

# BESIII Analysis Memo

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## Study of $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$

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## Abstract

Based on  $448.1 \times 10^6$   $\psi(2S)$  events collected from the BESIII detector, the decays  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$  ( $J = 0, 1, 2$ ) are studied and updated via the radiative transition  $\psi(2S) \rightarrow \gamma\chi_{cJ}$ . The decay branching fractions of  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$ ,  $\chi_{cJ} \rightarrow p\bar{p}\phi$ ,  $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520) + c.c.$  and  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  decay modes are improved, among them, the upper limit results of  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  are set to be  $B(\chi_{c1} \rightarrow p\bar{p}\phi) < 1.44 \times 10^{-5}$  and  $B(\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)) < 0.71 \times 10^{-4}$  at 90% confidence level, respectively. In addition, the evidence of the decay  $\chi_{c1} \rightarrow p\bar{p}\phi$  is observed for the first time and the corresponding branching fraction is measured to be  $(1.11 \pm 0.30 \pm 0.27) \times 10^{-5}$ . The uncertainties are statistical and systematic, respectively.

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# 1 Introduction

As we all know, the heavy-quark mass provides a natural boundary between the perturbative and non-perturbative regimes. Quarkonium systems are deemed as a distinctive laboratory to study the interplay between perturbative and non-perturbative effects in quantum chromodynamics(QCD). Charmonium states are mesons formed by a bound state of a charm quark and a charm anti-quark. In 1974, the first excited state of charmonium  $J/\psi$  boson was discovered in experiments [1–3]. Thereafter, more and more charmonium states have been discovered, etc.  $\psi(3686)$ (the so-called  $\psi(2S)$ ),  $\psi(3770)$  and so on. In the quark model, the  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) mesons are the  $3P_J$  charmonium states. Their decays are experimentally and theoretically not as well studied as the vector charmonium states  $J/\psi$  and  $\psi(2S)$ . Unlike the  $J/\psi$  and  $\psi(2S)$ , the P-wave charmonia states  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) are not directly produced in  $e^+e^-$  collisions, thus people don't have a deep understanding of this until today. Very clean environment for the study of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) states is important for a better understanding of their nature and decay mechanisms, as well as for testing QCD-based calculations because it can be generated in radiative decays  $\psi(2S) \rightarrow \gamma\chi_{cJ}$ . Recent theory indicates that the color octet mechanism [4] could have large contributions to the decays of the P-wave charmonium states. However, these calculations, as well as experimental measurements still have large errors, for example, branching fraction of  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  are the results of upper limit from particle data group (PDG) [5] till now. Therefore, more precise experimental data besides more theoretical efforts are crucial to deepen the dynamic mechanism of  $\chi_{cJ}$  decay. Getting more experimental data seems particularly important.

In this paper, by using the world largest  $\psi(2S)$  sample taken at the center-of-mass energy of  $\sqrt{s} = 3.686$  GeV with the Beijing Spectrometer III (BESIII) detector, we present the first measurement of the branching fraction of  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ . Besides, the branching fraction precision of other decay modes of  $\chi_{cJ}$  are improved.

## 2 BESIII detector, data and Monte Carlo

### 2.1 BESIII detector

The BESIII detector [6] at the double-ring Beijing Electron-Positron Collider (BEPCII) is a major upgrade of the BESII experiment at the single-ring Beijing Electron-Positron Collider (BEPC) [7] for studying physics in the  $\tau$ -charm energy region [8]. The designed peak luminosity of BEPCII is  $1.0 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$  at a beam current of 0.93 A which is optimized at the energy of  $\psi(3770)$  peak. The BESIII detector has a geometrical acceptance of 93% of  $4\pi$  solid angle and consists of the following main components.

- A small-celled main drift chamber (MDC) with 43 layers providing an average single-hit resolution of  $135 \mu\text{m}$  is used to track charged particles, and the momentum resolution of the charged particle in a 1.0 T magnetic field is 0.5% at 1 GeV/c.
- An electromagnetic calorimeter (EMC) made of 6240 CsI (Tl) crystals arranged in a cylindrical shape (barrel) plus two end caps is used to measure photon energy and position information. For 1.0 GeV photons, the energy resolution is 2.5% in the barrel and 5% in the end-caps, and the position resolution is 6 mm for the barrel and 9 mm for the end caps.
- A time-of-flight system (TOF) constructed of 5 cm thick plastic scintillators, with 176 detectors of 2.4 m length in two layers in the barrel and 96 fan-shaped detectors in the endcaps is used for particle identification (PID). The barrel (end-caps) time resolution of 80 (110) ps provided  $2\sigma K/\pi$  separation for momenta up to  $\sim 1.0 \text{ GeV}/c$ .
- A muon chamber system for muon detection made of resistive plate chambers arranged in 9 layers in the barrel and 8 layers in the end-caps provides 2 cm position resolution.

### 2.2 Data sets

The result reported in this analysis is based on the data sample taken at the  $\psi(2S)$  with the BESIII detector during 2009 and 2012. The total number of  $\psi(2S)$  events is  $(448.1 \pm 2.9) \times 10^6$ , which have been determined with inclusive hadronic events as described in Ref. [9].

### 2.3 Monte Carlo Simulation

#### 2.3.1 Simulation Tools and Simulation Environment

The optimization of event selection and estimation of physics backgrounds are performed through Monte Carlo (MC) simulations of background and signal. The GEANT4-based [10] simulation software BOOST [11]

1 includes the geometric and material description of the BESIII detectors, the detector response and dig-  
2 itization models, as well as the tracking of the detector running conditions and performance. The pro-  
3 duction of the  $\psi(2S)$  resonance is simulated by the MC event generator KKMC [12], while the decays  
4 are generated by EVTGEN [13] for known decay modes with branching ratios being set to the PDG [5]  
5 world average values, and by LUNDCHARM [14] for the remaining unknown decays.

6 This analysis is performed on the framework of the BESIII Offline Software System (BOSS) [15]  
7 and the version is BOSS 6. 6. 4. p03.

### 8 **2.3.2 Simulation for signal / data Events**

9 Several exclusive MC samples are used to determined detection efficiencies or study main background  
10 sources. For each decay,  $2.24 \times 10^5$  MC events are generated. The  $\psi(2S) \rightarrow \gamma\chi_{cJ}(J = 0, 1, 2)$  decays  
11 are modeled with P2GCJ( $J = 0, 1, 2$ ) models [16]. The decays  $\chi_{cJ} \rightarrow p\bar{p}\phi$ ,  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$ ,  $\chi_{cJ} \rightarrow$   
12  $pK^-\bar{\Lambda}(1520)$ ,  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$ ,  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ ,  $\Lambda(1520) \rightarrow pK^-$  and  $\bar{\Lambda}(1520) \rightarrow \bar{p}K^+$   
13 are generated using the phase space (PHSP) model. The decay  $\phi \rightarrow K^+K^-$  is generated using the VSS  
14 model.

15 About 506 million inclusive MC sample is used to investigate possible backgrounds, of which the  
16 branching fractions of the known physics processes are set to world average values according to PDG [5].

### 3 Event selection

The criteria of event selection for good candidates are discussed in the following aspects.

- For charged tracks, there are several general criteria. Charged tracks detected in the MDC must satisfy  $|\cos\theta| < 0.93$ , where  $\theta$  is the polar angle with respect to positron beam direction.  $V_r < 1.0$  cm and  $V_z < 10$  cm are performed, where  $V_r$  and  $V_z$  are the closest approach distance from the beam line to the reconstructed track in x-y and z direction, respectively. There should be two positive and two negative charged tracks which are selected for further selection. The total net charge is 0.
- A photon is marked as a photon candidate if it satisfies:  $0 \leq T_{TDC} \leq 14$  (with a unit of 50ns), TDC is the time information of EMC;  $E_\gamma \geq 25$  MeV for barrel EMC ( $|\cos\theta| < 0.80$ ),  $E_\gamma \geq 50$  MeV for endcap EMC ( $0.86 < |\cos\theta| < 0.92$ ). The opening angle between a shower and the nearest charged track should be no less than 10, the isolation angle is calculated by the angle between the position of the center crystal of the shower and the extrapolated position on EMC of the charged track.
- Particle identification (PID) is used to identify the charged tracking and the selection criteria are  $prob(p) > prob(K)$ ,  $prob(p) > prob(\pi)$  for proton and  $prob(K) > prob(p)$ ,  $prob(K) > prob(\pi)$  for kaon. Totally, we identify a proton, an anti-proton, two kaons in one event, and keep the total charges are zero.
- A vertex fitting is performed on the four tracks to ensure that they originate from the interaction point.
- The four-constraint energy-momentum conservation kinematic fit (4C fit) is used to the final state  $\gamma p \bar{p} K^+ K^-$ . All the tracks should be fulfill the 4C kinematic fit and the kinematic fit constrained by the initial  $\psi(2S)$  four-momentum is performed under the final state hypothesis  $\gamma K^+ K^- p \bar{p}$ . The  $\chi_{4c}^2$  is required to be less than 200. If there're more than one good photon, the combination with least  $\chi^2$  is chosen.

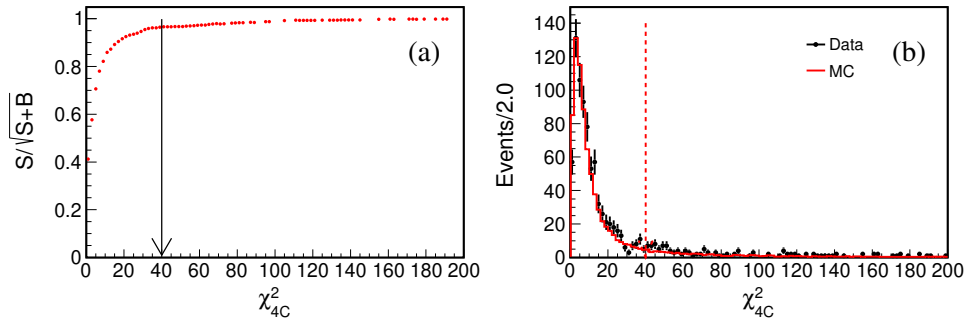


Figure 1: (a) The optimization of  $\chi_{4c}^2$  for  $\chi_{c0}$ . (b) The distribution of  $\chi_{4c}^2$ , where dots with error bars are data and the red histograms represent the signal MC for  $\chi_{c0}$ .

## 4 Analysis of $\chi_{cJ}$ decaying into $p\bar{p}K^+K^-$ final states

### 4.1 Analysis of $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$

#### 4.1.1 Final event selection

In this section, our description mainly focuses on  $\chi_{c0}$ , because procedures of analysis for three states  $\chi_{cJ}$  are similar.

In order to choose the proper  $\chi_{4c}^2$  requirement, we use the figure of merit (FOM) [17] to optimize this requirement. The  $\chi_{4c}^2$  cut is determined by optimizing the value of  $S/\sqrt{S+B}$  (FOM) and the optimized curve of  $\chi_{4c}^2$  is also shown in Fig. 1, where  $S$  refers to MC signal sample of  $\chi_{c0} \rightarrow p\bar{p}K^+K^-$  decay mode and  $S+B$  is that number of events candidate of data within  $3.365 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.455 \text{ GeV}/c^2$  for  $\chi_{c0}$ . The  $\chi_{4c}^2$  is required to be less than 40 as indicated in that figure.

To reject possible backgrounds with two photons in the final states, events with the  $\chi_{4c}^2$  value for the signal hypothesis should be less than those for the background hypotheses  $\chi_{\gamma\gamma K^+K^- p\bar{p}}^2$ .

The distributions of invariant mass of  $pK^-$ ,  $\bar{p}K^+$  and  $K^+K^-$  within  $\chi_{c0}$  mass region are shown in Fig. 2, where clusters of events corresponding to possible backgrounds with  $\Lambda(1520)$ ,  $\bar{\Lambda}(1520)$  and  $\phi$  production are evident. To suppress backgrounds containing  $\Lambda(1520)$ ,  $\bar{\Lambda}(1520)$  or  $\phi$ , the requirements  $|M(pK^-) - m(\Lambda(1520))| > 0.03 \text{ GeV}/c^2$ ,  $|M(\bar{p}K^+) - m(\bar{\Lambda}(1520))| > 0.03 \text{ GeV}/c^2$  and  $|M(K^+K^-) - m(\phi)| > 0.01 \text{ GeV}/c^2$  are applied, where  $m(\Lambda(1520))$ ,  $m(\bar{\Lambda}(1520))$  and  $m(\phi)$  are the nominal masses of  $\Lambda(1520)$ ,  $\bar{\Lambda}(1520)$  and  $\phi$ , respectively.

All the final selection criteria are listed here:

- $\chi_{4c}^2 < 40$
- $\chi_{4c}^2 < \chi_{\gamma\gamma K^+K^- p\bar{p}}^2$
- $|M(pK^-) - m(\Lambda(1520))| > 0.03 \text{ GeV}/c^2$



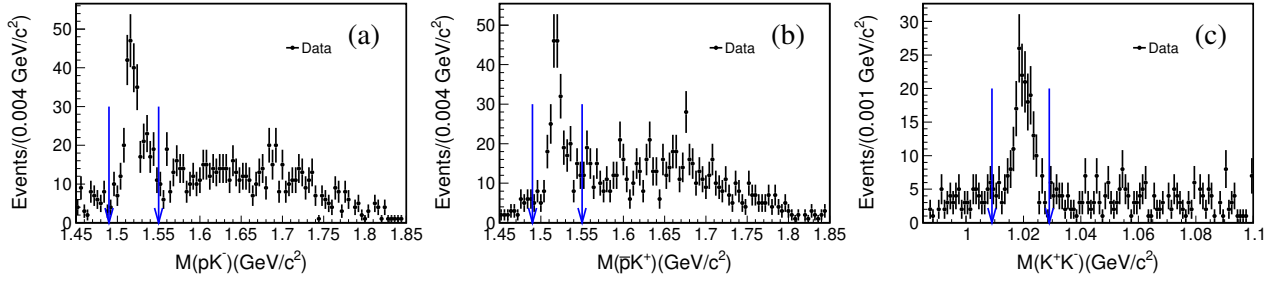


Figure 2: (a) Veto of  $\Lambda(1520)$ ; (b) Veto of  $\bar{\Lambda}(1520)$ ; (C) Veto of  $\phi$ . The dots with error bars are data.

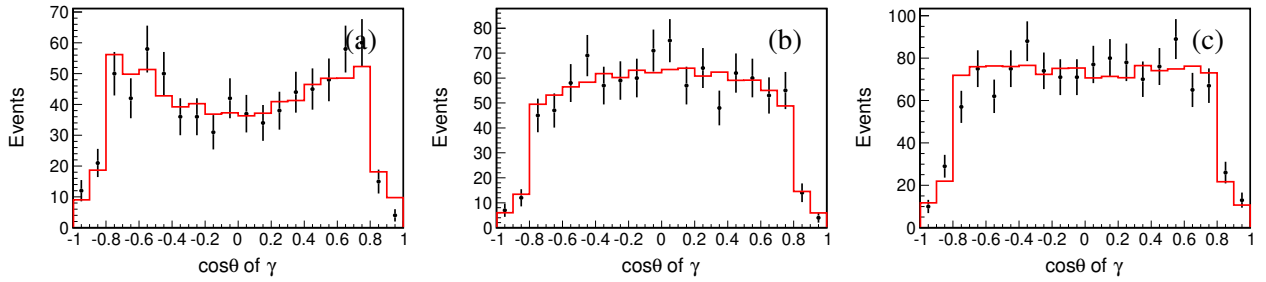


Figure 3: The  $\cos\theta$  distribution of  $\gamma$  in center-of-mass frame. (a)  $\chi_{c0}$  (b)  $\chi_{c1}$  (c)  $\chi_{c2}$ . The dots with error bars are data., the red histograms represent the signal MC of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

- 1 •  $|M(p\bar{p}K^+) - m(\bar{\Lambda}(1520))| > 0.03 \text{ GeV}/c^2$
- 2 •  $|M(K^+K^-) - m(\phi)| > 0.01 \text{ GeV}/c^2$

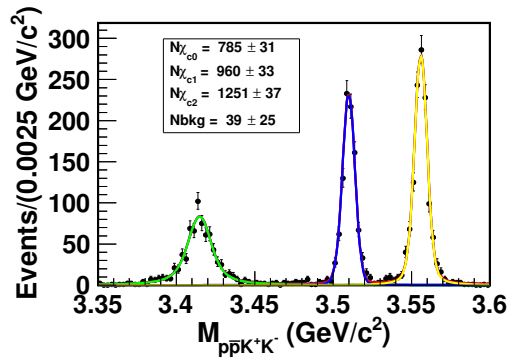
3 Besides, the  $\cos\theta$  distribution of  $\gamma$  within  $3.365 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.455 \text{ GeV}/c^2$  for  $\chi_{c0}$ ,  
 4  $3.490 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.530 \text{ GeV}/c^2$  for  $\chi_{c1}$ , and  $3.530 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.580 \text{ GeV}/c^2$   
 5 for  $\chi_{c2}$  are showed in Fig. 3, which indicate that the generated MC events could provide a good descrip-  
 6 tion of data.

#### 7 4.1.2 Background study and fitting result

8 After applying the previously mentioned selection criteria, the invariant mass distribution of  $p\bar{p}K^+K^-$   
 9 is displayed in Fig. 4 and the topology analysis of inclusive MC in  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$  process is listed  
 10 in Table 1. From the picture and table, we can see clear  $\chi_{c0}$ ,  $\chi_{c1}$  and  $\chi_{c2}$  signals events with low back-  
 11 grounds. The number of signal events can be obtained by fitting the invariant mass spectrum of  $p\bar{p}K^+K^-$ .  
 12 The observed invariant mass spectrum of  $p\bar{p}K^+K^-$  is fitted by a function of Breit-Wigner convolved with  
 13 Gaussian function for signal shape and a second order Chebychev polynomial function for background  
 14 shape, the natural width of Breit-Wigner function is fixed on the PDG [5] and other parameters are free.

Table 1: The topology of inclusive MC in  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$  process.

| rowNo | decay tree<br>(decay initial-final states)  | iDcyTr | iDcyIFSts | nEtr | nCEtr |
|-------|---|--------|-----------|------|-------|
| 1     | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow K^+K^-p\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )   | 1      | 0         | 1293 | 1293  |
| 2     | $\psi' \rightarrow \chi_{c1}\gamma, \chi_{c1} \rightarrow K^+K^-p\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )   | 2      | 0         | 933  | 2226  |
| 3     | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow K^+K^-p\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )   | 0      | 0         | 619  | 2845  |
| 4     | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow K^-p\bar{\Lambda}(1520)^0, \bar{\Lambda}(1520)^0 \rightarrow K^+\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 5      | 0         | 36   | 2881  |
| 5     | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow K^+\bar{p}\Lambda(1520)^0, \Lambda(1520)^0 \rightarrow K^-p$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 4      | 0         | 35   | 2916  |
| 6     | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow K^+\bar{p}\Lambda(1520)^0, \Lambda(1520)^0 \rightarrow K^-p$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 8      | 0         | 34   | 2950  |
| 7     | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow K^-p\bar{\Lambda}(1520)^0, \bar{\Lambda}(1520)^0 \rightarrow K^+\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 3      | 0         | 33   | 2983  |
| 8     | $\psi' \rightarrow \chi_{c1}\gamma, \chi_{c1} \rightarrow K^-p\bar{\Lambda}(1520)^0, \bar{\Lambda}(1520)^0 \rightarrow K^+\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 10     | 0         | 26   | 3009  |
| 9     | $\psi' \rightarrow \chi_{c1}\gamma, \chi_{c1} \rightarrow K^+\bar{p}\Lambda(1520)^0, \Lambda(1520)^0 \rightarrow K^-p$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 7      | 0         | 25   | 3034  |
| 10    | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow \phi p\bar{p}, \phi \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )   | 13     | 0         | 18   | 3052  |
| 11    | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow \phi p\bar{p}, \phi \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )   | 9      | 0         | 13   | 3065  |
| 12    | $\psi' \rightarrow \chi_{c1}\gamma, \chi_{c1} \rightarrow f_2(1270)p\bar{p}, f_2(1270) \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 15     | 0         | 13   | 3078  |
| 13    | $\psi' \rightarrow \chi_{c1}\gamma, \chi_{c1} \rightarrow a_2^0p\bar{p}, a_2^0 \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 11     | 0         | 10   | 3088  |
| 14    | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow f_2(1270)p\bar{p}, f_2(1270) \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 19     | 0         | 10   | 3098  |
| 15    | $\psi' \rightarrow \chi_{c2}\gamma, \chi_{c2} \rightarrow a_2^0p\bar{p}, a_2^0 \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 6      | 0         | 9    | 3107  |
| 16    | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow a_2^0p\bar{p}, a_2^0 \rightarrow K^+K^-$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ )  | 17     | 0         | 6    | 3113  |
| 17    | $\psi' \rightarrow \chi_{c0}\gamma, \chi_{c0} \rightarrow \Lambda(1520)^0\bar{\Lambda}(1520)^0, \Lambda(1520)^0 \rightarrow K^-p, \bar{\Lambda}(1520)^0 \rightarrow K^+\bar{p}$<br>( $\psi' \rightarrow K^+K^-p\bar{p}\gamma$ ) | 12     | 0         | 3    | 3116  |

Figure 4: The fitting result of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) on the invariant mass spectrum of  $M(p\bar{p}K^+K^-)$ . The dots with error bars are data, the green line represents the signal shape of  $\chi_{c0}$ , the blue line donates the signal shape of  $\chi_{c1}$ , and the yellow line stands for the signal shape of  $\chi_{c2}$ .

### 1 4.1.3 Branching fraction measurement

2 The branching fraction of the  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$  decay is determined to be

$$B = \frac{N_{obs}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times \epsilon}, \quad (1)$$

3 where  $N_{obs}$  is the number of observed signal candidates.  $N_{\psi(2S)}$  is total number of  $\psi(2S)$  events.  $B(\psi(2S) \rightarrow \gamma\chi_{cJ})$   
 4 is branching fractions of  $\psi(2S) \rightarrow \gamma\chi_{cJ}$  quoted from PDG [5].  $\epsilon$  represents detection efficiency of  
 5  $\psi(2S) \rightarrow \gamma\chi_{cJ} \rightarrow \gamma p\bar{p}K^+K^-$ . The results are listed in Table 2.

Table 2: The results of fitting and branching fractions of  $\chi_{cJ}$ .

| Decay mode                             | $N_{sig}$     | $\epsilon$ (%) | $B (\times 10^{-4})$ |
|--|---------------|----------------|----------------------|
| $\chi_{c0} \rightarrow p\bar{p}K^+K^-$ | $785 \pm 31$  | 10.1           | $1.77 \pm 0.07$      |
| $\chi_{c1} \rightarrow p\bar{p}K^+K^-$ | $960 \pm 33$  | 14.2           | $1.55 \pm 0.05$      |
| $\chi_{c2} \rightarrow p\bar{p}K^+K^-$ | $1251 \pm 37$ | 14.8           | $1.98 \pm 0.06$      |

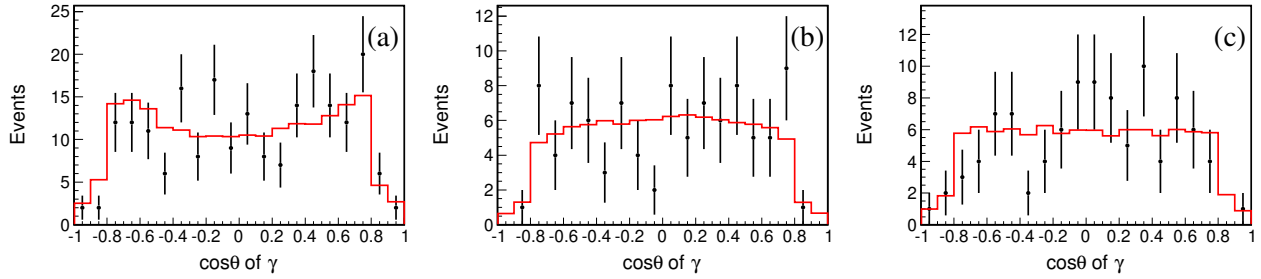


Figure 5: The  $\cos\theta$  distribution of  $\gamma$  in center-of-mass frame. (a)  $\chi_{c0}$  (b)  $\chi_{c1}$  (c)  $\chi_{c2}$ . The dots with error bars are data, the red histograms represent the signal MC of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

## 1 4.2 Analysis of $\chi_{cJ} \rightarrow p\bar{p}\phi$

### 2 4.2.1 Final event selection

3 The intermediate states from the  $\chi_{cJ}$  decaying to  $p\bar{p}K^+K^-$  can be studied by configuring the final parti-  
 4 cles. We investigate the  $pK^-$ ,  $\bar{p}K^+$  configurations as shown in Fig. 2 and veto backgrounds containing  
 5  $M(pK^-)$  and  $M(\bar{p}K^+)$  by applying the same cuts as Sec. 4.1.1. The mass ranges for studying the  $\chi_{cJ}$   
 6 ( $J = 0, 1, 2$ ) are as follows.

$$7 \quad \chi_{c0} : 3.365 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.455 \text{ GeV}/c^2$$

$$8 \quad \chi_{c1} : 3.490 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.530 \text{ GeV}/c^2$$

$$9 \quad \chi_{c2} : 3.530 \text{ GeV}/c^2 < M(p\bar{p}K^+K^-) < 3.580 \text{ GeV}/c^2$$

10 Besides, the  $\cos\theta$  distribution of  $\gamma$  for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) within  $|M(K^+K^-) - m(\phi)| < 0.01 \text{ GeV}/c^2$  are  
 11 showed in Fig. 5.

### 12 4.2.2 Fitting result

13 To extract the signal yields of  $\chi_{cJ} \rightarrow p\bar{p}\phi$  decay, we perform the fits to  $M(K^+K^-)$ , and the fitting results  
 14 are presented in Fig. 6 for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ), respectively. In the fits,  $M(K^+K^-)$  is described by a Breit-  
 15 Wigner function, due to the statistical limitation for  $\chi_{c1}$  and  $\chi_{c2}$  decays the natural width is fixed to PDG  
 16 value [5], while is free for  $\chi_{c0}$  decay process, and the mass in Breit-Wigner is free for all processes. For  
 17 backgrounds the third-order MyArgus function is used due to near the threshold. The MyArgus function  
 18 is defined as

$$19 \quad f(x) = (x^2/m_0^2 - 1.0)^p \cdot \exp(c(x^2/m_0^2 - 1.0)), \quad (2)$$

where  $c$  and  $p$  are free parameters and  $m_0$  is a threshold parameter.

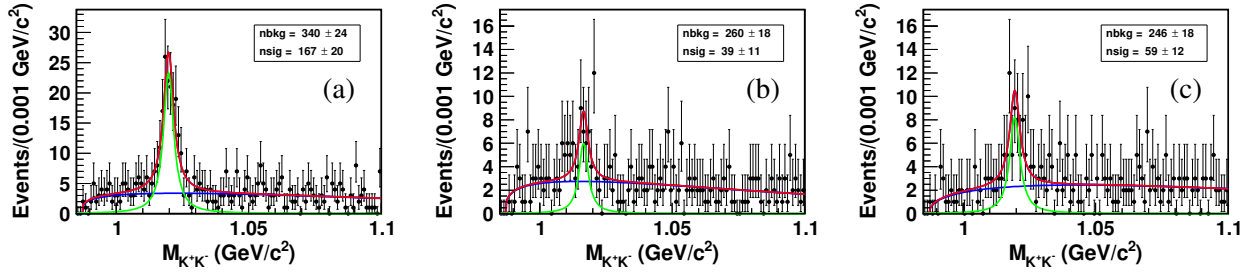


Figure 6: The distributions of  $M(K^+K^-)$  in the decays of (a)  $\chi_{c0}$ , (b)  $\chi_{c1}$  and (c)  $\chi_{c2}$ . The dots with error bars are data, the green lines represent the fitting result of signal shape and the blue lines denote the background shape.

### 1 4.2.3 Branching fraction measurement

2 The branching fraction of the  $\chi_{cJ} \rightarrow p\bar{p}\phi$  decay is determined to be

$$B = \frac{N_{obs}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\phi \rightarrow K^+K^-) \times \epsilon}, \quad (3)$$

3 where,  $N_{obs}$  is the number of observed signal candidates.  $N_{\psi(2S)}$  stands for the total number of  $\psi$  events.

4  $B(\psi(2S) \rightarrow \gamma\chi_{cJ})$  and  $B(\phi \rightarrow K^+K^-)$  are quoted from PDG [5].  $\epsilon$  is the detection efficiency of  $\psi(2S) \rightarrow$

5  $\gamma\chi_{cJ} \rightarrow \gamma p\bar{p}\phi \rightarrow \gamma p\bar{p}K^+K^-$ . The results are listed in Table 3. Since the signal of  $\phi$  in  $\chi_{c1}$  decay

6 is evident with a significance of  $3.6\sigma$ , the upper limit is also considered and the result is described in

7 Sec. 6.

Table 3: The results of fitting and branching fractions of  $\chi_{cJ}$ .

| Decay mode                           | $N_{sig}$    | $\epsilon$ (%) | $B (\times 10^{-5})$ |
|--------------------------------------|--------------|----------------|----------------------|
| $\chi_{c0} \rightarrow p\bar{p}\phi$ | $167 \pm 20$ | 11.5           | $6.72 \pm 0.80$      |
| $\chi_{c1} \rightarrow p\bar{p}\phi$ | $39 \pm 11$  | 16.2           | $1.11 \pm 0.30$      |
| $\chi_{c2} \rightarrow p\bar{p}\phi$ | $59 \pm 12$  | 16.5           | $1.69 \pm 0.35$      |

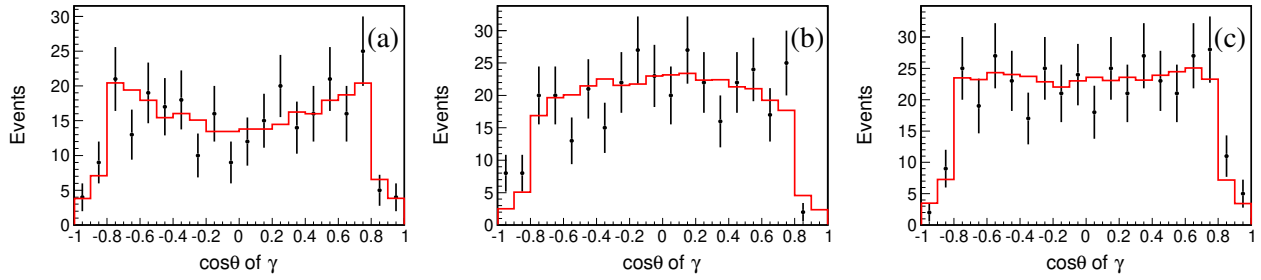


Figure 7: The  $\cos\theta$  distribution of  $\gamma$  in center-of-mass frame. (a)  $\chi_{c0}$  (b)  $\chi_{c1}$  (c)  $\chi_{c2}$ . The dots with error bars are data, the red histograms represent the signal MC of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

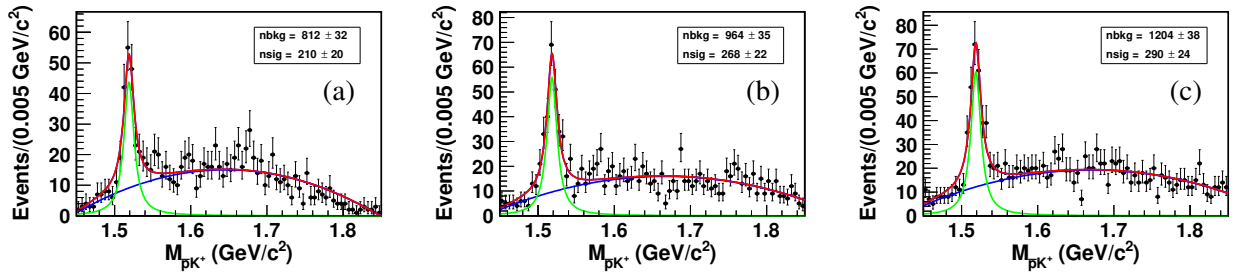


Figure 8: The fit result of  $\bar{\Lambda}(1520)$  on the invariant mass spectrum of  $M(\bar{p}K^+)$  in the decays of (a)  $\chi_{c0}$ , (b)  $\chi_{c1}$  and (c)  $\chi_{c2}$ . The dots with error bars are data, the green lines represent the fitting result of signal shape and the blue lines denote the background shape.

### 1 4.3 Analysis of $\chi_{cJ} \rightarrow pK^- \bar{\Lambda}(1520)$

#### 2 4.3.1 Final event selection

3 In order to suppress the backgrounds containing  $\Lambda(1520)$  and  $\phi$ , the criteria  $|M(pK^-) - m(\Lambda(1520))| >$   
 4  $0.03 \text{ GeV}/c^2$  and  $|M(K^+K^-) - m(\phi)| > 0.01 \text{ GeV}/c^2$  are required. The invariant mass distribution  $M(\bar{p}K^+)$   
 5 is presented in Fig. 8 for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ), respectively.  $\bar{\Lambda}(1520)$  signal events are observed clearly.

6 Besides, the  $\cos\theta$  distribution of  $\gamma$  for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) within  $|M(\bar{p}K^+) - m(\bar{\Lambda}(1520))| < 0.03 \text{ GeV}/c^2$   
 7 are showed in Fig. 7.

#### 8 4.3.2 Fitting result

9 The invariant mass distribution of  $M(\bar{p}K^+)$  is fitted by a Breit-Wigner function, plus a second order  
 10 Chebychev polynomial for the background. This is shown in Fig. 8. The natural width in Breit-Wigner  
 11 is fixed according to PDG [5] and the mass in Breit-Wigner is free. The parameters of the second order  
 12 Chebychev polynomial are also free.

### 1 4.3.3 Branching fraction measurement

2 The branching fraction of the  $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$  decay is determined to be

$$B = \frac{N_{obs}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\bar{\Lambda}(1520) \rightarrow \bar{p}K^+) \times \epsilon}, \quad (4)$$

3 where,  $N_{obs}$  is the number of observed signal candidates.  $N_{\psi(2S)}$  stands for the total number of  $\psi$  events.

4  $B(\psi(2S) \rightarrow \gamma\chi_{cJ})$  and  $B(\bar{\Lambda}(1520) \rightarrow \bar{p}K^+)$  are quoted from PDG [5].  $\epsilon$  is the detection efficiency of

5  $\psi \rightarrow \gamma\chi_{cJ} \rightarrow \gamma pK^-\bar{\Lambda}(1520) \rightarrow \gamma p\bar{p}K^+K^-$ . The results are listed in Table 4.

Table 4: The results of fitting and branching fractions of  $\chi_{cJ}$ .

| Decay mode                                      | $N_{sig}$    | $\epsilon$ (%) | $B$ ( $\times 10^{-4}$ ) |
|---|--------------|----------------|--------------------------|
| $\chi_{c0} \rightarrow pK^-\bar{\Lambda}(1520)$ | $210 \pm 20$ | 12.4           | $1.71 \pm 0.16$          |
| $\chi_{c1} \rightarrow pK^-\bar{\Lambda}(1520)$ | $268 \pm 22$ | 16.8           | $1.63 \pm 0.14$          |
| $\chi_{c2} \rightarrow pK^-\bar{\Lambda}(1520)$ | $290 \pm 24$ | 16.8           | $1.80 \pm 0.15$          |

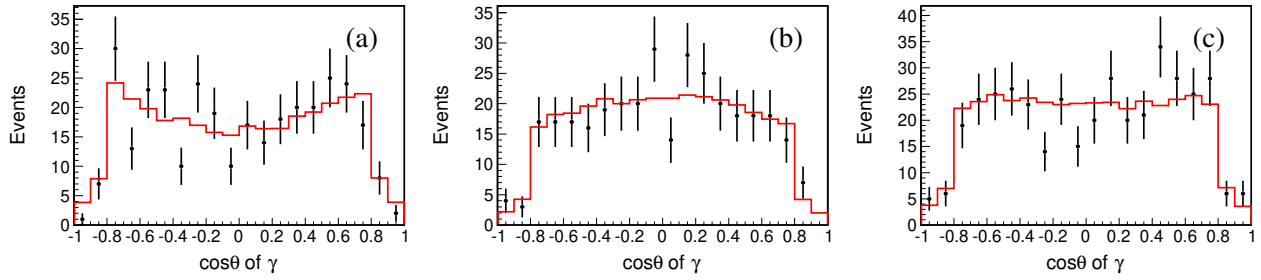


Figure 9: The  $\cos\theta$  distribution of  $\gamma$  in center-of-mass frame. (a)  $\chi_{c0}$  (b)  $\chi_{c1}$  (c)  $\chi_{c2}$ . The dots with error bars are data, the red histograms represent the signal MC of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

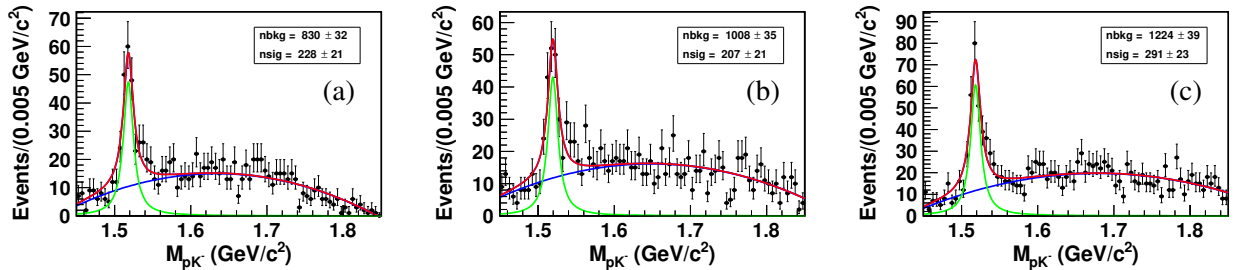


Figure 10: The fitting result of  $\Lambda(1520)$  on the invariant mass spectrum of  $M(pK^-)$  in the decays of (a)  $\chi_{c0}$ , (b)  $\chi_{c1}$  and (c)  $\chi_{c2}$ . The dots with error bars are data, the green lines represent the fitting result of signal shape and the blue lines denote the background shape.

#### 1 4.4 Analysis of $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$

##### 2 4.4.1 Final event selection

3 In order to suppress the backgrounds containing  $\bar{\Lambda}(1520)$  and  $\phi$ , the criteria  $|M(\bar{p}K^+) - m(\bar{\Lambda}(1520))| >$   
 4  $0.03 \text{ GeV}/c^2$  and  $|M(K^+K^-) - m(\phi)| > 0.01 \text{ GeV}/c^2$  are required. The invariant mass distribution  $M(pK^-)$   
 5 is presented in Fig. 10 for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ), respectively.  $\Lambda(1520)$  signal events are observed clearly.

6 Besides, the  $\cos\theta$  distribution of  $\gamma$  for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) within  $|M(pK^-) - m(\Lambda(1520))| < 0.03 \text{ GeV}/c^2$   
 7 are showed in Fig. 9.

##### 8 4.4.2 Fitting result

9 The invariant mass distribution of  $M(pK^-)$  is fitted by a Breit-Wigner function, plus a second order  
 10 Chebychev polynomial for the background. This is shown in Fig. 10. The natural width in Breit-Wigner  
 11 is fixed according to PDG [5] and the mass in Breit-Wigner is free. The parameters of the second order  
 12 Chebychev polynomial are also free.



### 1 4.4.3 Branching fraction measurement

2 The branching fraction of the  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$  decay is determined to be

$$B = \frac{N_{obs}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\Lambda(1520) \rightarrow pK^-) \times \epsilon}, \quad (5)$$

3 where,  $N_{obs}$  is the number of observed signal candidates.  $N_{\psi(2S)}$  stands for the total number of  $\psi$  events.

4  $B(\psi(2S) \rightarrow \gamma\chi_{cJ})$  and  $B(\Lambda(1520) \rightarrow pK^-)$  are quoted from PDG [5].  $\epsilon$  is the detection efficiency of

5  $\psi \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\bar{p}K^+\Lambda(1520) \rightarrow \gamma p\bar{p}K^+K^-$ . The results are listed in Table 5.

Table 5: The results of fitting and branching fractions of  $\chi_{cJ}$ .

| Decay mode                                      | $N_{sig}$    | $\epsilon$ (%) | $B$ ( $\times 10^{-4}$ ) |
|---|--------------|----------------|--------------------------|
| $\chi_{c0} \rightarrow \Lambda(1520)\bar{p}K^+$ | $228 \pm 21$ | 12.6           | $1.83 \pm 0.17$          |
| $\chi_{c1} \rightarrow \Lambda(1520)\bar{p}K^+$ | $207 \pm 21$ | 16.9           | $1.24 \pm 0.13$          |
| $\chi_{c2} \rightarrow \Lambda(1520)\bar{p}K^+$ | $291 \pm 23$ | 16.9           | $1.79 \pm 0.14$          |

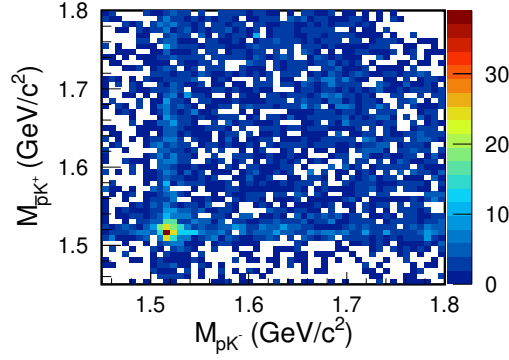


Figure 11: The scatter plot of  $M(pK^-)$  versus  $M(\bar{p}K^+)$  in the decays of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

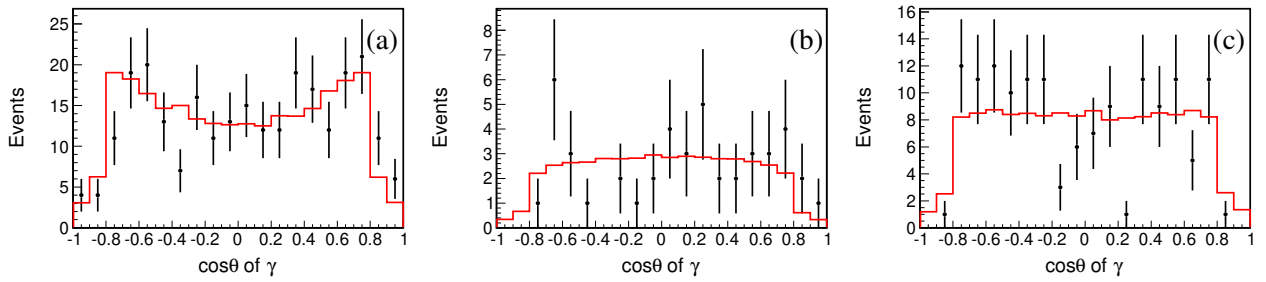


Figure 12: The  $\cos\theta$  distribution of  $\gamma$  in center-of-mass frame. (a)  $\chi_{c0}$  (b)  $\chi_{c1}$  (c)  $\chi_{c2}$ . The dots with error bars are data, the red histograms represent the signal MC of  $\chi_{cJ}$  ( $J = 0, 1, 2$ ).

## 1 4.5 Analysis of $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$

### 2 4.5.1 Final event selection

3 In Fig. 11, clear enhancement around  $1.52 \text{ GeV}/c^2$  indicates the existence of  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ .  
 4 Events are selected as signals that  $\phi$  has been rejected by the criterion  $|M(K^+K^-) - m(\phi)| > 0.01 \text{ GeV}/c^2$   
 5 , and a 2-Dimensional fit is used to estimate the signal and the background.

6 Besides, the  $\cos\theta$  distribution of  $\gamma$  for  $\chi_{cJ}$  ( $J = 0, 1, 2$ ) within  $|M(pK^-) - m(\Lambda(1520))| < 0.03 \text{ GeV}/c^2$   
 7 and  $|M(\bar{p}K^+) - m(\bar{\Lambda}(1520))| < 0.03 \text{ GeV}/c^2$  are showed in Fig. 12, which indicate that the generated MC  
 8 events could provide a good description of data.

### 9 4.5.2 Fitting result

10 An unbinned likelihood fit is performed to obtain the signal and background's yields and the 2-dimension  
 11 fit is applied. In the fit, A composite PDF is constructed as follow:

$$F = N_{sig} \times F_{sig} + N_{bkg} \times F_{bkg} + N_{bkg1} \times F_{bkg1} + N_{bkg2} \times F_{bkg2}, \quad (6)$$

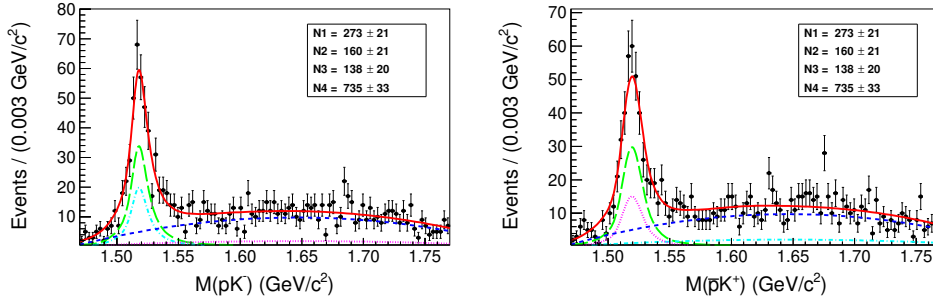


Figure 13: The 2-dimension fitting result for  $\chi_{c0}$ . The dots with error bars are data, the left plot is the projection of  $M(pK^-)$ , the right plot is the projection of  $M(\bar{p}K^+)$ , the green line is used to describe the shape of signal, the blue line is the shape of non-peaking background, the light blue line is the shape of background with  $\Lambda(1520)$ , the pink line is the shape of background with  $\bar{\Lambda}(1520)$ .

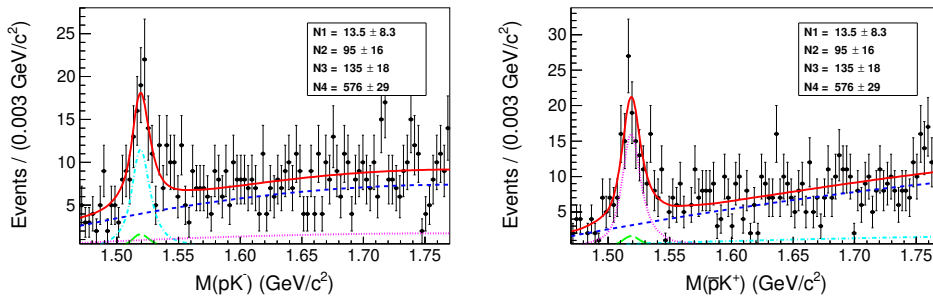


Figure 14: The 2-dimension fitting result for  $\chi_{c1}$ . The dots with error bars are data, the left plot is the projection of  $M(pK^-)$ , the right plot is the projection of  $M(\bar{p}K^+)$ , the green line is used to describe the shape of signal, the blue line is the shape of non-peaking background, the light blue line is the shape of background with  $\Lambda(1520)$ , the pink line is the shape of background with  $\bar{\Lambda}(1520)$ .

- 1 where,  $N_{sig}$  is the observed number of signal events,  $N_{bkg}$  is the observed number of background events,
- 2  $F_{sig}$  and  $F_{bkg}$ ,  $F_{bkg1}$ ,  $F_{bkg2}$  are signal *PDF* and background *PDF*, respectively.

- 3 The 2-dimensional signal and background *PDFs* are constructed by simply multiplying two 1-dimensional
- 4 *PDFs*, respectively,

$$\begin{aligned}
 F_{sig} &= PDF_{sig}(M(pK^-)) \times PDF_{sig}(M(\bar{p}K^+)), \\
 F_{bkg} &= PDF_{bkg}(M(pK^-)) \times PDF_{bkg}(M(\bar{p}K^+)), \\
 F_{bkg1} &= PDF_{sig}(M(pK^-)) \times PDF_{bkg}(M(\bar{p}K^+)), \\
 F_{bkg2} &= PDF_{bkg}(M(pK^-)) \times PDF_{sig}(M(\bar{p}K^+)),
 \end{aligned} \tag{7}$$

- 5 where,  $PDF_{sig}(M(pK^-))$  and  $PDF_{sig}(M(\bar{p}K^+))$  are the distribution of *PDF* shape obtained from
- 6 the MC simulation, respectively.  $PDF_{bkg}(M(pK^-))$  and  $PDF_{bkg}(M(\bar{p}K^+))$  are the *PDF* of background
- 7 represented by the second-order Chebychev Polynominal function with the free parameters. The fitting
- 8 results are shown in Fig. 13, Fig. 14 and Fig. 15, respectively.

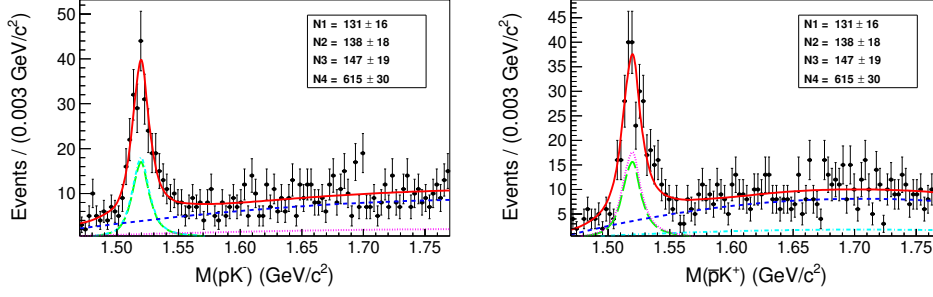


Figure 15: The 2-dimension fitting result for  $\chi_{c2}$ . The dots with error bars are data, the left plot is the projection of  $M(pK^-)$ , the right plot is the projection of  $M(\bar{p}K^+)$ , the green line is used to describe the shape of signal, the blue line is the shape of non-peaking background, the light blue line is the shape of background with  $\Lambda(1520)$ , the pink line is the shape of background with  $\bar{\Lambda}(1520)$ .

### 1 4.5.3 Branching fraction measurement

2 The branching fraction of the  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  decay is determined to be

$$B = \frac{N_{obs}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\Lambda(1520) \rightarrow pK^-)^2 \times \epsilon} \quad (8)$$

3 where,  $N_{obs}$  is the number of observed signal candidates.  $N_{\psi(2S)}$  stands for the total number of  $\psi(2S)$   
4 events.  $B(\psi(2S) \rightarrow \gamma\chi_{cJ})$ ,  $B(\bar{\Lambda}(1520) \rightarrow \bar{p}K^+)$  and  $B(\Lambda(1520) \rightarrow pK^-)$  are quoted from PDG [5].  $\epsilon$   
5 is the detection efficiency of  $\psi \rightarrow \gamma\chi_{cJ} \rightarrow \gamma\Lambda(1520)\bar{\Lambda}(1520) \rightarrow \gamma p\bar{p}K^+K^-$ . The results are listed in  
6 Table 6. Since the significance can be calculated to be  $1.8\sigma$  for the  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  decay, the  
7 upper limit is considered and the result is described in Sec. 6.

Table 6: The results of fitting and branching fractions of  $\chi_{cJ}$ .

| Decay mode   | $N_{sig}$    | $\epsilon$ (%) | $B$ ( $\times 10^{-4}$ ) |
|--|--------------|----------------|--------------------------|
| $\chi_{c0} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | $273 \pm 21$ | 16.8           | $7.33 \pm 0.56$          |
| $\chi_{c2} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | $131 \pm 16$ | 22.7           | $2.68 \pm 0.33$          |

## 5 Systematic Uncertainty

### 5.1 MDC track and PID

The MDC tracking efficiencies of charged protons and kaons are investigated using nearly background-free(clean) control samples of  $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$  and  $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$  [18, 19], respectively. The difference in tracking efficiencies between MC and data is 1% for each charged proton and kaon. In our work, there are four charged tracks of kaon and proton, so the systematic uncertainties from tracking is 4%. We have use the particle identification to identify the kaon and proton, the uncertainty of PID is 1% per track, so the uncertainty from this item is 4%.

### 5.2 Photon selection

The photon detection efficiency has been studied using a control sample of  $J/\psi \rightarrow \rho^0 \pi^0$  [20]. The results indicate that the difference between the detection efficiencies of data and MC is around 1% per photon. Thus, 1% is taken as the total systematic uncertainty for the detection of one photon in this analysis.

### 5.3 The number of $\psi(2S)$

The systematic uncertainty due to the number of  $\psi(2S)$  events is determined to be 0.6% according to Ref. [21].

### 5.4 Branching fraction of intermediate state

The systematic uncertainty due to branching fraction of intermediate states are from PDG [5], there are 2.0% for  $\psi(2S) \rightarrow \gamma\chi_{cJ}$  decay, 1.0% for  $\phi \rightarrow K^+ K^-$  decay and 2.0% for  $\Lambda(1520) \rightarrow pK^-$ .

### 5.5 The 4C kinematic fit

The systematic uncertainty due to 4C kinematic fit is estimated by correcting helix parameters of charged tracks described in Ref. [22], the efficiency differences of signal MC before and after correction are considered as the systematic uncertainties. Fig. 16 gives the comparison with and without tracking helix correction using  $\chi^2$  test for the decay mode  $\chi_{c0} \rightarrow p\bar{p}K^+K^-$ . The systematic uncertainties from the 4C kinematic fit are listed in the following Table 7.

### 5.6 Fitting range

The uncertainty from fitting range is estimated by varying the fitting range. The resulting largest differences of all decay modes are treated as the systematic uncertainties listed in Table 8.

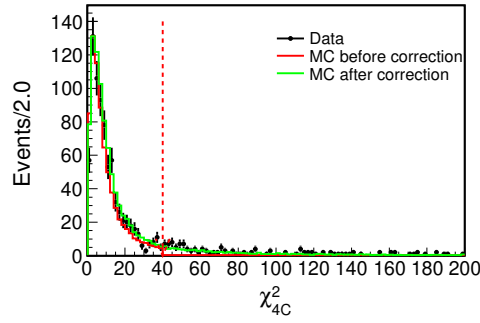


Figure 16: The decay mode  $\chi_{c0} \rightarrow p\bar{p}K^+K^-$  of  $\chi_{4C}^2$  distribution between track correction results and nominal results. Dots with error bars are experimental data, the red line denotes MC sample before track correction, the green line denotes the MC sample after track correction.

Table 7: The systematic uncertainty from the 4C kinematic fit.(%)

| Decay mode   | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ |
|--|-------------|-------------|-------------|
| $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$                   | 0.7         | 0.1         | 0.4         |
| $\chi_{cJ} \rightarrow p\bar{p}\phi$                     | 0.7         | 0.8         | 0.8         |
| $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$          | 0.8         | 0.8         | 1.0         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$          | 0.9         | 0.9         | 0.9         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | 0.6         | 0.7         | 0.6         |

Table 8: The systematic uncertainty from fitting range.(%)

| Decay mode   | Change (GeV/c <sup>2</sup> )  | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ |
|--|-------------------------------|-------------|-------------|-------------|
| $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$                   | [3.351,3.599] / [3.345,3.605] | 0.9         | 3.2         | 1.0         |
| $\chi_{cJ} \rightarrow p\bar{p}\phi$                     | [0.985,1.09] / [0.985,1.11]   | 1.6         | 0.3         | 4.0         |
| $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$          | [1.46,1.84] / [1.44,1.86]     | 1.4         | 2.6         | 3.5         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$          | [1.46,1.84] / [1.44,1.86]     | 3.4         | 5.5         | 2.7         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | [1.48,1.76] / [1.46,1.78]     | 3.6         | 17.0        | 5.9         |

## 1 5.7 Signal shape

2 In order to estimate the systematic uncertainty due to the difference of different signal shape, alterna-  
 3 tive fits are performed to determine the yields of signal events, replacing original shape with the other  
 4 function. The branching fraction differences are taken as the systematic uncertainties, and the values are  
 5 listed in Table 9.

Table 9: The systematic uncertainty from the signal shape.(%)

| Decay mode   | Change                  | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ |
|--|-------------------------|-------------|-------------|-------------|
| $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$                   | MC shape                | 8.5         | 1.3         | 2.7         |
| $\chi_{cJ} \rightarrow p\bar{p}\phi$                     | MC shape                | 1.6         | 12.1        | 3.5         |
| $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$          | MC shape                | 3.6         | 0.0         | 0.4         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$          | MC shape                | 5.1         | 1.9         | 0.6         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | a Breit-Wigner function | 7.3         | 20.4        | 9.9         |

## 6 5.8 Background shape

7 To estimate the systematic uncertainty from background shape, we perform a alternative fit by changing  
 8 original function to other function, and the differences of branching fraction are taken as the uncertainties  
 9 and the details can be found in Table 10.

Table 10: The systematic uncertainty from the background shape.(%)

| Decay mode   | Change                            | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ |
|--|-----------------------------------|-------------|-------------|-------------|
| $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$                   | 1st Polynomial function           | 0.8         | 0.3         | 0.6         |
| $\chi_{cJ} \rightarrow p\bar{p}\phi$                     | 3rd Chebychev Polynomial function | 1.9         | 19.9        | 3.4         |
| $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$          | 3rd Chebychev Polynomial function | 9.5         | 1.0         | 2.6         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$          | 3rd Chebychev Polynomial function | 4.1         | 2.2         | 0.2         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | 3rd Polynomial function           | 1.1         | 1.6         | 0.5         |

## 10 5.9 The mass window requirement

11 To estimate the systematic uncertainty of the veto of invariant mass, we change the cut value for data and  
 12 MC by enlarging and narrowing the mass window range, then perform the same fit with nominal analysis.  
 13 The maximum resulting differences of branching fractions are regarded as the systematic uncertainties  
 14 for each requirement. The values are shown in Table 11.

## 15 5.10 Total systematic uncertainty

16 The summaries of systematic uncertainty are listed in Table 12, Table 13, Table 14, Table 15 and Table 16  
 17 for above decay modes, respectively. The total systematic uncertainties are the square root of quadratic  
 18 sum of each item.

Table 11: The systematic uncertainty from veto of all decay modes.(%)

| Decay mode   | Mass window            | Change(MeV) | $\chi_{c0}$ | $\chi_{c1}$ | $\chi_{c2}$ |
|--|------------------------|-------------|-------------|-------------|-------------|
| $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$                   | $M(pK^-)$              | $\pm 5$     | 1.8         | 0.1         | 1.0         |
|  | $M(\bar{p}K^+)$        | $\pm 5$     | 1.4         | 2.3         | 2.1         |
|  | $M(K^+K^-)$            | $\pm 2$     | 1.9         | 1.4         | 0.9         |
| $\chi_{cJ} \rightarrow p\bar{p}\phi$                     | $M(pK^-)M(\bar{p}K^+)$ | $\pm 5$     | 2.9         | 5.1         | 4.6         |
| $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$          | $M(pK^-)$              | $\pm 5$     | 2.2         | 1.1         | 1.5         |
|  | $M(K^+K^-)$            | $\pm 2$     | 0.8         | 0.6         | 0.5         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$          | $M(\bar{p}K^+)$        | $\pm 5$     | 1.8         | 2.3         | 0.4         |
|  | $M(K^+K^-)$            | $\pm 2$     | 1.7         | 1.2         | 0.9         |
| $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | $M(K^+K^-)$            | $\pm 2$     | 0.3         | 0.1         | 0.4         |

Table 12: Systematic uncertainties on the branching fraction measurement for  $\chi_{cJ} \rightarrow p\bar{p}K^+K^-$  (%).

| Source                        | $\chi_{c0} \rightarrow p\bar{p}K^+K^-$ | $\chi_{c1} \rightarrow p\bar{p}K^+K^-$ | $\chi_{c2} \rightarrow p\bar{p}K^+K^-$ |
|-------------------------------|--|--|--|
| MDC track                     | 4.0                                    | 4.0                                    | 4.0                                    |
| PID                           | 4.0                                    | 4.0                                    | 4.0                                    |
| Photon selection              | 1.0                                    | 1.0                                    | 1.0                                    |
| Total Number of $\psi(2S)$    | 0.6                                    | 0.6                                    | 0.6                                    |
| The 4C Kinematic Fit          | 0.7                                    | 0.1                                    | 0.4                                    |
| Intermediate Decay            | 2.0                                    | 2.0                                    | 2.0                                    |
| Signal shape                  | 8.5                                    | 1.3                                    | 2.7                                    |
| Background shape              | 0.8                                    | 0.3                                    | 0.6                                    |
| Fitting range                 | 0.9                                    | 3.2                                    | 1.0                                    |
| Veto of $\Lambda(1520)$       | 1.8                                    | 0.1                                    | 1.0                                    |
| Veto of $\bar{\Lambda}(1520)$ | 1.4                                    | 2.3                                    | 2.1                                    |
| Veto of $\phi$                | 1.9                                    | 1.4                                    | 0.9                                    |
| Total                         | 11.0                                   | 7.5                                    | 7.2                                    |

Table 13: Systematic uncertainties on the branching fraction measurement for  $\chi_{cJ} \rightarrow p\bar{p}\phi$  (%).

| Source                                      | $\chi_{c0} \rightarrow p\bar{p}\phi$ | $\chi_{c1} \rightarrow p\bar{p}\phi$ | $\chi_{c2} \rightarrow p\bar{p}\phi$ |
|---|--------------------------------------|--------------------------------------|--------------------------------------|
| MDC track                                   | 4.0                                  | 4.0                                  | 4.0                                  |
| PID   | 4.0                                  | 4.0                                  | 4.0                                  |
| Photon selection                            | 1.0                                  | 1.0                                  | 1.0                                  |
| Total Number of $\psi(2S)$                  | 0.6                                  | 0.6                                  | 0.6                                  |
| The 4C Kinematic Fit                        | 0.7                                  | 0.8                                  | 0.8                                  |
| Intermediate Decay                          | 2.2                                  | 2.2                                  | 2.2                                  |
| Signal shape                                | 1.6                                  | 12.1                                 | 3.5                                  |
| Background shape                            | 1.9                                  | 19.9                                 | 3.4                                  |
| Fitting range                               | 1.6                                  | 0.3                                  | 4.0                                  |
| Veto of $\Lambda(1520)/\bar{\Lambda}(1520)$ | 2.9                                  | 5.1                                  | 4.6                                  |
| Total                                       | 7.5                                  | 24.6                                 | 10.0                                 |



Table 14: Systematic uncertainties on the branching fraction measurement for  $\chi_{cJ} \rightarrow pK^-\bar{\Lambda}(1520)$  (%).

| Source                     | $\chi_{c0} \rightarrow pK^-\bar{\Lambda}(1520)$ | $\chi_{c1} \rightarrow pK^-\bar{\Lambda}(1520)$ | $\chi_{c2} \rightarrow pK^-\bar{\Lambda}(1520)$ |
|----------------------------|---|---|---|
| MDC track                  | 4.0   | 4.0   | 4.0   |
| PID                        | 4.0   | 4.0   | 4.0   |
| Photon selection           | 1.0   | 1.0   | 1.0   |
| Total Number of $\psi(2S)$ | 0.6   | 0.6   | 0.6   |
| The 4C Kinematic Fit       | 0.8   | 0.8   | 1.0   |
| Intermediate Decay         | 2.0   | 2.0   | 2.0   |
| Signal shape               | 3.6   | 0.1   | 0.4   |
| Background shape           | 9.5   | 1.0   | 2.6   |
| Fitting range              | 1.4   | 2.6   | 3.5   |
| Veto of $\Lambda(1520)$    | 2.2   | 1.1   | 1.5   |
| Veto of $\phi$             | 0.8   | 0.6   | 0.5   |
| Total                      | 12.2  | 6.9   | 7.7   |

Table 15: Systematic uncertainties on the branching fraction measurement for  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{p}K^+$  (%).

| Source                        | $\chi_{c0} \rightarrow \Lambda(1520)\bar{p}K^+$ | $\chi_{c1} \rightarrow \Lambda(1520)\bar{p}K^+$ | $\chi_{c2} \rightarrow \Lambda(1520)\bar{p}K^+$ |
|-------------------------------|---|---|---|
| MDC track                     | 4.0   | 4.0   | 4.0   |
| PID                           | 4.0   | 4.0   | 4.0   |
| Photon selection              | 1.0   | 1.0   | 1.0   |
| Total Number of $\psi(2S)$    | 0.6   | 0.6   | 0.6   |
| The 4C Kinematic Fit          | 0.9   | 0.9   | 0.9   |
| Intermediate Decay            | 2.0   | 2.0   | 2.0   |
| Signal shape                  | 5.1   | 1.9   | 0.6   |
| Background shape              | 4.1   | 2.2   | 0.2   |
| Fitting range                 | 3.4   | 5.5   | 2.7   |
| Veto of $\bar{\Lambda}(1520)$ | 1.8   | 2.3   | 0.4   |
| Veto of $\phi$                | 1.7   | 1.2   | 0.9   |
| Total                         | 9.9   | 9.1   | 6.8   |

Table 16: Systematic uncertainties on the branching fraction measurement for  $\chi_{cJ} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  (%).

| Source                     | $\chi_{c0} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | $\chi_{c2} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ |
|----------------------------|--|--|--|
| MDC track                  | 4.0  | 4.0  | 4.0  |
| PID                        | 4.0  | 4.0  | 4.0  |
| Photon selection           | 1.0  | 1.0  | 1.0  |
| Total Number of $\psi(2S)$ | 0.6  | 0.6  | 0.6  |
| The 4C Kinematic Fit       | 0.6  | 0.7  | 0.6  |
| Intermediate Decay         | 3.5  | 3.5  | 3.5  |
| Signal shape               | 7.3  | 20.4   | 9.9  |
| Background shape           | 1.1  | 1.6  | 0.5  |
| Fitting range              | 3.6  | 17.0   | 5.9  |
| Veto of $\phi$             | 0.3  | 0.1  | 0.4  |
| Total                      | 10.7   | 27.5   | 13.4   |

## 6 Upper limit

To determine the upper limits of the branching fraction on  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ , a series of unbinned maximum likelihood fits are performed. In the fits, the fit models of signal and background are the same as their nominal ones, which are described in Sec. 4.2.2 and 4.5.2, respectively, while we fix the mean value of Breit-Wigner function to PDG [5] value for fits of  $\phi$  decay. To incorporate the multiplicative systematic uncertainty in the calculation of the upper limit, the likelihood distribution is smeared via [23,24]

$$L'(n') \propto \int_0^\infty L(n \frac{\epsilon}{\epsilon_0}) \exp[-\frac{(\epsilon - \epsilon_0)^2}{2\sigma_\epsilon^2}] d\epsilon, \quad (9)$$

where  $L(n)$  is the likelihood distribution as a function of the yield  $n$ ,  $\epsilon_0$  is the detection efficiency and  $\sigma_\epsilon$  is the multiplicative systematic uncertainty. Figure 17 shows the likelihood distributions with and without incorporating the systematic uncertainties. The number limits of the  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  are then determined to be  $N_{UL} = 50$  and 35, respectively.

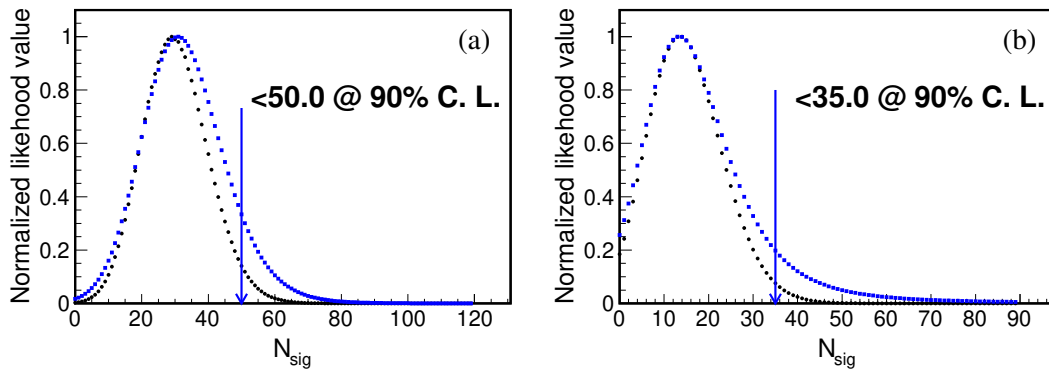


Figure 17: The normalized likelihood distributions with only one additive systematic uncertainty (background shape). (a)  $\chi_{c1} \rightarrow p\bar{p}\phi$ . (b)  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ . The black dots are the results with the nominal fits, the blue dots are obtained with incorporating the multiplicative and additive systematic uncertainties. The arrows are the position of the upper limit on the signal yields at 90% confidence level (C. L.).

With the detection efficiency of 16.2% obtained from the MC simulation of  $\chi_{c1} \rightarrow p\bar{p}\phi$ , the upper limits on the branching fraction is calculated with Eq. 10. The results of upper limit of the branching fraction are shown in Table 17.

$$B < \frac{N_{UL}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\phi \rightarrow K^+K^-) \times \epsilon}. \quad (10)$$

With the detection efficiency of 22.4% obtained from the MC simulation of  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ , the upper limits on the branching fraction is calculated with Eq. 11. The results of upper limit of the branching fraction are shown in Table 17.

$$B < \frac{N_{UL}}{N_{\psi(2S)} \times B(\psi(2S) \rightarrow \gamma\chi_{cJ}) \times B(\Lambda(1520) \rightarrow pK)^2 \times \epsilon}. \quad (11)$$

Table 17: The results of upper limit of the branching fraction.

| Decay mode   | $N_{UL}$ | $\epsilon$ (%) | Results               |
|--|----------|----------------|-----------------------|
| $\chi_{c1} \rightarrow p\bar{p}\phi$                     | 50       | 16.2           | $1.44 \times 10^{-5}$ |
| $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ | 35       | 22.4           | $0.71 \times 10^{-4}$ |

## 7 Summary

Based on  $448.1 \times 10^6$  events collected from BESIII detector at BEPCII storage ring at center-of-mass energy  $\sqrt{s} = 3.686$  GeV in 2009 and 2012, we study specific hadron decay modes of  $\chi_{cJ}$  listed in Table 18. The upper limit results of  $\chi_{c1} \rightarrow p\bar{p}\phi$  and  $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  are determined to be  $B(\chi_{c1} \rightarrow p\bar{p}\phi) < 1.44 \times 10^{-5}$  and  $B(\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)) < 0.71 \times 10^{-4}$  at 90% C. L., respectively. For the latter the branching fraction is also measured in the analysis because of the evident result with a significance level of  $3.6\sigma$  for the decay  $\chi_{c1} \rightarrow p\bar{p}\phi$ . Other hadronic decay modes of  $\chi_{cJ}$  can be improved considering simple clean environment and excellent detector performance. In this analysis, the branching fractions are consistent with the previous results within  $1\sigma$  standard deviation in  $\chi_{c1}/\chi_{c2} \rightarrow p\bar{p}K^+K^-$ ,  $\chi_{c0} \rightarrow p\bar{p}\phi$  and  $\chi_{c0}/\chi_{c2} \rightarrow pK^-\bar{\Lambda}(1520) + c.c.$ . The branching fractions are different in some decays. In the  $\chi_{c0} \rightarrow p\bar{p}K^+K^-$ ,  $\chi_{c2} \rightarrow p\bar{p}\phi$ ,  $\chi_{c1} \rightarrow pK^-\bar{\Lambda}(1520) + c.c.$  and  $\chi_{c2} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$ , the differences are covered by  $2\sigma$ , and in  $\chi_{c0} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  the difference is covered by  $3\sigma$ . The big difference in  $\chi_{c0} \rightarrow \Lambda(1520)\bar{\Lambda}(1520)$  may be caused by the difference of method of the signal yields extraction, because the 2-dimension fit is used in this analysis while the sideband method is performed in the previous work.

Table 18: Summary of the measured branching fraction.

| Decay mode  | This work                         | Previous measurement [25]        |
|---|-----------------------------------|----------------------------------|
| $\chi_{c0} \rightarrow p\bar{p}K^+K^- (\times 10^{-4})$                   | $1.77 \pm 0.07 \pm 0.20$          | $1.24 \pm 0.20 \pm 0.18$         |
| $\chi_{c1} \rightarrow p\bar{p}K^+K^- (\times 10^{-4})$                   | $1.55 \pm 0.05 \pm 0.12$          | $1.35 \pm 0.15 \pm 0.19$         |
| $\chi_{c2} \rightarrow p\bar{p}K^+K^- (\times 10^{-4})$                   | $1.98 \pm 0.06 \pm 0.14$          | $2.08 \pm 0.19 \pm 0.30$         |
| $\chi_{c0} \rightarrow p\bar{p}\phi (\times 10^{-5})$                     | $6.72 \pm 0.80 \pm 0.50$          | $6.12 \pm 1.18 \pm 0.86$         |
| $\chi_{c1} \rightarrow p\bar{p}\phi (\times 10^{-5})$                     | $1.11 \pm 0.30 \pm 0.27 (< 1.44)$ | $< 1.82$                         |
| $\chi_{c2} \rightarrow p\bar{p}\phi (\times 10^{-5})$                     | $1.69 \pm 0.35 \pm 0.17$          | $3.04 \pm 0.85 \pm 0.43$         |
| $\chi_{c0} \rightarrow pK^-\bar{\Lambda}(1520) (\times 10^{-4})$          | $1.71 \pm 0.16 \pm 0.21$          | $3.00 \pm 0.58 \pm 0.50 (+c.c.)$ |
| $\chi_{c1} \rightarrow pK^-\bar{\Lambda}(1520) (\times 10^{-4})$          | $1.63 \pm 0.14 \pm 0.11$          | $1.81 \pm 0.38 \pm 0.28 (+c.c.)$ |
| $\chi_{c2} \rightarrow pK^-\bar{\Lambda}(1520) (\times 10^{-4})$          | $1.80 \pm 0.15 \pm 0.14$          | $3.06 \pm 0.50 \pm 0.54 (+c.c.)$ |
| $\chi_{c0} \rightarrow \Lambda(1520)\bar{p}K^+ (\times 10^{-4})$          | $1.83 \pm 0.17 \pm 0.18$          | -                                |
| $\chi_{c1} \rightarrow \Lambda(1520)\bar{p}K^+ (\times 10^{-4})$          | $1.24 \pm 0.13 \pm 0.11$          | -                                |
| $\chi_{c2} \rightarrow \Lambda(1520)\bar{p}K^+ (\times 10^{-4})$          | $1.79 \pm 0.14 \pm 0.12$          | -                                |
| $\chi_{c0} \rightarrow \Lambda(1520)\bar{\Lambda}(1520) (\times 10^{-4})$ | $7.33 \pm 0.56 \pm 0.78$          | $3.18 \pm 1.11 \pm 0.53$         |
| $\chi_{c1} \rightarrow \Lambda(1520)\bar{\Lambda}(1520) (\times 10^{-4})$ | $< 0.71$                          | $< 1.00$                         |
| $\chi_{c2} \rightarrow \Lambda(1520)\bar{\Lambda}(1520) (\times 10^{-4})$ | $2.68 \pm 0.33 \pm 0.36$          | $5.05 \pm 1.29 \pm 0.93$         |

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