

$$\overline{|\mathcal{A}_{\eta' \rightarrow \pi^+ \pi^- l^+ l^-}|^2}(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_l, \phi) = \frac{e^2}{8k^2} |M(s_{\pi\pi}, s_{ll})|^2 \times \lambda(m_{\eta'}^2, s_{\pi\pi}, s_{ll}) \times [1 - \beta_1^2 \sin^2 \theta_l \sin^2 \phi] s_{\pi\pi} \beta_\pi^2 \sin^2 \theta_\pi$$

$$d\Gamma = \frac{|\mathcal{A}|^2 \beta_k \beta_p \lambda^{1/2}(m_P^2, s_{p+p-}, s_{k+k-})}{2^{15} \pi^6 m_P^3} \times ds_{p+p-} ds_{k+k-} \boxed{d \cos \theta_p d \cos \theta_k d\phi.}$$

积分

$$d\Gamma = \frac{e^2}{s_{ll}} |M|^2 \frac{1}{m_P^3 3^2 2^{13} \pi^5} s_{\pi\pi} \beta_\pi^3 \lambda^{\frac{3}{2}}(m_P^2, s_{\pi\pi}, s_{ll}) \beta_l \frac{3 - \beta_l^2}{2} ds_{\pi\pi} ds_{ll} = \text{part1} * VMD^2 ds_{\pi\pi} ds_{ll}$$

$$\square \text{ part1} = \frac{e^2}{s_{ll}} |\mathcal{M}|^2 \frac{1}{m_P^3 3^2 2^{13} \pi^5} s_{\pi\pi} \beta_\pi^3 \lambda^{\frac{3}{2}}(m_P^2, s_{\pi\pi}, s_{ll}) \beta_l \frac{3 - \beta_l^2}{2} = \frac{4\pi\alpha}{m_P^3 3^2 2^{14} \pi^5} |\mathcal{M}|^2 \frac{s_{\pi\pi}}{s_{ll}} \beta_\pi^3 \beta_l (3 - \beta_l^2) \lambda^{\frac{3}{2}}(m_P^2, s_{\pi\pi}, s_{ll})$$

- $e^2 = 4\pi\alpha$

- $\mathcal{M} = \frac{e}{8\pi^2 f_\pi^3 \sqrt{3}} \left(\frac{f_\pi}{f_8} \sin \theta_{mix} + 2\sqrt{2} \frac{f_\pi}{f_0} \cos \theta_{mix} \right)$

$$|\mathcal{M}|^2 = \frac{e^2}{3 \cdot 64 \pi^4 f_\pi^6} \left(\frac{f_\pi}{f_8} \sin \theta_{mix} + 2\sqrt{2} \frac{f_\pi}{f_0} \cos \theta_{mix} \right)^2$$

- $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$

$$\lambda^{\frac{3}{2}}(m_P^2, s_{\pi\pi}, s_{ll}) = \left(m_P^4 + s_{\pi\pi}^2 + s_{ll}^2 - 2(m_P^2 s_{\pi\pi} + s_{\pi\pi} s_{ll} + m_P^2 s_{ll}) \right)^{\frac{3}{2}}$$

- $\beta_1 = \sqrt{1 - \frac{4m_l^2}{s_{ll}}}, \beta_\pi = \sqrt{1 - \frac{4m_\pi^2}{s_{\pi\pi}}}$

$$d\Gamma = \text{part1} * VMD^2 ds_{\pi\pi} ds_{ll}$$

$$\square VMD(s_{\pi\pi}, s_{ll}) = -\frac{1}{2} + \frac{3}{2} \frac{m_V^2}{m_V^2 - s_{ll} - im_V \Gamma(s_{ll})} \frac{m_V^2}{m_V^2 - s_{\pi\pi} - im_V \Gamma(s_{\pi\pi})} (c_1 - c_2 = c_3 = 1)$$

- $m_V = 0.77549$
- $\Gamma s_{ll} = \Gamma s_{\pi\pi} = 0.1491$
- $\Gamma(\pi\pi ee): (4.1100039979885474e-05, 1.430926567635841e-08)$
- $\Gamma(\pi\pi\mu\mu): (4.0312879067525873e-07, 1.4854507720258601e-08)$
- $R = \frac{\Gamma(\pi\pi ee)}{\Gamma(\pi\pi\mu\mu)} = 101.95262886344851$

- $\Gamma(s) = g_{mv} \left(\frac{s}{m_V^2} \right) \left(\frac{1 - \frac{4m_l^2}{s}}{1 - \frac{4m_l^2}{m_V^2}} \right)^{\frac{3}{2}}$
- $\Gamma(\pi\pi ee): (7.606045511774495e-05, 1.4746857737388003e-08)$
- $\Gamma(\pi\pi\mu\mu): (4.7679124618134683e-07, 1.3088984323778775e-08)$
- $R = \frac{\Gamma(\pi\pi ee)}{\Gamma(\pi\pi\mu\mu)} = 159.52569542104277$

Decay mode	\mathcal{B}
$\eta' \rightarrow \pi^+ \pi^- e^+ e^-$	$(2.45 \pm 0.02 \pm 0.09) \times 10^{-3}$
$\eta' \rightarrow \pi^+ \pi^- \mu^+ \mu^-$	$(2.16 \pm 0.12 \pm 0.06) \times 10^{-5}$

$$R = \frac{\mathcal{B}(\eta' \rightarrow \pi^+ \pi^- e^+ e^-)}{\mathcal{B}(\eta' \rightarrow \pi^+ \pi^- \mu^+ \mu^-)} = 113.43 \pm 6.37$$