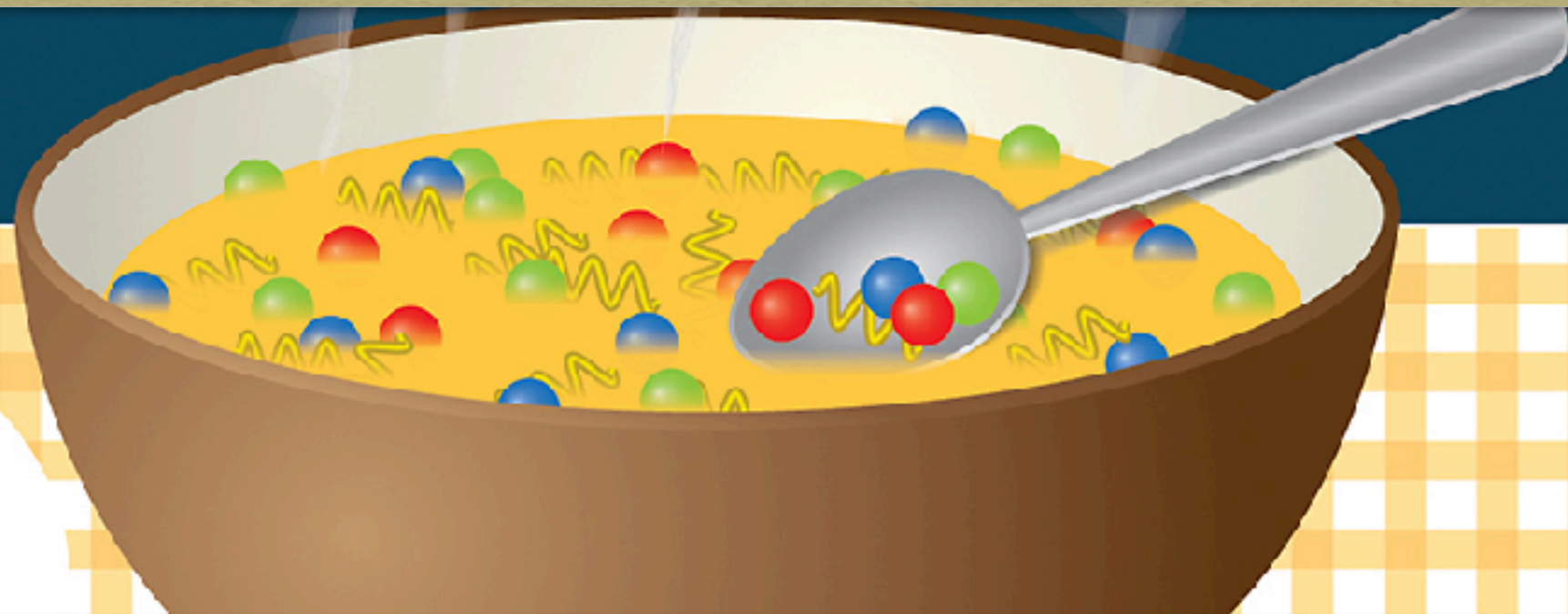




Illinois Center for Advanced Studies of the Universe

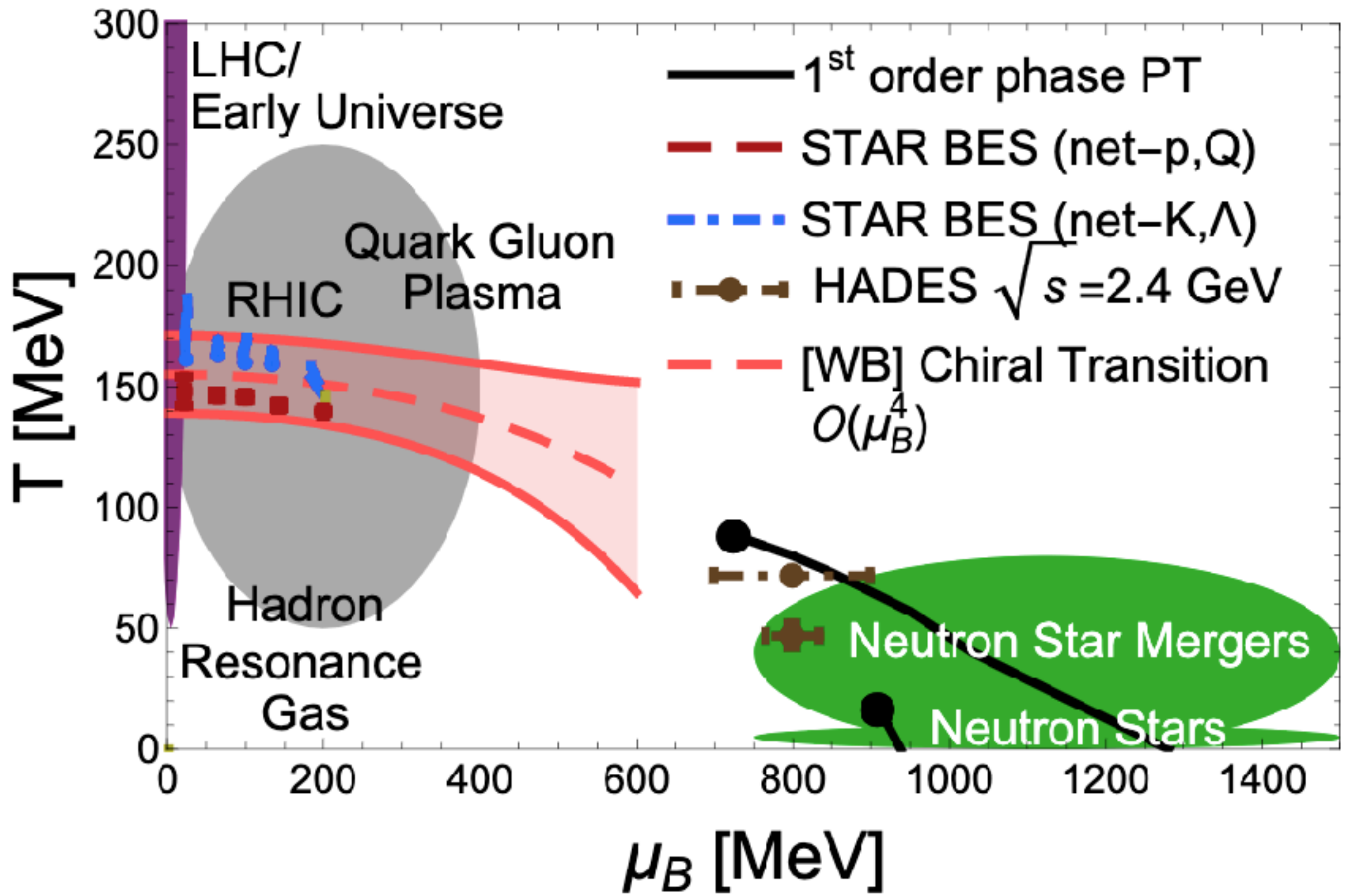


Hydrodynamics, QGP, and the QCD phase diagram



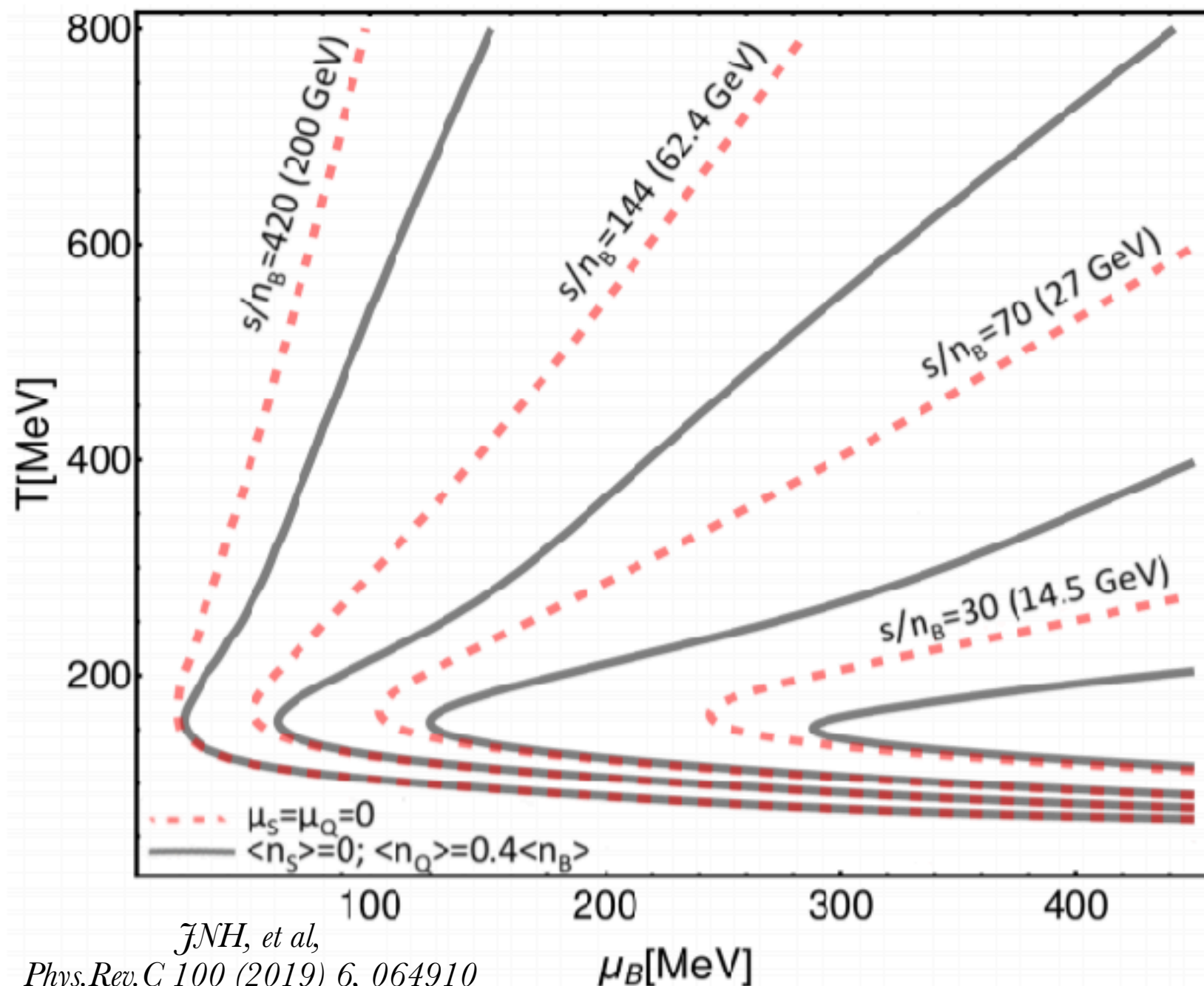
Jacquelyn Noronha-Hostler
University of Illinois Urbana-Champaign

RHIC-BES on-line seminar series



Light transition *Phys.Lett. B738 (2014) 305-310*; **Strange Transition** Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, *arXiv:1805.00088* ;
Neutron Star (mergers) V. Dexheimer *arXiv:1708.08342*; **Holography** Critelli, JNH, et al, *Phys.Rev. D96 (2017) no.9, 096026*

Ideal hydrodynamics: Isentrope trajectories



JNH, et al,
Phys.Rev.C 100 (2019) 6, 064910

Ideal hydro =
 no entropy
 production

$$S/N_B = \text{const}$$

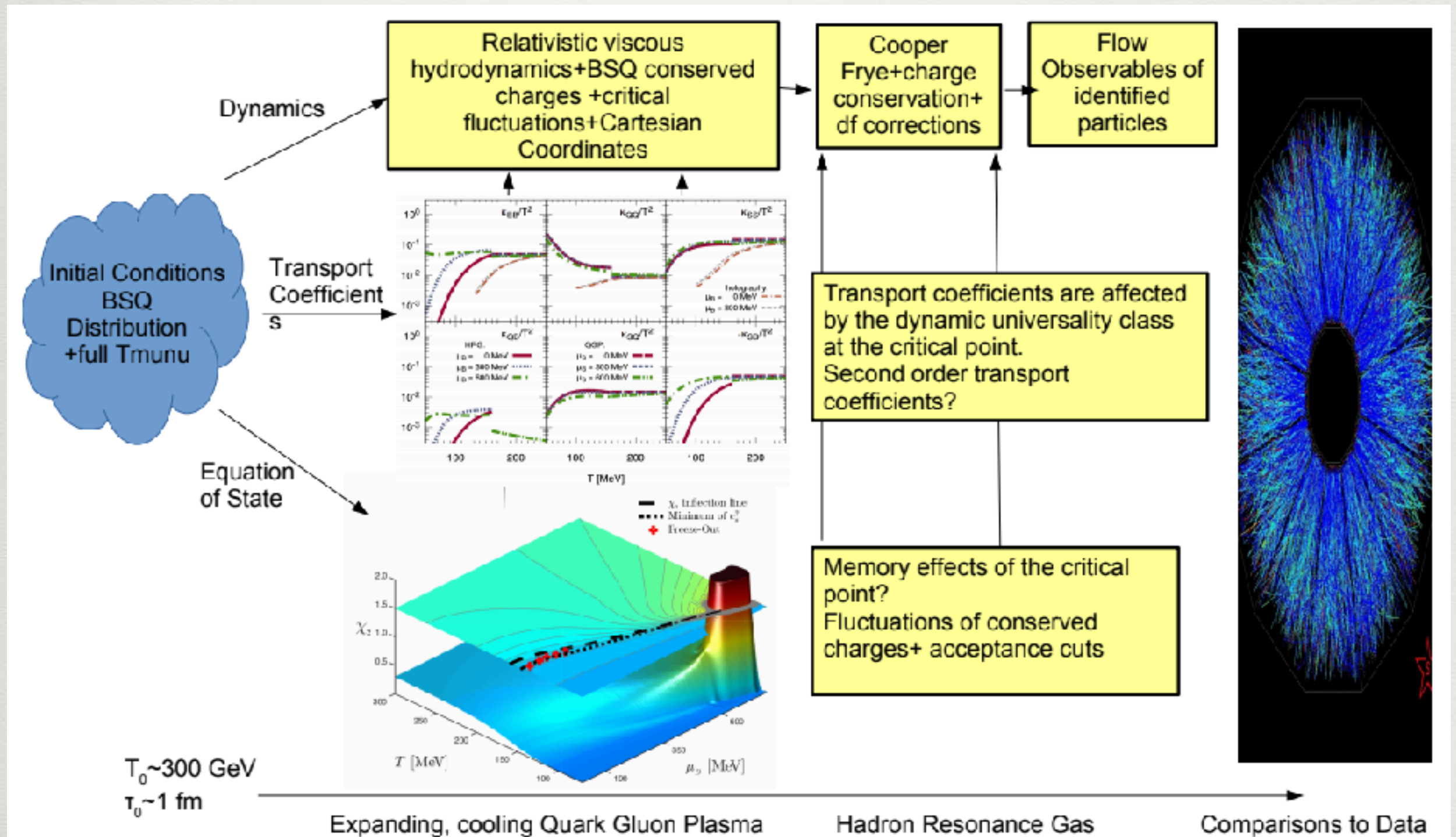
How does out-of-equilibrium effects (entropy production) influence the trajectories?



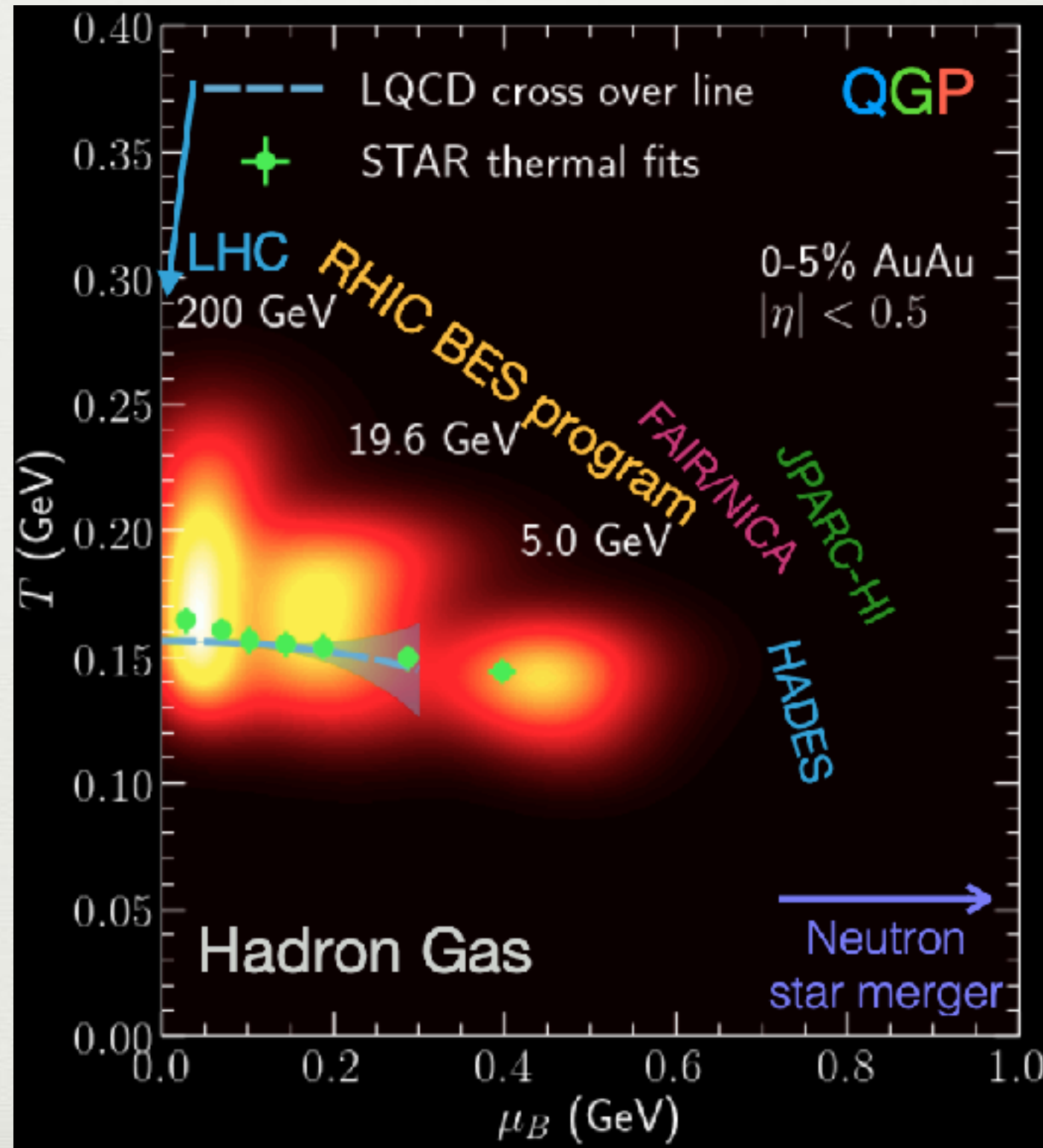
Need relativistic viscous hydrodynamics + 3 conserved charges (BSQ)

Review: Dexheimer, Noronha, JNH, Ratti, Yunes, *J.Phys.G* 48 (2021) 7, 073001

BEST: 2108.13867 [nucl-th]



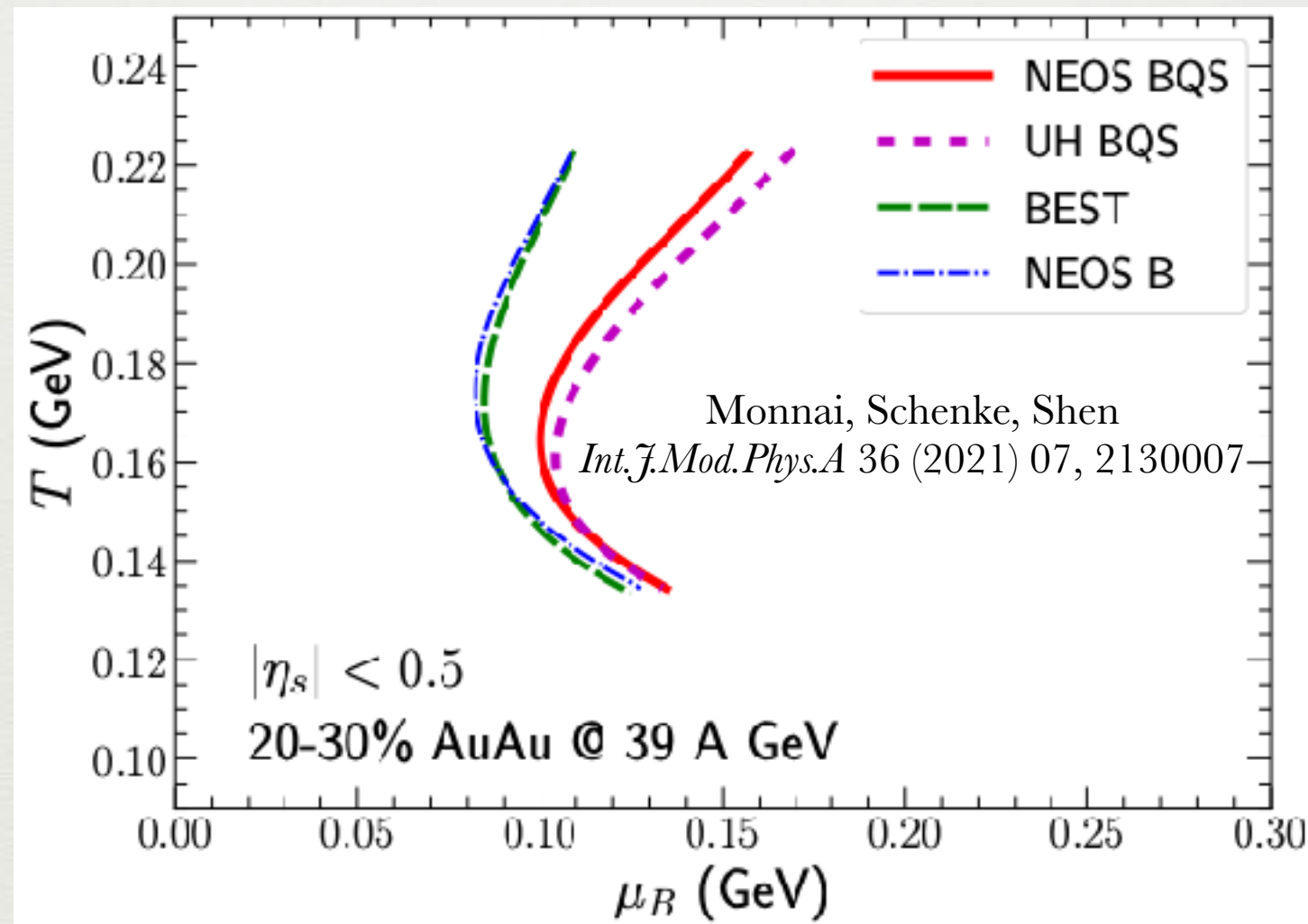
Hydrodynamics is a field, don't just probe one trajectory in (T, μ_B)



Courtesy Chun Shen

Averaged trajectories vs. EOS

Differences between only baryon conservation vs. BSQ influence the trajectories through the QCD phase diagram. Shown with $\eta/s = 0.08$



0+1D Hydrodynamics+baryon current

Dore, et al, *Phys.Rev.D* 102 (2020) 7, 074017

Energy density: $\dot{e} = -\frac{1}{\tau} \left[e + p + \Pi + \pi_\eta^\eta \right]$

Baryon density: $\rho_B = \frac{\rho_0}{\tau}$

Diffusion vanishes
in Bjorken flow.
1+1D future study.

Shear Viscosity

DMNR: $\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{1}{\tau} \left[\frac{4\eta}{3} - \pi_\eta^\eta (\delta_{\pi\pi} + \tau_{\pi\pi}) + \lambda_{\pi\Pi} \Pi \right]$

Israel-Stewart: $\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{1}{\tau} \left[\frac{4\eta}{3} - \frac{\eta T \pi_\eta^\eta}{2} (\beta_\pi + \tau \dot{\beta}_\pi) \right]$

Bulk Viscosity

DNMR: $\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau} \left(\zeta + \delta_{\Pi\Pi} \Pi + \frac{2}{3} \lambda_{\Pi\pi} \pi_\eta^\eta \right)$

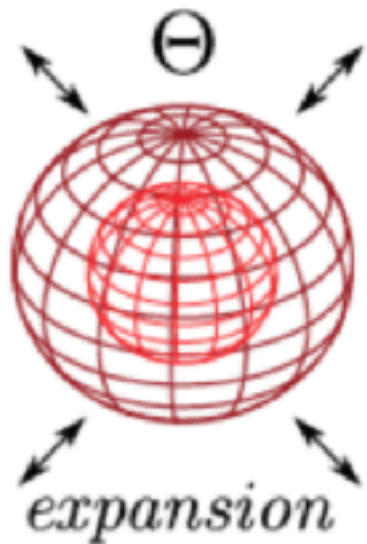
Israel-Stewart: $\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau} \left[\zeta + \frac{\zeta T \Pi}{2} (\beta_\Pi + \tau \dot{\beta}_\Pi) \right]$

Transport coefficients/viscosities

Transport coefficient: Perturb the fluid from equilibrium- how quickly does it return to equilibrium?

Bulk

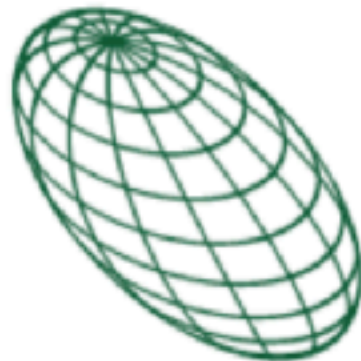
$$\Pi \sim -\zeta \Theta$$



Shear

$$\pi^{\mu\nu} \sim 2\eta\sigma^{\mu\nu}$$

$\sigma_{\mu\nu}$



shear

Diffusion

$$q_{\perp}^{\mu} \sim \kappa \nabla_{\perp}^{\mu} (\mu/T)$$

↳ QCD conserved charges (B,S,Q)



Diffusion

Vorticity

ω_{μ}



vorticity

Viscosity - resistance to deformation or “thickness” of liquid

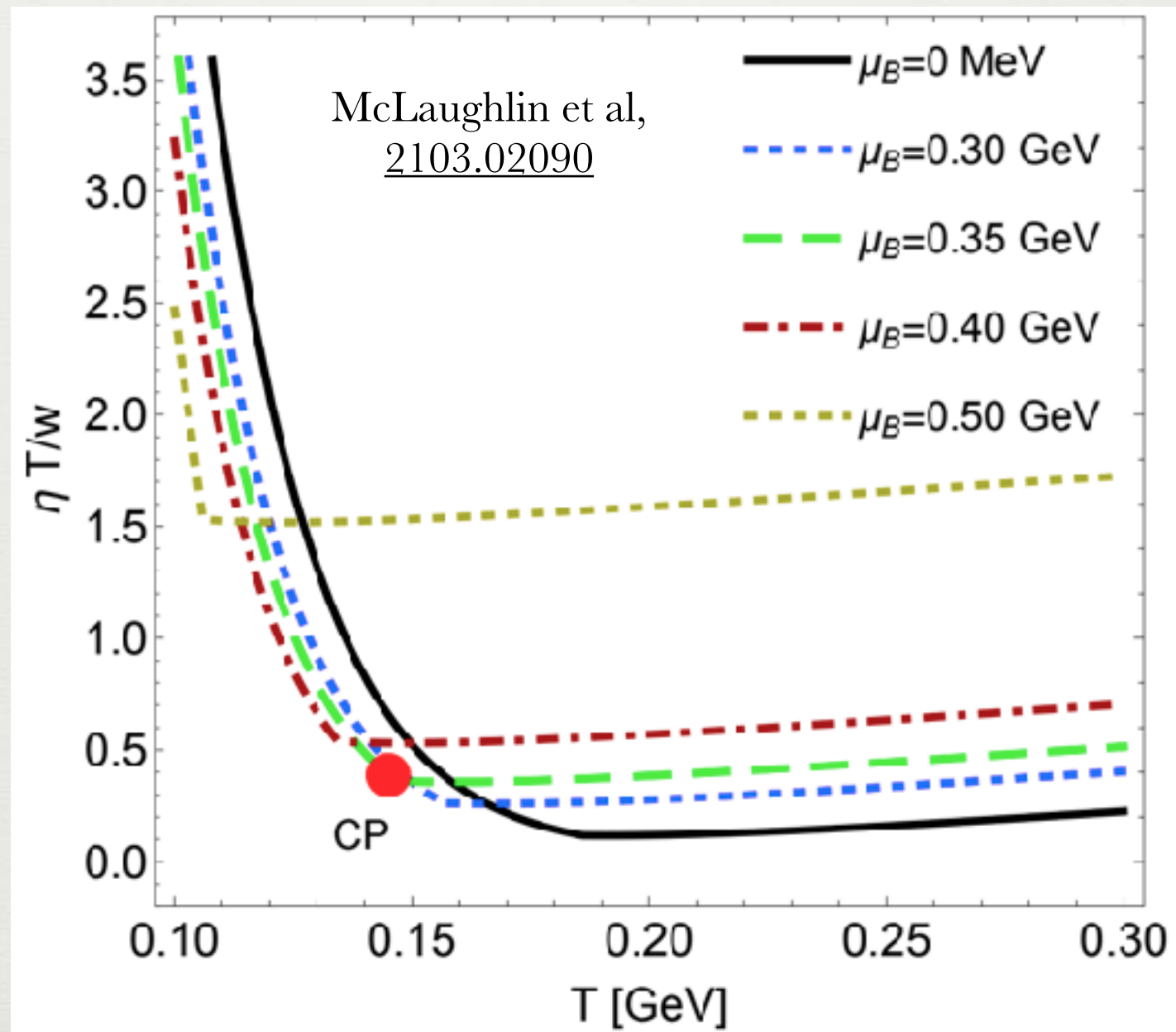
Shear viscosity at $\mu_B > 0$

Dore, et al, *Phys.Rev.D* 102 (2020) 7, 074017


Hadron resonance gas + Parameterized QGP phase

JNH, Noronha, Greiner *Phys.Rev. C* 86 (2012) 024913

Dubla et al, *Nucl.Phys. A* 979 (2018) 251-264



Bulk Viscosity at $\mu_B > 0$

Non-critical bulk: $\frac{\zeta T}{w} = 36 \times \frac{1/3 - c_s^2}{8\pi}$ 

Critical Scaling: $\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w} \left[1 + \left(\frac{\xi}{\xi_0}\right)^3 \right]$

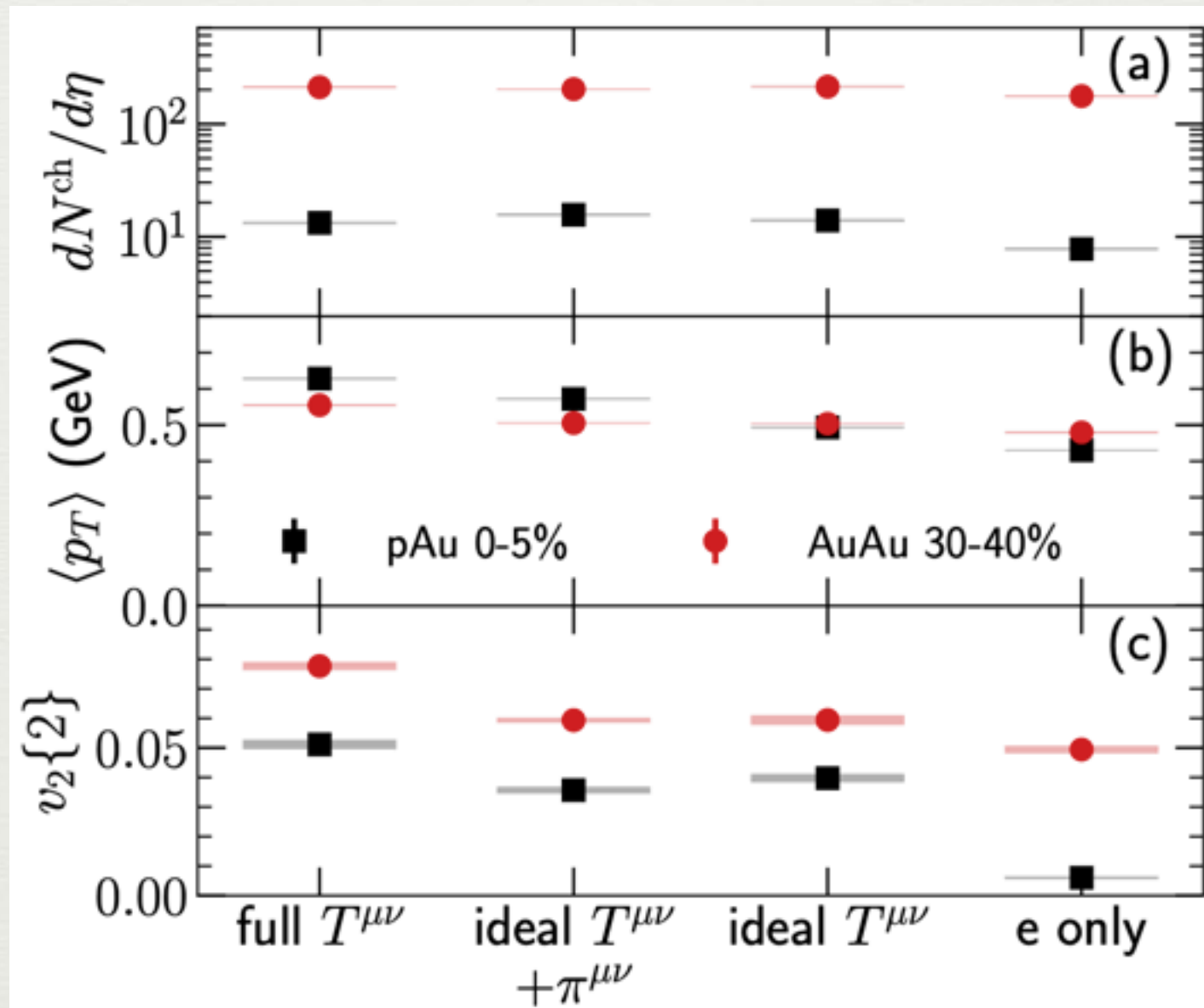
Critical Scaling: Monnai,
Mukherjee, Yin; Phys.Rev.C 95
(2017) 3, 034902

Correlation length: $\xi^2 = \frac{1}{H_0} \left(\frac{\partial M(r, h)}{\partial h} \right)_r$

Correlation Length to $\mathcal{O}(\theta^5)$: Dore, et al,
Phys.Rev.D 102 (2020) 7, 074017

See also: Martinez et al Phys.Rev.D 100 (2019) 7,
074017; Rajagopal et al Phys.Rev.D 102 (2020) 9,
094025

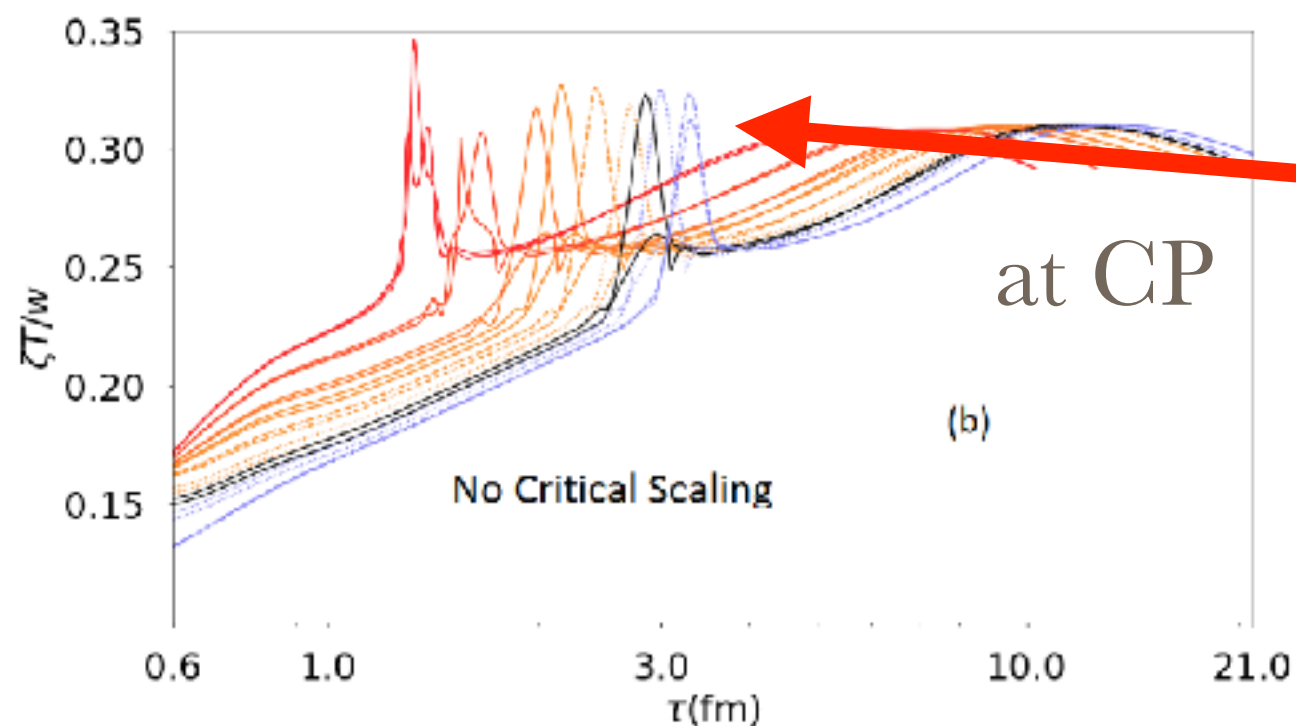
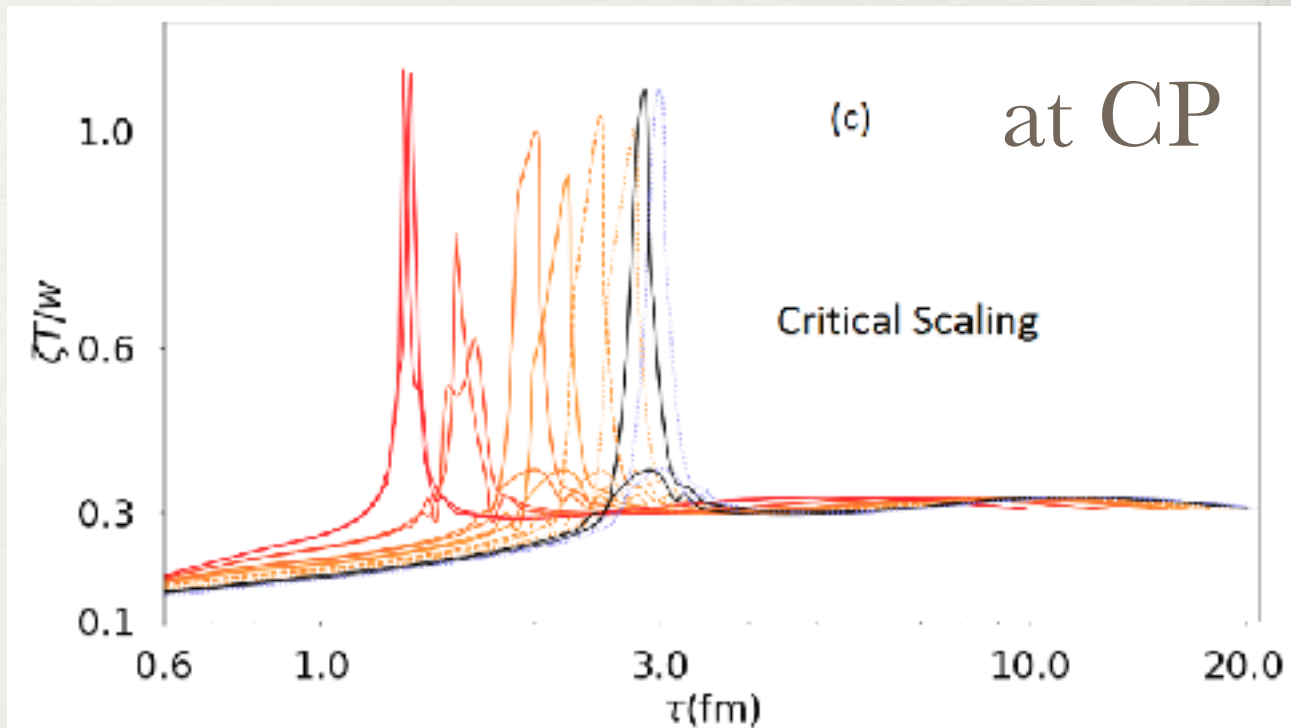
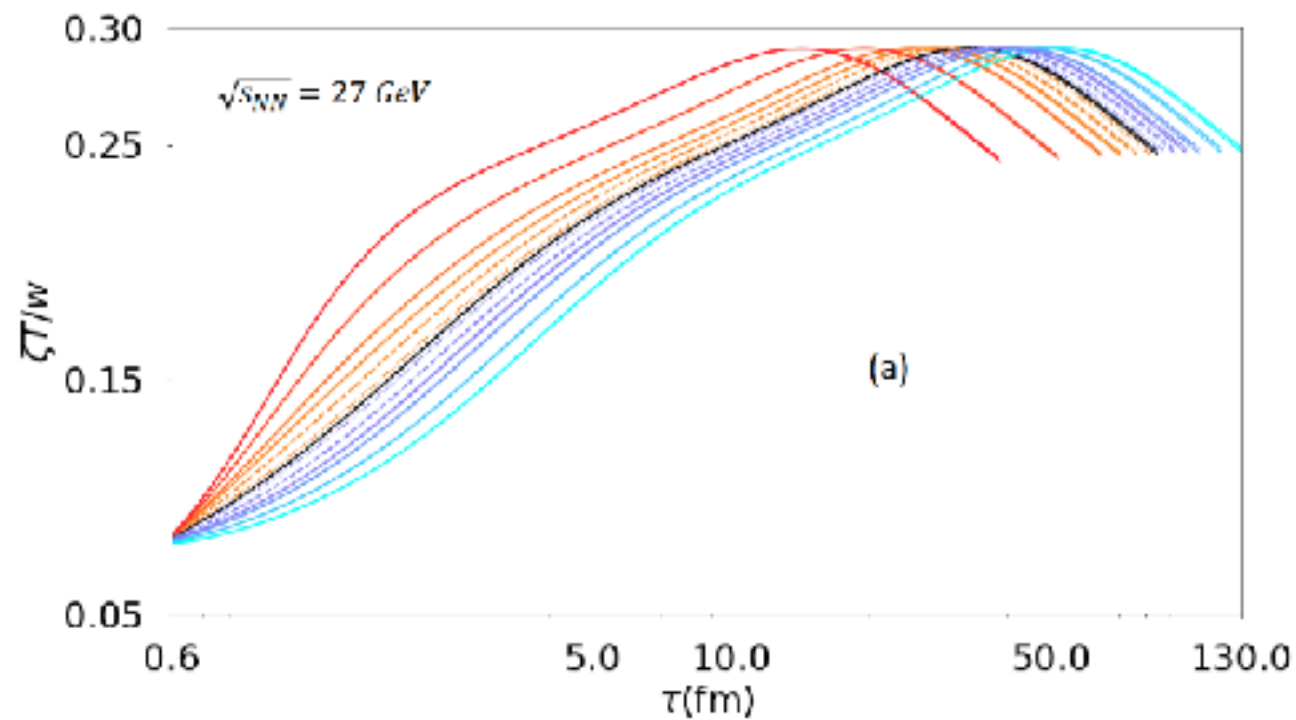
Simulations: full $T^{\mu\nu}$ small effect



Schenke et al, *Phys.Lett.B* 803 (2020) 135322

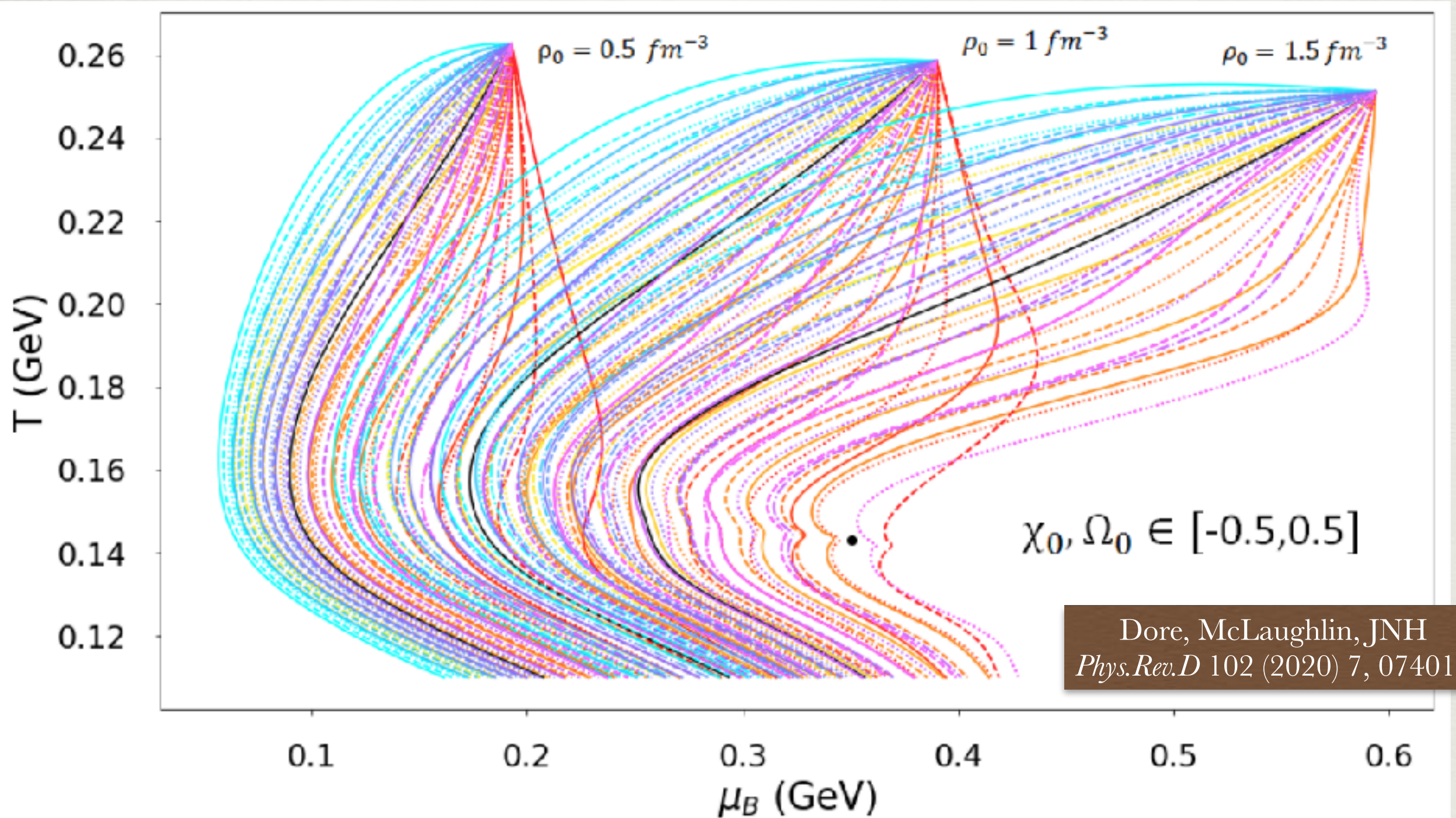
Bulk viscosity, varying $\left(\pi_{\eta}^{\eta}, \Pi_0\right)$

Dore, et al, *Phys.Rev.D* 102 (2020) 7, 074017



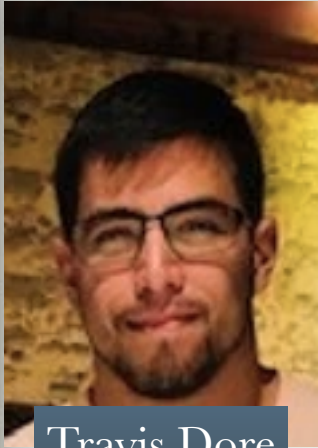
Effect from c_s^2 alone

Fixed initial condition $(\varepsilon_0, \rho_B^0)$, vary (π_η^η, Π_0)

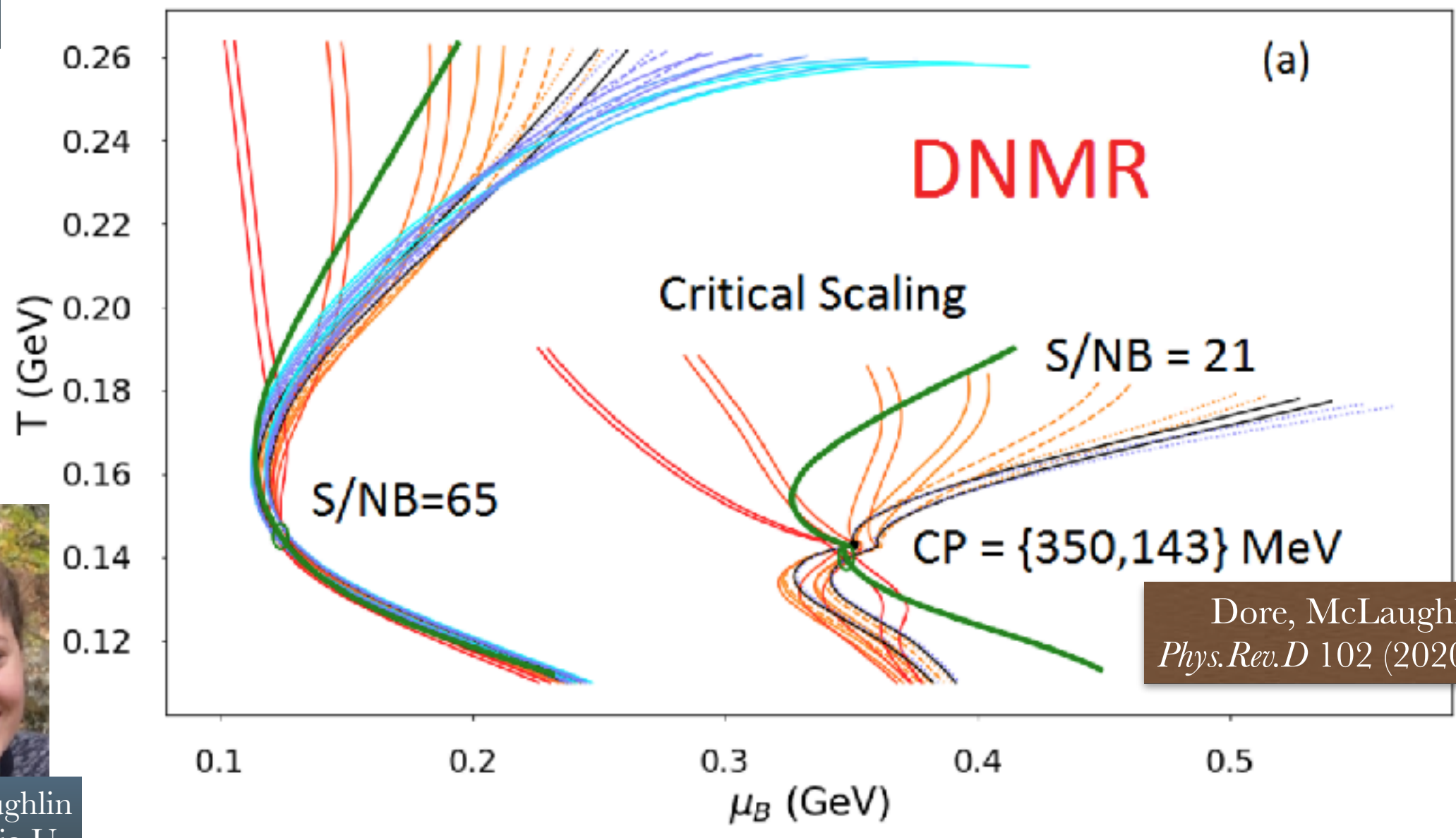


Dore, McLaughlin, JNH
Phys.Rev.D 102 (2020) 7, 074017

Far from equilibrium causes further deviations from ideal



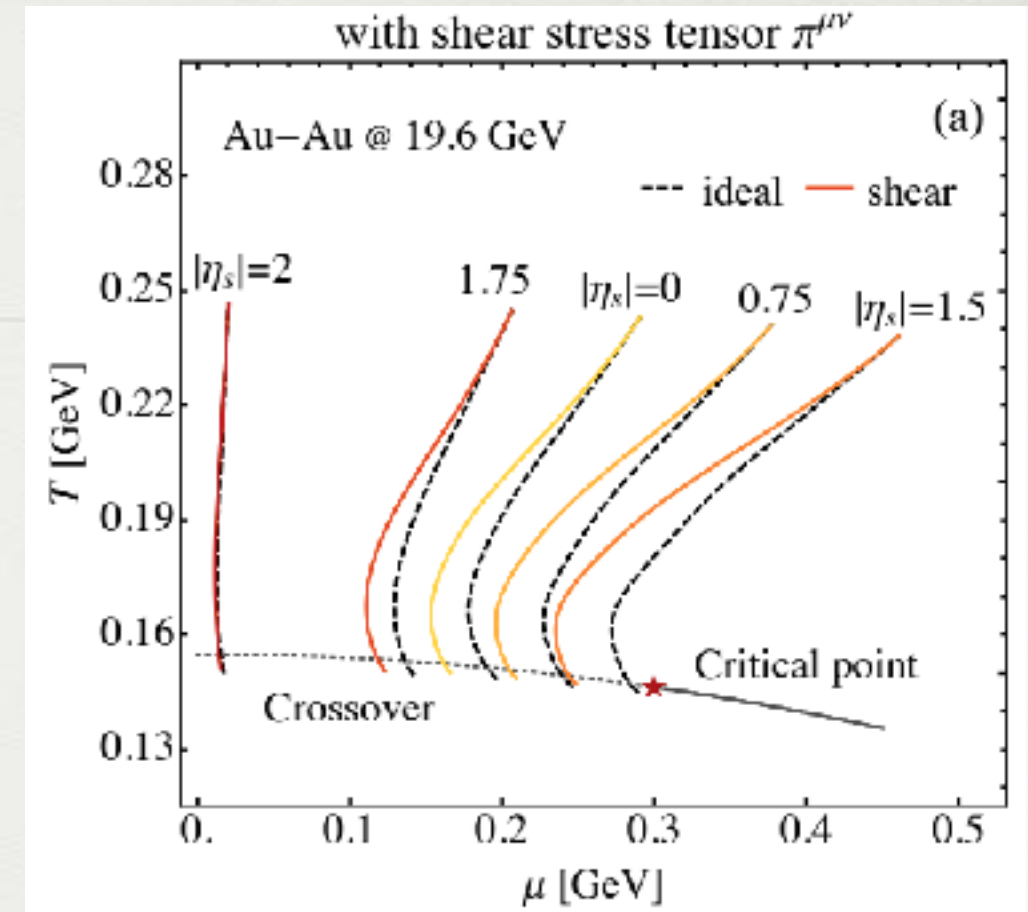
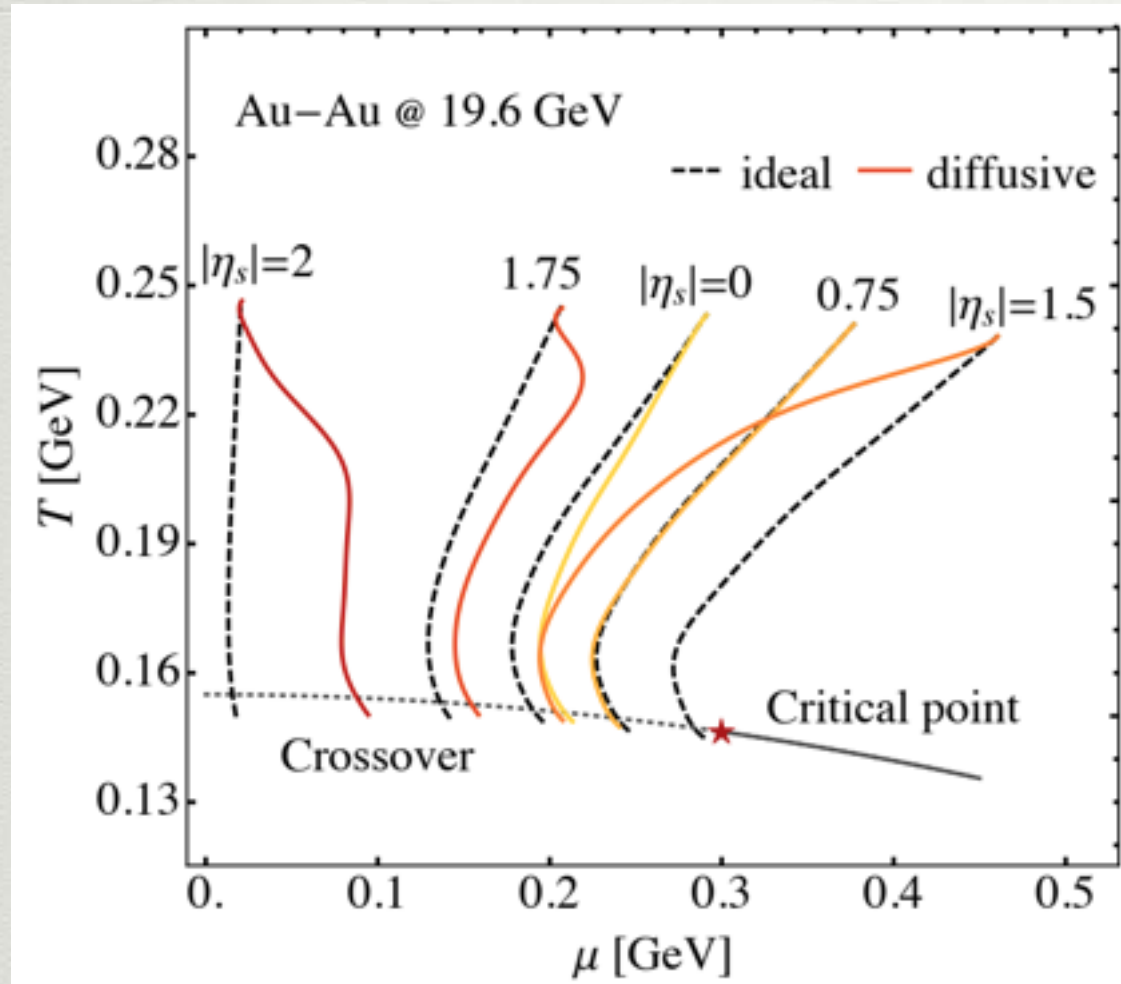
Travis Dore
PhD UIUC



Emma McLaughlin
PhD Columbia U.

Assuming we know the point of freeze-out (from thermal models/fluctuations), initial conditions then unknown

Beyond 0+1D studies

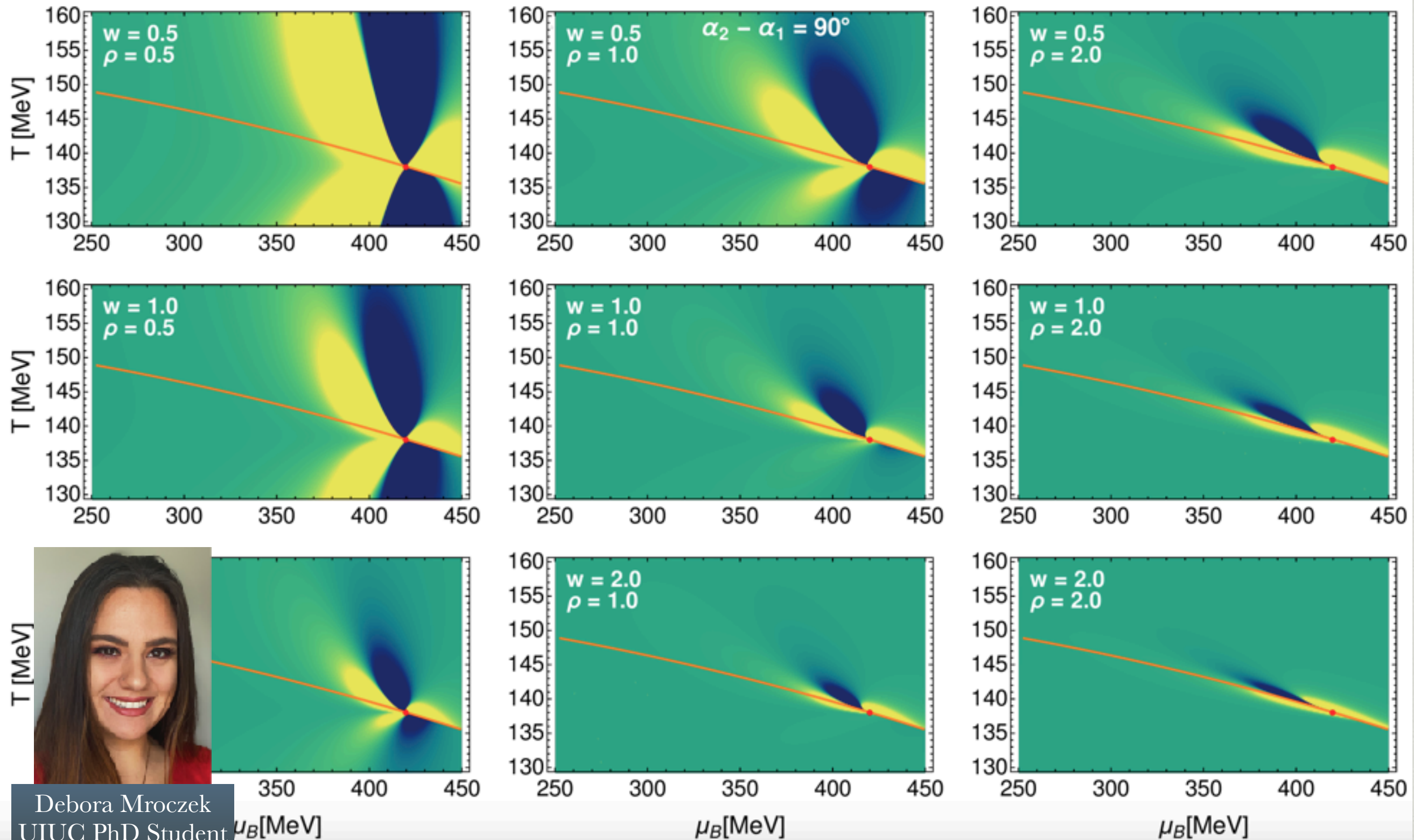


1+1D study, if initial start in equilibrium, diffusion and shear play biggest role

See also Du & Schlichting *Phys.Rev.Lett.* 127 (2021) 12, 122301

All critical points not created equally

Mroczek et al, *Phys.Rev.C* 103 (2021) 3, 034901



Debora Mroczek
UIUC PhD Student

Effect of critical region on hydro?

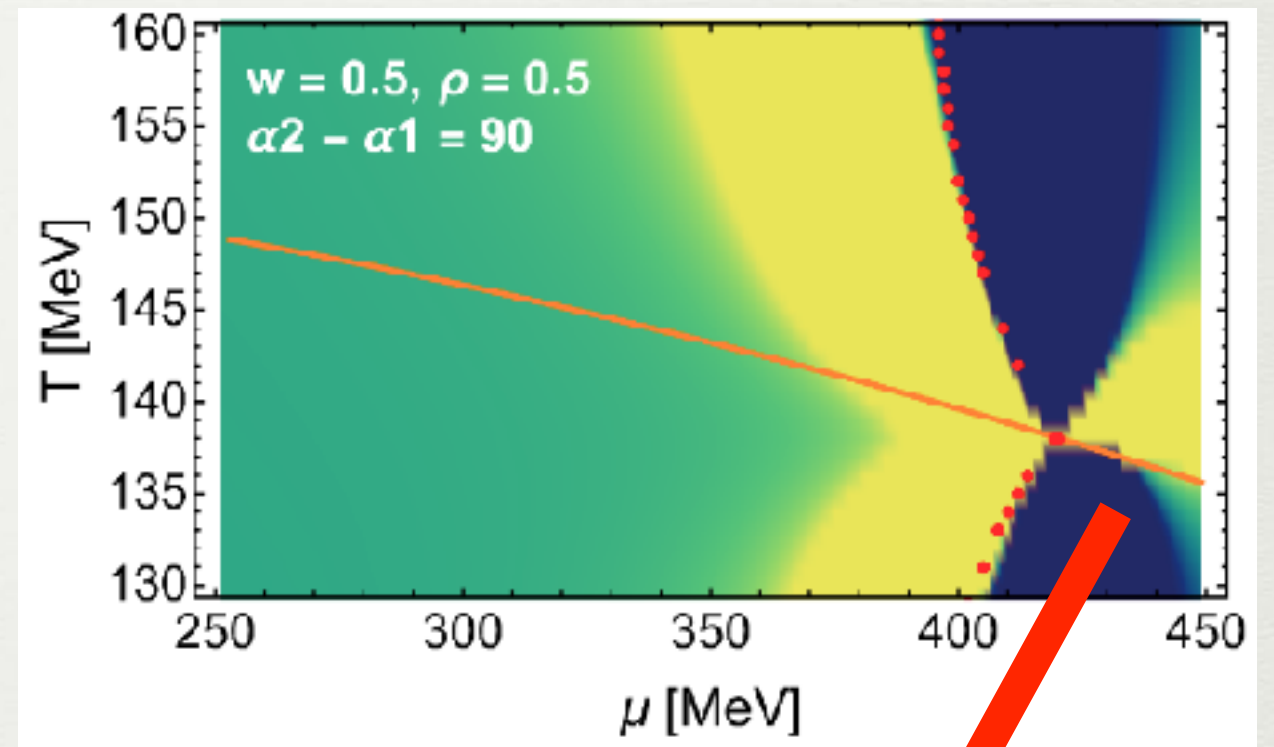
Travis Dore, Isaac Long, Debora Mroczek, Yukari Yamauchi, Paolo Parotto, Claudia Ratti, JNH, to appear soon



Travis Dore
PhD UIUC



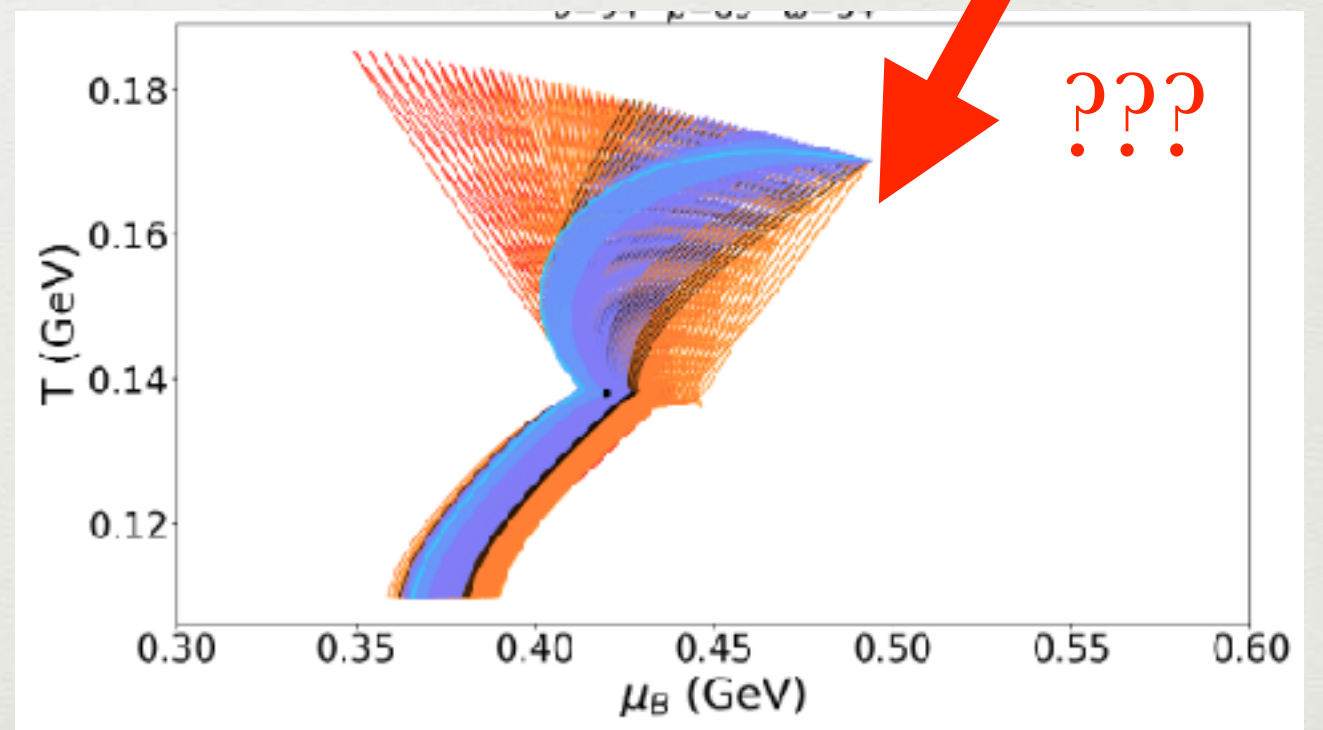
Isaac Long
UIUC REU



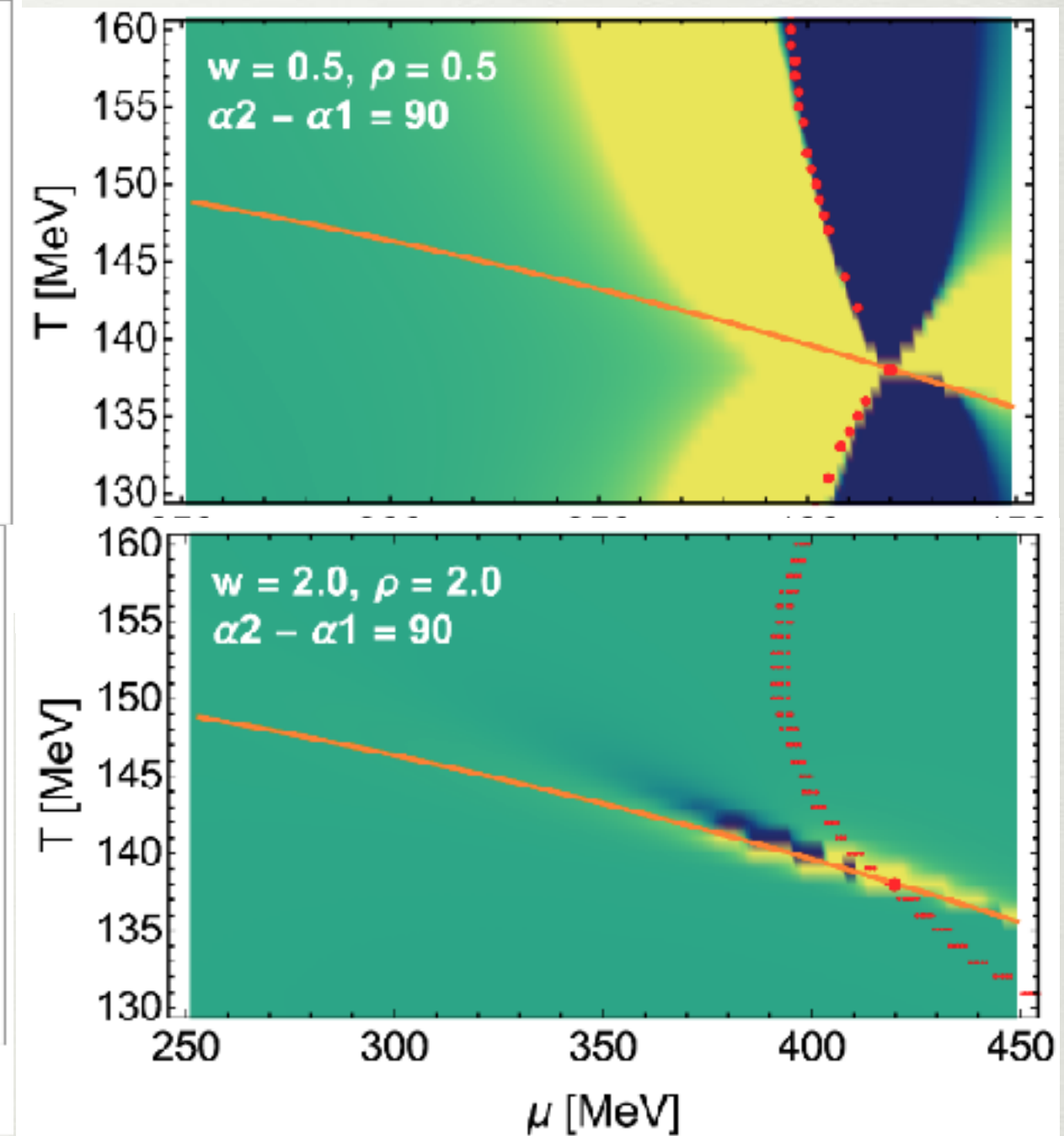
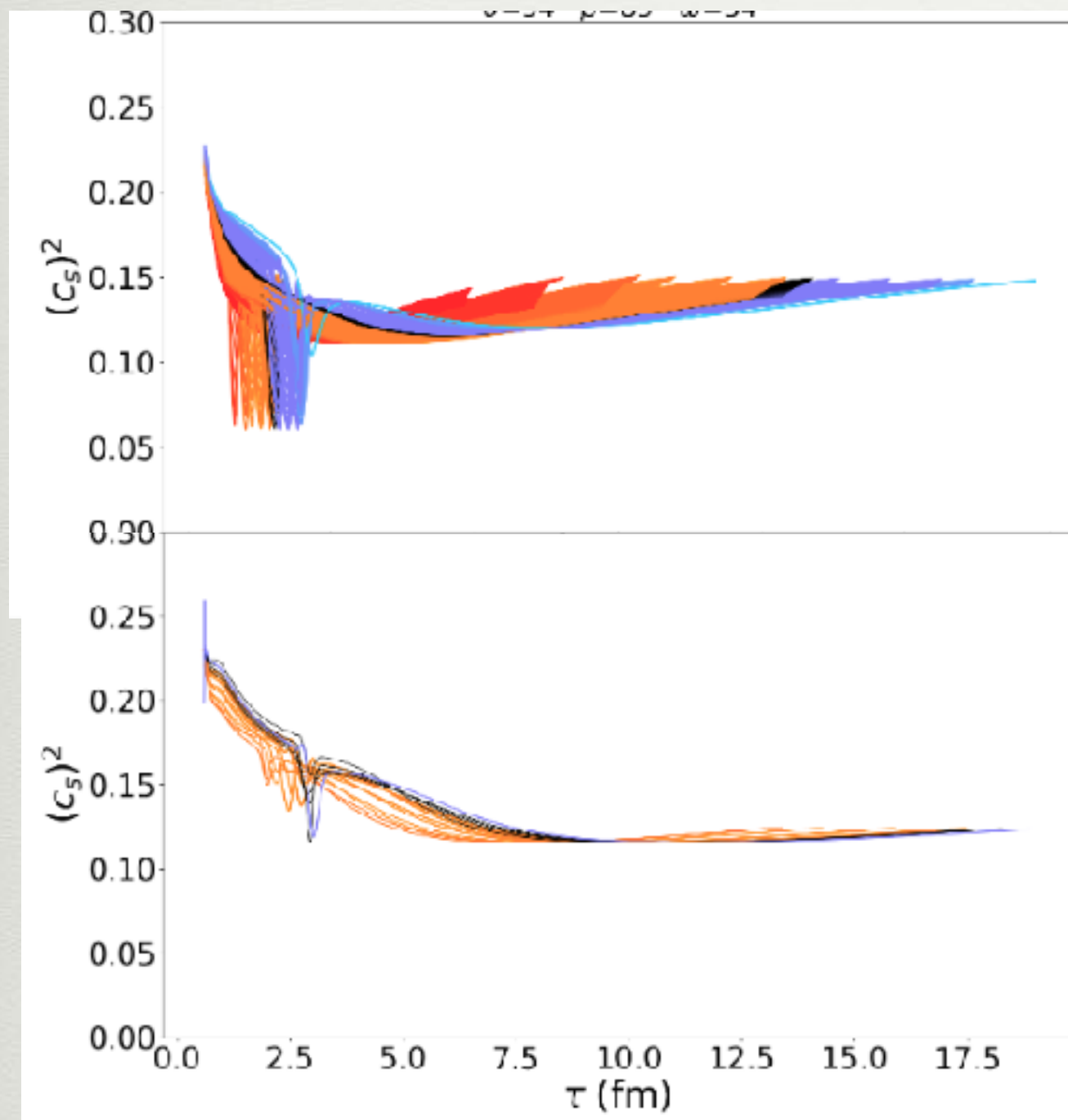
Debora Mroczek
UIUC PhD Student



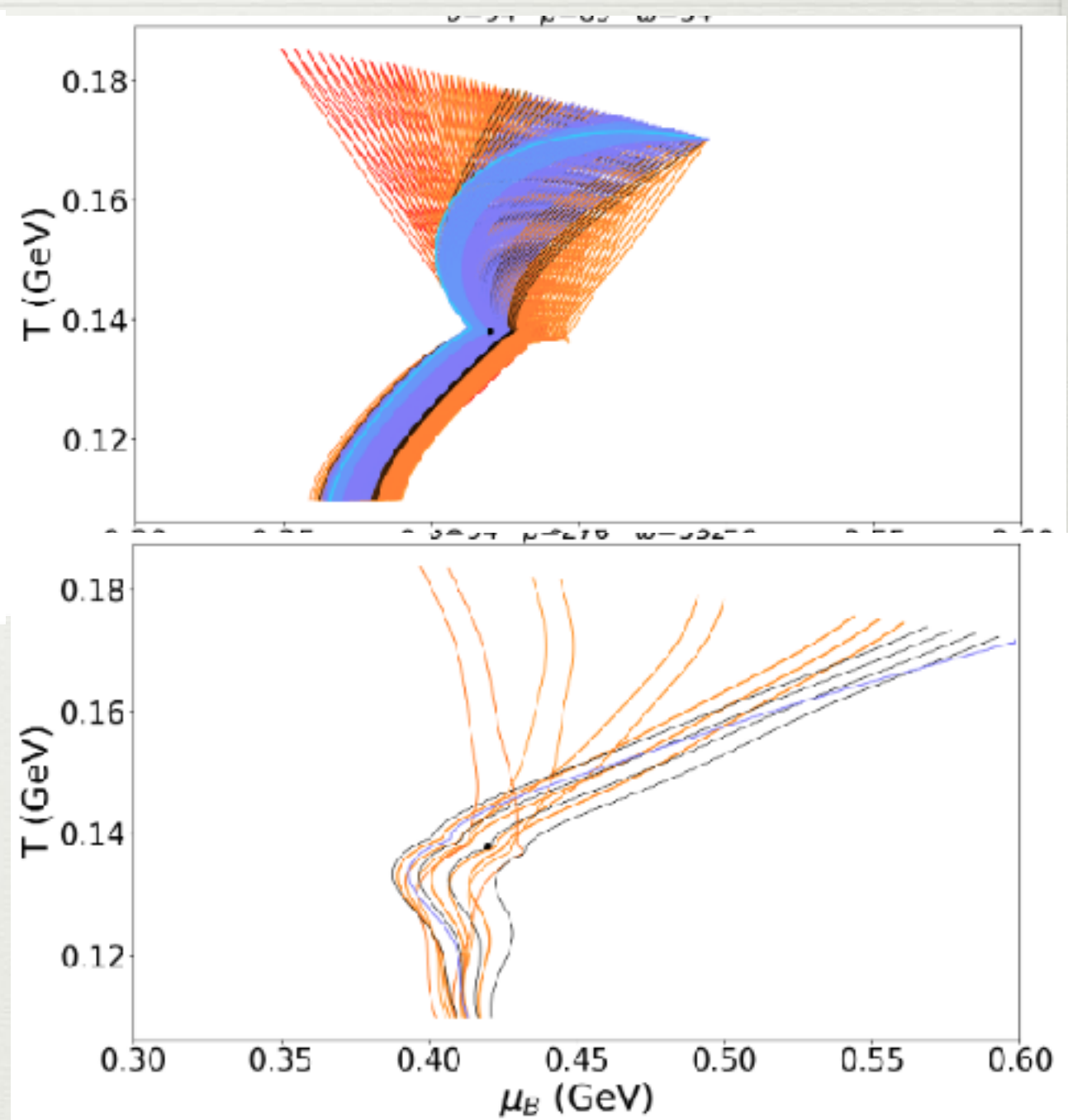
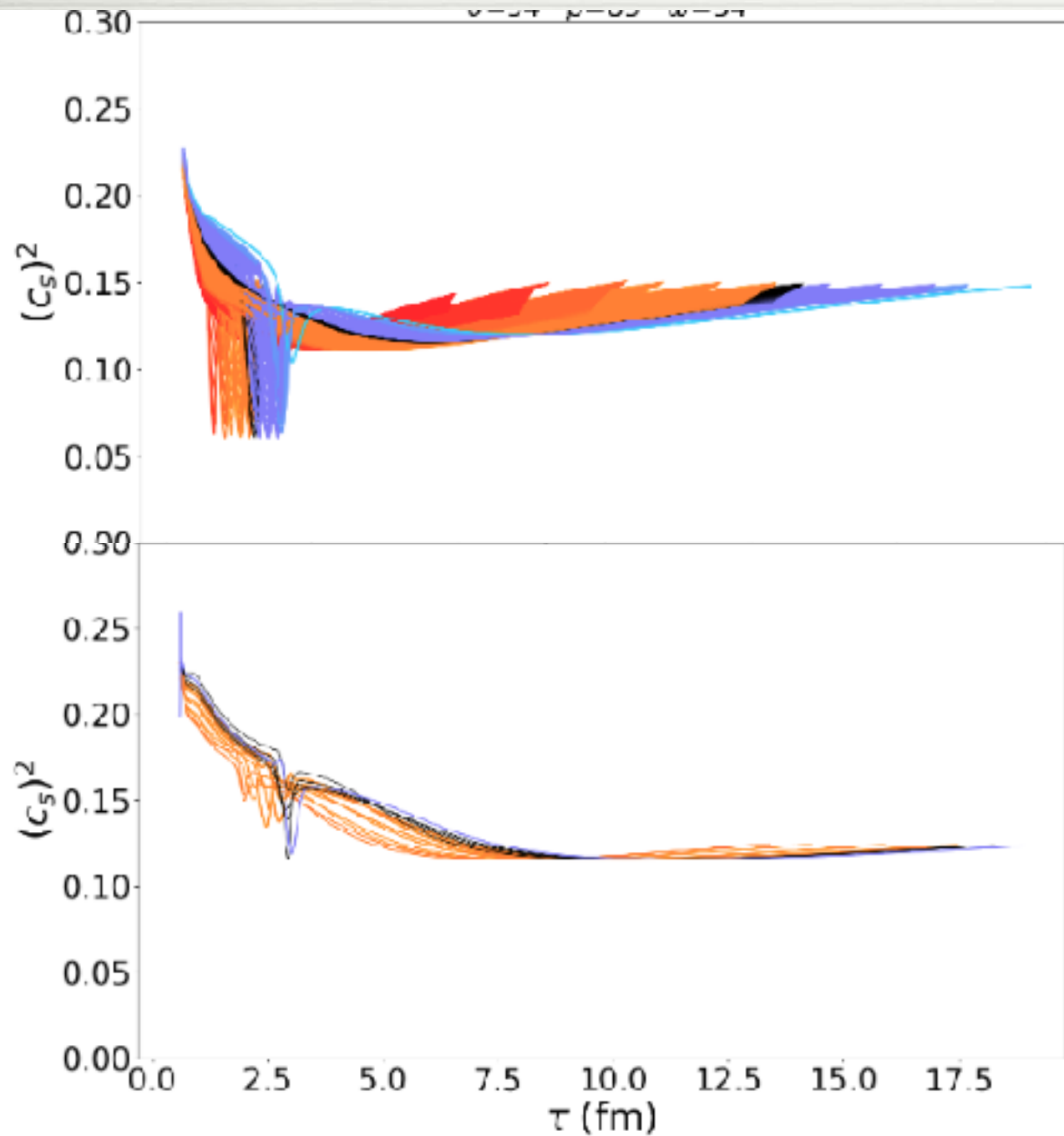
Yukari Yamauchi
Maryland PhD Student



Spread in critical region vs dip in c_s^2



c_s^2 dip \rightarrow critical point trajectories attractor

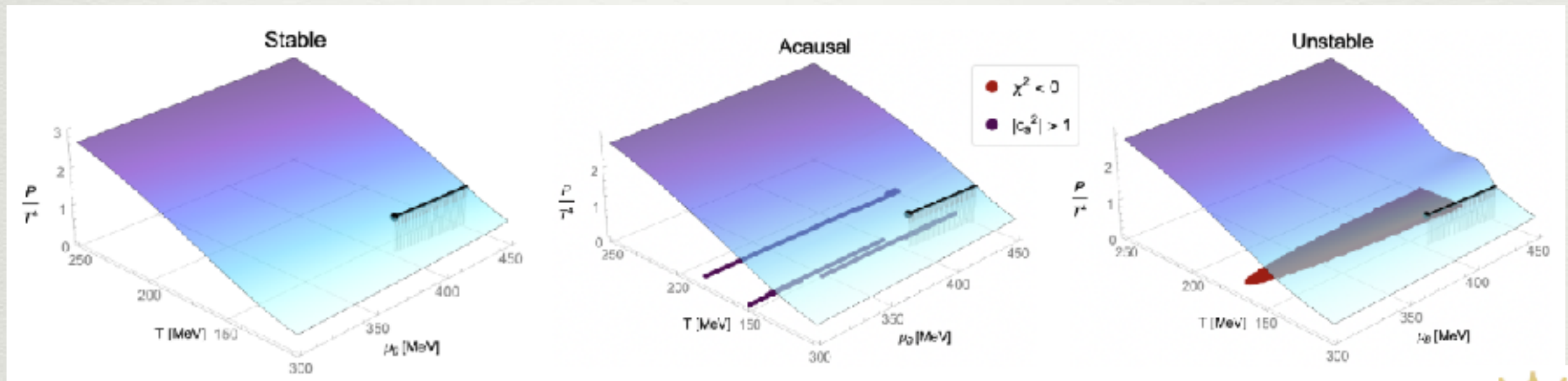


Orientation of critical region affects observability of critical region

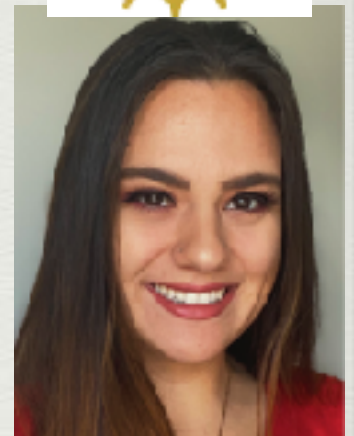
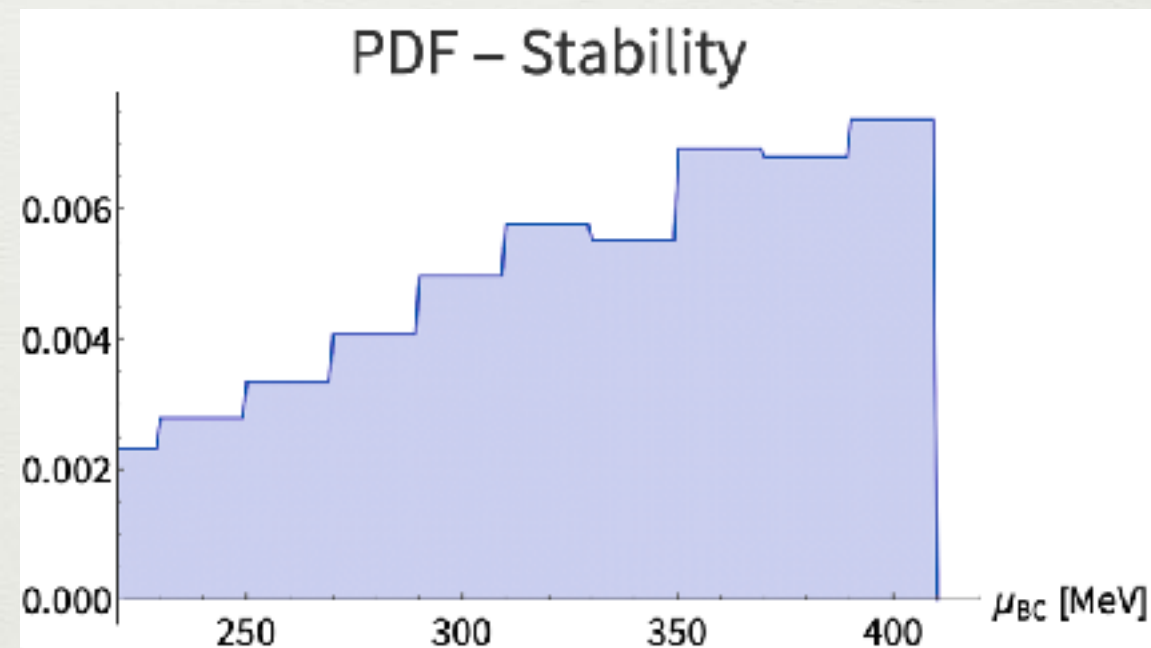
EoS: Constraining the 4 free parameters

Mroczek, Parotto, Hjorth-Jensen, JNH, Ratti, Vilalta to appear soon

Using active learning+machine learning to label viable EOS

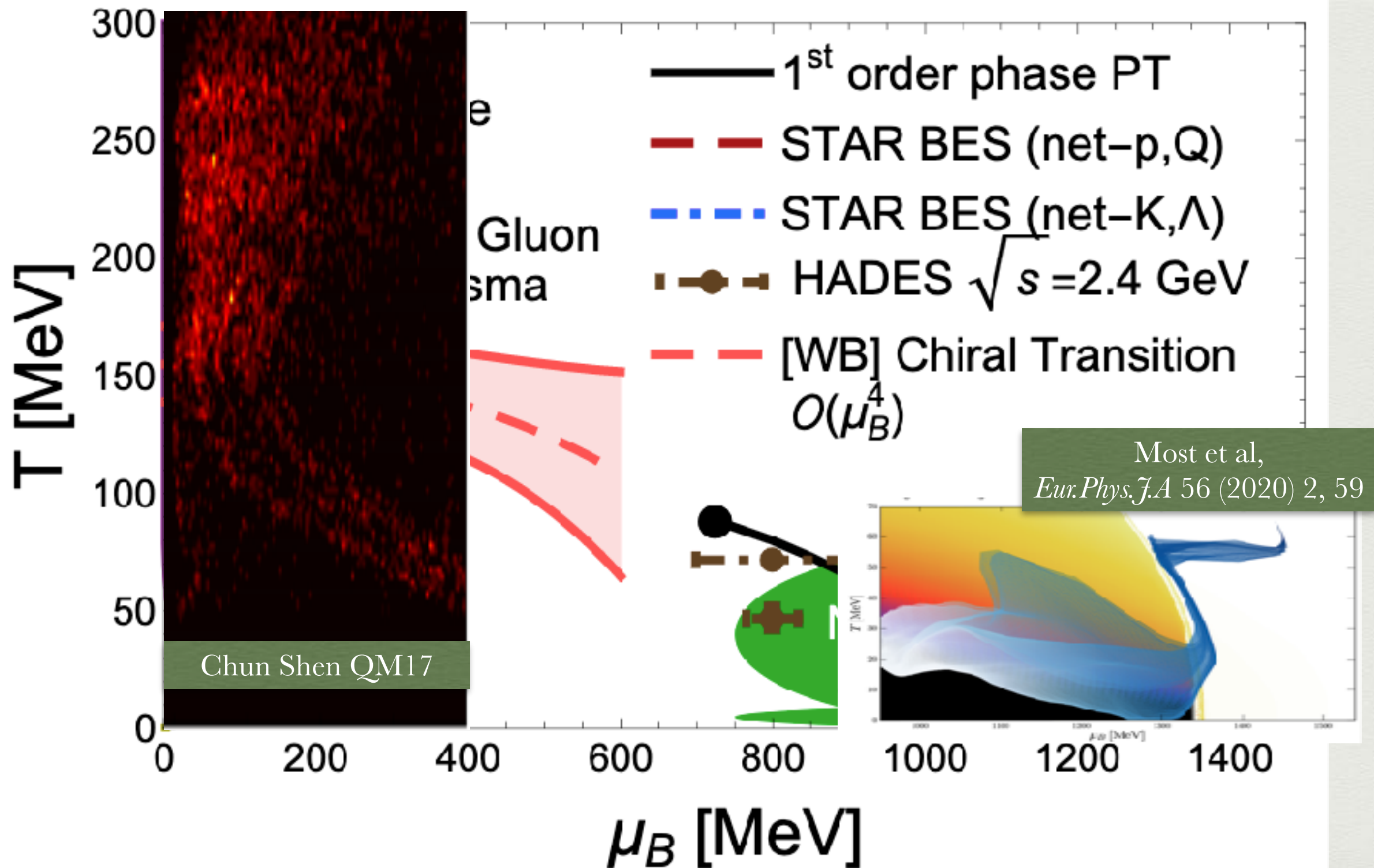


Most efficient method:
random forrest with
margin sampling



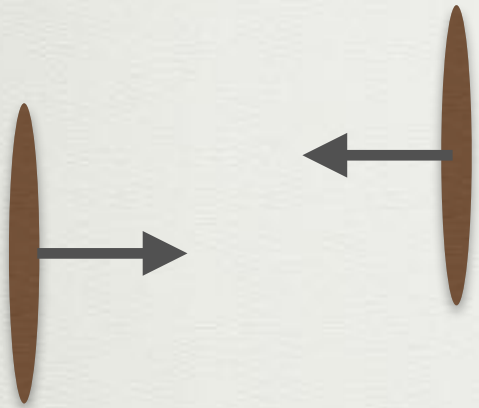
Debora Mroczek
UIUC PhD Student

Full 2+1 & 3+1 BSQ hydro



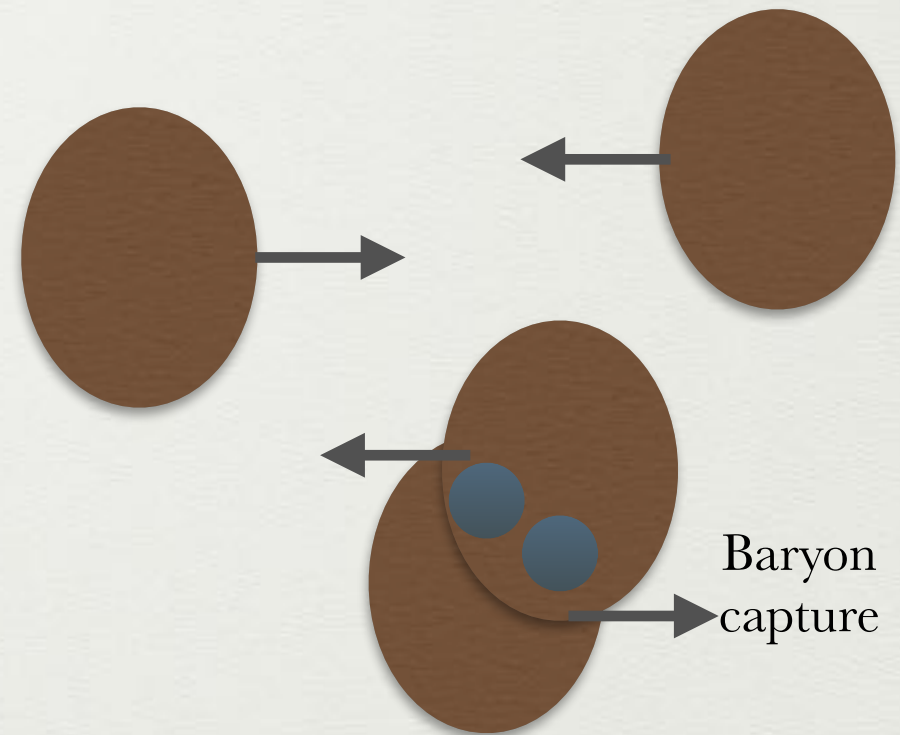
Heavy-ion Collisions: $\sqrt{s_{NN}}$ vs ρ_B

Large $\sqrt{s_{NN}}$



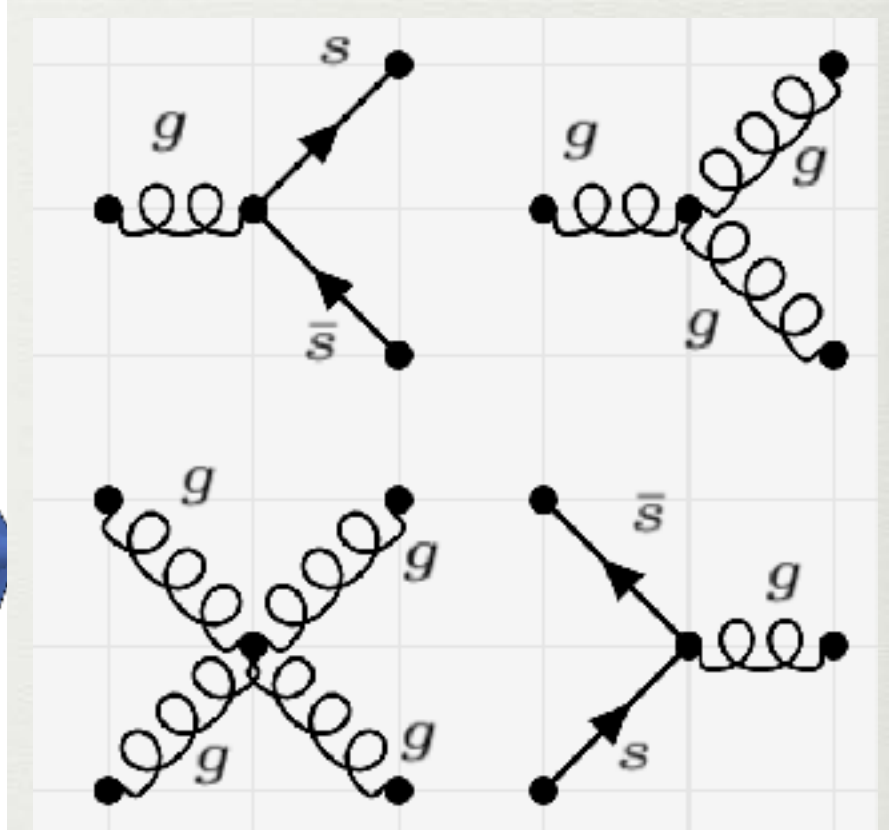
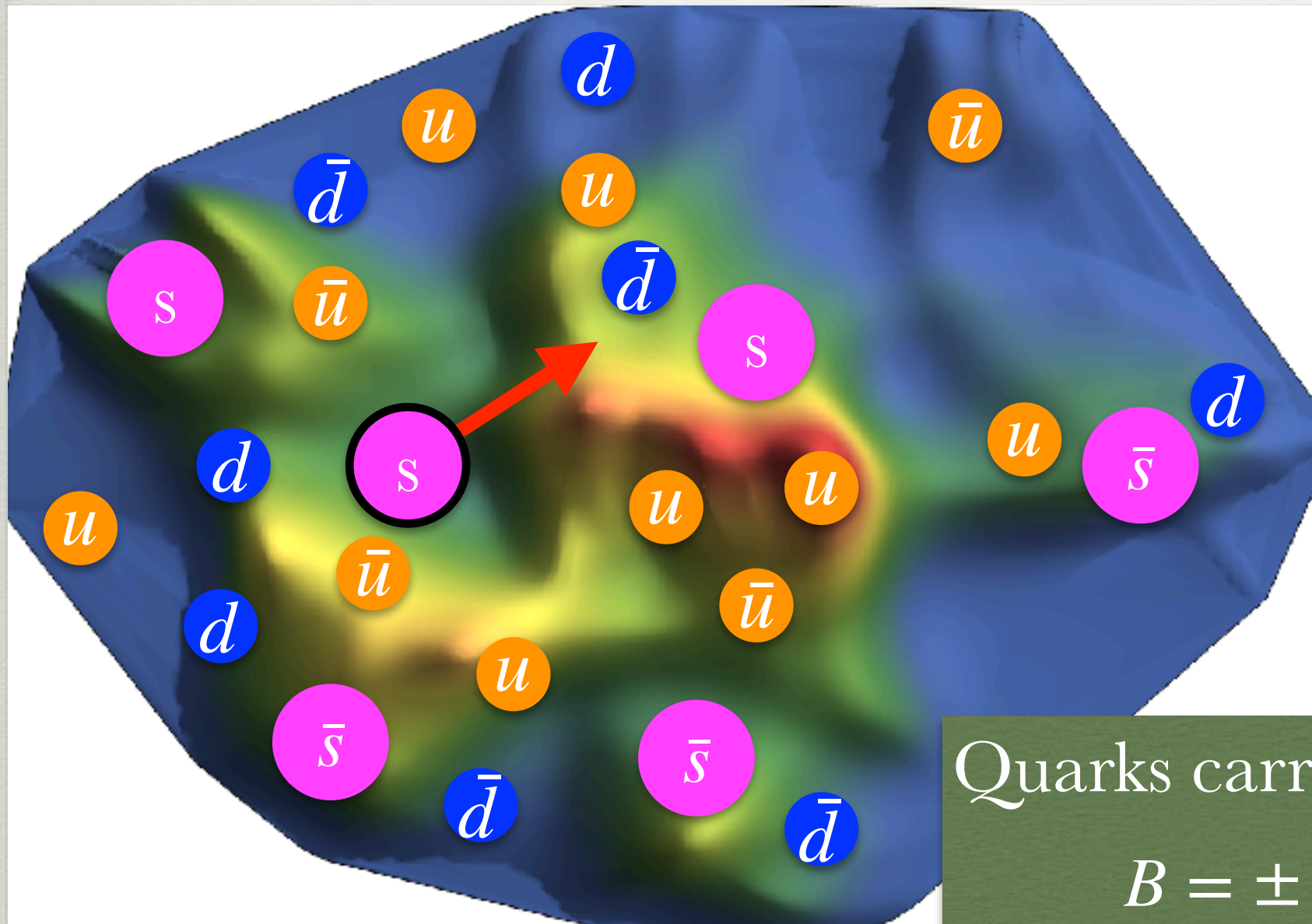
- Lorentz contracted (2D)
- Nuclei pass through instantaneously
- Too quick to capture baryons

Small $\sqrt{s_{NN}}$



- 3D nuclei pass slowly
- Time to capture baryons

Initial conditions: Local charge fluctuations



Each grid point in an initial condition needs to initialize

$$\left\{ T, \rho_B, \rho_S, \rho_Q \right\}$$

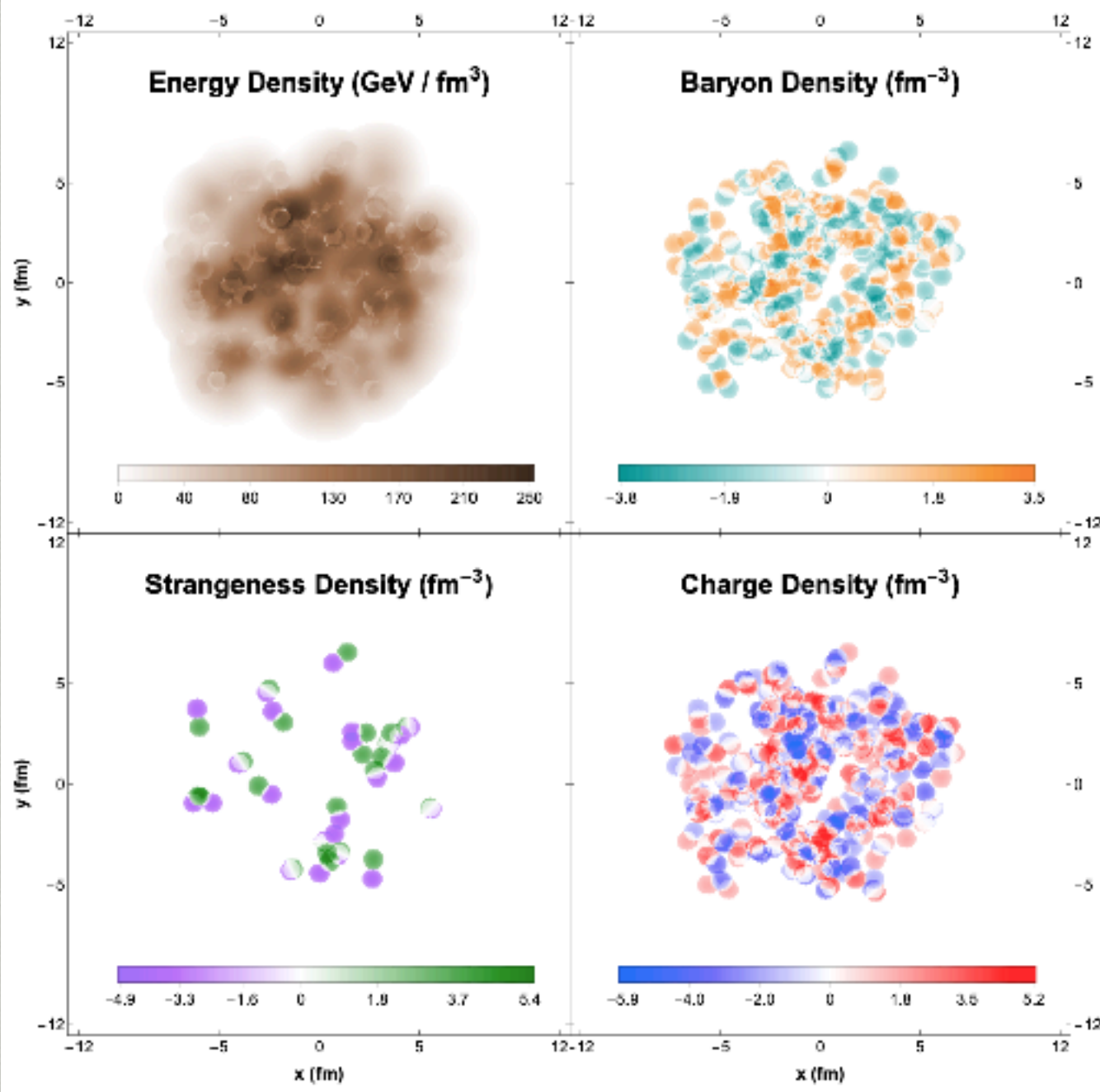
Quarks carry ~ 3 conserved charges:

$$B = \pm \frac{1}{3} \text{ Baryon number}$$

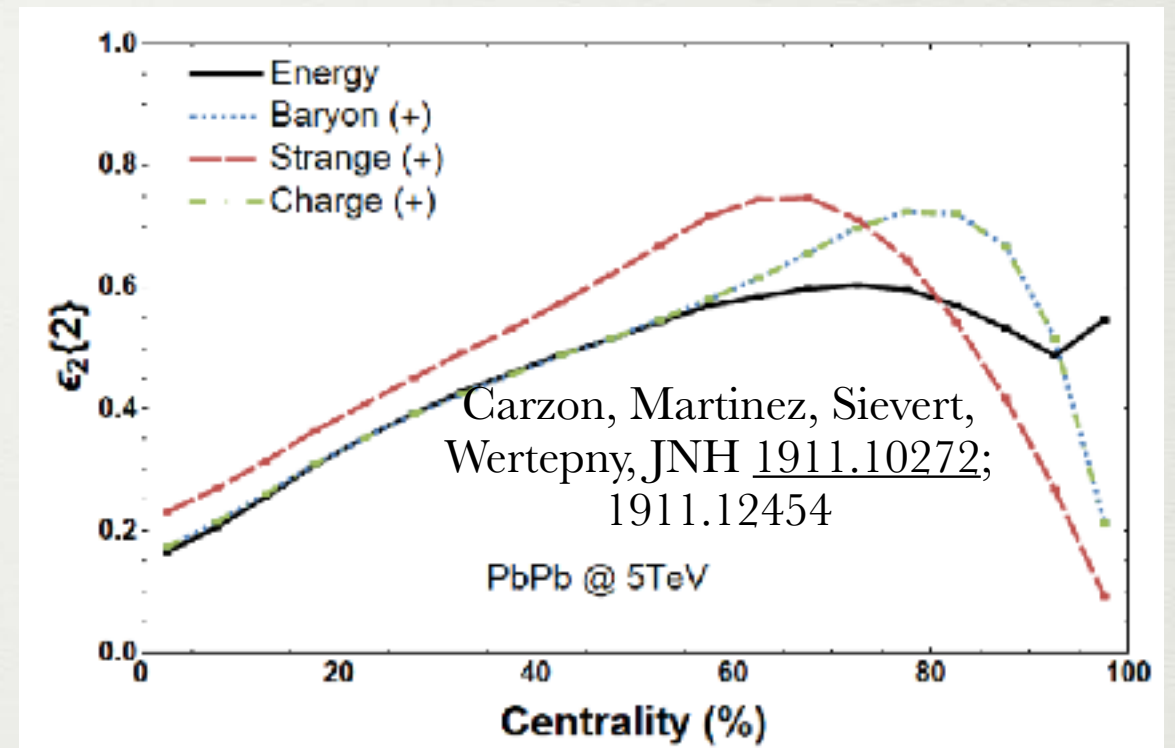
$$S = \pm 1 \text{ Strangeness}$$

$$Q = \pm \frac{1}{3} \text{ (d,s) or } Q = \pm \frac{2}{3} \text{ (u)}$$

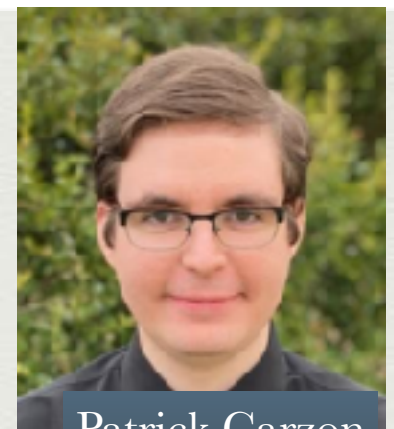
Initial conditions: ICCING- Initializing Conserved Charges in Nuclear Geometries



$$g \leftrightarrow q\bar{q}$$



Matt Sievert
Former Postdoc UIUC
Faculty NMSU



Patrick Carzon
PhD UIUC

Theoretical development

Martinez, Sievert, Wertepny

JHEP 02 (2019) 024; JHEP 1807 (2018) 003

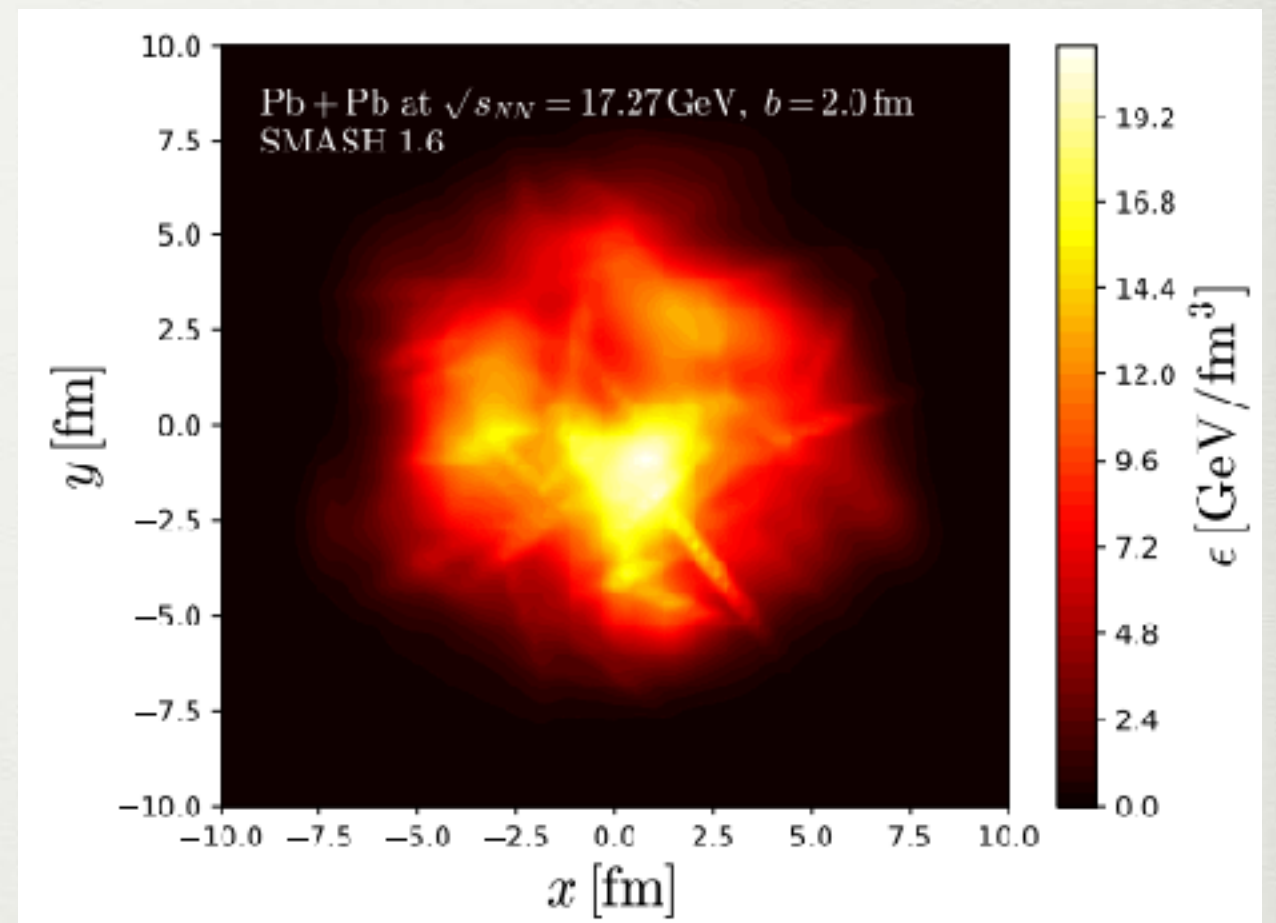
Initializing finite baryon density with nucleons

Strings between nucleons to simulate baryon stopping

Werner, Phys.Rept. 232 (1993) 87-299 ; Jeon and Kapusta, Phys.Rev. C56 (1997) 468-480 ; Bialis, Bzdak, Bozek, Acta Phys.Polon. B49 (2018) 103; Schenke and Shen Phys.Rev. C97 (2018) no.2, 024907

Transport (UrQMD/SMASH)

Karpenko, et al; Phys.Rev. C91 (2015) no.6, 064901



Mohs et al, J.Phys.G 47 (2020) 6, 065101

What about at the quark/gluon level?

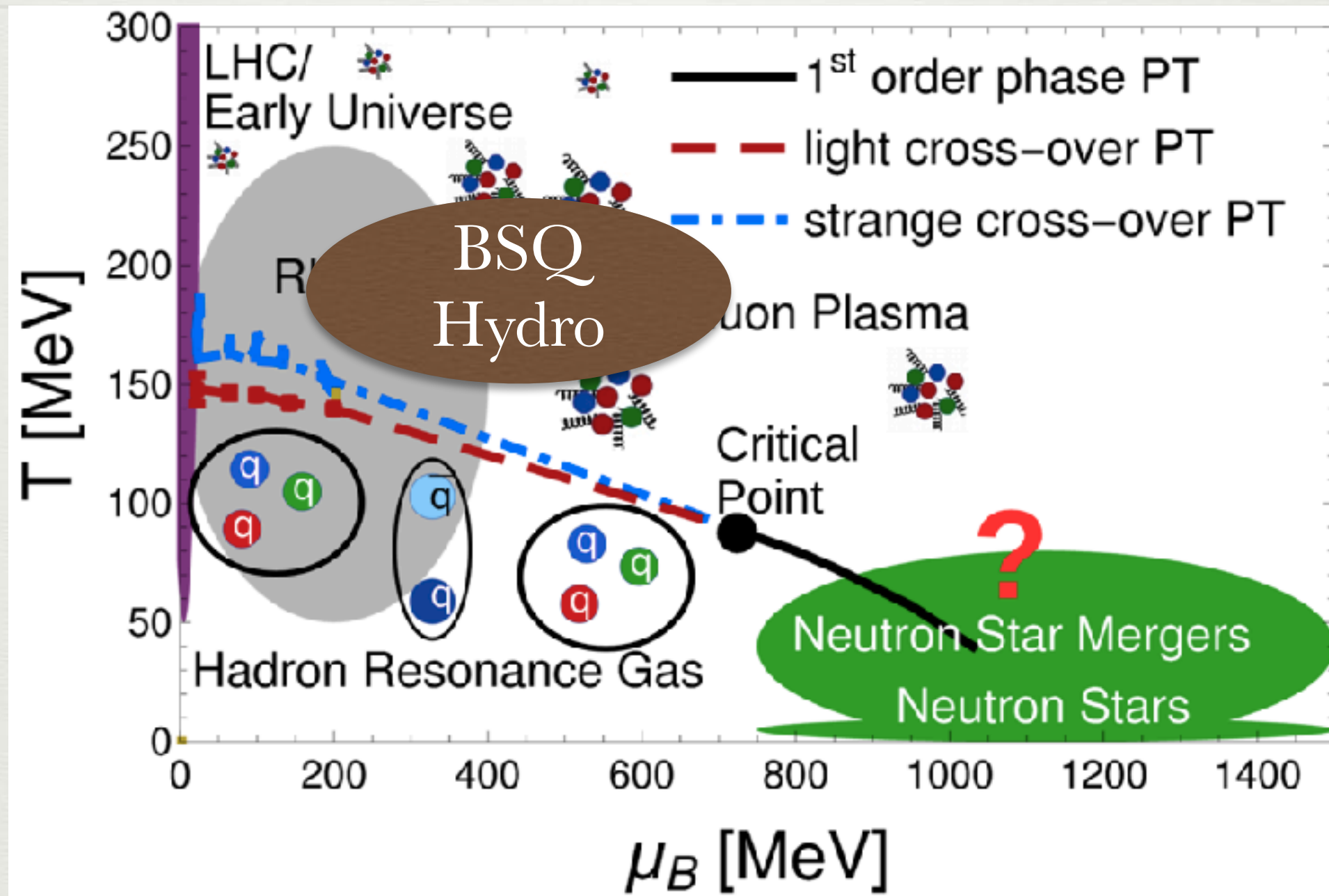
Initial Conditions: What are we missing?

- Definition of eccentricities for BSQ: what is the eccentricity of the initial baryon density? How does that translate to a v_2 of protons?
 - Eccentricities are calculated in the center of mass frame, can't do a center of charge frame when $\rho_B \rightarrow 0$
- Full $T^{\mu\nu}$ at finite μ_B . Need initial q^μ as well
- What are the right degrees of freedom? Just nucleons?
- Should we be modeling the initial conditions as fluids?

Initial state: $\{e, u_0, \pi^{\mu\nu}, \Pi, q^\mu\}$

- Current simulations at finite μ_B include initial conditions for $\{e, u_0\}$
- Very common to still run ideal hydro at low $\sqrt{s_{NN}}$
- *Some* include transport coefficients: $\eta T/w, \zeta T/w, \kappa_{XY}$ where $X, Y = B, S, Q$. *Some* include 2nd order
E.g. Du & Heinz Comput.Phys.Commun. 251 (2020) 107090
- None include initial conditions from $\{\pi^{\mu\nu}, \Pi, q^\mu\}$

QCD Phase Diagram



Light transition *Phys.Lett. B738 (2014) 305-310*; **Strange Transition** Bellwied, JNH, Parotto, Vazquez, Ratti, Stafford, *arXiv:1805.00088* ;
Neutron Star (mergers) V. Dexheimer *arXiv:1708.08342*; **Holography** Critelli, JNH, et al, *Phys.Rev. D96 (2017) no.9, 096026*

Upgrades to Hydrodynamics

Equation of State
Baryon, Strangeness,
Electric Charge
EOS+CP

Transport Coefficients

Shear and bulk viscosity

$$\eta T/w(T, \mu_B, \mu_S, \mu_Q)$$

BSQ Diffusion

$$\kappa_{BB}, \kappa_{BS}, \dots$$

BSQ Hydro

Ideal BSQ

$$\partial_\mu B^\mu = 0, B^\mu = \rho_B u^\mu + n_B^\mu$$

Diffusion

$$\kappa_{BB} \Delta^{\mu\nu} \partial_\nu (\mu_B/T)$$

BSQ cross terms

$$\kappa_{BS} \Delta^{\mu\nu} \partial_\nu (\mu_B/T)$$

Full BSQ hydro simulations



Lydia Spychalla
REU UIUC



Dekra Almaalol
PhD Kent
Soon UIUC postdoc

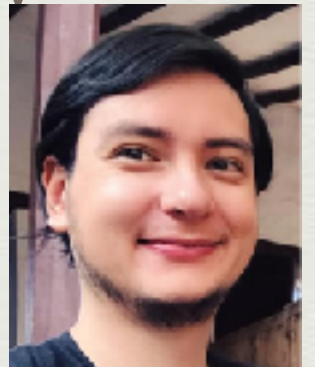
2+1 BSQ relativistic viscous hydrodynamics+ICcing

Coming Soon!

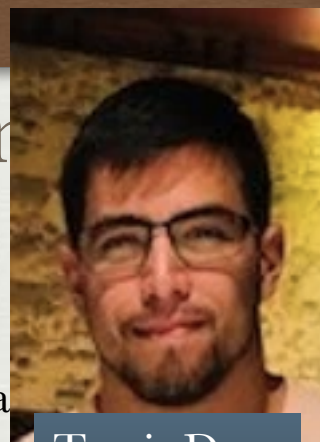
Almaalol, Carzon, Cruz Camacho, Dore, Mroczek, Plumberg, Spychalla, Sievert, JNH



Christopher Plumberg
UIUC postdoc



Nikolas Cruz Camacho
PhD UIUC



Travis Dore
PhD student



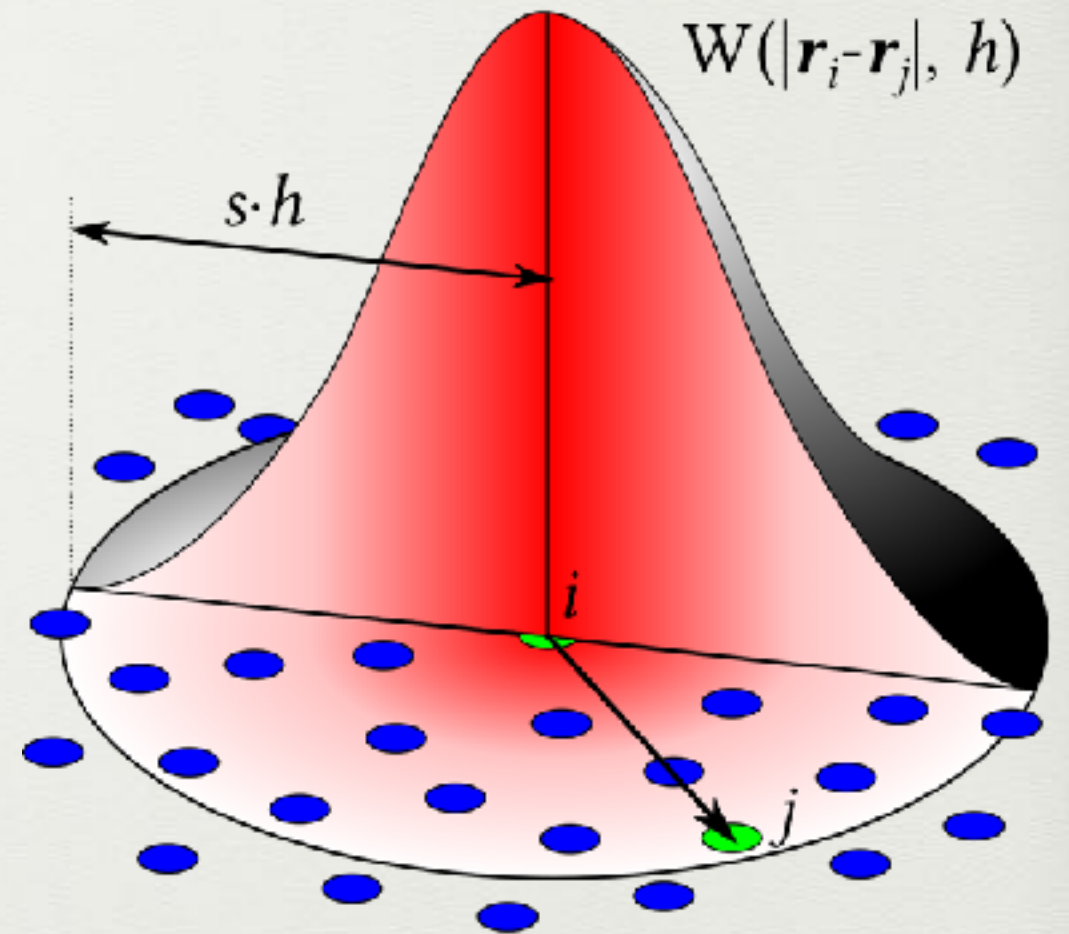
Debora Mroczek
UIUC PhD Student

Dynamics: Smoothed Particle Hydrodynamics (SPH) for BSQ

With BSQ diffusion, the charge current becomes

$$N_{B,S,Q}^\mu = \underbrace{\rho_{B,S,Q} u^\mu}_{\text{ideal}} + \underbrace{n_{B,S,Q}^\mu}_{\text{diffusion}}$$

However, SPH requires one to divide by ρ_{BSQ} , in heavy-ions ρ_{BSQ} may be 0!



Define $\rho_{BSQ}^* \equiv \rho_{B,S,Q} u^\mu + n_{B,S,Q}^\mu$ where $\rho_{BSQ}^* = \sqrt{J^\mu J_\mu} > 0$

BSQ hydrodynamic simulations coming soon!

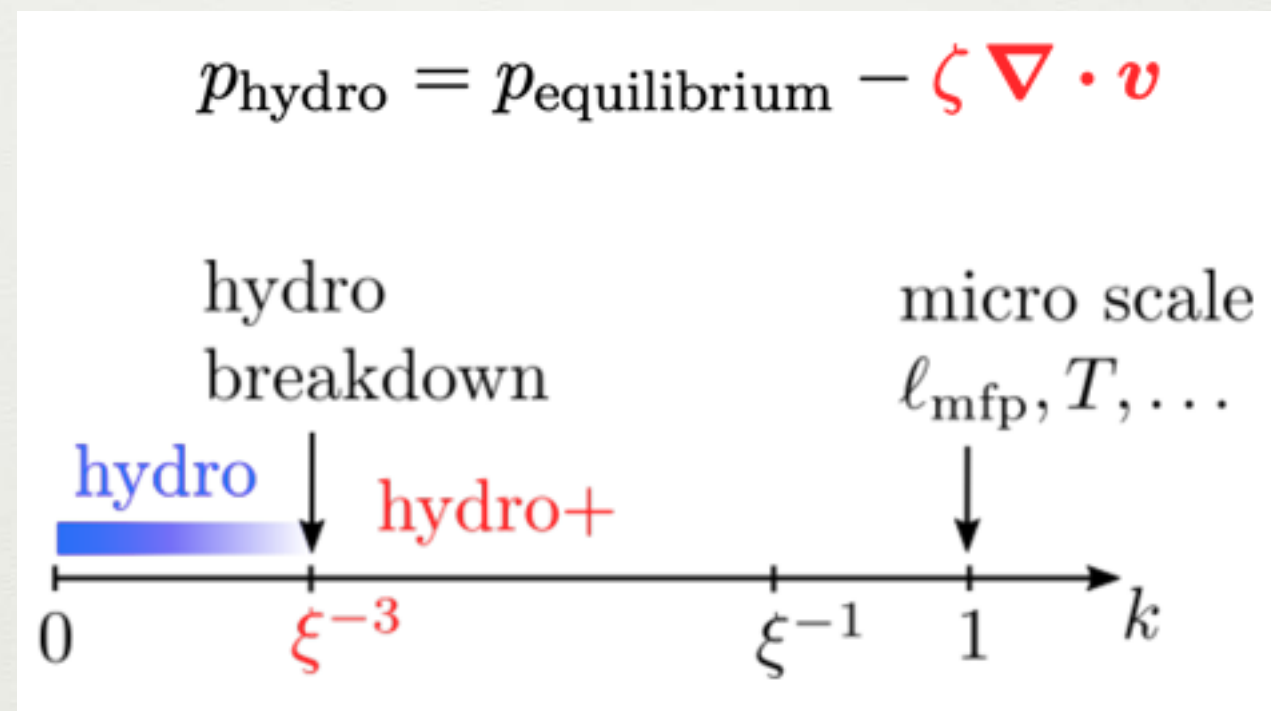
Hydro \neq Hydro \neq Hydro

How are the hydrodynamic equations of motion derived?

- Phenomenological Israel-Stewart: derived from the 2nd law of thermodynamics i.e. entropy *always* \uparrow
- DNMR: power counting in Kn and Re^{-1}
 - “Standard in the field”
- BDNK: frame choice neither Eckhart nor Landau, first-order hydrodynamics
 - Fewer transport coefficients
 - Near equilibrium

Hydro as we approach a critical point

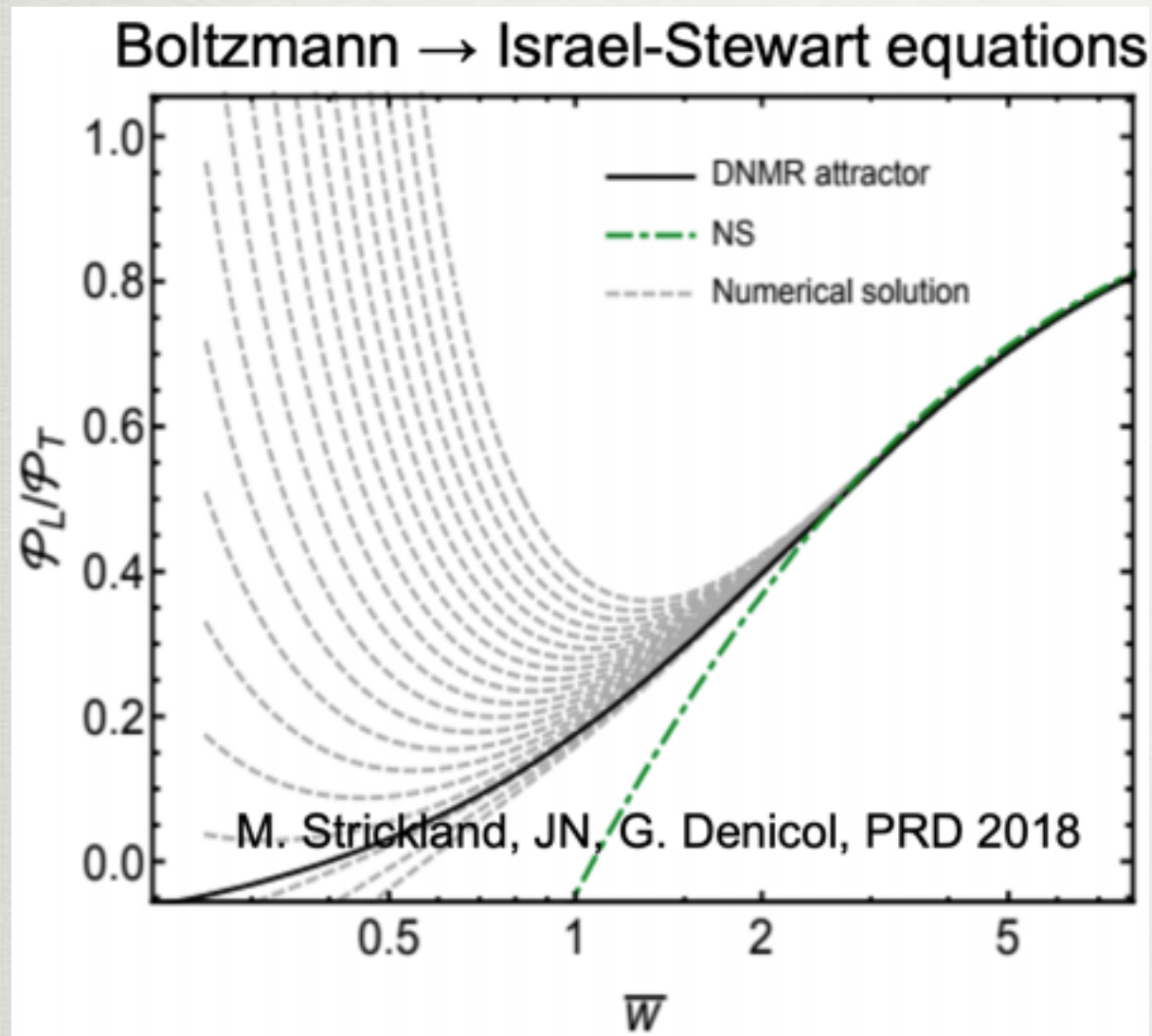
Hydro breaks down as \rightarrow critical point



Stephanov, Yin Phys. Rev. D 98, 036006 (2018)

However, what theory works best for as long as possible?

How well-behaved in hydro near a CP? Do attractors exist?



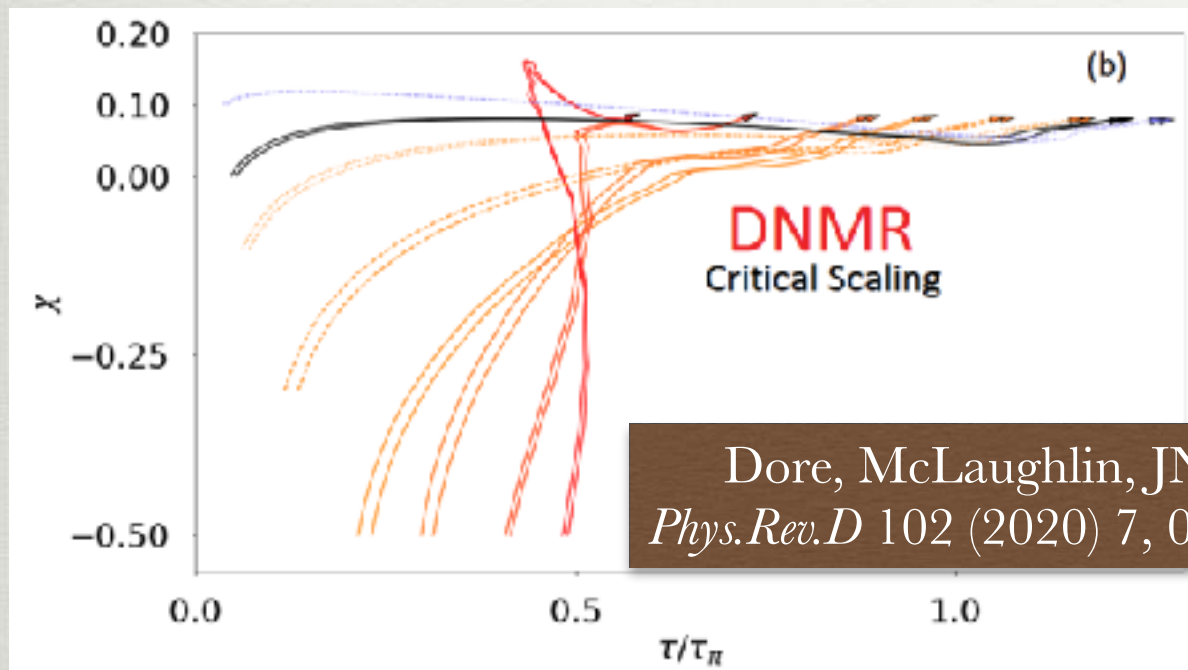
- Attractors at $\mu_B = 0$ imply out-of-equilibrium contributions to the initial conditions are washed out
- Mostly only ε_n 's matter

First work from Heller & Spalinski
Phys.Rev.Lett. 115 (2015) 7, 072501

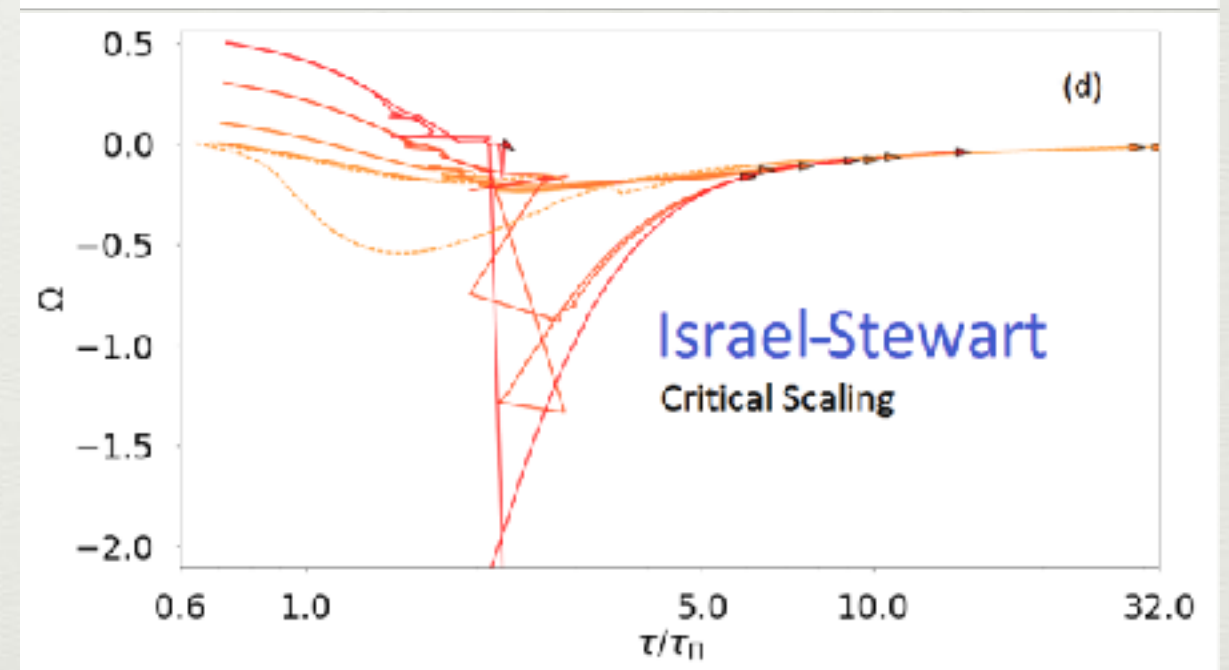
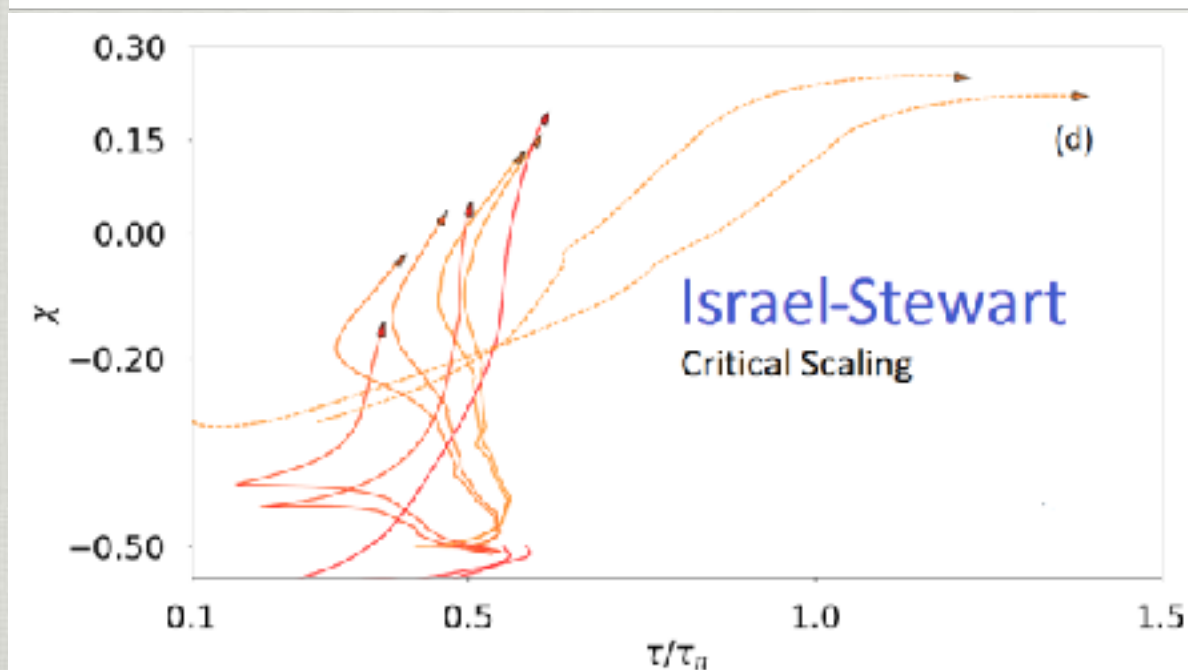
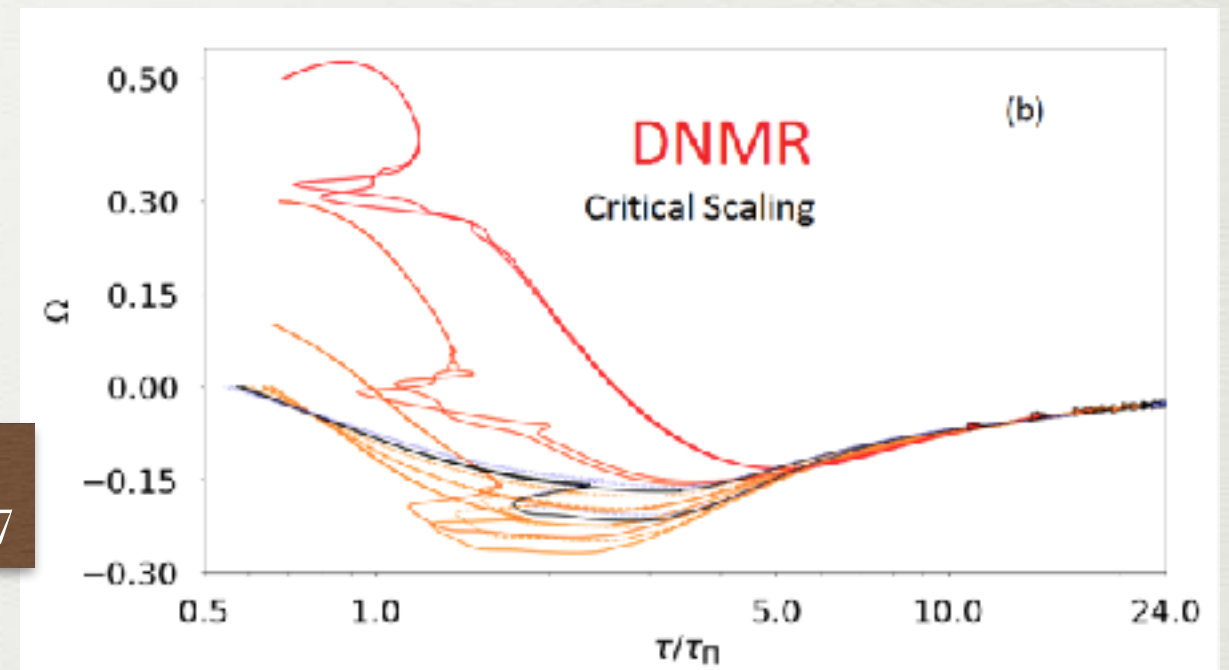
Critical point and equations of motion and attractors, varying (π_η^η, Π_0)

Shear stress tensor

Bulk pressure



Dore, McLaughlin, JNH
Phys.Rev.D 102 (2020) 7, 074017



Dynamics: BSQ diffusion in relativistic hydrodynamics

Ensure only positive entropy production

Many new terms that couple shear, bulk, and BSQ diffusion!

$$\tau_{\pi} \dot{\pi}^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \frac{\tau_{\pi}\pi^{\mu\nu}}{2}\theta - \frac{\tau_{\pi}}{2\beta_{\pi}}\dot{\beta}_{\pi}\pi^{\mu\nu} - \frac{2\eta}{\beta} \left(\gamma_1^q \nabla^{\langle\mu} n_q^{\nu\rangle} + \frac{1}{2} n_q^{\langle\mu} \nabla^{\nu\rangle} \gamma_1^q \right)$$

$$\tau_{\Pi} \dot{\Pi} + \Pi = - \left(\zeta + \frac{\tau_{\Pi}}{2}\Pi \right) \theta - \frac{\tau_{\Pi}}{2\beta_{\Pi}} \dot{\beta}_{\Pi} \Pi - \frac{\zeta}{\beta} \left(\gamma_0^q D_{\mu} n_q^{\mu} + \frac{1}{2} n_q^{\mu} \nabla_{\mu} \gamma_0^q \right)$$

$$\tau_{qq'} \dot{n}_{q'}^{\mu} + n_q^{\mu} = - \kappa_{qq'} \nabla^{\mu} \alpha_{q'} + \frac{\tau_{qq'} n_{q'}^{\mu}}{2} \theta - \frac{\tau_{qq'}}{2\beta_{qq'}} \dot{\beta}_{qq'} n_{qq'}^{\mu} - \frac{\kappa_{qq'}}{\beta} \left(\gamma_0^{qq'} \nabla^{\mu} \Pi - \frac{\Pi}{2} \nabla^{\mu} \gamma_0^{qq'} \right) - \frac{\kappa_{qq'}}{\beta} \left(\gamma_1^{qq'} \nabla_{\nu} \pi^{\mu\nu} + \frac{\pi^{\mu\nu}}{2} \nabla_{\nu} \gamma_1^{qq'} \right)$$

Dore, Mroczek, Almaalol, Sievert, JNH to appear soon

BSQ diffusion from DNMR/DBNK

DNMR

Fotakis, Niemi, Denicol,
Greiner, Greif

DBNK

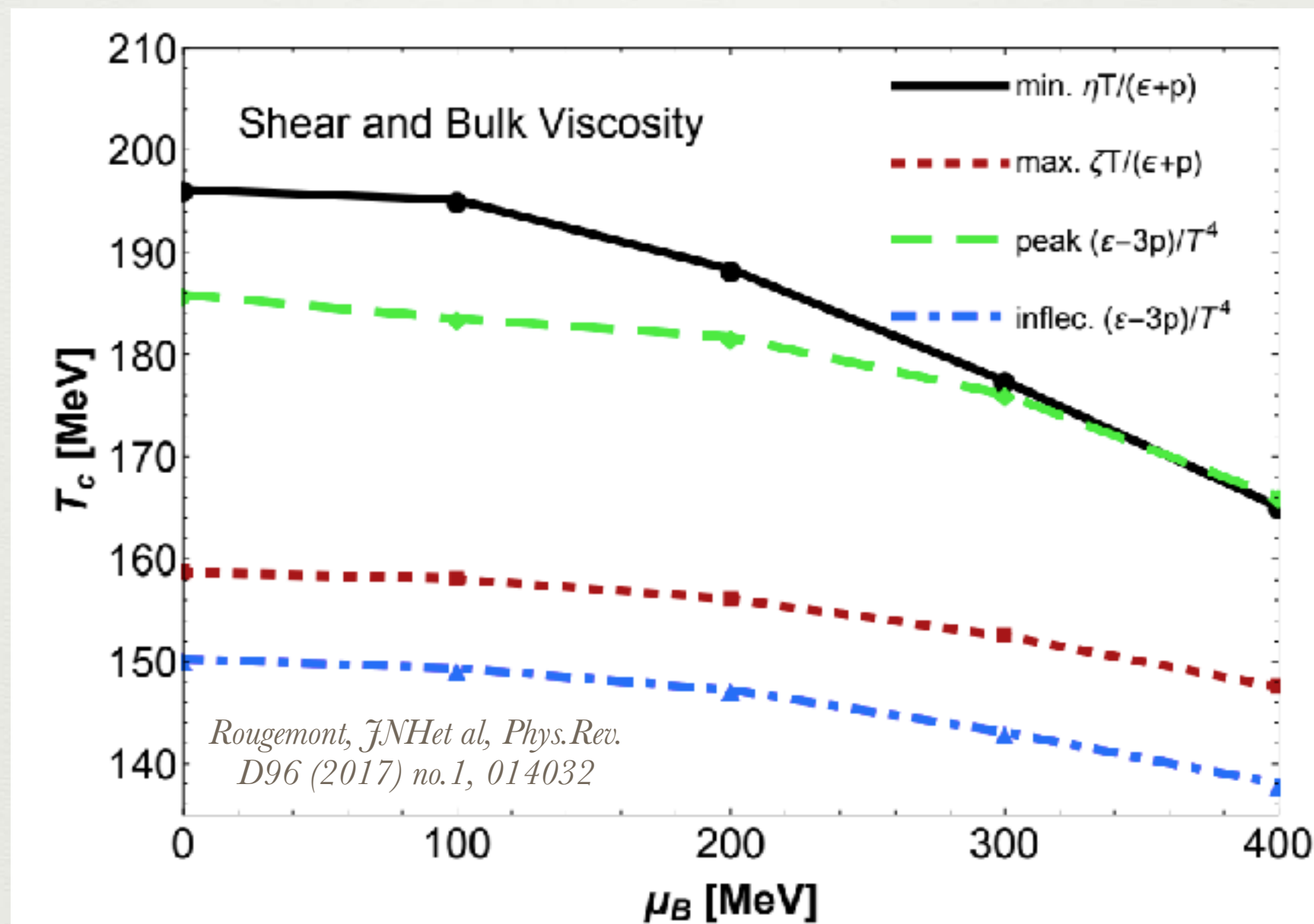
Disconzi, Bemfica,
Noronha, Kovtun



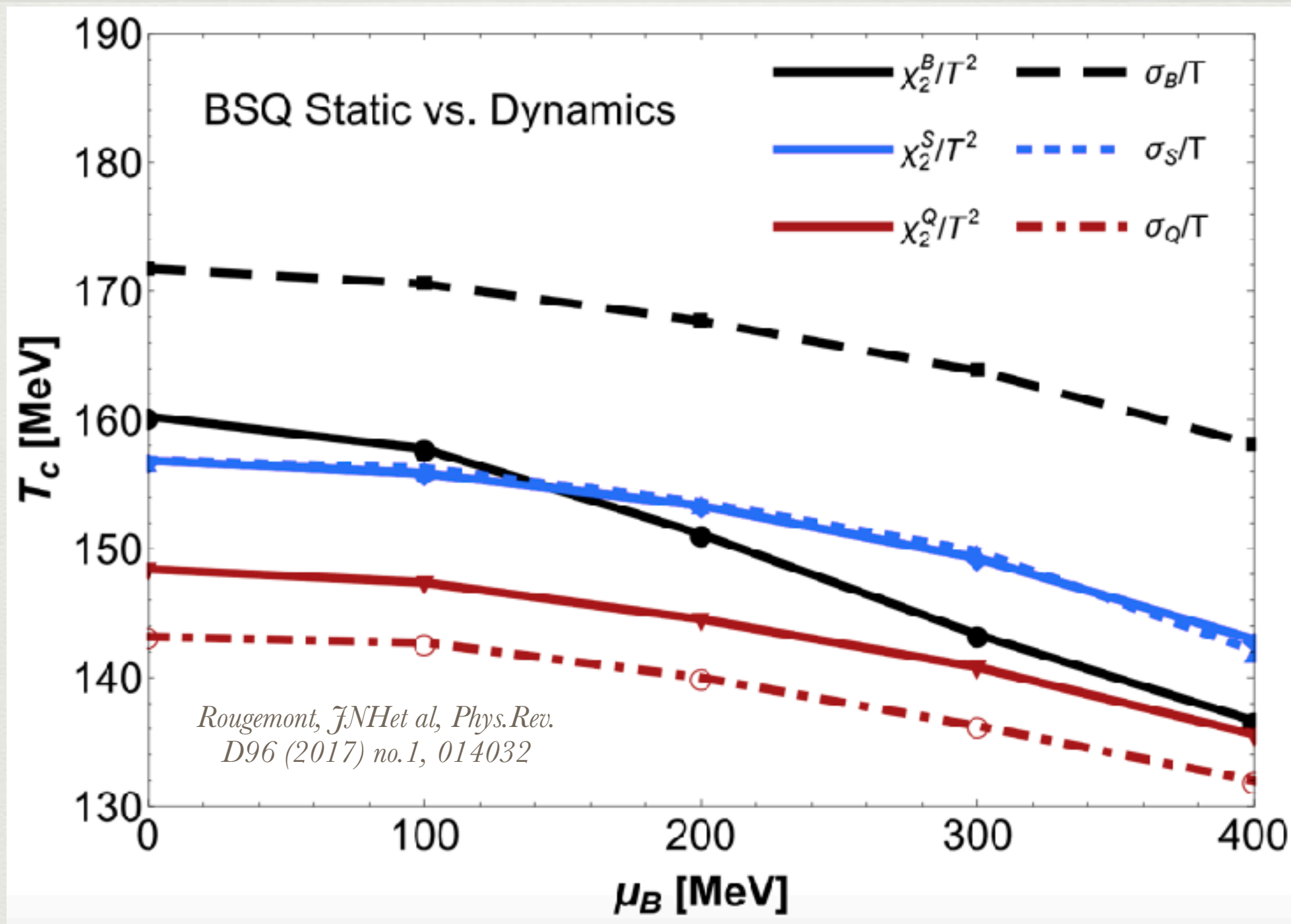
Pseudo-critical temperatures

Example: T at $(\eta/s)_{min}$

Guidance from holography



BSQ T_{pc} in and out of equilibrium



Pseudo critical temperatures of transport coefficients

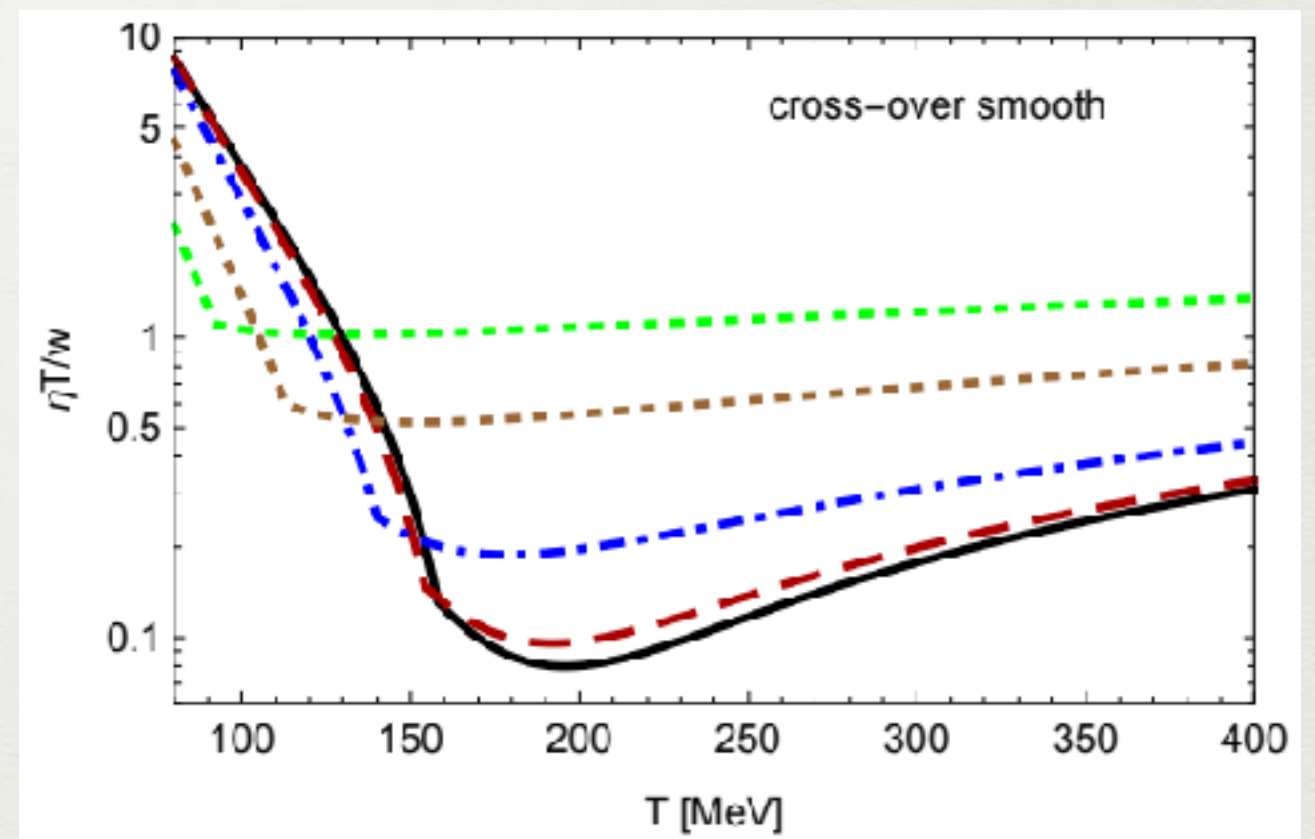
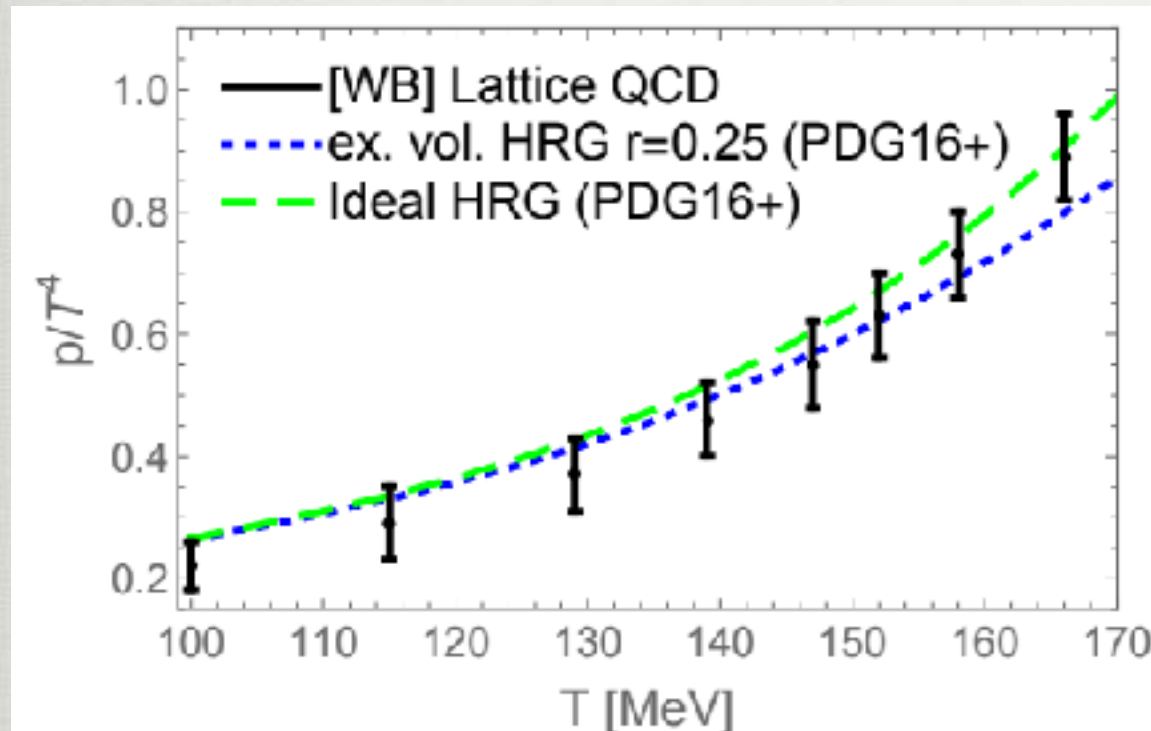
- Should all converge all the critical point
- Large spread at low μ_B
- Equilibrium ($\chi_{B,S,Q}$) vs. Out-of-Equilibrium ($\sigma_{B,S,Q}$) quantities do not, necessarily, have the same pseudo critical temperatures
- How does this affect simulations????

Testable shear viscosity $\eta T/w(T, \mu_B)$

McLaughlin, JNH, et al, [2103.02090](#)

Constrain the Excluded volume from Lattice QCD

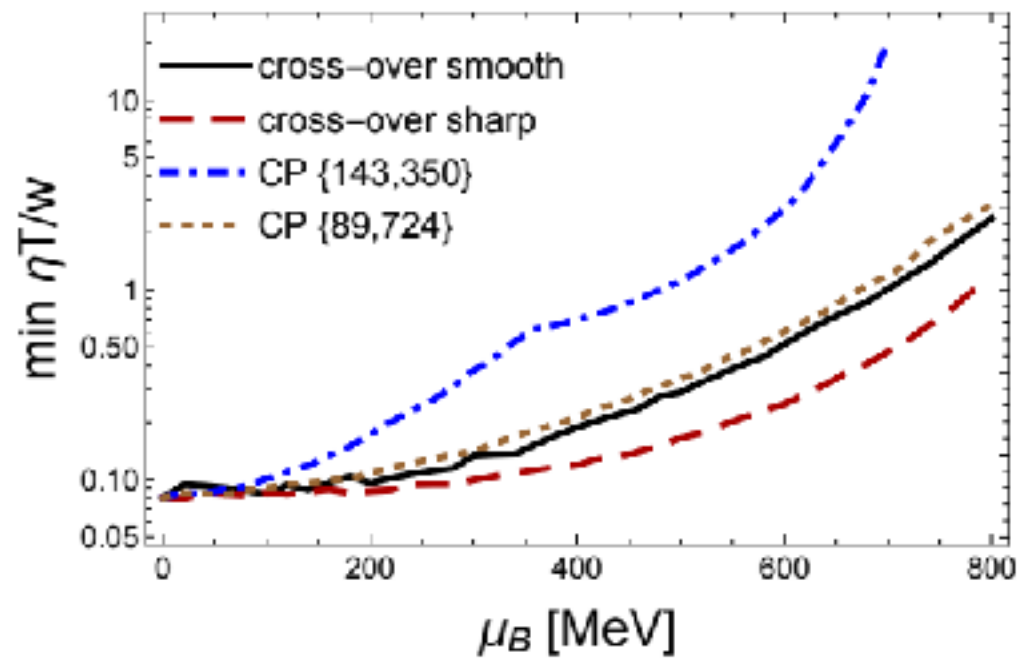
Matched to parameterized QGP shear using a variety of methods



Include critical point (different locations) and sharp vs. smooth cross-over

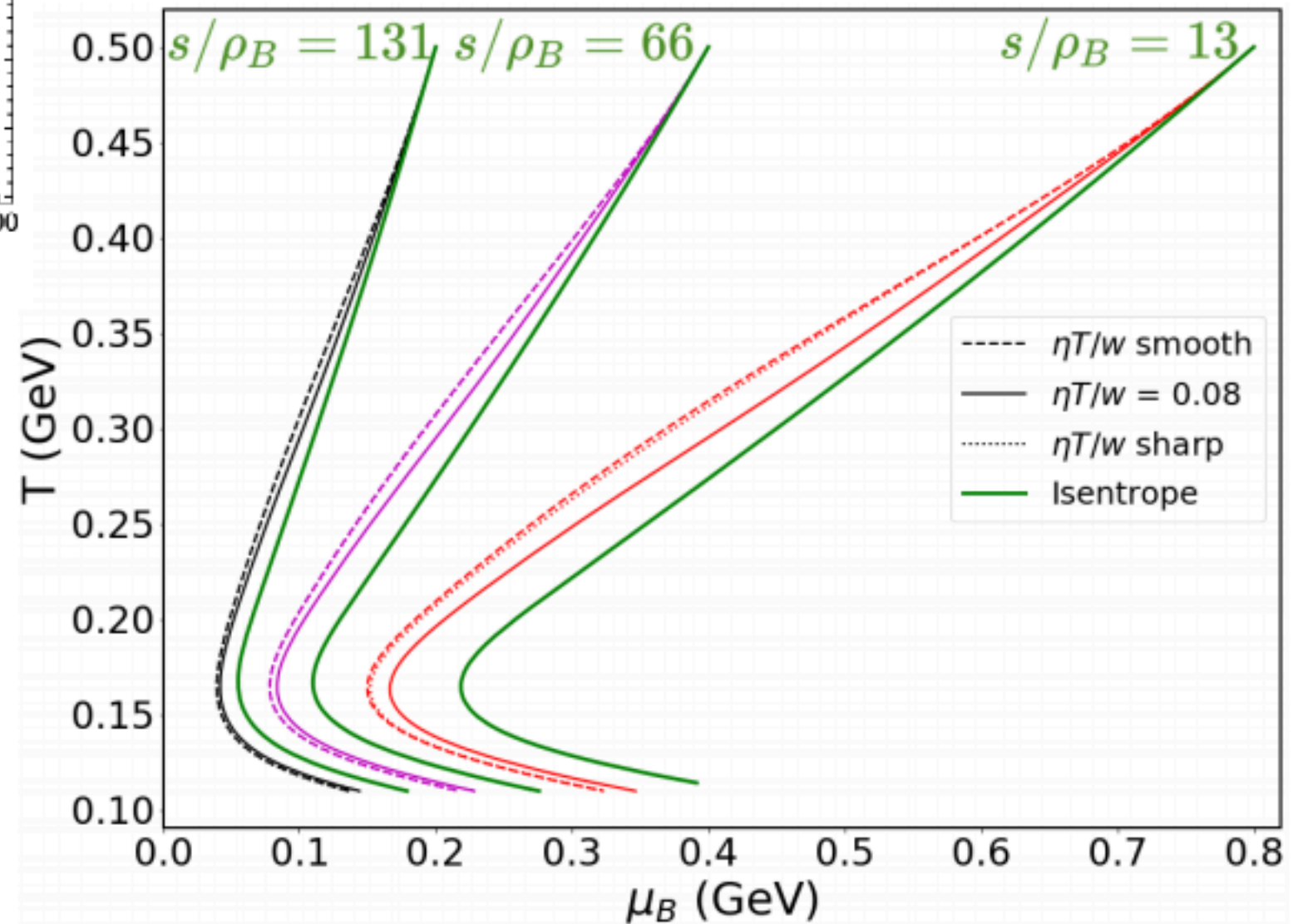
Consequences of $\eta T/w$ across μ_B

McLaughlin, JNH, et al, [2103.02090](#)

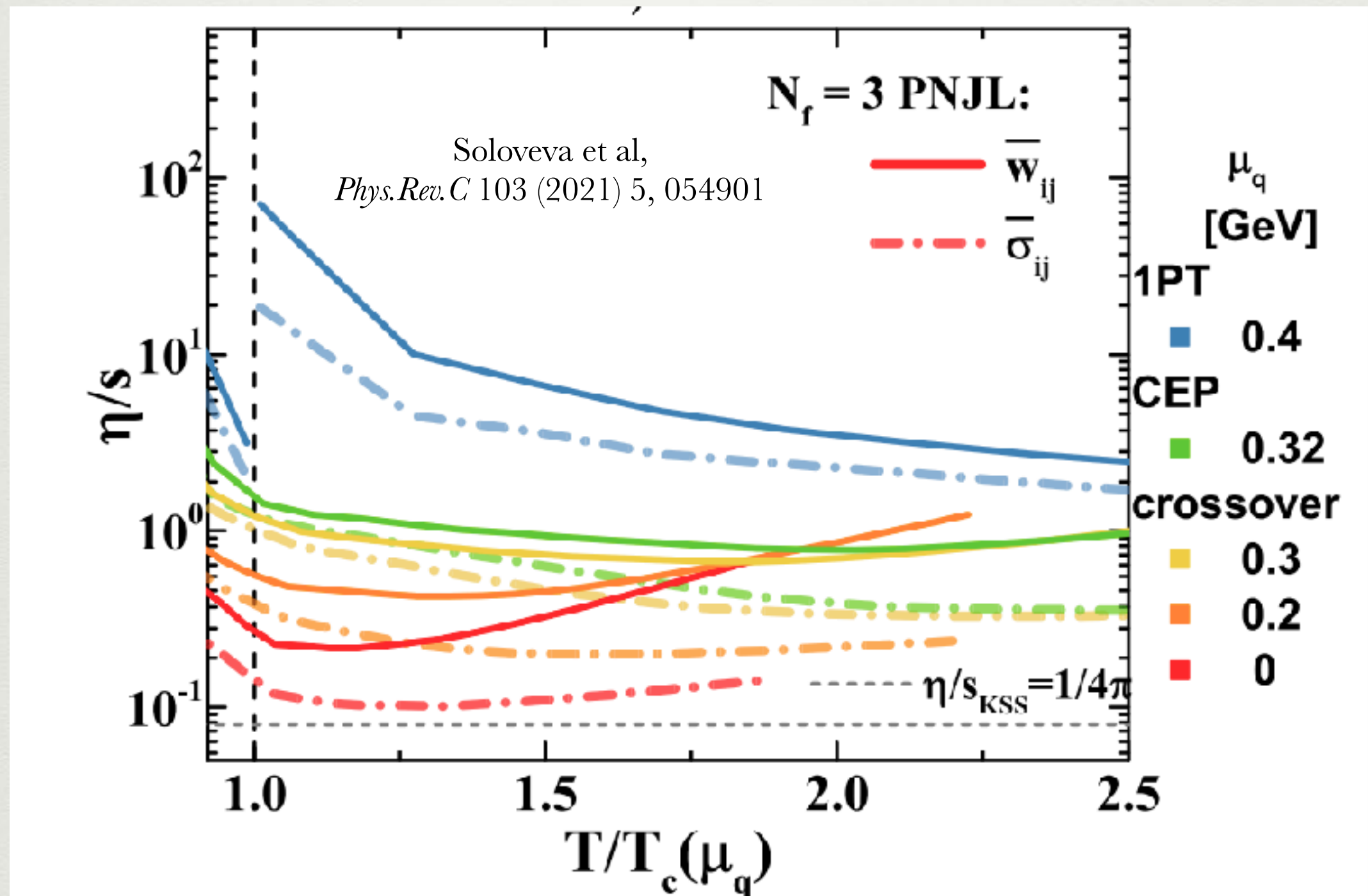


Transition line affects magnitude of $\eta T/w$

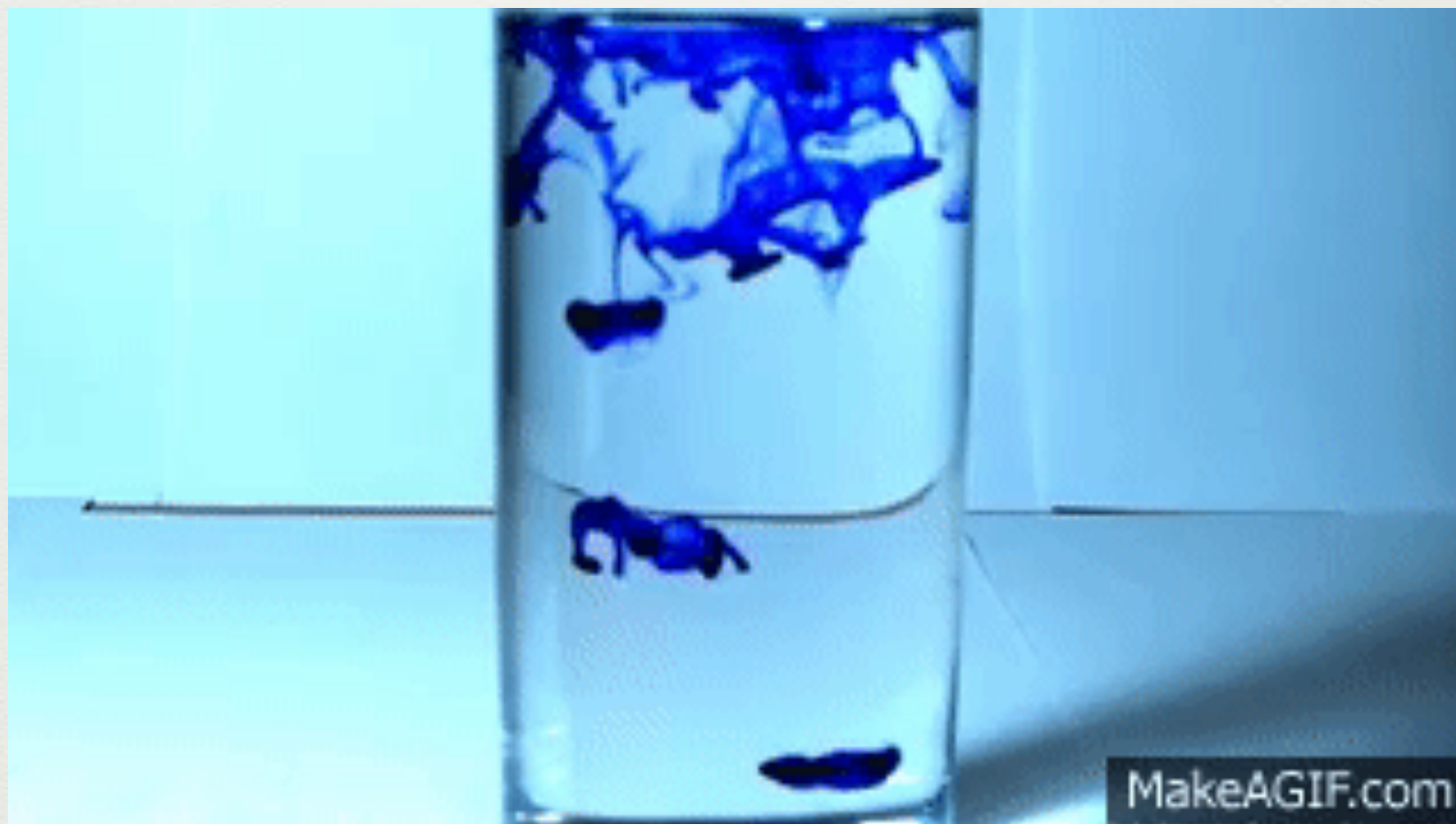
Viscous effects more prevalent at low beam energies



Jump in η/s across a 1st-order phase transition?

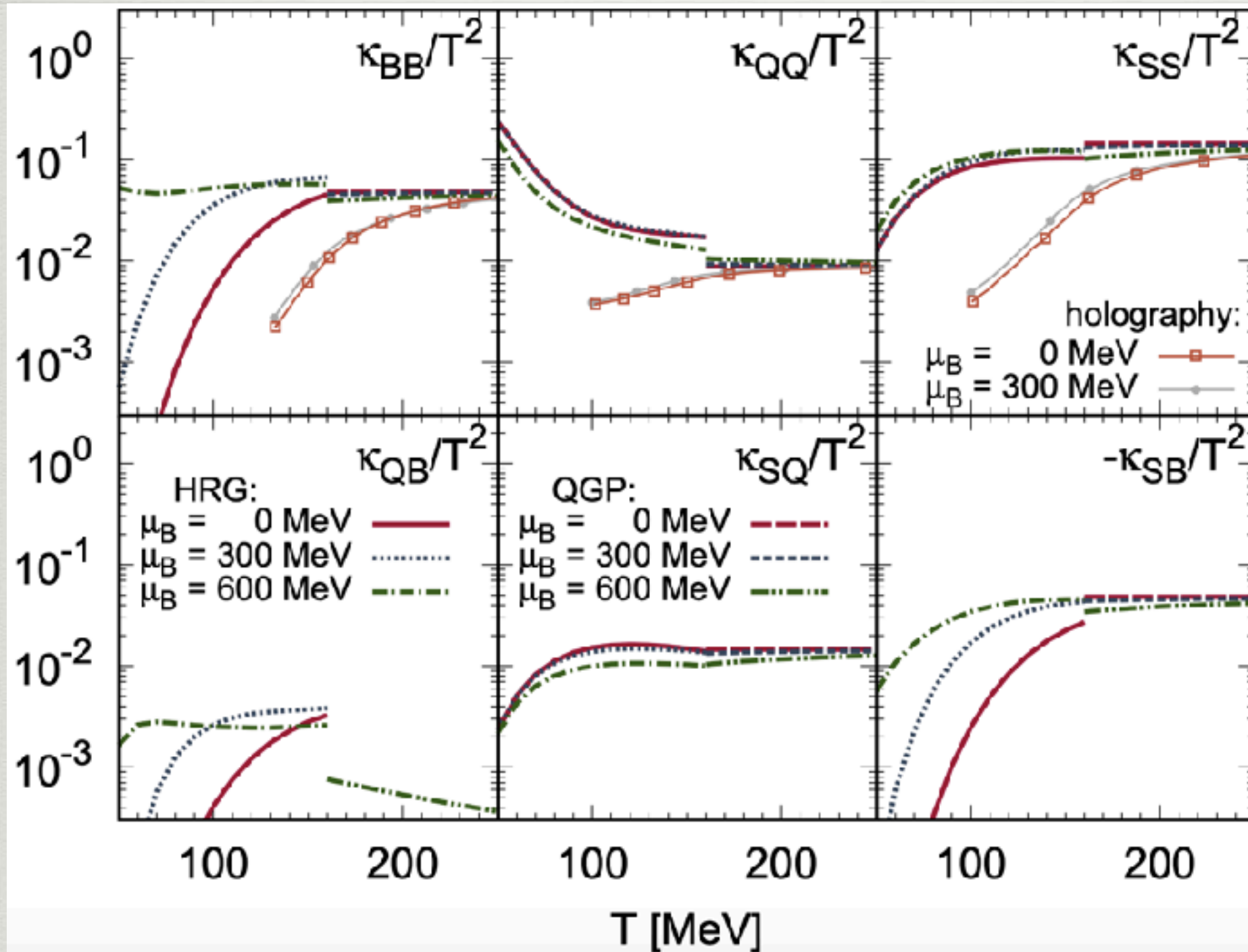


BSQ diffusion



How do quarks spread out over time? How quickly do u , d , s quarks diffuse through out the QGP since they carry multiple conserved charges?

BSQ Diffusion



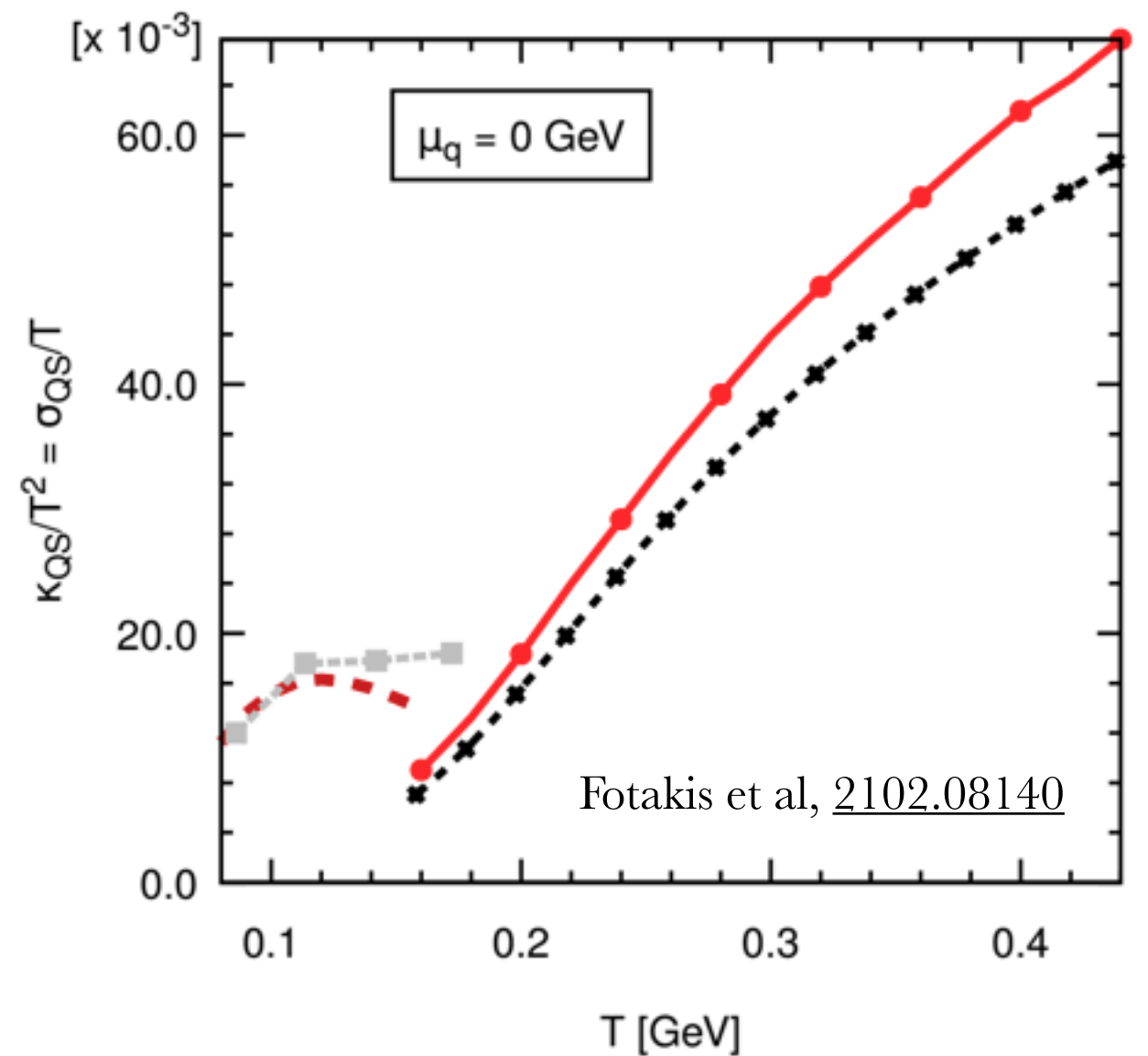
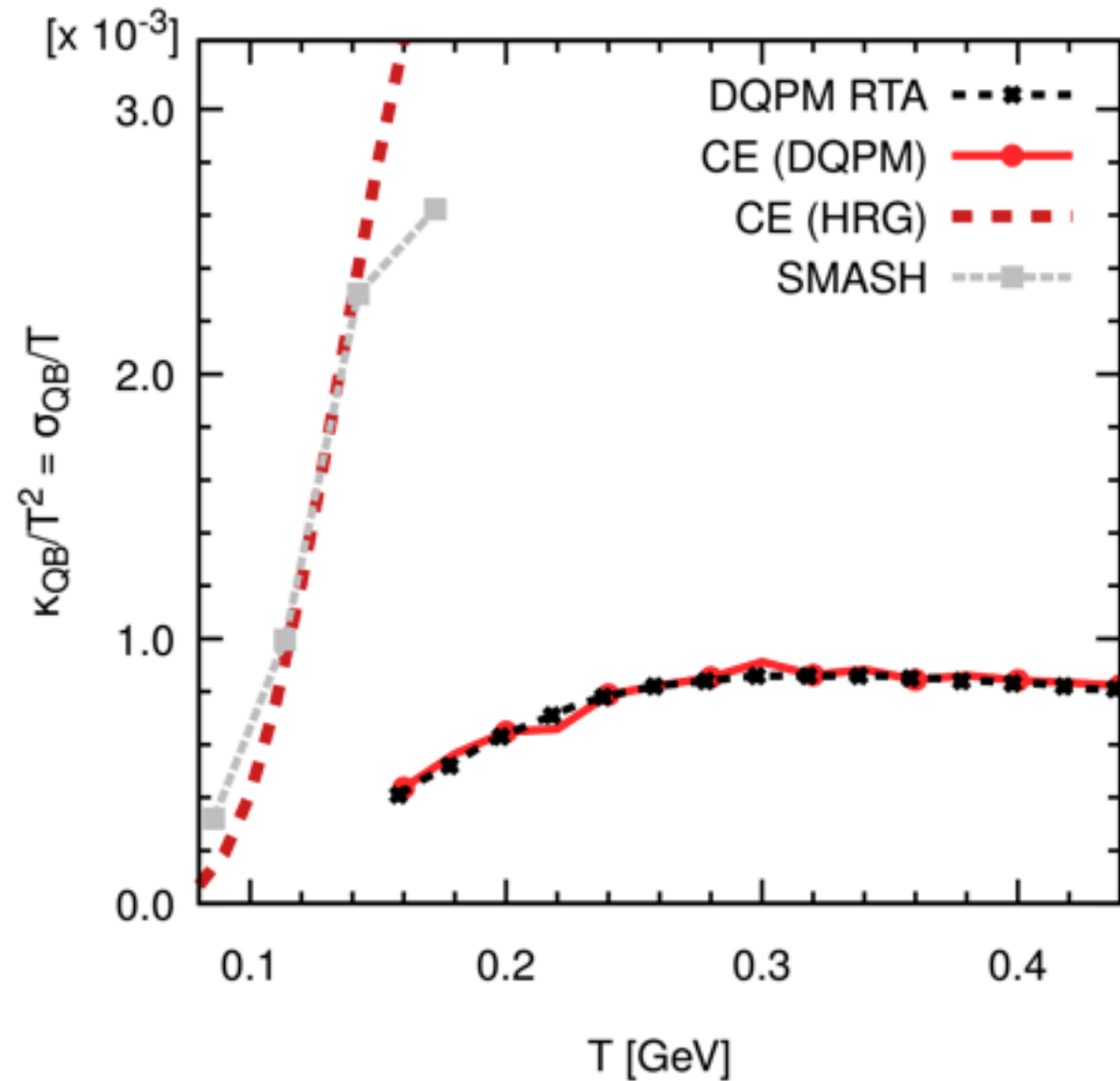
Kinetic Theory

*Greif, Fotakis, Denicol, Greiner
Phys.Rev.Lett. 120 (2018) no.24,
242301*

Holography
(black hole
engineering)

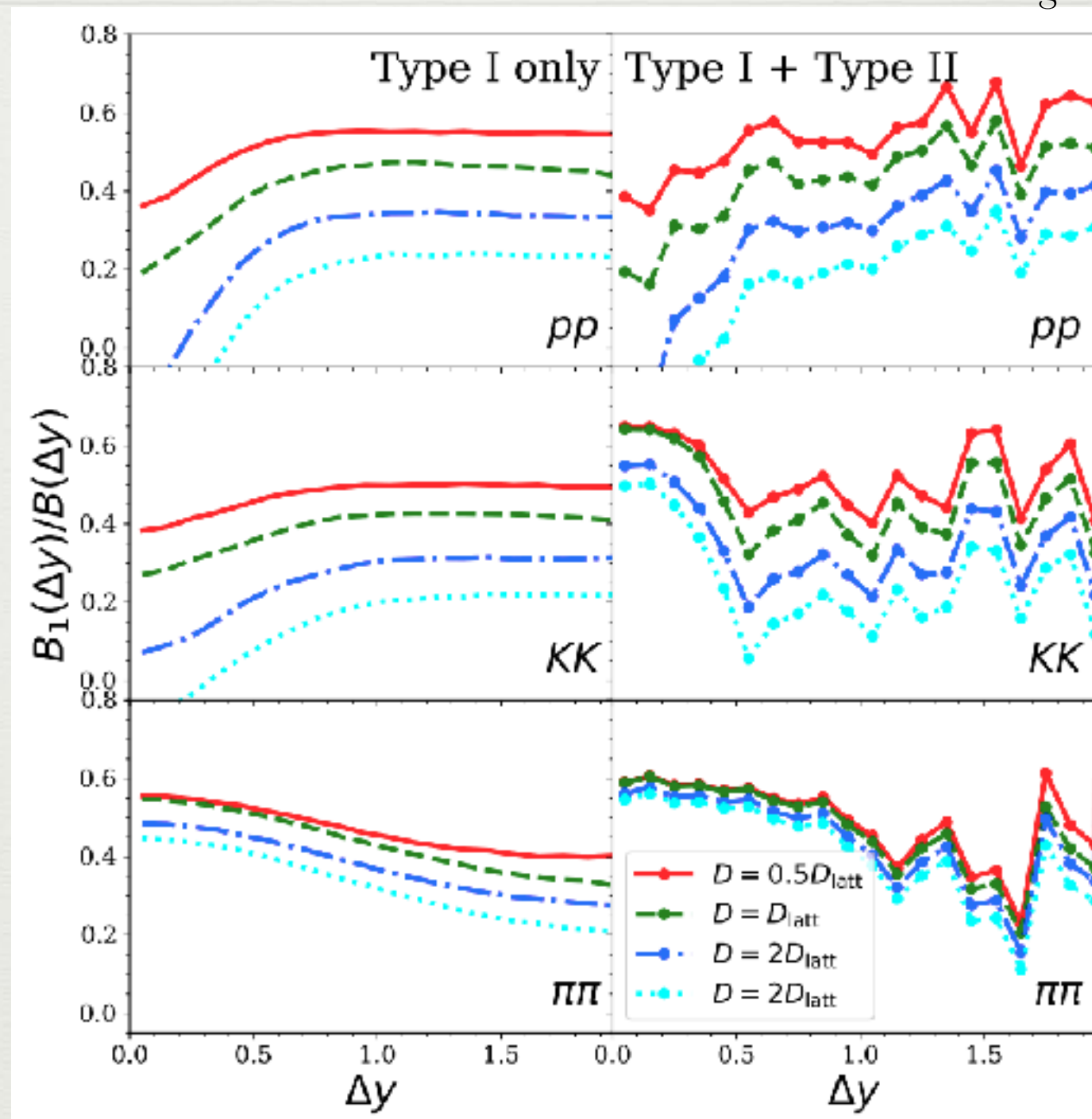
*Rougemont, Critelli, JNH, Noronha,
Ratti Phys.Rev. D96 (2017) no.1,
014032*

Diffusion: HRG to QGP?



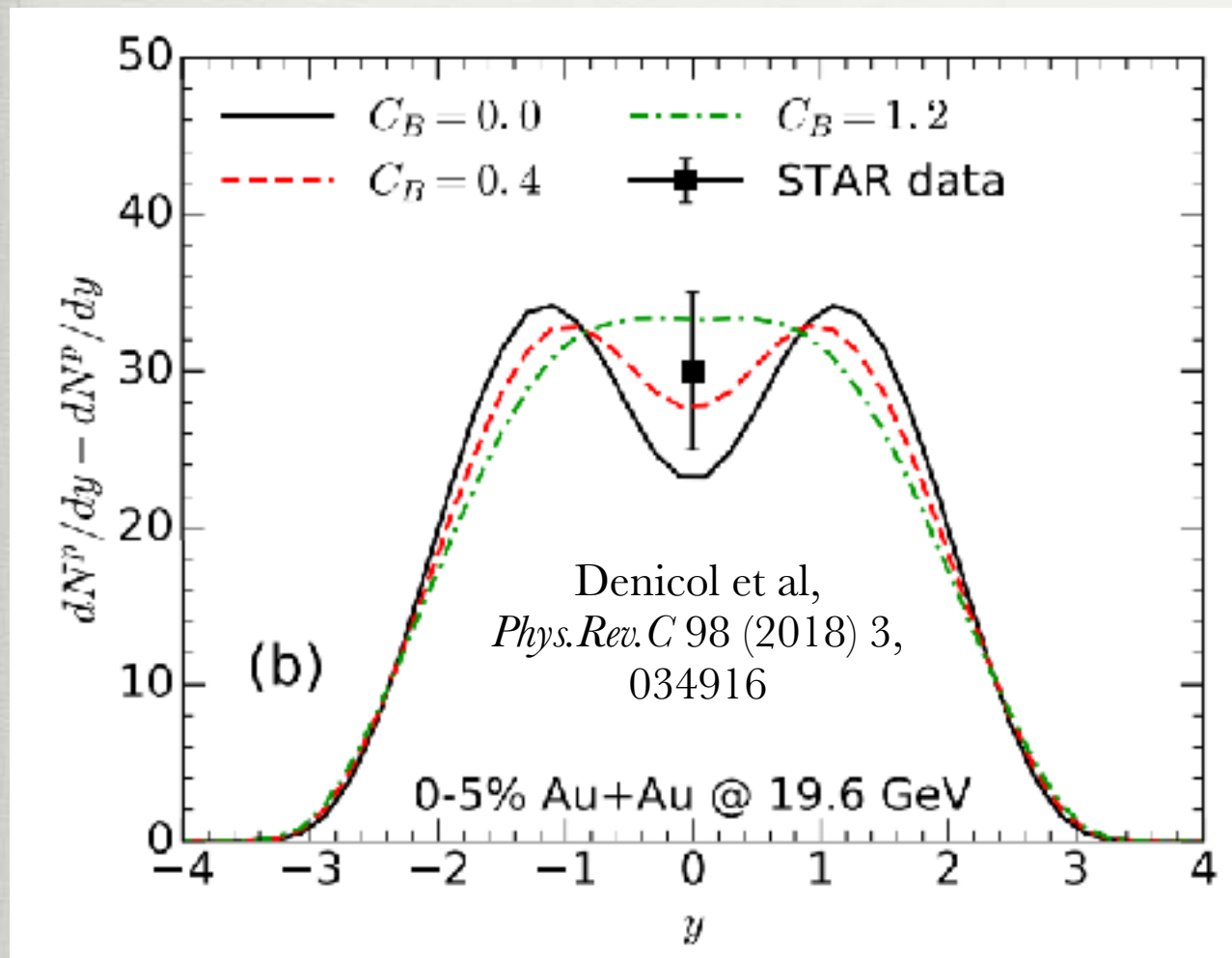
Charge Balance functions: potential probe of diffusion?

Pratt and Plumberg *Phys.Rev.C* 104 (2021) 1, 014906

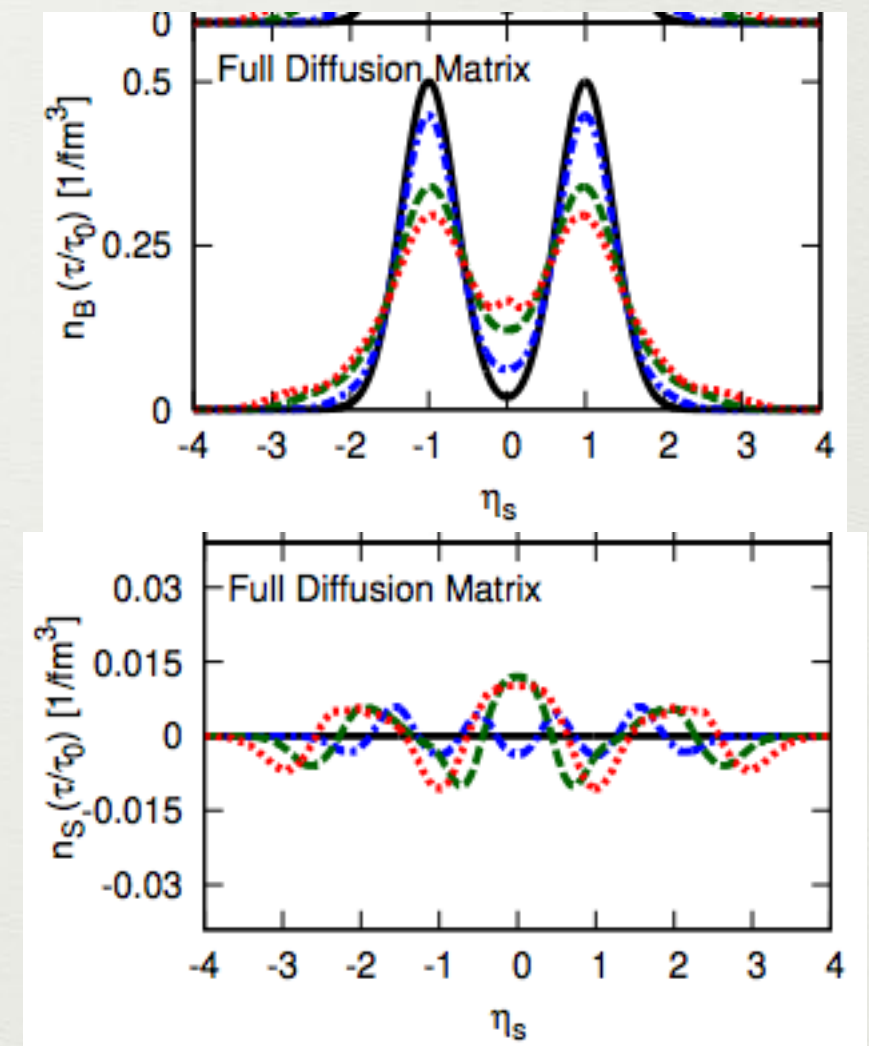


Initial hydro results

3+1 B hydro

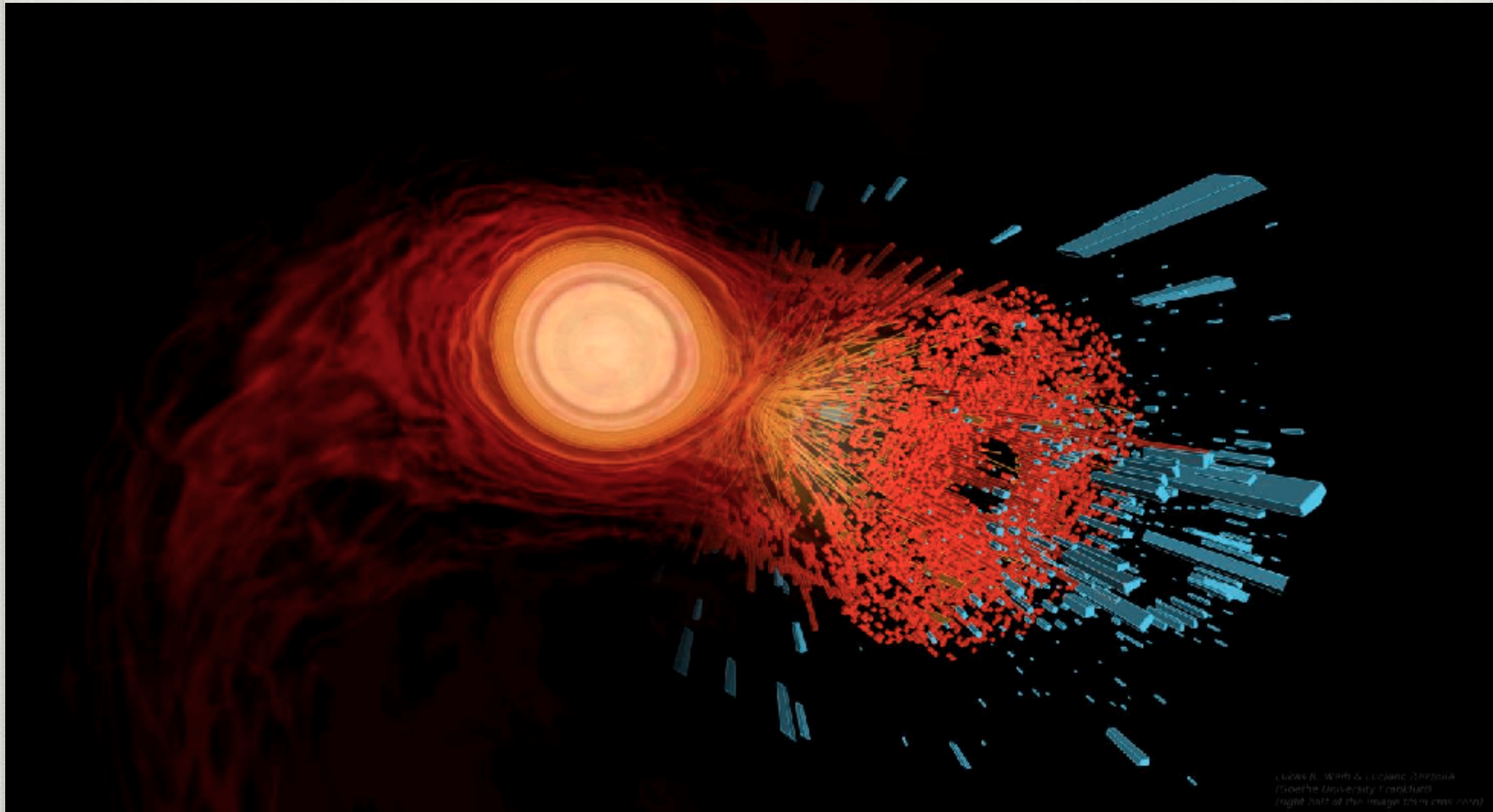


1+1 BS hydro



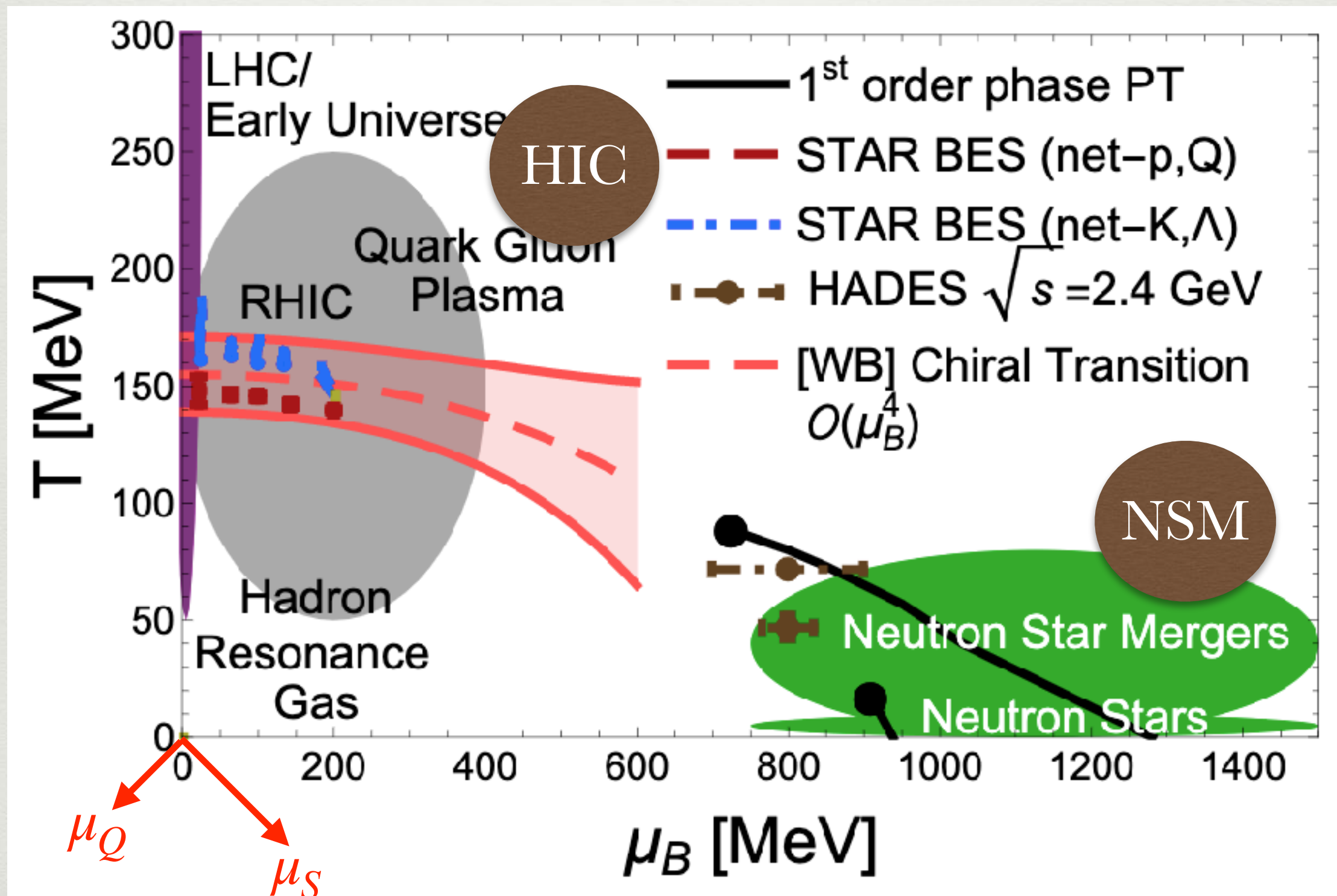
See also, Du, Heinz *Comput.Phys.Commun.* 251 (2020) 107090

Nucleus-Nucleus collisions = mini-neutron star mergers?

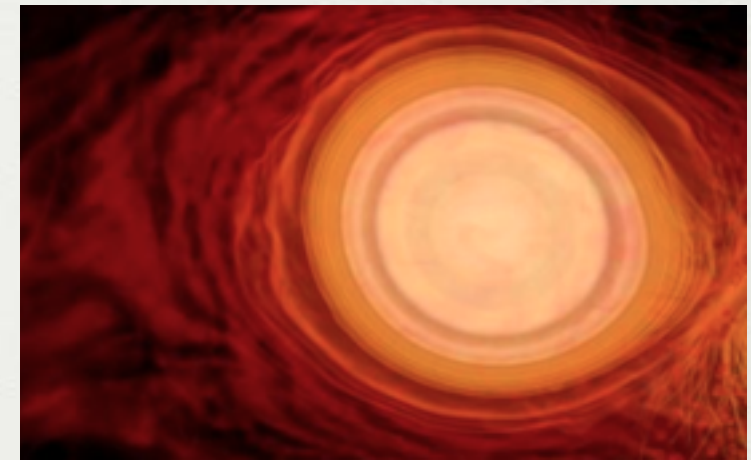
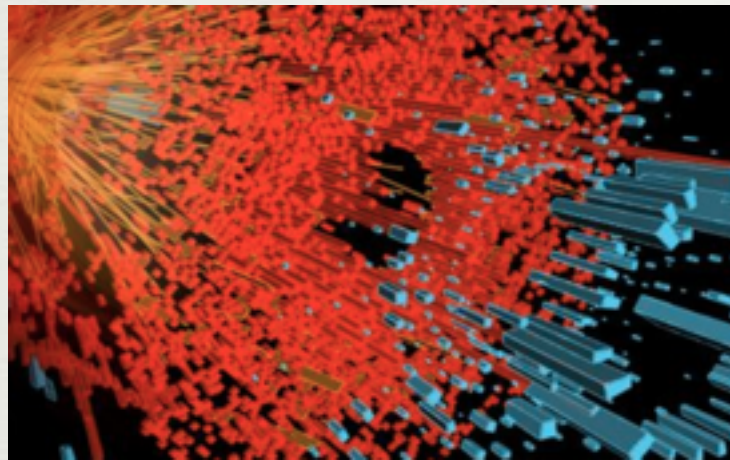


Ulrich H. Wirth & Volker Jurek
Goethe University Frankfurt
(right half of the image from [1])

How do we connection the heavy-ion and neutron star EOS?



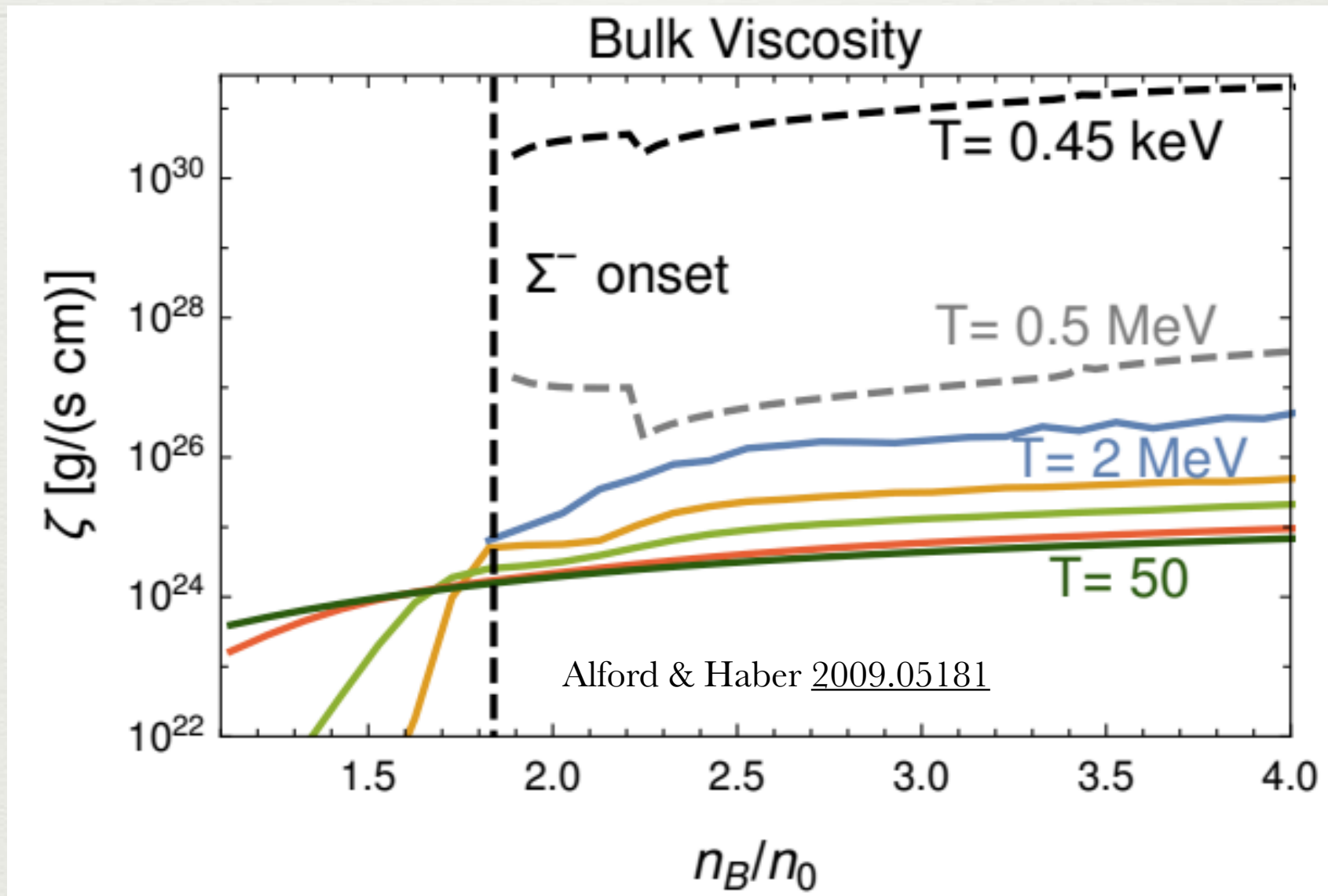
Key differences between heavy-ions and neutron star (mergers)



- Strangeness neutrality
 $\langle S \rangle = N_S - N_{\bar{S}} = 0$
- Short lifetime,
strangeness conserved
- Charge: p vs. n in ions
 $0.4\langle Q \rangle \sim \langle B \rangle$

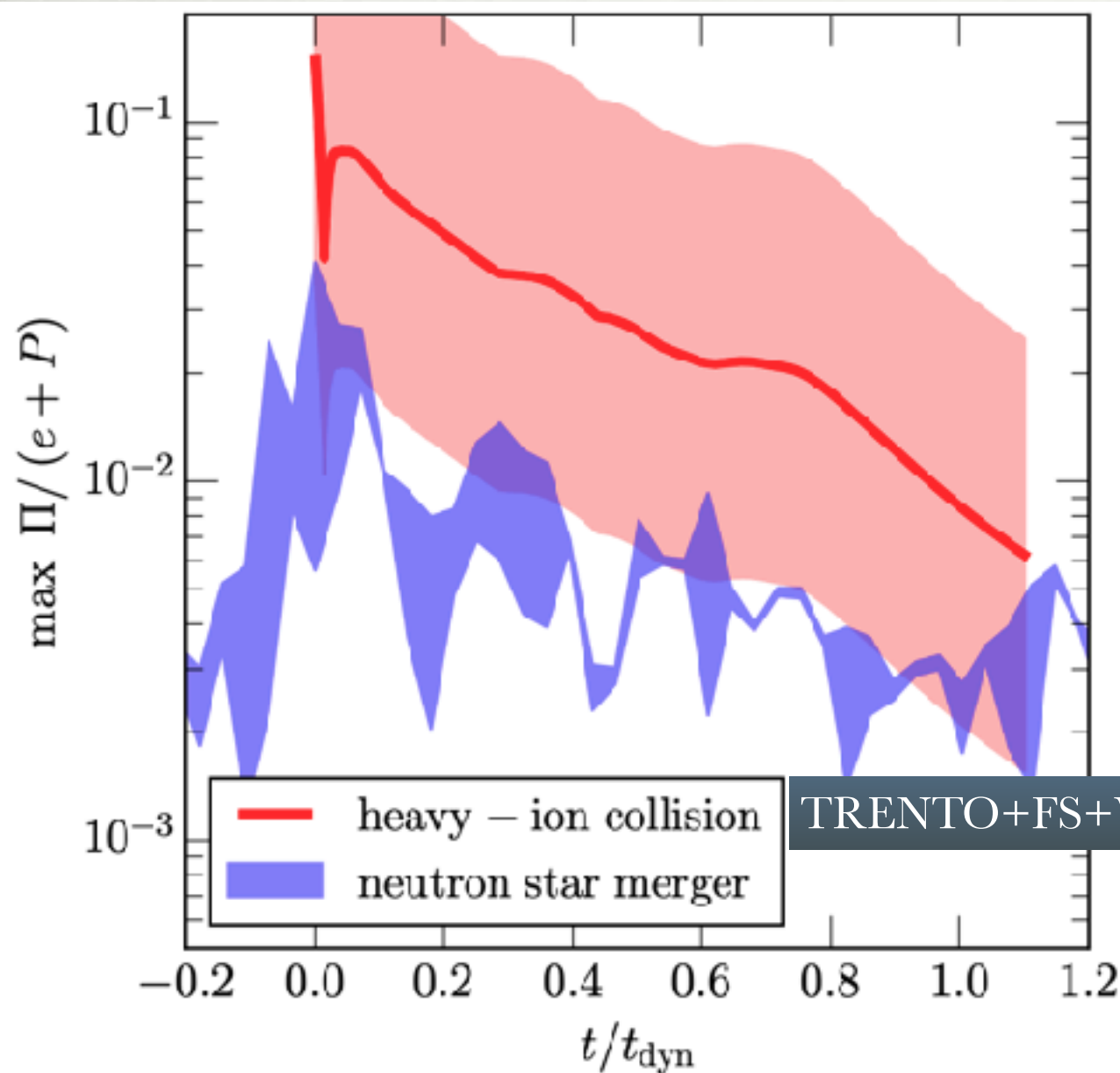
- Long lifetime, weak
decay: $s \rightarrow u + W^-$
- Strangeness most likely
not in equilibrium
- Electrically neutral for
stability $\langle Q \rangle = 0$

Bulk viscosity in neutron star mergers



New approach to understand the microscopic degrees of freedom in neutron stars

Bulk viscosity in neutron star mergers, comparable to heavy-ions!



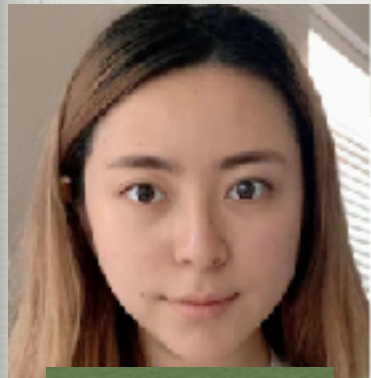
- Collaborations with relativistic viscous hydrodynamics needed
- +GR, what new observables?
- Effects on lifetime? QCD phase transition?

Alford, Harris, Most, Noronha, JNH, Pretorius, Plumberg, Witek, Yunes 2107.05094 [astro-ph.HE]

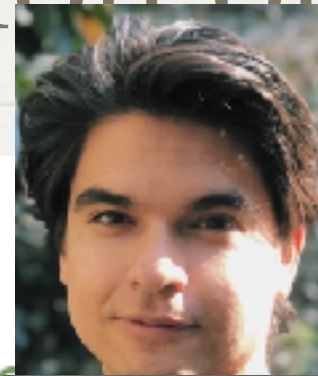
Viscosity in heavy-ions to neutron stars

- Very different sources.
 - Heavy-ions: directly from QCD
 - Neutron star mergers: neutrino oscillations, turbulence
- Differences in codes.
 - Heavy-ions: focus on fast codes, short timescales, negative densities
 - Neutron star mergers: precision, adaptive hydrodynamics

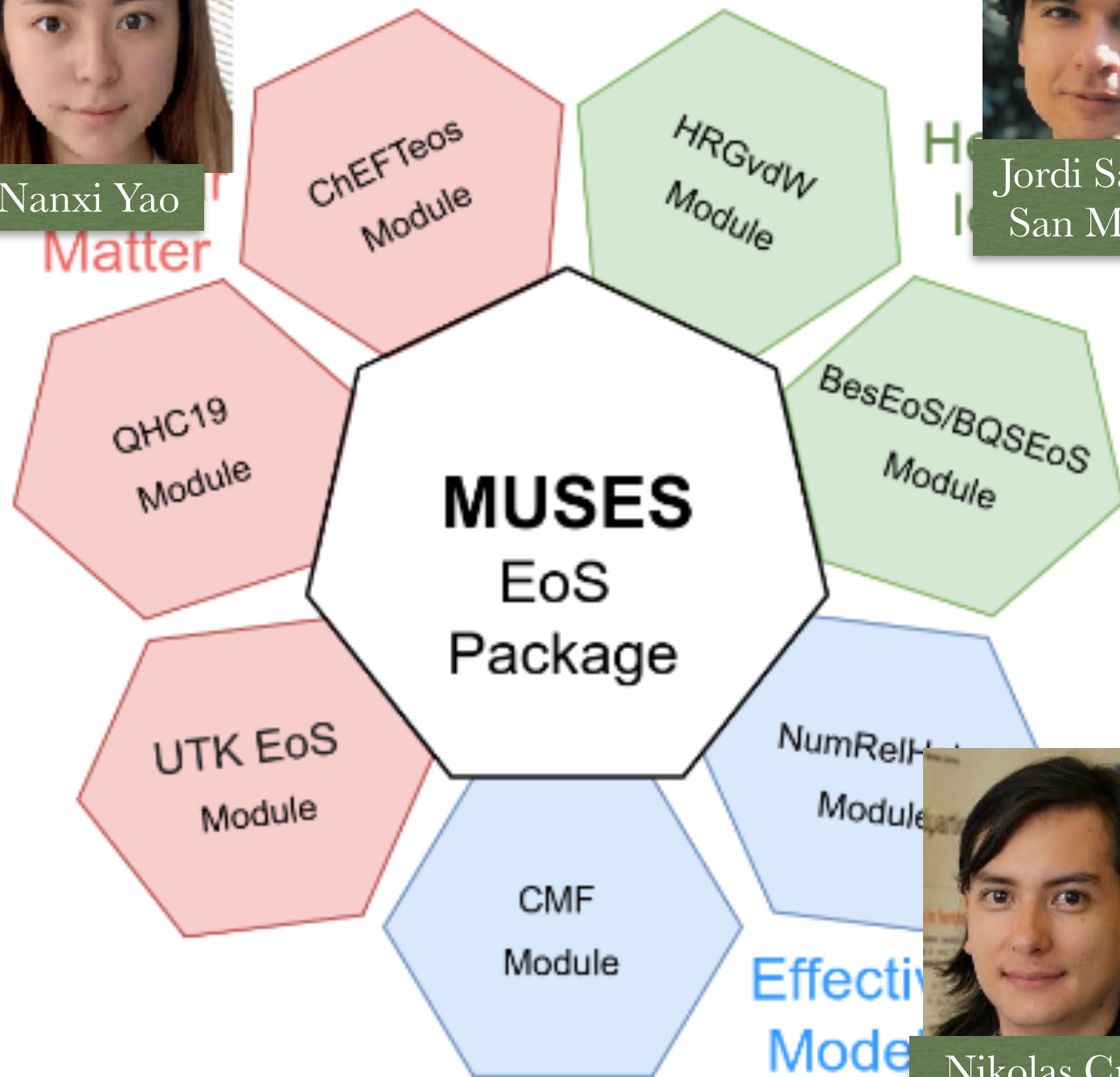
MUSES: Modular Unified Solver of the Equation of State



Nanxi Yao
Matter



Jordi Salinas
San Martín



Nikolas Cruz
Camacho



Recommended for
funding by the NSF

16+ institutions, combines
nuclear/computer science/
gravity/astro/particle

Conclusions and Outlook

- Critical region along T axis creates a trajectory attractor
- BSQ charges likely will affect final flow harmonics
- Studies of BSQ viscous hydrodynamics underway
- Unanswered questions about transport coefficients at finite μ_B
- Overlap in heavy-ion collisions and neutron star mergers, but need viscosity in future simulations