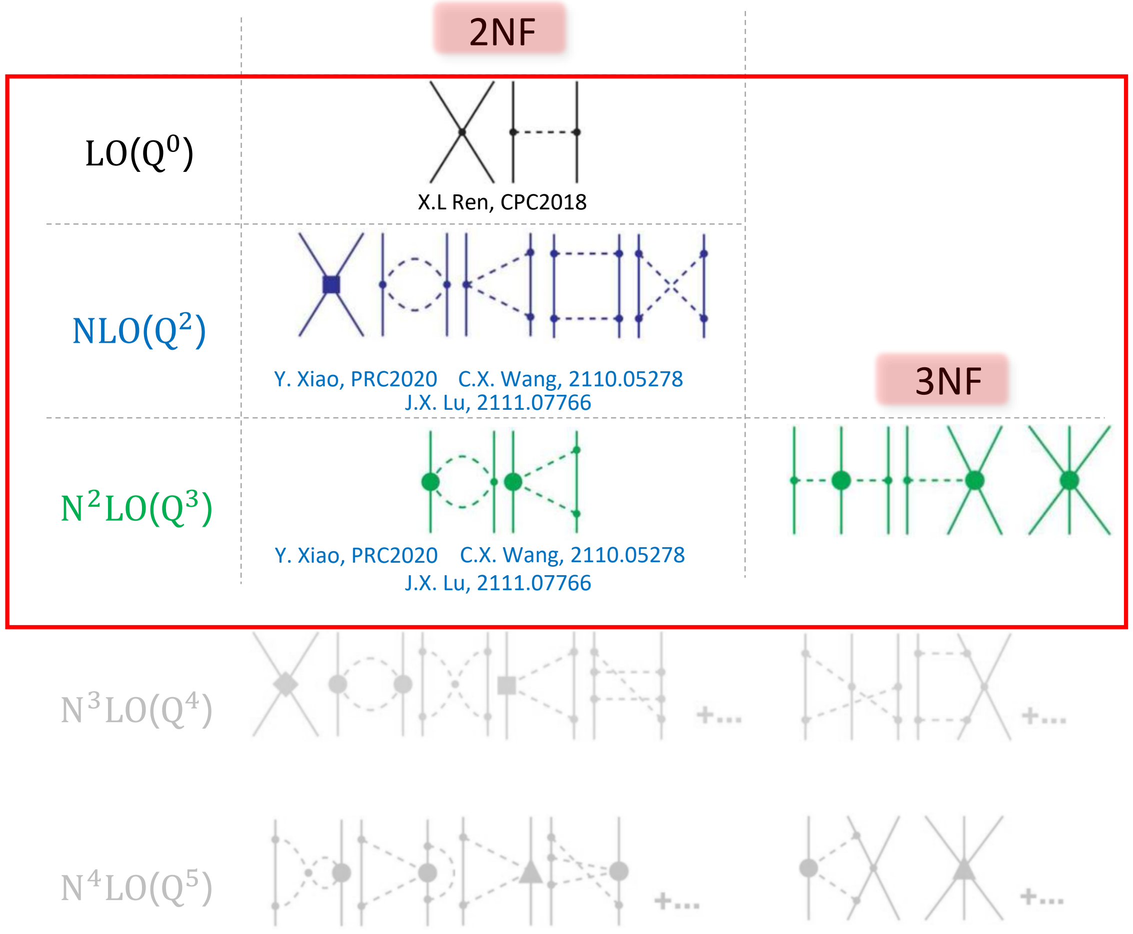
- **Covariant scattering equation (instead of Lippmann-Schwinger eq.)**

$$\mathcal{T}(p', p \mid W) = \mathcal{A}(p', p \mid W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p', k \mid W) G(k \mid W) \mathcal{T}(k, p \mid W)$$

$$G(k \mid W) = \frac{i}{[\gamma^\mu (W+k)_\mu - m_N + i\epsilon]^{(1)} [\gamma^\mu (W-k)_\mu - m_N + i\epsilon]^{(2)}}$$

- **Covariant power counting (vs. the Weinberg PC)**

Accurate relativistic chiral NN interaction up to NNLO



Three key ingredients

1. **Four nucleon (baryon) vertices:**
   - ✓ **PRC99(2019)024004**
2. **Meson-baryon vertices:**
   - ✓ **PRD99(2019)054024**
3. **Two-meson exchanges**
   - ✓ **PRC102(2020)054001**
   - ✓ **PRC105(2022)014003**

□ NR leading order BoX diagram

$$V_{\rm NLO}^{\rm TPEP} = -\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{384\pi^2 f_\pi^4}L(q)\left\{4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right\}-\frac{3g_A^4}{64\pi^2f_\pi^4}L(q)\{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}-q^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\}+P(\boldsymbol{k},\boldsymbol{q}),$$

$$L(q)=\frac{1}{q}\sqrt{4M_\pi^2+q^2}\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{2M_\pi}$$

□ Leading order BoX diagram

$\mathcal{A} =$ Loop integral × Bilinear

- Bilnear (**114 项**)

$$\bar u_3 u_1\cdot\bar u_4 u_2 \quad \bar u_3\not p_i u_1\cdot\bar u_4 u_2 \quad \bar u_3\not p_i\not p_j u_1\cdot\bar u_4 u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\gamma_\mu u_2 \quad \bar u_3\not p_i\gamma^\mu u_1\cdot\bar u_4\not p_j\gamma_\mu u_2$$

$$\bar u_3 u_1\cdot\bar u_4\not p_i u_2 \quad \bar u_3 u_1\cdot\bar u_4\not p_i\not p_j u_2 \quad \bar u_3\not p_i u_1\cdot\bar u_4\not p_j u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\not p_i\gamma_\mu u_2 \quad \bar u_3\gamma^\mu u_1\cdot\bar u_4\not p_i\not p_j\gamma_\mu u_2$$

and more...

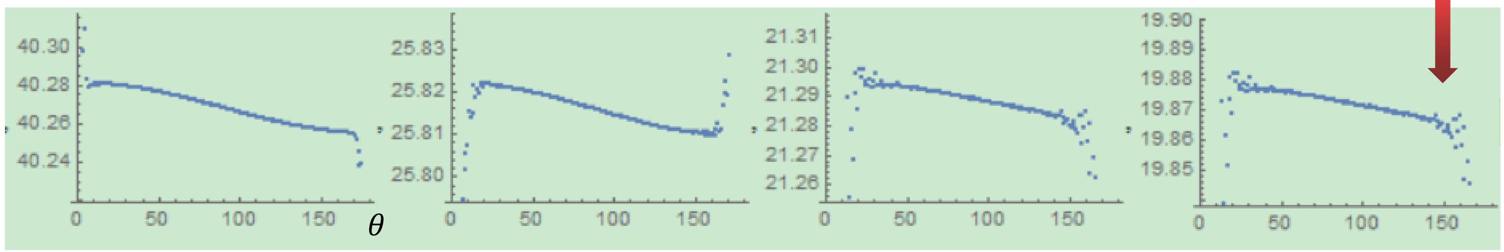
- Loop integrals—— scalar and tensor (**29 terms**)

$$A_0, B_0, B_{00}, C_0, C_1, C_2, C_{00}, C_{11}, C_{12}, C_{22}, D_0, D_1, D_2, D_{00}, D_{11}, D_{12}, D_{22}, D_{23}$$

- Numerical accuracy and singularity
  - ➢ $\frac{1}{1-\cos\theta^2}$
  - ➢ $(\frac{p_{\rm off}}{M_N})^n \approx 10^{-3n}$

Hardware accuracy →

$D_{11}, D_{12}, D_{22}, D_{23}$ coefficients diverge



Uncertainty Estimates

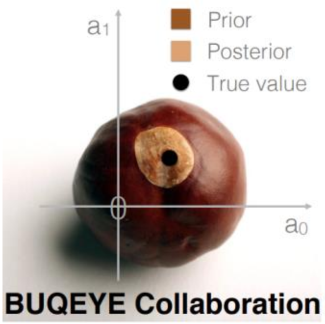
PRA editorial: Uncertainty estimates—PRA83(2011)040001

- ✓ If the authors claim high accuracy, or improvements on the accuracy of previous work.
- ✓ If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
- ✓ If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

Bayesian Uncertainty quantification

What is BUQEYE? - BUQEYE Collaboration

Dick Furnstahl



Daniel Phillips

BAND
Bayesian Analysis of Nuclear Dynamics

https://bandframework.github.io/

NNLO high precision relativistic chiral force

- 19 LECs fitted to the phase shifts of all the partial waves with $J \leq 2$ at $E_{lab} = 1,5,10,25,50,100,150,200\ \mathrm{MeV}$

$$\tilde{\chi}^2 = \sum (\delta^i - \delta^i_{\mathrm{PWA93}})^2,$$

- **TPE&OPE fixed**

c_1	c_2	c_3	c_4	f_π	g_A
-1.39	4.01	-6.61	3.92	92.4	1.29

- **Fit results**

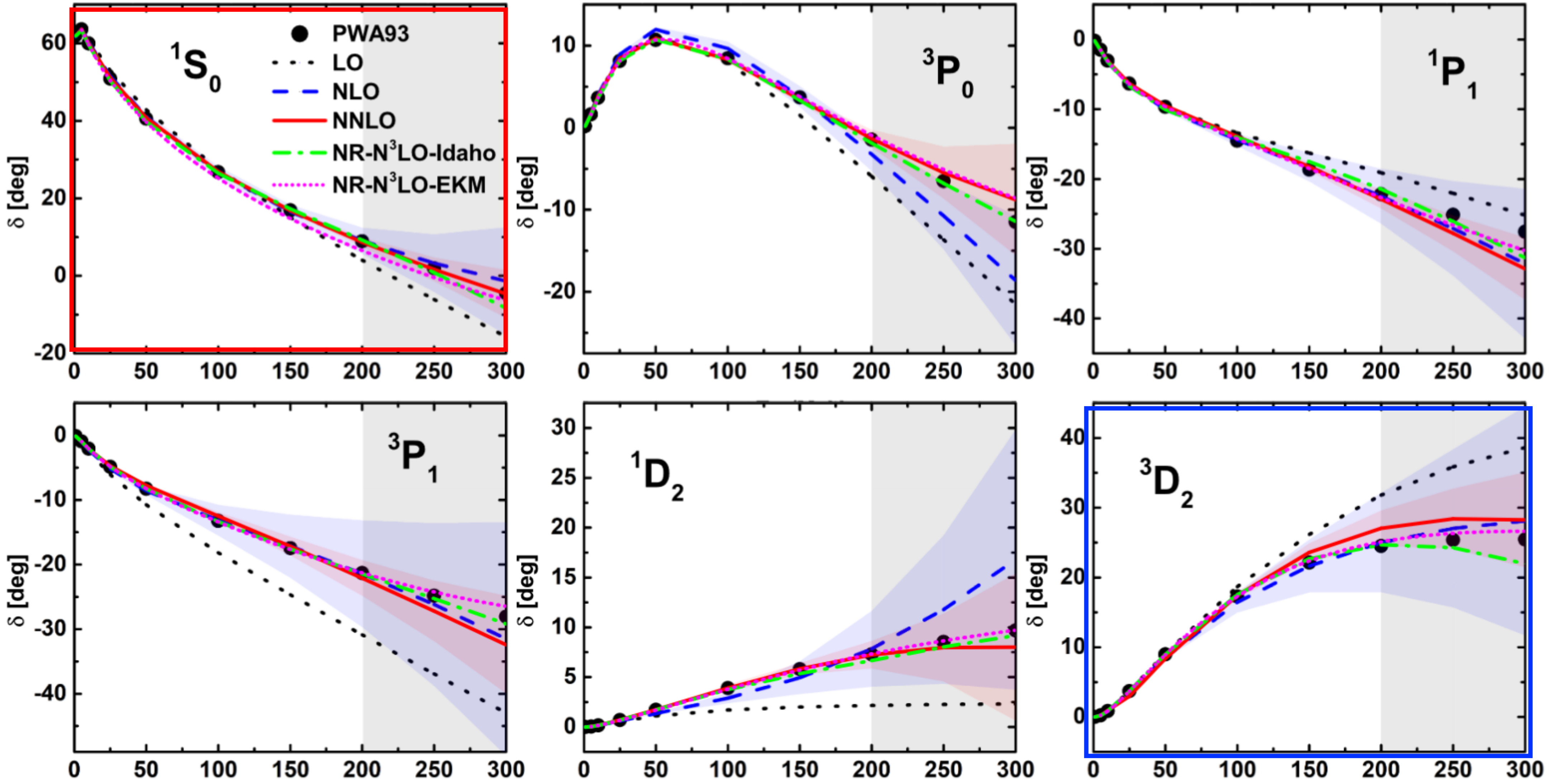
	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9	O_{10}	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	O_{16}	O_{17}	D_1	D_2
LO	-13.23	-2.06	-9.34	3.14															
NLO	-2.62	9.45	-5.42	-6.05	30.09	9.02	-9.19	8.74	4.74	7.02	3.52	11.42	-6.03	-20.55	-4.99	-12.80	6.30	0.42	0.28
NNLO	-14.83	-2.25	-4.85	6.24	-0.82	1.96	-6.89	7.19	1.44	3.50	-8.10	-9.38	-4.33	-12.89	-12.26	-11.69	3.86	-1.88	-0.63

comparable to best NR-N3LO

TABLE III. $\tilde{\chi}^2 = \sum_i (\delta^i - \delta^i_{\mathrm{PWA93}})^2$ of different chiral forces for partial waves up to $J \leq 2$.

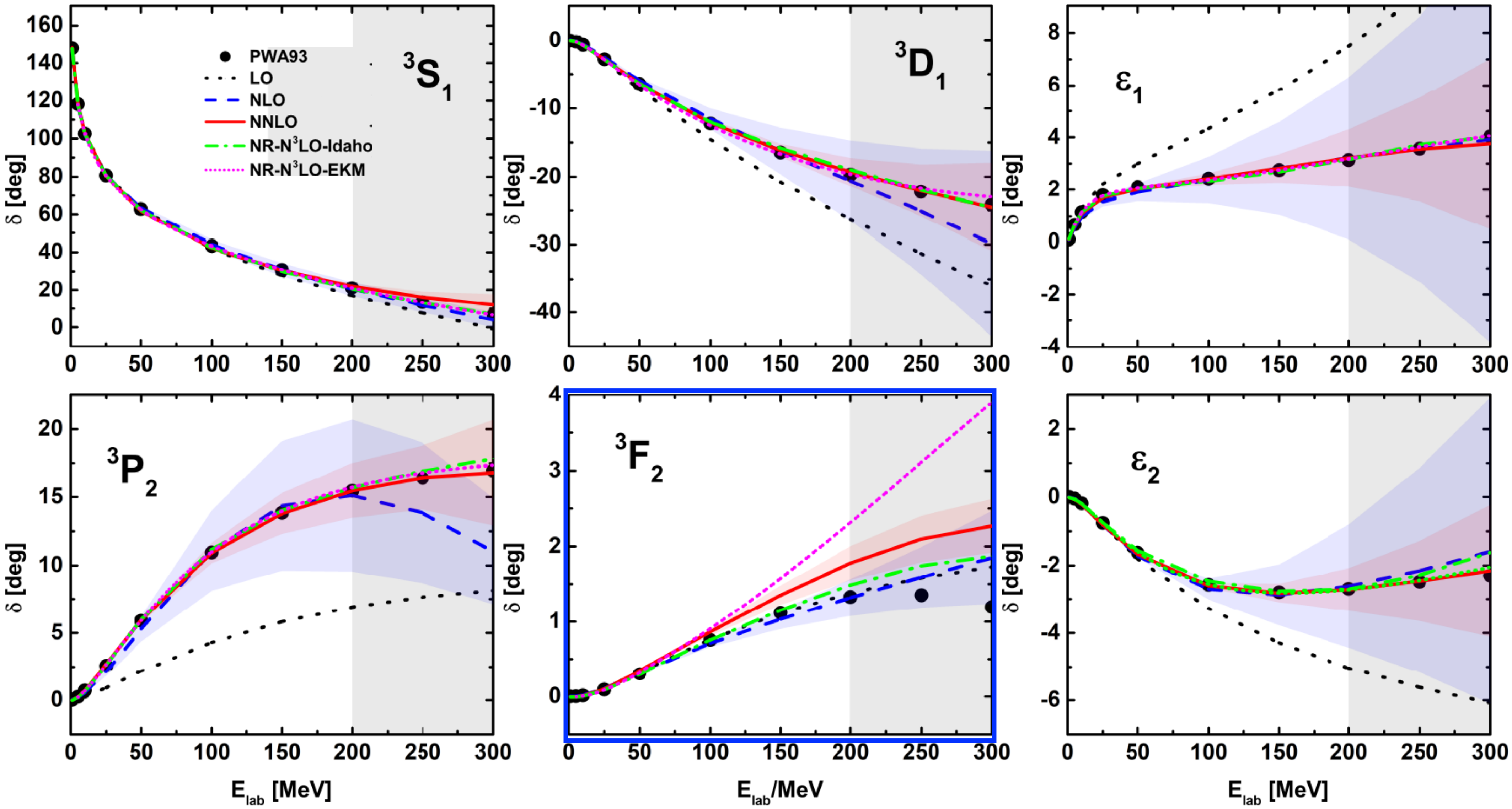
	Total	$^1S_0$	$^3P_0$	$^1P_1$	$^3P_1$	$^3S_1$	$^3D_1$	ϵ_1	$^1D_2$	$^3D_2$	$^3P_2$	$^3F_2$	ϵ_2
NLO	17.02	1.02	7.04	0.46	0.33	1.80	1.69	0.15	2.18	1.35	0.95	0.01	0.04
NNLO	16.61	0.18	0.30	1.07	1.55	3.36	0.26	0.03	0.01	9.56	0.01	0.27	0.01
NR-N^{3}LO-Idaho	8.84	1.53	0.30	2.41	0.04	2.33	1.00	0.02	0.57	0.42	0.17	0.03	0.02
NR-N^{3}LO-EKM	16.08	13.45	0.29	0.34	0.06	0.01	0.13	0.01	0.02	0.43	0.12	1.22	0.00

Fit results for $J \leq 2$ partial waves



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

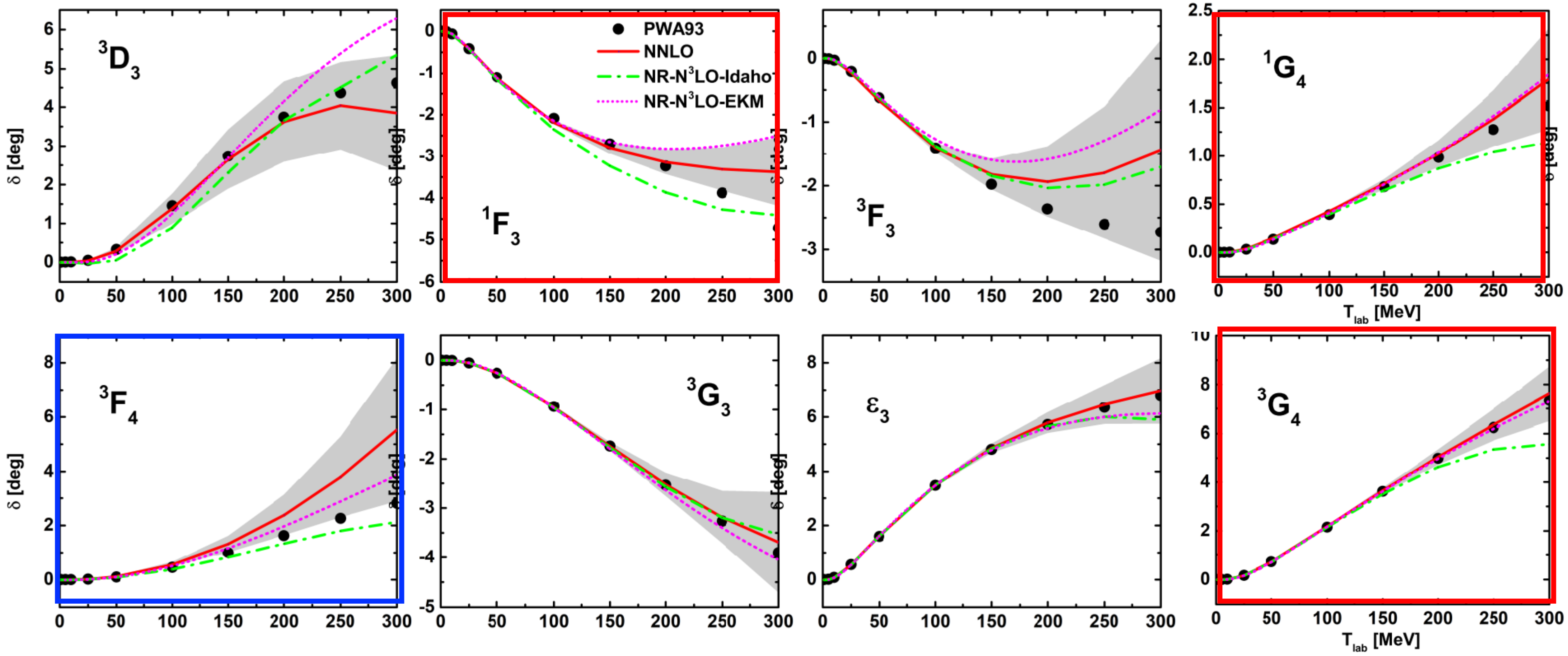
Fit results for $J \leq 2$ partial waves



NR-N3LO-Idaho: Machleidt and D. R. Entem, Phys. Rept.503, 1 (2011); NR-N3LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur. Phys. J.A51,53 (2015)

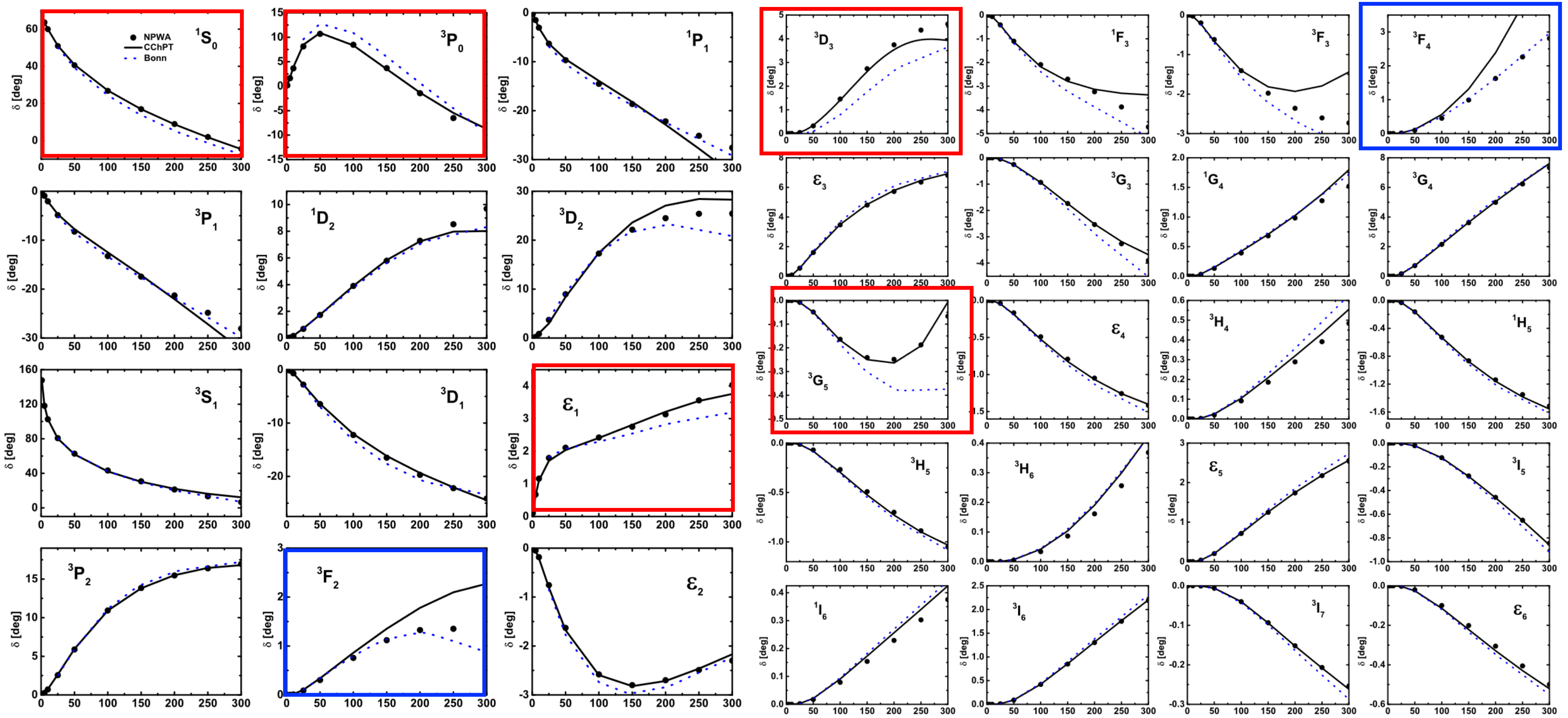
Better predictions for higher partial waves

	Total	$^3D_3$	$^1F_3$	$^3F_3$	$^3F_4$	$^3G_3$	ϵ_3	$^1G_4$	$^3G_4$
NNLO	0.98	0.03	0.03	0.21	0.70	0.00	0.01	0.00	0.00
NR-N^{3}LO-Idaho	1.73	0.58	0.73	0.13	0.12	0.00	0.01	0.01	0.15
NR-N^{3}LO-EKM	3.00	0.56	1.44	0.28	0.54	0.01	0.01	0.03	0.13

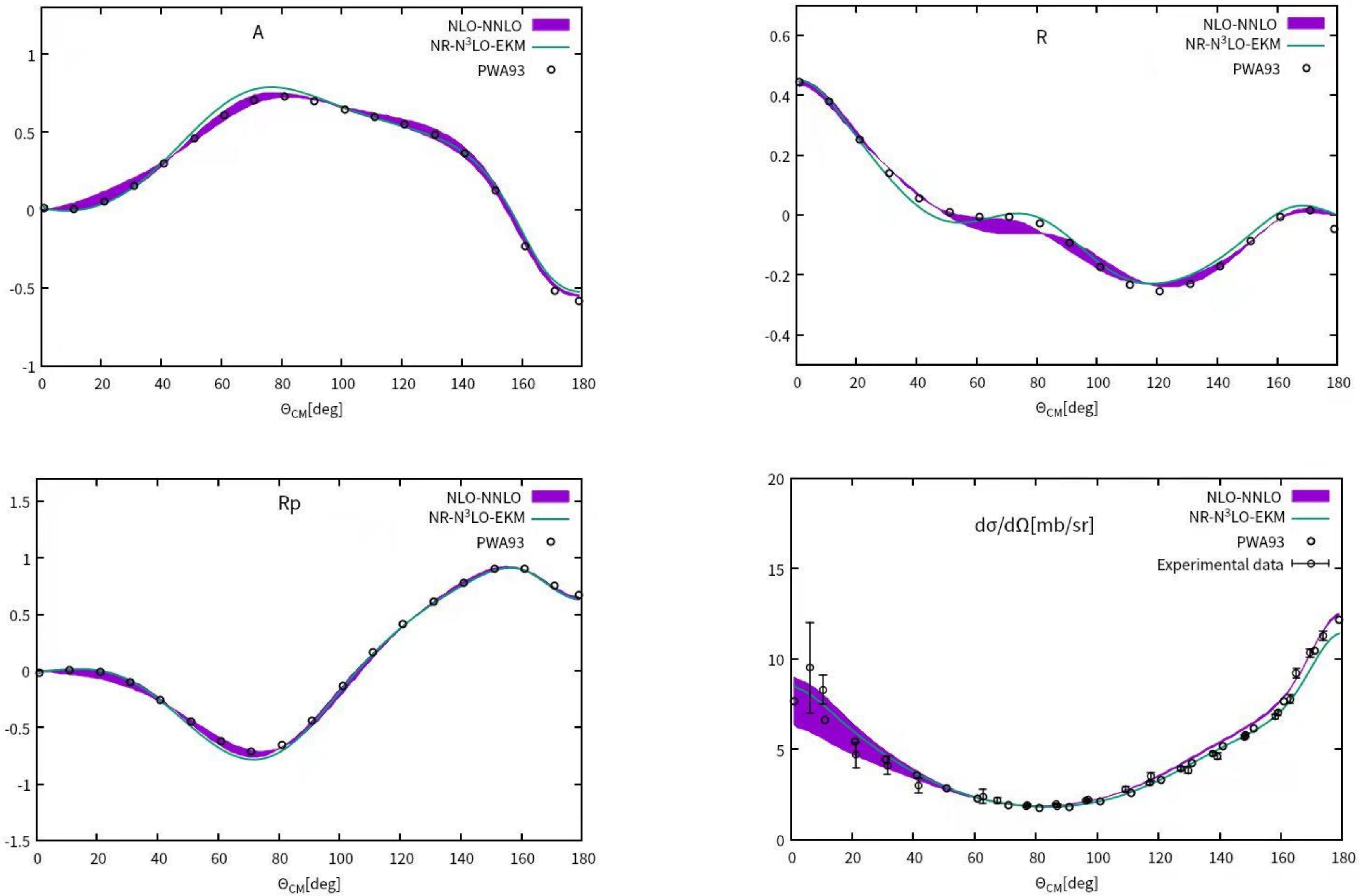


Much better than the Bonn potential

R. Machleidt, K. Holinde, C. Elster, Phys.Rept. 149 (1987) 1-89



Predictions for observables at Tlab=200 MeV



Contents

Summary and outlook

We have constructed the first high-precision relativistic chiral nuclear force

1990/91 Weinberg

1994, Van Kolck

2003, Machleidt

2015, Epelbaum

Beijing 2022

非相对论手征核力

相对论手征核力

LO NN, Weinberg, PLB251 (1990) 288

NN potential with Delta at **N2LO**, Ordonez, Ray, **U. van Kolck**, PRL72 (1994) 1982

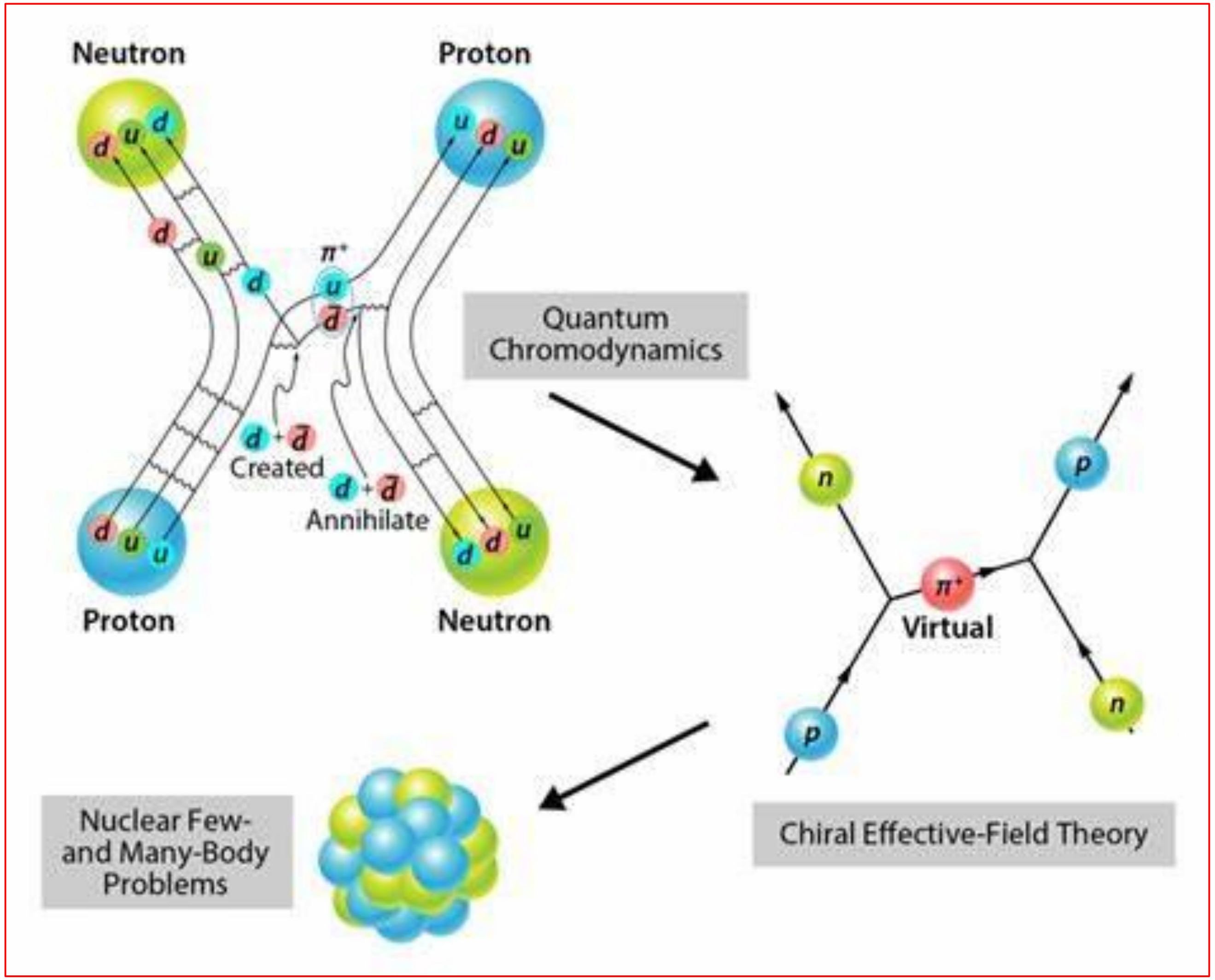
Accurate NN potential at **N3LO**, Entem and **Machleidt**, PRC68(2003)041001

PHYSICAL REVIEW C

Precision NN potential at **N4LO**, **Epelbaum**, Krebs, Meißner, **PRL115 (2015)122301**

Relativistic NN potential at **N2LO**, Lu, Wang, Xiao, **Geng***, Meng, Ring, **PRL128 (2022) 142002**

Summary and outlook



- ☐ Three-nucleon forces
- ☐ Hyperon-nucleon interactions
- ☐ Antinucleon-nucleon scattering
- ☐ Ab initio nuclear structure and reaction studies
- ☐ ……

Collaborators

- **Beihang University:** Jun-Xu Lu, Yang-Xiao, Chun-Xuan Wang, Kai-Wen Li, Xiu-Lei Ren, Zhi-Wei Liu, Jing Song, Qian-Qian Bai, **Pavon Valderrama**
- **Peking University: Jie Meng**
- **Sichuan University: Bing-Wei Long**
- **Institute of Modern Physics: Ju-jun Xie**
- **Technical University of Munich: Peter Ring**

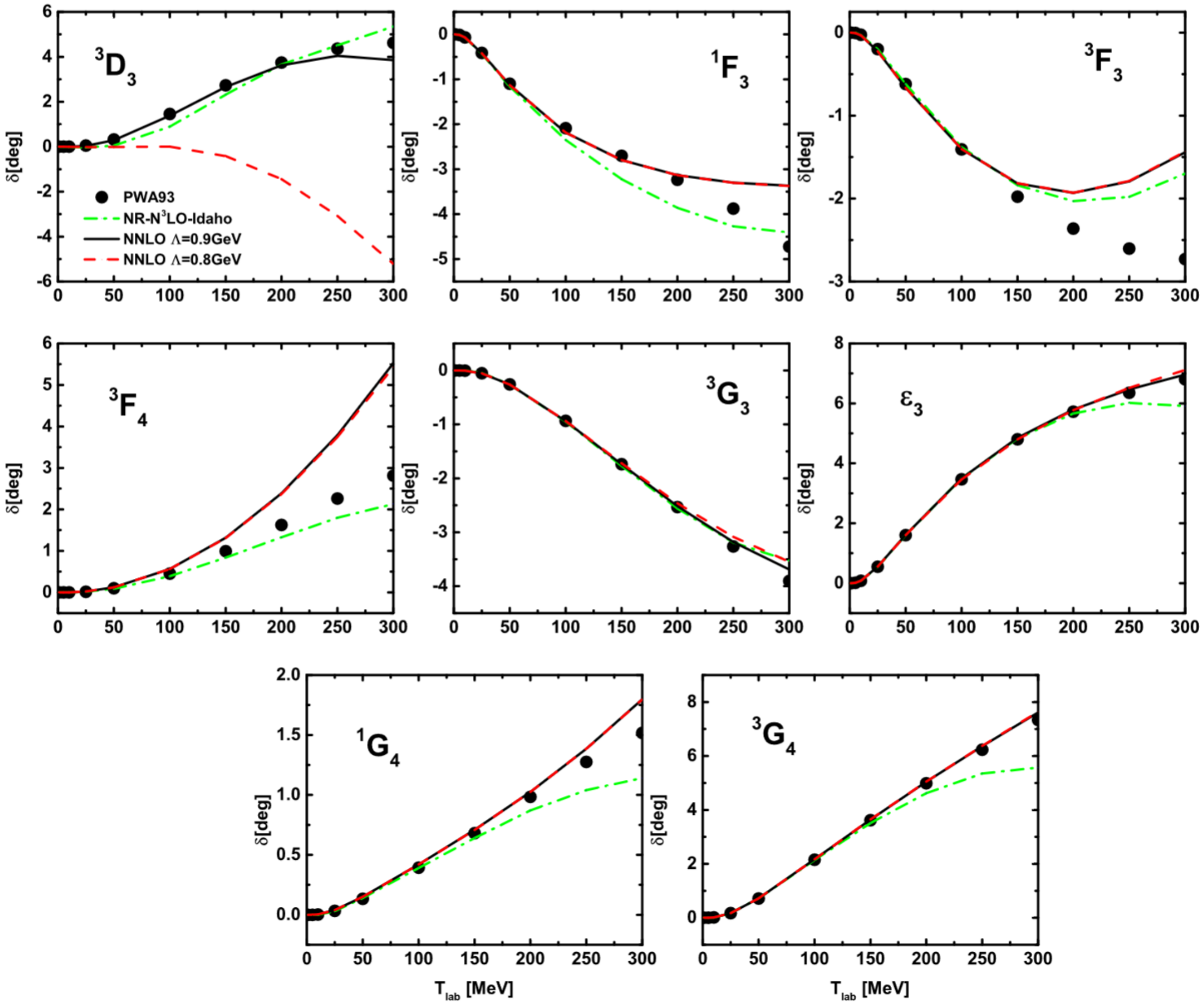
Funding agencies

NSFC
National Natural Science Foundation of China

中华人民共和国教育部
Ministry of Education of the People's Republic of China

北京航空航天大学
BEIHANG UNIVERSITY
1952

Cutoff dependence



Family of chiral two- plus three-nucleon interactions for accurate nuclear structure studies

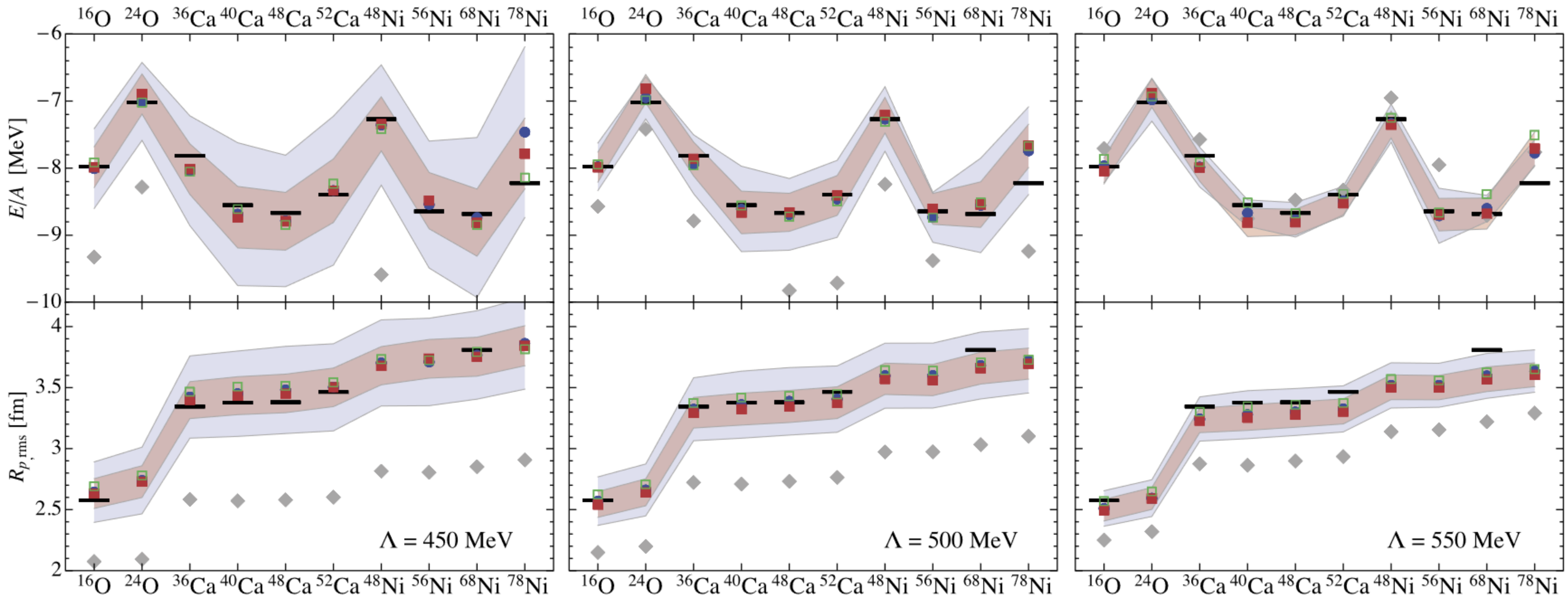
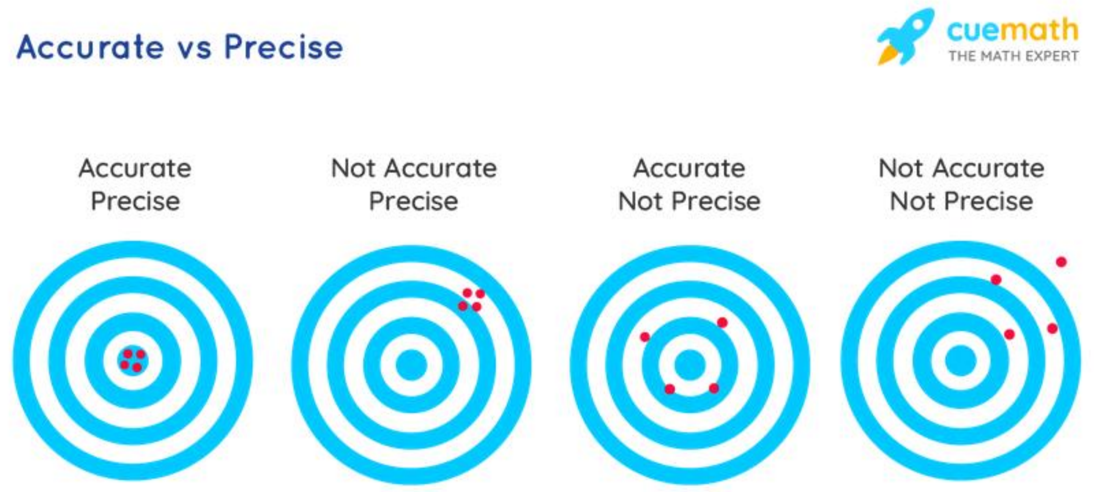


Fig. 4. Ground-state energies (top panels) and point-proton rms radii (bottom panels) obtained in IM-SRG calculations for the NLO (solid gray diamonds), N^{2}LO (blue circles), N^{3}LO (red boxes), and N^{3}LO' (open green boxes) interactions with $\Lambda = 450$ MeV (left), 500 MeV (center), and 550 MeV (right). The error bands for N^{2}LO (blue) and N^{3}LO (red) are derived from the order-by-order behavior and include the many-body uncertainties (see text). Experimental data is indicated by black bars [5,37–39].

Thomas Hüther, Klaus Vobig, Kai Hebeler, Ruprecht Machleidt, Robert Roth, Physics Letters B 808 (2020) 135651

Accurate vs. Precise

Accuracy	Precision
The degree of correctness to the true or exact value.	The degree of exactness to the values obtained each time.
The closeness of the measured value to a standard or true value.	The closeness of two or more measured values, to each other.
Single measurement	Multiple measurements
For something to be accurate consistently, it must be precise.	Precision is independent of accuracy.



Non-perturbative treatment

- Blankenbecler-Sugar equation(BbS quation)

$$\mathcal{T}(p',p|W) = \mathcal{A}(p',p|W) + \int \frac{d^4k}{(2\pi^4)} \mathcal{A}(p',k|W)G(k|W)\mathcal{T}(k,p|W),$$

Solution: difficult

  - 3D reduction

$$\mathcal{T} = \mathcal{V} + \mathcal{V}g\mathcal{T}, \quad \mathcal{V} = \mathcal{A} + \mathcal{A}(G-g)\mathcal{V}$$

$$g = \frac{\pi i \delta(k^0)\Lambda_+^1(\boldsymbol{k})\Lambda_+^2(-\boldsymbol{k})}{2E_k(E_k^2 - s/4 - i\epsilon)}$$

3D propagator

$$T_{l'l}^{sj}(p',p|\sqrt{s}) = V_{l'l}^{sj}(p',p|\sqrt{s}) + \sum_{l''} \int \frac{k^2 dk}{(2\pi)^3} V_{l'l''}^{sj}(p',k|\sqrt{s}) \frac{M^2}{E_k} \frac{1}{p^2 - k^2 + i\varepsilon} T_{l''l}^{sj}(k,p|\sqrt{s})$$

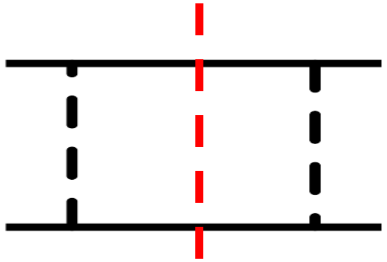
  - Ultraviolet divergence: **Regulator**

$$V_{l'l}^{sj}(p',p|\sqrt{s}) = f_R(p)V(p',p|\sqrt{s})f_R(p')$$

$$f_R(p) = f_R^{\text{sharp}}(p) = \theta(\Lambda^2 - p^2)$$

- Iterated OPE

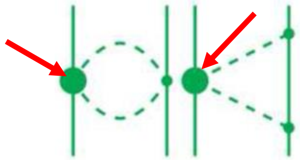
  - Planar box diagram and once-iterated OPE: double counting



$$V_{\text{IOPE}}^{l'l,sj}(p',p|\sqrt{s}) = \sum_{l''} \int \frac{k^2 dk}{(2\pi)^3} V_{\text{OPE}}^{l'l'',sj}(p',k|\sqrt{s}) G_{\text{BbS}}(k|\sqrt{s}) V_{\text{OPE}}^{l''l,sj}(k,p|\sqrt{s}),$$

Fitting details

- n-p phase shifts for partial waves with $J \leq 2$ and $T_{\rm lab} \in \{1,5,10,25,50,100,150,200\}$ MeV
- Function to be minimized: $\tilde{\chi}^2 = \sum (\delta^i - \delta^i_{\rm PWA93})^2$ PWA93: V. G. J. Stoks et al, PRC1993
- Fixed input: LECs $c_{1,2,3,4}$ from $\mathcal{L}^{(2)}_{MB}$



NNLO

Covariant NNLO πN scattering

Y.-H. Chen et al. PRD2013

c_1	c_2	c_3	c_4
-1.39	4.01	-6.61	3.92

- Parameters and regulators

CO-NNLO	**19**	**4(LO) + 13(NLO) + 2(promoted)**	**Sharp cutoff**
NR-N^{3}LO-Idaho	**29**	**2(LO) + 7(NLO) + 15(N^{3}LO) + 2(Charge) + 3($c_{2,3,4}$ semi-free)**	**$e^{-p^{n_1}}$(S), $e^{-p^{n_2}}$(L)**
NR-N^{3}LO-EKM	**26**	**2(LO) + 7(NLO) + 15(N^{3}LO) + 2(Charge)**	**$e^{-p^{n_1}}$(S), $(1-e^{-r^2})^{n_2}$(L)**

- NR-N^{3}LO-Idaho: R. Machleidt and D. R. Entem, Phys.Rev.C(2003), Phys.Rept.(2011)
- NR-N^{3}LO-EKM: E. Epelbaum, H. Krebs, and U. G. Meißner, Eur.Phys.J.A(2015), Phys.Rev.Lett. (2015).

TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [38] below 290 MeV by various *np* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

TABLE III. χ^2/datum for the reproduction of the 1999 *pp* database [38] below 290 MeV by various *pp* potentials.

Bin (MeV)	# of data	N^3LO^a	$NNLO^b$	NLO^b	$AV18^c$
0–100	795	1.05	6.66	57.8	0.96
100–190	411	1.50	28.3	62.0	1.31
190–290	851	1.93	66.8	111.6	1.82
0–290	2057	1.50	35.4	80.1	1.38

R. Machleidt et al. PRC2003

N^3LO: Non-relativistic Chiral NF reached the level of most refined phenomenological forces

NUMBER OF PARAMETERS
for the *np* potential

	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^3LO Q^4 (N^4LO)	N^5LO Q^6
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$-$^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$-$^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	9	24	50

An Accurate nucleon-nucleon potential with charge independence breaking

#16

Robert B. Wiringa (Argonne, PHY), V.G.J. Stoks (Flinders U.), R. Schiavilla (Jefferson Lab and Old Dominion U.) (Aug, 1994)

Published in: *Phys.Rev.C* 51 (1995) 38-51 • e-Print: nucl-th/9408016 [nucl-th]

pdf links DOI cite

2,827 citations

The High precision, charge dependent Bonn nucleon-nucleon potential (CD-Bonn)

#4

R. Machleidt (Idaho U.) (Jun, 2000)

Published in: *Phys.Rev.C* 63 (2001) 024001 • e-Print: nucl-th/0006014 [nucl-th]

pdf DOI cite

1,630 citations

Accurate charge dependent nucleon nucleon potential at fourth order of chiral perturbation theory

#6

D.R. Entem (Idaho U. and Salamanca U.), R. Machleidt (Idaho U.) (Apr, 2003)

Published in: *Phys.Rev.C* 68 (2003) 041001 • e-Print: nucl-th/0304018 [nucl-th]

pdf DOI cite

The Two-nucleon system at next-to-next-to-next-to-leading order

#4

E. Epelbaum (Jefferson Lab), W. Glockle (Ruhr U., Bochum), Ulf-G. Meissner (Bonn U., HISKP and Julich, Forschungszentrum) (May, 2004)

Published in: *Nucl.Phys.A* 747 (2005) 362-424 • e-Print: nucl-th/0405048 [nucl-th]

pdf links DOI cite

660 citations

Precision nucleon-nucleon potential at fifth order in the chiral expansion

#1

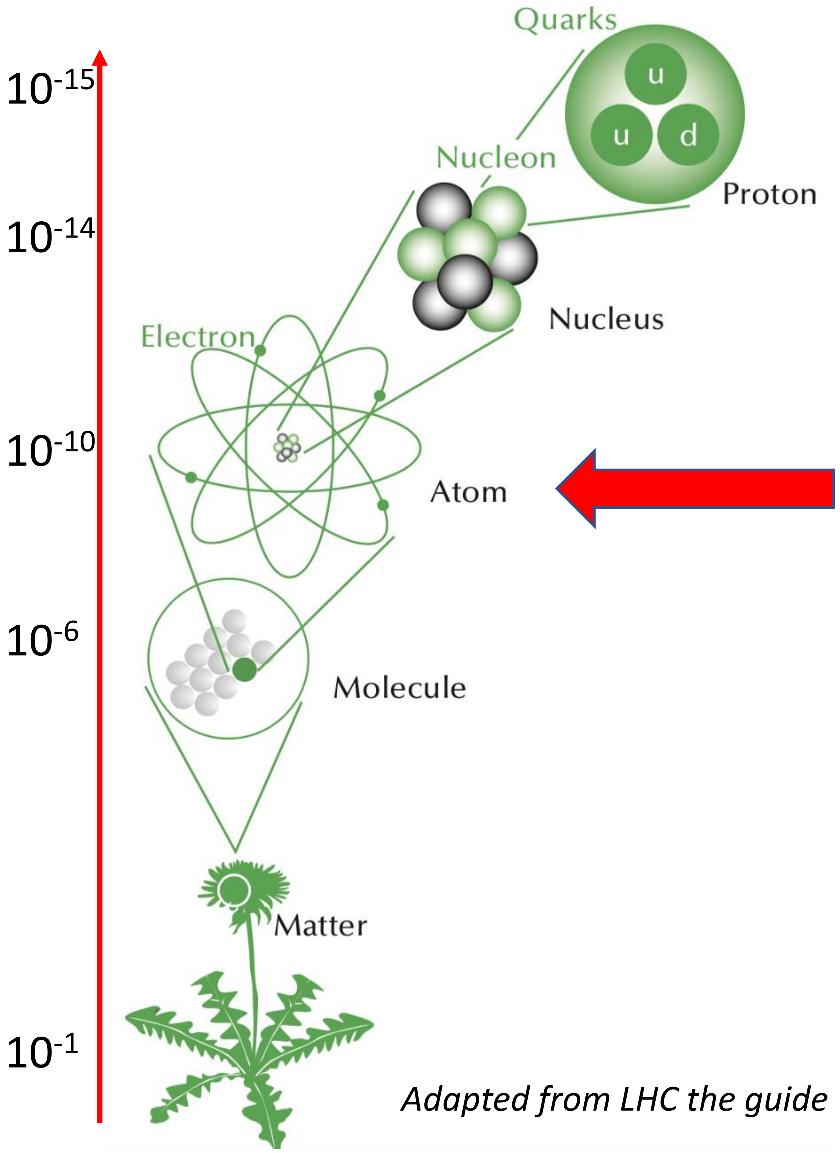
E. Epelbaum (Ruhr U., Bochum), H. Krebs (Ruhr U., Bochum), U.G. Meißner (Bonn U., HISKP and IAS, Julich and JCHP, Julich and Julich, Forschungszentrum) (Dec 15, 2014)

Published in: *Phys.Rev.Lett.* 115 (2015) 12, 122301 • e-Print: 1412.4623 [nucl-th]

pdf DOI cite

317 citations

Nucleons are the essential building blocks of Matter!



Adapted from LHC the guide

IUPAC Periodic Table of the Elements

Key: atomic number / **Symbol** / name / conventional atomic weight / standard atomic weight

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** hydrogen 1.008 [1.0078, 1.0082]																	2 **He** helium 4.0026
3 **Li** lithium 6.94 [6.938, 6.997]	4 **Be** beryllium 9.0122											5 **B** boron 10.81 [10.806, 10.821]	6 **C** carbon 12.011 [12.009, 12.012]	7 **N** nitrogen 14.007 [14.006, 14.008]	8 **O** oxygen 15.999 [15.999, 16.000]	9 **F** fluorine 18.998	10 **Ne** neon 20.180
11 **Na** sodium 22.990	12 **Mg** magnesium 24.305 [24.304, 24.307]											13 **Al** aluminium 26.982	14 **Si** silicon 28.085 [28.084, 28.086]	15 **P** phosphorus 30.974	16 **S** sulfur 32.06 [32.059, 32.076]	17 **Cl** chlorine 35.45 [35.446, 35.457]	18 **Ar** argon 39.95 [39.792, 39.963]
19 **K** potassium 39.098	20 **Ca** calcium 40.078(4)	21 **Sc** scandium 44.956	22 **Ti** titanium 47.867	23 **V** vanadium 50.942	24 **Cr** chromium 51.996	25 **Mn** manganese 54.938	26 **Fe** iron 55.845(2)	27 **Co** cobalt 58.933	28 **Ni** nickel 58.693	29 **Cu** copper 63.546(3)	30 **Zn** zinc 65.38(2)	31 **Ga** gallium 69.723	32 **Ge** germanium 72.630(8)	33 **As** arsenic 74.922	34 **Se** selenium 78.971(8)	35 **Br** bromine 79.904 [79.901, 79.907]	36 **Kr** krypton 83.798(2)
37 **Rb** rubidium 85.468	38 **Sr** strontium 87.62	39 **Y** yttrium 88.906	40 **Zr** zirconium 91.224(2)	41 **Nb** niobium 92.906	42 **Mo** molybdenum 95.95	43 **Tc** technetium	44 **Ru** ruthenium 101.07(2)	45 **Rh** rhodium 102.91	46 **Pd** palladium 106.42	47 **Ag** silver 107.87	48 **Cd** cadmium 112.41	49 **In** indium 114.82	50 **Sn** tin 118.71	51 **Sb** antimony 121.76	52 **Te** tellurium 127.60(3)	53 **I** iodine 126.90	54 **Xe** xenon 131.29
55 **Cs** caesium 132.91	56 **Ba** barium 137.33	57-71 lanthanoids	72 **Hf** hafnium 178.49(2)	73 **Ta** tantalum 180.95	74 **W** tungsten 183.84	75 **Re** rhenium 186.21	76 **Os** osmium 190.23(3)	77 **Ir** iridium 192.22	78 **Pt** platinum 195.08	79 **Au** gold 196.97	80 **Hg** mercury 200.59	81 **Tl** thallium 204.38 [204.38, 204.39]	82 **Pb** lead 207.2	83 **Bi** bismuth 208.98	84 **Po** polonium	85 **At** astatine	86 **Rn** radon
87 **Fr** francium	88 **Ra** radium	89-103 actinoids	104 **Rf** rutherfordium	105 **Db** dubnium	106 **Sg** seaborgium	107 **Bh** bohrium	108 **Hs** hassium	109 **Mt** meitnerium	110 **Ds** darmstadtium	111 **Rg** roentgenium	112 **Cn** copernicium	113 **Nh** nihonium	114 **Fl** flerovium	115 **Mc** moscovium	116 **Lv** livermorium	117 **Ts** tennessine	118 **Og** oganesson

57 **La** lanthanum 138.91	58 **Ce** cerium 140.12	59 **Pr** praseodymium 140.91	60 **Nd** neodymium 144.24	61 **Pm** promethium	62 **Sm** samarium 150.36(2)	63 **Eu** europium 151.96	64 **Gd** gadolinium 157.25(3)	65 **Tb** terbium 158.93	66 **Dy** dysprosium 162.50	67 **Ho** holmium 164.93	68 **Er** erbium 167.26	69 **Tm** thulium 168.93	70 **Yb** ytterbium 173.05	71 **Lu** lutetium 174.97
89 **Ac** actinium	90 **Th** thorium 232.04	91 **Pa** protactinium 231.04	92 **U** uranium 238.03	93 **Np** neptunium	94 **Pu** plutonium	95 **Am** americium	96 **Cm** curium	97 **Bk** berkelium	98 **Cf** californium	99 **Es** einsteinium	100 **Fm** fermium	101 **Md** mendelevium	102 **No** nobelium	103 **Lr** lawrencium

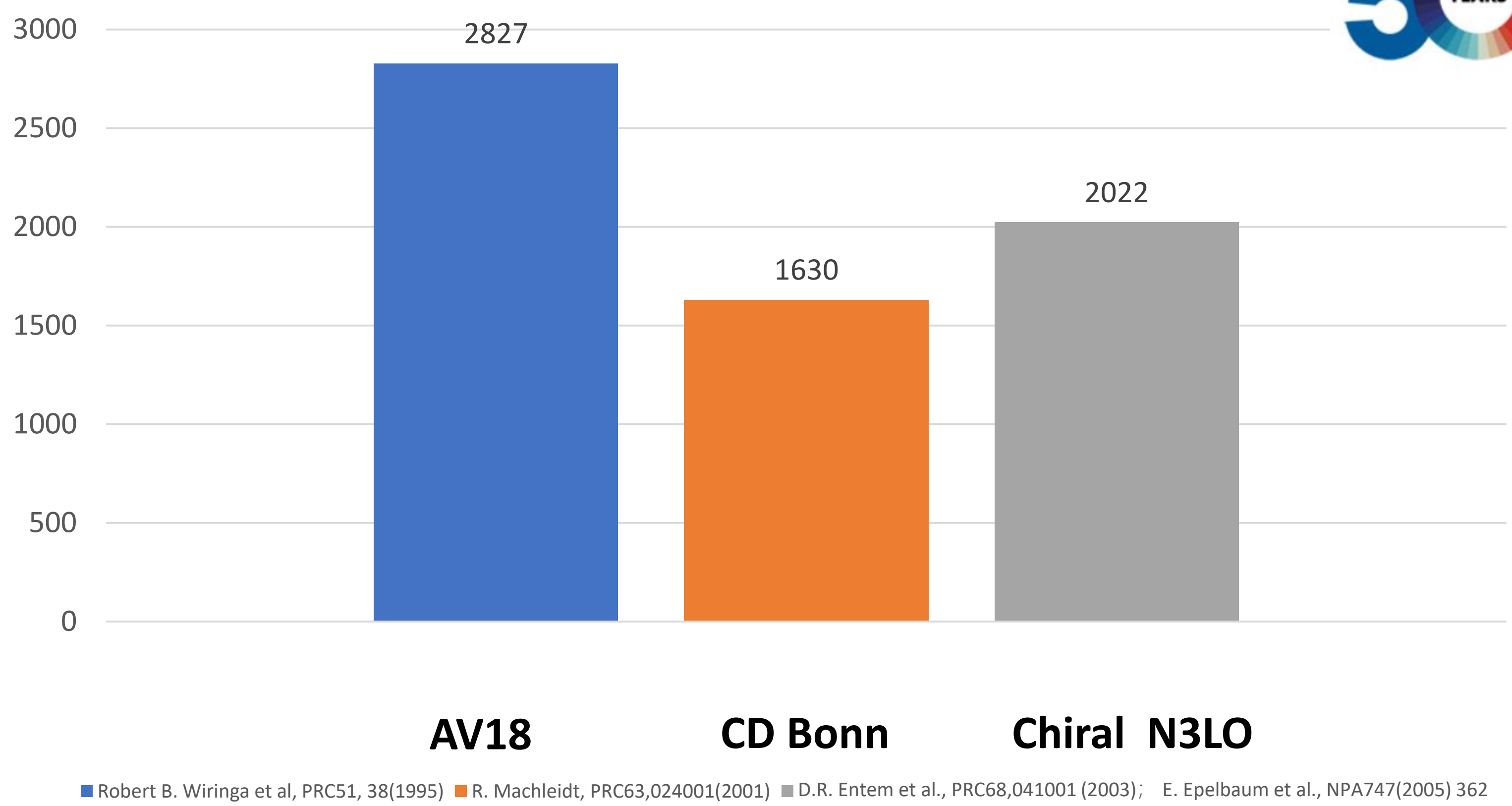
INTERNATIONAL UNION OF PURE AND APPLIED CHEMISTRY

For notes and updates to this table, see www.iupac.org. This version is dated 1 December 2018.

United Nations Educational, Scientific and Cultural Organization

International Year of the Periodic Table of Chemical Elements

A brief account of the long history: accurate/precise

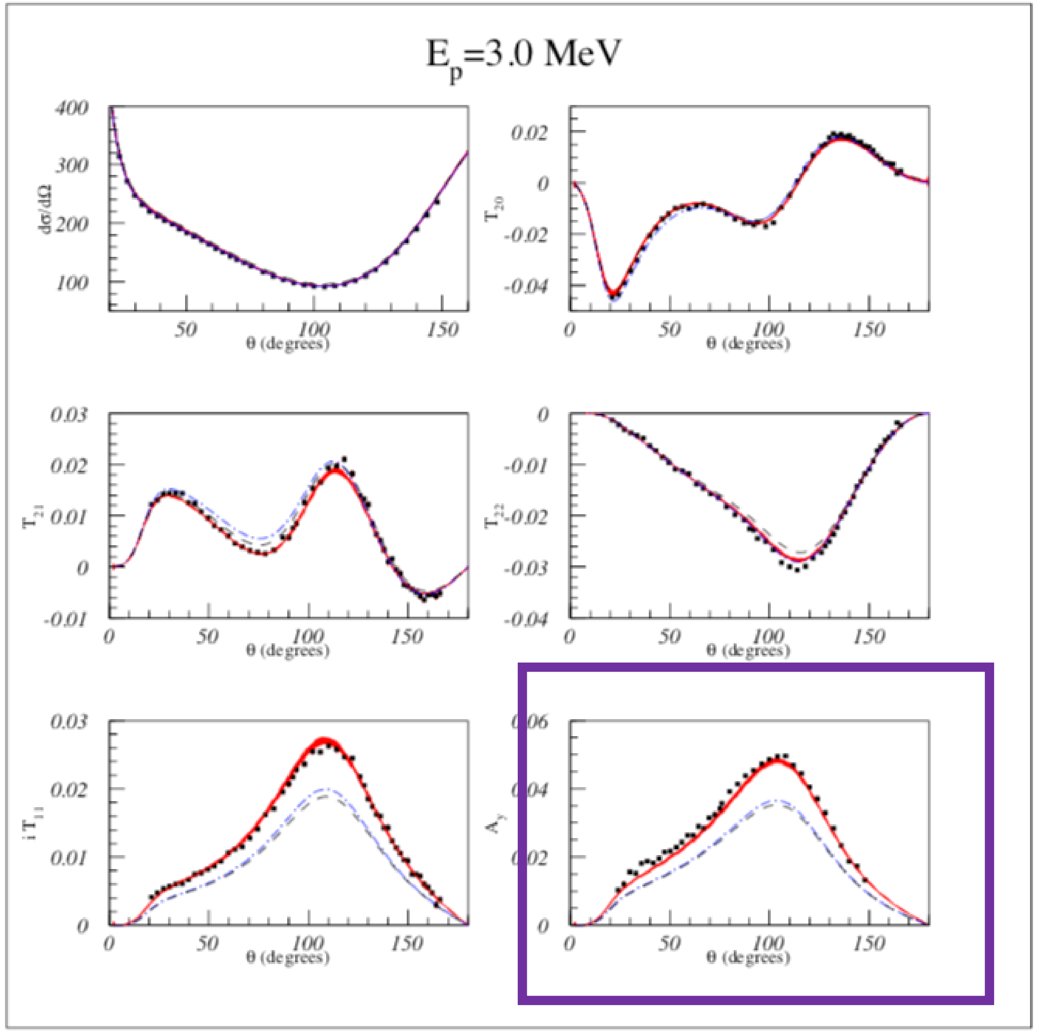


PHYSICAL REVIEW C

7/41

In 2020, to celebrate the 50th anniversary of PRC, "we are putting together a collection of milestone papers that **remain central to developments in the field of nuclear physics**. These papers **announce major discoveries or open up new avenues of research**"

Promising three-body potential



1811.09398

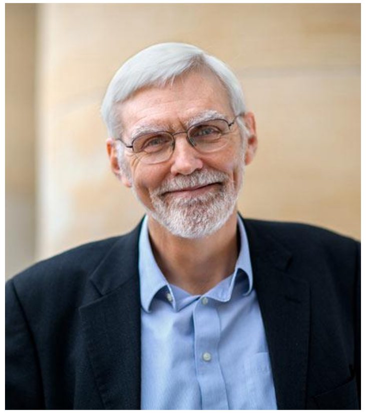
L. Girlanda, A. Kievsky, M. Viviani and L.E. Marcucci

"Fit results in the leading order of the relativistic counting to a set of cross section and polarization p − d observables at 3 MeV proton energy, for Λ = 200 − 500 MeV (red bands) as compared to the purely two-body AV18 interaction (dashed, black lines) and to the AV18+UIX two- and three-nucleon interaction (dashed-dotted, blue lines). "

Ay puzzle– a 32 year old persistent problem!

Renormalization Group Invariance

- **A fundamental feature of EFT** that physics at low-energy/long-distance scales is **insensitive to** the details at high-energy/short-distance scales



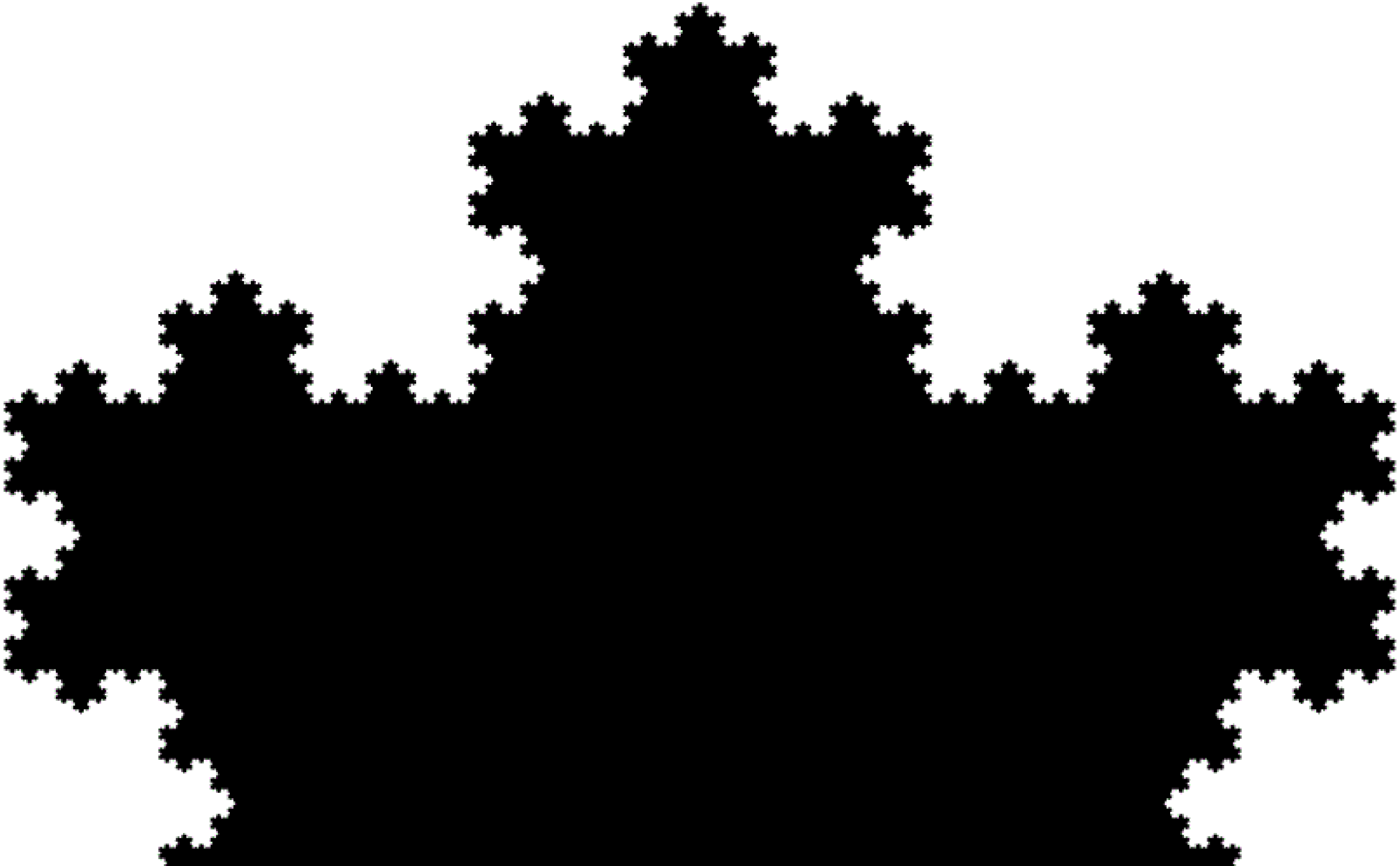
Peter Lepage

- The NN interaction can be considered properly renormalized **when the calculated observables are independent of the cutoff scale within the range of validity of the ChEFT** or involves a small residual cutoff dependence due to the truncation of the chiral expansion.

- In the language of **Wilson's renormalization group**, this means that **the LECs must run with the cutoff** scale in such a way that the scattering amplitude becomes renormalization group invariant (RGI)

Kenneth G. Wilson

RGI vs. self-similarity



RGI-a highly debated topic

Kaplan

van Kolck

Epelbaum

Machleidt

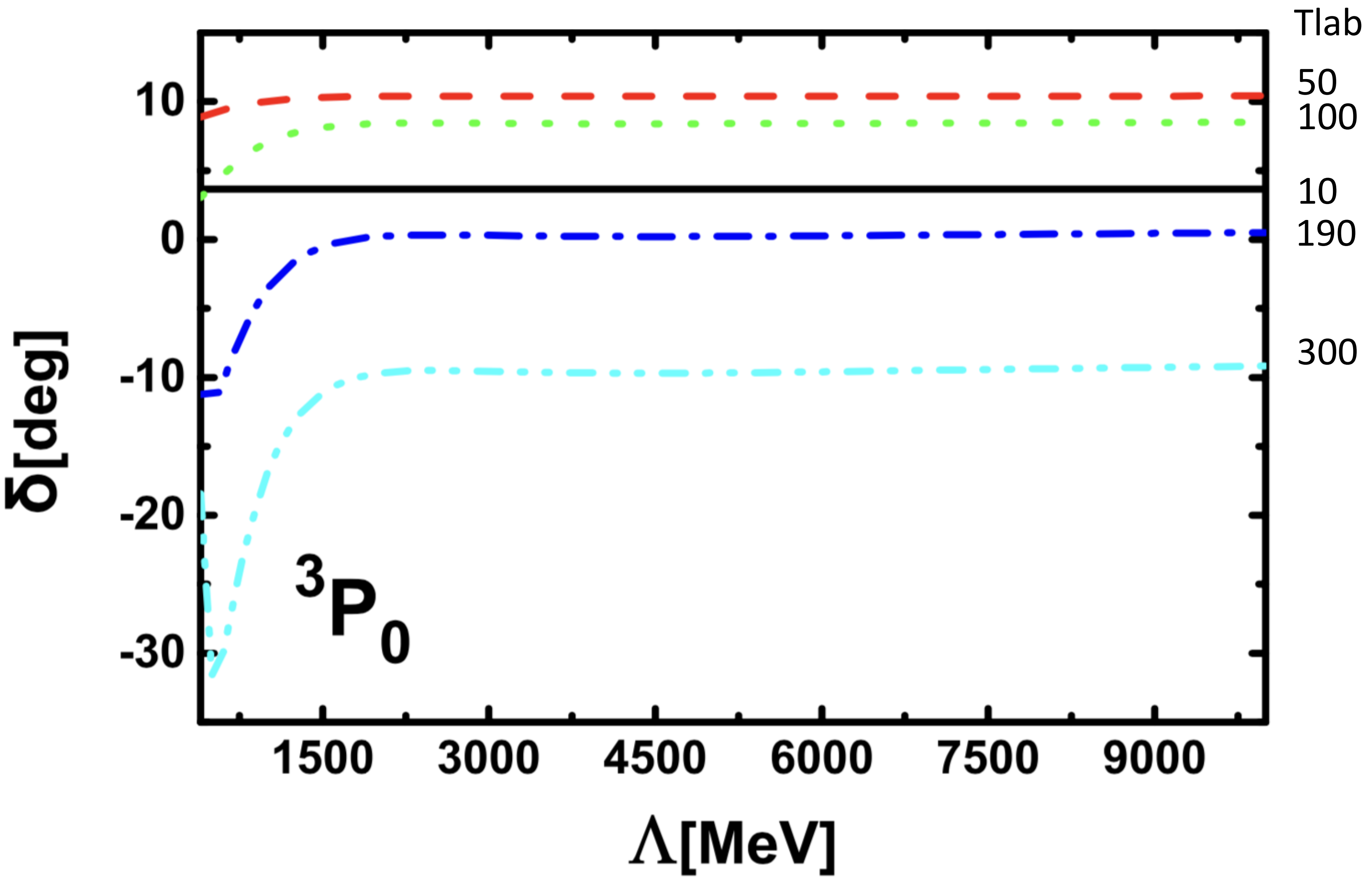
Key issues

- ☐ **Whether RGI is essential?**
- ☐ **How to realize it?**

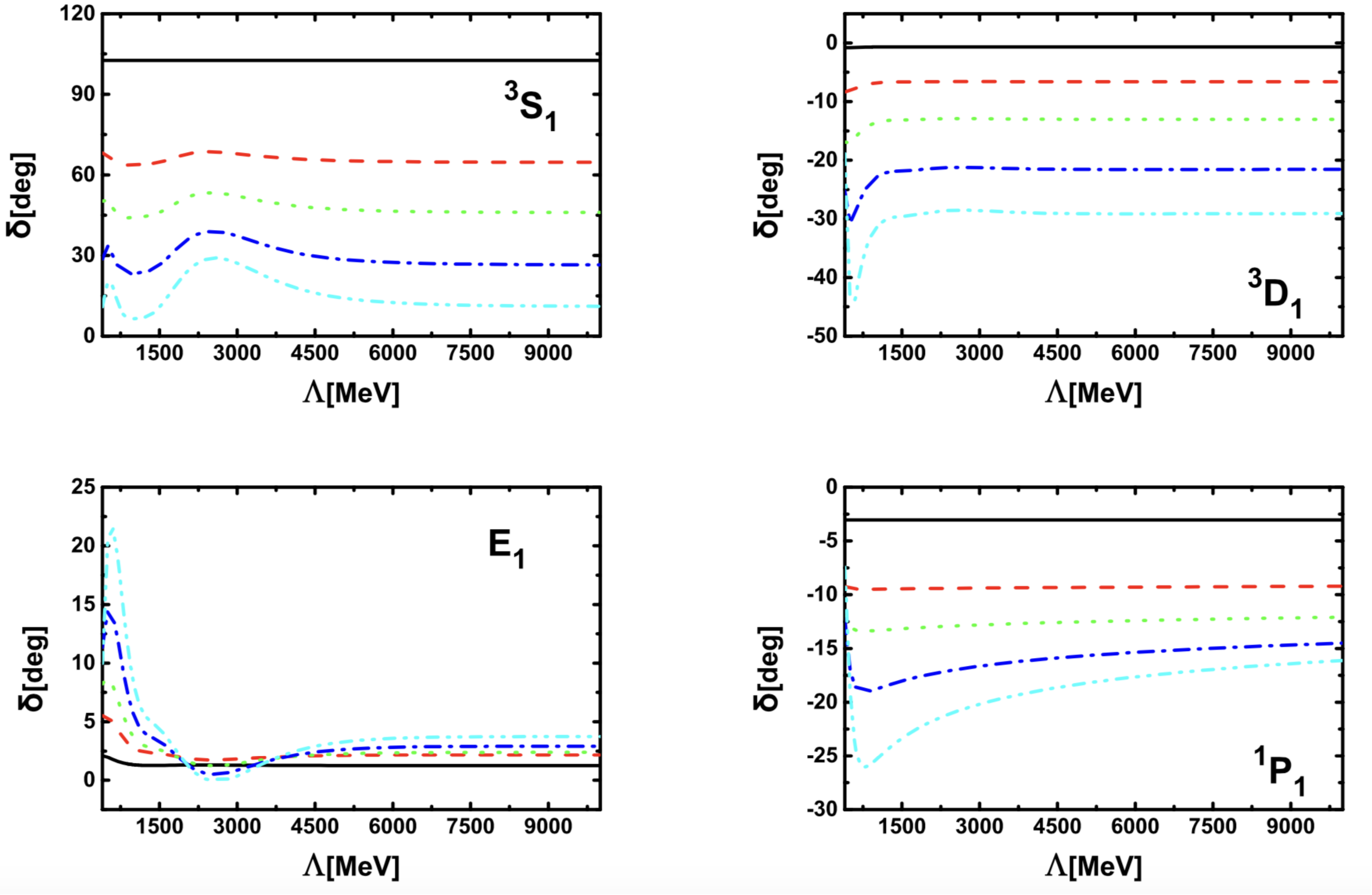
Key references

- ✓ *A New expansion for nucleon-nucleon interactions, David B. Kaplan et al., Phys.Lett. B424 (1998) 390*
- ✓ *Towards a perturbative theory of nuclear forces, S.R. Beane et al., Nucl.Phys. A700 (2002) 377*
- ✓ *How to renormalize the Schrodinger equation, P. Lepage,* nucl-th/9706029

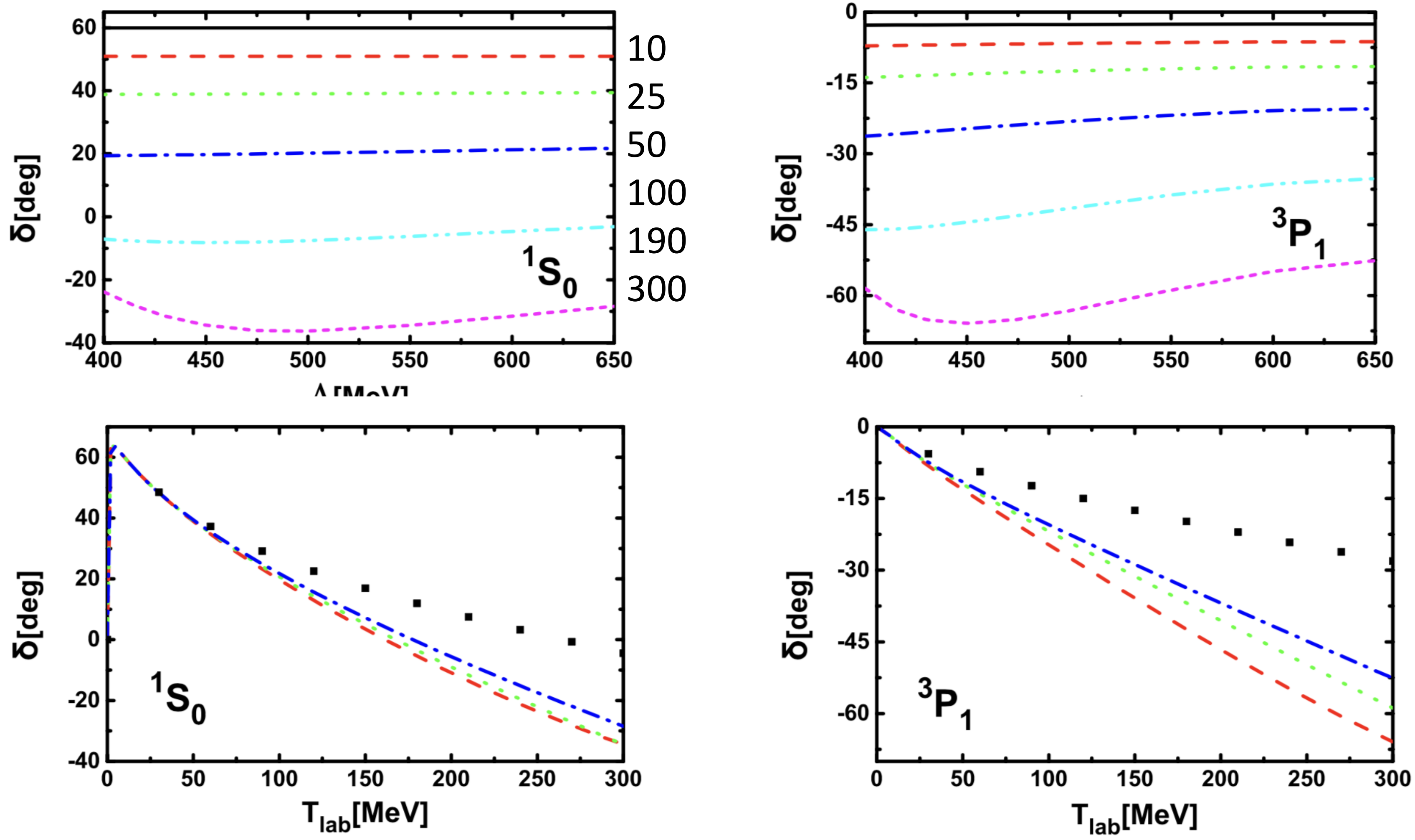
3P0 issue nicely solved



3S1, 3D1, E1, and 1P1 as well



Interesting correlation 1s0 & 3P1



- Pure contact interactions cannot describe 1S0 – large cutoff limit
- NLO 3P1 in the Weinberg case loses RGI

核结构第一性原理方法

◆ **Realistic nuclear forces:**

Nijmegen, Paris, JISP, Argonne, Bonn, Chiral EFT (NN, NNN)

◆ **Renormalization: full space (bare) → finite space (effective)**

G-Matrix, UCOM, OLS, $V_{low\text{-}k}$, SRG

◆ ***ab initio many-body* methods**

No Core Shell Model (NCSM)

Coupled Cluster (CC)

In-medium Similarity Renormalization Group (IM-SRG)

Configuration interaction shell model

Selfconsistent Green's Function

Diffusion Monte Carlo

Many-Body Perturbation Theory (MBPT)

Lattice Nuclear Chiral EFT

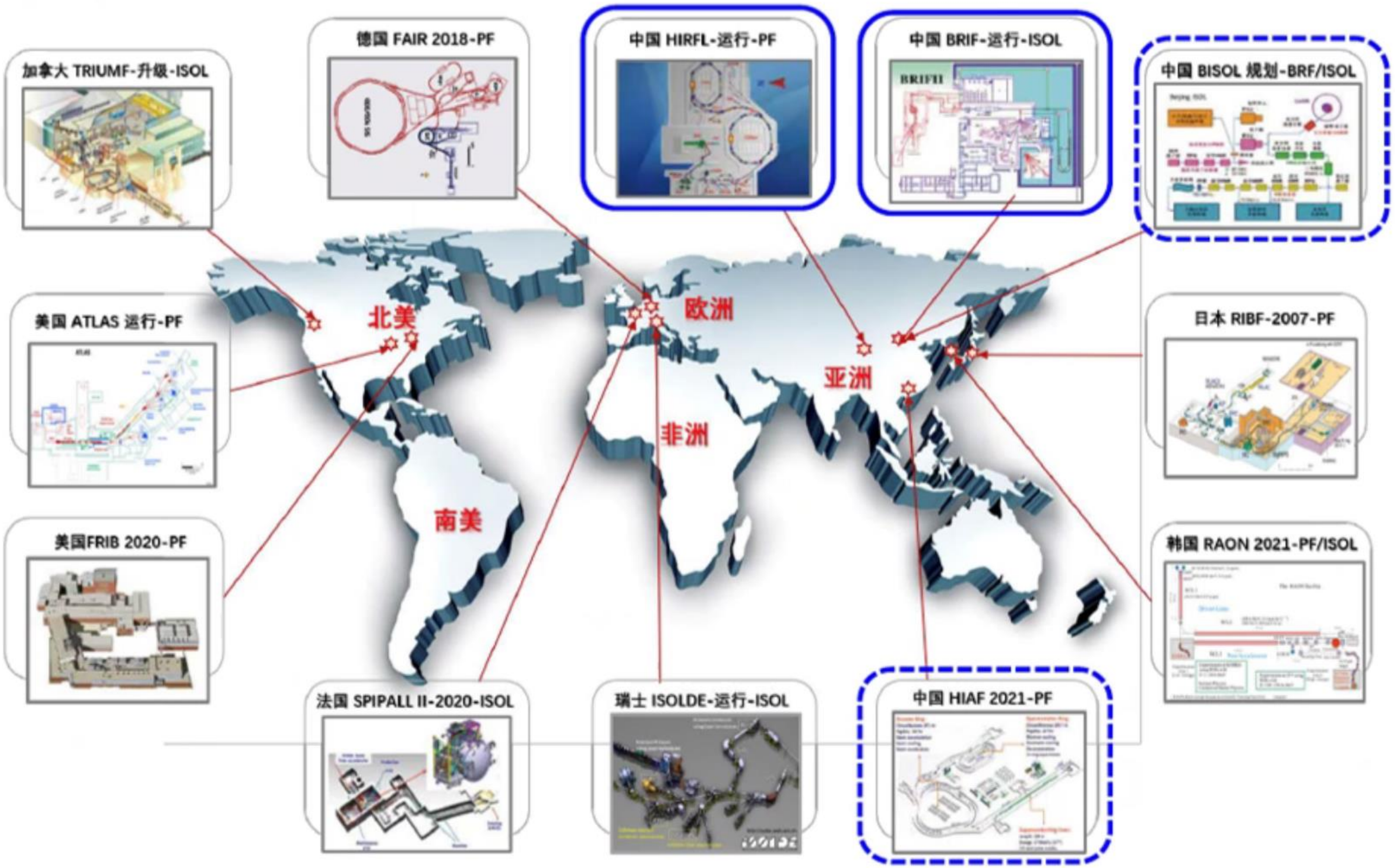
(Dirac-) Bruckner-Hartree-Fock …

Number of parameters

NUMBER OF PARAMETERS
for the np potential

	Nijmegen PWA93	CD-Bonn "high precision"	NLO Q^2 (NNLO)	N^3LO Q^4 (N^4LO)	N^5LO Q^6
$^1S_0$	3	4	2	4	6
$^3S_1$	3	4	2	4	6
$^3S_1$-$^3D_1$	2	2	1	3	6
$^1P_1$	3	3	1	2	4
$^3P_0$	3	2	1	2	4
$^3P_1$	2	2	1	2	4
$^3P_2$	3	3	1	2	4
$^3P_2$-$^3F_2$	2	1	0	1	3
$^1D_2$	2	3	0	1	2
$^3D_1$	2	1	0	1	2
$^3D_2$	2	2	0	1	2
$^3D_3$	1	2	0	1	2
$^3D_3$-$^3G_3$	1	0	0	0	1
$^1F_3$	1	1	0	0	1
$^3F_2$	1	2	0	0	1
$^3F_3$	1	2	0	0	1
$^3F_4$	2	1	0	0	1
$^3F_4$-$^3H_4$	0	0	0	0	0
$^1G_4$	1	0	0	0	0
$^3G_3$	0	1	0	0	0
$^3G_4$	0	1	0	0	0
$^3G_5$	0	1	0	0	0
Total	35	38	9	24	50

放射性核束物理(RIB)大科学装置



更高流强、更远离稳定线

5

HIAF简介

2010	May, 2011	December, 2015	April, 2017	2018
提交*HIAF*项目建议书	*HIAF*进入“十二五”候选项目名单（共16项）	建设地点最终确定，*HIAF*项目得到政府批准立项	*HIAF*项目可研报告获批	12月23号，HIAF建设正式启动

强流重离子加速器装置（HIAF）

国际上脉冲束流强度最高的重离子加速器装置

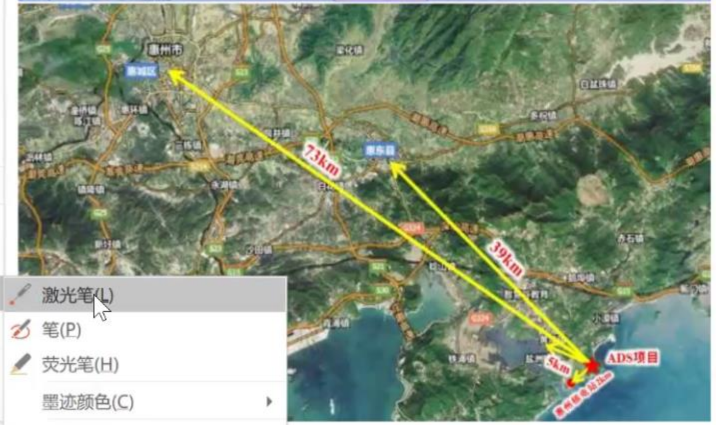
“十二五”
国家重大科技基础设施

定位于重离子物理及应用研究

科学目标：

1）认识原子核中的有效相互作用
2）理解宇宙中从铁到铀重元素的来源
3）研究高能量密度物理性质
4）空间辐射环境地面模拟
5）……

■坐落地点：广东省惠州



下一张(N)
上一张(P)
上次查看的位置(V)
查看所有幻灯片(A)
放大(Z)
自定义放映(W)
显示演示者视图(R)
屏幕(C)
指针选项(O)
帮助(H)
暂停(S)
结束放映(E)
激光笔(L)
笔(P)
荧光笔(H)
墨迹颜色(C)
橡皮擦(R)

Covariant NN contact Lagrangians (N4LO)

$\widetilde{O}_1$	$(\bar\psi\psi)(\bar\psi\psi)$	$\widetilde{O}_{21}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^2\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_2$	$(\bar\psi\gamma^\mu\psi)\left(\bar\psi\gamma_\mu\psi\right)$	$\widetilde{O}_{22}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\alpha}\psi)\partial^2\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_3$	$(\bar\psi\gamma_5\gamma^\mu\psi)\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$	$\widetilde{O}_{23}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^\beta\partial_\nu\left(\bar\psi\sigma_{\alpha\beta}i\overleftrightarrow{\partial}_{\mu}\psi\right)$
$\widetilde{O}_4$	$(\bar\psi\sigma^{\mu\nu}\psi)\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{24}$	$\frac{1}{16m^4}(\bar\psi\psi)\partial^4(\bar\psi\psi)$
$\widetilde{O}_5$	$(\bar\psi\gamma_5\psi)(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{25}$	$\frac{1}{16m^4}(\bar\psi\gamma^\mu\psi)\partial^4\left(\bar\psi\gamma_\mu\psi\right)$
$\widetilde{O}_6$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{26}$	$\frac{1}{16m^4}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^4\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$
$\widetilde{O}_7$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{27}$	$\frac{1}{16m^4}(\bar\psi\sigma^{\mu\nu}\psi)\partial^4\left(\bar\psi\sigma_{\mu\nu}\psi\right)$
$\widetilde{O}_8$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}\psi\right)\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{28}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5 i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_5$
$\widetilde{O}_9$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\alpha}\psi)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{29}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\alpha i\overleftrightarrow{\partial}_{\mu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_6$
$\widetilde{O}_{10}$	$\frac{1}{4m^2}(\bar\psi\psi)\partial^2(\bar\psi\psi)$	$\widetilde{O}_{30}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_7$
$\widetilde{O}_{11}$	$\frac{1}{4m^2}(\bar\psi\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_\mu\psi\right)$	$\widetilde{O}_{31}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\mu}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial^\alpha\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_8$
$\widetilde{O}_{12}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\gamma^\mu\psi)\partial^2\left(\bar\psi\gamma_5\gamma_\mu\psi\right)$	$\widetilde{O}_{32}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\partial_\alpha\partial^\nu\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\beta}\psi\right)-\widetilde{O}_9$
$\widetilde{O}_{13}$	$\frac{1}{4m^2}(\bar\psi\sigma^{\mu\nu}\psi)\partial^2\left(\bar\psi\sigma_{\mu\nu}\psi\right)$	$\widetilde{O}_{33}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{10}$
$\widetilde{O}_{14}$	$\frac{1}{4m^2}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_1$	$\widetilde{O}_{34}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{11}$
$\widetilde{O}_{15}$	$\frac{1}{4m^2}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_2$	$\widetilde{O}_{35}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{12}$
$\widetilde{O}_{16}$	$\frac{1}{4m^2}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_3$	$\widetilde{O}_{36}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_{13}$
$\widetilde{O}_{17}$	$\frac{1}{4m^2}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}\psi\right)-\widetilde{O}_4$	$\widetilde{O}_{37}$	$\frac{1}{16m^4}\left(\bar\psi i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{14}-\widetilde{O}_1$
$\widetilde{O}_{18}$	$\frac{1}{4m^2}(\bar\psi\gamma_5\psi)\partial^2(\bar\psi\gamma_5\psi)$	$\widetilde{O}_{38}$	$\frac{1}{16m^4}\left(\bar\psi\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{15}-\widetilde{O}_2$
$\widetilde{O}_{19}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\nu}\psi\right)\partial^2\left(\bar\psi\gamma_5\gamma_\nu i\overleftrightarrow{\partial}_{\mu}\psi\right)$	$\widetilde{O}_{39}$	$\frac{1}{16m^4}\left(\bar\psi\gamma_5\gamma^\mu i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\gamma_5\gamma_\mu i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{16}-\widetilde{O}_3$
$\widetilde{O}_{20}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}\psi\right)\partial^2\left(\bar\psi\sigma_{\mu\alpha}i\overleftrightarrow{\partial}_{\nu}\psi\right)$	$\widetilde{O}_{40}$	$\frac{1}{16m^4}\left(\bar\psi\sigma^{\mu\nu}i\overleftrightarrow{\partial}^{\alpha}i\overleftrightarrow{\partial}^{\beta}\psi\right)\left(\bar\psi\sigma_{\mu\nu}i\overleftrightarrow{\partial}_{\alpha}i\overleftrightarrow{\partial}_{\beta}\psi\right)-2\widetilde{O}_{17}-\widetilde{O}_4$

Relativistic: 40
vs.
Non-relativistic: 24

Non-relativistic reduction

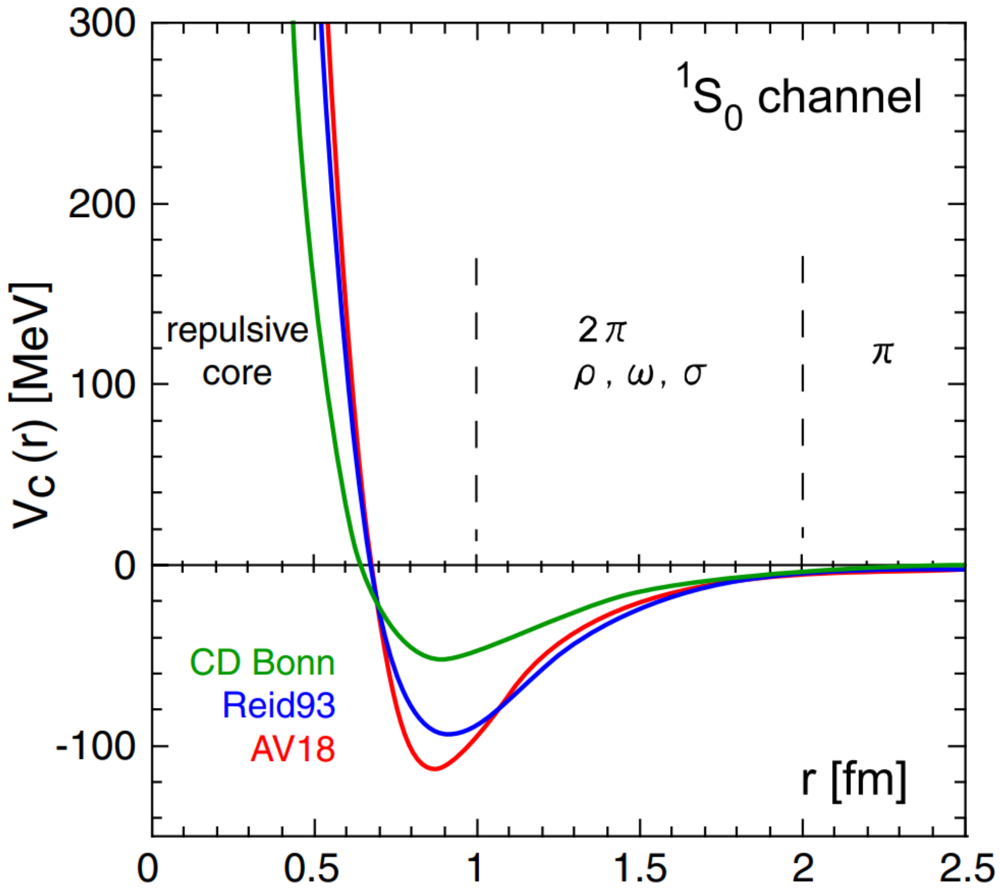
- **Non-relativistic expansion:** $\psi \to N$, expand Lagrangians in terms of **1/m**
  - ✓ **Relativistic nucleon field operator:** $\psi(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tilde{b}_s(\mathbf{p})\, u^{(s)}(\mathbf{p})\, \mathrm{e}^{-ip\cdot x},$
  - ✓ **Non-relativistic nucleon field operator:** $N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3} b_s(\mathbf{p})\, \chi_s\, \mathrm{e}^{-ip\cdot x}$
  - ✓ **Expansion of field operator**
  - ✓ **Dirac matrices expressed in term of pauli matrices**

$$\psi(x) = \left[\begin{pmatrix}1\\0\end{pmatrix} - \frac{i}{2m}\begin{pmatrix}0\\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\end{pmatrix} + \frac{1}{8m^2}\begin{pmatrix}\boldsymbol{\nabla}^2\\0\end{pmatrix} - \frac{3i}{16m^3}\begin{pmatrix}0\\ \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\nabla}^2\end{pmatrix} + \frac{11}{128m^4}\begin{pmatrix}\boldsymbol{\nabla}^4\\0\end{pmatrix}\right] N(x) + \mathcal{O}(Q^5)\,.$$

$$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ \gamma^5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \vec{\gamma} = \begin{pmatrix}0 & \vec{\sigma}\\ -\vec{\sigma} & 0\end{pmatrix},\ \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

- **After expansion and keeping only appropriate powers of $1/m_N$, we can reduce the 40 relativistic terms into the 2+7+15 non-relativistic terms**

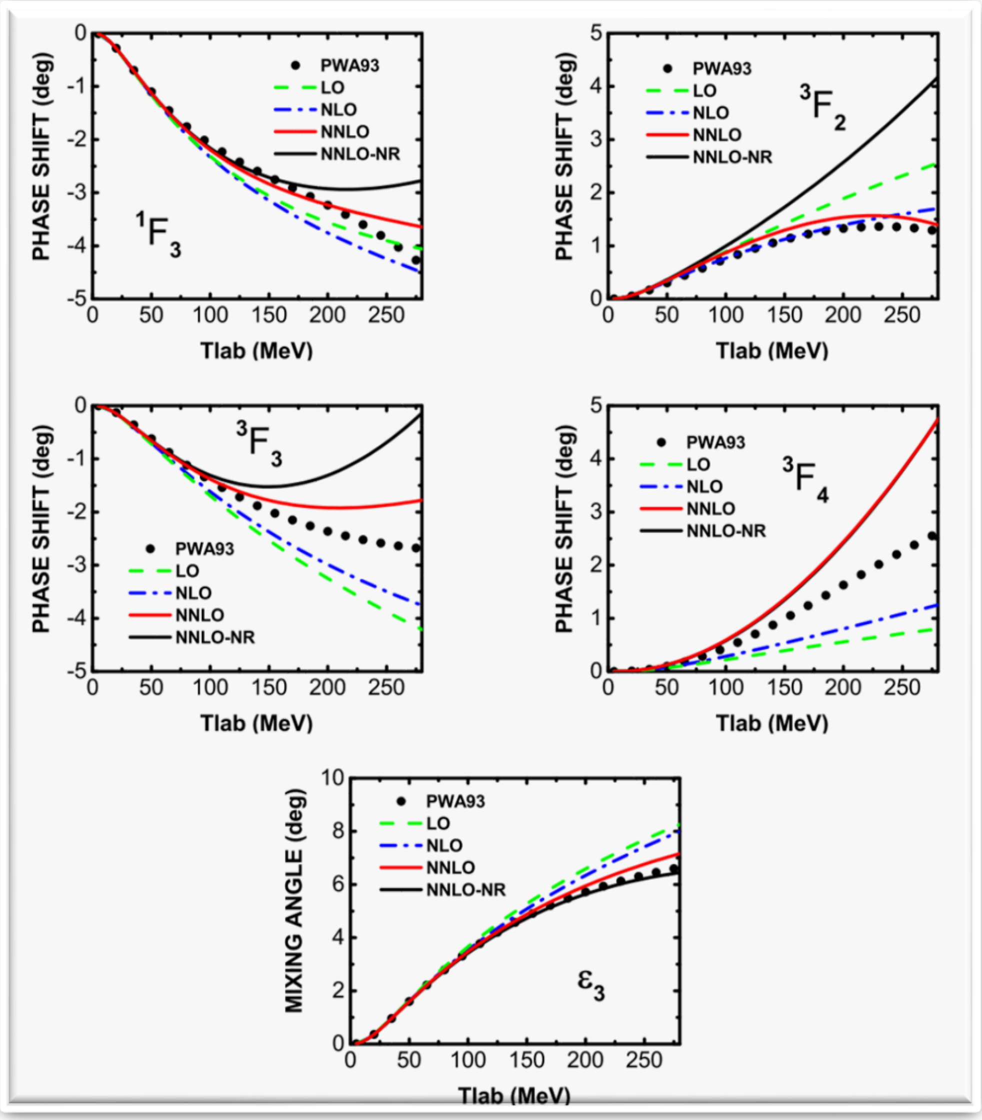
Highly nontrivial



*PRL*99(2007)022001

- There are no unknown LECs
- Contribute to all the partial waves, but almost saturate partial waves of $L \geq 3$
- Perfect candidates to check chiral corrections and the convergence of chiral expansions

F-waves



- Agreement with data is better than D-wave
- Relativistic TPE more moderate than non-relativistic TPE, and agree better with data (except 3F4)
- Improvement still needed

TPE contributions in a word

- Perturbative relativistic corrections are relatively small
- **Amazingly,** they **improve** the NR results
- **Non-perturbative summation improves further the description**