Probing proton structure at LHCb

尹航

味物理讲座

Jan. 6th, 2022

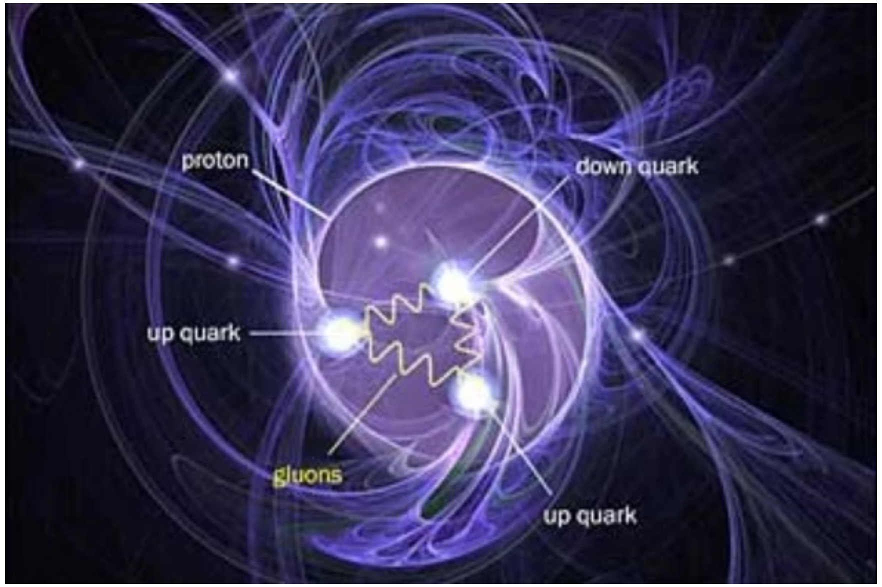
Outline

- Introduction
- Recent LHCb results on the probing of proton:
  - Precision measurement of Z boson production cross-section
    arXiv:2112.07458 Submitted to JHEP
  - Hints of the intrinsic charm (IC)
    arXiv:2109.08084 Submitted to PRL
- Prospects
- Summary

The proton in the spotlight

THE SCIENCES

Proton Spin Mystery Gains a New Clue



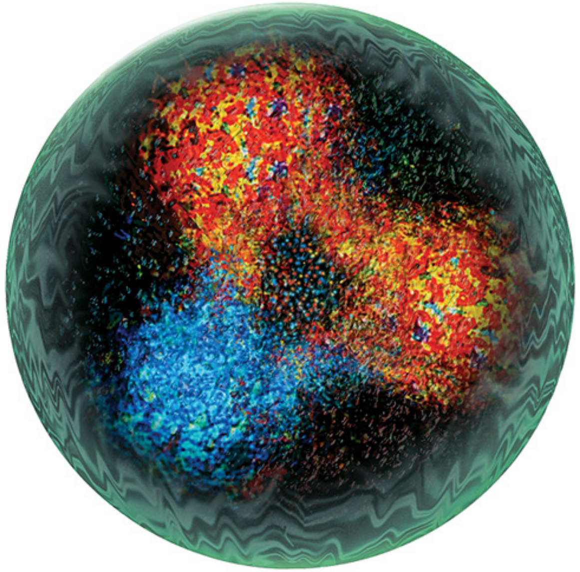
Non-zero gluon polarisation
Scientific American (2014)

Science

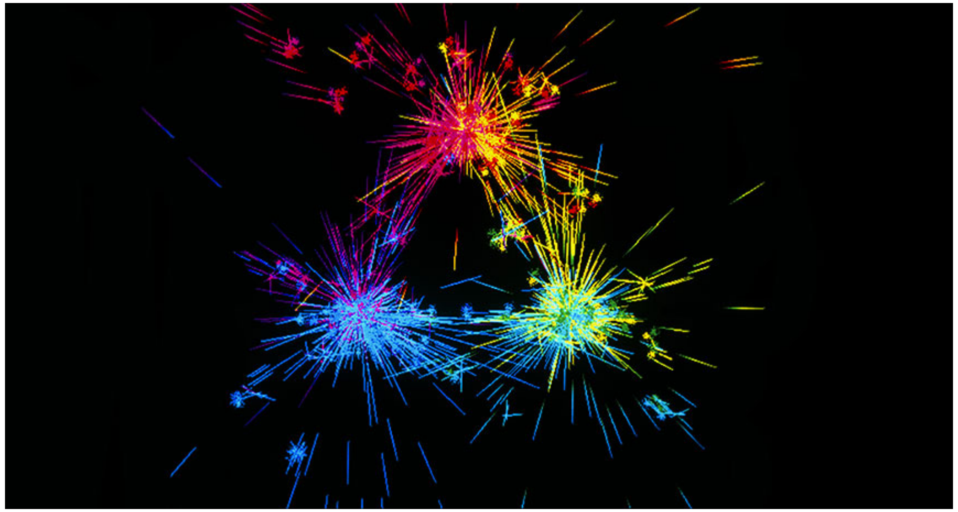
HOME > SCIENCE > VOL. 347, NO. 6220 > PROBING THE PROTON

FEATURE

Probing the proton



The inside of a proton endures more pressure than anything else we've seen



ScienceNew 2018

About proton

N BARYONS
($S = 0$, $I = 1/2$)
$p, N^+ = u\,u\,d$; $n, N^0 = u\,d\,d$

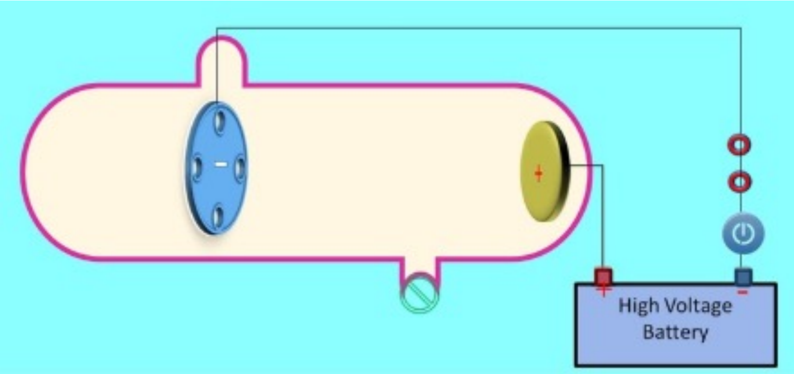
INSPIRE search

p $I(J^P) = 1/2(1/2^+)$

PDG 2021

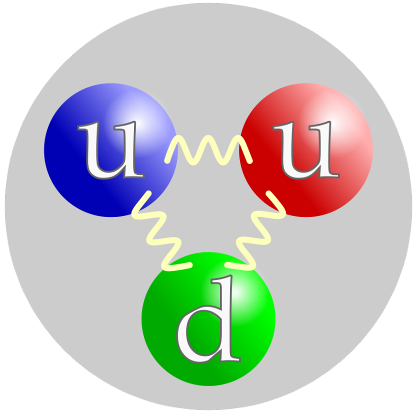
p MASS (atomic mass units u)	$1.00727646663 \pm 0.00000000009$ u (S = 2.9)
p MASS (MeV)	938.272081 ± 0.000006 MeV
$\lvert m_p - m_{\overline{p}}\rvert / m_p$	$< 7 \times 10^{-10}$ CL=90.0%
$\overline{p}/p$ CHARGE-TO-MASS RATIO, $\lvert \frac{q_{\overline{p}}}{m_{\overline{p}}} \rvert / (\frac{q_p}{m_p})$	$1.00000000000 \pm 0.00000000007$

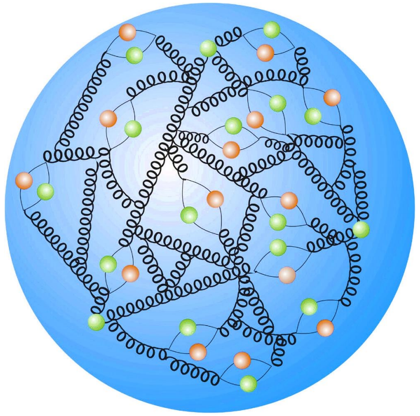
- **Canal ray**: 1886, Eugen GoldStein
- **Discovered** by **Ernest Rutherford**, **1917**



- QCD bound state of quarks and gluons
  - Origin of mass?
  - Origin of spin?
  - Gluon dominated matter?
  - 3D imaging?
  - Heavy quark content?

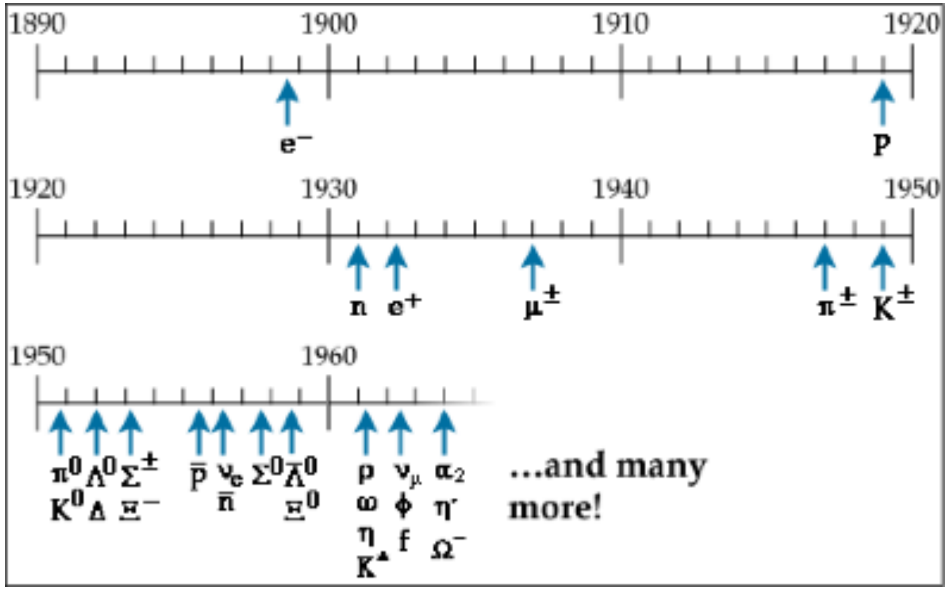
….



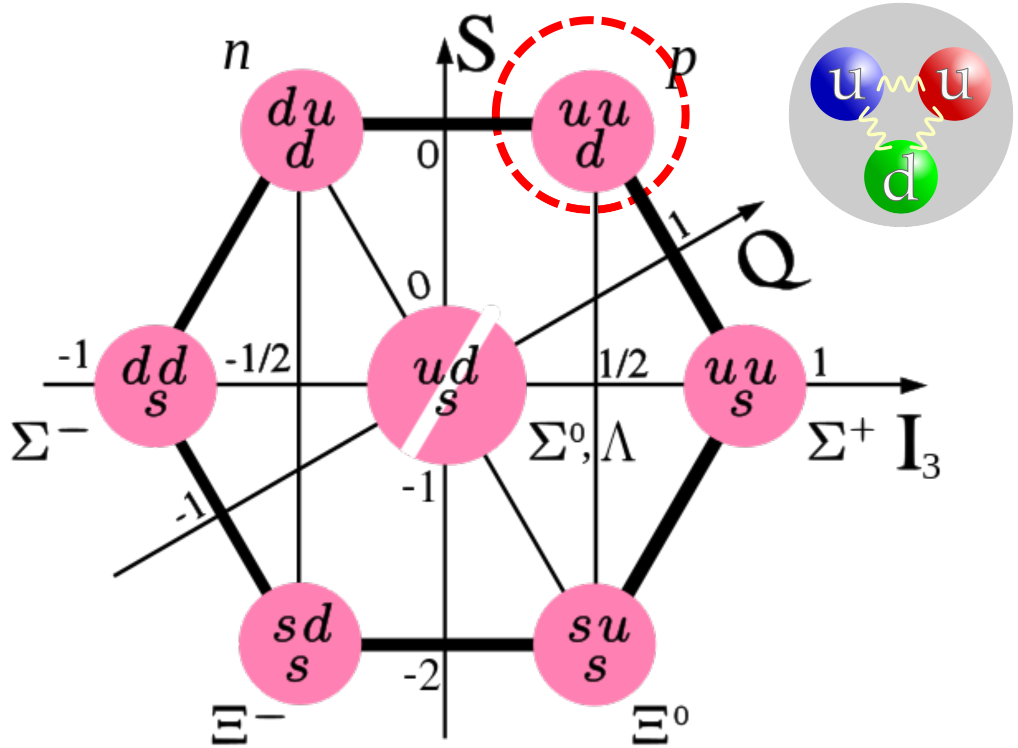


Quark model

- Gell-Mann and Zweig proposed a way to construct the numerous hadrons using three fundamental particles (1964): quarks



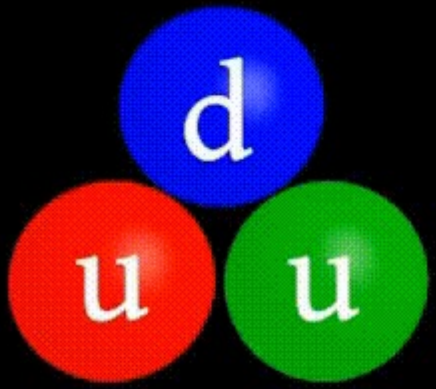
The particle Zoo



Quark model

Murray Gell-Mann and George Zweig in 1964

proton



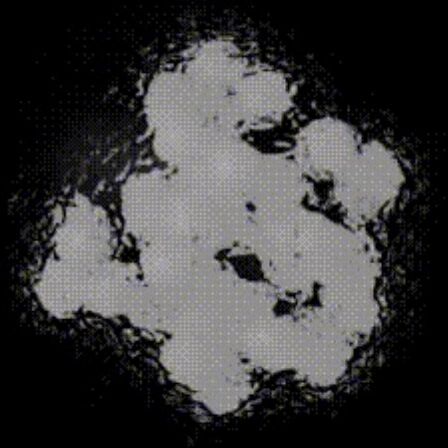
Parton model

Richard Feynman in 1968

parton
or fraction

momentum
charge
spin

partons



Quark parton model (QPM)

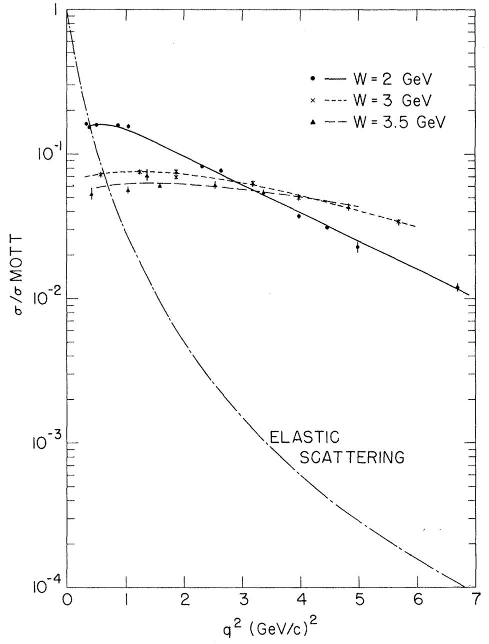
- Point-like structures within the proton: partons (quarks)
  - With sufficiently high energies, the partons behave like free particles
- The distribution of the fraction of the protons momentum (x) over the partons is described by Parton Distribution Functions (PDFs)
  - The gluon contribution is taken into account
  - The QCD improved QPM

VOLUME 23, NUMBER 16 PHYSICAL REVIEW LETTERS 20 OCTOBER 1969

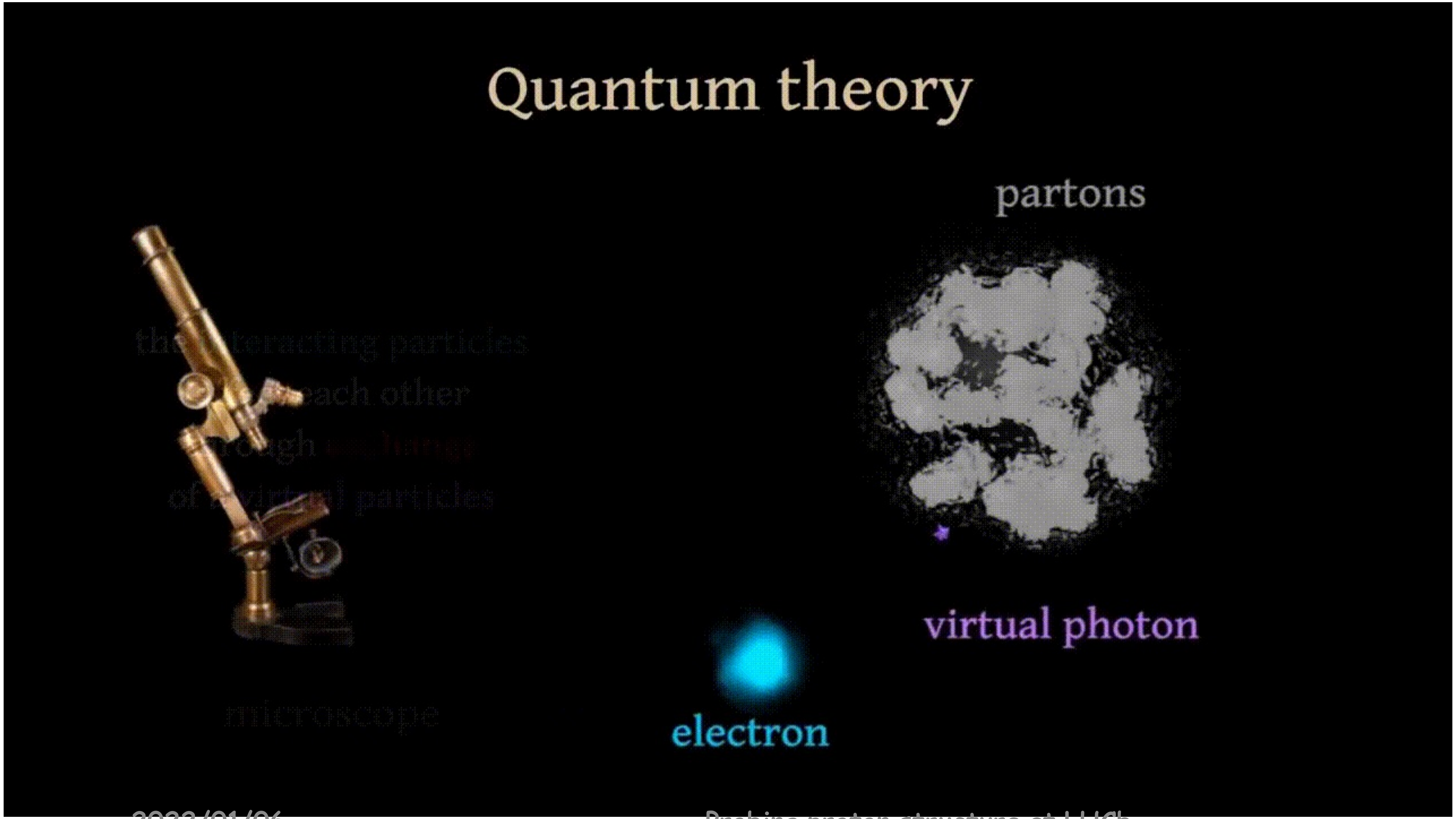
OBSERVED BEHAVIOR OF HIGHLY INELASTIC ELECTRON-PROTON SCATTERING

M. Breidenbach, J. I. Friedman, and H. W. Kendall
Department of Physics and Laboratory for Nuclear Science,*
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

SLAC-MIT experiment



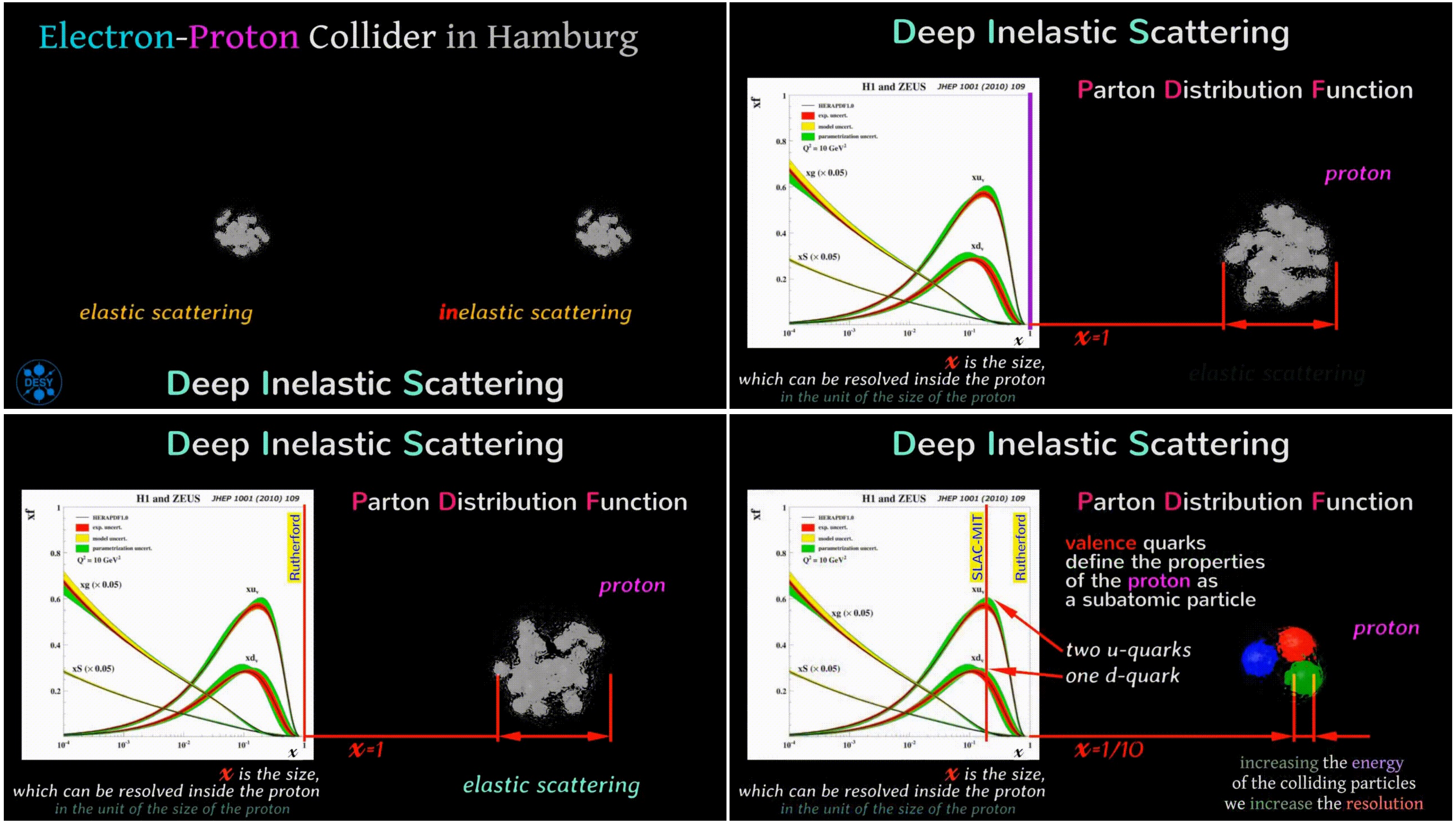
Deep Inelastic Scattering (DIS)



Use virtual particle (photon) as probe

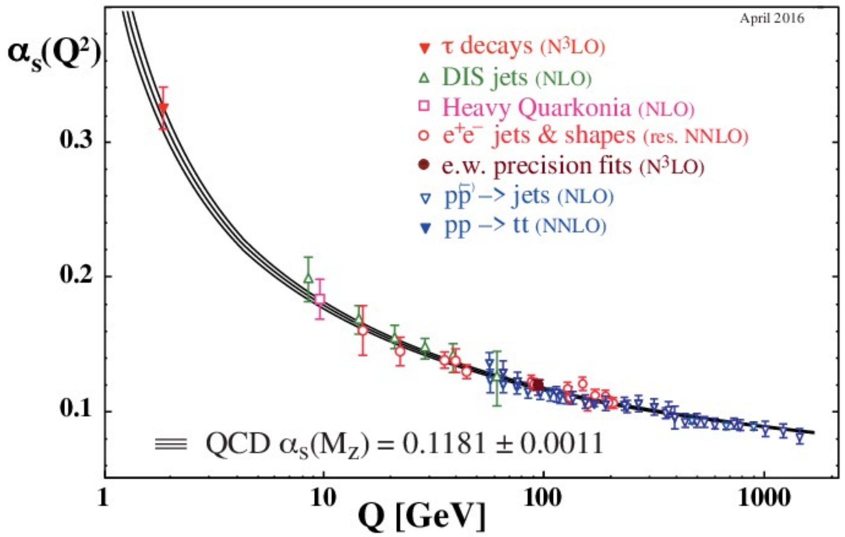
Key point:
The wavelenght of probe (λ)

How to probe proton?



The paradox

- The QPM is very success in the past decades
- But, there is a paradox:
  - Quarks are strongly bound into hadrons
  - Quarks act as free "partons" in DIS

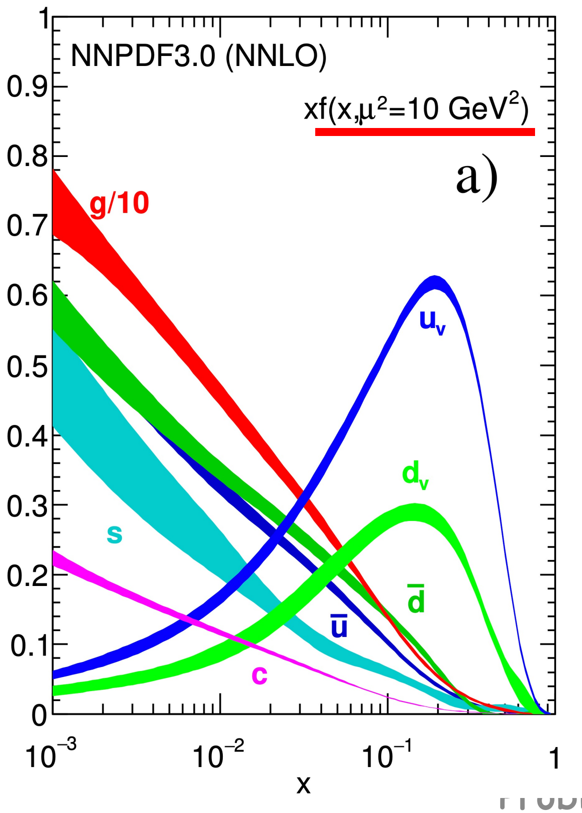


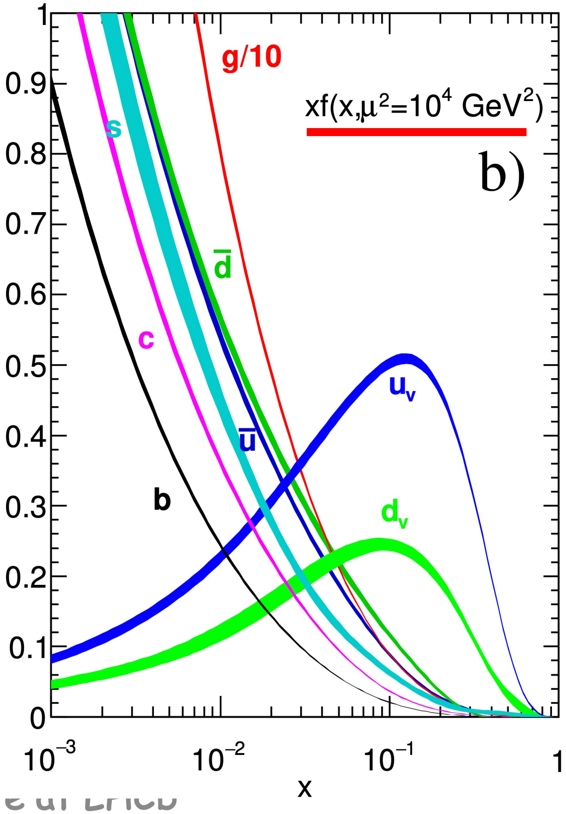
To slove this paradox: running of α_S

- **Low Q region: confinement**
- **High Q region: asymptotic freedom**

Proton PDFs

- Universal distributions containing long-distance structure of hadron: the QCD factorisation theorem
- At least 11 PDFs (5 quarks + anti-quarks + gluon)





PDFs parameterisations

EPJC 73, 2318 (2013)

$$xf(x, Q_0^2) = A(1-x)^{\eta} x^{\delta}\left(1 + \sum_{i=1}^{n} a_i T_i^{Ch}(y(x))\right)$$

- $Q_0^2 := 1\ GeV$, is the input scale
- T_i^{Ch}: Chebyshev polynomials in y, $y = 1 - 2x^k$ $(k = 0.5, n = 4)$
- Global fit to determine the values of A, δ, η, a_i for each PDF
  - For gluon: (a second term with a different small x power)

$$xf(x, Q_0^2) = A_g(1-x)^{\eta_g} x^{\delta_g}\left(1 + \sum_{i=1}^{2} a_{g,i} T_i^{Ch}(y(x))\right) + A_{g'}(1-x)^{\eta_{g'}} x^{\delta_{g'}}$$

- Constraints:
  - Valence quarks: $\int_0^1 dx u_V(x, Q_0^2) = 2$, $\int_0^1 dx d_V(x, Q_0^2) = 1$
  - Sea quarks: $\int_0^1 dx(s(x, Q_0^2) - \bar{s}(x, Q_0^2)) = 0$
  - Sum over: $\int_0^1 dx\, x[u_V(x, Q_0^2) + d_V(x, Q_0^2) + S(x, Q_0^2) + g(x, Q_0^2)] = 1$

DGLAP function?

- The logic is very similar to running coupling, we have "running functions"

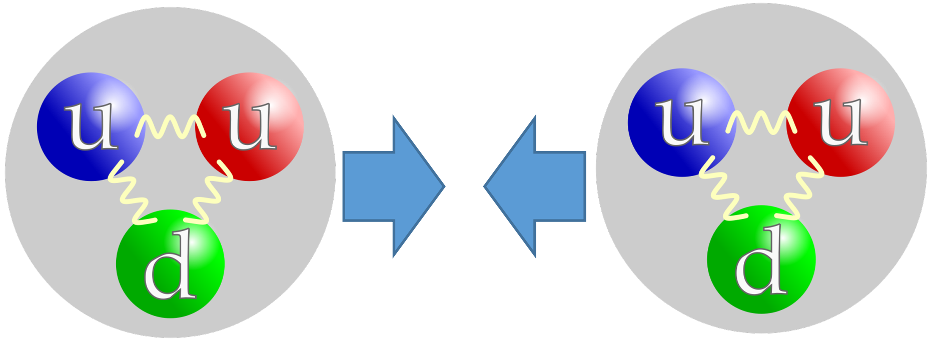
$$\mu \frac{d}{d\mu}\phi_{i/H}(x,\mu) = \int_x^1 \frac{dz}{z} P_{ij}\big(Z,\alpha_S(\mu)\big)\phi_{j/H}(\frac{x}{z},\mu) \equiv \big[P_{ij}\otimes\phi_{j/H}\big](x,\mu)$$

  - P_{ij} are the aplitting functions: parton evolution kernels

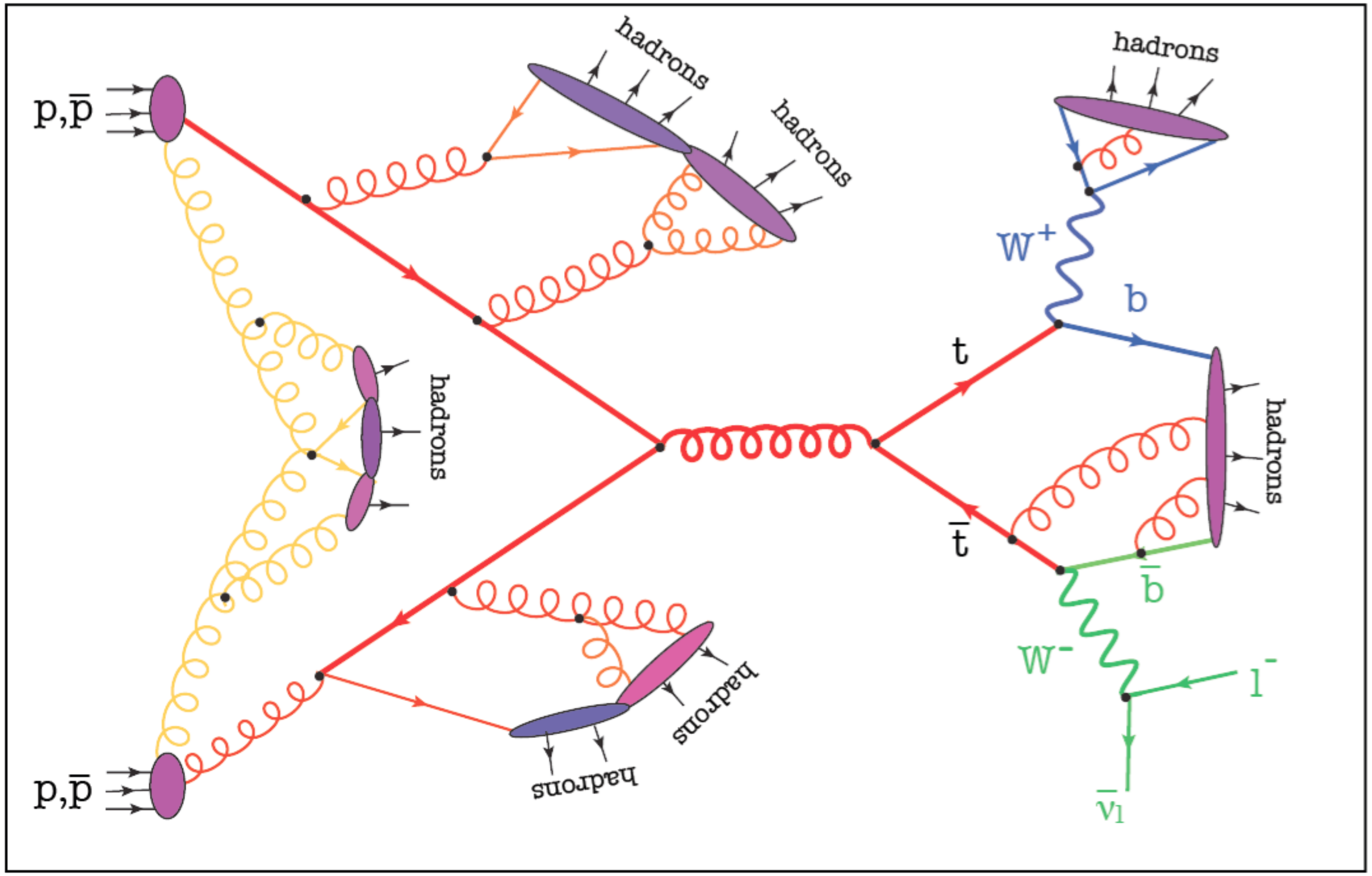
- Determine the PDFs at some scale $\boldsymbol{Q}$

- Compute them at all other scales by sloving the DGLAP equations

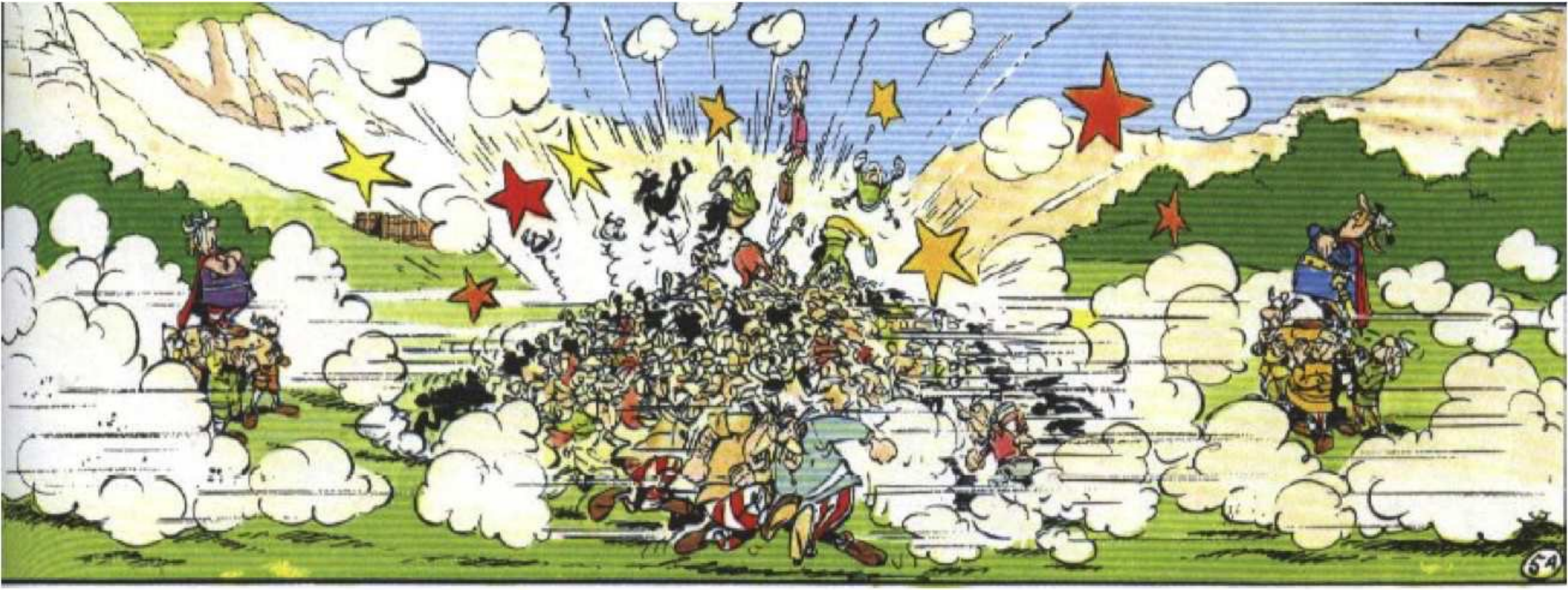
Hadron collider



Proton-proton collision

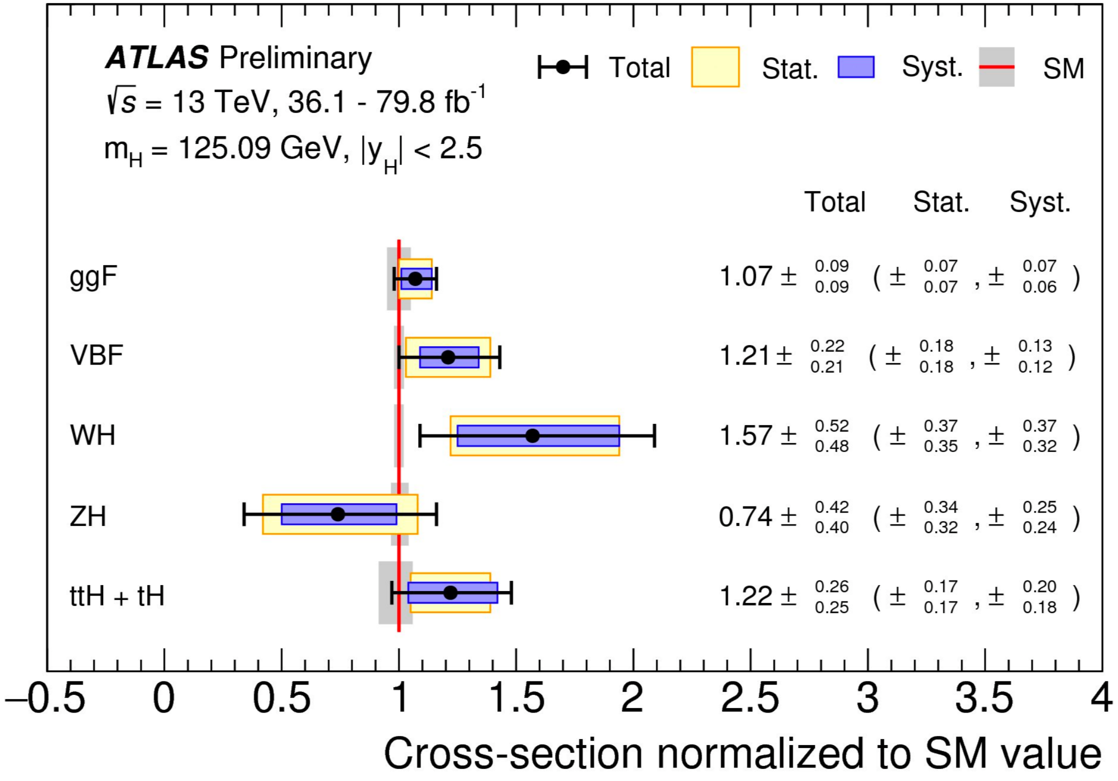


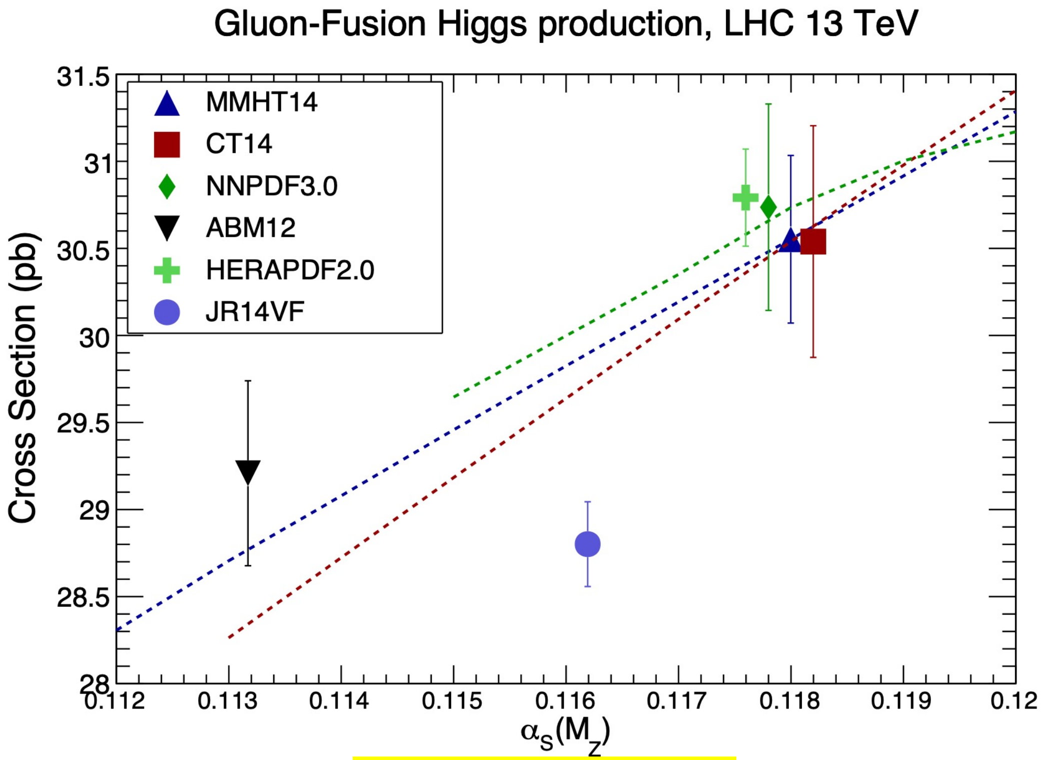
Hadron collider



Impacts from PDFs

- Higgs boson production cross-section
  - PDFs uncertainty is one of dominat uncertainties



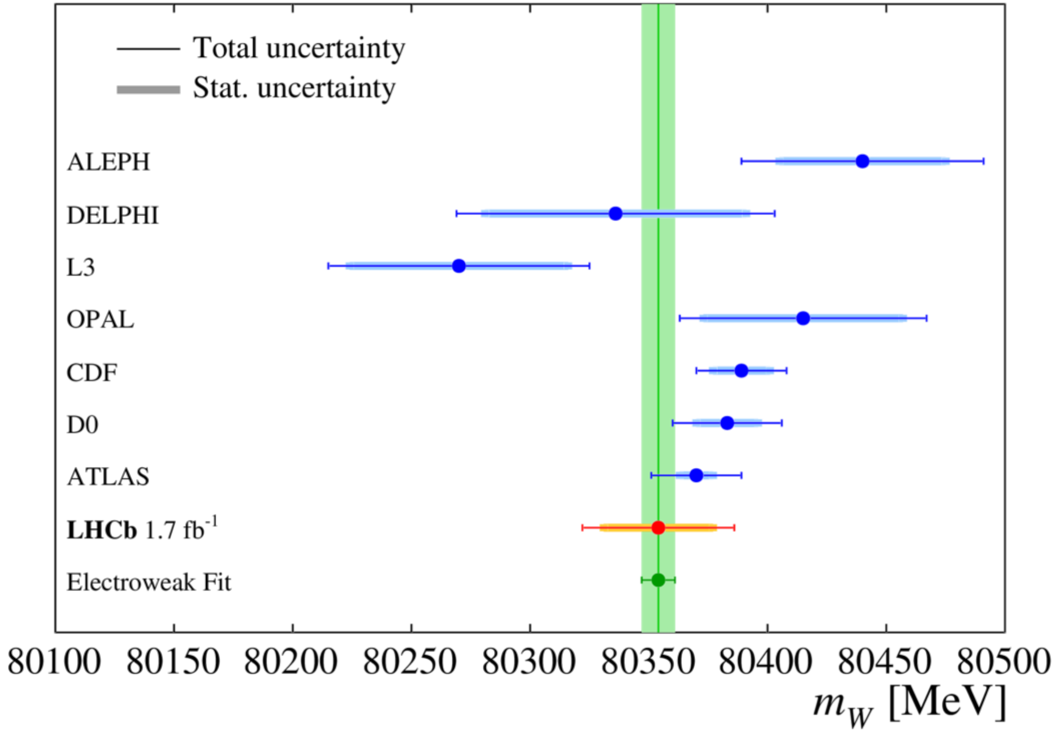


arXiv:1610.07922

Impacts from PDFs

arXiv:2109.01113
Submitted to JHEP

- Recently, LHCb published the first measurement of the *W* boson mass
  - PDFs uncertainty is one of dominat uncertainties

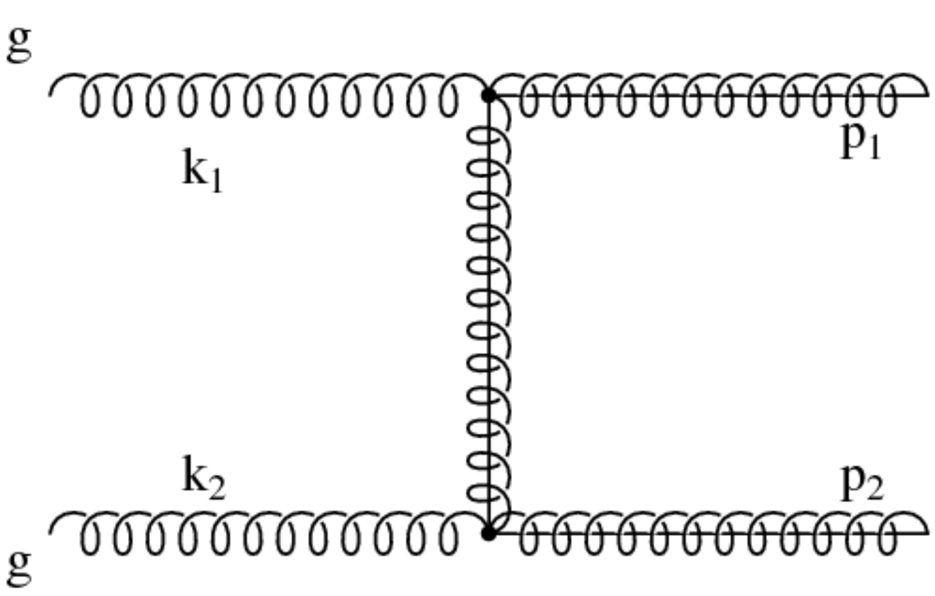


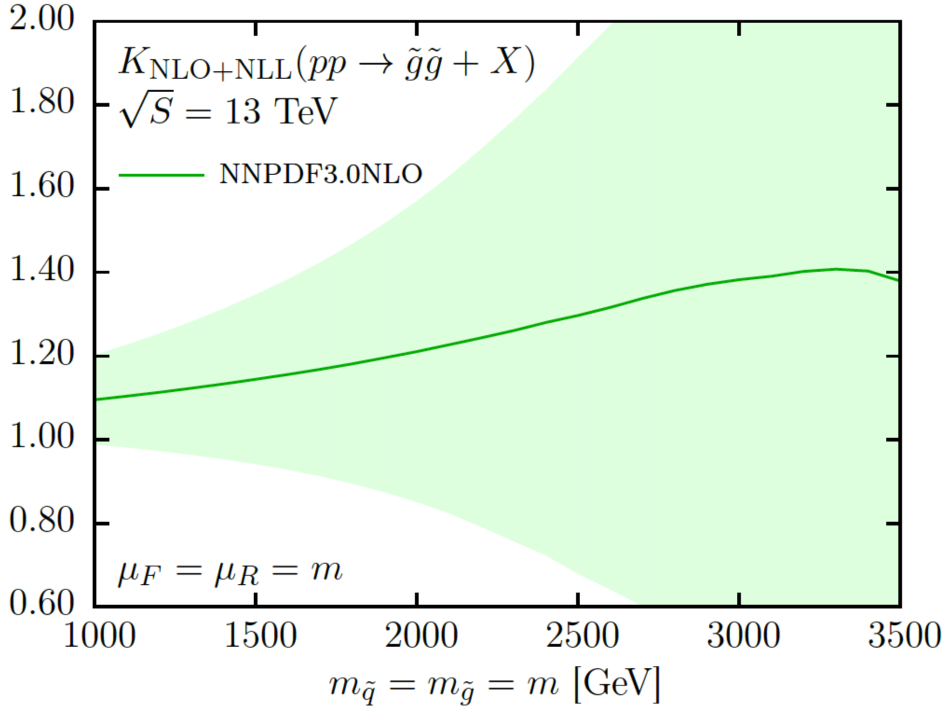
Source	Size [MeV]
Parton distribution functions	9
Theory (excl. PDFs) total	17
Transverse momentum model	11
Angular coefficients	10
QED FSR model	7
Additional electroweak corrections	5
Experimental total	10
Momentum scale and resolution modelling	7
Muon ID, trigger and tracking efficiency	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total	32

Impacts from PDFs

EPJC 76, 53 (2016)

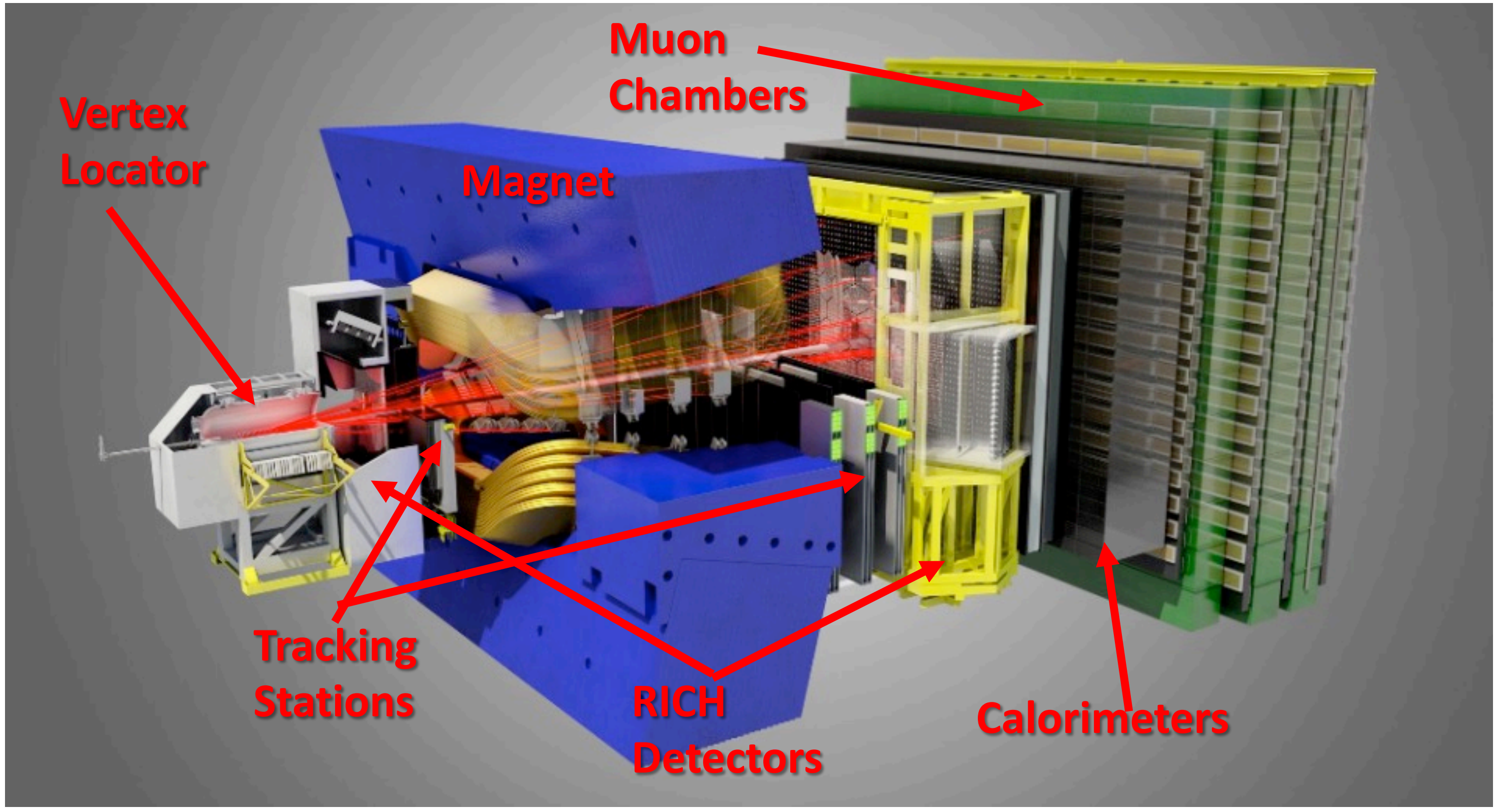
- PDFs uncertainty in the production of NP heavy resonances up to 100%
- Limited knowledge of gluon PDFs in the large x region





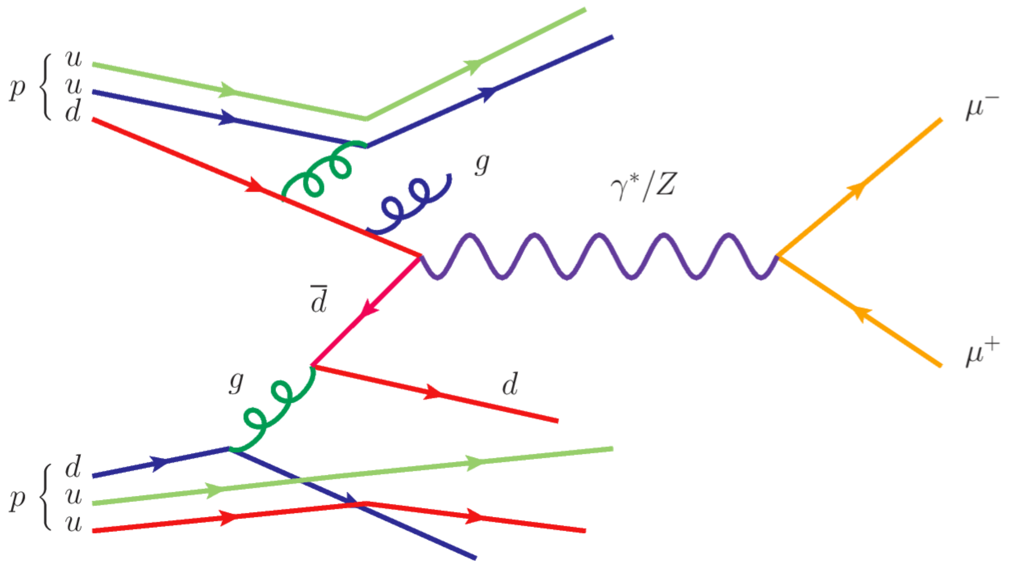
LHCb detector

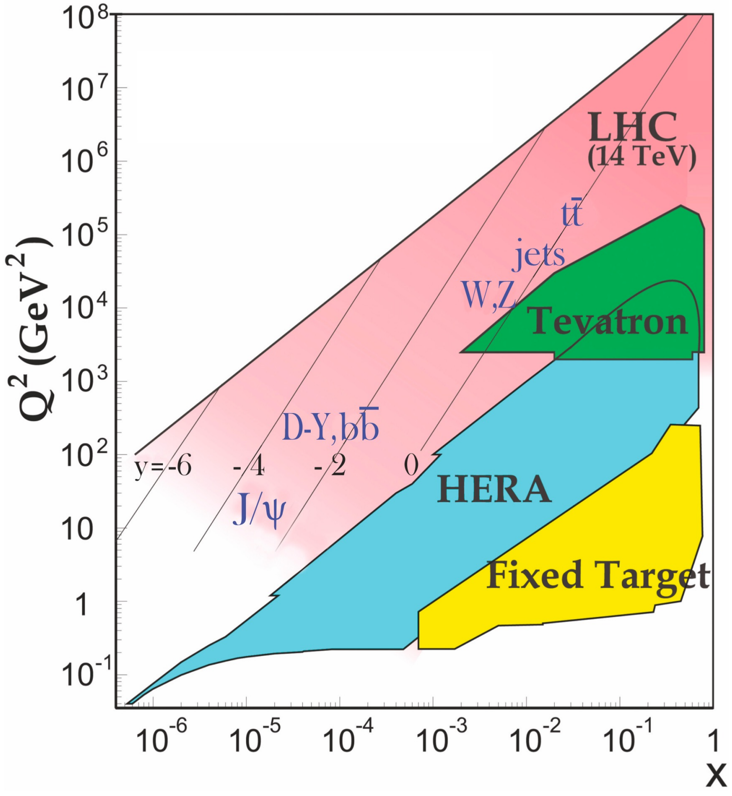
- The LHCb detector is single-arm forward spectrometer
  - Designed for the heavy flavour physics, with $2 < \eta < 5$
  - Extended to EW measurements: excellent performance of tracking and muon detector



EW physics @ LHCb

- The x value of interacting partons are correlated with the boson production
  - Rapidity: $y = \frac{1}{2} ln \frac{x_1}{x_2}$
  - Large rapidity: either very large x or very small x





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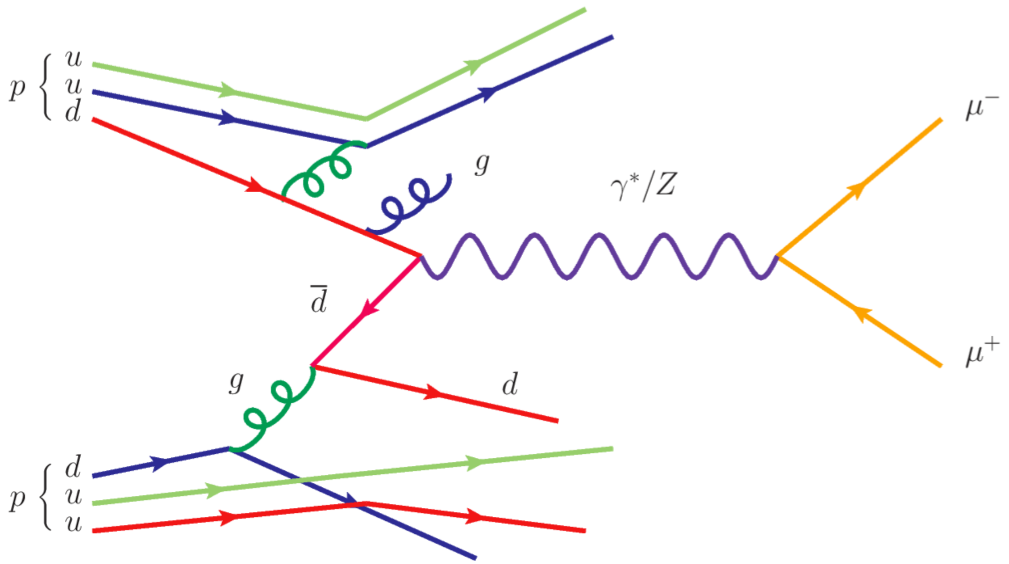
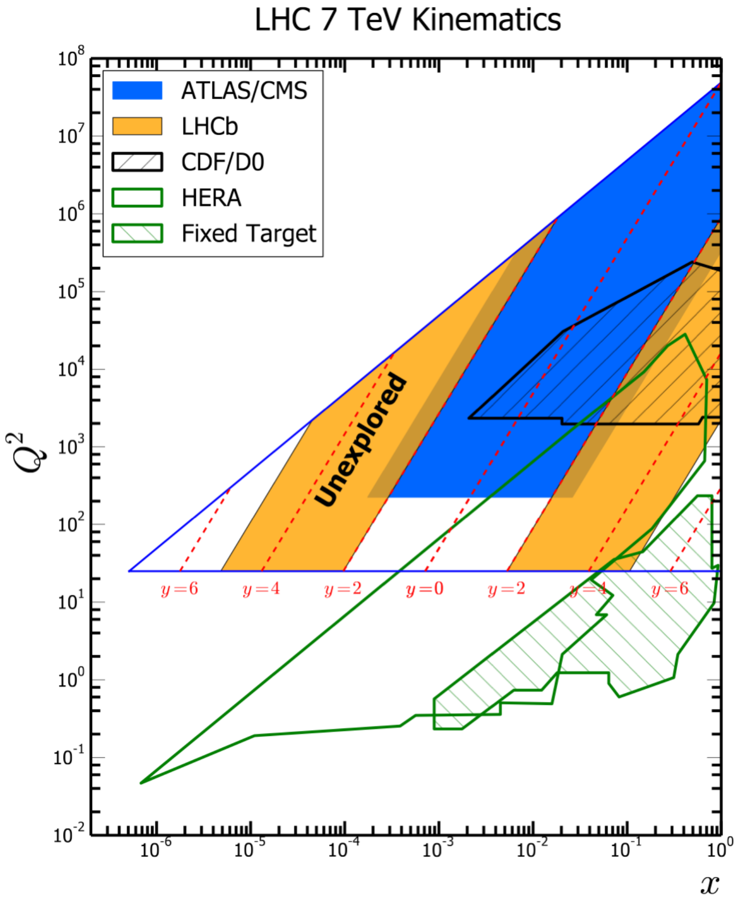
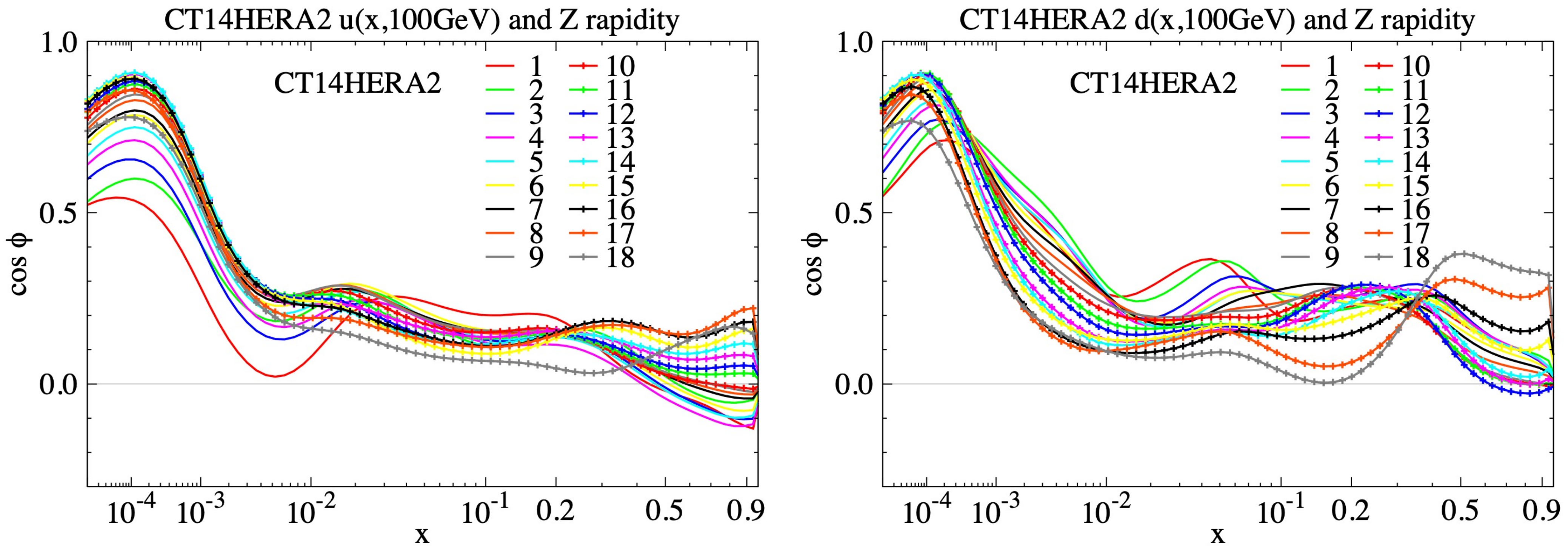




LHCb data and PDFs

Chinese Phys. C **45** (2021) 023110

- Strong correlations between LHCb Z boson rapidity and high-x PDFs are seen
  - For both u and d quarks



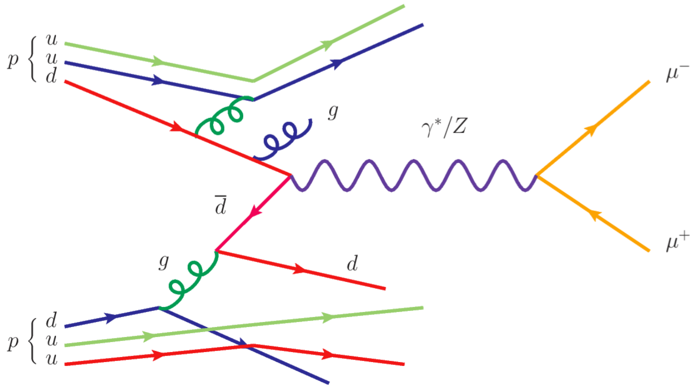
Z boson production cross-section measurement

Motivation

- Z boson Cross-sections can be used to test the SM

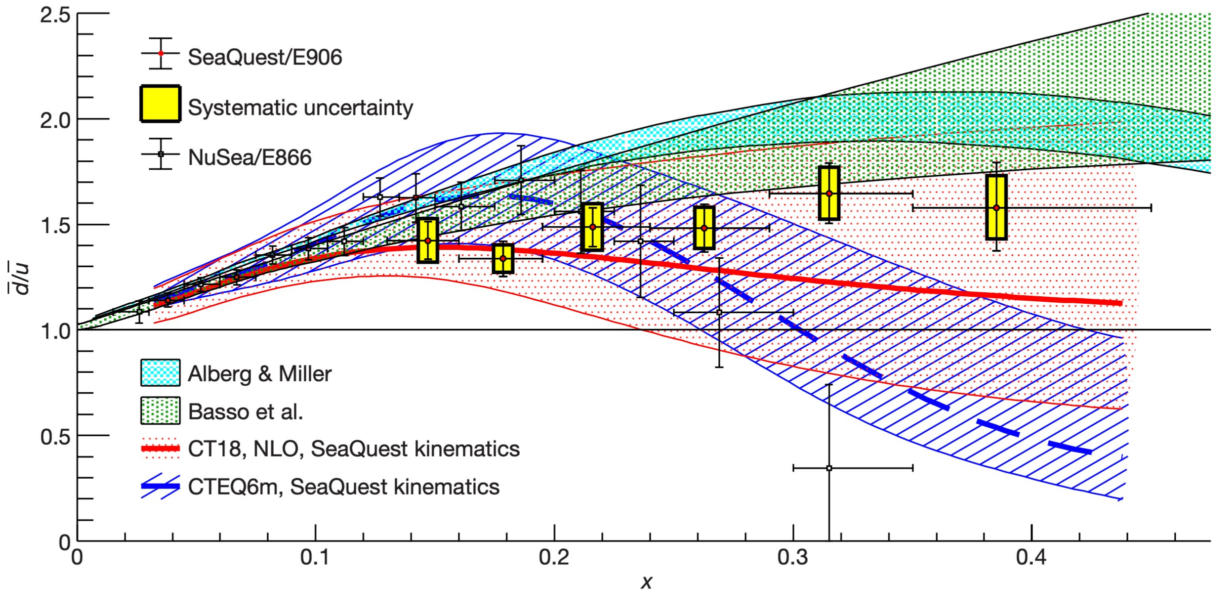
- Potential to improve constraints on parton distribution functions (PDFs)

- Measurements of the Z boson rapidity are particularly important for constraining u-, d-quark PDFs

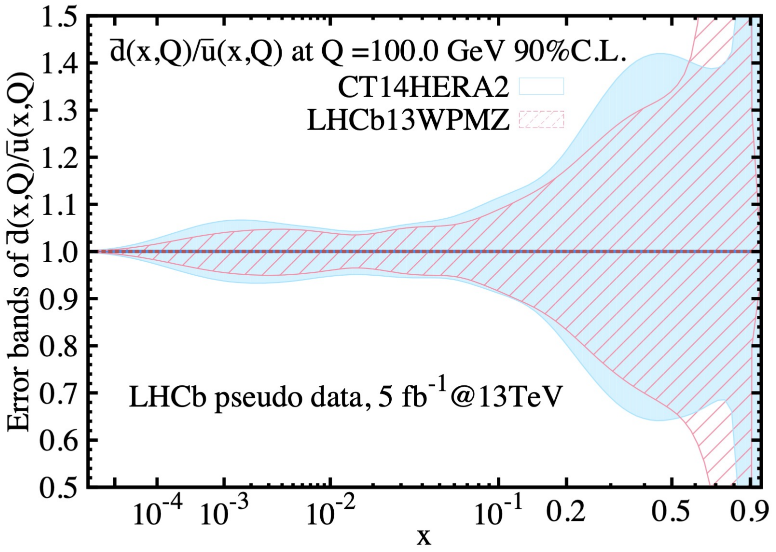


Results from SeaQuest

- Recently, the SeaQuest Collaboration report the results on the $\bar{d}/\bar{u}$ PDFs
  - Tensions between SeaQuest and NuSea results are seen
- The LHCb data will be the only clean data to constraint the $\bar{d}/\bar{u}$ PDFs



Nature 590, 561 (2021)

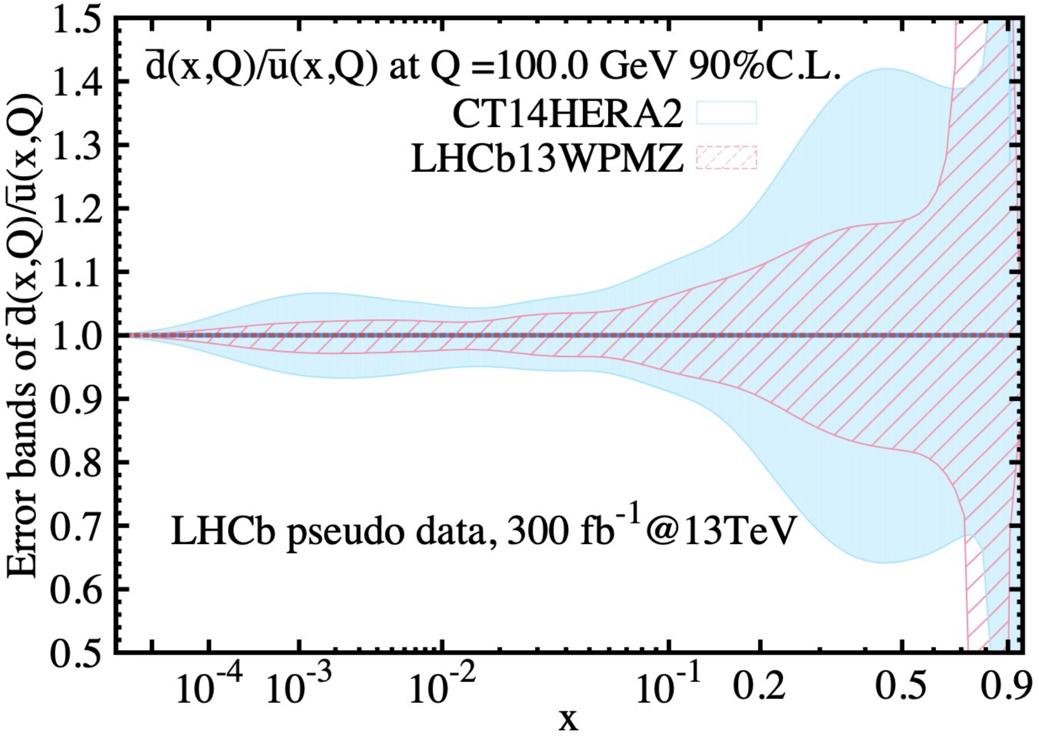


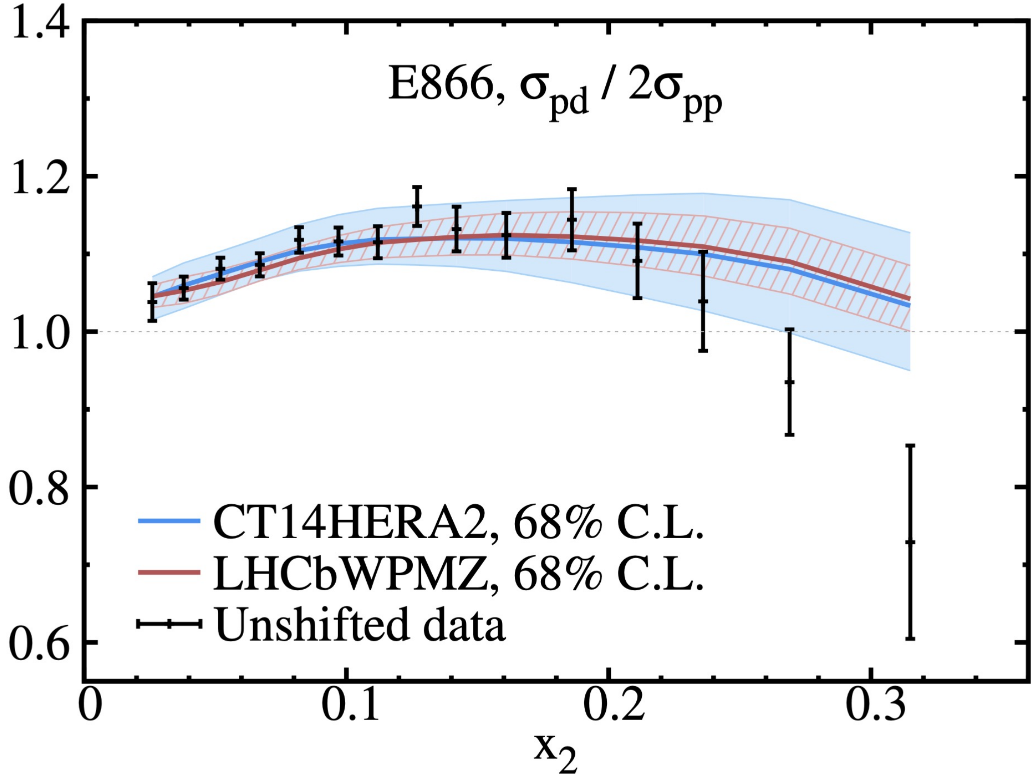
Chinese Phys. C **45** (2021) 023110

A confirmation from the LHCb?

Chinese Phys. C **45** (2021) 023110

- With 300 $\mathbf{fb}^{-1}$ LHCb, pp collision data @ 13 TeV in future



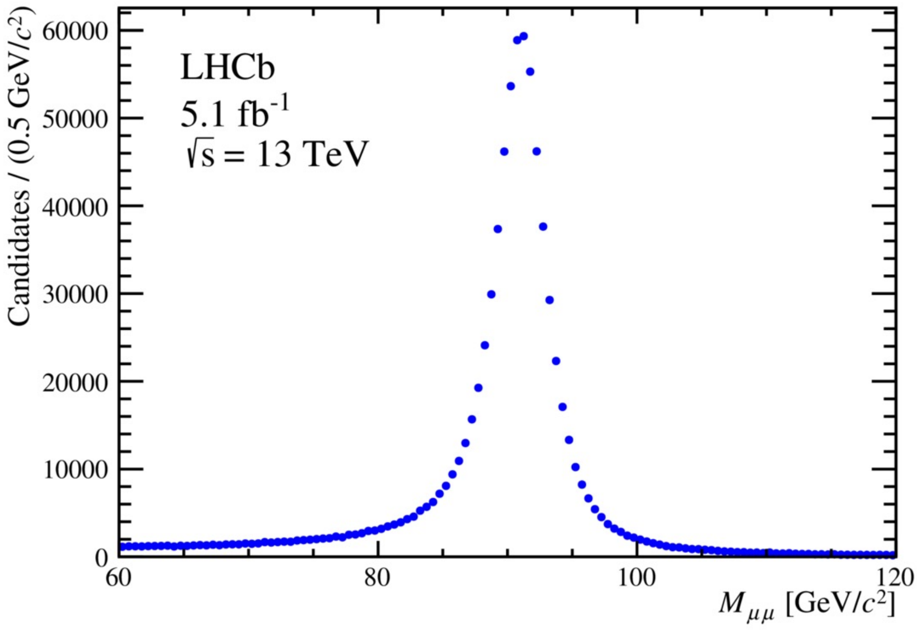


Event selection

Very simple selection

- Trigger: One muon fires single muon trigger path
- Background contribution: ~2%

μ	Z
$p_{\mathrm{T}} > 20\,\mathrm{GeV}/c$	$60 < M_{\mu^+\mu^-} < 120\,\mathrm{GeV}/c^2$
$2 < \eta < 4.5$	
$\sigma_P/P < 10\%$	



Strategy: cross-section measurement

- Fiducial region: Muons with $2 < \eta < 4.5$, $P_T > 20\ \mathrm{GeV}/c$, $60 < M_{\mu\mu} < 120\ \mathrm{GeV}/c^2$
- Cross-section measured in bin of $Z\ y$, P_T and ϕ^*_η is given by

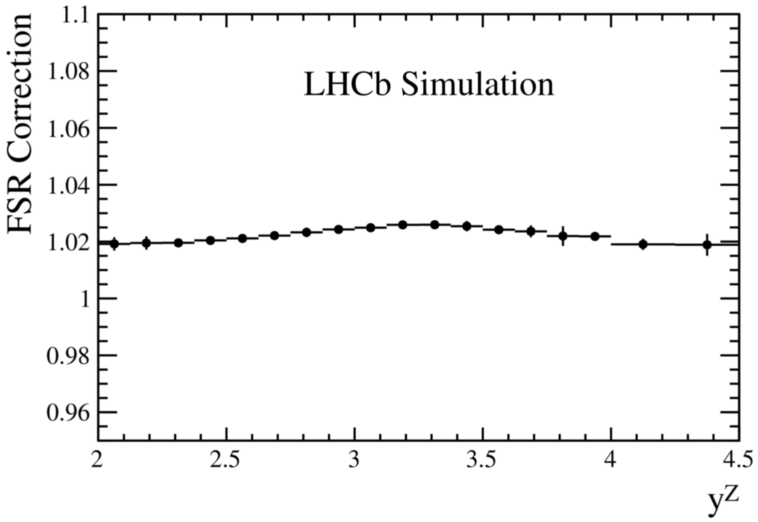
$$\frac{d\sigma_{Z\to\mu^+\mu^-}}{dy}(i) = \frac{N_Z(i)\cdot f^Z_{FSR}(i)}{\mathcal{L}\cdot\varepsilon^Z_{REC}(i)\cdot\Delta y(i)}$$

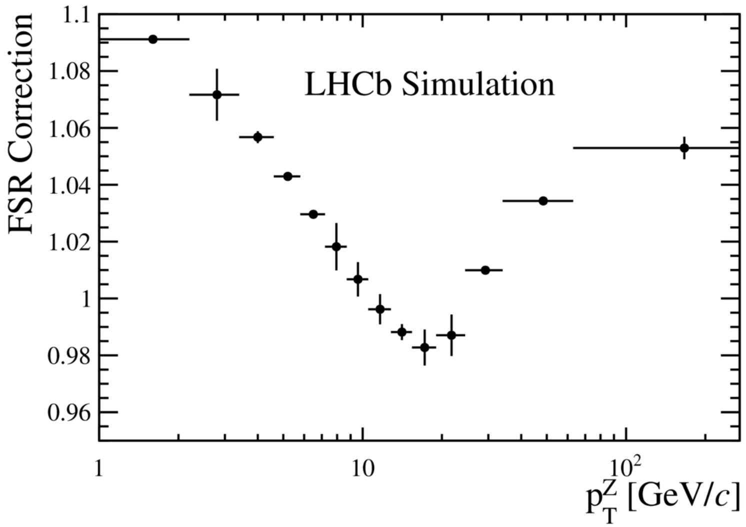
$$\begin{aligned}\varepsilon^Z_{REC}(i) =&(\varepsilon^\mu_{TRK}(\eta_i^+)\cdot\varepsilon^\mu_{TRK}(\eta_i^-))\cdot\\ &(\varepsilon^\mu_{ID}(\eta_i^+)\cdot\varepsilon^\mu_{ID}(\eta_i^-))\cdot\\ &(\varepsilon^\mu_{TRG}(\eta_i^+)+\varepsilon^\mu_{TRG}(\eta_i^-)-\varepsilon^\mu_{TRG}(\eta_i^+)\cdot\varepsilon^\mu_{TRG}(\eta_i^-)\ ),\end{aligned}$$

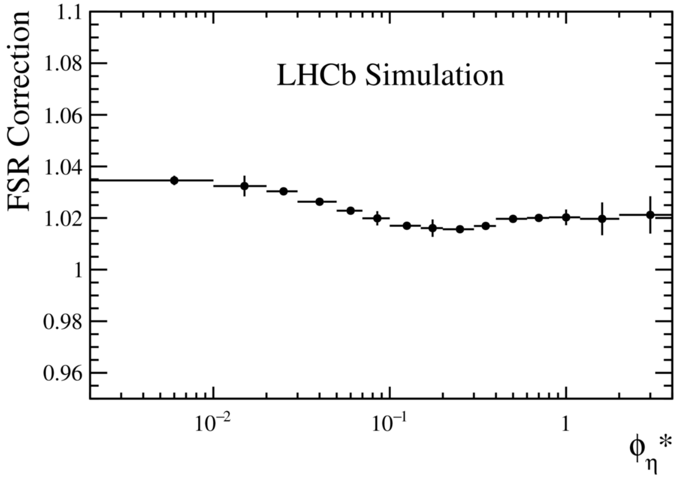
- Integrated cross-section obtained by summing the differential cross-section

Corrections

- Detector level alignment
- Efficiency correction
- Unfolding
- Final-state radiation: back to Born level







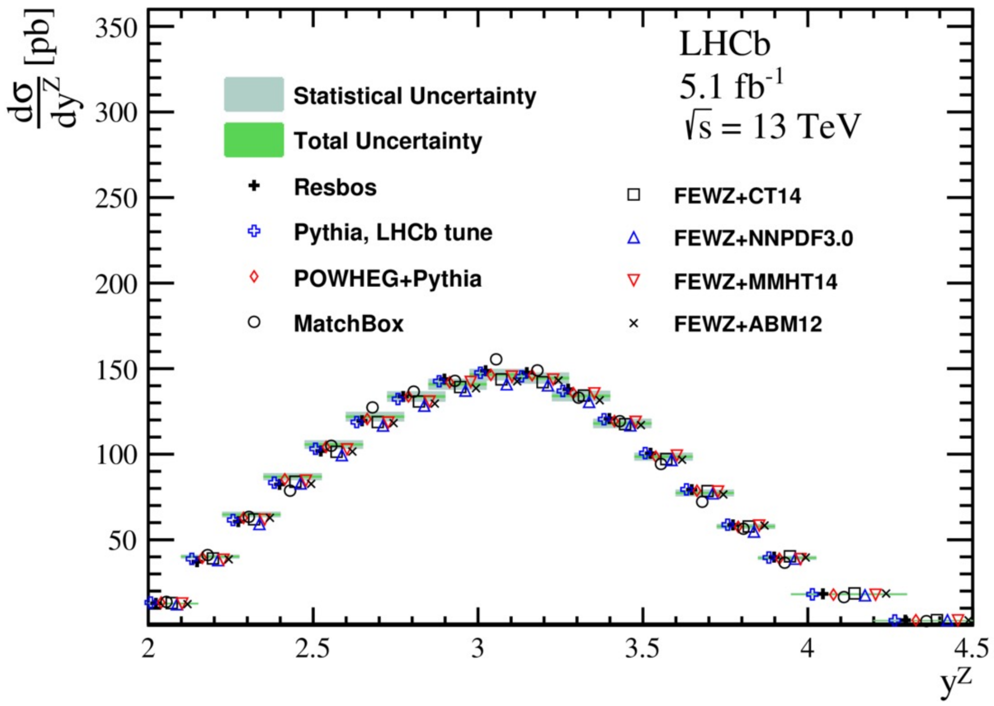
Systematic uncertainties

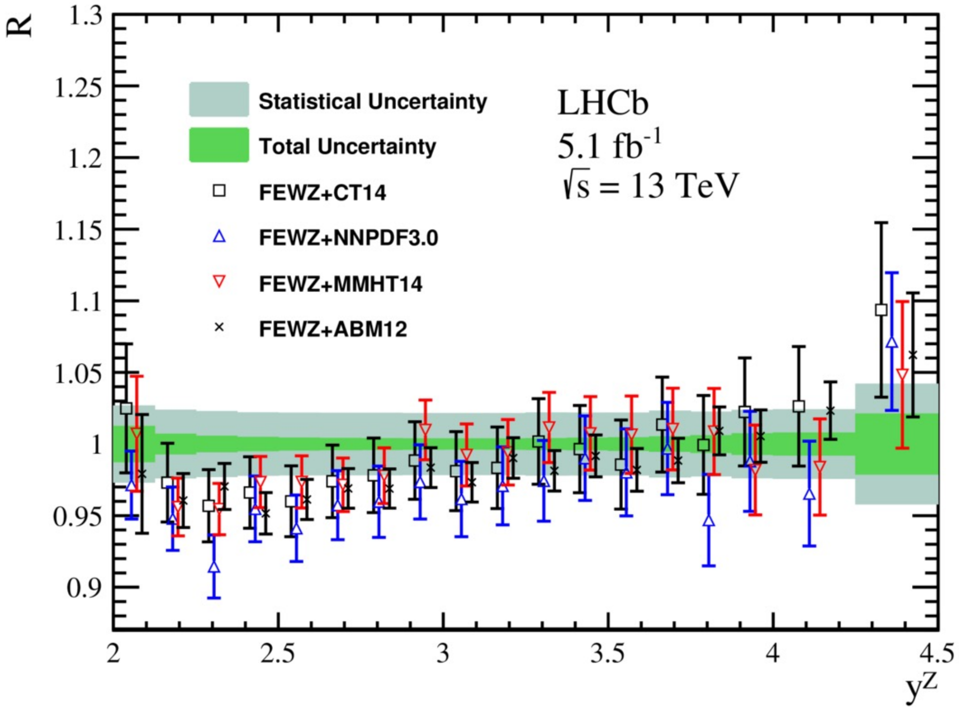
- Luminosity determination precision: 2%
- Tracking Systematic
  - The systematic uncertainty on each muon track is determined to be 0.47%

Source	$\Delta\sigma/\sigma$ [%]
Statistical	0.11
Background	0.03
Alignment & calibration	-
Efficiency	0.77
Closure	0.06
FSR	0.04
Total Systematic (excl. lumi.)	0.77
Luminosity	2.00
Total	2.15

Differential cross section: Z-rapidity

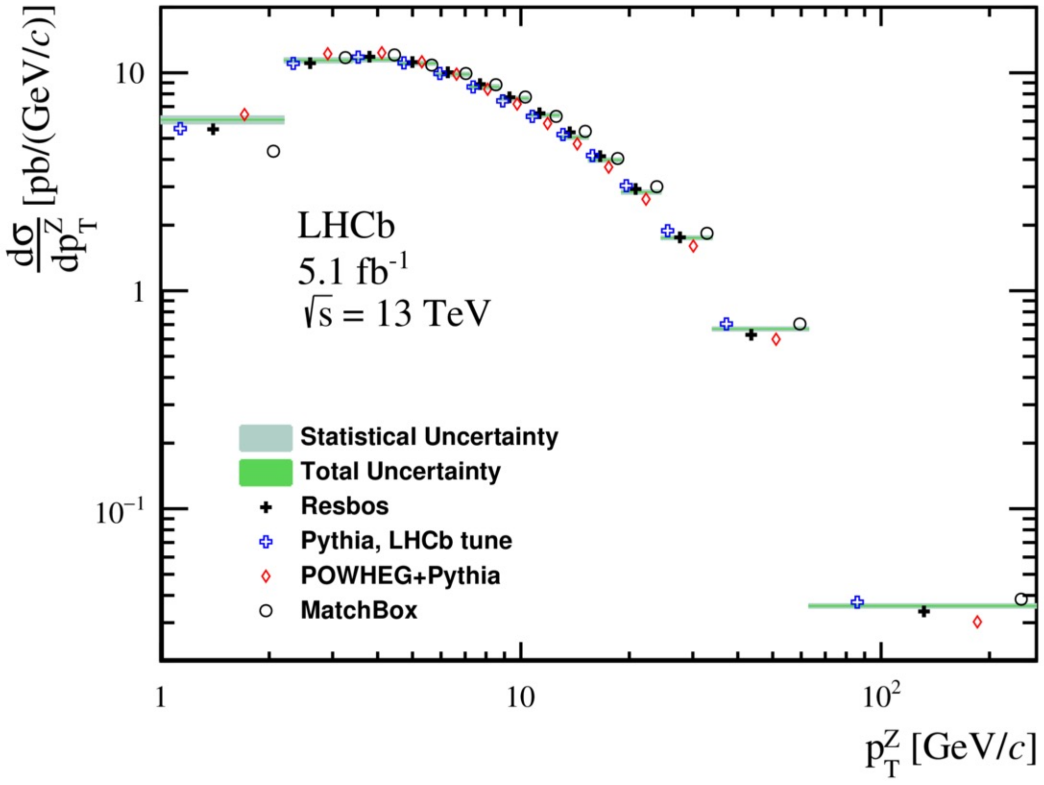
- Reasonable agreeement between data and predictions
- Some trends in the lower rapidity region

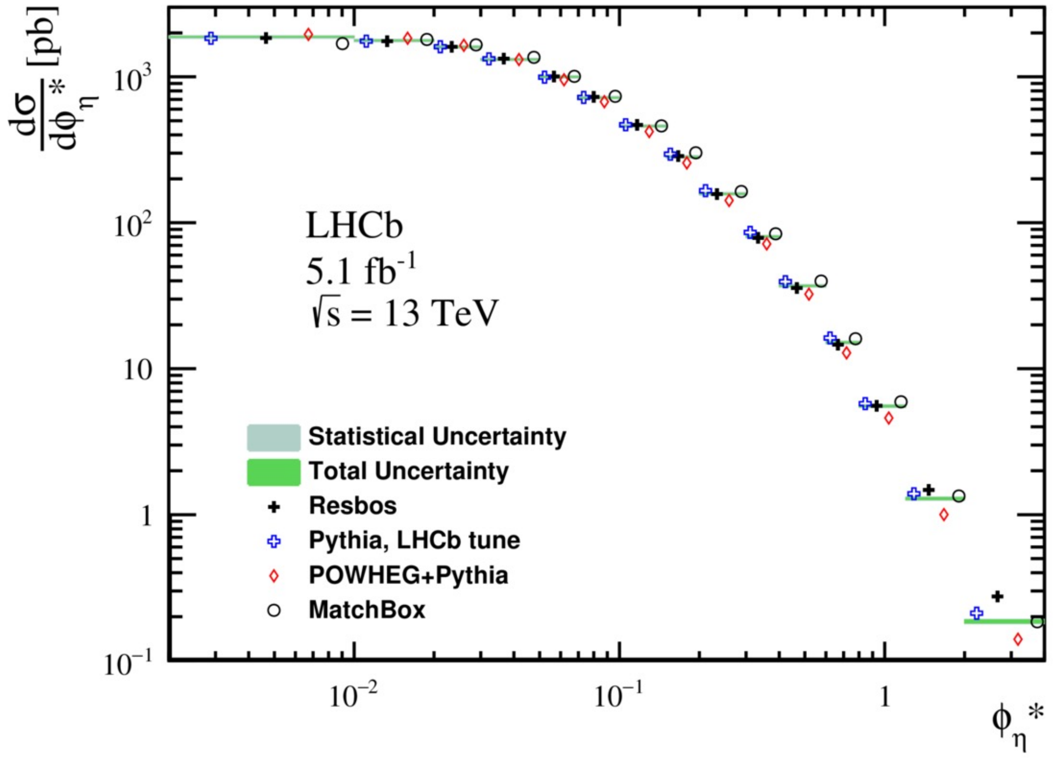




Differential cross section: $Z - p_T$ and ϕ_η^*

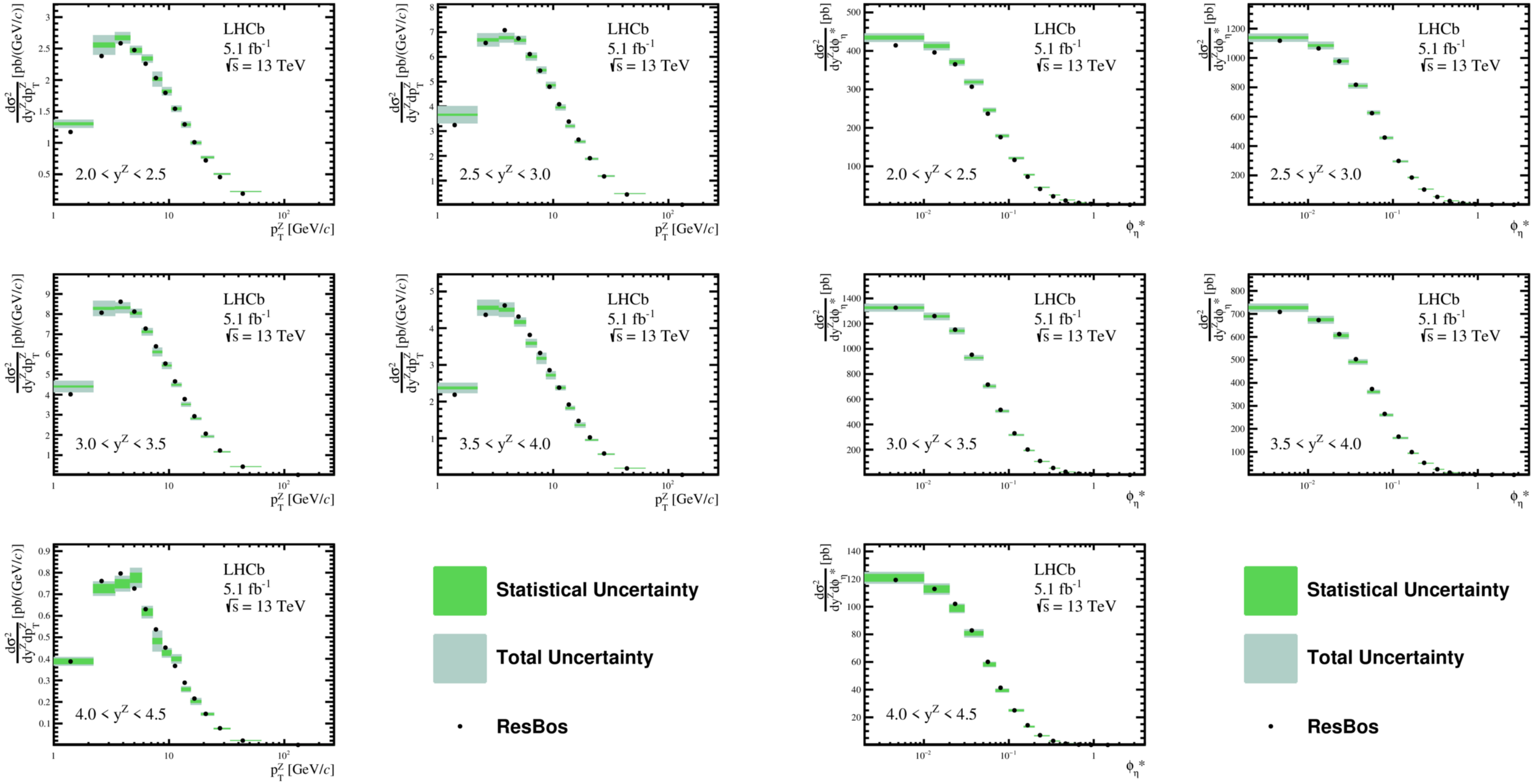
- Reasonable agreement between data and predictions
- Provide a stringent test on different QCD calculations



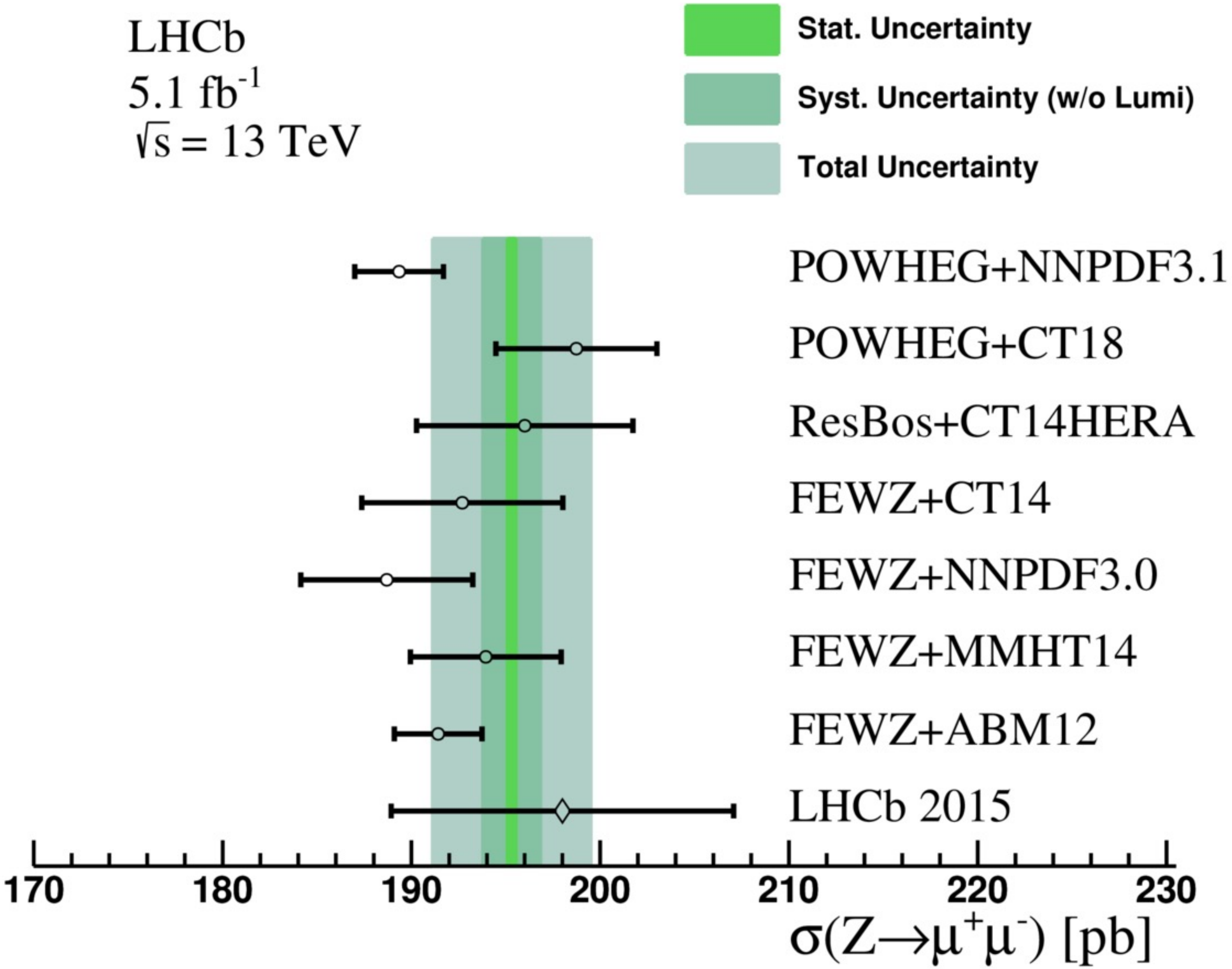


Double differential cross-section

- The first double differential cross-section measurement in the forward region



Integrated cross section



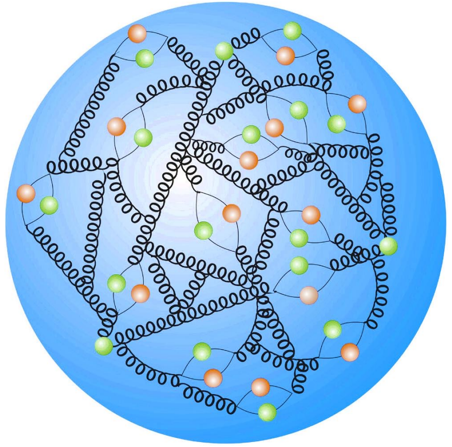
arXiv:2112.07458
Submitted to JHEP

The most precise measurement in the Forward region @ 13 TeV

Year	$\sigma(Z \to \mu^+\mu^-)$ [pb]
2016	$194.0 \pm 0.4 \pm 1.9 \pm 3.9$
2017	$196.0 \pm 0.4 \pm 1.9 \pm 3.9$
2018	$196.2 \pm 0.4 \pm 1.8 \pm 3.9$
Run II	$195.3 \pm 0.2 \pm 1.5 \pm 3.9$

Intrisic charm

Intrinsic charm

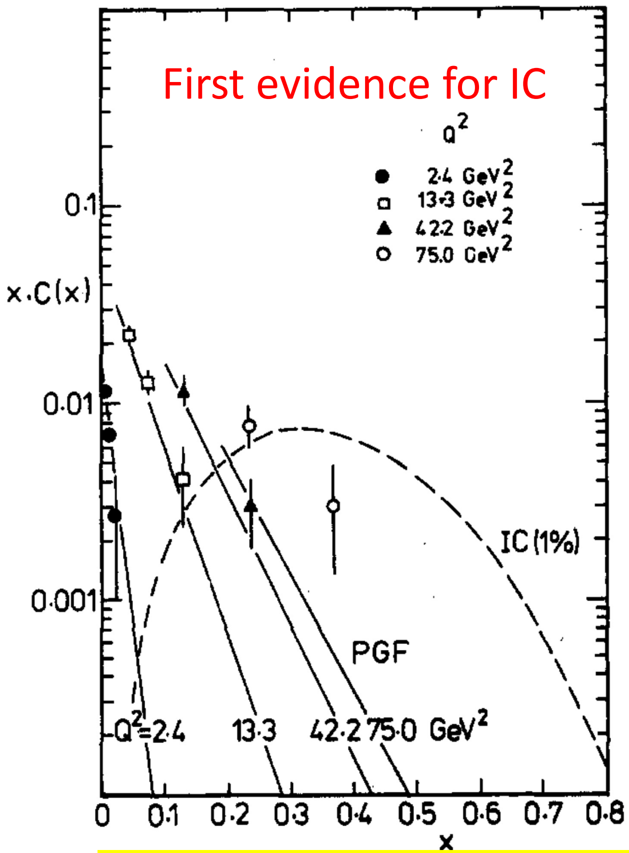


- Two types of parton contributions
- The extrinsic quarks/gluons:
  - Generated on a short time scale in association with a large transverse-momentum reaction
- The intrinsic quarks/gluons:
  - Exist over a time scale independent of any probe momentum, they are associated with the found state hadron dynamics
- Intrinsic charm:
  - Long-lived charm component: longer than the interaction time of the probe
  - BHPS model: Brodsky, Hoyer, Peterson, and Sakai (1980)
    - non-negligible $uudc\bar{c}$ component (independen of PDFs)

Intrisic charm: Experiments

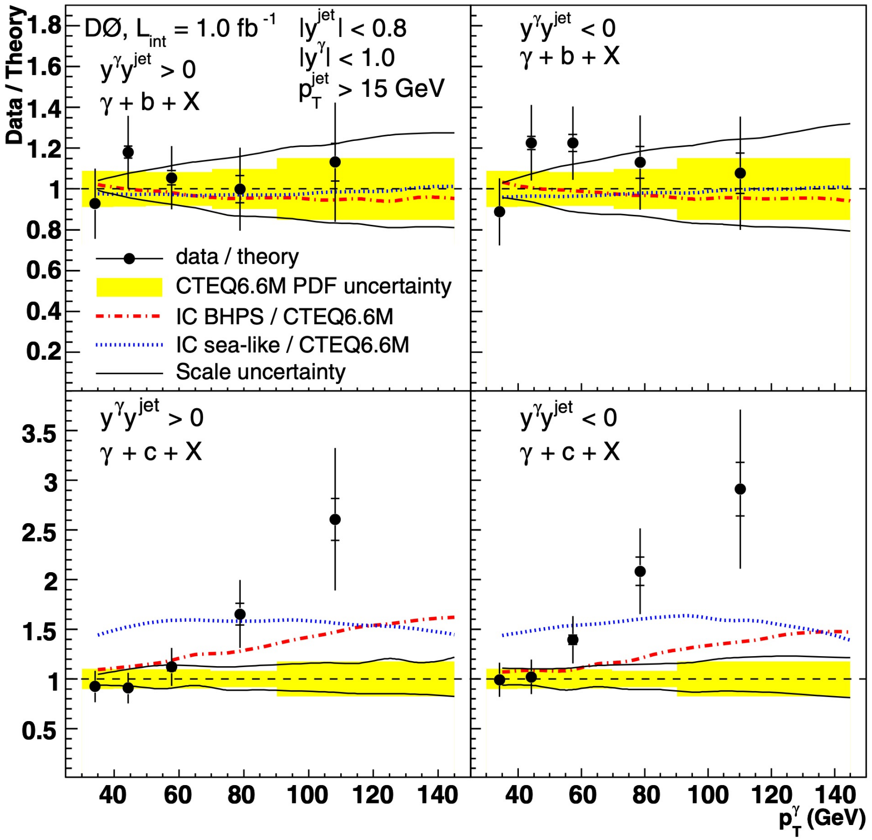
PRODUCTION OF CHARMED PARTICLES IN 250 GeV μ^+-IRON INTERACTIONS

The European Muon Collaboration



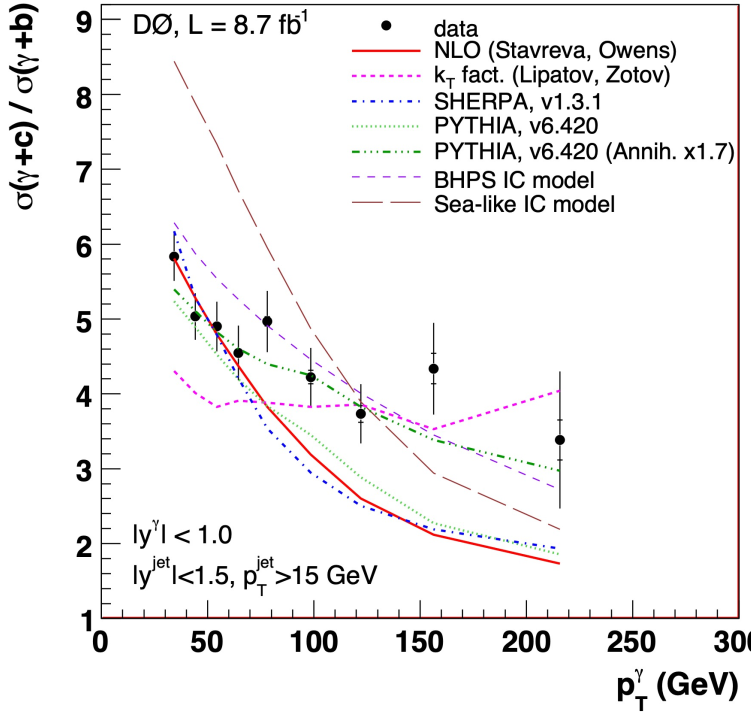
Nuc. Phys. B 213, 31 (1983)

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV



PRL 102, 192002 (2009)

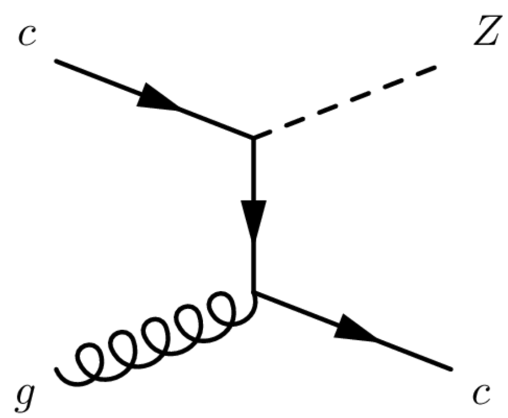
Measurement of the differential $\gamma + c$-jet cross section and the ratio of differential $\gamma + c$ and $\gamma + b$ cross sections in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV

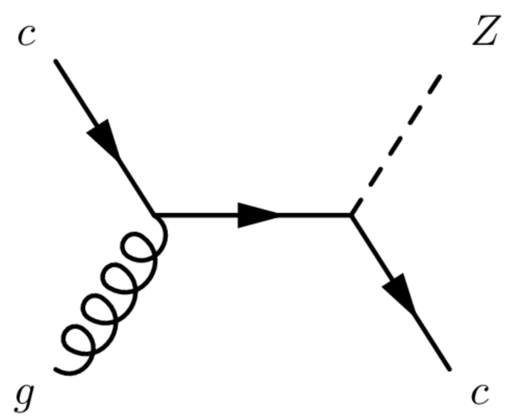


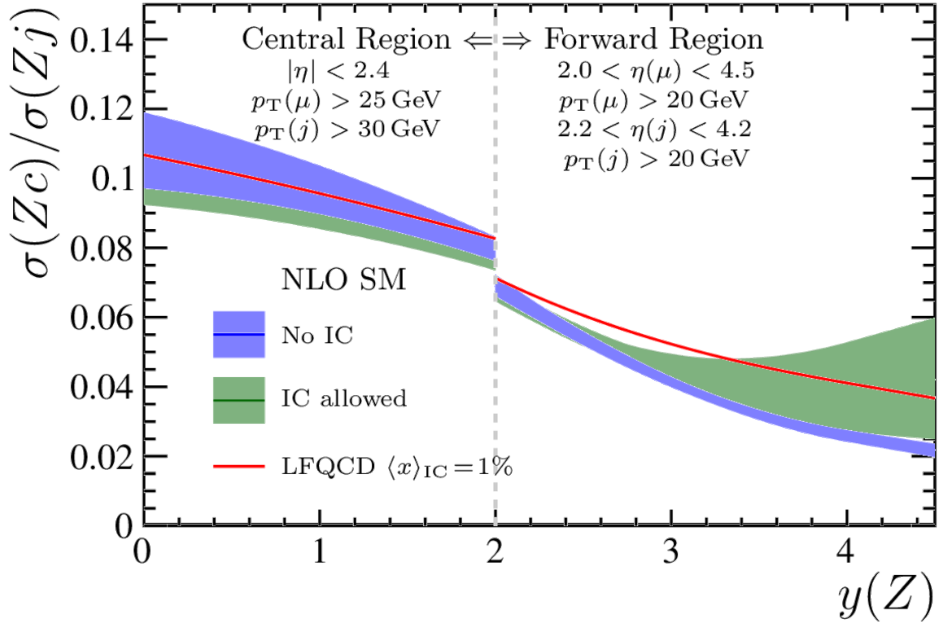
Phys. Lett. B 719 (2013) 354-361

How to probe intrinsic charm at LHCb?

- Ratio measurement: $Z_j^c \equiv \sigma(Zc)/\sigma(Zj) = \dfrac{N_c^{\mathrm{tag}}}{N_j \times \epsilon_c^{\mathrm{tag}}}$,
- IC may enhance c-jet production at high $y(Z)$
- Challenges: c-jet tagging and its efficiency



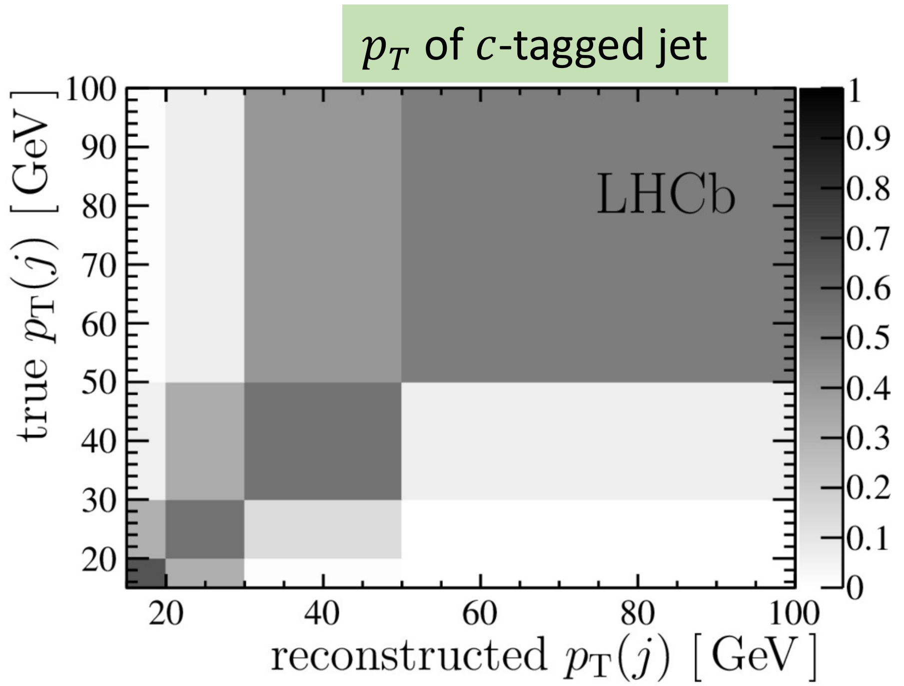




Event selection

Z bosons	$p_{\mathrm{T}}(\mu) > 20\,\mathrm{GeV},\ 2.0 < \eta(\mu) < 4.5,\ 60 < m(\mu^+\mu^-) < 120\,\mathrm{GeV}$
Jets	$20 < p_{\mathrm{T}}(j) < 100\,\mathrm{GeV},\ 2.2 < \eta(j) < 4.2$
Charm jets	$p_{\mathrm{T}}(c\text{ hadron}) > 5\,\mathrm{GeV},\ \Delta R(j, c\text{ hadron}) < 0.5$
Events	$\Delta R(\mu, j) > 0.5$

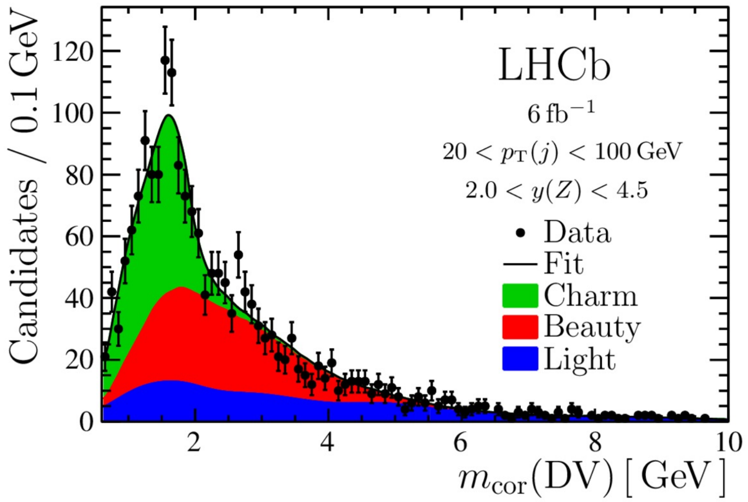
- After selection, 69k Z+jet candidates
- Binning scheme:
  - Three η bins [2.00, 2.75, 3.50, 4.50]
  - Four $p_T(jet)$ bins [15, 20, 30, 50, 100] GeV
- Unfolding to remove detector response

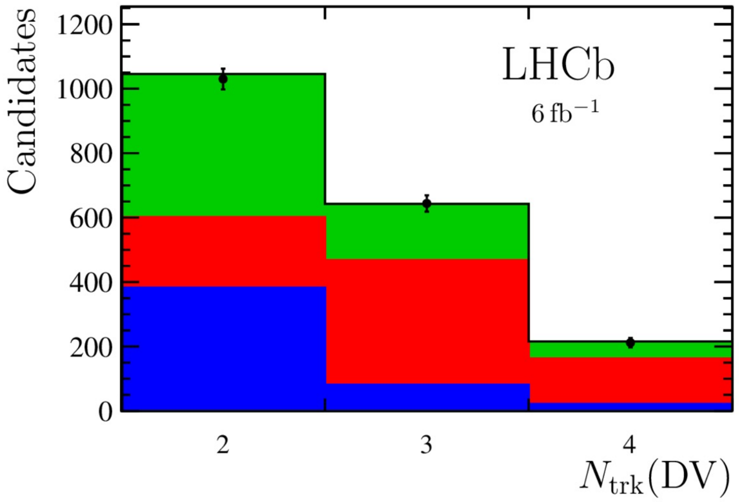


Jet and c-jet

- Jet reconstuction: particle-flow algorithm using the anti-k_T clustering algorithm (FastJet)
- Displaced-Vertex (DV) tagger: to separate c-jet from b-jet and light jets
  - Inputs: mass, momentum, position, direction
  - ≤ 4 tracks: to reject b-hadron decays

Simultaneously fit of m_{cor} and N_{trk}





c-jet tagger efficiency

- **Source of dominant systematic uncertainty**
- **Cannot fully trust simulation, must perform a data-driven calibration**
  - Use tag-and-probe method: a data/MC correction
  - Numerator: DV-tagged c-jet yield in probe sample, get from fitting
  - Denominator: the number of c-jet with exclusive decay mode, such as $D^0 \to K^-\pi^+$ and $D^+ \to K^-\pi^+\pi^+$
  - Corrections: detector response, branching fraction, fragmentation fraction
- **Topically efficiency: $(\mathbf{23.9 \pm 1.4})\%$ in the 20-30 GeV $\boldsymbol{p_T}$(jet) bin**

$$\epsilon_{\mathrm{tag,\,Q}} = \frac{N_{\mathrm{SV},\,Q}}{N_{\mathrm{total},\,Q}} \qquad N_{\mathrm{total},\,Q} = \frac{N_{D^0,\,Q}}{\epsilon_{D^0} f(Q \to D^0)\mathcal{B}(D^0 \to K^-\pi^+)},$$

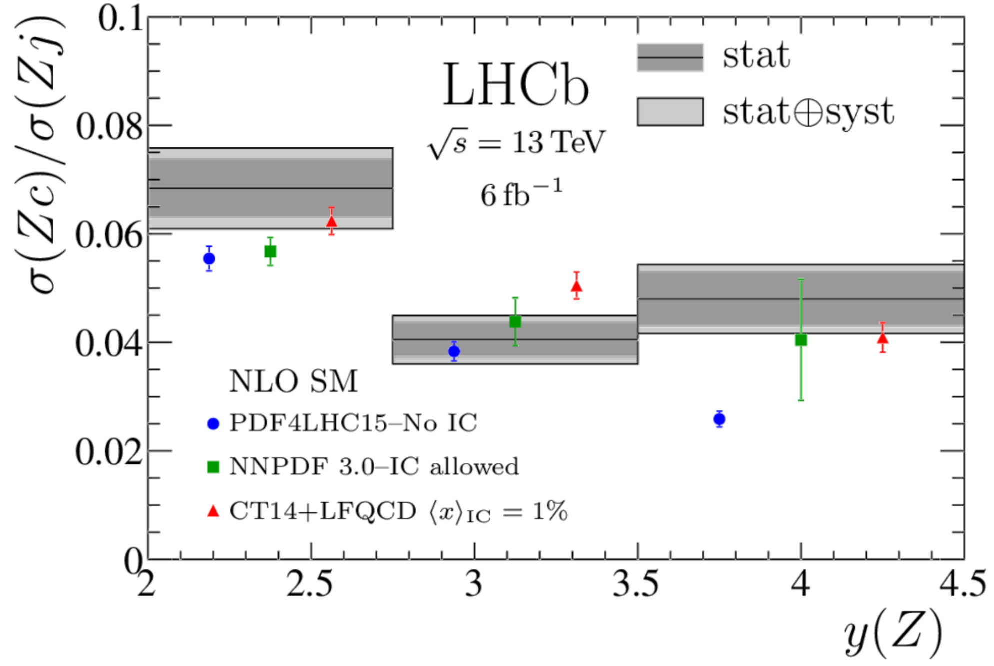
Uncertainty

Source	Relative Uncertainty
c tagging	6–7%
DV-fit templates	3–4%
Jet reconstruction	1%
Jet p_{T} scale & resolution	1%
Total	8%

Results

arXiv:2109.08084
Submitted to PRL

- The measured result has a sizeable enhancement at forward Z rapidity region
  - Consistent with LFQCD + IC
  - Indicate a valence-like IC component in the proton wave function



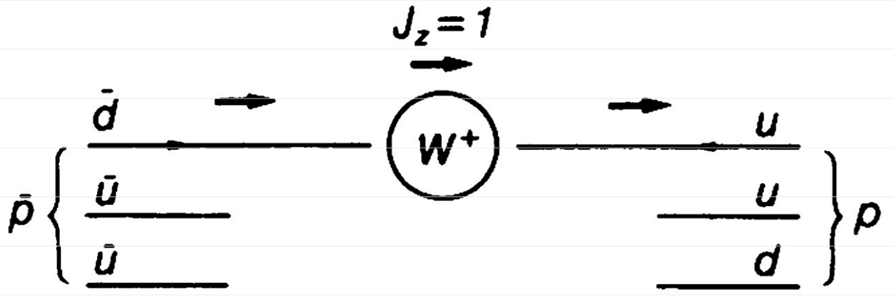
Prospects

What can be done further at LHCb?

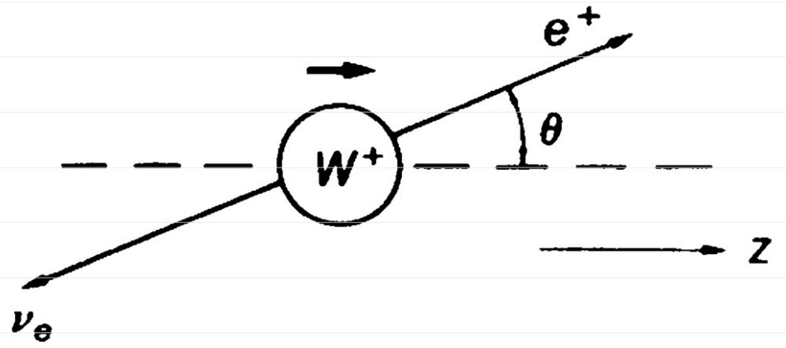
- W charge asymmetry
- W cross section measurement
- Z angular coefficient measurement
- V+jet:
  - $V = W, Z$
  - Jet: light hadron, c-jet, b-jet

W boson charge asymmetry

- Probe valence quark: u and d quarks
- Asymmetries:
  - W boson charge asymmetry: $A = \frac{\frac{d\sigma}{dy_W^+} - \frac{d\sigma}{dy_W^-}}{\frac{d\sigma}{dy_W^+} + \frac{d\sigma}{dy_W^-}}$



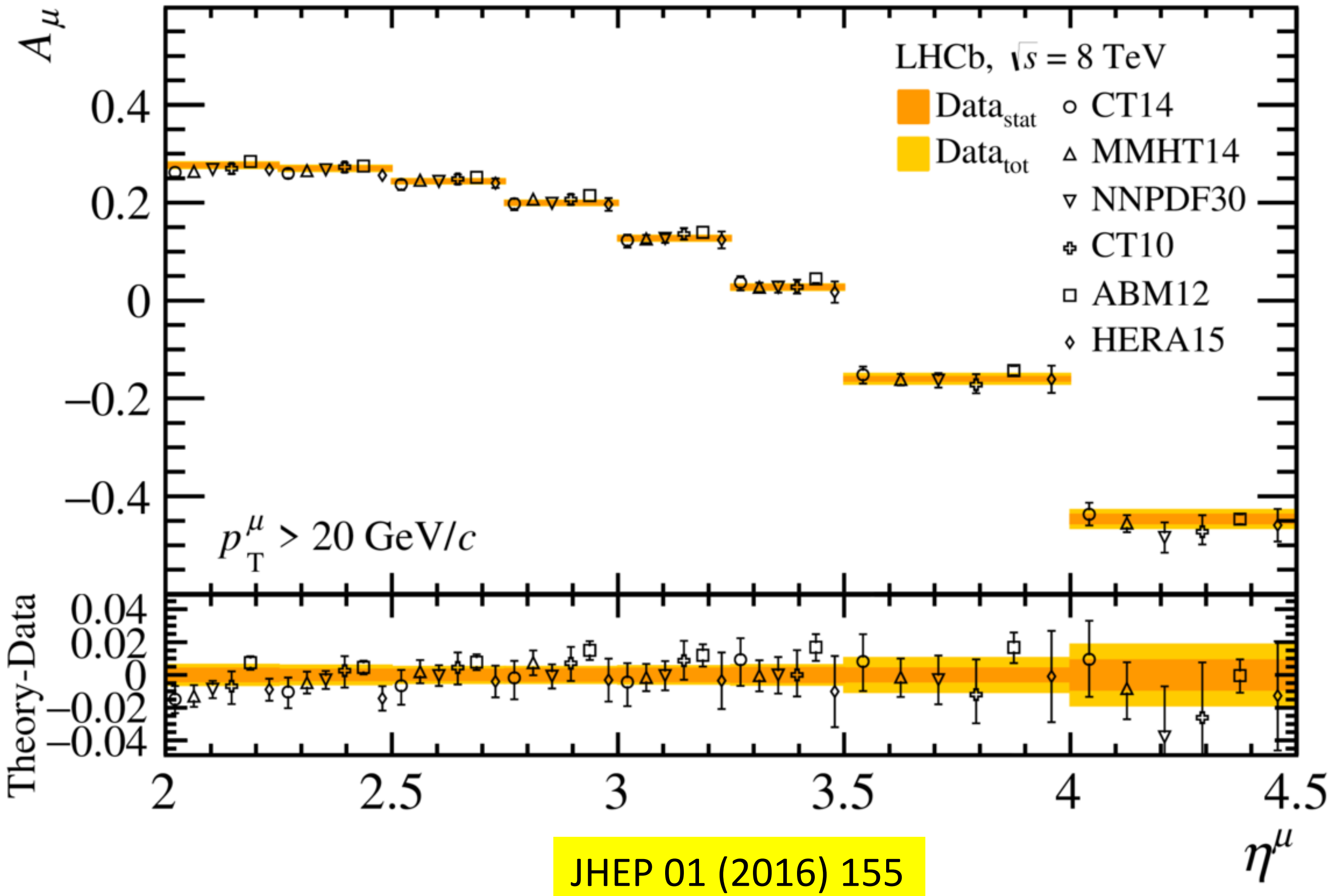
  - However, there is a neutrino in the final state



  - Lepton charge asymmetry: $A = \frac{\frac{d\sigma}{dy_l^+} - \frac{d\sigma}{dy_l^-}}{\frac{d\sigma}{dy_l^+} + \frac{d\sigma}{dy_l^-}}$

W boson charge asymmetry

- Probe va
- Asymme
  - W bos
  - Howev
  - Leptor



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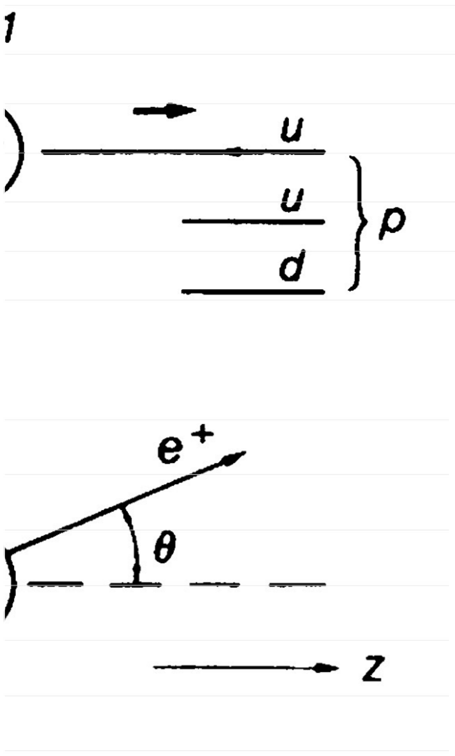
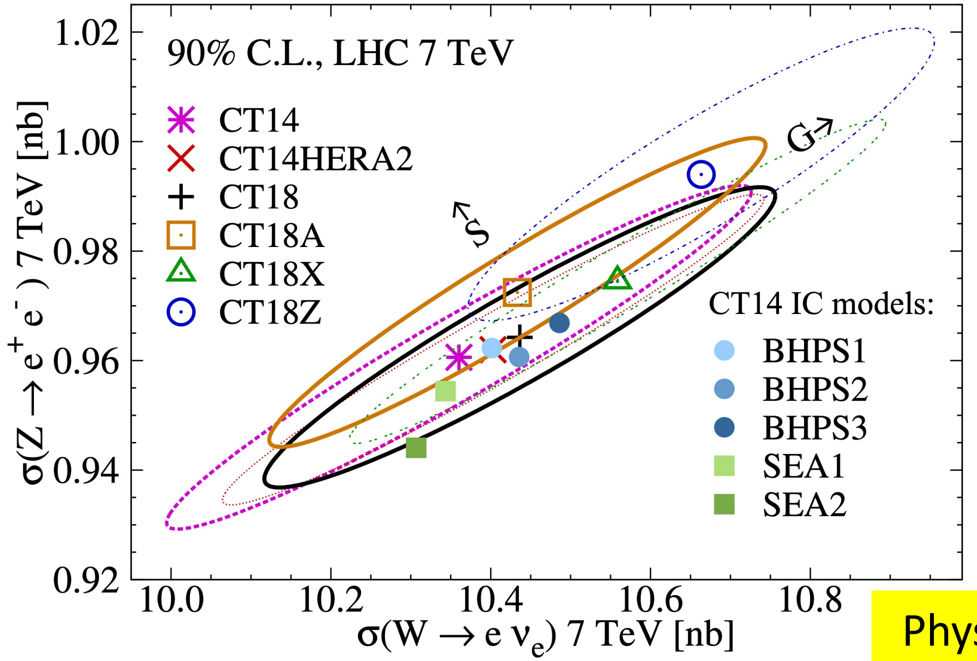
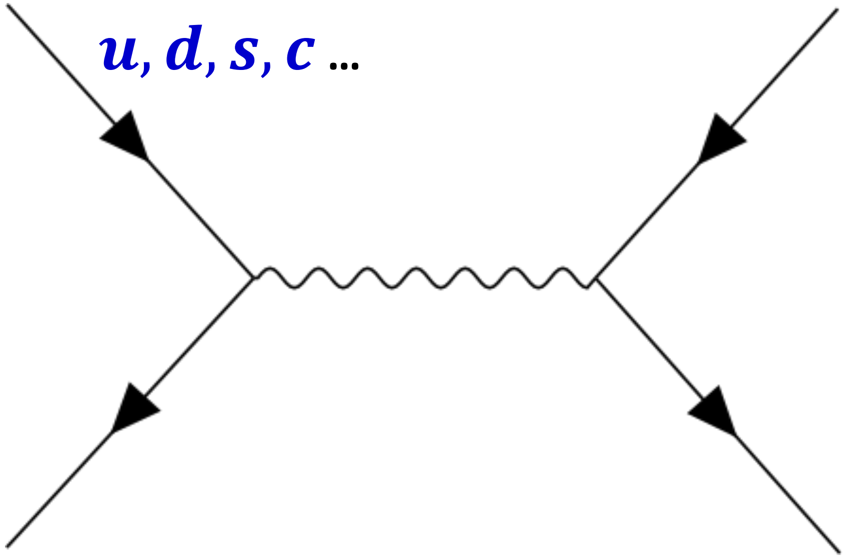


W cross section measurement

- Probe the *s* quark PDFs
- Use *W* and *Z* boson cross-section ratio
  - $R = \frac{\sigma(W^+)+\sigma(W^-)}{\sigma(Z)}$
  - In the first order: $R \propto \frac{u\bar{s}+\bar{u}s}{u\bar{u}+d\bar{d}} \approx \frac{s+\bar{s}}{\bar{u}+\bar{d}}$





Phys. Rev. D 103, 014013 (2021)

Z angular coefficient measurement

$$\frac{d\sigma}{dP_T^2\,dy\,d\cos\theta\,d\phi} \propto (1+\cos^2\theta)$$ → *LO term*

$$+\frac{1}{2}A_0(1-3\cos^2\theta)$$ → *$\cos^2\theta$: higher order term*

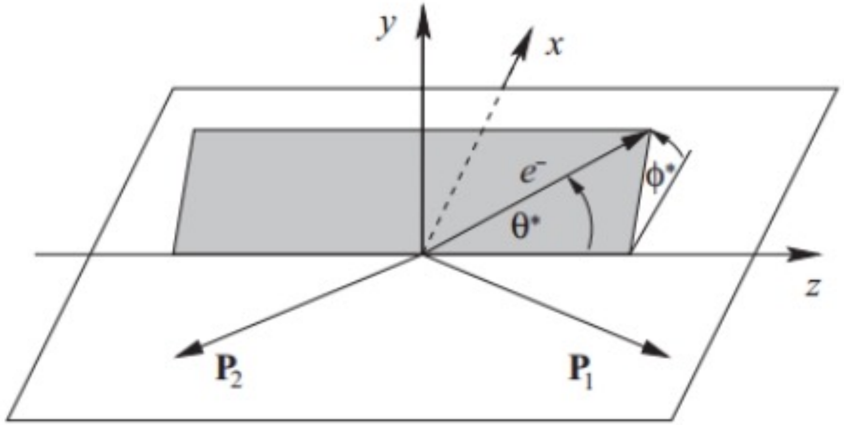
$$+A_1\sin 2\theta\cos\phi+\frac{1}{2}A_2\sin^2\theta\cos 2\phi+A_3\sin\theta\cos\phi$$ → *(θ, ϕ) terms*

$$+A_4\cos\theta$$ → *LO term : determine A_{fb}*

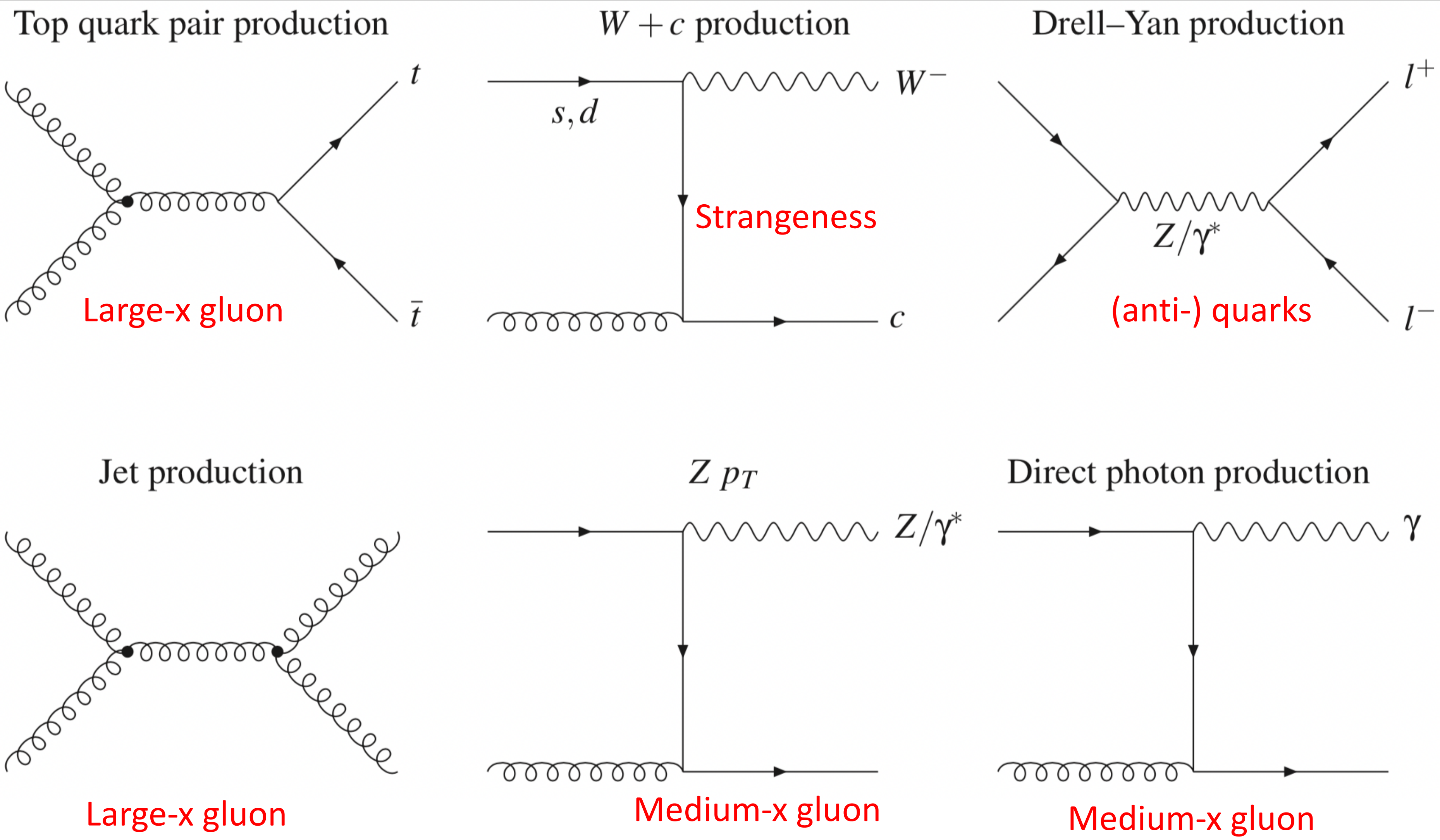
$$+A_5\sin^2\theta\sin 2\phi+A_6\sin 2\theta\sin\phi+A_7\sin\theta\sin\phi$$ → *very small terms*

- A_2 is sensitive to the Boer-Mulders transverse momentum dependent PDFs (TMD)

- LHCb Run-2 result: close to submission
  - Low mass, middle mass and high mass region
  - Study A_2 vs. $Z\ p_T$



V+Jet and other measurements



Summary

- The knowledge of the PDFs is crucial for precision measurements at hadron colliders

- With detector instrumented in the forward region, the LHCb results could provide unique information for the PDFs fitting
  - Very large or small x region
  - Sea quark PDFs
  - Instrisic charm PDFs
  - ….

- The EW processes play a key role in the LHCb proton structure studies

Stay tuned for new results !

Physics @LHCb

Better understanding of the SM

Challenging the SM

QCD

QED and Weak

Backup

Parton-Parton interactions

- Interaction between the partons which constitute the hadrons
  - Not well defined parton energy, but energy distribution
- PDFs are parameterizations of the partonic content of the proton
- At hadron colliders, the cross-section calculation is a convolution of the cross-section at parton level and PDFs:
  - $\sigma_X = \sum_{a,b} \int_0^1 dx_a dx_b f(x_a, flav_a, Q^2) f(x_b, flav_b, Q^2) \cdot \sigma_{ab\to X}(x_a, x_b, Q^2)$

Sum over initial partonic states a,b

Parton Distribution Function

Hard scattering cross-section

PDFs analyses

- Predictions are calculated with NNLO (QCD) and NLO (EW) accuracy using either analytic formulae for NC/CC DIS processes, or K-factor techniques for other processes
- Global fit: adjusting agreement between predictions (T) and data (D)
  - $$\chi^2 = \sum_i \frac{\left[D_i - T_i\left(1 - \sum_j \gamma_{ij} b_j\right)\right]^2}{\delta^2_{i,\mathrm{uncor}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i} + \sum_j b_j^2 + \sum_i log \frac{\delta^2_{i,\mathrm{uncor}} T_i^2 + \delta^2_{i,\mathrm{stat}} D_i T_i}{\delta^2_{i,\mathrm{uncor}} D_i^2 + \delta^2_{i,\mathrm{stat}} D_i^2}$$

    Partial χ^2 Correlated χ^2 Log penalty

  - Includes uncorrelated, statistical uncertainties, also the correlated one (γ) using nuisance parameters b

DGLAP function

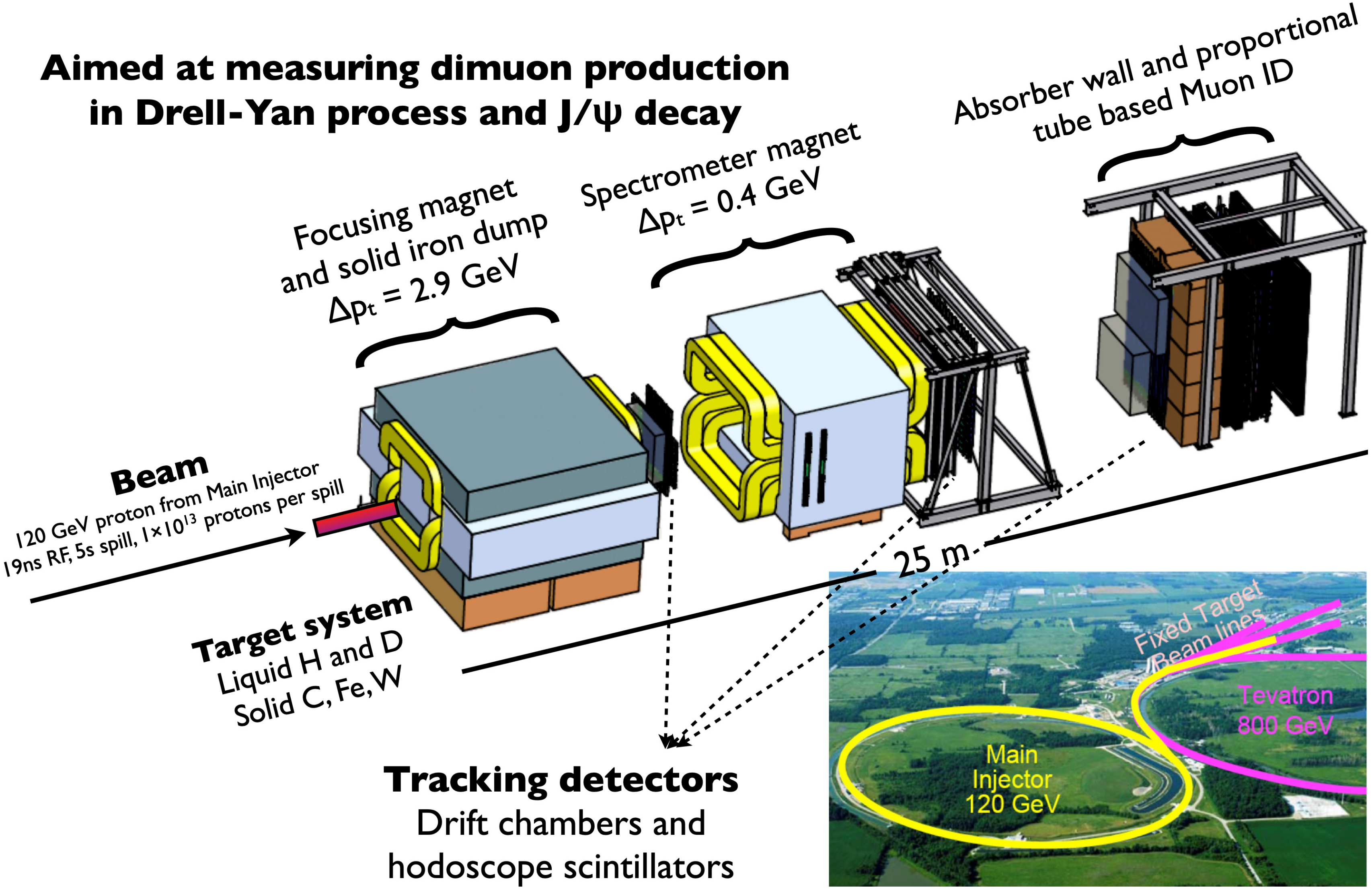
- Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations
  - Vladimir Gribov and Lev Lipatov in 1972 [Sov. J. Nucl. Phys. 15:438 (1972)]
  - Yuri Dokshitzer in 1977 [Sov. Phys. JETP 46:641 (1977)]
  - Guido Altarelli and Giorgio Parisi in 1977 [Nuclear Physics B. 126 298–318]

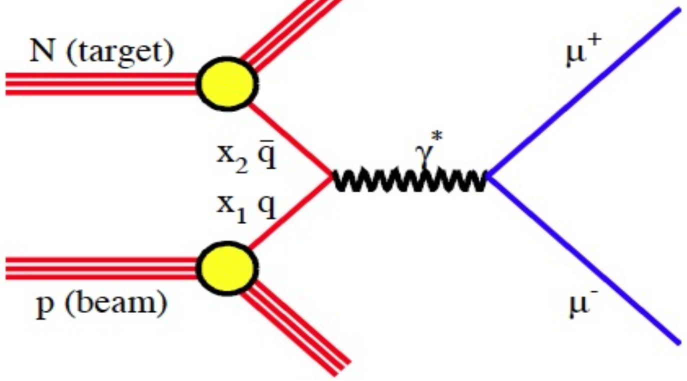
2015 High Energy and Particle Physics Prize (EPS)

Giorgio Parisi
The Nobel Prize in Physics 2021

E906/SeaQuest experiment @Fermilab

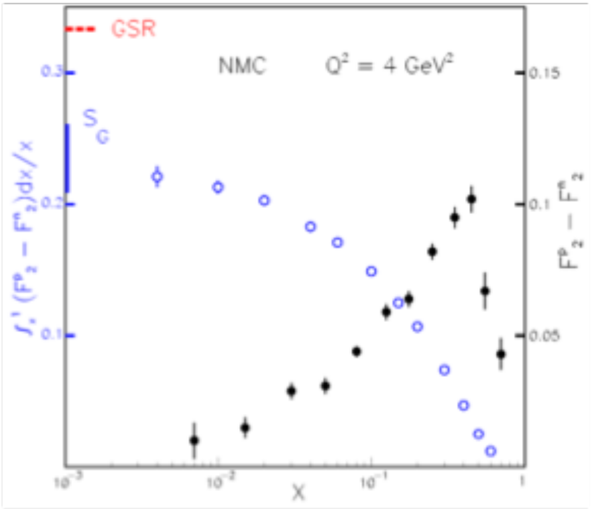
Aimed at measuring dimuon production in Drell-Yan process and J/ψ decay



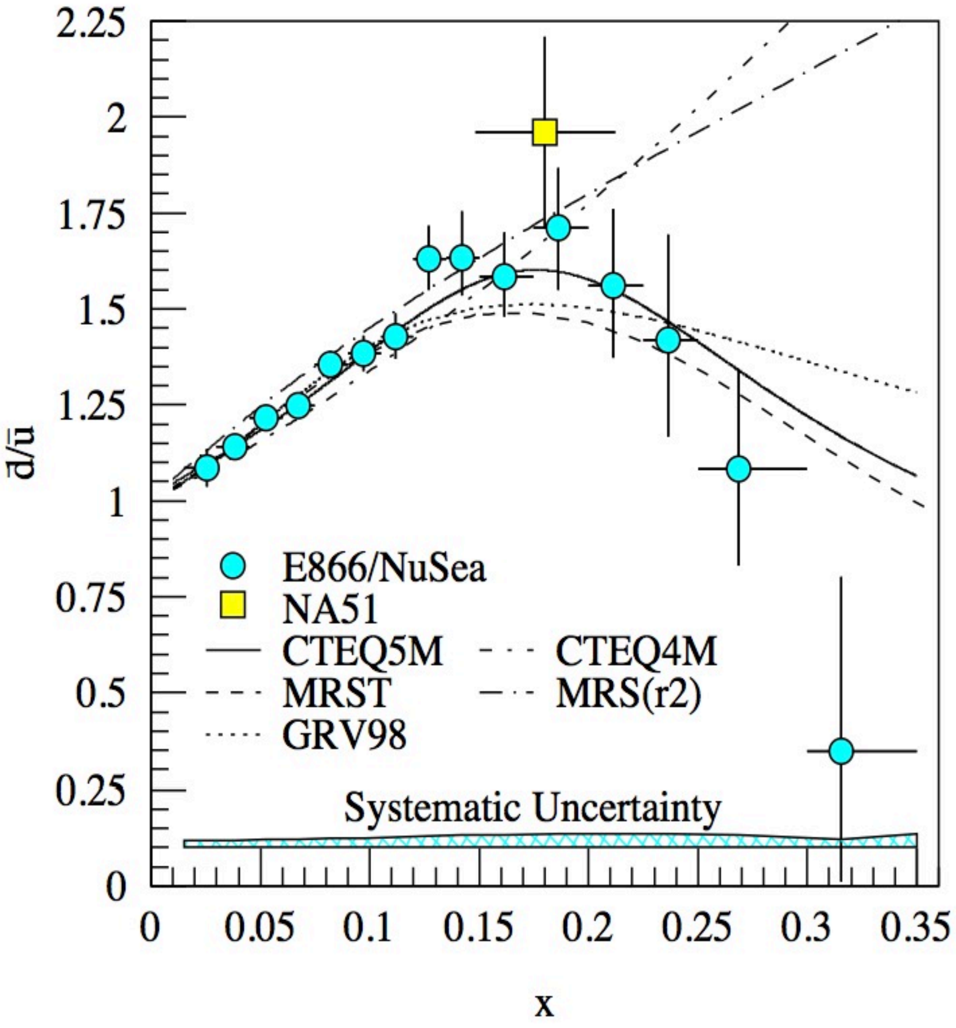


New Muon Collaboration (NMC)

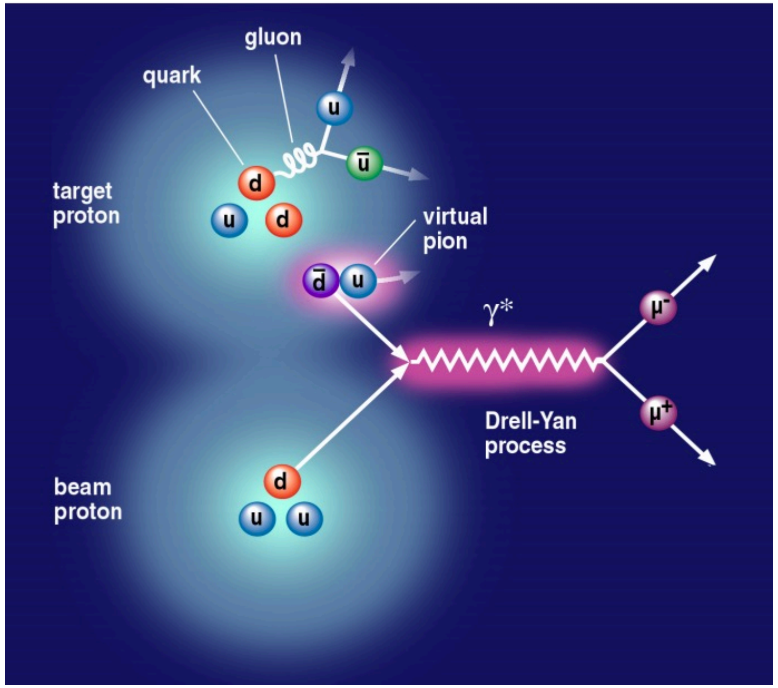
- Gottfried Sum Rule
  - $S_G = \frac{1}{3} + \int_0^1 \frac{2}{3}\left(\bar{u}_p(x) - \bar{d}_p(x)\right) dx$
- Symmetric sea implies $S_G = \frac{1}{3}$
- NMC experiment (LD2, LH2, 90 GeV and 280 GeV μ-beam)
  - $S_G = \int_0^1 \frac{(F_2^p - F_2^n)dx}{x} = 0.235 \pm 0.026$
  - Violation of symmetric sea quark contributions!



Flavor asymmetry in light quark sea

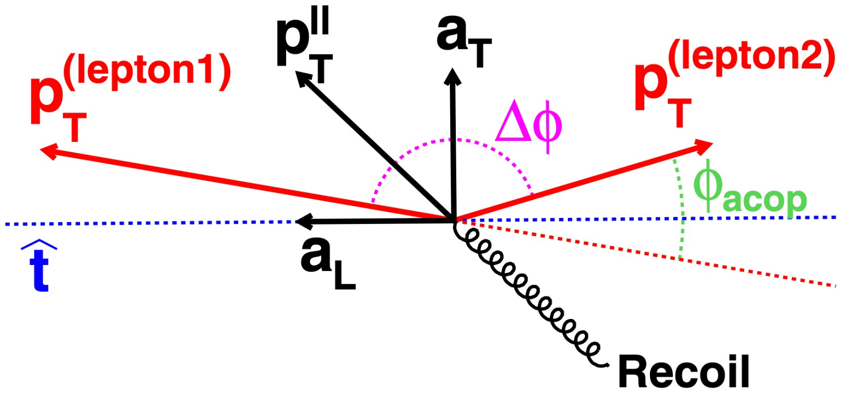


Asumming charge symmetry, ignoring nuclear effects of deuterium
And heavy quark contributions
Naively, we should expect flavour symmetry between $\bar{u}$ and $\bar{d}$
E866/NuSea experiment reveals a striking asymmetry in the sea
Distribution at moderate x

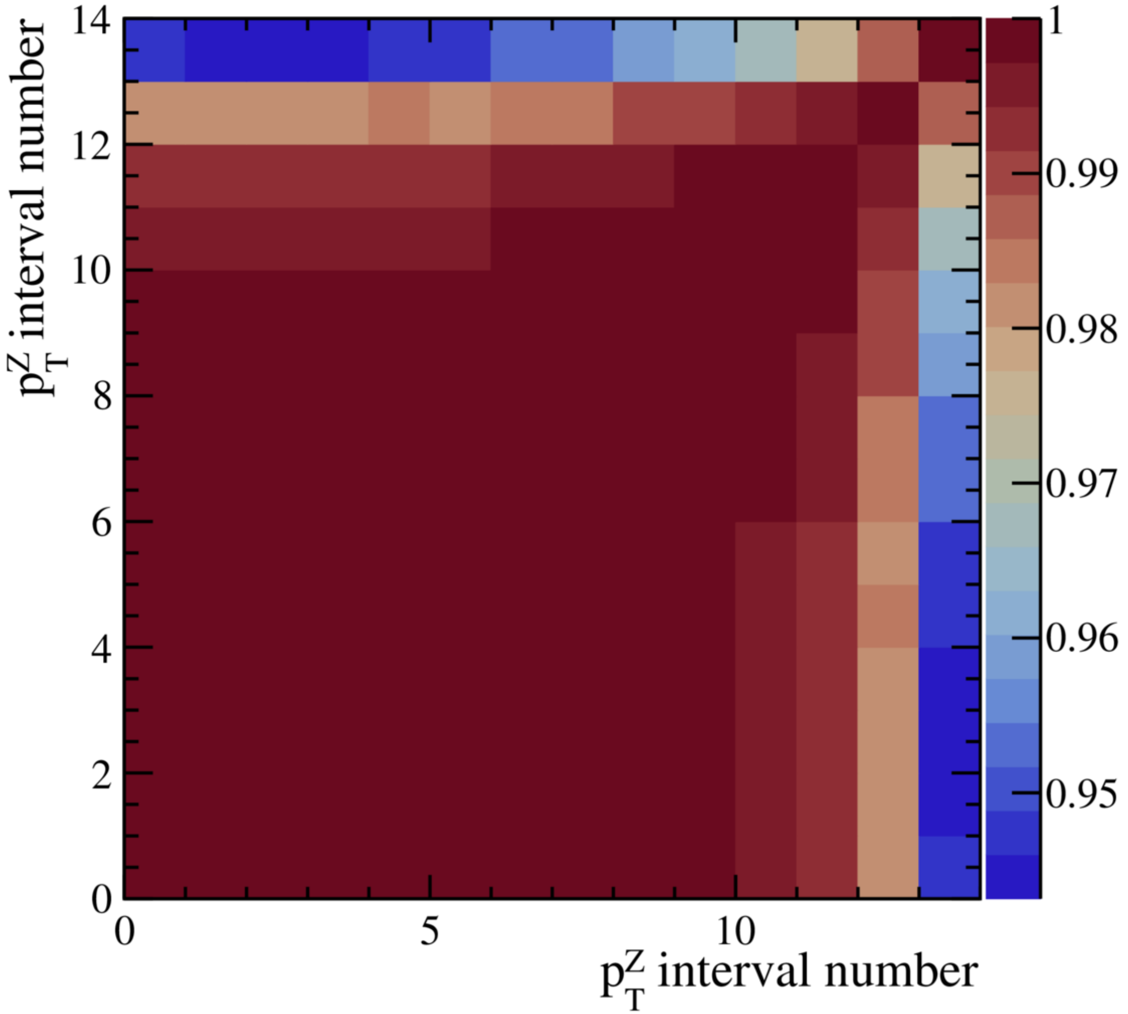


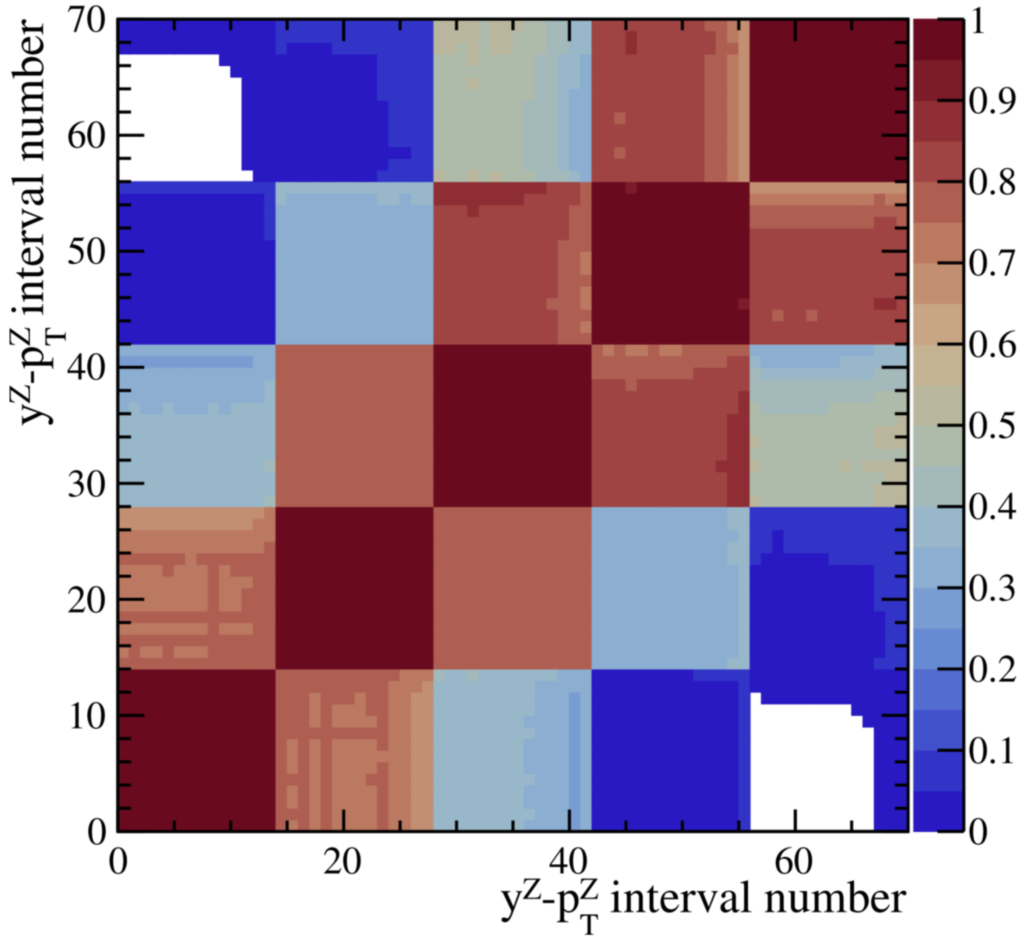
ϕ_η^* angle

- Probe same physics Z boson p_T, but smaller uncertainty
- $\phi_\eta^* = \tan(\phi_{acop}/2)\sin(\theta_\eta^*)$
  - ϕ_{acop}: acoplanarity angle, $\pi - \Delta\phi_{ll}$
  - $\Delta\phi_{ll}$: the difference in azimuthal angle, ϕ, between two lepton candidates
  - θ_η^*: scattering angle of the leptons with respect to the proton beam in the rest frame of dilepton system:
    - $\cos(\theta_\eta^*) = \tanh[\eta^- - \eta^+/2]$



Correlation matrix





TMD

- The genenral PDFs describes the parton inside a proton
- Only have one longitudinal freedom, x
  - quarks are perfectly collinear
- Transverse Momentum Dependent PDFs: TMD
  - Admit a finite quark transverse momentum k_T
  - Provide 3D image of hadrons in momentum space
  - Correlation between parton momentum and hadron spin

leading twist		quark polarization		
		unpolarized [U]	longitudinal [L]	transverse [T]
nucleon polarization	U	f_1 = unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − helicity	$h_{1L}^\perp$ = − worm gear 1
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − worm gear 2	h_1 = − transversity $h_{1T}^\perp$ = − pretzelosity

Boer-Mulders function

- By D. Boer, P. J. Mulders (1997):
  - Phys. Rev. D 57 (1998), 5780

$$h_1^{\perp[\mathcal{C}]}(x, k_T^2)\, \epsilon_T^{ij} k_{Tj} = \frac{M}{2}\, \text{F.T.}\, \langle P|\, \overline{\psi}(0)\, \mathcal{L}_{\mathcal{C}}(0,\xi)\gamma^i\gamma^+\gamma_5\psi(\xi)\, |P\rangle\big|_{\xi^+=0}$$

- Represents a correlation between quark $\boldsymbol{k_T}$ and transverse spin in an unpolarized hadron
  - A time-reversal odd, chiral-odd TMD PDFs
  - Lead to an azimuthal $\cos(2\phi)$ dependence in Drell-Yan

Time-reversal odd distribution functions in leptoproduction

Boer-Mulders function

A correlation between quark k_T and transverse spin in an unpolarized hadron!

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Transverse momentum

Parton momentum as a fraction of proton momentum

Time-reversal odd distribution functions in leptoproduction

Boer-Mulders function

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$$h_1^{\perp[\mathcal{C}]}(x, k_T^2)\,\epsilon_T^{ij} k_{Tj} = \frac{M}{2}\,\mathrm{F.T.}\,\langle P|\,\overline{\psi}(0)\,\mathcal{L}_{\mathcal{C}}(0,\xi)\gamma^i\gamma^+\gamma_5\psi(\xi)\,|P\rangle|_{\xi^+=0}$$

A proton with a momentum P

Time-reversal odd distribution functions in leptoproduction

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A three dimensional Fourier transform

Time-reversal odd distribution functions in leptoproduction

Boer-Mulders function

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$$h_1^{\perp[\mathcal{C}]}(x, k_T^2)\, \epsilon_T^{ij} k_{Tj} = \frac{M}{2}\, \text{F.T.}\, \langle P|\, \overline{\psi}(0)\, \mathcal{L}_{\mathcal{C}}(0,\xi)\gamma^i\gamma^+\gamma_5\psi(\xi)\, |P\rangle\big|_{\xi^+=0}$$

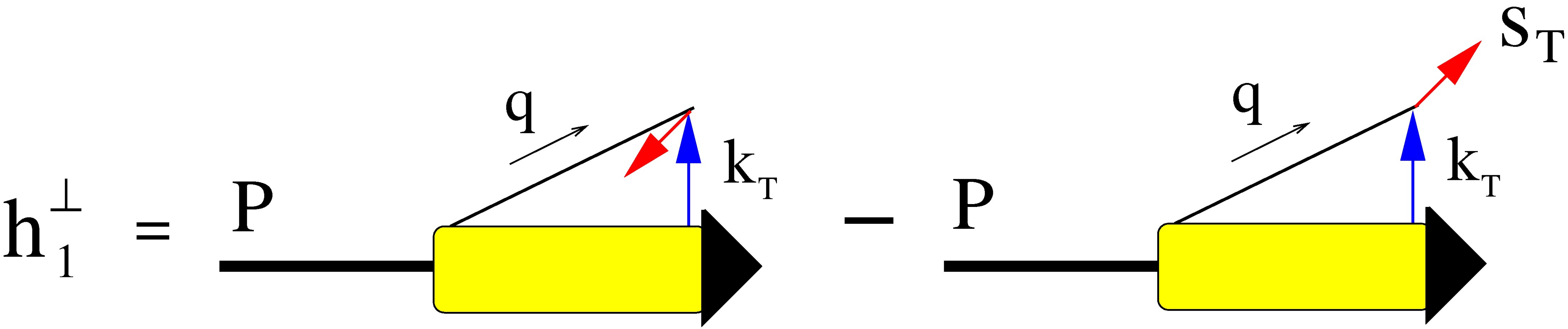
A specific path of the Wilson line L, depends on the process in a calculable and predictable way

Time-reversal odd distribution functions in leptoproduction

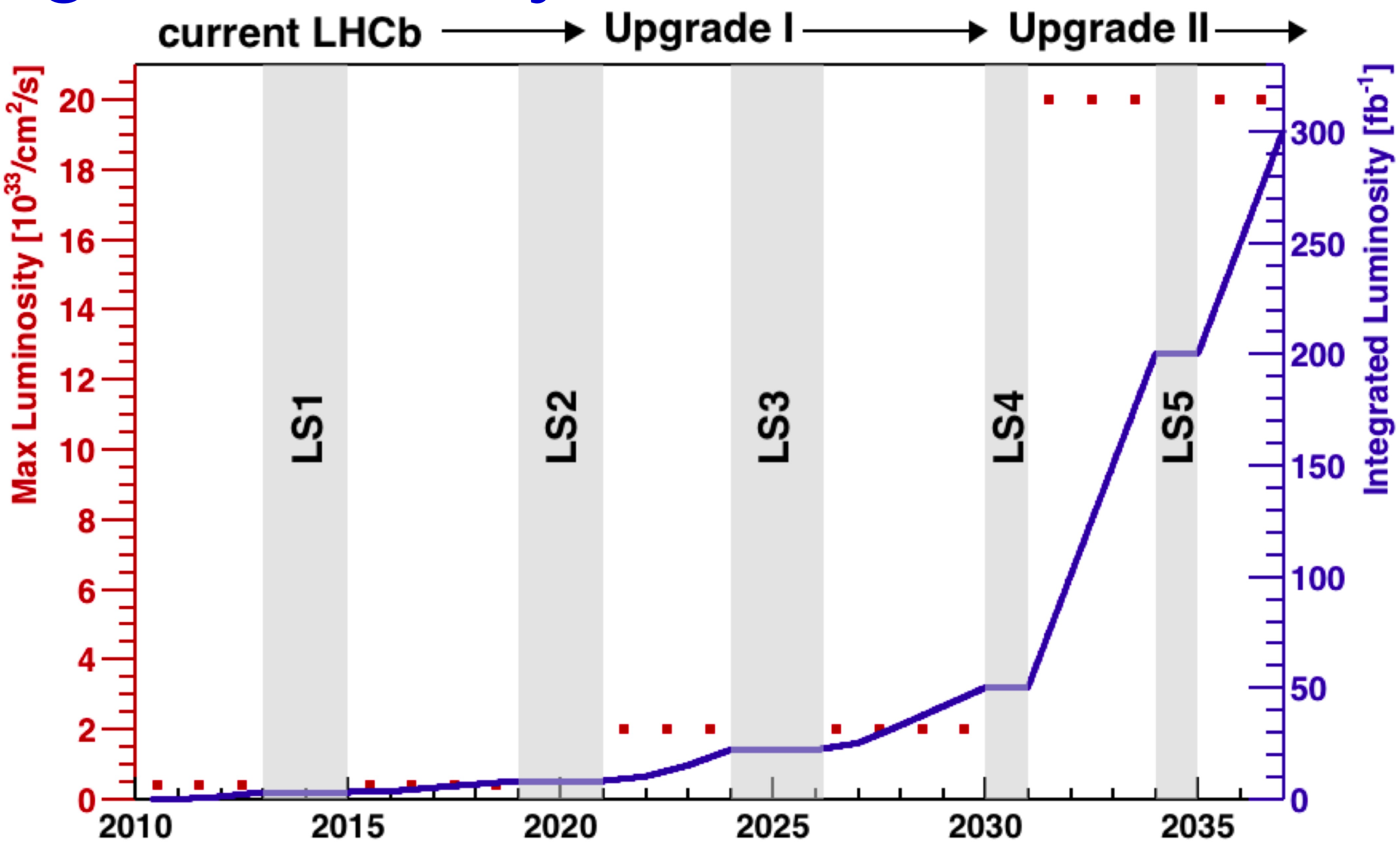
Boer-Mulders function

- By D. Boer, P. J. Mulders (1997):
  - Phys. Rev. D 57 (1998), 5780

$$h_1^{\perp[\mathcal{C}]}(x, k_T^2)\,\epsilon_T^{ij} k_{Tj} = \frac{M}{2}\,\text{F.T.}\,\langle P|\,\overline{\psi}(0)\,\mathcal{L}_{\mathcal{C}}(0,\xi)\gamma^i\gamma^+\gamma_5\psi(\xi)\,|P\rangle\big|_{\xi^+=0}$$



Target luminosity

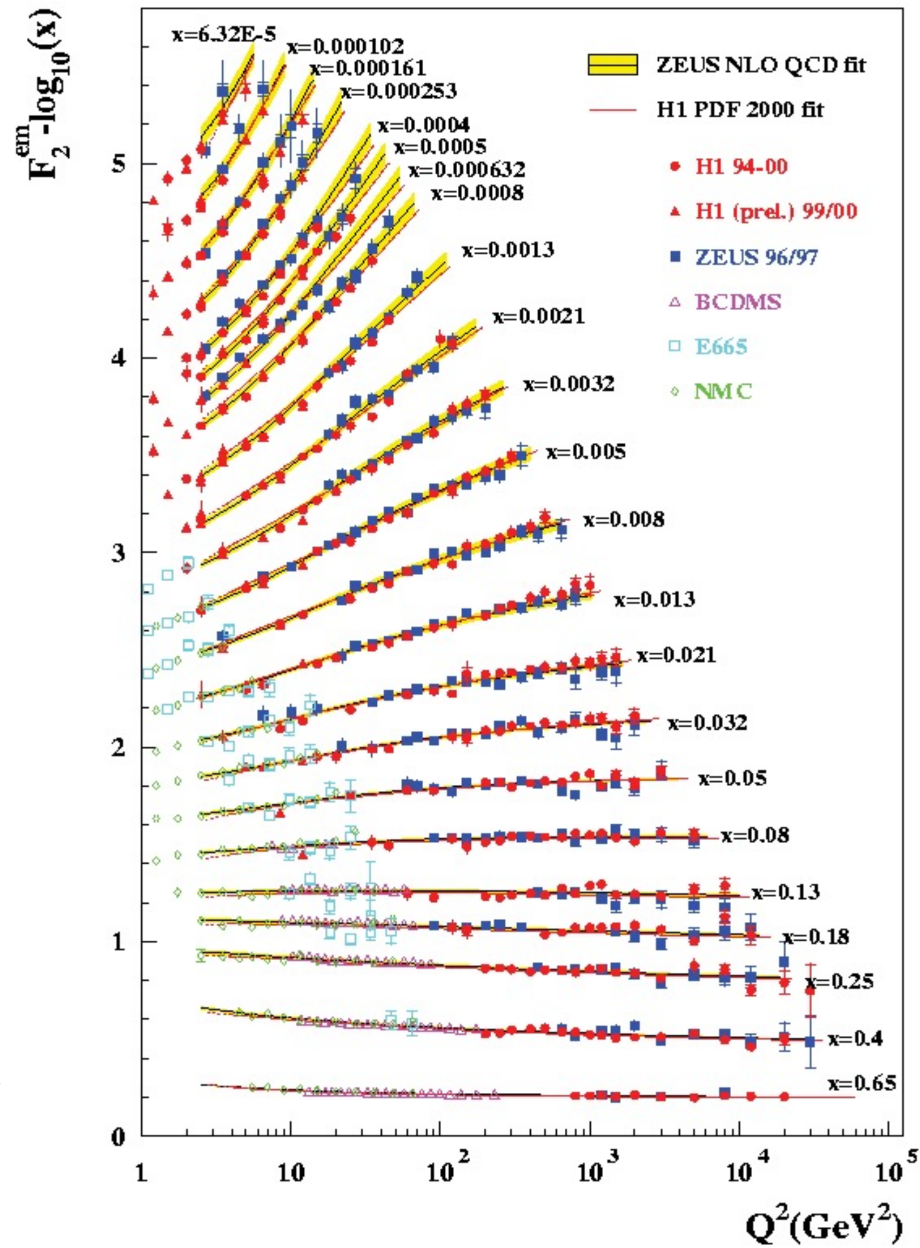


$F_2(x, Q^2)$ results

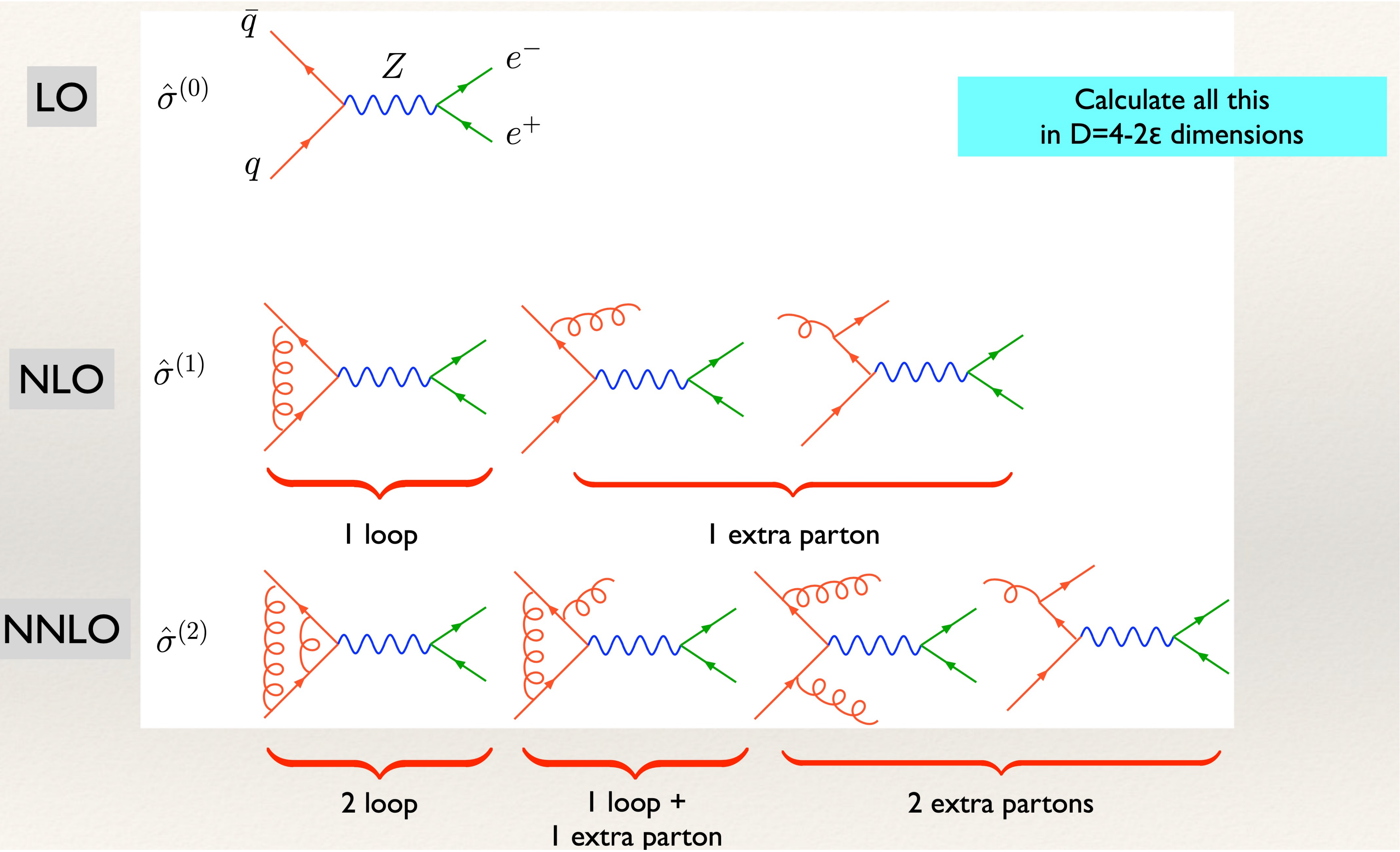
- For the $e^- p \to e^- X$ inelastic scattering

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + yF_1(x,Q^2)\right]$$

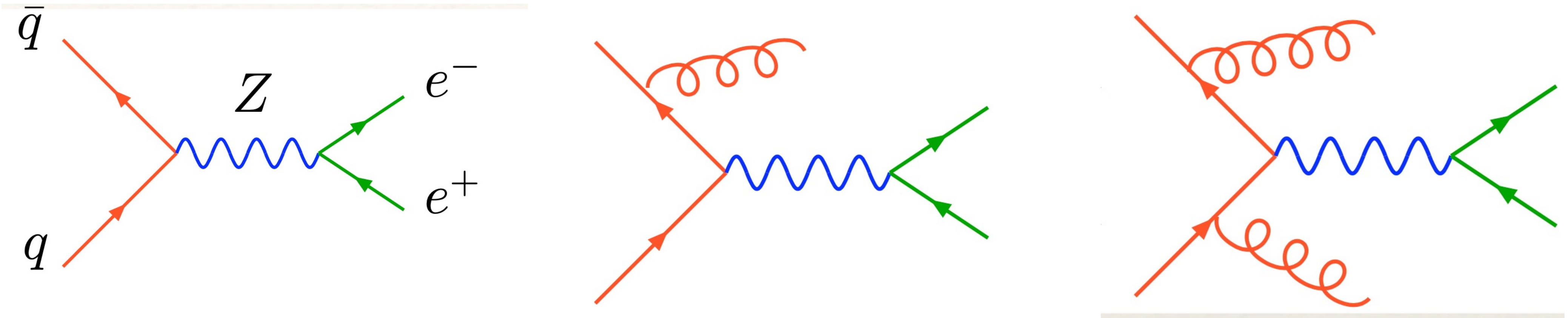
  - $F_1(x,Q^2)$, $F_2(x,Q^2)$: structure functions
  - $F_1(x,Q^2)$: pure magnetic structure function
  - $F_2(x,Q^2)$: electromagnetic structure function
  - Independent of Q^2: Bjorken scaling
  - Callan-Gross relation: $F_2(x) = 2xF_1(x)$
- In the low x region,
  - Clear violations of scaling
  - At high Q^2 (shorter wave-length) resolve finer structure



Leading-Order and Higher-Order amplitudes



Higher-Order calculation



$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right),$$ with $L \equiv ln\left(\frac{Q^2}{q_T^2}\right)$

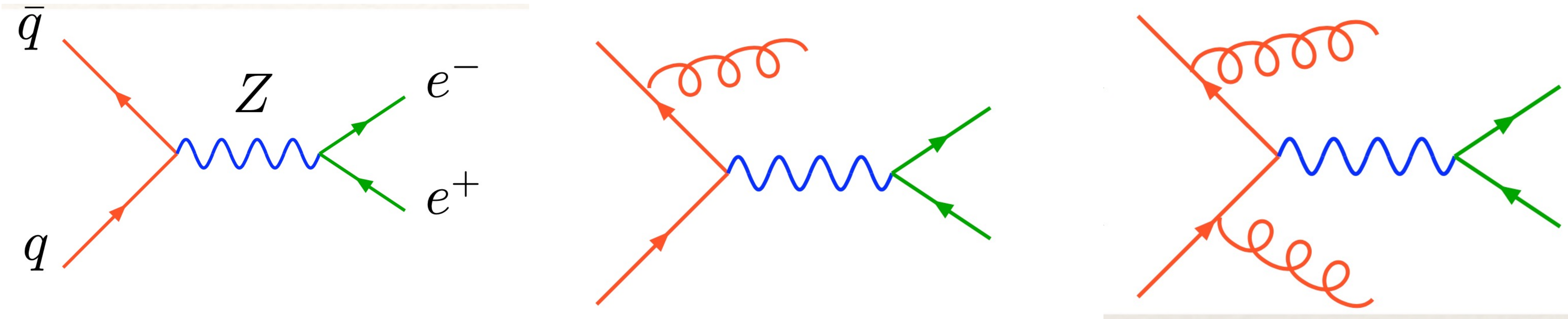
$$\sim \frac{1}{q_T^2}\{\alpha_S(L+1)$$
$$+\alpha_S^2(L^3+L^2+L+1)$$
$$+\alpha_S^3(L^5+L^4+L^3+L^2+L+1)$$
$$+\cdots\}$$

In the low q_T region, $\alpha_S \ln(\frac{Q^2}{q_T^2}) \approx 1$

For example, $0.1 \times \ln(\frac{90^2}{3^2}) \sim 1$!!

Even larger than 1.

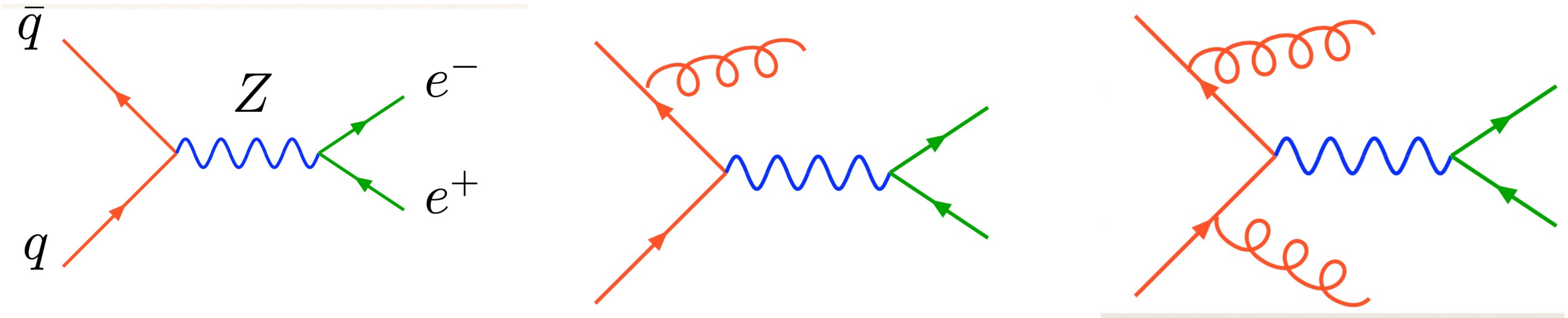
Higher-Order calculation: resummation



$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right), \text{ with } L \equiv ln\left(\frac{Q^2}{q_T^2}\right)$$

$$\begin{aligned}
\sim \frac{1}{q_T^2}\{ & [\alpha_S(L+1) + \alpha_S^2(L^3+L^2) + \alpha_S^3(L^5+L^4) + \cdots] \\
& +[\quad + \alpha_S^2(L+1) \quad + \alpha_S^3(L^3+L^2) + \cdots] \\
& +[\quad + \alpha_S^3(L+1) \quad + \cdots] \\
& + \cdots \}
\end{aligned}$$

Higher-Order calculation: Collins-Soper-Sterman



$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right), \text{ with } L \equiv ln\left(\frac{Q^2}{q_T^2}\right)$$

$$\sim \alpha_S^n (C_0 + C_1 \alpha_S + \cdots) exp[(\sum_{n=1} \alpha_S^n L^{n+1} c_n) + (\sum_{n=1} \alpha_S^n L^n d_n) + (\sum_{n=1} \alpha_S^n L^{n-1} e_n) + \cdots]$$

LL,NLL

NNLL

LL

NLL

NNLL

Fixed-Order vs. Resummation

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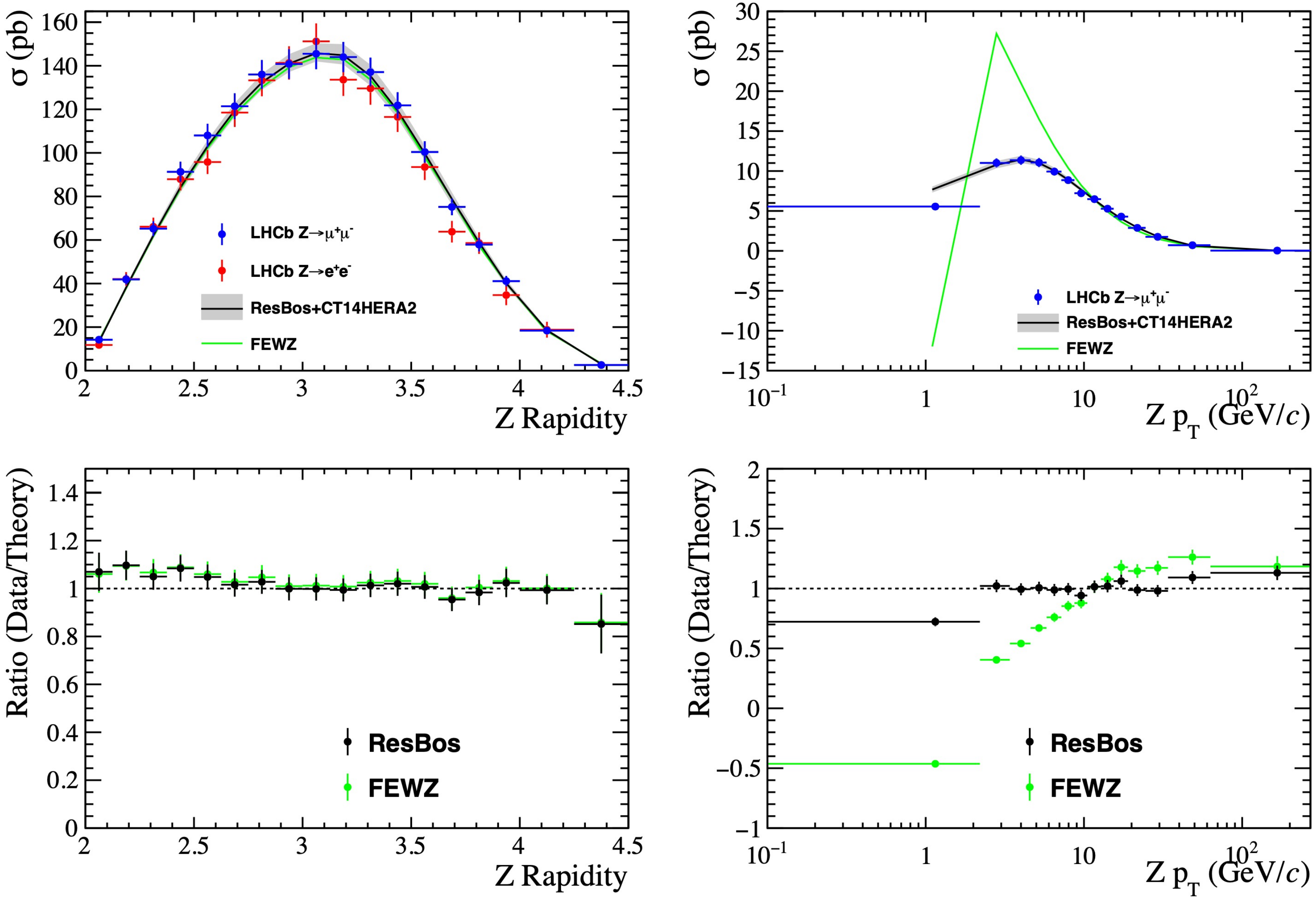

Impacts from the future LHCb data

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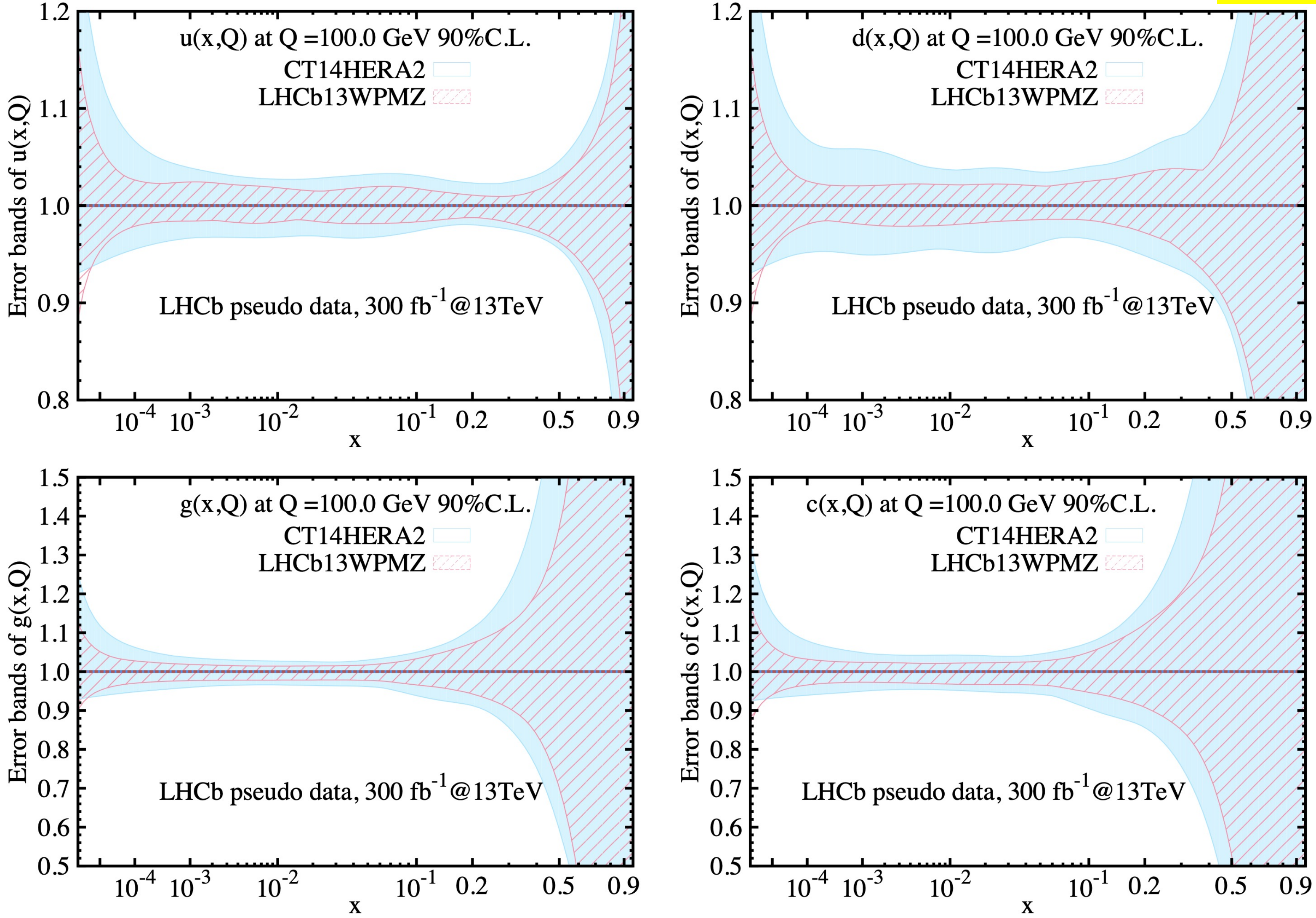

Impacts from the future LHCb data

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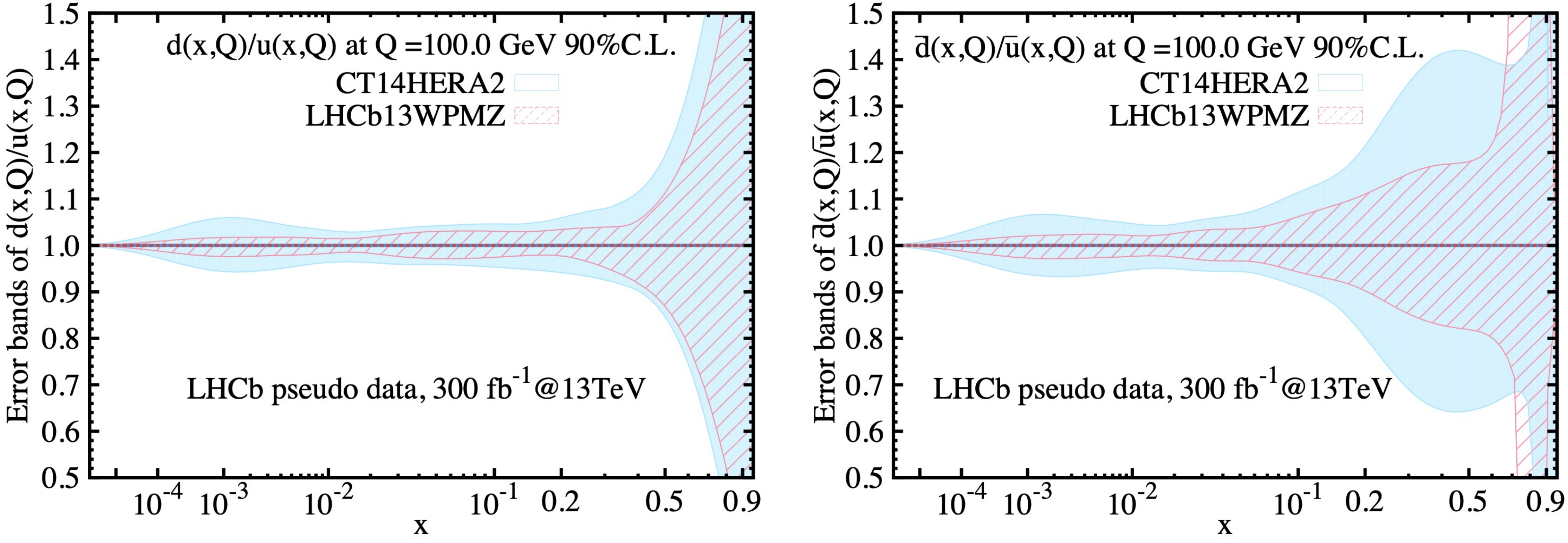

Impacts from the future LHCb data

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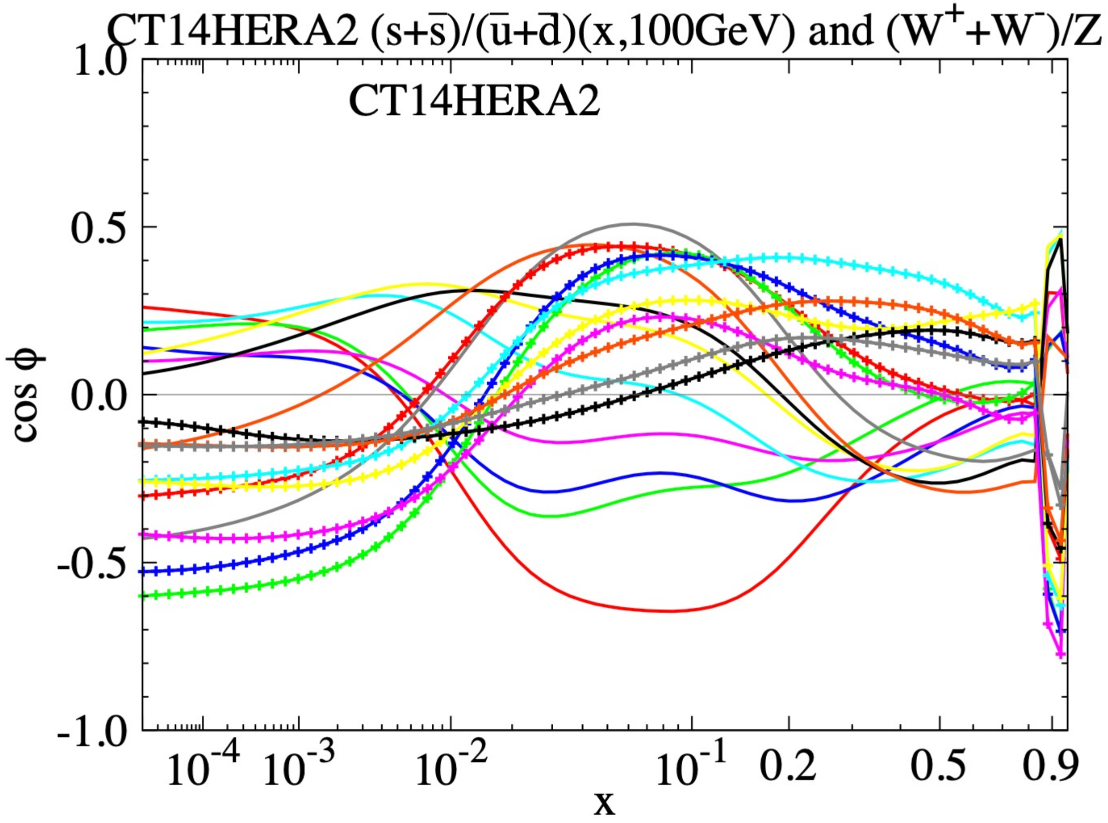
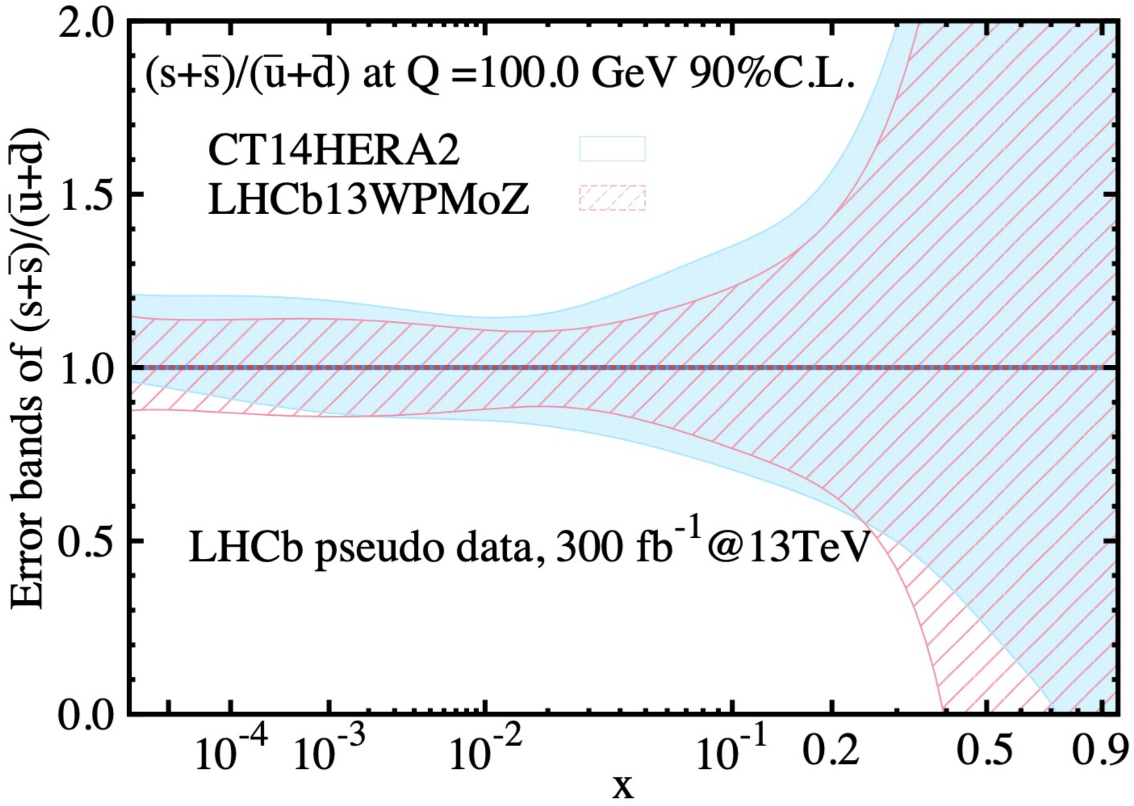
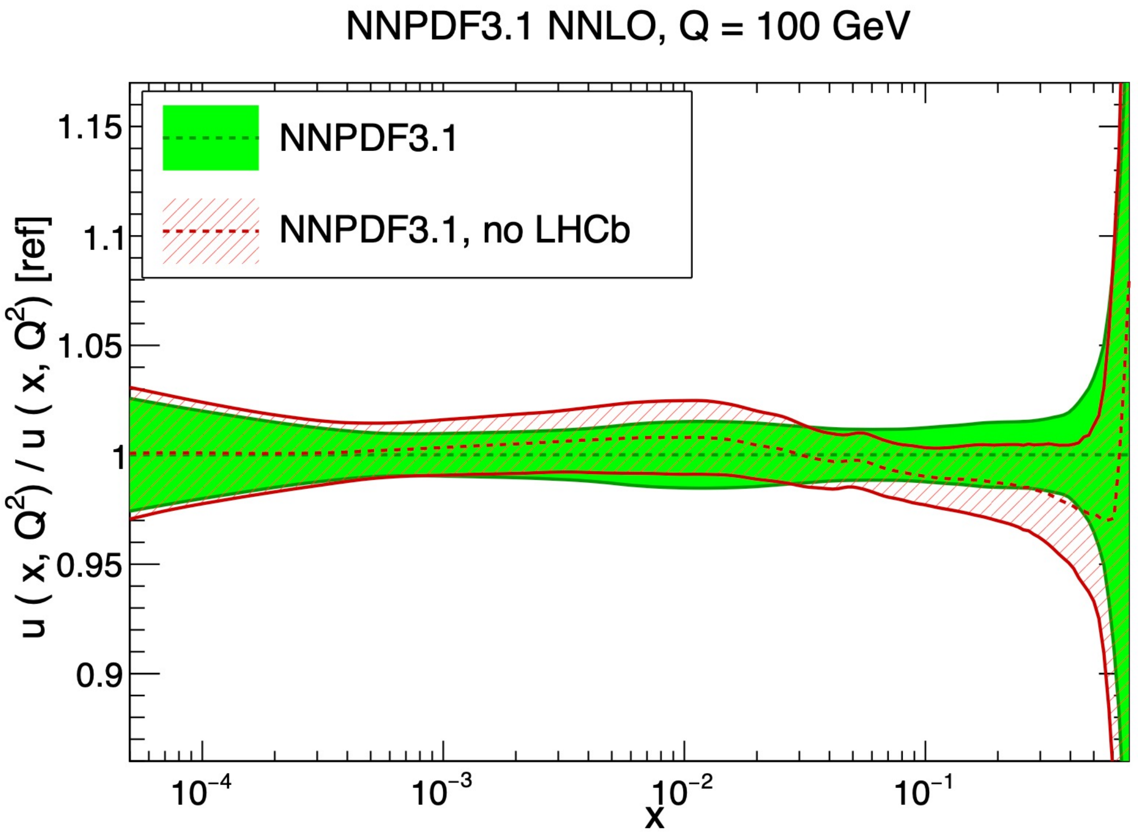
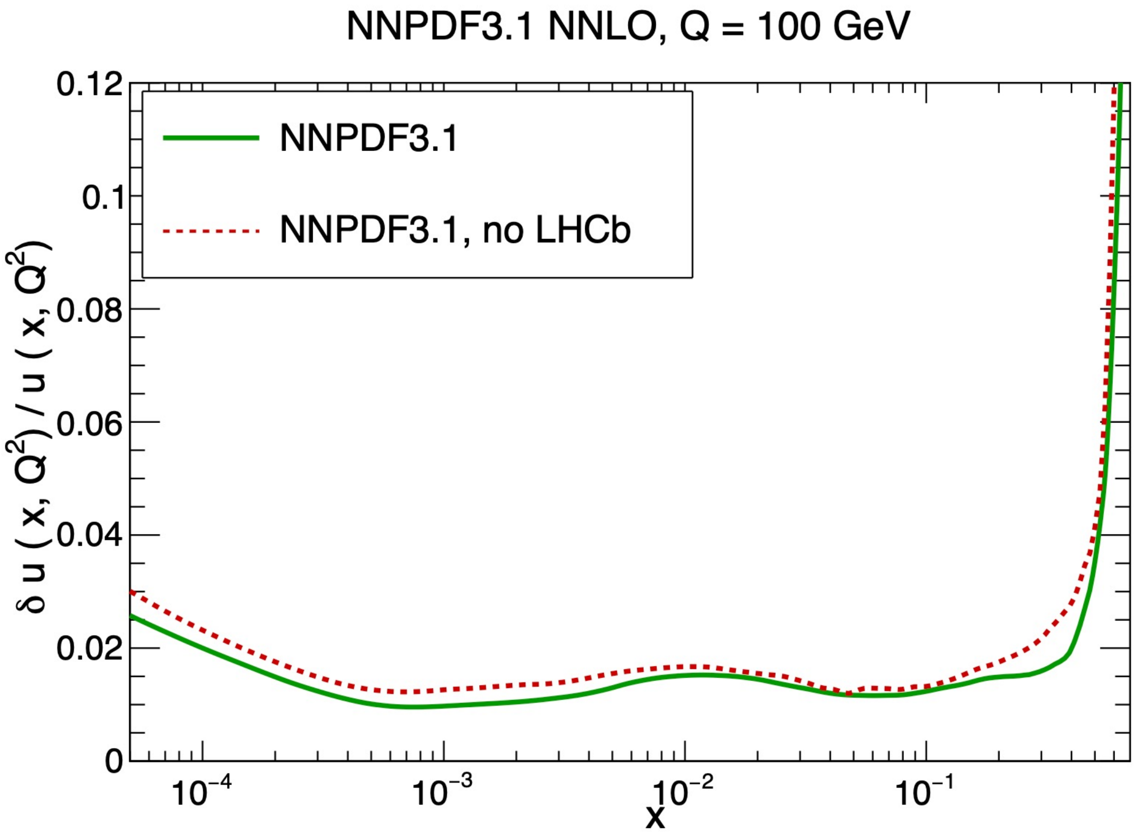




LHCb previous results

arXiv:1705.04468





LHCb previous results

arXiv:1705.04468