On η_c and $\eta_c(2S)$ decays

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on line

Based on Hongpeng Wang （王宏鹏）& CZY, arXiv:2112.08584

The “12% rule” in vector charmonium decays

M. Appelquist and H. D. Politzer, PRL34, 43 (1975)

$$\Gamma_h = |M_h|^2 |\Psi(0)|^2$$

$$= (2/9\pi)(\pi^2 - 9)\tfrac{5}{18}\alpha_s^3(\tfrac{4}{3}\alpha_s)^3 m_{\psi'}. \qquad (3)$$

The leptonic width via one photon into $\bar{l}l$ is

$$\Gamma_l = |M_l|^2 |\Psi(0)|^2 = \tfrac{1}{2}(\tfrac{2}{3}\alpha)^2(\tfrac{4}{3}\alpha_s)^3 m_{\psi'}, \qquad (4)$$

where $\alpha \approx \frac{1}{137}$. Although separately these calculations are not trustworthy, the ratio

$$\frac{\Gamma_l}{\Gamma_h} = \frac{\frac{2}{9}\alpha^2}{(2/9\pi)(\pi^2 - 9)5/\alpha_s^3} \qquad (5)$$

is independent of wave-function effects.

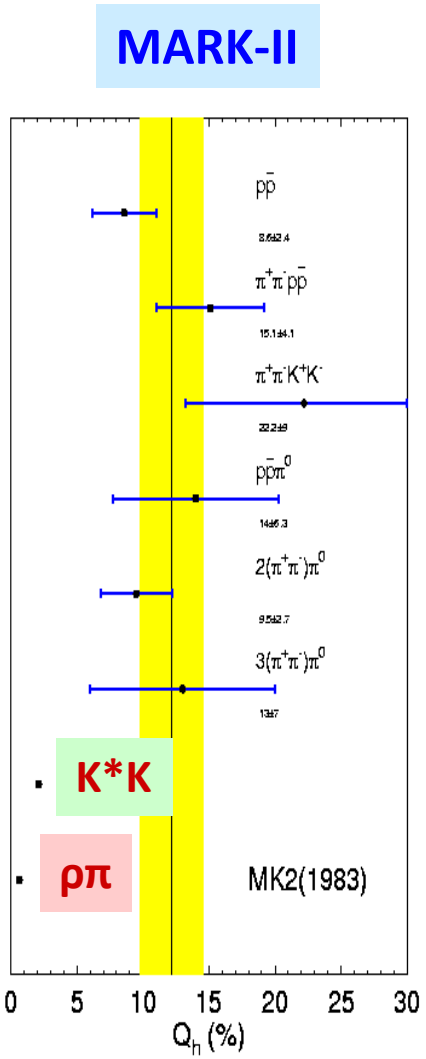
$$Q^V = \frac{B_{\psi(2S)\to X}}{B_{J/\psi\to X}} = \frac{B_{\psi(2S)\to e^+e^-}}{B_{J/\psi\to e^+e^-}} = 12\%$$

$\approx 13.3\%$ [PDG2021 data]

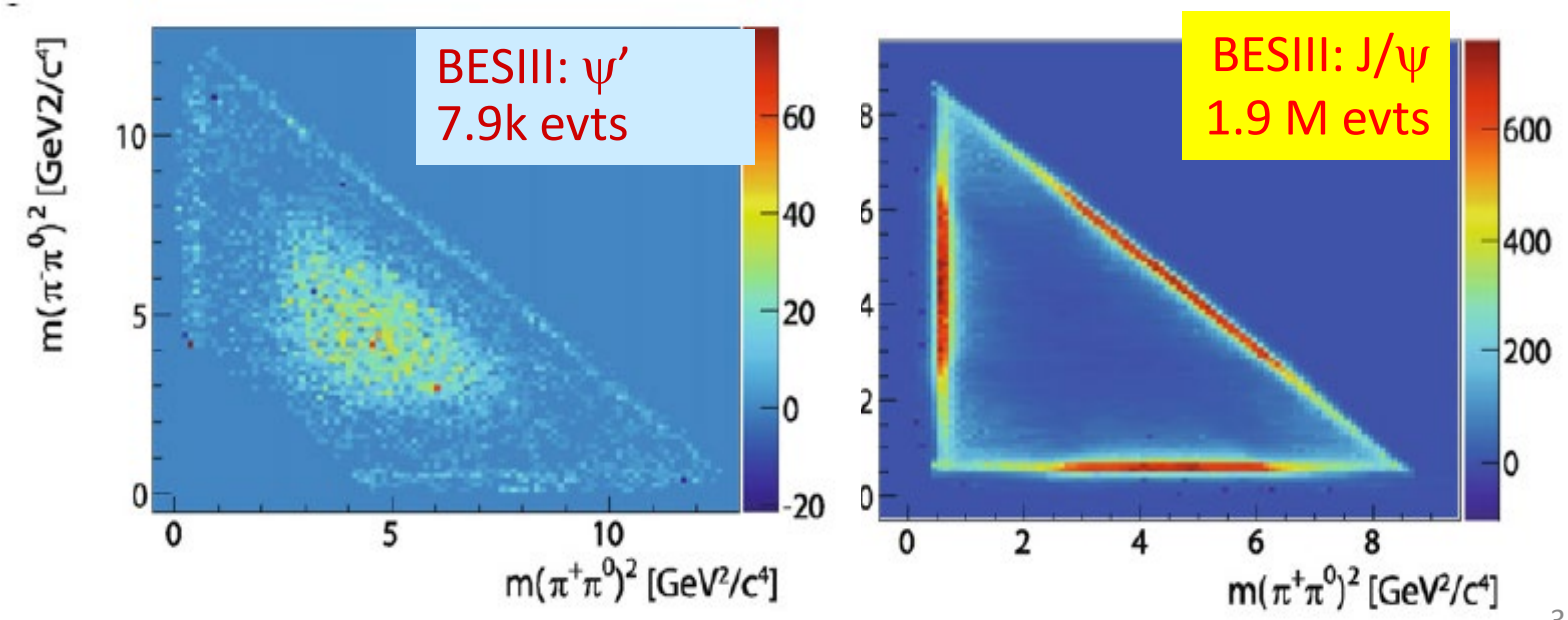
$Q^V = 13.3\%$

“12% rule” and “ρπ puzzle”

- Violation found by Mark-II , confirmed by BESI at higher sensitivity.
- Extensively studied by BESI/BESII/CLEOc/BESIII
- Dozens of models, none satisfactory



$\psi' / J/\psi \rightarrow \pi^+\pi^-\pi^0$



Extension of “12% rule” to pseudoscalar charmonia

- Mauro Anselmino, Marco Genovese, and Enrico Predazzi, Phys. Rev. D 44, 1597 (1991),

$$\frac{\mathrm{BR}(\eta_c' \to h)}{\mathrm{BR}(\eta_c \to h)} \approx \frac{\mathrm{BR}(\psi' \to h)}{\mathrm{BR}(J/\psi \to h)} = 0.128$$

“12% rule”

$Q^P = Q^V$

- Kuang-Ta Chao, Yi-Fan Gu, and S.F. Tuan, Commun. Theor. Phys. 25 (1996) 471-478,

In contrast to Anselmino *et al.*, we argue that, unlike the ψ', J/ψ case (2), the branching ratio relationship for η_c' and η_c to a light hadronic channel h is

$$\frac{\mathrm{BR}(\eta_c' \to h)}{\mathrm{BR}(\eta_c \to h)} \approx 1\,. \qquad (8)$$

$Q^P=1$

- Qian Wang, Xiao-Hai Liu, and Qiang Zhao, Physics Letters B 711 (2012) 364–370,

$$R_{\eta_c\eta_c'} \equiv \frac{BR(\eta_c' \to 2g)}{BR(\eta_c \to 2g)} = \frac{BR(\eta_c' \to \gamma\gamma)}{BR(\eta_c \to \gamma\gamma)}.$$

$Q^P=1.18\pm0.81$ [PDG2021]

Experimental data [PDG2021]

	Mode	Fraction (Γ_i/Γ)	Confidence level
	$\eta_c(1S)$ DECAY MODES		
Γ_1	$\eta'(958)\pi\pi$	(4.1 ± 1.7) %	
Γ_2	$\rho\rho$	(1.8 ± 0.5) %	
Γ_3	$K^*(892)^0 K^-\pi^+ +$ c.c.	(2.0 ± 0.7) %	
Γ_4	$K^*(892)\overline{K}^*(892)$	$(6.9 \pm 1.3) \times 10^{-3}$	
Γ_5	$K^*(892)^0\overline{K}^*(892)^0\pi^+\pi^-$	(1.1 ± 0.5) %	
Γ_{27}	$K\overline{K}\pi$	(7.3 ± 0.4) %	
Γ_{28}	$K\overline{K}\eta$	(1.36 ± 0.15) %	
Γ_{29}	$\eta\pi^+\pi^-$	(1.7 ± 0.6) %	
Γ_{30}	$\eta 2(\pi^+\pi^-)$	(4.4 ± 1.6) %	
Γ_{31}	$K^+K^-\pi^+\pi^-$	$(6.6 \pm 1.1) \times 10^{-3}$	
Γ_{32}	$K^+K^-\pi^+\pi^-\pi^0$	(3.5 ± 0.6) %	
Γ_{33}	$K^0K^-\pi^+\pi^-\pi^+ +$c.c.	(5.6 ± 1.9) %	
Γ_{34}	$K^+K^-2(\pi^+\pi^-)$	$(7.5 \pm 2.4) \times 10^{-3}$	
Γ_{35}	$2(K^+K^-)$	$(1.43 \pm 0.30) \times 10^{-3}$	
Γ_{36}	$\pi^+\pi^-\pi^0$	$< 5 \times 10^{-4}$	
Γ_{37}	$\pi^+\pi^-\pi^0\pi^0$	(4.7 ± 1.4) %	
Γ_{38}	$2(\pi^+\pi^-)$	$(9.1 \pm 1.2) \times 10^{-3}$	
Γ_{39}	$2(\pi^+\pi^-\pi^0)$	(15.8 ± 2.3) %	
Γ_{40}	$3(\pi^+\pi^-)$	(1.7 ± 0.4) %	
Γ_{41}	$p\overline{p}$	$(1.44 \pm 0.14) \times 10^{-3}$	
Γ_{42}	$p\overline{p}\pi^0$	$(3.6 \pm 1.5) \times 10^{-3}$	
Γ_{43}	$\Lambda\overline{\Lambda}$	$(1.06 \pm 0.23) \times 10^{-3}$	
Γ_{44}	$K^+\overline{p}\Lambda +$ c.c.	$(2.5 \pm 0.4) \times 10^{-3}$	
Γ_{45}	$\overline{\Lambda}(1520)\Lambda +$ c.c.	$(3.1 \pm 1.3) \times 10^{-3}$	
Γ_{46}	$\Sigma^+\overline{\Sigma}^-$	$(2.1 \pm 0.6) \times 10^{-3}$	
Γ_{47}	$\Xi^-\overline{\Xi}^+$	$(9.0 \pm 2.6) \times 10^{-4}$	
Γ_{48}	$\pi^+\pi^-p\overline{p}$	$(5.3 \pm 2.1) \times 10^{-3}$	

$\eta_c(2S)$ DECAY MODES

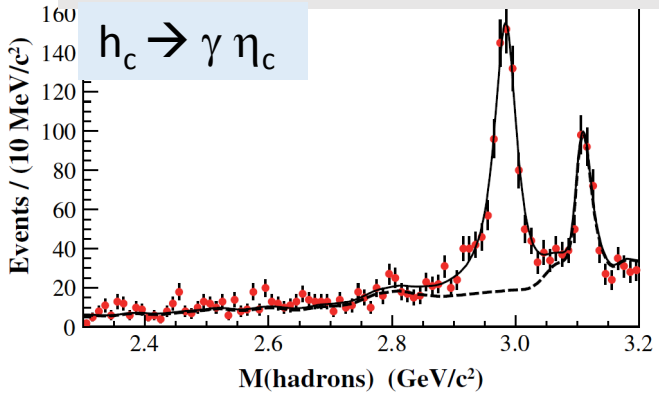
	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	hadrons	not seen	
Γ_2	$K\overline{K}\pi$	(1.9 ± 1.2) %	
Γ_3	$K\overline{K}\eta$	$(5 \pm 4) \times 10^{-3}$	
Γ_4	$2\pi^+2\pi^-$	not seen	
Γ_5	$\rho^0\rho^0$	not seen	
Γ_6	$3\pi^+3\pi^-$	not seen	
Γ_7	$K^+K^-\pi^+\pi^-$	not seen	
Γ_8	$K^{*0}\overline{K}^{*0}$	not seen	
Γ_9	$K^+K^-\pi^+\pi^-\pi^0$	(1.4 ± 1.0) %	
Γ_{10}	$K^+K^-2\pi^+2\pi^-$	not seen	
Γ_{11}	$K^0_S K^-2\pi^+\pi^- +$ c.c.	seen	
Γ_{12}	$2K^+2K^-$	not seen	
Γ_{13}	$\phi\phi$	not seen	
Γ_{14}	$p\overline{p}$	seen	
Γ_{15}	$p\overline{p}\pi^+\pi^-$	seen	
Γ_{16}	$\gamma\gamma$	$(1.9 \pm 1.3) \times 10^{-4}$	
Γ_{17}	$\gamma J/\psi(1S)$	< 1.4 %	90%
Γ_{18}	$\pi^+\pi^-\eta$	not seen	
Γ_{19}	$\pi^+\pi^-\eta'$	not seen	
Γ_{20}	$\pi^+\pi^-\eta_c(1S)$	< 25 %	90%

- PDG only listed a few modes with large errors in BFs.
- Experimental measurements were not used properly.
- Q^P was not calculated for any mode. ➔ Let's do it!

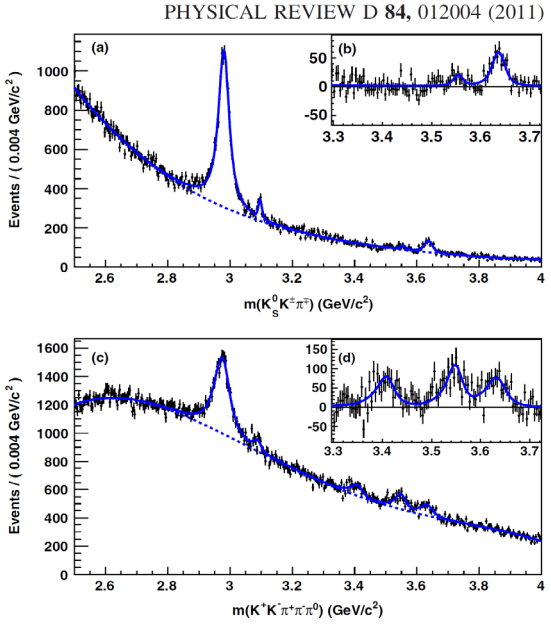
Data on η_c & $\eta_c(2S)$ production and decays

$\eta_c \rightarrow$ ...	$\eta_c(2S) \rightarrow$ ...	experiments
$\gamma\gamma \rightarrow \eta_c \rightarrow$ hadrons	$\gamma\gamma \rightarrow \eta_c(2S) \rightarrow$ hadrons	BaBar, Belle, Delphi, AMY, CLEO
$p\bar{p} \rightarrow \eta_c \rightarrow \gamma\gamma$	$p\bar{p} \rightarrow \eta_c(2S) \rightarrow \gamma\gamma$	E760, E835
$B^+ \rightarrow \eta_c K^+$	$B^+ \rightarrow \eta_c(2S) K^+$	BaBar, Belle, LHCb
$\psi(2S) \rightarrow \pi^0 h_c \rightarrow \pi^0 \gamma \eta_c$	$\psi(2S) \rightarrow \gamma \eta_c(2S)$	BESIII, CLEO

BESIII, PRD 86, 092009 (2012)



BaBar



E835, PLB 566 (2003) 45

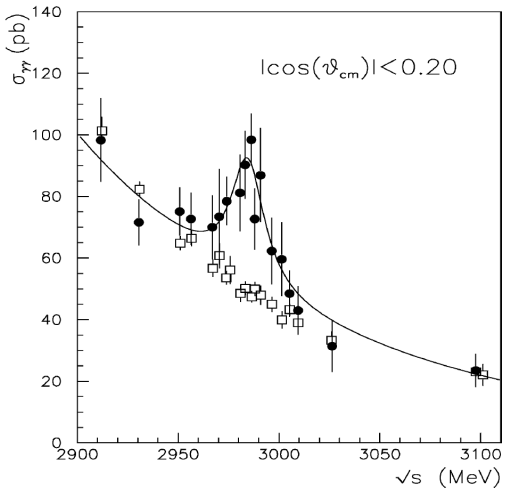
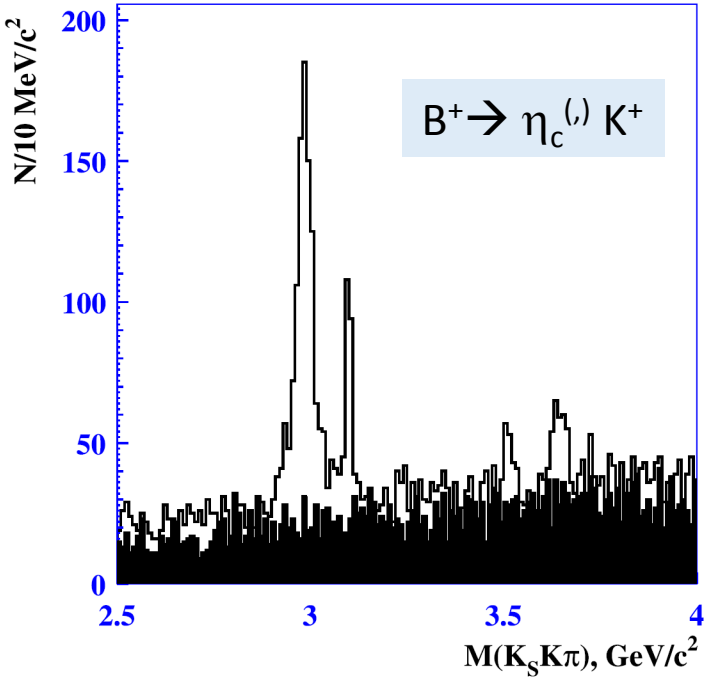
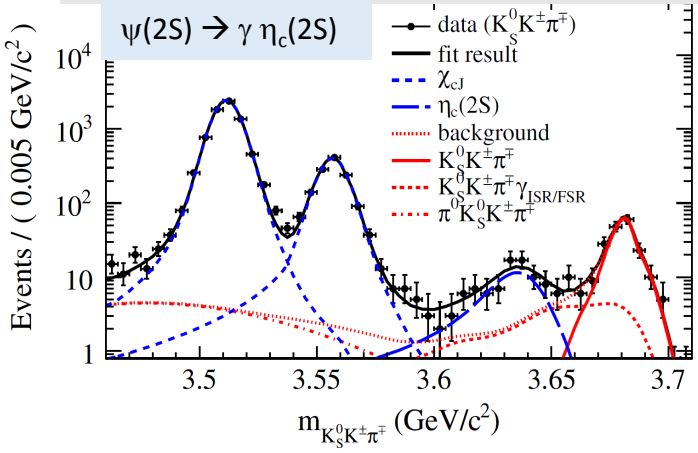


Fig. 2. Measured $\gamma\gamma$ cross sections for $\cos(\theta_{cm})_{max} = 0.20$ (solid circles). The open squares are the calculated feed-down cross section. The curve represents the best fit to a Breit–Wigner resonance on a power law background (see text).

Belle, PLB 706 (2011) 139



BESIII, PRL 109, 042003 (2012)



Usually both $\eta_c(2S)$ & η_c are measured in the same analysis, many systematic bias cancels out.

Data on η_c & $\eta_c(2S)$ production and decays

- Data from $J/\psi \rightarrow \gamma\, \eta_c$ are not used [BES, DM2, MK3, CBAL, ...]
  - Interference with non-η_c amplitude not treated properly
  - Strong model dependence in η_c line shape
  - Early experiments used wrong (too narrow) η_c width

Jorge Segovia and Jaume Tarrús Castella, PRD 104, 074032 (2021)

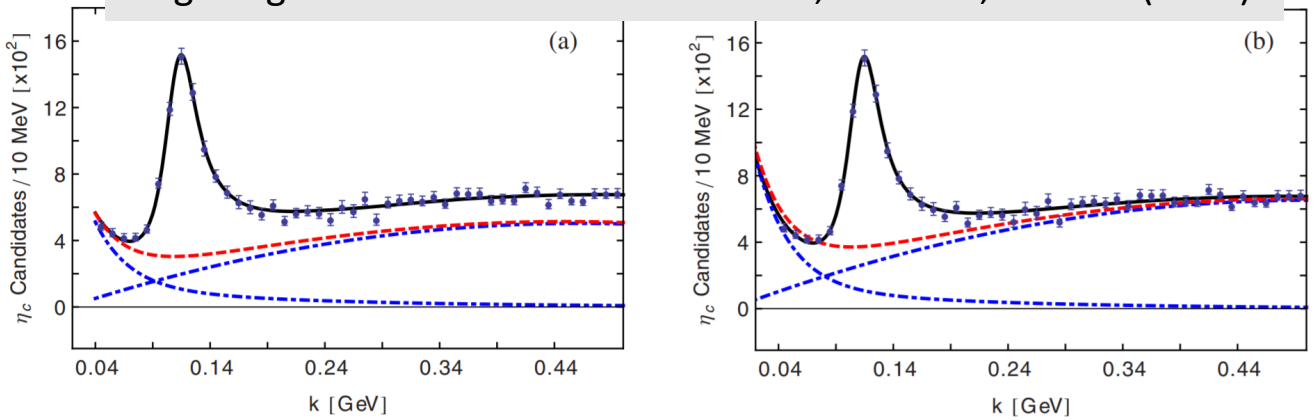


FIG. 2. The (black) solid line is the fit to the photon spectrum in exclusive $J/\psi \rightarrow \gamma\eta_c \rightarrow \gamma X$ decays using the unsubtracted pNRQCD line shape of Eq. (3) (a) and using the subtracted pNRQCD line shape of Eq. (6) (b). Experimental data points are taken from Ref. [7].

	m_{η_c} (MeV)	Γ_{η_c} (MeV)	$\mathcal{B}_{1S}^{\rm exp}$ (%)
pNRQCD	2985.8 ± 0.6	29.7 ± 1.7	(3.86 ± 0.33)
pNRQCD$_{\rm sub}$	2985.8 ± 0.6	29.7 ± 1.7	(2.17 ± 0.18)
CLEO	2982.2 ± 0.6	31.5 ± 1.5	$(1.98 \pm 0.09 \pm 0.30)$
KEDR	$2983.5 \pm 1.4^{+1.6}_{-3.6}$	$27.2 \pm 3.1^{+5.4}_{-2.6}$	(3.40 ± 0.33)

BES Collaboration / Physics Letters B 578 (2004) 16–22

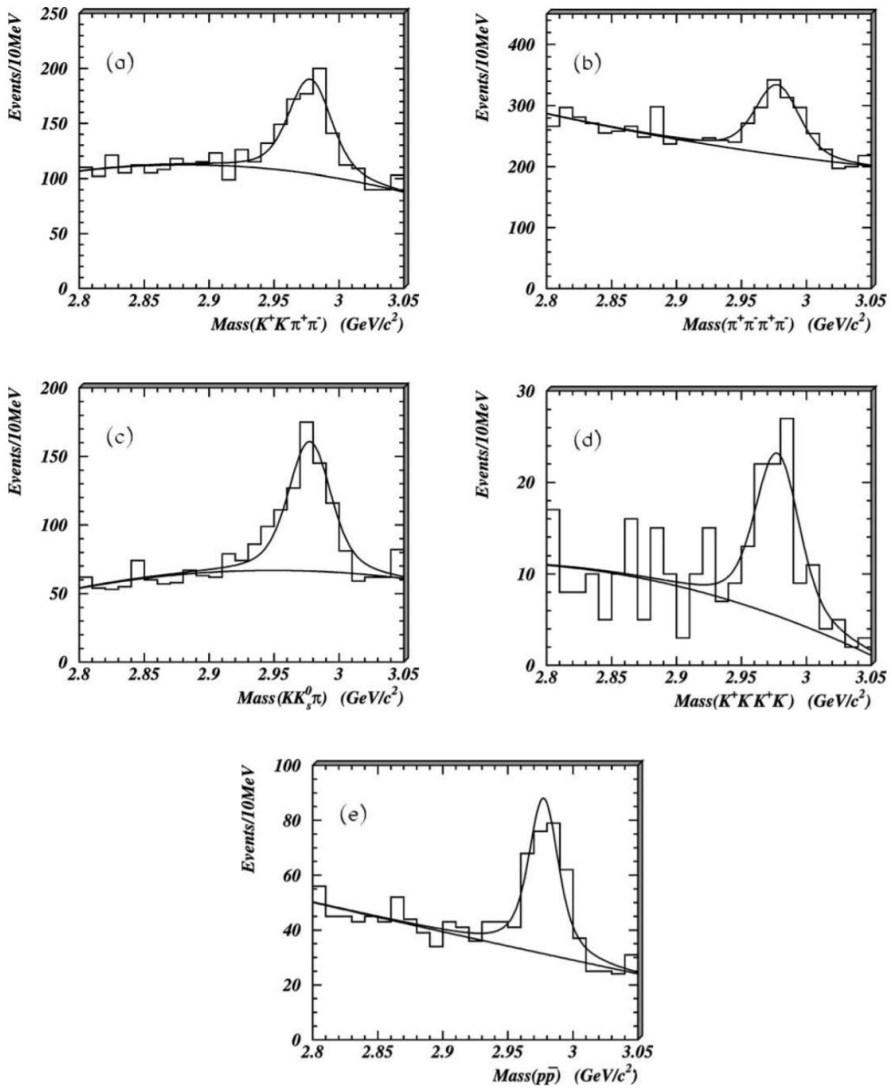


Fig. 1. Invariant mass distributions in the η_c region (a) $m_{K^+K^-\pi^+\pi^-}$, (b) $m_{\pi^+\pi^-\pi^+\pi^-}$, (c) $m_{K^\pm K^0_S\pi^\mp}$, (d) $m_{\phi\phi}$ and (e) $m_{p\bar{p}}$. The histograms correspond to the data; the curves are the fit result.

Global analysis to extract η_c & $\eta_c(2S)$ decay BFs

- Measurements used in the analysis
  - all measured $\eta_c(2S)$ decay modes & corresponding η_c decay modes
  - A few additional modes with large η_c decay BFs
  - η_c & $\eta_c(2S)$ widths — for $\Gamma_{\gamma\gamma}$ determination
- 97 input data from the AMY, BaBar, Belle, BESIII, CLEO, DELPHI, E760, E835, and LHCb experiments
- 29 free parameters
- Least square fit assuming Gaussian errors

$$\chi^2 = \sum_{i=1}^{97} \frac{(xi - \widehat{xi})^2}{\sigma_{xi}^2}$$

Experimental measurements

TABLE III: Data used in the analysis: absolute branching fractions and the ratios of the branching fractions for η_c and $\eta_c(2S)$.

Index	quantity	Value	Experiment
	Branching fraction	(%)	
1	$\mathcal{B}(\eta_c \to K^+K^-\pi^0)$	$1.15 \pm 0.12 \pm 0.10$	BESIII [29]
2	$\mathcal{B}(\eta_c \to K_S^0 K^\pm \pi^\mp)$	$2.60 \pm 0.21 \pm 0.20$	BESIII [29]
3	$\mathcal{B}(\eta_c \to p\bar{p})$	$0.120 \pm 0.026 \pm 0.015$	BESIII [29]
4	$\mathcal{B}(\eta_c \to 2(\pi^+\pi^-\pi^0))$	$15.3 \pm 1.8 \pm 1.8$	BESIII [29]
5	$\mathcal{B}(B^+ \to \eta_c K^+)$	$0.120 \pm 0.008 \pm 0.007$	Belle [30]
6	$\mathcal{B}(B^+ \to \eta_c K^+)$	$0.096 \pm 0.012 \pm 0.006$	BaBar [31]
7	$\mathcal{B}(B^+ \to \eta_c(2S) K^+)$	$0.048 \pm 0.011 \pm 0.003$	Belle [30]
8	$\mathcal{B}(B^+ \to \eta_c(2S) K^+)$	$0.035 \pm 0.017 \pm 0.005$	BaBar [31]
	Ratio of the branching fractions		
9	$\frac{\mathcal{B}(\eta_c \to \phi\phi)}{\mathcal{B}(\eta_c \to p\bar{p})}$	$1.79 \pm 0.14 \pm 0.32$	LHCb [32]
10	$\frac{\mathcal{B}(\eta_c \to \phi\phi)}{\mathcal{B}(\eta_c \to K\bar{K}\pi)}$	$0.032^{+0.014}_{-0.010} \pm 0.009$	Belle [33]
11	$\frac{\mathcal{B}(\eta_c \to K^+K^-\eta)}{\mathcal{B}(\eta_c \to K^+K^-\pi^0)}$	$0.571 \pm 0.025 \pm 0.051$	BaBar [34]
12	$\frac{\mathcal{B}(\eta_c \to \phi K^+K^-)}{\mathcal{B}(\eta_c \to K\bar{K}\pi)}$	$0.052^{+0.016}_{-0.014} \pm 0.014$	Belle [33]
13	$\frac{\mathcal{B}(\eta_c \to 2(K^+K^-))}{\mathcal{B}(\eta_c \to K\bar{K}\pi))}$	$0.026^{+0.009}_{-0.007} \pm 0.007$	Belle [33]
14	$\frac{\mathcal{B}(\eta_c(2S) \to K^+K^-\eta)}{\mathcal{B}(\eta_c(2S) \to K^+K^-\pi^0)}$	$0.82 \pm 0.21 \pm 0.27$	Belle [34]
15	$\frac{\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)\cdot\mathcal{B}(B^+ \to \eta_c(2S)K^+)}{\mathcal{B}(\eta_c \to K\bar{K}\pi)\cdot\mathcal{B}(B^+ \to \eta_c K^+)}$	$0.096^{+0.020}_{-0.019} \pm 0.025$	BaBar [35]
16	$\frac{\mathcal{B}(\eta_c(2S) \to K_S^0 K^\pm\pi^\mp)\cdot\Gamma_{\eta_c(2S)\to\gamma\gamma}}{\mathcal{B}(\eta_c \to K_S^0 K^\pm\pi^\mp)\cdot\Gamma_{\eta_c\to\gamma\gamma}}$	$0.18 \pm 0.05 \pm 0.02$	CLEO [36]

Experimental measurements

TABLE IV: Data used in the analysis: product branching fractions measured in B decays and charmonium decays.

Index	quantity	Value ($\times 10^{-6}$)	Experiment
17	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$0.65 \pm 0.19 \pm 0.10$	BESIII [37]
18	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(\eta_c \to \gamma\gamma)$	$0.224^{+0.038}_{-0.037} \pm 0.020$	E835 [38]
19	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(\eta_c \to \gamma\gamma)$	$0.336^{+0.080}_{-0.070}$	E760 [39]
20	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$1.64 \pm 0.41^{+0.17}_{-0.24}$	Belle [40]
21	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$1.79 \pm 0.68^{+0.19}_{-0.25}$	Belle [40]
22	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$1.8^{+0.3}_{-0.2} \pm 0.2$	BaBar [41]
23	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$1.42 \pm 0.11^{+0.16}_{-0.20}$	Belle [42]
24	$\mathcal{B}(\eta_c \to \gamma\gamma) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$0.22^{+0.09}_{-0.07}{}^{+0.04}_{-0.02}$	Belle [43]
25	$\mathcal{B}(\eta_c \to \phi\phi) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$4.7 \pm 1.2 \pm 0.5$	BaBar [44]
26	$\mathcal{B}(\eta_c \to \phi\phi) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$2.4 \pm 1.4 \pm 0.3$	BaBar [44]
27	$\mathcal{B}(\eta_c \to K\bar{K}\pi) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$74.0 \pm 5.0 \pm 7.0$	BaBar [44]
28	$\mathcal{B}(\eta_c \to K\bar{K}\pi) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$64.8 \pm 8.5 \pm 7.1$	BaBar [44]
29	$\mathcal{B}(\eta_c \to K^+K^-\pi^0) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$4.54 \pm 0.76 \pm 0.48$	BESIII [37]
30	$\mathcal{B}(\eta_c \to K^+K^-\pi^0) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$11.4 \pm 2.5^{+1.1}_{-1.8}$	Belle [40]
31	$\mathcal{B}(\eta_c \to K^+K^-\pi^0) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$16.6 \pm 5.0 \pm 1.8$	Belle [40]
32	$\mathcal{B}(\eta_c \to K^0_S K^\pm\pi^\mp) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$11.35 \pm 1.25 \pm 1.50$	BESIII [37]
33	$\mathcal{B}(\eta_c \to K^0_S K^\pm\pi^\mp) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$24.0 \pm 1.2^{+2.1}_{-2.0}$	Belle [45]
34	$\mathcal{B}(\eta_c \to K^0_S K^\pm\pi^\mp) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$20.1 \pm 4.7^{+3.0}_{-4.5}$	Belle [40]
35	$\mathcal{B}(\eta_c \to \pi^+\pi^-\eta) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$7.22 \pm 1.47 \pm 1.11$	BESIII [37]
36	$\mathcal{B}(\eta_c \to K^+K^-\eta) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$2.11 \pm 1.01 \pm 0.32$	BESIII [37]
37	$\mathcal{B}(\eta_c \to 2(\pi^+\pi^-)) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$7.51 \pm 0.85 \pm 1.11$	BESIII [37]
38	$\mathcal{B}(\eta_c \to 2(K^+K^-)) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$0.94 \pm 0.37 \pm 0.14$	BESIII [37]
39	$\mathcal{B}(\eta_c \to 2(K^+K^-)) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$2.0 \pm 0.6 \pm 0.4$	BaBar [44]
40	$\mathcal{B}(\eta_c \to 2(K^+K^-)) \cdot \mathcal{B}(B^0 \to \eta_c K^0)$	$0.9 \pm 0.9 \pm 0.4$	BaBar [44]
41	$\mathcal{B}(\eta_c \to \pi^+\pi^-p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c K^+)$	$3.94^{+0.41}_{-0.39}{}^{+0.22}_{-0.18}$	Belle [46]
42	$\mathcal{B}(\eta_c \to \pi^+\pi^-p\bar{p}) \cdot \mathcal{B}(B^0 \to \eta_c K^0_S)$	$1.90^{+0.32}_{-0.29}{}^{+0.13}_{-0.47}$	Belle [46]
43	$\mathcal{B}(\eta_c \to \pi^+\pi^-p\bar{p}) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$2.30 \pm 0.65 \pm 0.36$	BESIII [37]
44	$\mathcal{B}(\eta_c \to K^+K^-\pi^+\pi^-) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$4.16 \pm 0.76 \pm 0.59$	BESIII [37]
45	$\mathcal{B}(\eta_c \to K^0_S K^\pm\pi^\mp\pi^+\pi^-) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$12.01 \pm 2.22 \pm 2.04$	BESIII [37]
46	$\mathcal{B}(\eta_c \to 2(\pi^+\pi^-\pi^0)) \cdot \mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$75.13 \pm 7.42 \pm 9.99$	BESIII [37]
47	$\mathcal{B}(\eta_c(2S) \to p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c(2S)K^+)$	$0.0342 \pm 0.0071 \pm 0.0021$	LHCb [47]
48	$\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi) \cdot \mathcal{B}(\psi(2S) \to \gamma\eta_c(2S))$	$13.0 \pm 2.0 \pm 3.0$	BESIII [48]
49	$\mathcal{B}(\eta_c(2S) \to K^0_S K^\pm\pi^\mp) \cdot \mathcal{B}(B^+ \to \eta_c(2S)K^+)$	$3.1 \pm 0.8 \pm 0.2$	Belle [45]
50	$\mathcal{B}(\eta_c(2S) \to \pi^+\pi^-p\bar{p}) \cdot \mathcal{B}(B^+ \to \eta_c(2S)K^+)$	$1.12^{+0.18}_{-0.16}{}^{+0.05}_{-0.07}$	Belle [46]
51	$\mathcal{B}(\eta_c(2S) \to \pi^+\pi^-p\bar{p}) \cdot \mathcal{B}(B^0 \to \eta_c(2S)K^0_S)$	$0.42^{+0.14}_{-0.12} \pm 0.03$	Belle [46]
52	$\mathcal{B}(\eta_c(2S) \to K^0_S K^\pm\pi^\mp\pi^+\pi^-) \cdot \mathcal{B}(\psi(2S) \to \gamma\eta_c(2S))$	$7.03 \pm 2.10 \pm 0.70$	BESIII [49]

Experimental measurements

TABLE V: Data used in the analysis: product of $\gamma\gamma$ partial width and branching fraction of η_c and $\eta_c(2S)$ decays measured in two-photon processes.

Index	quantity	Value (eV)	Experiment
53	$\mathcal{B}(\eta_c \to \phi\phi) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$7.75 \pm 0.66 \pm 0.62$	Belle [50]
54	$\mathcal{B}(\eta_c \to \phi\phi) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$6.8 \pm 1.2 \pm 1.3$	Belle [19]
55	$\mathcal{B}(\eta_c \to p\bar{p}) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$7.20 \pm 1.53^{+0.67}_{-0.75}$	Belle [51]
56	$\mathcal{B}(\eta_c \to K\bar{K}\pi) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$386 \pm 8 \pm 21$	BaBar [52]
57	$\mathcal{B}(\eta_c \to K\bar{K}\pi) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$374 \pm 9 \pm 31$	BaBar [53]
58	$\mathcal{B}(\eta_c \to K\bar{K}\pi) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$600 \pm 120 \pm 90$	DELPHI [17]
59	$\mathcal{B}(\eta_c \to K^0_S K^\pm \pi^\mp) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$490 \pm 290 \pm 90$	AMY [54]
60	$\mathcal{B}(\eta_c \to K^0_S K^\pm \pi^\mp) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$142 \pm 4 \pm 14$	Belle [55]
61	$\mathcal{B}(\eta_c \to \pi^+\pi^-\eta') \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$65.4 \pm 2.6 \pm 7.8$	Belle [56]
62	$\mathcal{B}(\eta_c \to 2(\pi^+\pi^-)) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$40.7 \pm 3.7 \pm 5.3$	Belle [19]
63	$\mathcal{B}(\eta_c \to K^+K^-\pi^+\pi^-) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$25.7 \pm 3.2 \pm 4.9$	Belle [19]
64	$\mathcal{B}(\eta_c \to K^+K^-\pi^+\pi^-) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$280 \pm 100 \pm 60$	DELPHI [17]
65	$\mathcal{B}(\eta_c \to 2(K^+K^-)) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$5.6 \pm 1.1 \pm 1.6$	Belle [19]
66	$\mathcal{B}(\eta_c \to 2(K^+K^-)) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$350 \pm 90 \pm 60$	DELPHI [17]
67	$\mathcal{B}(\eta_c \to K^+K^-\pi^+\pi^-\pi^0) \cdot \Gamma_{\eta_c \to \gamma\gamma}$	$190 \pm 6 \pm 28$	BaBar [52]
68	$\mathcal{B}(\eta_c(2S) \to K\bar{K}\pi) \cdot \Gamma_{\eta_c(2S) \to \gamma\gamma}$	$41 \pm 4 \pm 6$	BaBar [52]
69	$\mathcal{B}(\eta_c(2S) \to K^0_S K^\pm \pi^\mp) \cdot \Gamma_{\eta_c(2S) \to \gamma\gamma}$	$11.2 \pm 2.4 \pm 2.7$	Belle [55]
70	$\mathcal{B}(\eta_c(2S) \to \pi^+\pi^-\eta') \cdot \Gamma_{\eta_c(2S) \to \gamma\gamma}$	$5.6^{+1.2}_{-1.1} \pm 1.1$	Belle [56]
71	$\mathcal{B}(\eta_c(2S) \to K^+K^-\pi^+\pi^-\pi^0) \cdot \Gamma_{\eta_c(2S) \to \gamma\gamma}$	$30 \pm 6 \pm 5$	BaBar [52]

Experimental measurements

η_c width

Index	Process	Width (MeV)	Experiment
72	$\gamma\gamma \to \eta_c, \eta_c \to \eta'\pi^+\pi^-$	$30.8^{+2.3}_{-2.2} \pm 2.9$	Belle [56]
73	$\gamma\gamma \to \eta_c, \eta_c \to K^+K^-\eta$	$34.8 \pm 3.1 \pm 4.0$	BaBar [34]
74	$\gamma\gamma \to \eta_c, \eta_c \to K^+K^-\pi^0$	$25.2 \pm 2.6 \pm 2.4$	BaBar [34]
75	$\gamma\gamma \to \eta_c, \eta_c \to K^0_S K^\pm\pi^\mp$	$32.1 \pm 1.1 \pm 1.3$	BaBar [52]
76	$\gamma\gamma \to \eta_c, \eta_c \to K^0_S K^\pm\pi^\mp$	$24.8 \pm 3.4 \pm 3.5$	CLEO [36]
77	$\gamma\gamma \to \eta_c, \eta_c \to K^0_S K^\pm\pi^\mp$	$36.6 \pm 1.5 \pm 2.0$	Belle [55]
78	$\gamma\gamma^* \to \eta_c, \eta_c \to K^0_S K^\pm\pi^\mp$	$31.7 \pm 1.2 \pm 0.8$	BaBar [53]
79	$\gamma\gamma \to \eta_c, \eta_c \to K^+K^-\pi^+\pi^-\pi^0$	$36.2 \pm 2.8 \pm 3.0$	BaBar [52]
80	$\gamma\gamma \to \eta_c, \eta_c \to hadrons$	$28.1 \pm 3.2 \pm 2.2$	Belle [19]
81	$B^+ \to \eta_c K^+, \eta_c \to p\bar{p}$	$34.0 \pm 1.9 \pm 1.3$	LHCb [47]
82	$B^+ \to \eta_c K^+, \eta_c \to p\bar{p}$	$48^{+8}_{-7} \pm 5$	Belle [42]
83	$B^+ \to \eta_c K^+, \eta_c \to \Lambda\bar{\Lambda}$	$40 \pm 19 \pm 5$	Belle [42]
84	$B^+ \to \eta_c K^+, \eta_c \to K^0_S K^\pm\pi^\mp$	$35.4 \pm 3.6^{+3.0}_{-2.1}$	Belle [45]
85	$B \to \eta_c K^{(*)}, \eta_c \to K\bar{K}\pi$	$36.3^{+3.7}_{-3.6} \pm 4.4$	BaBar [35]
86	$b \to \eta_c X, \eta_c \to \phi\phi$	$31.4 \pm 3.5 \pm 2.0$	LHCb [32]
87	$b \to \eta_c X, \eta_c \to p\bar{p}$	$25.8 \pm 5.2 \pm 1.9$	LHCb [57]
88	$p\bar{p} \to \eta_c, \eta_c \to \gamma\gamma$	$20.4^{+7.7}_{-6.7} \pm 2.0$	E835 [38]
89	$p\bar{p} \to \eta_c, \eta_c \to \gamma\gamma$	$23.9^{+12.6}_{-7.1}$	E760 [39]
90	$\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c, \eta_c \to hadrons$	$32.0 \pm 1.2 \pm 1.0$	BESIII [58]
91	$\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c, \eta_c \to hadrons$	$36.4 \pm 3.2 \pm 1.7$	BESIII [58]

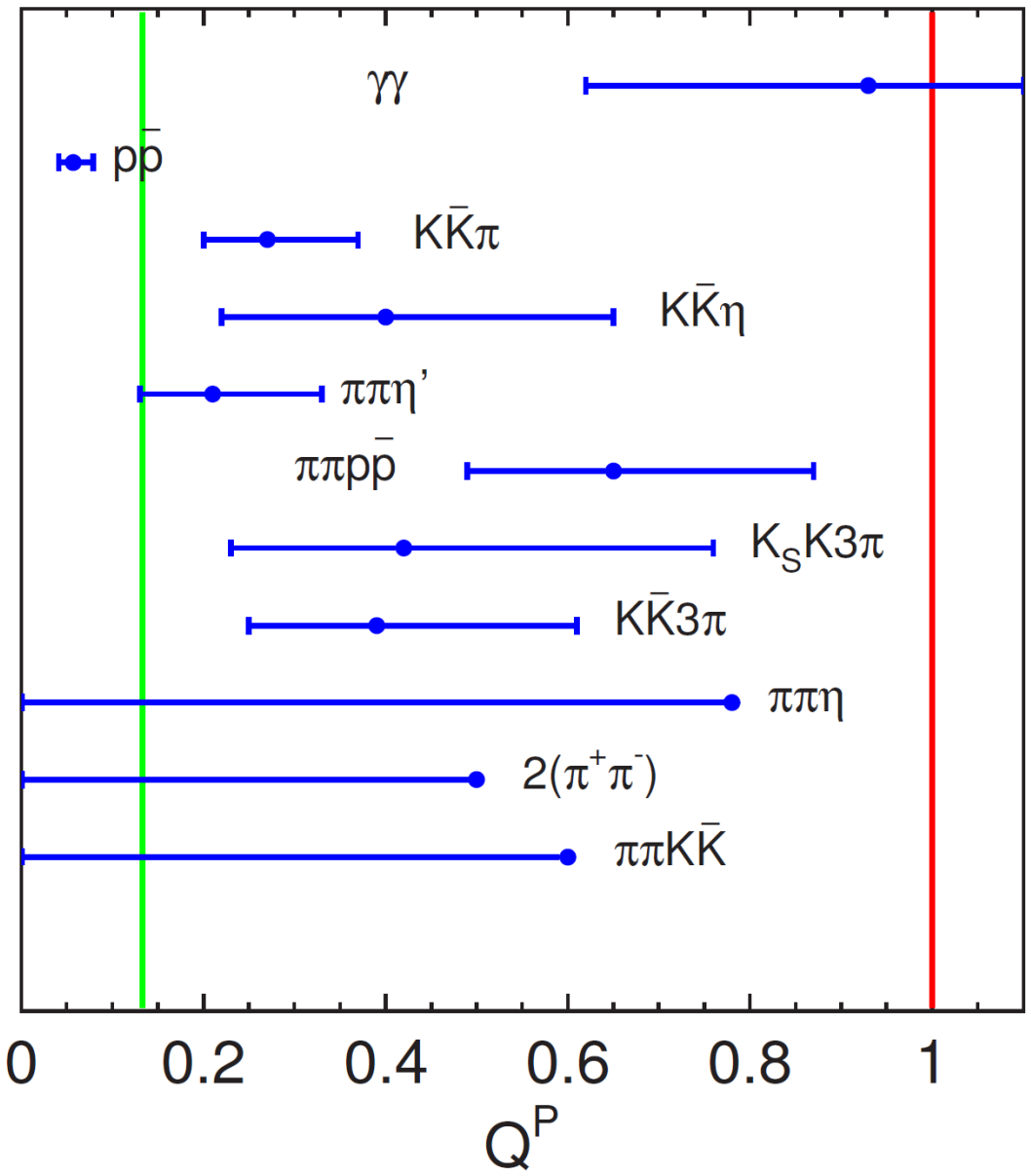
$\eta_c(2S)$ width

92	$\gamma\gamma \to \eta_c(2S), \eta_c(2S) \to K^0_S K^\pm\pi^\mp$	$13.4 \pm 4.6 \pm 3.2$	BaBar [52]
93	$\gamma\gamma \to \eta_c(2S), \eta_c(2S) \to K^0_S K^\pm\pi^\mp$	$6.3 \pm 12.4 \pm 4.0$	CLEO [36]
94	$\gamma\gamma \to \eta_c(2S), \eta_c(2S) \to K^0_S K^\pm\pi^\mp$	$19.1 \pm 6.9 \pm 6.0$	Belle [55]
95	$B^+ \to \eta_c(2S)K^+, \eta_c(2S) \to K^0_S K^\pm\pi^\mp$	$41.0 \pm 12.0^{+6.4}_{-10.9}$	Belle [45]
96	$\psi(2S) \to \gamma\eta_c(2S), \eta_c(2S) \to K\bar{K}\pi$	$16.9 \pm 6.4 \pm 4.8$	BESIII [48]
97	$\psi(2S) \to \gamma\eta_c(2S), \eta_c(2S) \to K^0_S K^\pm\pi^\mp\pi^+\pi^-$	$9.9 \pm 4.8 \pm 2.9$	BESIII [49]

Fit results (hadronic decays)

- χ^2/ndf=86/68, C.L.=5.7% ➔ reasonable fit

decay mode (h)	$\mathcal{B}(\eta_c \to h)$ (%)	$\mathcal{B}(\eta_c(2S) \to h)$ (%)	Q_h^P
$p\bar{p}$	0.136 ± 0.012	$0.0077^{+0.0028}_{-0.0021}$	$0.057^{+0.022}_{-0.016}$
$K\bar{K}\pi$	$6.90^{+0.44}_{-0.42}$	$1.86^{+0.68}_{-0.49}$	$0.27^{+0.10}_{-0.07}$
$K\bar{K}\eta$	$1.27^{+0.15}_{-0.14}$	$0.51^{+0.31}_{-0.23}$	$0.40^{+0.25}_{-0.18}$
$\pi^+\pi^-\eta'$	$1.20^{+0.18}_{-0.17}$	$0.25^{+0.14}_{-0.09}$	$0.21^{+0.12}_{-0.08}$
$\pi^+\pi^-p\bar{p}$	$0.365^{+0.042}_{-0.039}$	$0.236^{+0.076}_{-0.052}$	$0.65^{+0.22}_{-0.16}$
$K_S^0K^\pm\pi^\mp\pi^+\pi^-$	$2.39^{+0.67}_{-0.62}$	$1.00^{+0.69}_{-0.42}$	$0.42^{+0.34}_{-0.19}$
$K^+K^-\pi^+\pi^-\pi^0$	$3.50^{+0.60}_{-0.57}$	$1.36^{+0.70}_{-0.48}$	$0.39^{+0.22}_{-0.14}$
$\pi^+\pi^-\eta$	$1.43^{+0.41}_{-0.38}$	< 0.96 [18]	< 0.78
$2(\pi^+\pi^-)$	$0.86^{+0.13}_{-0.12}$	< 0.41 [19]	< 0.50
$K^+K^-\pi^+\pi^-$	0.57 ± 0.10	< 0.32 [19]	< 0.60
$2(K^+K^-)$	$0.135^{+0.028}_{-0.027}$	< 0.14 [19]	< 1.5
$3(\pi^+\pi^-)$	1.75 ± 0.48 [20]	< 2.9 [18]	< 2.0
$K^+K^-2(\pi^+\pi^-)$	0.72 ± 0.37 [20]	< 2.2 [18]	< 5.4
$\phi\phi$	$0.155^{+0.018}_{-0.017}$	—	—
ϕK^+K^-	$0.36^{+0.15}_{-0.14}$	—	—
$2(\pi^+\pi^-\pi^0)$	$15.1^{+2.0}_{-1.9}$	—	—



A few observations & questions:

- Q<1 for all the modes with significant signals and some modes with upper limits
- $Q^{p\bar{p}} \ll 1$ (>40σ below 1), very strange
- Problem in theoretical assumptions?
- Problem in experimental data (neglecting interference, …)?
- Glueball mixing in η_c or/and η'_c ?
- Exotic decays of η_c or/and η'_c ?
  - Relevant to B.3 "… heavy meson loops"
- Are there dominant decay modes of η'_c ?

$$\frac{\Gamma_{\eta'_c\to h}}{\Gamma_{\eta_c\to h}} \neq \frac{\Gamma_{\eta'_c\to 2g}}{\Gamma_{\eta_c\to 2g}} \; ? \qquad \frac{\Gamma_{\eta'_c\to h}}{\Gamma_{\eta_c\to h}} \neq \frac{\Gamma_{\eta'_c}}{\Gamma_{\eta_c}} \; ?$$

Fit results & discussions

2.6σ lower than LQCD result of 6.57±0.17 keV [Meng, Feng, Liu, Wang, Zou, arXiv:2109.09381v2]

PDG average is 3.7σ lower than LQCD result of 6.57±0.17 keV

First result!

		PDG2021
Γ_{η_c}	32.2 ± 0.7 MeV	32.0±0.7 MeV
$\Gamma_{\eta_c(2S)}$	14.1 ± 3.1 MeV	$11.3^{+3.2}_{-2.9}$ MeV
$\Gamma_{\eta_c \to \gamma\gamma}$	$5.43^{+0.41}_{-0.38}$ keV	5.15±0.35 keV
$\Gamma_{\eta_c(2S) \to \gamma\gamma}$	$2.21^{+0.88}_{-0.64}$ keV	Not available
$\mathcal{B}(B^+ \to \eta_c K^+)$	$(10.8 \pm 0.6) \times 10^{-4}$	$(10.8\pm0.8)\times10^{-4}$
$\mathcal{B}(B^+ \to \eta_c(2S) K^+)$	$(4.42 \pm 0.96) \times 10^{-4}$	$(4.4\pm1.0)\times10^{-4}$
$\mathcal{B}(\psi(2S) \to \gamma\eta_c(2S))$	$(7.0^{+3.4}_{-2.5}) \times 10^{-4}$	$(7\pm5)\times10^{-4}$
$\mathcal{B}(\psi(2S) \to \pi^0 h_c \to \pi^0\gamma\eta_c)$	$(5.03^{+0.52}_{-0.49}) \times 10^{-4}$	$(4.3\pm1.0)\times10^{-4}$

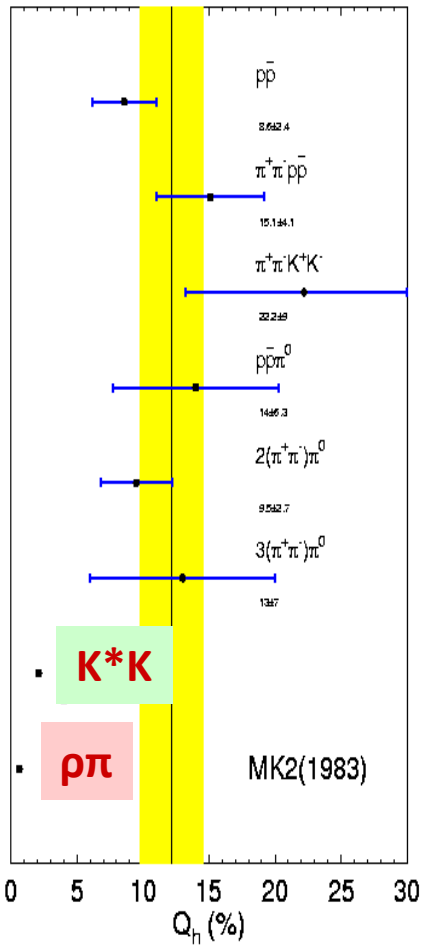
This study: $\Gamma_{\eta_c(2S)}/\Gamma_{\eta_c} = 0.44 \pm 0.10$

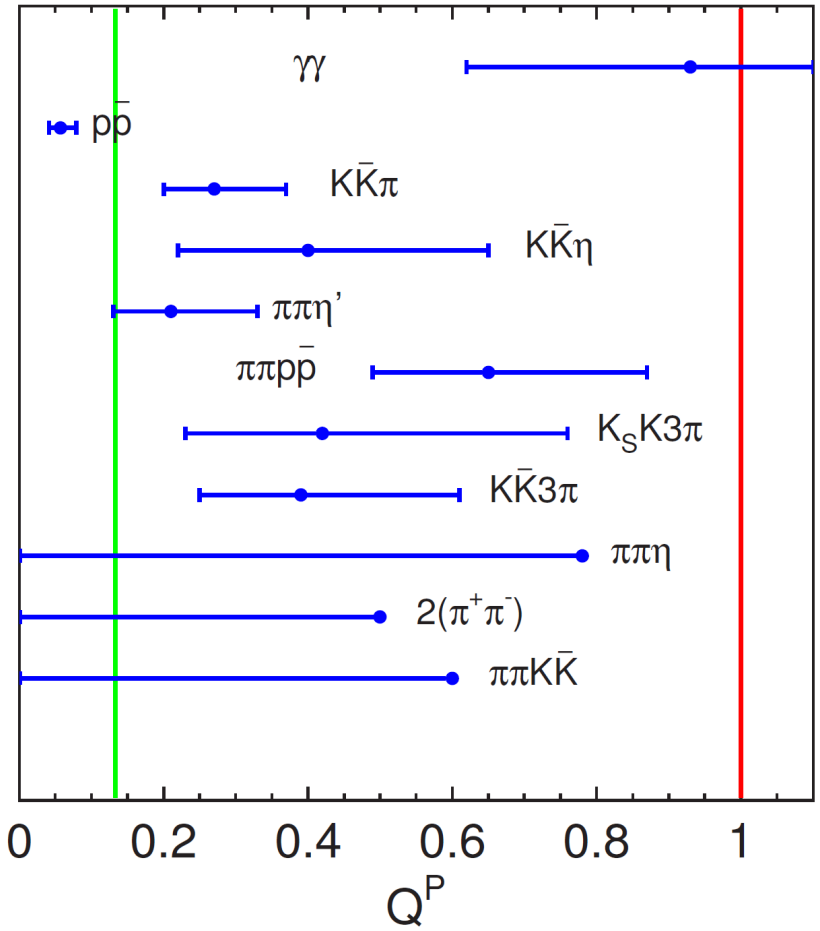
$$\frac{\mathcal{B}(\eta_c(2S) \to \gamma\gamma)}{\mathcal{B}(\eta_c \to \gamma\gamma)} = \frac{\Gamma_{\eta_c(2S)\to\gamma\gamma}/\Gamma_{\eta_c(2S)}}{\Gamma_{\eta_c\to\gamma\gamma}/\Gamma_{\eta_c}} = 0.93^{+0.48}_{-0.31} \approx 1$$

Chao, Gu, Tuan: $\frac{\Gamma_{\eta_c(2S)}}{\Gamma_{\eta_c}} \approx \frac{\Gamma_{\eta_c(2S)\to hadrons}}{\Gamma_{\eta_c\to hadrons}} \approx \frac{\Gamma_{\psi(2S)\to e^+e^-}}{\Gamma_{J/\psi\to e^+e^-}} = 0.42 \pm 0.01$

Summary & Discussions

- BESIII with 10B J/ψ, 3B ψ' events, Belle II & LHCb with large B-decay samples, Belle II with more two-photon collision data will supply more information on these decays.
- We should try to understand these puzzles.





- Anselmino, Genovese, Predazzi (1991), trigluonium-charmonium mixing
- Chao, Gu, Tuan (1996) : bigluonium-charmonium mixing
- Qiang Zhao et al.,
  - 1110.6235, η_c *mixing effects on charmonium and B meson decays*
  - 1712.02550, *Revisiting the pseudoscalar meson and glueball mixing and key issues in the search for a pseudoscalar glueball state*
- Qian, Xiaohai and Qiang, 1202.3026, *Updated study of the* η_c *and* η_c' *decays into light vector mesons*
- Ying Chen et al., 2107.12749, the glueball content of ηc
- …

On Q^V and Q^P, with minimal assumption

Assuming only $\frac{\Gamma(2S\to h)}{\Gamma(1S\to h)} = \frac{\sum_n \Gamma(2S\to ng)}{\sum_n \Gamma(1S\to ng)}$, one reaches $Q^V\approx 14\%$ and $Q^P\approx 1$

Mo, Yuan, Wang, CPC 31 (2007) 686-701, hep-ph/0611214

Channel	$\mathscr{B}(J/\psi)$	$\mathscr{B}(\psi')$	PDG21
$\gamma^* \to$ hadrons	(13.50±0.30)%	(1.66±0.18)%	1.73%
e^+e^-	(5.94±0.06)% 5.97	$(7.35\pm0.18)\times10^{-3}$	7.93E-3
$\mu^+\mu^-$	(5.93±0.06)% 5.96	$(7.3\pm0.8)\times10^{-3}$	8.0E-3
$\tau^+\tau^-$	–	$(2.8\pm0.7)\times10^{-3}$	3.1E-3
$\gamma^* \to X$	(25.37±0.35)% 25.43	(3.41±0.27)%	3.63%
$\gamma\eta_c$	(1.3±0.4)%	$(2.6\pm0.4)\times10^{-3}$	3.4E-3
$\pi^+\pi^- J/\psi$	(1.7±0.4)%	(31.8 ±0.6)%	34.68%
$\pi^0\pi^0 J/\psi$		(16.46±0.35)%	18.24%
$\eta J/\psi$		(3.09 ±0.08)%	3.37%
$\pi^0 J/\psi$		$(1.26\pm0.13)\times10^{-3}$	1.27E-3
$\gamma\chi_{c0}$		(9.2±0.4)%	9.79
$\gamma\chi_{c1}$		(8.7±0.4)%	9.75
$\gamma\chi_{c2}$		(8.1±0.4)%	9.52
$c\bar{c}X$	(1.3±0.4)%	(77.7±1.2)%	85.97%

$\gamma^* + c\bar{c}X = 27.13\%$

$B_{3g}^{\psi} + B_{\gamma gg} = 72.87\%$

$\gamma^* + c\bar{c}X = 89.60\%$

$\Rightarrow B_{3g}^{\psi'} + B_{\gamma gg} = 10.4\%$

$\gamma\eta_c'$ 7×10^{-4}

$\pi^0 h_c$ 8.6×10^{-4}

was ~26%

no big difference now!

$R = \frac{\psi'}{\psi} = \frac{10.4}{72.8} = 14.3\%$ $\quad R_{ee} = \frac{7.93}{59.7} = 13.3\%$

Kuang-Ta Chao, Yi-Fan Gu, S.F. Tuan, Commun. Theor. Phys. 25 (1996) 471-478

$$\Gamma_{tot}(\eta_c) \approx \Gamma(\eta_c \to 2g)$$

$$\Gamma_{tot}(\eta_c') \approx \Gamma(\eta_c' \to 2g)$$

$$\Gamma_{tot}(\eta_c') = \Gamma(\eta_c' \to \eta_c + \pi + \pi) + \Gamma(\eta_c' \to \psi(^1P_1) + \gamma) + \Gamma(\eta_c' \to J/\psi + \gamma) + \Gamma(\eta_c' \to 2g)$$

(i) $\Gamma(\eta_c' \to \eta_c + \pi + \pi)$ and $\Gamma(\psi' \to J/\psi + \pi + \pi)$ both preserve parity and C-parity of their $c\bar{c}$ piece in transition, and have comparable phase space. Hence $\Gamma(\eta_c' \to \eta_c + \pi + \pi) \approx \Gamma(\psi' \to J/\psi + \pi + \pi)$,[17] and is thus equal to 140 keV.[7]

(ii) The $E1$ transition width $\Gamma(\eta_c' \to \psi(^1P_1) + \gamma) = (4/3)e_c^2\alpha k^3|\langle^1P_1|r|\eta_c'\rangle|^2$. Taking $M(^1P_1)$ at the c.o.g. of the $M(^3P_J)$ states, $M_{\eta_c'}$ at 3600 MeV, $k = 80$ MeV, and the dipole transition matrix element $|\langle^1P_1|r|2S\rangle| = 2$ to 3 GeV^{-1}, it is then estimated[18] that $\Gamma(\eta_c' \to \psi(^1P_1) + \gamma)$ is about 11 keV, certainly in the range $O(10)$ keV.

(iii) The $M1$ transition width $\Gamma(\eta_c' \to J/\psi + \gamma)$ has also been estimated,[18] and is found to be about 5 keV hence in the range $O(1)$ keV.

$$Q^P = \frac{B(\eta_c(2S)\to h)}{B(\eta_c\to h)} = \frac{\Gamma(\eta_c(2S)\to h)/\Gamma_{\eta_c(2S)}}{\Gamma(\eta_c\to h)/\Gamma_{\eta_c}} = 1$$

Thank you!

Kuang-Ta Chao, Yi-Fan Gu, and S.F. Tuan, Commun. Theor. Phys. 25 (1996) 471-478

The argument is as follows. First, analogous to J/ψ, ψ', we have

$$\frac{\Gamma(\eta_c' \to h)}{\Gamma(\eta_c \to h)} \approx \frac{[|\psi(0)|/M]^2_{\eta_c'}}{[|\psi(0)|/M]^2_{\eta_c}}$$

via the two-gluon intermediary to any normal light hadron final state h.

$$\Gamma_{\rm tot}(\eta_c) \approx \Gamma(\eta_c \to 2g) = \frac{8\alpha_s^2|R(0)|^2_{\eta_c}}{3M^2_{\eta_c}}\left(1 + 4.8\frac{\alpha_s}{\pi}\right)$$

$$\Gamma_{\rm tot}(\eta_c') \approx \Gamma(\eta_c' \to 2g) = \frac{8\alpha_s^2|R(0)|^2_{\eta_c'}}{3M^2_{\eta_c'}}\left(1 + 4.8\frac{\alpha_s}{\pi}\right)$$

Justify $\Gamma_{tot}(\eta_c') \approx \Gamma(\eta_c' \to 2g)$

$$\Gamma_{tot}(\eta_c') = \Gamma(\eta_c' \to \eta_c + \pi + \pi) + \Gamma(\eta_c' \to \psi(^1P_1) + \gamma) + \Gamma(\eta_c' \to J/\psi + \gamma) + \Gamma(\eta_c' \to 2g)$$

(i) $\Gamma(\eta_c' \to \eta_c + \pi + \pi)$ and $\Gamma(\psi' \to J/\psi + \pi + \pi)$ both preserve parity and C-parity of their $c\bar{c}$ piece in transition, and have comparable phase space. Hence $\Gamma(\eta_c' \to \eta_c + \pi + \pi) \approx \Gamma(\psi' \to J/\psi + \pi + \pi)$,[17] and is thus equal to 140 keV.[7]

(ii) The $E1$ transition width $\Gamma(\eta_c' \to \psi(^1P_1) + \gamma) = (4/3)e_c^2 \alpha k^3 |\langle {}^1P_1 | r | \eta_c' \rangle|^2$. Taking $M(^1P_1)$ at the c.o.g. of the $M(^3P_J)$ states, $M_{\eta_c'}$ at 3600 MeV, $k = 80$ MeV, and the dipole transition matrix element $|\langle {}^1P_1 | r | 2S \rangle| = 2$ to 3 GeV^{-1}, it is then estimated[18] that $\Gamma(\eta_c' \to \psi(^1P_1) + \gamma)$ is about 11 keV, certainly in the range $O(10)$ keV.

(iii) The $M1$ transition width $\Gamma(\eta_c' \to J/\psi + \gamma)$ has also been estimated,[18] and is found to be about 5 keV hence in the range $O(1)$ keV.

(iv) $\Gamma(\eta_c' \to 2g)$ is estimated to be about 4 MeV by noting[18] that in the nonrelativistic limit with the hyperfine splitting effects neglected

$$\frac{\Gamma(\eta_c' \to 2g)}{\Gamma(\eta_c \to 2g)} = \frac{[|R(0)|/M]^2_{\eta_c'}}{[|R(0)|/M]^2_{\eta_c}} \approx \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} = \frac{2.33 \pm 0.04 \text{ keV}}{5.53 \pm 0.10 \text{ keV}} = 0.42 \pm 0.01$$

and using $\Gamma(\eta_c \to 2g) \simeq \Gamma_{tot}(\eta_c) = 10.3^{+3.8}_{-3.4}$ MeV[7] and the experimental values for the J/ψ and ψ' leptonic widths.

$$\boxed{\Gamma_{\eta_c} = 32.2 \pm 0.7 \text{ MeV} \Rightarrow \Gamma_{\eta_c'} \approx 13 \text{ MeV}}$$