



Hadronic atoms in the charmonium energy region

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B. 3 Workshop of CRC110

2022.02.16-17

Based on

X atom: Z.-H. Zhang, F.-K. Guo. Phys. Rev. Lett. 127, 012002 (2021)

Dionium: P.-P. Shi, Z.-H. Zhang, F.-K. Guo, Z. Y. arXiv: 2111.13496



X atom: Introduction

$$\delta = m_{D^0} + m_{D^{*0}} - m_X = (0.00 \pm 0.18) \text{ MeV}$$

Typical size of the $X(3872)$ at long distance: $r_X \simeq \frac{1}{\sqrt{2\mu_0\delta}} \gtrsim 10 \text{ fm}$

Typical size (Bohr radius) of the $D^+ D^{*-}$ bound state: $r_B = \frac{1}{\alpha\mu_c} = 27.86 \text{ fm}$

$$\mu_0 = \frac{m_{D^0}m_{D^{*0}}}{\Sigma_0} \quad \mu_c = \frac{m_D m_{D^*}}{\Sigma_c} \quad \Sigma_0 = m_{D^0} + m_{D^{*0}} \quad \Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07) \text{ MeV}$$

Coulomb binding energies: $-E_n = -\frac{\alpha^2\mu_c}{2n^2} = \frac{-E_1}{n^2} = -\frac{25.81 \text{ keV}}{n^2}$

X atom: The ground state $\frac{1}{\sqrt{2}}(|D^+ D^{*-}\rangle - |D^- D^{*+}\rangle)$ atom with $C = +$

Scale separation: $r_B \Lambda_{\text{QCD}} \gg 1$, strong interaction between $D^+ D^{*-}$ is a correction

Effects of strong interaction at LO:

(a) **Energy level shift:** $\Delta E_n^{\text{str}} \sim \mathcal{O}(\alpha^3)$ (b) **Decay modes:** $D^0 \bar{D}^{*0}, D^0 \bar{D}^0 \pi^0, J/\psi \pi\pi, \dots$

The strong interaction is nonperturbative due to the existence of the $X(3872)$

Only hadronic atoms with light quarks have been studied

Gasser, Lyubovitskij, Rusetsky, *Phys. Rept.* 456 (2008)



X atom: Introduction

The X atom is related to the $X(3872)$ (as a hadronic molecule) by **isospin symmetry**

$D^+ D^{*-}$ threshold: $\Sigma_c = m_D + m_{D^*} = (3879.91 \pm 0.07) \text{ MeV}$, **no signal near the threshold**

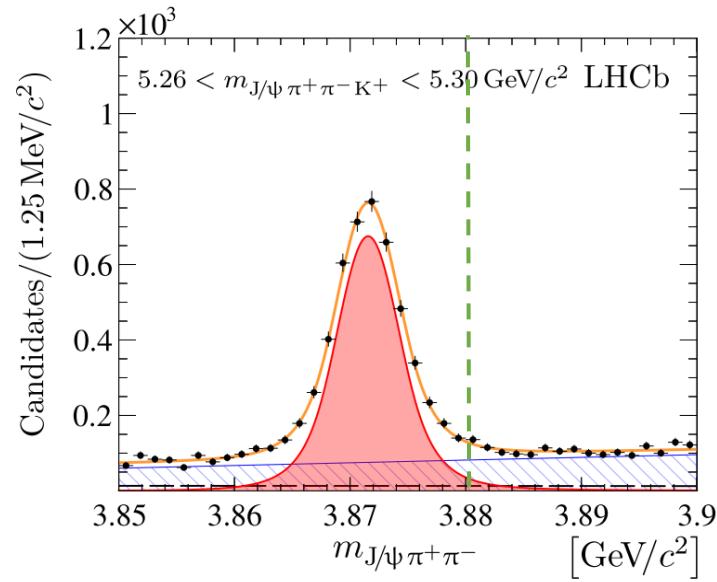
Make use of the zero signal to:

- Put a lower bound on the $X(3872)$ binding energy
- Give a criterion on the $X(3872)$ nature

Scale separation: $r_B \Lambda_{\text{QCD}} \gg 1$; Nonrelativistic effective field theory (**NREFT**) applicable

Approximation: Isospin-1 strong interaction neglected

- No isovector state was found
- Isospin breaking in the couplings is small $\frac{g_{X\rho}}{g_{X\omega}} = 0.26^{+0.08}_{-0.05}$



LHCb, *J. High Energy Phys.* 08 (2020) 123

Hanhart et al., *Phys. Rev. D* 85 (2012) 011501



X atom: NREFT

Coupled channel: **CH 1** : $D^+ D^{*-} \rightarrow D^+ D^{*-}$ **CH 2** : $D^0 \bar{D}^{*0} \rightarrow D^0 \bar{D}^{*0}$

Non-relativistic effective Lagrangian: **Galilean, Gauge invariant; C, P, T**

Around threshold, LO Lagrangian: constant contact terms for strong interactions

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\phi=D^\pm, D^0, \bar{D}^0} \phi^\dagger \left(iD_t - m_\phi + \frac{\nabla^2}{2m_\phi} \right) \phi + \sum_{\phi=D^{*\pm}, D^{*0}, \bar{D}^{*0}} \phi^\dagger \left(iD_t - m_\phi + i\frac{\Gamma_\phi}{2} + \frac{\nabla^2}{2m_\phi} \right) \phi \\ & - \frac{C_0}{2} (D^+ D^{*-} - D^- D^{*+})^\dagger (D^+ D^{*-} - D^- D^{*+}) - \frac{C_0}{2} \left[(D^+ D^{*-} - D^- D^{*+})^\dagger (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0}) + \text{h. c.} \right] \\ & - \frac{C_0}{2} (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0})^\dagger (D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0}) + \dots \end{aligned}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad D_t \phi = \partial_t \phi \mp i Q A_0 \phi$$

Constant width approximation for D^*

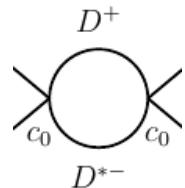
Hanhart, Kalashnikova, Nefediev, Phys. Rev. D 81 (2010) 094028



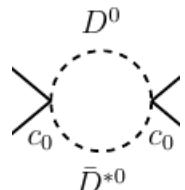
X atom: NREFT

S-wave T-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel: $T(E) = V[1 - G(E)V]^{-1}$

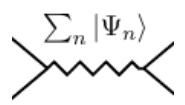
Strong contact term: $V = C_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ **Green's function:** $G(E) = \begin{pmatrix} J_c(E) + J_{|\Psi\rangle}(E) & 0 \\ 0 & J_0(E) \end{pmatrix}$



$$J_c(E) = \frac{\mu_c}{2\pi} \left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_c(E + i\Gamma_c/2)} \right) \quad E = \sqrt{s} - \Sigma_c$$



$$J_0(E) = \frac{\mu_0}{2\pi} \left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_0(E + \Delta + i\Gamma_0/2)} \right) \quad \Delta = \Sigma_c - \Sigma_0$$

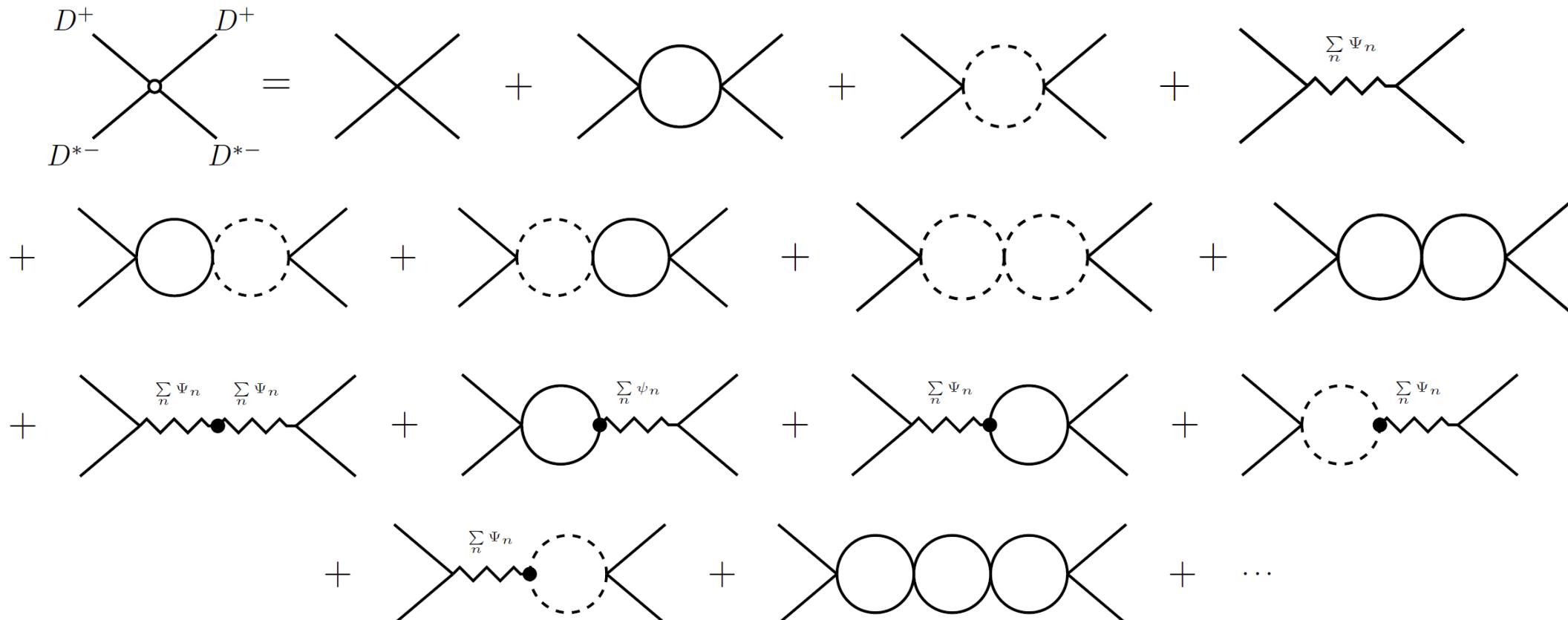


$$J_{|\Psi\rangle}(E) = \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{E + E_n + i\Gamma_c/2} \quad \Gamma_c \equiv \Gamma_{D^*}, \quad \Gamma_0 \equiv \Gamma_{D^{*0}}$$

X atom: NREFT

S-wave T-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:

$$T(E) = \frac{1}{C_0^{-1} - [J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$





X atom: Strong Energy Level Shift

S-wave T-matrix for $I^G J^{PC} = 0^+(1^{++})$ coupled channel:

$$T(E) = \frac{1}{C_0^{-1} - [J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Renormalization: $C_{0R}^{-1} = C_0^{-1} + \Lambda(\mu_0 + \mu_c)/\pi^2$

The $X(3872)$ and hadronic atoms appear as **poles of the T-matrix**

$$\text{X(3872) pole: } E = -\Delta - \delta - i\frac{\Gamma_0}{2} \quad \delta\Gamma = \Gamma_c - \Gamma_0$$

$$C_{0R}^{-1} = \frac{\mu_0}{2\pi} \sqrt{2\mu_0\delta} + \frac{\mu_c}{2\pi} \sqrt{2\mu_c \left(\Delta + \delta - i\frac{\delta\Gamma}{2} \right)} - \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{\Delta + \delta - E_n - i\delta\Gamma/2} = \frac{\mu_c}{2\pi} \sqrt{2\mu_c\Delta} \left[1 + \mathcal{O}\left(\frac{\delta}{\Delta}, \frac{\delta\Gamma}{\Delta}, \frac{\alpha^3 \mu_c^{3/2}}{\Delta^{3/2}}\right) \right]$$

$$\text{S-wave hadronic atom poles: } E = -E_{An} - i\frac{\Gamma_c}{2}$$

$$0 = C_{0R}^{-1} + i\frac{\mu_0}{2\pi} \sqrt{2\mu_0 \left(\Delta - E_{An} - i\frac{\delta\Gamma}{2} \right)} - \frac{\mu_c}{2\pi} \sqrt{2\mu_c E_{An}} - \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{-E_{An} + E_n}$$



X atom: Strong Energy Level Shift

Strong energy level shift: $\Delta E_n = E_{An} - E_n$

$$\Delta E_n = \frac{2\alpha^3 \mu_c^2}{n^3 \sqrt{2\mu_c \Delta}} \left[-1 - i + \mathcal{O}\left(\alpha \sqrt{\frac{\mu_c}{\Delta}}\right) \right]^{-1}$$

S-wave hadronic atom poles: $E = -E_{An} - i\frac{\Gamma_c}{2} = -E_n - \Delta E_n - i\frac{\Gamma_c}{2}$

Ground state: $n = 1$

Binding energy: $\text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV} \quad M_{A1} = (3879.89 \pm 0.07) \text{ MeV}$

Decay width: $\Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV}$

$D^* \rightarrow D\pi, D\gamma, \dots \quad \Gamma_c = (83.4 \pm 1.8) \text{ keV}$

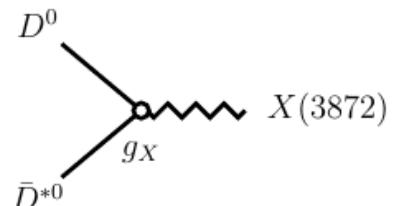
$A \text{ (X atom)} \rightarrow D^0 \bar{D}^{*0} (\bar{D}^0 D^{*0}) \quad \Gamma_s = 2\text{Im}E_{A1} = 5.8 \text{ keV}$



X atom: Effective Coupling

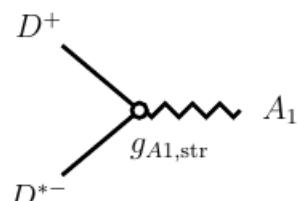
The effective coupling squared is the residue of the T -matrix at the pole

$$D^0 \bar{D}^{*0} \rightarrow X(3872) \quad \text{X(3872) pole: } E = -\Delta - \delta - i \frac{\Gamma_0}{2}$$



$$g_X^2 = \lim_{E \rightarrow -\Delta - \delta - i \frac{\Gamma_0}{2}} \left(E + \Delta + \delta + i \frac{\Gamma_0}{2} \right) T_{22}(E) = \frac{2\pi}{\mu_0^2} \sqrt{2\mu_0\delta} \left[1 + \mathcal{O}\left(\frac{\delta^{1/2}}{\Delta^{1/2}}\right) \right]^{-1}$$

$$D^+ D^{*-} \rightarrow A_1 \quad \text{Hadronic atom poles: } E = -E_{An} - i \frac{\Gamma_c}{2}$$



$$g_{A1,str}^2 = \lim_{E \rightarrow -E_{A1} - i \frac{\Gamma_c}{2}} \left(E + E_{A1} + i \frac{\Gamma_c}{2} \right) T_{11}(E) = -i \frac{\pi \alpha^3}{\Delta} \left[1 + \mathcal{O}\left(\frac{\alpha^2 \mu_c}{\Delta}\right) \right]^{-1}$$

X atom: Production

Production in exclusive B decays:

$$\begin{aligned} \text{---} \otimes \text{---} &= \text{---} + \text{---} + \text{---} + \dots \\ &= \text{---} + \text{---} \otimes \text{---} \end{aligned}$$

$$\mathcal{A}_{B^+ \rightarrow A_1 K^+} = \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{(\Lambda)} G_C(\Lambda, E) g_{A_1, \text{str}}$$

$$G_C(\Lambda, E) = -\frac{\mu_c \Lambda}{\pi^2} - \frac{\alpha \mu_c^2}{\pi} \left[\ln \frac{\Lambda}{\alpha \mu_c} + \ln(x) + \frac{1}{2x} - \psi(-x) - \gamma_E \right]$$

$$B^0 \rightarrow (DD^*)_+^0 K^0 \rightarrow X K^0$$

$$\mathcal{A}_{B^0 \rightarrow X K^0} = \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{(\Lambda)} G_0(\Lambda, E) g_X$$

$$G_0(\Lambda, E) = -\frac{\mu_c^0 \Lambda}{\pi^2} + \frac{\mu_c^0}{2\pi} \left(\sqrt{-2\mu_c^0 E - i\epsilon} \right)$$

$$\begin{aligned} |(DD^*)_+^0\rangle &= \frac{1}{\sqrt{2}} \left(|D^0 \bar{D}^{*0}\rangle - |\bar{D}^0 D^{*0}\rangle \right) & x &= \frac{\alpha \mu_c}{\sqrt{-2\mu_c(E + i\frac{\Gamma_c}{2})}} & \psi(x) &= \frac{\Gamma'(x)}{\Gamma(x)} & \text{Kong, Ravndal, } \textit{Nucl. Phys. A 665 (2000)} \\ |(DD^*)_+\rangle &= \frac{1}{\sqrt{2}} \left(|D^+ D^{*-}\rangle - |D^- D^{*+}\rangle \right) \end{aligned}$$

Factorized amplitudes: $\mathcal{A}_{B^+ \rightarrow A_1 K^+} = \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}} g_{A_1, \text{str}}$ $\mathcal{A}_{B^0 \rightarrow X K^0} = \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} g_X$

Braaten, Kusunoki, *Phys. Rev. D 72 (2005) 014012*

Isospin symmetry:

Zhen-Hua Zhang, Hadronic atoms

$$\left| \mathcal{A}_{B^+ \rightarrow (DD^*)_+ K^+}^{\text{s.d.}} \right| = \left| \mathcal{A}_{B^0 \rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} \right|$$

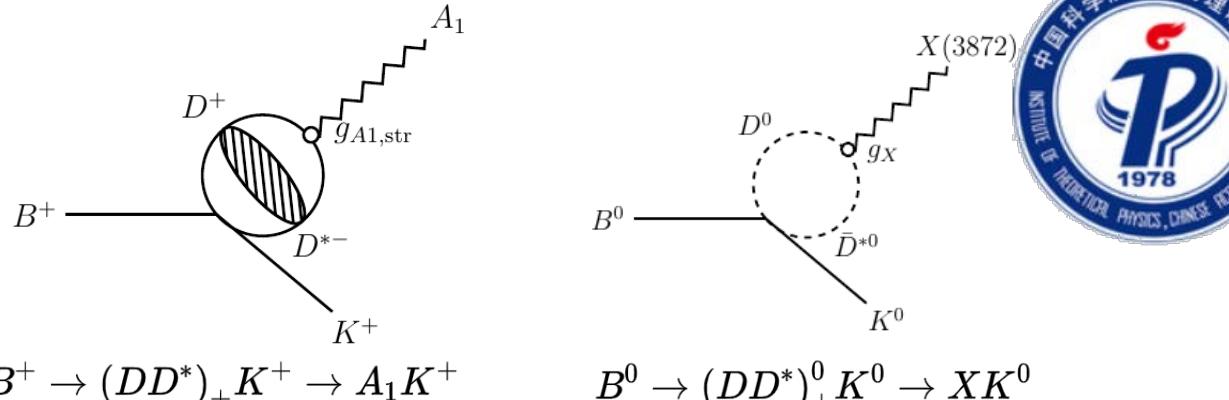


X atom: Production

Production in exclusive B decays:

$$|(DD^*)_+^0\rangle = \frac{1}{\sqrt{2}}(|D^0\bar{D}^{*0}\rangle - |\bar{D}^0D^{*0}\rangle)$$

$$|(DD^*)_+^-\rangle = \frac{1}{\sqrt{2}}(|D^+D^{*-}\rangle - |D^-D^{*+}\rangle)$$



Factorized amplitudes:

$$\mathcal{A}_{B^+\rightarrow A_1 K^+} = \mathcal{A}_{B^+\rightarrow (DD^*)_+ K^+}^{\text{s.d.}} g_{A1,\text{str}}$$

$$\mathcal{A}_{B^0\rightarrow X K^0} = \mathcal{A}_{B^0\rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} g_X$$

Isospin symmetry:

$$\left| \mathcal{A}_{B^+\rightarrow (DD^*)_+ K^+}^{\text{s.d.}} \right| = \left| \mathcal{A}_{B^0\rightarrow (DD^*)_+^0 K^0}^{\text{s.d.}} \right|$$

Lower bound on the $X(3872)$ binding energy:

$$R_\Gamma \equiv \frac{\Gamma_{B^+\rightarrow A_1 K^+}}{\Gamma_{B^0\rightarrow X K^0}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2} \quad \delta \simeq \frac{0.25 \text{ eV}}{R_\Gamma^2}$$

Production in inclusive pp collisions:

$$R_\sigma \equiv \frac{d\sigma_{pp\rightarrow A_1+y}}{d\sigma_{pp\rightarrow X+y}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2} \quad \delta \simeq \frac{0.25 \text{ eV}}{R_\sigma^2} \quad R_\Gamma \simeq R_\sigma \gtrsim 1 \times 10^{-3}$$





X atom: Decay

Constituent D^* decay: $D^* \rightarrow D\pi, D\gamma, \dots$ $\Gamma_c = (83.4 \pm 1.8) \text{ keV}$

Decay into neutral pair: A (X atom) $\rightarrow D^0 \bar{D}^{*0} (\bar{D}^0 D^{*0})$ $\Gamma_s = 2\text{Im}E_{A1} = 5.8 \text{ keV}$

Decay into $J/\psi\pi\pi$ & $J/\psi\pi^+\pi^-\pi^0$ (like the $X(3872)$) $A \rightarrow J/\psi\pi\pi, J/\psi\pi^+\pi^-\pi^0$

Ratio of branchings for the $X(3872)$: $\frac{\text{Br}_{[X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0]}^{\text{exp}}}{\text{Br}_{[X(3872) \rightarrow J/\psi\pi^+\pi^-]}^{\text{exp}}} = 1.1 \pm 0.4$

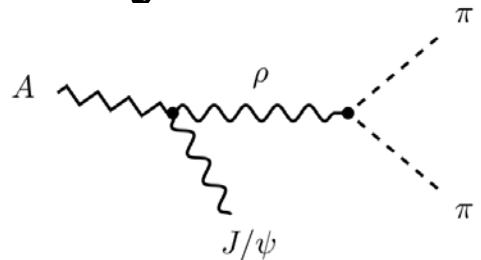
Isospin breaking: $R_X = \frac{g_{[X(3872) \rightarrow J/\psi\rho]}}{g_{[X(3872) \rightarrow J/\psi\omega]}} = 0.26$ C. Hanhart et al., *Phys. Rev. D* 85 (2012) 011501

$D^+ D^{*-}$ atom (A): $m_A = 3879.89 \pm 0.07 \text{ MeV}$

Isospin breaking negligible: $|D^+ D^{*-}\rangle = \frac{1}{\sqrt{2}}(|I=1\rangle + |I=0\rangle)$ $R_A = \frac{g_{[A \rightarrow J/\psi\rho]}}{g_{[A \rightarrow J/\psi\omega]}} = 1$

The phase space of the $D^+ D^{*-}$ atom is larger than the phase space of the $X(3872)$

X atom: Decay

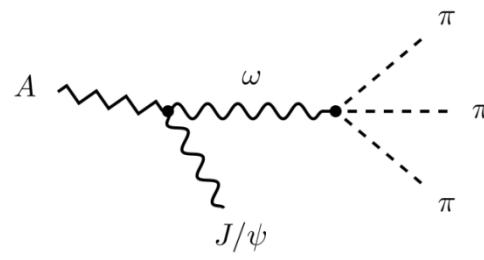


$$A \rightarrow J/\psi \pi \pi$$

C. Hanhart et al., *Phys. Rev. D* **85** (2012) 011501

O. Kaymakcalan, S. Rajeev, and J. Schechter, *Phys. Rev. D* **30**, 594 (1984)

E. A. Kuraev and Z. K. Silagadze, *Phys. At. Nucl.* **58**, 1589 (1995)



$$A \rightarrow J/\psi \pi^+ \pi^- \pi^0$$

$$\frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]} / \text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^-]}}{= 1.09}$$

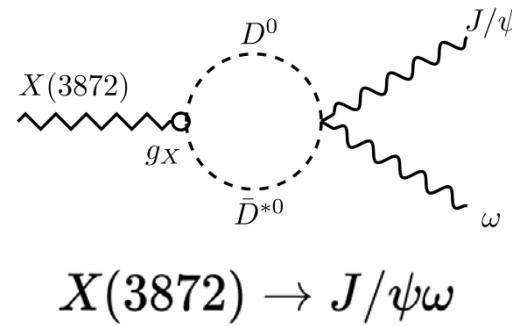
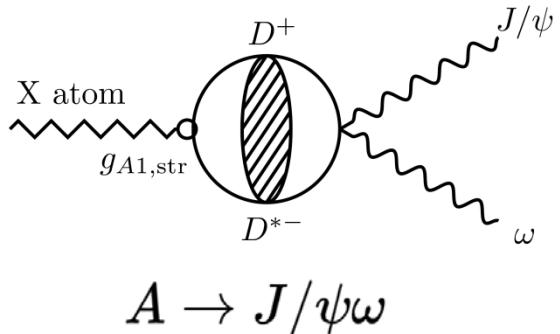
$$\frac{\text{Br}^{\text{exp}}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]} / \text{Br}^{\text{exp}}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^-]}}{= 1.1 \pm 0.4}$$

Effective couplings: $R_A = \frac{g_{[A \rightarrow J/\psi \rho]}}{g_{[A \rightarrow J/\psi \omega]}} = 1$ $R_X = \frac{g_{[X(3872) \rightarrow J/\psi \rho]}}{g_{[X(3872) \rightarrow J/\psi \omega]}} = 0.26$

Ratio of branchings: $\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]} / \text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}{= 3.34} \quad \frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]} / \text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}{= 0.91}$

$$\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]} / \text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}{\simeq 3.65} \quad \frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]} / \text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}$$

X atom: Decay



Factorized amplitudes : $\mathcal{A}_{[A \rightarrow J/\psi\omega]} = g_{A1,str} \mathcal{A}_{[(DD^*)_+ \rightarrow J/\psi\omega]}^{\text{s.d.}}$ $\mathcal{A}_{[X(3872) \rightarrow J/\psi\omega]} = g_X \mathcal{A}_{[(DD^*)_+^0 \rightarrow J/\psi\omega]}^{\text{s.d.}}$

Ratio of phase spaces :

$$\frac{\Phi_{[A \rightarrow J/\psi\pi^+\pi^-\pi^0]}}{\Phi_{[X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0]}} = 3.76$$

Ratio of decay widths : $\frac{\Gamma_{[A \rightarrow J/\psi\pi^+\pi^-\pi^0]}}{\Gamma_{[X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0]}} = \frac{|g_{A1,str}|^2}{|g_X|^2} \frac{\Phi_{[A \rightarrow J/\psi\pi^+\pi^-\pi^0]}}{\Phi_{[X(3872) \rightarrow J/\psi\pi^+\pi^-\pi^0]}} \gtrsim 3.76 \times 10^{-3}$

$$\frac{\text{Br}_{[A \rightarrow J/\psi\pi\pi]}}{\text{Br}_{[A \rightarrow J/\psi\pi^+\pi^0\pi^-]}} \simeq 3.65 \frac{\text{Br}_{[X(3872) \rightarrow J/\psi\pi\pi]}}{\text{Br}_{[X(3872) \rightarrow J/\psi\pi^+\pi^0\pi^-]}}$$

$$\frac{\Gamma_{[A \rightarrow J/\psi\pi\pi]}}{\Gamma_{[X(3872) \rightarrow J/\psi\pi\pi]}} \gtrsim 1.37 \times 10^{-2}$$

X atom: Results

(a) Binding Energy and Decay Width for the X Atom

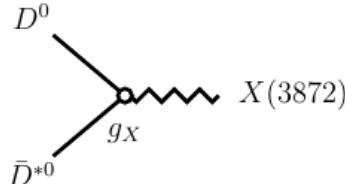
$$\text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV}$$

$$M_{A1} = (3879.89 \pm 0.07) \text{ MeV}$$

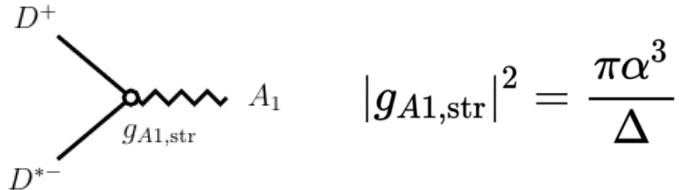
$$\Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV}$$

$$\Gamma_c = (83.4 \pm 1.8) \text{ keV}$$

(b) LO Effective Couplings



$$g_X^2 = \frac{2\pi}{\mu_0^2} \sqrt{2\mu_0 \delta}$$



$$|g_{A1,\text{str}}|^2 = \frac{\pi \alpha^3}{\Delta}$$

(c) Lower bound on the $X(3872)$ binding energy

$$\delta \simeq \frac{0.25 \text{ eV}}{R_{\Gamma(\sigma)}^2} \quad R_\Gamma \equiv \frac{\Gamma_{B^+ \rightarrow A_1 K^+}}{\Gamma_{B^0 \rightarrow X K^0}} \quad R_\sigma \equiv \frac{d\sigma_{pp \rightarrow A_1 + y}}{d\sigma_{pp \rightarrow X + y}} \quad \delta = m_{D^0} + m_{D^{*0}} - m_X \quad R_\Gamma \simeq R_\sigma \gtrsim 1 \times 10^{-3}$$

(d) Ratio of decay widths

$$\frac{\text{Br}_{[A \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[A \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}} \simeq 3.65 \frac{\text{Br}_{[X(3872) \rightarrow J/\psi \pi \pi]}}{\text{Br}_{[X(3872) \rightarrow J/\psi \pi^+ \pi^0 \pi^-]}}$$

$$\frac{\Gamma_{[A \rightarrow J/\psi \pi^+ \pi^- \pi^0]}}{\Gamma_{[X(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0]}} \gtrsim 3.76 \times 10^{-3}$$

$$\frac{\Gamma_{[A \rightarrow J/\psi \pi \pi]}}{\Gamma_{[X(3872) \rightarrow J/\psi \pi \pi]}} \gtrsim 1.37 \times 10^{-2}$$



Dionium: Introduction

S. Prelovsek et al., *JHEP* 06 (2021) 035

LQCD calculation of the $D\bar{D}$ & $D_s\bar{D}_s$ scattering indicates a possible $D\bar{D}$ bound state with $J^{PC} = 0^{++}$ and a binding energy of $E_B \equiv 2m_D - M_{D\bar{D}} = 4.0^{+5.0}_{-3.7}$ MeV

Dionium $A_{D^+D^-}$: The ground state $|D^+D^-\rangle$ atom with $C = +$ $\mu_0 = \frac{m_{D^0}}{2}$ $\mu_c = \frac{m_D}{2}$

Coulomb binding energies: $-E_n = -\frac{\alpha^2 \mu_c}{2n^2} = \frac{-E_1}{n^2} = -\frac{24.9 \text{ keV}}{n^2}$

Bohr radius of the D^+D^- bound state: $r_B = \frac{1}{\alpha \mu_c} = (6.82 \text{ MeV})^{-1} = 28.91 \text{ fm}$

Scale separation: $r_B \Lambda_{\text{QCD}} \gg 1$, strong interaction between D^+D^- is a correction

Effects of strong interaction at LO:

(a) **Energy level shift:** $\Delta E_n^{\text{str}} \sim \mathcal{O}(\alpha^3)$ (b) **Decay modes:** $D^0\bar{D}^0, \dots$

The decay width of the dionium is much smaller than the X atom and the dionium is more likely to be detected.



Dionium: NREFT

E. Braaten, E. Johnson, and H. Zhang, *JHEP* 02 (2018) 150

$D^0\bar{D}^0 - D^+D^-$ Coupled-channel system: thresholds $\Sigma_0 = 2m_{D^0}$, $\Sigma_c = 2m_D$;

difference between thresholds $\Delta = \Sigma_c - \Sigma_0$; initial energy relative to Σ_c $E = \sqrt{s} - \Sigma_c$

LO strong scattering amplitude: $\mathbf{T}_s(E) = \mathbf{V}_{\text{LO}}^\Lambda + \mathbf{V}_{\text{LO}}^\Lambda \mathbf{G}^\Lambda(E) \mathbf{T}_s(E)$

Contact potential: $\mathbf{V}_{\text{LO}} = \frac{1}{2} \begin{pmatrix} C_{0a} + C_{1a} & C_{0a} - C_{1a} \\ C_{0a} - C_{1a} & C_{0a} + C_{1a} \end{pmatrix}$. Green's functions: $\mathbf{G}^\Lambda(E) = \begin{pmatrix} G_0^\Lambda(E) & 0 \\ 0 & G_c^\Lambda(E) \end{pmatrix}$,

C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, *Phys. Rev. D* 87 (2013) 076006

$$G_0^\Lambda(E) = -\frac{\mu_0 \Lambda}{\pi^2} + J_0(E), \quad J_0(E) = \frac{\mu_0}{2\pi} \sqrt{-2\mu_0(E + \Delta)},$$

$$G_c^\Lambda(E) = -\frac{\mu_c \Lambda}{\pi^2} - \frac{\alpha \mu_c^2}{\pi} \left(\ln \frac{\Lambda}{\alpha \mu_c} - \gamma_E \right) + J_c(E), \quad J_c(E) = -\frac{\alpha \mu_c^2}{\pi} \left[\ln \eta + \frac{1}{2\eta} - \psi(-\eta) \right]$$

Kong, Ravndal, *Nucl. Phys. A* 665 (2000)

$$\eta = \frac{\alpha \mu_c}{\sqrt{-2\mu_c E}} \quad \psi(x) = \frac{d}{dx} \ln \Gamma(x)$$



Dionium: NREFT

LO strong scattering amplitude:

$$\begin{aligned}\mathbf{T}_s^{-1}(E) &= \left(\mathbf{V}_{\text{LO}}^{\Lambda}\right)^{-1} - \mathbf{G}^{\Lambda}(E) = \frac{1}{2} \begin{pmatrix} \frac{1}{C_{0a}} + \frac{1}{C_{1a}} & \frac{1}{C_{0a}} - \frac{1}{C_{1a}} \\ \frac{1}{C_{0a}} - \frac{1}{C_{1a}} & \frac{1}{C_{0a}} + \frac{1}{C_{1a}} + \delta \end{pmatrix} - \begin{pmatrix} G_0^{\Lambda}(E) & \\ & G_C^{\Lambda}(E) \end{pmatrix} \\ &= \begin{pmatrix} w_+ - J_0(E) & -w_- \\ -w_- & w_+ - J_c(E) + \mathcal{O}\left(\frac{\alpha\mu_c}{\Lambda_{\text{QCD}}}\right) \end{pmatrix}\end{aligned}$$

δ counts the isospin breaking effect and absorbs the logarithmic UV divergence

$$\mathbf{T}_s(E) = \frac{1}{\det} \begin{pmatrix} w_+ - J_c(E) & w_- \\ w_- & w_+ - J_0(E) \end{pmatrix}, \quad \det = [w_+ - J_0(E)][w_+ - J_c(E)] - w_-^2.$$

Renormalized constants:

$$w_+ = \frac{1}{2C_{1a}^R} + \frac{1}{2C_{0a}^R}, \quad w_- = \frac{1}{2C_{1a}^R} - \frac{1}{2C_{0a}^R}$$

$D\bar{D}$ S-wave scattering lengths:

$$\mathbf{T}_{s00}(E = -\Delta) \equiv -\frac{2\pi}{\mu_0}(a_0 + a_1) = \frac{w_+ - J_c(-\Delta)}{w_+[w_+ - J_c(-\Delta)] - w_-^2},$$

$$\mathbf{T}_{s01}(E = -\Delta) \equiv -\frac{2\pi}{\mu_0}(a_0 - a_1) = \frac{w_-}{w_+[w_+ - J_c(-\Delta)] - w_-^2},$$

$$C_{1a}^R \simeq \frac{-\frac{4\pi}{\mu_0}a_1}{1 + \frac{2\pi}{\mu_c}(a_0 - a_1)J_c(-\Delta)},$$

$$C_{0a}^R \simeq \frac{-\frac{4\pi}{\mu_0}a_0}{1 - \frac{2\pi}{\mu_c}(a_0 - a_1)J_c(-\Delta)}.$$



Dionium: Ground energy level shift

The dionium with a binding energy E_A is the lowest pole of $T_s(E)$: $\det|_{E=-E_A} = 0$.

Separating the ground Coulomb pole from $J_c(E)$:

$$J_c(E) = -\frac{\alpha\mu_c^2}{\pi} \left[\ln \eta - \frac{1}{2\eta} - \frac{1}{\eta+1} + 2 - \psi(2-\eta) \right] + \frac{\alpha^3\mu_c^3}{\pi[E - (-E_1)]}$$

LO ground energy level shift : $\Delta E_1^{\text{str}} - i\frac{\Gamma_1}{2} = -E_A + E_1$

$$R \equiv \frac{w_-^2}{w_+^2 + \left(\frac{\mu_0}{2\pi}\right)^2 2\mu_0\Delta}.$$

Expressed in $D\bar{D}$ scattering lengths:

Decay width of $A_{D^+D^-} \rightarrow D^0\bar{D}^0$:

$$\begin{aligned} \Delta E_1^{\text{str}} &= \frac{\alpha^3\mu_c^3}{\pi} \frac{w_+(1-R) - i\frac{\mu_0}{2\pi}\sqrt{2\mu_0\Delta}R}{w_+^2(1-R)^2 + \left(\frac{\mu_0}{2\pi}\right)^2 2\mu_0\Delta R^2} + O\left(\frac{\alpha^5\mu_c^{5/2}}{\Delta^{3/2}}\right), \\ w_+ &= -\frac{\frac{\mu_0}{2\pi}(a_0 + a_1) + (a_0 - a_1)^2 J_c(-\Delta)}{4a_0 a_1}, \\ w_- &= -\frac{\frac{\mu_0}{2\pi}(a_0 - a_1) + (a_0^2 - a_1^2) J_c(-\Delta)}{4a_0 a_1}, \end{aligned}$$

$$\Gamma_1 = \frac{2\alpha^3\mu_c^3}{\pi} \frac{\frac{\mu_0}{2\pi}R\sqrt{2\mu_0\Delta}}{w_+^2(1-R)^2 + \left(\frac{\mu_0}{2\pi}\right)^2 2\mu_0\Delta R^2}.$$



Dionium: Ground energy level shift

In the presence of a isoscalar $D\bar{D}$ bound state while no isovector bound state,

the isovector coupling is significantly weak, $|C_{0a}^R| \gg |C_{1a}^R|$ M. Albaladejo et al., *Eur. Phys. J. C* 75 (2015) 547

LO strong scattering amplitude for $C_{1a}^R = 0$: $\mathbf{T}_s(E) = \frac{1}{\frac{2}{C_{0a}^R} - J_0(E) - J_c(E)} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$

$D\bar{D}$ bound state pole at $E = -\Delta - E_B \equiv -\Delta_B$: $\frac{2}{C_{0a}^R} - J_0(E = -\Delta_B) - J_c(E = -\Delta_B) = 0$

LO strong energy level shift: $\Delta E_1^{\text{str}} - i\frac{\Gamma_1}{2} = -E_A + E_1 \simeq \frac{\alpha^3 \mu_c^3}{\pi} \frac{\frac{\mu_0}{2\pi} \sqrt{2\mu_0 E_B} + J_c(-\Delta_B) - i\frac{\mu_0}{2\pi} \sqrt{2\mu_0 \Delta}}{\left[\frac{\mu_0}{2\pi} \sqrt{2\mu_0 E_B} + J_c(-\Delta_B) \right]^2 + \left(\frac{\mu_0}{2\pi} \right)^2 2\mu_0 \Delta}$

Binding energy of the $D\bar{D}$ bound state from LQCD: $E_B = 4.0^{+5.0}_{-3.7}$ MeV

Binding energy and mass of the dionium: $\text{Re}E_A = E_1 - \Delta E_1^{\text{str}} \simeq 22.9^{+0.3}_{-0.4}$ keV, $m_{A_{D^+D^-}} = (3739.3 \pm 0.1)$ MeV

Decay width of $A_{D^+D^-} \rightarrow D^0\bar{D}^0$: $\Gamma_1 \simeq 1.8^{+1.4}_{-0.6}$ keV

Effective couplings: $D^+D^- \rightarrow A_{D^+D^-}$ $g_{\text{str}}^2 = \lim_{E \rightarrow -E_A} (E + E_A) \mathbf{T}_{s22}(E) \simeq [3.2^{+2.1}_{-2.8} - (3.5^{+4.0}_{-1.5})i] \times 10^{-8} \text{ MeV}^{-1}$

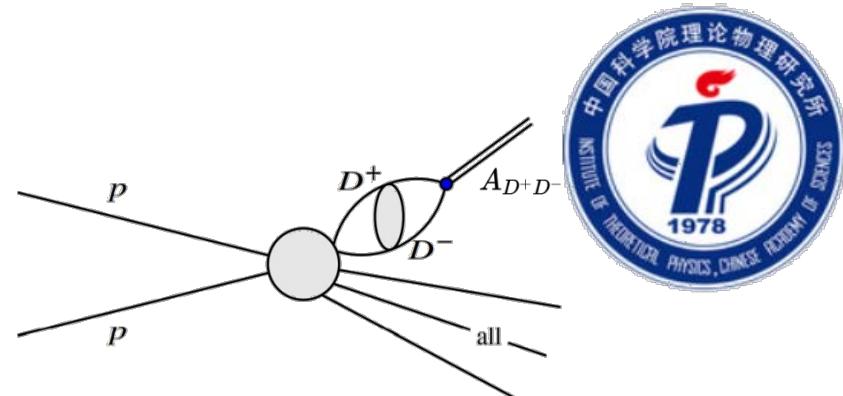
$D^0\bar{D}^0 \rightarrow (D\bar{D})_{\text{molecule}}$ $g_0^2 = \lim_{E \rightarrow -\Delta_B} (E + \Delta_B) \mathbf{T}_{s11}(E) \simeq (3.9^{+1.4}_{-2.5}) \times 10^{-4} \text{ MeV}^{-1}$

Dionium: Production in pp collisions

The cross section for dionium production

F.-K. Guo et al., *Eur. Phys. J. C* 74 (2014) 3063

$$\sigma[A_{D^+D^-} + \text{all}] \simeq \frac{m_{A_{D^+D^-}}}{m_{D^+}m_{D^-}} |G_c^\Lambda(E)g_{\text{str}}|^2 \left(\frac{d\sigma[D^+D^- + \text{all}]}{dk} \right)_{\text{MC}} \frac{4\pi^2\mu_c}{k^2}$$



P. Artoisenet, E. Braaten, *Phys. Rev. D* 81 (2010) 114018

The general differential Monte Carlo cross section for the D^+D^- pair production:

$$d\sigma[D^+D^- + \text{all}]_{\text{MC}} = \frac{1}{2} K_{D^+D^-} \frac{1}{\text{flux}} \sum_{\text{all}} \int d\phi_{[D^+D^-+\text{all}]} |\mathcal{M}[D^+D^- + \text{all}]|^2 \frac{d^3\mathbf{k}_{D^+D^-}}{(2\pi)^3 2\mu_c}$$

$K_{D^+D^-} \simeq 1$: Difference between MC and experimental data

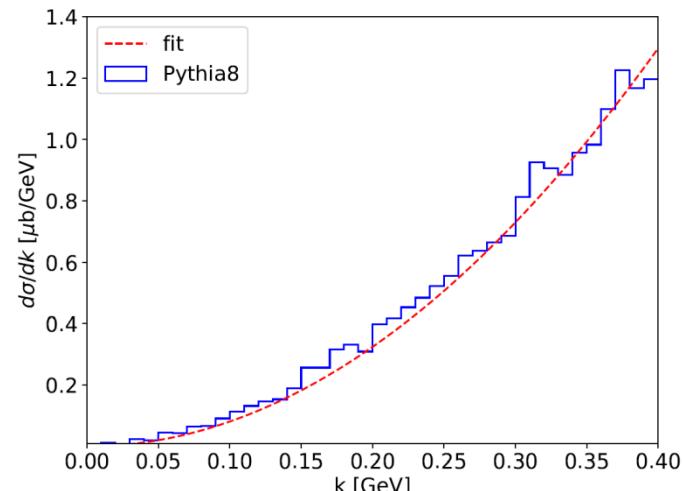
$\left(\frac{d\sigma[D^+D^- + \text{all}]}{dk} \right)_{\text{MC}}$ from MC generator - PYTHIA8 with CMS parameters

P. Artoisenet et al.(CMS), *JHEP* 04 (2013) 154

$$\left(\frac{d\sigma[D^+D^-]}{dk} \right)_{\text{MC}} \simeq 3.34 \times 10^6 \frac{(k/\mu_c)^2}{s_{pp}(m_{D^+} + m_{D^-})}$$

- pp collisions with $\sqrt{s_{pp}} = 7 \text{ TeV}$
- Full QCD $2 \rightarrow 2$ process
- Transverse momentum

$10 < p_T < 50 \text{ GeV}$





Dionium: Production in pp collisions

The cross sections for $pp \rightarrow A_{D^+D^-} + \text{all}$ with different cutoffs in $G_c^\Lambda(E)$ at CMS

TABLE I. Order-of-magnitude estimates of the integrated cross section for $pp \rightarrow A_{D^+D^-} + \text{all}$ at $\sqrt{s_{pp}} = 7 \text{ TeV}$ with $10 < p_T < 50 \text{ GeV}$. In the first row, Λ is the sharp cutoff used to regularize the Green's function in Eq. (9).

Λ (GeV)	0.5	1.0
$\sigma(pp \rightarrow A_{D^+D^-} + \text{all})$ (pb)	1^{+7}_{-1}	49^{+76}_{-33}

At CMS, $\sqrt{s_{pp}} = 7 \text{ TeV}$, $10 < p_T < 50 \text{ GeV}$, $\sigma[pp \rightarrow A_{D^+D^-} + \text{all}] \sim \mathcal{O}(10 \text{ pb})$, $\sigma[pp \rightarrow X(3872) + \text{all}] \sim \mathcal{O}(10 \text{ nb})$

$$\frac{\sigma[pp \rightarrow A_{D^+D^-} + \text{all}]}{\sigma[pp \rightarrow X(3872) + \text{all}]} \sim \frac{\sigma[pp \rightarrow X_{\text{atom}} + \text{all}]}{\sigma[pp \rightarrow X(3872) + \text{all}]} \sim \mathcal{O}(10^{-3}) \quad \sigma_{X(3872)} \text{Br}[X(3872) \rightarrow J/\psi \pi^+ \pi^-] = (1.06 \pm 0.11 \pm 0.15) \text{ nb}$$

P. Artoisenet et al.(CMS), *JHEP* 04 (2013) 154

At LHCb, $\sqrt{s_{pp}} = 7 \text{ TeV}$, $5 < p_T < 20 \text{ GeV}$, **rapidity** $2.5 < y < 4.5$

$$\sigma_{X(3872)} \text{Br}[X(3872) \rightarrow J/\psi \pi^+ \pi^-] = (5.4 \pm 1.3 \pm 0.8) \text{ nb}$$

Long-distance part of the production of both the $X(3872)$ and dionium is universal

$$\left(\frac{\sigma_{A_{D^+D^-}}}{\sigma_{X(3872)}} \right)_{\text{CMS}} \sim \left(\frac{\sigma_{A_{D^+D^-}}}{\sigma_{X(3872)}} \right)_{\text{LHCb}} \quad \sigma[pp \rightarrow A_{D^+D^-} + \text{all}] \sim \mathcal{O}(0.1 \text{ nb})$$

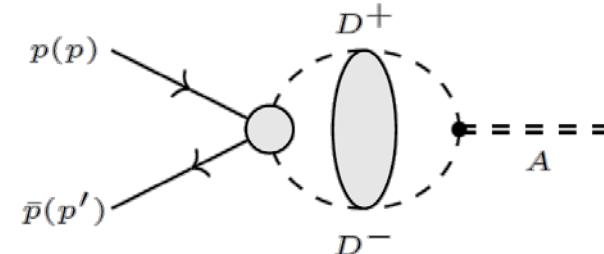


Dionium: Production in $p\bar{p}$ collisions

Scattering amplitude for $p\bar{p} \rightarrow A_{D^+D^-}$:

$$i\mathcal{M}_{A_{D^+D^-}} = i\mathcal{M}_{\text{ISI}}\mathcal{M}_{p\bar{p} \rightarrow D^+D^-} \frac{\sqrt{8m_{A_{D^+D^-}} m_{D^+} m_{D^-}} G_c^\Lambda g_{\text{str}}}{4m_{D^+} m_{D^-}}$$

$p\bar{p}$ initial state interaction: $|\mathcal{M}_{\text{ISI}}|^2 \approx 0.2$



Y. Dong et al., *Phys. Rev. D* **90** (2014) 094001

Estimate of the $p\bar{p} \rightarrow D^+D^-$ amplitude:

$$i\mathcal{M}_{p\bar{p} \rightarrow D^+D^-} \simeq -\bar{v}(p') g_s \gamma^5 \frac{i F_{p\Sigma_c D}^2}{m_{\Sigma_c}} g_s \gamma^5 u(p) + \mathcal{O}(m_{\Sigma_c}^{-2})$$

$N\Sigma_c D$ coupling from the flavor SU(4) model: $g_s = 2.69$ R. Shyam, H. Lenske, *Phys. Rev. D* **93** (2016) 034016

Form factor: $F_{p\Sigma_c D}^2 = \frac{\Lambda_1^4}{[(p-k)^2 - m_{\Sigma_c}^2]^2 + \Lambda_1^4}$, $\Lambda_1 = 2.0$ GeV S. Sakai, H.-J. Jing, F.-K Guo, *Phys. Rev. D* **102** (2020) 114041

Cross section at $\sqrt{s_{p\bar{p}}} = m_{A_{D^+D^-}}$:

$$\sigma[p\bar{p} \rightarrow A_{D^+D^-}] = \frac{1}{\Gamma_{A_{D^+D^-}} \sqrt{s\lambda(s, m_p^2, m_p^2)}} |\mathcal{M}_{A_{D^+D^-}}|^2 = \frac{(E_p^2 - m_p^2) |\mathcal{M}_{\text{ISI}}|^2}{\Gamma_{A_{D^+D^-}} \sqrt{s\lambda(s, m_p^2, m_p^2)}} \frac{m_{A_{D^+D^-}} g_s^4 F_{p\Sigma_c D}^4}{m_{D^+} m_{D^-} m_{\Sigma_c}^2} |G_c^\Lambda g_{\text{str}}|^2$$

$$\text{Br}[A_{D^+D^-} \rightarrow D^0 \bar{D}^0] \simeq 1$$

$$\Gamma_{A_{D^+D^-}} \simeq \Gamma_1 \simeq 1.8 \text{ keV}$$

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

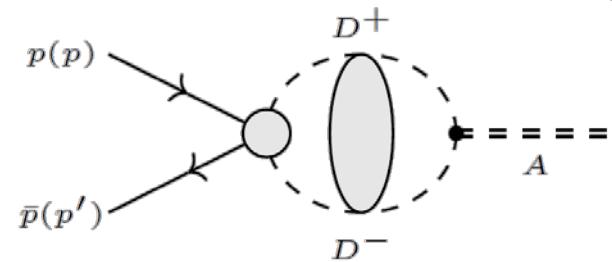


Dionium: Production in $p\bar{p}$ collisions

The cross sections for $p\bar{p} \rightarrow A_{D^+D^-}$ with different cutoffs in $G_c^\Lambda(E)$ at PANDA:

TABLE II. Order-of-magnitude estimates of the integrated cross section for $p\bar{p} \rightarrow A_{D^+D^-}$. In the first row, the cutoff Λ originated from the regularized Green function in Eq. (9).

Λ (GeV)	0.5	1.0
$\sigma(p\bar{p} \rightarrow A_{D^+D^-})$ (μb)	$0.002^{+0.013}_{-0.002}$	$0.1^{+0.2}_{-0.1}$
PANDA events at $\sqrt{s_{p\bar{p}}} = 3739.3$ MeV	0.04×10^8	2.0×10^8



The integrated luminosity of PANDA at $\sqrt{s_{p\bar{p}}} = 3872$ MeV is about 2 fb^{-1} in five month.

G. Barucca et al.(PANDA), *Eur. Phys. J. A* 55 (2019) 42

Assume the integrated luminosity is same at $\sqrt{s_{p\bar{p}}} = 3739.3$ MeV

Reconstructing through the reaction chain $p\bar{p} \rightarrow A_{D^+D^-} \rightarrow D^0\bar{D}^0 \rightarrow K^-\pi^+K^+\pi^-$:

$$\sigma[p\bar{p} \rightarrow A_{D^+D^-} \rightarrow D^0\bar{D}^0] = \sigma[p\bar{p} \rightarrow A_{D^+D^-}] \text{Br}[A_{D^+D^-} \rightarrow D^0\bar{D}^0] \simeq \sigma[p\bar{p} \rightarrow A_{D^+D^-}] \quad \text{Br}[A_{D^+D^-} \rightarrow D^0\bar{D}^0] \simeq 1$$

$$\text{Br}[D^0\bar{D}^0 \rightarrow K^-\pi^+K^+\pi^-] \simeq 4\% \quad \mathcal{O}(10^3 \sim 10^5) \text{ events at PANDA!}$$

P. Zyla et al.(PDG), *Prog. Theor. Exp. Phys.* 2020 (2020) 083C01



Summary

- We show that the null signal of the X atom can be used to put a low limit on the binding energy of the $X(3872)$.
- We estimate that the cross section of the production of the dionium should be of $\mathcal{O}(0.1 \text{ nb})$, and thus the observation of the dionium at LHCb is promising.
- We expect the dionium can be detected at PANDA, because there can be $\mathcal{O}(10^3 \sim 10^5)$ events of the reaction chain $p\bar{p} \rightarrow A_{D^+D^-} \rightarrow D^0\bar{D}^0 \rightarrow K^-\pi^+K^+\pi^-$.

Thank you for your attention!

Back Up

Line Shape of the X atom

Binding energy of the X atom: $E_{XA} \sim \text{Re } E_{A1} = E_1 - \frac{\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} \simeq 22.92 \text{ keV}$

Decay width of the X atom: $\Gamma_{XA} \sim \Gamma_c + 2 \text{Im } E_{A1} = \Gamma_c + \frac{2\alpha^3 \mu_c^2}{\sqrt{2\mu_c \Delta}} = (89.2 \pm 1.8) \text{ keV} \gg E_{XA}$

The line shape of the X atom is more like the line shape of the Toponium.

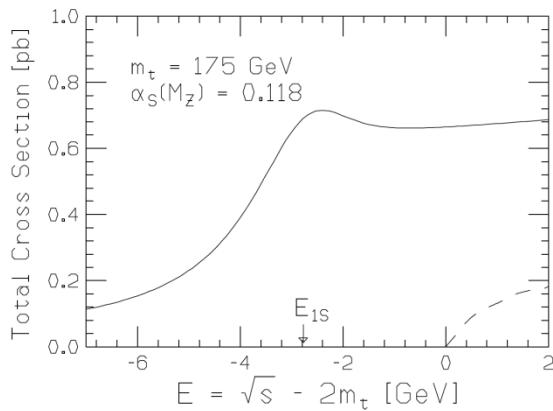


Fig. 4. The total cross section vs. energy, $E = \sqrt{s} - 2m_t$. The solid curve is calculated from the Green function. The dashed curve shows the tree-level total cross section for a stable top quark.

Total cross section of the Toponium.

Y. Sumino, Adv. Ser. Direct. High Energy Phys. 19, 135(2005)

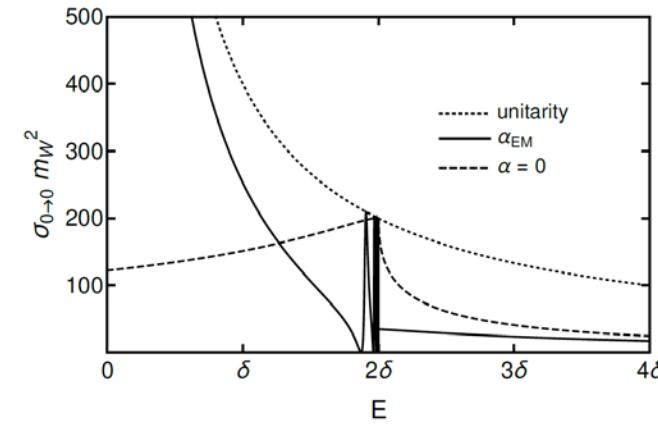


Figure 4. Neutral-wino elastic cross section $\sigma_{0 \rightarrow 0}$ as a function of the energy E . The cross section for $M_* = 2.39 \text{ TeV}$ is shown for $\alpha = 1/137$ (solid curve) and for $\alpha = 0$ (dashed curve). The S-wave unitarity bound is shown as a dotted curve.

Total cross section of the Neutral-wino.

E. Braaten, E. Johnson and H. Zhang, J. High Energy Phys. 02(2018) 150

3-body treatment for the $X(3872)$

V. Baru et al., *Phys. Rev. D* **84** (2011) 074029

V. BARU *et al.*

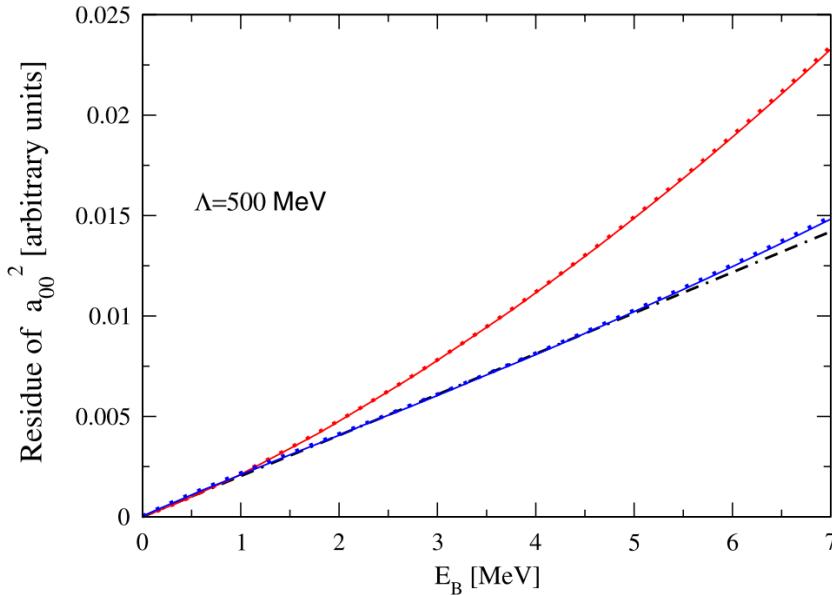


FIG. 3 (color online). Residue of the $D^0\bar{D}^{*0}$ scattering amplitude squared versus the binding energy in the $D^0\bar{D}^{*0}$ system. The upper, red (lower, blue) dotted curve corresponds to the solution of the single(two)-channel $D^0\bar{D}^{*0}$ problem with the contact $D\bar{D}^*$ interaction. Solutions of the full three-body equation with dynamical pions are given by the solid lines: upper, red line—for the single-channel case and lower, blue line—for the two-channel case. The straight dot-dashed line (black) is shown to guide the eye.

PHYSICAL REVIEW D **84**, 074029 (2011)

Three-body $D\bar{D}\pi$ dynamics for the $X(3872)$

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(Received 2 September 2011; published 20 October 2011)

Single Channel $D^0\bar{D}^{*0}$

Coupled Channel $D^0\bar{D}^{*0}$ D^+D^{*-}

In addition, we found that the residue for $X \rightarrow D\bar{D}^*$ is weakly dependent on the kind of pion dynamics included. Especially, the dependence of the residue on the X binding energy is very close for a fully dynamical calculation and for a calculation with a contact-type interaction only. A deviation between the coupled-channel and the single-channel treatment is clearly observed but with the larger effect for binding energies beyond 1 MeV.

3-body treatment for the $X(3872)$

V. Baru et al. , *Phys. Rev. D* 84 (2011) 074029

THREE-BODY $D\bar{D}\pi$ DYNAMICS FOR THE $X(3872)$

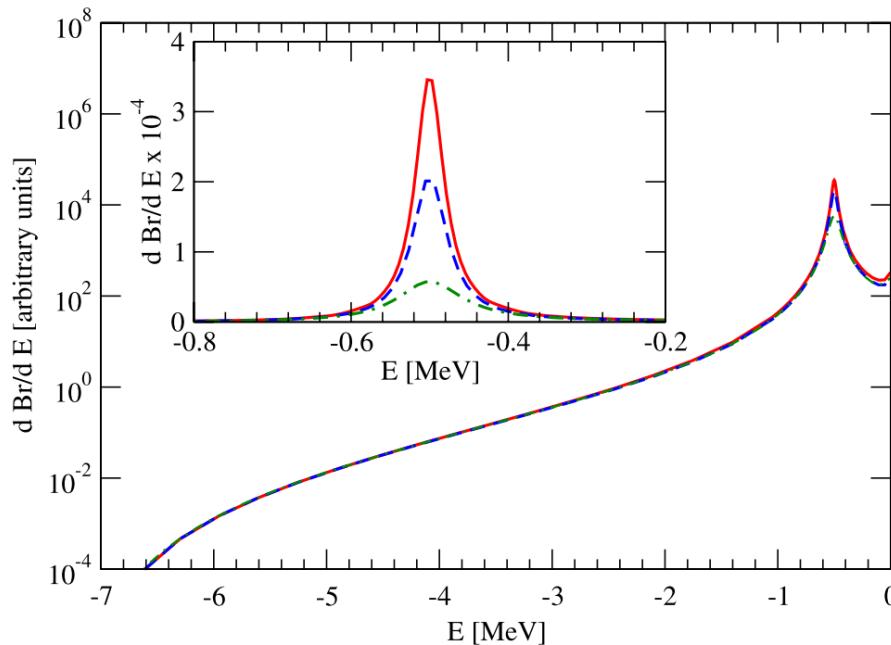


FIG. 5 (color online). Production rate (in logarithmic scale) for the three calculations as described in the text: (i) solution of the single-channel problem in the static limit—(green) dot-dashed line; (ii) solution of the single-channel dynamical calculation—(blue) dashed line; (iii) solution of the full two-channel dynamical problem—(red) solid line. All curves are normalized near the $D^0\bar{D}^0\pi^0$ threshold, located at $E = -7$ MeV. The inlay shows a zoom into the peak region in linear scale.

The most striking effect of dynamical pions is observed in their impact on the X line shapes: in the fully dynamical calculation the width from the $D\bar{D}\pi$ intermediate states appears to be reduced by about a factor of 2, from 102 keV down to 44 keV, assuming that the $X(3872)$ corresponds to a resonance state with a peak at 0.5 MeV below the $D\bar{D}^*$ threshold. Stated differently, by using the naive static approximation for the $D\bar{D}\pi$ intermediate states one overestimates substantially their effect on the X width.

On the contrary, the effect of the coupled-channel dynamics on the X width turned out to be rather moderate, which can be attributed to the fact that both the real part of the resonance pole E_B and the X width Γ_X are small as compared to the separation ΔM between the neutral and the charged thresholds.